2014–2015
MATHCOUNTS®
School Handbook

Contains 300 creative math problems
that meet NCTM standards for grades 6-8.

For questions about your local MATHCOUNTS program, please contact your chapter (local) coordinator. Coordinator contact information is available through the Find My Coordinator link on www.mathcounts.org/competition.

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A contribution to the MATHCOUNTS Foundation will help us continue to make its worthwhile programs available to middle school students nationwide.

The MATHCOUNTS Foundation will use your contribution for programwide support to give thousands of students the opportunity to participate.

To become a supporter of MATHCOUNTS, send your contribution to:

MATHCOUNTS Foundation
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For more information regarding contributions, call 703-299-9006, ext. 107 or email info@mathcounts.org.

The MATHCOUNTS Foundation is a 501(c)3 organization. Your gift is fully tax deductible.

The National Association of Secondary School Principals has placed this program on the NASSP Advisory List of National Contests and Activities for 2014–2015.

TABLE OF CONTENTS

CRITICAL 2014-2015 DATES

2014

Aug. 18 - Dec. 12 — Send in your school's Competition Series Registration Form to participate in the Competition Series and to receive the 2014-2015 School Competition Kit, with a hard copy of the *2014-2015 MATHCOUNTS School Handbook*. Kits begin shipping shortly after receipt of your form, and mailings continue every two weeks through December 31.

Mail, e-mail or fax the MATHCOUNTS Competition Series Registration Form with payment to:

MATHCOUNTS Registration, P.O. Box 441, Annapolis Junction, MD 20701
E-mail: reg@mathcounts.org
Fax: 240-396-5602

Questions? Call 301-498-6141 or confirm your registration via www.mathcounts.org/competitionschools. (Please allow 10 days before confirming your registration online.)

Nov. 3 — The 2015 School Competition will be available online. With a username and a password, a registered coach can download the competition from www.mathcounts.org/competitioncoaches.

Nov. 14 — **Deadline to register for the Competition Series *at reduced registration rates*** ($90 for a team and $25 for each individual). After November 14, registration rates will be $100 for a team and $30 for each individual.

Dec. 12 (postmark) — **Competition Series Registration Deadline**
In some circumstances, late registrations might be accepted at the discretion of MATHCOUNTS and the local coordinator. ***Late fees will also apply. Register on time to ensure your students' participation.***

2015

Early Jan. — If you have not been contacted with details about your upcoming competition, call your local or state coordinator!

Late Jan. — If you have not received your School Competition Kit, contact the MATHCOUNTS national office at 703-299-9006.

Jan. 31 - Feb. 28 — **Chapter Competitions**

Mar. 1-31 — **State Competitions**

May 8 — **2015 Raytheon MATHCOUNTS National Competition in Boston, MA**

MATHCOUNTS PROGRAM OVERVIEW

MATHCOUNTS was founded in 1983 as a way to provide new avenues of engagement in math to middle school students. MATHCOUNTS programs help students of all levels reach their full potential—whether they love math or fear it. We help expand students' academic and professional opportunities through three unique, but complementary, programs: the **MATHCOUNTS Competition Series**, **The National Math Club** and the **Math Video Challenge**. This *School Handbook* supports each program in different ways.

The **MATHCOUNTS Competition Series** is a national program that provides bright students the opportunity to compete head-to-head against their peers from other schools, cities and states in four levels of competition: school, chapter (local), state and national. MATHCOUNTS provides preparation and competition materials and, with the leadership of the National Society of Professional Engineers, hosts more than 500 Chapter Competitions, 56 State Competitions and the National Competition each year. This year, the top four students from each U.S. state and territory will compete at the 2015 Raytheon MATHCOUNTS National Competition in Boston, MA. Students win hundreds of thousands of dollars in scholarships each year at the local, state and national levels. There is a registration fee for students to participate in this program and registration is limited only to schools. Participation beyond the school level is limited 10 students per school. More information about the Competition Series can be found on pg. 40-52 of this *School Handbook* and at **www.mathcounts.org/competition**.

Working through the* School Handbook *and past competitions is the best way to prepare for MATHCOUNTS competitions.

The National Math Club is a free enrichment program that provides teachers and club leaders with resources to run a math club. The materials provided through The National Math Club are designed to engage students of all ability levels—not just the top students—and are a great supplement for classroom teaching. This program emphasizes collaboration and provides students with an enjoyable, pressure-free atmosphere in which they can learn math at their own pace. Active clubs can also earn rewards by having a minimum number of club members participate (based on school/organization/group size). There is *no cost to sign up* for The National Math Club, and registration is open to schools, organizations and groups that consist of at least four students in 6th, 7th or 8th grade and have regular in-person meetings. More information can be found at **www.mathcounts.org/club**.

The* School Handbook *is supplemental to The National Math Club. Resources in the* Club Activity Book *will be better suited for more collaborative and activity-based club meetings.

The **Math Video Challenge** is an innovative program that challenges students to work in teams to create a video explaining the solution to a MATHCOUNTS problem and demonstrating its real-world application. This project-based activity builds math, communication and collaboration skills. Students post their videos to the contest website, where the general public votes for the best videos. The 100 videos with the most votes advance to judging rounds, in which 20 semifinalists and, later, four finalists are selected. This year's finalists will present their videos to the students competing at the 2015 Raytheon MATHCOUNTS National Competition, and those 224 Mathletes will vote to determine the winner. Members of the winning team receive college scholarships. Registration is open to all 6th, 7th and 8th grade students. More information can be found at **videochallengemathcounts.org**.

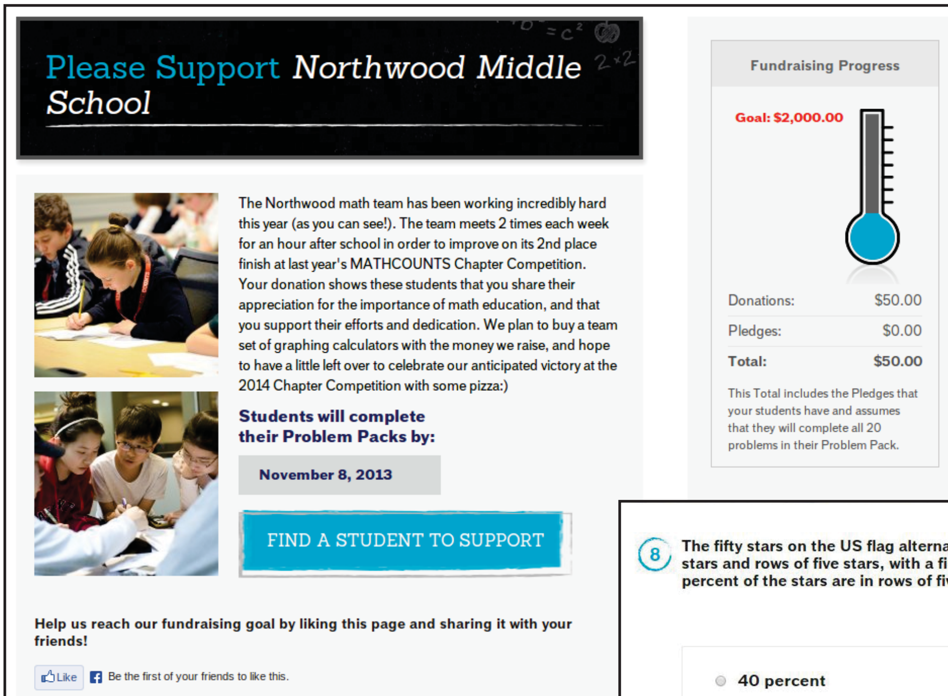
***Students must base their video for the 2014-2015 Math Video Challenge on a problem they select from the 2014-2015* School Handbook.**

HELPFUL RESOURCES

THE MATHCOUNTS SOLVE-A-THON

Solve-A-Thon is a fund-raiser that empowers students and teachers to use math to raise money for the math programs at their school. All money raised goes to support math programming that benefits the students' local communities, with 60% going directly back to the school. Launched last year, Solve-A-Thon was designed with teachers in mind; participating is free, and getting started takes just a couple of minutes. Here's how the fund-raiser works:

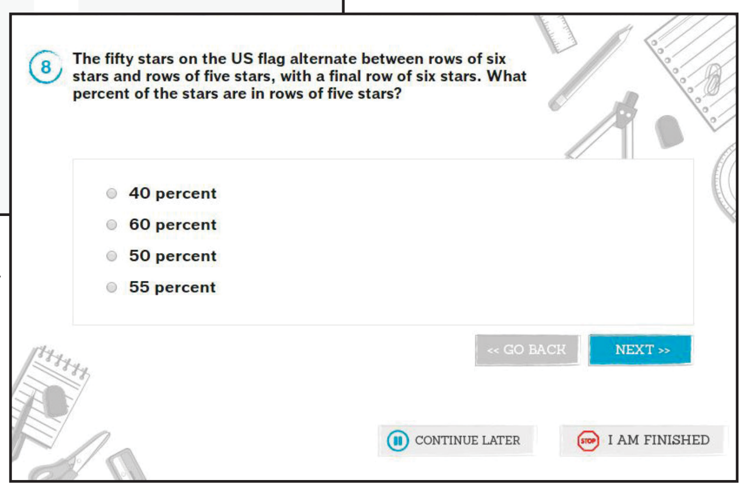
1. Teachers and students sign up and **create** online fund-raising pages (example below) that explain why they value math and why they are raising money for their school's math program.



2. Students **share** the links to their fund-raising pages with friends, family and local community members to earn donations and pledges.

3. Students **solve** an online Problem Pack with 20 math problems covering topics from the NCTM Standards for grades 6-8 (example below).

4. Students, teachers, schools and local communities **win** critical funding, prizes and improved math programs.

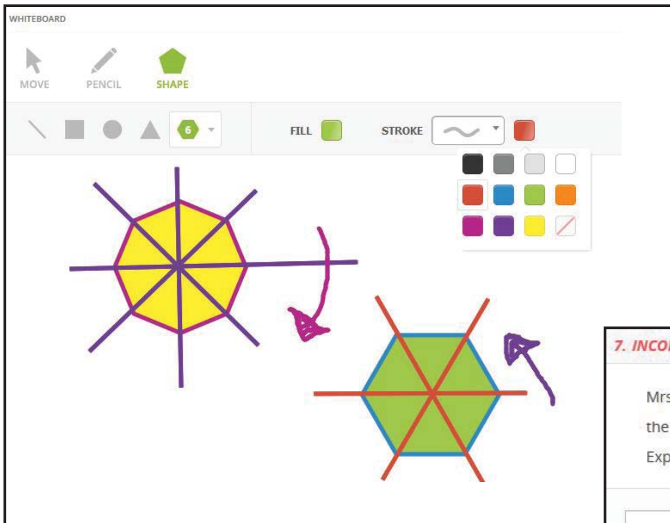


Unlike traditional fund-raisers, there is no door-to-door selling and no tracking of sales or money raised. Students participate in a meaningful learning activity to raise money, and money earned is tracked online automatically. Learn more and sign up at **solveathon.mathcounts.org**.

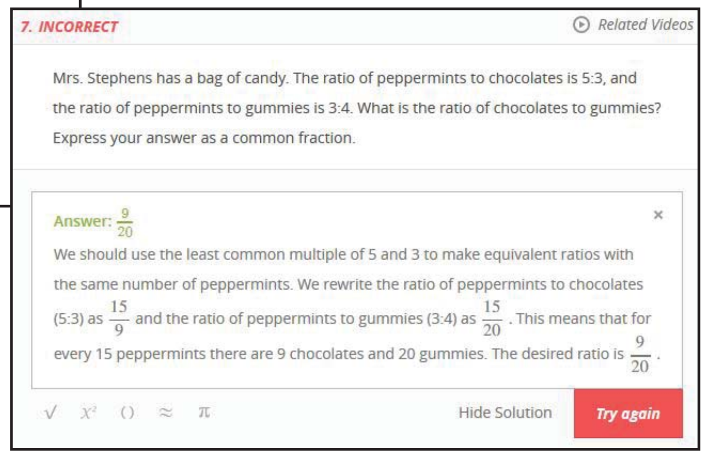
INTERACTIVE MATHCOUNTS PLATFORM

The **Interactive MATHCOUNTS Platform** provides a unique forum where members of the MATHCOUNTS community can collaborate, chat and take advantage of innovative online features as they work on problems from MATHCOUNTS handbooks and School, Chapter and State competitions.

Powered by NextThought, the Interactive MATHCOUNTS Platform continues to grow, with more problems and features being added every year. Currently this resource includes problems from *MATHCOUNTS School Handbooks* from 2011-2012, 2012-2013, 2013-2014 and 2014-2015, as well as School, Chapter and State Competitions from 2012, 2013 and 2014. Users will enjoy a truly engaging and interactive experience. Here are just a few of the features on the platform.



- **Digital whiteboards** (example at left) enable students to highlight problems, add notes and questions and show their work.
- **Interactive problems** (example below) can be used to assess student or team performance.
- **Advanced search tools** make it easy to find MATHCOUNTS content and notes.
- **Collaborative forums** allow users to chat and share with the global MATHCOUNTS community.



See how this incredible tool can add to your MATHCOUNTS experience. Create a free account and try out the Interactive MATHCOUNTS Platform at **mathcounts.nextthought.com**.

THE MATHCOUNTS OPLET

The **Online Problem Library and Extraction Tool** is an online database of over 13,000 problems and over 5,000 step-by-step solutions. OPLET subscribers can create personalized worksheets/quizzes, flash cards and Problems of the Day with problems pulled from the past 14 years of MATHCOUNTS handbooks and competitions.

With OPLET, creating original resources to use with your competition team and in your classroom is easy. You can personalize the materials you create in the following ways.

- **Format**: worksheet/quiz, flash cards or Problem of the Day
- **Range of years** of MATHCOUNTS materials
- **Difficulty level**: five levels from easy to difficult
- **Number of questions**
- **Solutions included/omitted** for select problems
- **MATHCOUNTS usage**: filters by competition rounds or handbook problem types
- **Math concept:** including arithmetic, algebra, geometry, counting/probability, number theory

OPLET Subscriptions can be purchased at **www.mathcounts.org/oplet**. A 12-month subscription costs $275, and schools registering students in the MATHCOUNTS Competition Series receive a $5 discount per registered student (up to $50 off). Plus, if you purchase OPLET by October 17, 2014, you can save an additional $25, for a total savings of up to $75 (see coupon at right).

GET A $25 DISCOUNT!

Save $25 on your OPLET subscription if you purchase or renew by **October 17, 2014**.

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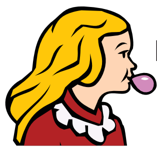
www.mathcounts.org/oplet

Warm-Up 1

1. ________ What is the sum of the two-digit multiples of 11?

2. ________ If $a \# b = a + 2b$, for integers a and b, what is the value of $3 \# 4$?

3. ________ times



Kimba chewed a piece of gum 42 times in one minute. If she continued to chew at the same rate, how many times would she chew her gum in 100 seconds?

4. ________ miles Carver jogs 4 times a week. Last week, his distances were $4\frac{1}{3}$, $3\frac{1}{2}$, $3\frac{5}{6}$ and $4\frac{1}{6}$, all measured in miles. What was Carver's average distance for the 4 days? Express your answer as a mixed number.

5. ________ socks Rob has 10 white, 8 red and 6 blue socks in his drawer. If he selects socks from the drawer randomly, without looking, what is the least number of socks Rob must select to guarantee that he has removed a pair of white socks?

6. ________ hamburgers At a particular restaurant, hamburgers are priced \$3 each, 2 for \$5 and 5 for \$9. What is the maximum number of hamburgers that can be purchased for \$48?

7. Statement ________

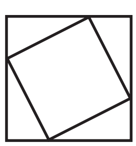
Statement ________

Which two of the following four statements, labeled A through D, are true statements?

A: Statement B is false, but statement C is true.
B: Statement C is true, but statement D is false.
C: Statement D is false, and statement A is false.
D: Statement A is true, and statement B is true.

8. ________ seconds How many seconds are in 3.14 hours?

9. ________



The vertices of the smaller square in the figure are at trisection points of the sides of the larger square. What is the ratio of the area of the smaller square to the area of the larger square? Express your answer as a common fraction.

10. ________ students Students at Central School were surveyed regarding lunch choices. Of the students that responded, exactly $\frac{1}{3}$ wanted more fresh fruits and vegetables as choices. Of those students not wanting more fresh fruits and vegetables, exactly $\frac{1}{8}$ wanted more seafood. What is the minimum number of students that responded to the survey?

Warm-Up 2

11.__________ Two cylinders are equal in volume. The radius of one is doubled, and the height of the other cylinder is increased to k times its original height. If the two new cylinders are equal in volume, what is the value of k?

12.__________ What positive integer must be included in the set {1, 2, 4, 8} so that the new set of five integers has a median that is equal to its mean?

13.__________ coins

Debbie has equal numbers of dimes and quarters with a total value of \$1.40. How many coins does she have altogether?

14.__________ What is the average of the first 99 counting numbers?

15.__________ minutes How many fewer minutes does it take to drive 35 miles at 30 mi/h than to drive the same distance at 25 mi/h?

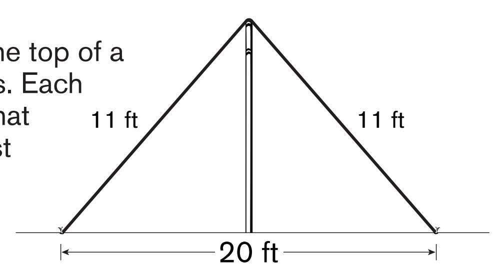
16. \$__________ After a 20% reduction, the price of a computer was \$800. What was the price of the computer before the reduction?

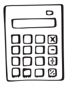
17.__________ ounces Koka-Kola is sold in packages of eight 12-ounce bottles. Pepsy-Kola is sold in packages of six 16-ounce bottles. In ounces, what is the absolute difference in the total number of ounces in a package of Koka-Kola and a package of Pepsy-Kola?

18.__(,)__ Point A lies at the intersection of $y = x$ and $y = -\frac{2}{3}x + 5$. What are the coordinates of A? Express your answer as an ordered pair.

19.__________ When rolling two fair, standard dice, what is the probability that the sum of the numbers rolled is a multiple of 3 or 4? Express your answer as a common fraction.

20.__________ feet Two wires are connected by stakes from the ground to the top of a vertical pole that is anchored halfway between the stakes. Each wire is 11 feet long, and the stakes are 20 feet apart. What is the height of the pole? Express your answer in simplest radical form.



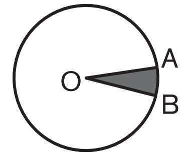


Workout 1

21.__________ What is the sum of the prime factors of 2015?

22.__________ What is $(3.5 \times 10^4)^2$ when written in scientific notation with four significant digits?

23.__________ meters The area of the shaded region of circle O is 9π m^2, and the measure of $\angle AOB$ is 22.5 degrees. What is the length of the radius of circle O?



24. $__________ In Fuelville, the cost of gas averaged \$3.50 per gallon at the start of April, then rose 6% during April and dropped 10% during May. What was the cost of a gallon of gas at the end of May?

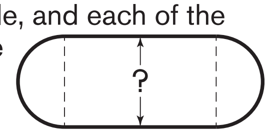
25.__________ ft^2 A square residential lot is measured to be 100 feet on each side, with a possible measurement error of 1% in each of the length and width. What is the absolute difference between the largest and smallest possible measures of the area given this possible error?

26.__________ years old Currently, the sum of the ages of Yumi, Rana and Victoria is 42 years. Four years ago, the sum of the ages of Rana and Victoria was equal to the current age of Yumi. What is Yumi's current age?

27.__________ students

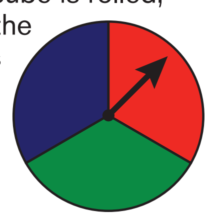
Of 600 students at Goodnight Middle School in Texas, 85% are not native Texans. Of those non-native students, 60% have lived in Texas more than 10 years, and 30 students have lived in Texas less than a year. How many non-native students have lived in Texas for at least 1 year but not more than 10 years?

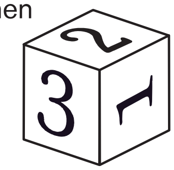
28.__________ miles The track at Dividend Middle School, depicted here, has two semicircular ends joined by two parallel sides. The total distance around the track is 1/4 mile, and each of the semicircular ends is 1/4 of the total distance. What is the distance between the two parallel sides of the track? Express your answer as a decimal to the nearest hundredth.



29.__________ What is the sum of the coordinates of the point at which $y = x - 3$ and $y = -2x + 9$ intersect?

30.__________ A spinner is divided into three congruent sections colored red, blue and green. The numbers 1 through 6 appear on the faces of a fair number cube. When the pointer on the spinner is spun and the number cube is rolled, what is the probability that the pointer lands within the blue section and an even number is rolled? Express your answer as a common fraction.





Warm-Up 3

31. ________ What is the value of $\dfrac{6!}{5! + 4!}$?

32. ________ The same digit A occupies both the thousands and tens places in the five-digit number 1A,2A2. For what value of A will 1A,2A2 be divisible by 9?

33. ________ pounds In Lewis Carroll's *Through the Looking-Glass*, this conversation takes place between Tweedledee and Tweedledum. Tweedledum says, "The sum of your weight and twice mine is 361 pounds." Tweedledee answers, "The sum of your weight and twice mine is 362 pounds." What is the absolute difference in the weights of Tweedledee and Tweedledum?

34. ________ pieces A 63-inch long string is cut into pieces so that their lengths form a sequence. First, a 1-inch piece is cut from the string, and each successive piece that is cut is twice as long as the previous piece cut. Into how many pieces can the original length of string be cut in this way?

35. ________ Two fair, six-sided dice are rolled. They are marked so one die has the numbers 1, 3, 5, 7, 9, 11 and the other has the numbers 2, 4, 6, 8, 10, 12. What is the probability that the sum of the numbers rolled is divisible by 5? Express your answer as a common fraction.

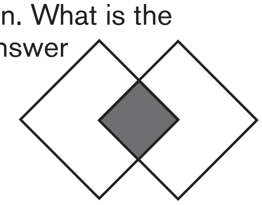
36. $________ A shirt company charges a one-time setup fee plus a certain price per shirt. An order of 10 shirts costs $84. An order of 20 shirts costs $159. How much does an order of 30 shirts cost?

37. ________ On a number line, what is the nearest integer to π^2?

38. ________ minutes Cara needs to drive to Greenville. She can drive 50 mi/h along a 200-mile highway, or she can take a different route, which requires her to drive 150 miles at 60 mi/h and then 50 miles at 40 mi/h. How many minutes would Cara save by taking the faster route?

39. $________ Square tiles with 4-inch sides are 20¢ each, and square tiles with 6-inch sides are 40¢ each. How much will Jerry save tiling a 4-foot by 6-foot floor with the 6-inch tiles laid side by side instead of the 4-inch tiles laid side by side?

40. ________ units^2 Two unit squares intersect at the midpoints of two adjacent sides, as shown. What is the area of the shaded intersection of the two square regions? Express your answer as a common fraction.



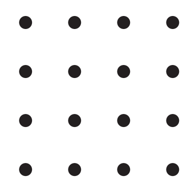
Warm-Up 4

41. ________ If $a - b = 0$, what is the value of $a \times b$? Express your answer in terms of a.

42. ________ When rolling two fair, eight-sided dice, each with faces numbered 1 through 8, what is the probability that the two numbers rolled have a sum of 9? Express your answer as a common fraction.

43. ________ If $2015 = 101a + 19b$, for positive integers a and b, what is the value of $a + b$?

44. ________ squares How many squares can be drawn using only dots in this grid of 16 evenly spaced dots as vertices?



45. ________ ounces A cup contains 6 ounces of milk. Two ounces of chocolate syrup are added to the cup and thoroughly mixed. Then 2 ounces of that mixture are poured out. How many ounces of chocolate syrup are in the remaining mixture? Express your answer as a mixed number.

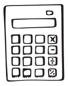
46. ________ If $f(x) = \sqrt{x+4}$, for what value of x does $f(x) = 3$?

47. ________ What is the product of the greatest common factor and least common multiple of 48 and 72?

48. ________ trips There are 13 stations along the Cheshire Railroad, which runs in a straight line from east to west. A “trip” is defined by its starting and ending stations (regardless of intermediate stops) and must always go westward. How many different trips are possible along the Cheshire Railroad?

49. ________ units^2 What is the area of $\triangle ABC$ with vertices A(2, 3), B(17, 11) and C(17, 3) ?

50. ________ If x is positive, what is the result when $8x$ is doubled and then divided by one half of $8x$?



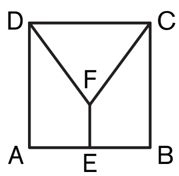
Workout 2

51. ________ points In a basketball game, Aisha and Britney scored 23 points in all; Aisha and Courtney scored 21 points in all; and Britney and Courtney scored 20 points in all. How many points did the three girls score altogether?

52. ________ % The peak of volcano Mauna Kea is 13,803 feet above sea level. When measured from its oceanic base, it measures 33,100 feet vertically to its peak. What percent of Mauna Kea's altitude is below sea level? Express your answer to the nearest whole number.

53. ________ inches A pizza of diameter 16 inches has an area large enough to serve 4 people. What is the diameter of a pizza with an area large enough to serve 12 people? Express your answer as a decimal to the nearest tenth.

54. ________ cm



Square ABCD has sides of length 12 cm. The three interior segments divide the square, as shown, into two congruent trapezoids and an isosceles triangle, all with equal areas. What is the length of segment CF?

55. ________ Given the following facts about the integers *a*, *b*, *c*, *d*, *e* and *f*, what is the value of *a* if $0 \le a \le 60$?

a is odd. $\qquad c = \dfrac{b}{2}$ is even. $\qquad e = \dfrac{d-1}{2}$ is odd.

$b = \dfrac{a-1}{2}$ is even. $\qquad d = \dfrac{c}{2}$ is odd. $\qquad f = \dfrac{e-1}{2}$ is even.

56. $ ________ The box office staff at a theater sets the ticket price so that total ticket sales will be $7200 if tickets are sold for all 240 seats. By what amount should they increase ticket prices if they want to keep total ticket sales at $7200 but they sell 15 fewer tickets?

57. ________ minutes Lindsey's e-mail account was bombarded with 34,000 spam messages. The web interface allows Lindsey to delete batches of 50 messages at one time, a process that takes 0.5 second to complete. What is the fewest number of minutes will it take Lindsey to delete all 34,000 messages if she deletes batches of 50 messages at a time? Express your answer to the nearest whole number.

58. ________ Let n represent the smallest positive integer such that $2015 + n$ is a perfect square. Let m represent the smallest positive integer such that $2015 - m$ is a perfect square. What is the value of $n + m$?

59. ________ Emilio started a job on July 1, 2000 at a salary of $40,000. If he got an increase of 10% each year, in what year did his salary first exceed $80,000?

60. ________ inches Anne is trying out for the track team, and she had three triple jump attempts. The distance of each triple jump attempt is measured to the nearest inch, and her first two attempts measured 36' 4" and 38' 4", respectively. If Anne's third attempt measured no less than her first attempt and no more than her second attempt, what is the difference between the greatest and least possible averages for the three triple jump attempts?

Warm-Up 5

61. $________ Rachel, Siriana, Tanya and Ursula each contributed to a lottery pool. Rachel contributed \$12, Siriana \$15, Tanya \$20 and Ursula \$8. One of their tickets was a winner, paying \$1100. If the winnings were distributed in proportion to each person's contribution, what was Siriana's share of the winnings?

62. ________ ways The Statesville Middle School basketball team has 8 players. If a player can play any position, in how many different ways 5 starting players be selected?

63. ________ What is 5 times the sum of all the distinct positive factors of 144?

64. ________ The denominator of a fraction is 2 more than its numerator. The reciprocal of this fraction is equal to the fraction itself. What is the sum of its numerator and denominator?

65. ________ Julius has some spare change consisting of quarters, dimes and nickels. If the ratio of quarters to dimes is 3:4 and the ratio of quarters to nickels is 4:5, what is the ratio of dimes to nickels? Express your answer as a common fraction.

66. ________ miles When Mr. Tesla drives 60 mi/h, the commute from home to his office takes 30 minutes less than it does when he drives 40 mi/h. What is the distance of Mr. Tesla's commute from home to his office?

67. ________ hours Jack Frost makes 18 snowballs every hour, but 2 snowballs melt each 15 minutes. How many hours will it take Jack to accumulate 2 dozen snowballs? Express your answer as a decimal to the nearest tenth.

68. ________ years old A mother was 26 years old when her daughter was born. The mother is now 6 years less than 3 times as old as her daughter. How old is the daughter now?

69. ________ red marbles A bag contains a total of 36 marbles colored either red or blue. Twice the number of red marbles is 6 less than the number of blue marbles. How many red marbles are in the bag?

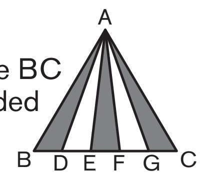
70. ________ When $a > 5$ and $b \leq 5$, $a @ b = (a + b)(a - b)$, and when $a \leq 5$ and $b > 5$, $a @ b = (b + a)(b - a)$. What is the value of $2 \times ((2 @ 6) @ -4) - 1$?

Warm-Up 6

71. ________ If $\frac{1}{n} + \frac{1}{2n} + \frac{1}{3n} = k$, what is the value of nk? Express your answer as a common fraction.

72. ________ minutes Ellie can paint a room in 2 hours, and Diedre can paint the same room in 4 hours. How many minutes will it take Ellie and Diedre, working together, to paint the room?

73. ________ cm^2 Equilateral triangle ABC has sides of length 10 cm. If D, E, F and G divide base BC into five congruent segments, as shown, what is the total area of the three shaded regions? Express your answer in simplest radical form.



74. ________ degrees Given that the Earth is turning at a constant rate around its axis and it makes exactly one complete rotation every 24 hours, how many degrees does the center of home plate at Fenway Park rotate around Earth's axis from 6 a.m. on May 8th to noon on May 10th of the same year?

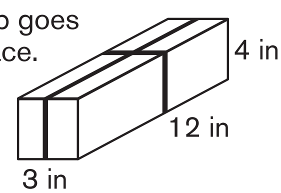
75. ________ sides If the difference in the degree measure of an interior and exterior angle of a regular polygon is 100°, how many sides does the polygon have?

76. ________ What positive four-digit integer has its thousands and hundreds digits add up to the tens digit, its hundreds and tens digits add up to its ones digit and its tens and ones digits add up to the two-digit number formed by the thousands and hundreds digits?

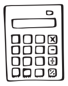
77. ________ meters The surface area of a sphere, in square meters, and its volume, in cubic meters, are numerically equal. What is the length of the radius of the sphere?

78. ________ slips Ben won a writing contest at school. To determine his prize, he draws slips of paper from a bag containing 16 slips, with the names of 4 different prizes written on 4 slips each. He continues to draw slips without replacement until he has selected 4 slips for the same prize. What is the maximum number of slips that Ben can draw without knowing his prize?

79. ________ inches The wrapped present shown has two loops of ribbon around it. Each loop goes completely around the box once and always runs down the middle of a face. The two loops overlap each other at a right angle, but the ends of the ribbon do not overlap when making a loop. How many inches of ribbon were used?



80. ________ times On a 12-hour digital clock, at how many times during a 24-hour day will all of the digits showing the time be the same?



Workout 3

81. ________ If $(x + y)^2 = x^2 + y^2$, what is the value of xy?

82. ________ % Kiera and Aubrey were the only candidates for SGA President. Kiera received 44% of the 7th graders' votes, and Aubrey received 42% of the 8th graders' votes. The student body consists of 325 7th graders and 350 8th graders, and every student voted for one of these two candidates. What percent of the students' votes did the winner receive? Express your answer to the nearest whole number.

83. ________ inches Enoch has a photo that is 8 inches high and 6 inches wide. He wants to enlarge it so that it can fit into a frame that is 10 inches by 7 inches. How much will he need to crop from the width to exactly fill the entire frame? Express your answer as a decimal to the nearest tenth.

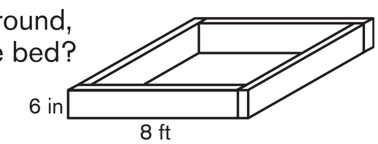
84. ________ miles At 1:00 p.m. an airplane left the local airport and flew due east at 300 mi/h. At 3:00 p.m. a second plane left the same airport and flew due north at 400 mi/h. Assuming the curvature of the earth is negligible, how many miles apart are the planes at 8:00 p.m.?

85. ________ If the mean of the integers 7, 3, 11, 13, 5 and x is 4 more than the mode of the six integers, what is the value of x?

86. ________ students After 6 new students entered the Happy Hearts Childcare Center and 2 students exited, there were 3 times as many students as before. How many students were present before students entered and exited the center?

87. ________ The product of three consecutive prime numbers is 2431. What is their sum?

88. ________ ft^3 Boards that are 8 feet long, 2 inches thick and 6 inches wide are used to make the raised garden bed shown here. If the boards are placed on level ground, how many cubic feet of soil are needed to completely fill the bed? Express your answer to the nearest whole number.



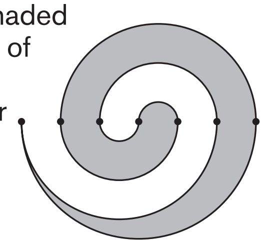
89. ________ dollars Bennie is ordering a new computer. A 6.25% sales tax will be added to the price of the computer, and then an $11 delivery charge will be added to that total. If Bennie has $500 to spend, what is the maximum price of a computer that he can afford? Express your answer to the nearest whole number.

90. ________ teams Each team in the softball league plays each of the other teams exactly once. If 21 games are played, how many teams are in the league?

Warm-Up 7

91.__________ % Mrs. Smith's 1st period class of 28 students averaged 84% on the last test. Her 2nd period class of 24 students averaged 86% on the same test. What must her 3rd period class of 32 students average so that all of the students in her three classes have a combined average of 80%?

92.__________ units² In the figure the collinear dots are equally spaced 2 units apart, and the shaded region is formed from two semicircles of diameter 2 units, two semicircles of diameter 6 units, two semicircles of diameter 10 units and one semicircle of diameter 12 units. What is the area of this shaded region? Express your answer in terms of π.



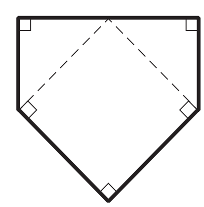
93.__________ If $5a - b - c = 36$ and $b = c = \frac{1}{2}a$, what is the value of a?

94.__________ seats A bus stopped at Main Street and boarded passengers, after which half of its seats were filled. At its next stop, at Oak Street, 2 passengers got off and 7 got on, and then 60% of the seats were filled. How many seats are there on the bus?

95.__________ Three fractions are inserted between $\frac{1}{4}$ and $\frac{1}{2}$ so that the five fractions form an arithmetic sequence. What is the sum of these three new fractions? Express your answer as a common fraction.

96.__________ In the equation $123 \times 4A6 = 5B548$, what is the value of $A \times B$, the product of the two missing digits?

97.__________ ft² Home plate at a school's baseball field was constructed by adding two right isosceles triangles to a 1-foot by 1-foot square, as shown. What is the area of this home plate? Express your answer as a common fraction.



98.__________ goldfish Elisa had a new fishbowl but no fish, so Kendell gave her half of his goldfish. Elisa's fishbowl was not very large, so she gave half of her new goldfish to Rocky. Rocky kept 8 of the goldfish he was given and gave the remaining 10 goldfish to Aster. What is the difference between Kendell's starting number of goldfish and the number of goldfish Elisa kept for her new fishbowl?

99.__________ teachers In a school that has 20 teachers, 10 teach mathematics, 8 teach social studies and 6 teach science. Two teach both mathematics and social studies, but none teach both social studies and science. How many teach both mathematics and science?

100.__________ A line segment has endpoints $(-5, 10)$ and (a, b). If the midpoint of the segment is $(13, -2)$, what is the absolute difference between a and b?

Warm-Up 8

101. ________ integers For how many positive integers n is it possible to have a triangle with side lengths 5, 12 and n?

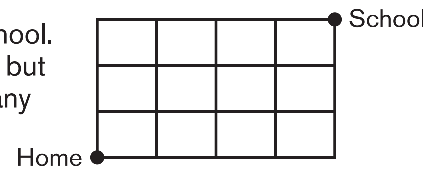
102. ________ laps By the time Shana had completed $\frac{3}{8}$ of her first lap in a race, she had also completed $\frac{1}{32}$ of the entire race. How many laps were there in the race?

103. ________

1	2	3	4	5	6
12	11	10	9	8	7
13	14	15	16	17	18
24	23	22	21	20	19
25	…	…	…	…	…

The counting numbers are written in a table with six columns so that in each successive row the numbers alternate between increasing from left to right and increasing from right to left, as shown. What is the first number in the 15th row?

104. ________ days Alvin lives 4 blocks west and 3 blocks south of his school. He wants to take a different route to school each day, but each route must be exactly 7 blocks long. For how many days can he do this without repeating any route?



105. ________ If x and y are positive integers, and the mean of 4, 20 and x is equal to the mean of y and 16, what is the smallest possible value of $x + y$?

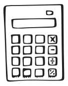
106. ________ cm^2 Five rectangles are arranged in a row. Each rectangle is half as tall as the previous one. Also, each rectangle's width is half its height. The first rectangle is 32 cm tall. What is the sum of the areas of all five rectangles?

107. ________ If one-half of a number is 8 less than two-thirds of the number, what is the number?

108. ________ coins Colton's soda cost $1.95. He paid for it with nickels, dimes and quarters only. He used 2 more dimes than twice the number of nickels. The number of quarters was 1 more than the number of nickels and dimes combined. How many coins did he use to pay for the soda?

109. $ ________ Hiro started a new job with a salary of $50,000 per year. He received increases of 10%, 20% and 30% at the end of his first, second and third years of employment, respectively. How much did Hiro's salary increase after working three years at his new job?

110. ________ A positive integer plus 4 times its reciprocal is equal to the product of the integer and 4 times its reciprocal. What is the integer?

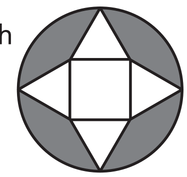


Workout 4

111.________ cm^2 What is the area of a circle that circumscribes a 6-cm by 8-cm rectangle? Express your answer to the nearest whole number.

112.________ For positive integer n, $n? = n! \cdot (n-1)! \cdot \ldots \cdot 1!$ and $n\# = n? \cdot (n-1)? \cdot \ldots \cdot 1?$. What is the value of $4\# \cdot 3\# \cdot 2\# \cdot 1\#$?

113.________ % An equilateral triangle is placed on the outside of each side of a square of length 6 cm, and a circle is then drawn through four vertices of the triangles as shown. What percent of the area of the circle is the unshaded region? Express your answer to the nearest whole number.



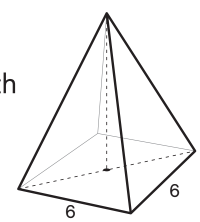
114.________ If 21 is written as a sum of n consecutive positive integers, what is the greatest possible value of n?

115.________ minutes Normally, the hose in Elena's garden will fill her small pool in 15 minutes. However, a leak in the hose allows $\frac{1}{3}$ of the water flowing through the hose to spill into the flower bed. How many minutes will it take to fill the pool? Express your answer as a decimal to the nearest tenth.

116.________ people How many people must be in a group to guarantee that at least 3 of them share the same first initial and last initial?

117.________ inches The ratio of width to height for Carl's TV screen is 16:9. Carl's TV screen has a 37-inch diagonal. What is the width of his TV screen? Express your answer as a decimal to the nearest tenth.

118.________ m^3 A pyramid with a square base of side length 6 m has a height equal to the length of the diagonal of the base. What is the volume of the pyramid? Express your answer in simplest radical form.



119.________ in^3 Boynton's sheet cake measures 18 × 24 inches and has a height of 4 inches. However, these measurements include a $\frac{3}{4}$-inch thick layer of frosting on the top and sides. What is the volume of Boynton's cake excluding the frosting? Express your answer to the nearest whole number.

120.________ What is the smallest positive integer value of x for which $54x$ is a perfect square?

Warm-Up 9

121. ________ degrees The ratio of the angles of a quadrilateral is 3:4:5:6. What is the number of degrees in the largest of the angles?

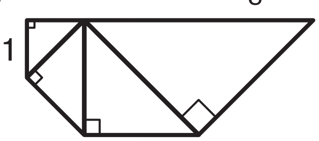
122. ________ Six positive integers have a mean of 6. If the median of these six integers is 8, what is the largest possible value of one of these six integers?

123. $ ________ The prize money for the Math County Science Fair was divided among the top three projects so that the 1st place winner got as much as the 2nd and 3rd place winners combined, and the 2nd place winner got twice as much as the 3rd place winner. If the total prize money awarded was \$2400, how much did the 3rd place winner receive?

124. ________ For the function defined as $f(x)=\begin{cases} x+4 \text{ when } x<-1 \\ x^2-6 \text{ when } x\ge -1 \end{cases}$, what is the value of $f(f(2))$?

125. ________ What is the value of $\dfrac{20^2-15^2}{18^2-17^2}$?

126. ________ units Starting with an isosceles right triangle with legs of length 1 unit, a second isosceles right triangle is built using the hypotenuse of the first triangle as a leg. A third isosceles right triangle is then built using the second triangle's hypotenuse as a leg, and so on, as demonstrated in the figure. If this pattern continues, what will be the number of units in the length of the hypotenuse of the 20th isosceles right triangle?

1


127. ________ miles Tony's Towing Service charges \$30.00 to hook a vehicle to the tow truck and \$1.75 for each mile the vehicle is towed. Mr. Alman's car broke down at school and was towed to his house. If the total amount charged by Tony's Towing was \$59.75, what is the distance Mr. Alman's car was towed from the school to his house?

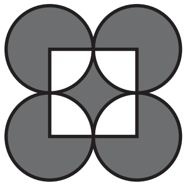
128. $ ________ A car's present value is \$20,000, and its value decreases by the same percentage every year. At the end of one year, it will be worth \$18,000. What will it be worth at the end of 3 years?

129. ________ What is the least possible sum of two positive integers whose product is 182?

130. ________ colors Chin-Chin is constructing a tetrahedron and a cube using gumdrops for vertices and toothpicks for edges. She wants to have different-colored gumdrops on the two ends of each toothpick, and no gumdrop used for the cube should be the same color as any gumdrop used for the tetrahedron. What is the least number of colors Chin-Chin needs?

Warm-Up 10

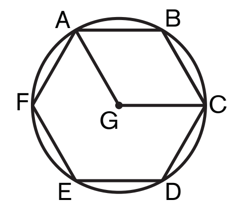
131. ________ mm^2



In the figure shown, four circles of radius 4 mm are centered at the corners of a square of side length 8 mm. What is the total area of the shaded regions? Express your answer in terms of π.

132. ________ Consider any positive three-digit integer that has all of its digits distinct and none equal to zero. What is the largest possible difference between such an integer and any integer that results from rearranging its digits?

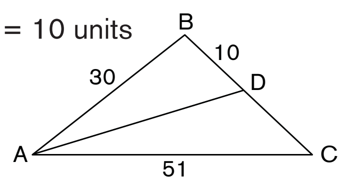
133. ________ $units^2$ A regular hexagon ABCDEF is inscribed in a unit circle with center G, as shown. What is the area of quadrilateral ABCG? Express your answer as a common fraction in simplest radical form.



134. ________ inches The amount of paint required to cover a surface is directly proportional to the area of that surface. The amount of paint needed to cover five spheres of radius 10 inches is the same as the amount of paint required to cover a solid right circular cylinder with radius 20 inches. What is the height of the cylinder?

135. ________ If $x + 2y + 3z = 6$, $2x + 3y + z = 8$ and $3x + y + 2z = 10$, what is the value of $x + y + z$?

136. ________ units In triangle ABC, segment AD bisects angle A. If AB = 30 units, BD = 10 units and AC = 51 units, what is the length of segment BC?



137. ________

1
2
3,4
5,6,7,8,9,10,11

The positive integers are written in order in rows of various lengths. The first row contains the number 1. For every row after the first, the number of entries in the row is the sum of the numbers in the previous row. The first four completed rows are shown. What is the last number in the sixth row?

138. ________ If x and y are negative integers and $x - y = 1$, what is the least possible value for xy?

139. ________ $units^2$ ABCD is a square with side length 4 units, and AEFC is a rectangle with point B on side EF. What is the area of AEFC?

140. ________ liters One hundred liters of a salt and water solution contains 1% salt. After some of the water has evaporated, the solution contains 5% salt. How many liters of water evaporated?

Workout 5

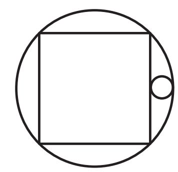
141. ________ The mean of a set of n numbers is 12, and the mean of a set of $3n$ numbers is 6. What is the mean when the two sets are combined? Express your answer as a decimal to the nearest tenth.

142. ________ units² The diagonals of convex quadrilateral ABCD intersect at X. If the areas, in square units, of $\triangle AXB$, $\triangle BXC$ and $\triangle CXD$ are 8, 15 and 5, respectively, what is the area of $\triangle DXA$? Express your answer as a decimal to the nearest tenth.

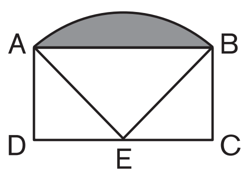
143. ________ What is the sum of all of the four-digit numbers whose digits are permutations of 1, 2, 3 and 4?

144. ________ Duncan picks a number and will subtract 6 each second. Taz picks a different number and will add 8 each second. They begin at the same time, and after 14 seconds, both arrive at the number 25. What is the sum of Duncan's and Taz's starting numbers?

145. ________ units A square is inscribed in a circle of radius 2 units, and then the largest possible circle is inscribed between the square and the original circle, as shown. What is the radius of the inscribed circle? Express your answer as a decimal to the nearest tenth.



146. ________ cm²



In rectangle ABCD, shown here, AB = 4 cm and BC = 2 cm. If E is the midpoint of side DC and also is the center of a circle that contains points A and B, what is the area of the shaded segment of the circle determined by chord AB? Express your answer as a decimal to the nearest tenth.

147. ___:___ a.m. Ellie and Emma live 1.04 miles from each other. They decide to meet by walking toward each other, Ellie at 2.4 mi/h and Emma at 2.8 mi/h. If they both leave at 8:00 a.m., at what time in the morning will they meet?

148. ________ What is the smallest absolute difference between the squares of two distinct positive four-digit integers?

149. ________ bowls At the school fair, MATHCOUNTS parents sold chocolate and vanilla ice cream as a fund-raiser. Forty bowls of chocolate ice cream were sold for \$2.15 per bowl. Bowls of vanilla ice cream sold for \$1.90 each. How many bowls of ice cream were sold if the total amount of money collected was \$158.20?

150. ________ Driving at an average speed of 66 mi/h, Dusty traveled m miles in n minutes. What is the least possible whole-number value of n such that the value of m also is a whole number?

Warm-Up 11

151. ________ times The indicator lights on two different pieces of machinery blink at different intervals, one every 4 seconds and the other every 7 seconds. If they blink together at 10:00 p.m., how many more times will they blink together before 10:15 p.m.?

152. ________ base 10 What is the absolute difference, expressed in base 10, between the largest three-digit base 5 number and the smallest four-digit base 4 number?

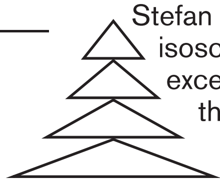
153. ________ The Chug-A-Long Train Company boxes toy trains with either 5 cars or 7 cars per box. The trains in stock have a total of 53 cars. If Charles selects a box at random, what is the probability that the box contains 7 cars? Express your answer as a common fraction.

154. ________ Given the following facts about the numbers a, b, c and d, what is the value of $a + b$? Express your answer as a mixed number.

$ab = 1$
$bc = -9$
$b + c + d = 0$
$b = -c$
$c < -a$

155. ________ units² Twenty-seven unit cubes are arranged to form a $3 \times 3 \times 3$ cube. The center unit cube from each face is then removed. What is the surface area of the resulting solid?

156. ________

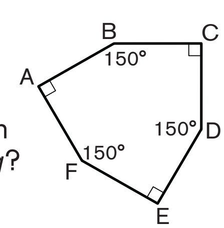


Stefan created this tree design in his computer drawing class. It consists of four isosceles triangles of the same height arranged vertically as shown. With the exception of the top triangle, the apex of each triangle is the midpoint of the base of the triangle above it, and the base of each triangle is 50% larger than the base of the triangle above it. What is the ratio of the area of the smallest triangle to the area of the largest? Express your answer as a common fraction.

157. (___ , ___) What are the coordinates of the point at which the line through the points (2, 6) and (5, 9) intersects the line through the points (−1, −1) and (5, −7)? Express your answer as an ordered pair.

158. ________ If a and b are positive integers such that $ab = 48$ and $a - b = 8$, what is the value of $a + b$?

159. ________ As shown, convex hexagon ABCDEF has right angles at A, C and E, and 150-degree angles at B, D and F. If each side is 2 inches long, its area can be expressed in simplest radical form as $p + q\sqrt{3}$. What is the value of pq?



160. ________ integers How many integers from 100 to 999 have three different digits?

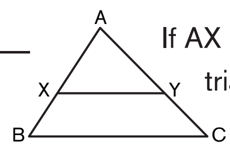
Warm-Up 12

161. ________ What is the value of $\dfrac{2 \times 4 \times 6 \times 8 \times 10 \cdots \times 20}{10!}$?

162. ________ What is the sum of all positive integers less than 20 that cannot be written as the sum of two prime numbers?

163. ________ cm A circle has a diameter of 8 cm. A chord perpendicular to its diameter divides the diameter into segments of lengths 1 cm and 7 cm. What is the length of the chord? Express your answer in simplest radical form.

164. ________

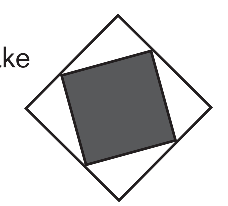


If AX and AY are $\dfrac{2}{3}$ of AB and AC, respectively, what is the ratio of the area of triangle AXY to trapezoid XYCB? Express your answer as a common fraction.

165. ________ integers The *digital sum* of a number is the sum of its digits. How many positive three-digit integers have a digital sum of 5?

166. ________ mi/h The Zed Zee courier drove to a location 180 miles away to deliver an urgent package. The courier's average speed driving from the delivery location back to her starting location was 20 mi/h less than her average speed driving to the delivery location. If the entire trip took 7.5 hours, what was her average speed driving to the delivery location?

167. ________ One square is inscribed in another so that the sides of the inner square make 30-degree and 60-degree angles with the sides of the outer square. Each side of the inner square is 4 units, and the area of the outer square, in simplest radical form, is $a + b\sqrt{3}$. What is $a + b$?



168. ________ A set is said to be *closed* under multiplication if the product of elements in the set also is an element in the set. For what number k is the set $\{0, -1, k\}$ closed under multiplication?

169. ________ units³ A certain box of width w has a length that is twice its width, and its height is three times its width. What is the total volume of 24 of these boxes? Express your answer in terms of w.

170. ________ If $a_1 = 13$ and $a_n = 77$, for an arithmetic sequence of integers, $a_1, a_2, a_3, \ldots, a_n$, with n terms, what is the median of all possible values of n?

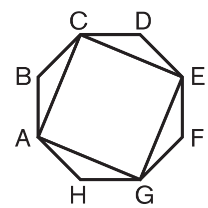
Workout 6

171.________ 1s Becky tries an experiment. She writes some numbers on the blackboard and then applies the following rule: she picks any number on the board that is greater than 1, erases it and replaces it with the list of its proper divisors. For example, if the number 6 was on the board, she would apply the rule by erasing the 6 and replacing it with the numbers 1, 2 and 3. The experiment ends when there are only 1s left on the board. If Becky begins with just the number 72 on the board, how many 1s will be on the board when she is finished?

172.________ mm A bug is walking on the ticking second hand of a clock, starting from the center and walking outward. Every second, the bug walks 1 mm along the stationary second hand, and then the hand ticks while the bug stands still. If the bug starts at the very center of the clock and proceeds for exactly 60 seconds, what is the total distance that the bug will travel? Express your answer as a decimal to the nearest tenth.

173.________ seconds A Ferris wheel has the same height as a building with 60 floors of identical height. After boarding at the bottom of the Ferris wheel, Courtney used a stopwatch to find that it took 8 minutes 26 seconds to rise to the top of the 45th floor of the building. How many seconds will it take from there for the Ferris wheel to bring her back around to where she started, assuming the wheel rotates at a constant rate?

174.________ units^2 Regular octagon ABCDEFGH has side-length 1 unit. What is the area of square ACEG? Express your answer as a decimal to the nearest tenth.



175.________ strings How many distinct three-letter strings can be formed using three of the five letters in the word *SILLY*?

176.________ bunnies Sammie took his little sister to the petting zoo. His sister really liked the area with chicks and bunnies. Altogether, in the group of 27 chicks and bunnies, Sammie counted 78 legs. Assuming every chick had two legs and every bunny had four legs, how many bunnies were in the group?

177.________ % A path crosses a rectangular field on a diagonal. If someone travels across the field on the diagonal, instead of walking along the sides, what is the greatest possible percent reduction in total distance traveled? Express your answer to the nearest whole number.

178.________ cm^3 For a particular rectangular solid with integer dimensions, the sum of its length, width and height is 50 cm. What is the absolute difference between the greatest possible volume and the least possible volume of the solid?

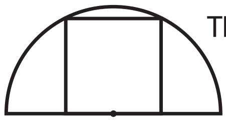
179.________ A jar contains 28 red jelly beans, 14 black jelly beans and 6 green jelly beans. What is the probability that two jelly beans selected at random, and without replacement, from this jar are the same color? Express your answer as a common fraction.

180.________ What is the units digit of the product $2^{2015} \times 7^{2015}$?

Warm-Up 13

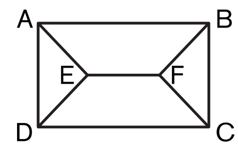
181. ________ perfect squares How many perfect squares are factors of 12!?

182. ________



The figure shows a square inscribed in a semicircle. What is the ratio of the radius of the semicircle to the side length of the square? Express your answer as a common fraction in simplest radical form.

183. ________ units^2 In the figure shown, AE = ED = EF = BF = CF = 1 unit, and $m\angle AED = m\angle BFC = 90$ degrees. What is the area of rectangle ABCD? Express your answer in simplest radical form.



184. ________ What is the decimal difference between 1111_3 and 1111_2?

185. ________ inches A candle that burns at a uniform rate was 11 inches tall after burning for 4 hours and 8 inches tall after burning for a total of 6 hours. How many inches tall was the candle before it was lit?

186. ________ minutes The express train from Addington to Summit travels the 18-mile route at an average speed of 72 mi/h, stopping only in Summit. The local train stops for 1.5 minutes at each of 6 stops between these two locations, and it averages 54 mi/h while it is in motion. How many minutes more does the local train take for this trip than the express train?

187. ________ Jack randomly chooses one of the positive integer divisors of 20, and Jill randomly picks one of the positive integer divisors of 30. What is the probability that Jack and Jill pick the same number? Express your answer as a common fraction.

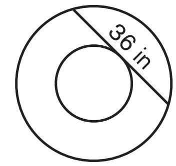
188. ________ miles One dragonfly flew in a straight path at a rate of 36 mi/h for 45 minutes. Meanwhile, a second dragonfly rode for 45 minutes on the windshield of a car that was driving in a straight path at 60 mi/h. How many more miles did the second dragonfly travel than the first dragonfly?

189. ________ The sum of two numbers is 1, and the absolute difference of the two numbers is 2. What is the product of the two numbers? Express your answer as a common fraction.

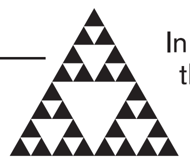
190. ________ If $(2a - 3b)^{\sqrt{x}} = 16a^4 - 96a^3b + 216a^2b^2 - 216ab^3 + 81b^4$, what is the value of x?

Warm-Up 14

191. ________ in^2 Two concentric circles and a line segment tangent to the smaller circle are shown. If the length of the line segment is 36 inches, what is the area of the region between the circles? Express your answer in terms of π.



192. ________

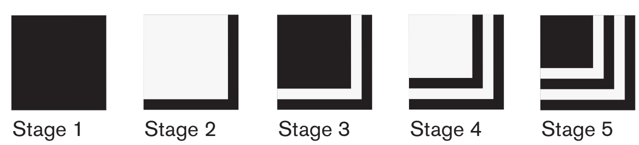


In the equilateral triangle, each downward facing white triangle has its vertices at the midpoints of the sides of the larger upward facing triangle that just contains it. What fraction of the entire figure is white? Express your answer as a common fraction.

193. ________ For non-negative integers m and n, $\frac{m+n}{m-n} = \frac{25}{4}\left(\frac{m-n}{m+n}\right)$ and $m > n$. What is the value of $\frac{m}{n}$? Express your answer as a common fraction.

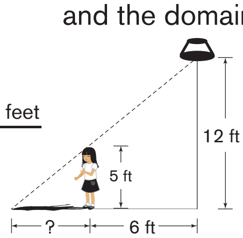
194. ________ cm^3 How many cubic centimeters of silver are needed to cover just the faces of a cube with a layer of silver that is 1 mm thick if the cube has edges of length 6 cm? Express your answer to the nearest whole number.

195. ________ Stage 1 of a pattern is a black square of side length 2015 units. At stage 2, a white square of side length 2014 units is placed on top of the square in stage 1 and positioned so that the upper left vertices of the squares coincide. The pattern continues with alternating white and black squares placed and positioned in this manner at each stage. Each new square placed has sides 1 unit shorter than the square placed at the previous stage. The first five stages are shown. At stage 2015, what portion of the figure will be black? Express your answer as a common fraction.



196. ________ integers The domain of $f(x) = x^2 - 3$ is $\{-4, -3, ..., 3, 4\}$. How many integers are in both the range and the domain of f?

197. ________ feet



The lamp of a streetlight is 12 feet above the street below. A girl who is 5 feet tall stands at a point 6 feet from the spot directly below the light. How long is her shadow? Express your answer as a mixed number.

198. ________ integers How many positive integers have the same digits in the same order when written in base 7 and in base 13?

199. ________ Maggie writes the numbers 1, 2, 3, …, 10 on separate slips of paper and tosses the 10 slips into a hat. She then randomly pulls three slips from the hat at the same time. What is the probability that the arithmetic mean of the three selected values is in fact written on one of those same three slips? Express your answer as a common fraction.

200. ________ If $2015 + a = b$ for positive integers a and b, both of which are palindromes, what is the smallest possible value of a?

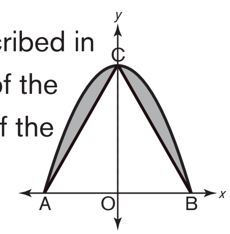
Workout 7

201. ________ If a, b, c and d are positive integers such that $a^b c^d = 2^{10} \times 7^9$, what is the least possible value of $a + b + c + d$?

202. ________ What common fraction is equivalent to $0.2\overline{36}$?

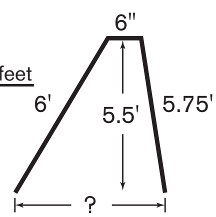
203. ________ units A 3-4-5 right triangle made of paper is cut along the altitude from the right angle, resulting in 2 smaller right triangles. These 2 triangles are then cut along the altitudes from their right angles, then the 4 resulting right triangles are cut the same way and finally, 8 triangles are cut the same way, resulting in 16 smaller right triangles. What is the sum of the perimeters of these 16 triangles? Express your answer as a decimal to the nearest tenth.

204. ________ units^2 The figure shows $\triangle ABC$, with base length b = AB and height h = CO, inscribed in the region bounded by the curve $y = -\frac{1}{4}x^2 + 16$ and the x-axis. If the area of the entire region bounded by the curve and the x-axis is $\frac{2}{3}bh$, what is the area of the shaded region? Express your answer as a decimal to the nearest tenth.



205. ________ What is the value of $1^2 - 2^2 + 3^2 - 4^2 + \ldots + 99^2 - 100^2$?

206. ________ feet



A ladder is 5.5 feet high when opened. The platform at the top is 6 inches wide and parallel to the floor, and the two supports are 6 feet and 5.75 feet, as shown. How many feet long apart are the two supports on the floor? Express your answer as a decimal to the nearest hundredth.

207. ________ The product of three consecutive odd integers is between 64,000,000 and 65,000,000. What is the greatest of the three integers?

208. ________ What is the 2015th digit after the decimal point in the decimal representation of $\frac{1}{13}$?

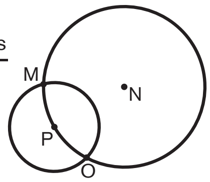
209. ________ sprinklers Jason has a square garden with an area of $\frac{1}{10}$ acre. He is installing sprinklers that each water a circular area with a radius of 11 feet. Jason does not want water from the sprinklers to extend beyond the perimeter of the garden. He also does not want any portion of the garden to receive water from more than one sprinkler, though he realizes that by doing so some portions of the garden will not be watered. Given that an acre is equivalent to 43,560 ft^2, what is the maximum number of sprinklers Jason can install?

210. ________ years A certain computer program will take 2000 years to run using current technology. Every year, advances in technology make it possible to run the program in half the time it would have taken starting in the previous year. However, once the program is started it cannot be interrupted to apply newer technology. Including the years spent waiting to start, what is the least number of years it will take to finish running the program? Express your answer to the nearest whole number.

Warm-Up 15

211. ______ ways In how many ways can the vertices of an equilateral triangle be colored with four available colors? Two colorings are considered the same if they can be obtained from each other by any combination of rotations and reflections.

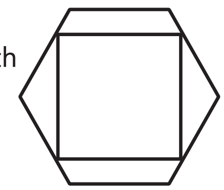
212. ______ degrees



Circle P has its center on circle N. The central angle formed by the radii of circle N that intersect M and O, the points of intersection of the circles, has measure 64 degrees. What is the measure of the central angle formed by the radii of circle P that intersect points M and O?

213. ______ Given that $f(x) = 3x - 7$ and $g(x) = x^2 - 4$, what is the value of $f(g(f(3)))$?

214. ______ cm A square has its four vertices on the sides of a regular hexagon with side length 1 cm. What is the side length of the square? Express your answer in simplest radical form.

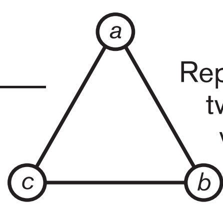


215. ______ % The original price for a pair of shoes was increased by 150%, and then this new price was decreased by 75%. By what percent must the current price be increased to return to the original price?

216. ______ Let P(n) denote the probability that a randomly selected n-digit number contains the digits 42, adjacent and in that order, among its digits. Two 4-digit examples are $3\underline{42}2$ and $\underline{42}05$. What is the absolute difference between P(2) and P(3)? Express your answer as a common fraction.

217. ______ % Each year for the first five years of life, a baby elephant's weight increases by 10%. By what percent of its birth weight does an elephant's weight increase during these five years? Express your answer to the nearest whole number.

218. ______



Replace a, b and c with three different positive integers so that the sum of the two numbers along each side is a perfect square. What is the smallest possible value of the sum $a + b + c$?

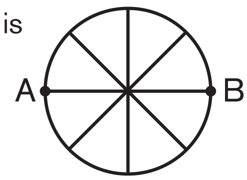
219. $ ______ Ronny had 9 oranges, and Donny had 15 oranges. They met up with Lonny, who had no oranges. Lonny gave $8 to Ronny and Donny, and the three of them shared the oranges equally. If Ronny and Donny split the $8 in proportion to the number of oranges each contributed, how much of the $8 should Ronny receive?

220. ______ The point (8, k) in the first quadrant is the same distance from the point (0, 4) as it is from the x-axis. What is the value of k?

Warm-Up 16

221. ________ If the counting numbers are written in order, what is the value of the 2015th digit written?

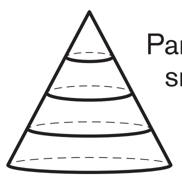
222. ________ cm The circle shown has four equally-spaced diameters of length 2 cm. What is the length of the longest path that can be drawn in one continuous pen stroke from A to B without retracing and without having the path cross itself? (The path may meet itself only at the center but may not cross over itself.) Express your answer in terms of π.



223. ________ cm Two chords of a circle intersect. The point of intersection divides the first chord into two segments of length 5 cm and 8 cm and divides the second chord into two segments, one which has length 4 cm. How long is the second chord?

224. ________ vertices A soccer ball is a polyhedron comprised of 12 pentagons and 20 hexagons. How many vertices does a soccer ball have?

225. ________



Parallel planes divide a cone into a smaller cone and three frustums so that the smaller cone and the three frustums have equal heights, as shown. What is the ratio of the volume of the smallest frustum to the volume of the largest frustum? Express your answer as a common fraction.

226. ________ All points with coordinates (x, y) that are equidistant from the points (1, 3) and (7, 11) lie along a single line. When the equation of the line is written in the form $y = mx + b$, what is the value of b?

227. ________ A sequence begins 1, 2, …, and each term after the second term is the sum of all preceding terms. What is the 15th term of this sequence?

228. ________ Abe chooses a number from Group A, Bob chooses a number from Group B and Charlie chooses a number from Group C.

Group A:	741	345	624	813
Group B:	519	825	717	456
Group C:	134	260	503	152

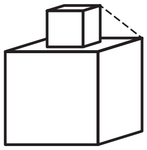
Then Abe chooses a digit X from his number, Bob chooses a digit Y from his number and Charlie chooses a digit Z from his number. The digits are arranged to form the three-digit number XYZ. Abe, Bob and Charlie, in that order, then choose different digits from their selected numbers to form a second three-digit number. Finally, in the same order, they use the remaining digits to form a third three-digit number. What is the sum of the three numbers that are formed?

229. ________ If $6^{12} = 6(6^n + 6^n + 6^n + 6^n + 6^n + 6^n)$, what is the value of n?

230. ________ ways Donald has nine one-day passes to Dizzyworld. He can go alone and use one, or he can take a friend and use two. If he visits every day until he uses all the passes, in how many different ways can he use them? Using two a day for four days and then going alone at the end (2, 2, 2, 2, 1) is different from reversing the order (1, 2, 2, 2, 2).

Workout 8

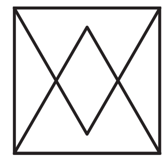
231. ______ inches



A cube with 2-inch edges is centered on a cube with 4-inch edges so that corresponding edges are parallel, as shown. What is the distance from a top vertex of the smaller cube to the nearest vertex of the larger one? Express your answer in simplest radical form.

232. $ ______ Writing down the four-digit dollar amount of a deposit, a bank teller accidentally transposed two adjacent digits. What is the greatest possible value of the resulting difference between the correct amount and the incorrect amount?

233. ______ Two equilateral triangles are drawn in a square. Two opposite sides of the square coincide with one side of each of the triangles, as shown. What is the ratio of the area of the center rhombus to the area of the entire square? Express your answer as a decimal to the nearest thousandth.



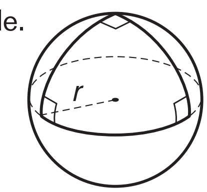
234. ______ paths

B	A	N
A	N	A
N	A	N

How many paths through this grid spell the word *BANANA*? A path consists of a series of moves from one cell to any cell with which it shares a side and may include multiple moves to a single cell.

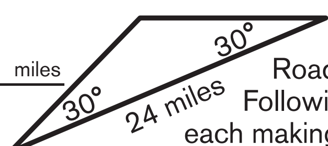
235. ______ On average, Josiah allows one hit for every five pitches. If he were to throw nine pitches, what is the probability that he would allow exactly three hits? Express your answer as a decimal to the nearest hundredth.

236. ______ The figure shows a triangular region bounded by three congruent arcs drawn on the surface of a sphere so that the intersection of any two arcs is a right angle. The triangular region's perimeter and surface area have numerical values A and B, respectively. If $\frac{A^2}{B} = \pi k$, what is the value of k? Express your answer as a decimal to the nearest tenth.



237. ______ The two solutions of the equation $x^2 + 7x + 5 = 0$ each can be expressed as a common fraction in simplest radical form as $\frac{l + u\sqrt{m}}{p}$. What is the least value of the product *plum*?

238. ______ miles



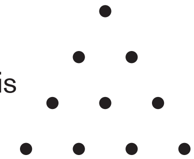
Road repairs prevented Anita from taking the 24-mile direct route home. Following a detour, she traveled on two different roads of equal distance, each making a 30-degree angle with the main road, as illustrated. As a result of the detour, how many more miles did Anita travel? Express your answer as a decimal to the nearest tenth.

239. ______ units^2 Points A(−1, 2), B(−2, −5) and C(7, −2) are on a circle. What is the area of the circle? Express your answer as a decimal to the nearest tenth.

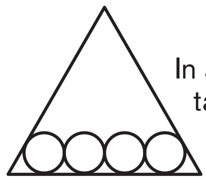
240. ______ The sum of the pairwise products of three consecutive natural numbers is 8111. What is the largest of the three numbers?

Warm-Up 17

241. ________ In the sum $4 + 2\sqrt{2} + 2 + \sqrt{2} + \ldots$ each term is obtained by dividing the previous term by $\sqrt{2}$. If the sum of the series, in simplest radical form, is $m + n\sqrt{2}$, what is the value of $m + n$?



242. ________ circles How many different circles can be drawn that intersect exactly four points in this triangular grid, made up of 10 points equally spaced?



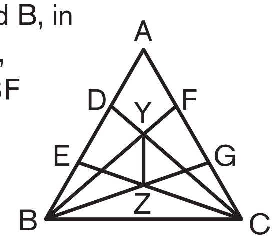
243. ________ cm In an equilateral triangle with edge length 12 cm, four congruent circles are tangent to each other and at least one side of the triangle as shown. What is the radius of each circle? Express your answer in simplest radical form.

244. ________ Isosceles triangle ACB has a right angle at C and shares a leg with equilateral triangle BCD of side length 2 in. The triangles, otherwise, do not intersect. Segments BC and AD intersect at E. What is the value of BE/EC? Express your answer in simplest radical form.

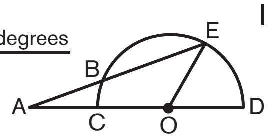
245. ________ What is the value of $a^2(b + c) + b^2(a + c) + c^2(a + b)$ if $a + b + c = 6$, $a^2 + b^2 + c^2 = 40$ and $a^3 + b^3 + c^3 = 200$?

246. ________ days Kevin and Devin each make one hat per day for charity, but they started on different days. Today, Kevin made his 520th hat, and Devin made his 50th. A celebration is planned for the next day that Kevin's hat count is evenly divisible by Devin's hat count. In how many days from today will they celebrate?

247. ________ In equilateral triangle ABC with side length 6 inches, points A, D, E and B, in that order, are equally spaced along side AB, and points A, F, G and C, in that order, are equally spaced along side AC as shown. Segments BF and CD intersect at Y, and segments BG and CE intersect at Z. When expressed as a common fraction in simplest radical form, the length of segment YZ is $\frac{r\sqrt{3}}{s}$ inches. What is the value of $r + s$?



248. ________ degrees In the figure, $\overset{\frown}{CBD}$ is a semicircle with center O and diameter CD. If AB = OD and the measure of angle EOD is 60 degrees, what is the measure of angle A?



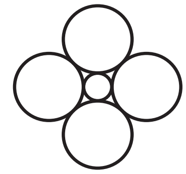
249. ________ times During a game of paintball, ten friends were positioned in a field so that no two of them stood the same distance apart. Each person aimed at his or her closest opponent, and at the signal everyone fired. What is the maximum number of times one player could have been hit?

250. ________ members In a certain state legislature, a proposed bill was defeated with 6 fewer votes for the bill than against it. After the bill was amended, 9 members who had previously voted against the bill were now for it. This resulted in 60% of the legislature now being in favor of the bill. If all those who previously voted for the bill remained in favor of it, how many members are in this state legislature, assuming every member voted each time?

Warm-Up 18

251. ______ units² An equiangular hexagon has side lengths 3, 4, 5, 3, 4 and 5 units in that order. What is its area? Express your answer as a common fraction in simplest radical form.

252. ______ mm Each of the four large circles shown here is tangent to two other circles of equal size and is tangent to the center circle, which has radius 1mm. What is the radius of each of the large circles? Express your answer in simplest radical form.



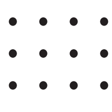
253. ______ units² If point Q lies on side AB of square ABCD such that QC = $\sqrt{10}$ units and QD = $\sqrt{13}$ units, what is the area of square ABCD?

254. ______ If $45_a = 54_b$ for positive integers a and b, what is the smallest possible value of $a + b$?

255. ______ If $\sqrt{2\sqrt[3]{2\sqrt[4]{2}}} = 2^c$, what is the value of c? Express your answer as a common fraction.

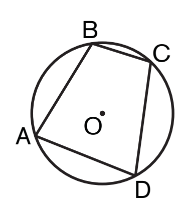
256. ______ Connecting the centers of the four faces of a regular tetrahedron creates a smaller regular tetrahedron. What is the ratio of the volume of the smaller tetrahedron to the volume of the original one? Express your answer as a common fraction.

257. ______ Daniel began painting a room at 9:00 a.m. Yeong, who can paint twice as fast as Daniel, started helping Daniel at 9:20 a.m., and they worked together until the room was fully painted at 10:00 a.m. What fraction of the room had been painted by 9:30 a.m.? Express your answer as a common fraction.

258. ______  Three different dots are randomly chosen from the 16 equally spaced dots in the grid shown. What is the probability that the three dots are collinear? Express your answer as a common fraction.

259. ______ pounds To weigh an object by using a balance scale, Brady places the object on one side of the scale and places enough weights on each side to make the two sides of the scale balanced. Brady's set of weights contains the minimum number necessary to measure the whole-number weight of any object from 1 to 40 pounds, inclusive. What is the greatest weight, in pounds, of a weight in Brady's set?

260. ______ degrees Quadrilateral ABCD is inscribed in circle O as shown. Arc AB = 100 degrees, and arc BC = 50 degrees. What is the measure of angle ADC?

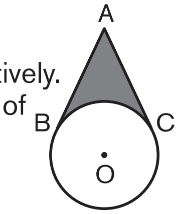


Workout 9

261. ________ If a, b, c and d represent four different positive integers such that $a^2 + d^2 = b^2 + c^2$, what is the least possible value of $|d - a|$?

262. ________ units^2 The line with equation $ax + by = c$, where a, b and c are positive, forms a right triangle with legs on the x- and y-axes. What is the area of the triangle? Express your answer as a common fraction in terms of a, b and c.

263. ________ in^2 In the figure, segments AB and AC are tangent to circle O at B and C, respectively. If AB = 12 inches, and minor arc BC measures 120 degrees, what is the area of the shaded region? Express your answer as a decimal to the nearest tenth.



264. ________ Each morning, Khalid and Jack arrive at the dining hall separately, at a random time from 8:00 to 9:00 a.m., and each remains there for exactly 15 minutes while eating. What is the probability they will see each other in the dining hall tomorrow morning? Express your answer as a common fraction.

265. ________ Colby is practicing his basketball shots. He knows that he has a 0.7 chance of making each 2-point shot and a 0.4 chance of making each 3-point shot. If he takes five 2-point and five 3-point shots, what is his probability of earning 20 points or more? Express your answer as a decimal to the nearest thousandth.

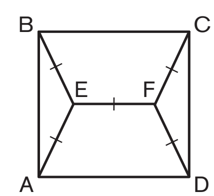
266. ________ in^2 One circular base of a cylinder with radius 2 inches and height 4 inches is glued flush to the center of each face of a 4-inch cube. What is the surface area of the resulting solid? Express your answer as a decimal to the nearest tenth.

267. ________ fish Sam, Taylor and Pat counted the number of fish in each of their fish tanks. They noticed that Sam's tank had exactly 25% more fish than Taylor's tank, and Pat's tank had exactly 24% more fish than Sam's tank. If each tank had at least one fish, what is the minimum combined number of fish that could have been in the three tanks?

268. ________ degrees

The lateral surface of a right circular cone of radius 48 feet and height 14 feet is cut along a line drawn from the apex perpendicular to the base and unfurled to form the two-dimensional shape shown. What is the central angle measure of the sector that is missing from the circle? Express your answer as a decimal to the nearest tenth.

269. ________ cm^2 Inside square ABCD, shown here, points E and F are chosen in such a way that AE = BE = CF = DF = EF = 4 cm. What is the area of triangle ABE?



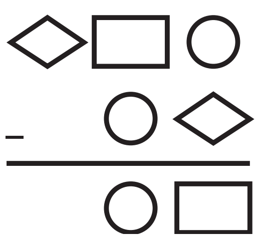
270. ________ Each digit 0 through 9 is used exactly once to create two five-digit numbers. What is the sum of the digits of the greatest product of two such numbers?

Logic Stretch

271.__________ Celia, Desi and Everett are each wearing a hat that displays a different whole number from 1 to 9, inclusive. Each number cannot be seen by the person wearing it, but that number is visible to the other two individuals. Everett says, "The sum of the numbers I see is 6." Celia says, "The product of the numbers I see is 10." What is the sum of the numbers that Everett could possibly have on his hat?

272.__________ people In a survey, 30 people reported that they enjoy some combination of walking, hiking and jogging. The number who enjoy only walking, the number who enjoy only hiking and the number who enjoy only jogging are all equal. Likewise, the number who enjoy only walking and hiking, the number who enjoy only walking and jogging and the number who enjoy only hiking and jogging are equal. In addition, the survey showed that half as many people enjoy exactly two of these activities as those who enjoy only one activity. If three people enjoy all three activities, how many people enjoy jogging?

273.__________



In the subtraction problem shown, the shapes ◇, □ and ○ each represent a different digit. What is the value of □◇ ÷ ○?

274.Box__________ Three identical boxes contain tennis balls, baseballs or both. A label is affixed to each box. The three labels correctly describe the three boxes, but none of the labels is on the correct box. Box 1 is labeled "Tennis Balls." Box 2 is labeled "Baseballs." Box 3 is labeled "Tennis Balls & Baseballs." Devon reaches into Box 3 and pulls out a baseball. Which box contains only tennis balls?

275.Page__________ Drew purchased a used 50-page book at the book fair. Drew later realized that the book, in which left-hand pages contained even page numbers and right-hand pages contained odd page numbers, did not contain all 50 pages. The sum of the page numbers on the pages that Drew's book did contain was 1242. What is the greatest page number that could be on a page missing from Drew's book?

276.__________ In the addition problem shown, each letter stands for a different digit. If T = 3, what is the value of the four-digit number MATH?

$$\begin{array}{cccc} & G & E & T \\ + & T & H & E \\ \hline M & A & T & H \end{array}$$

277.______ seconds Starting at the lower landing of a staircase, Porscha goes up the steps by repeating a three-step sequence: moving two steps up and then moving one step down. Starting at the upper landing of the same staircase, Micah goes down the steps by repeating a different three-step sequence: moving two steps down and then moving one step up. After simultaneously moving to their first steps, Porscha and Micah both move to another step every 3 seconds. To go from the upper landing to the lower landing of the staircase involves a net movement of 12 steps. How many seconds after moving to the starting steps will Porscha and Micah reach the same step?

278.__________ If the six-digit number 3D6,D92 is divisible by 11, what is the value of D?

279.__________ A special deck of cards contains cards numbered 1 through 4 for each of four suits. Each of the 16 cards has a club, diamond, heart or spade on one side and the number 1, 2, 3 or 4 on the other side. After a dealer mixed up the cards, three were selected at random. What is the probability that of these three randomly selected cards, displayed here, one of the cards showing the number 2 has a heart printed on the other side? Express your answer as a common fraction.

280.__________ The units digit of a three-digit number, ABC, is moved to the left of the remaining two digits to make a new three-digit number, CAB. If CAB − ABC = 162, what is the sum of the least and greatest possible values of ABC?

Solving Inequalities Stretch

Quick Review of Inequality Properties

For any numbers a, b and c,

- if $a > b$, then $a + c > b + c$ and $a - c > b - c$. (applies to $>$, $\geq$, $<$ and $\leq$)
- if $a > b$ and $c > 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$. (applies to $>$, $\geq$, $<$ and $\leq$)
- if $a > b$ and $c < 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$. (applies to $>$, $\geq$, $<$ and $\leq$)
- if $|a| < b$, then $a < b$ and $a > -b$. (applies to $<$ and $\leq$)
- if $|a| > b$, then $a > b$ or $a < -b$. (applies to $>$ and $\geq$)

Solve each inequality, and graph the solution on the number line provided.

281.__________ $3 - \frac{x}{3} \leq 5$

282.__________ $3 - \frac{x}{3} \geq -5$

283.__________ $\left|3 - \frac{x}{3}\right| \leq 5$

284.__________ $3 - \frac{x}{3} < x - 5$

285.__________ $3 - \frac{x}{3} > 5 - x$

286.__________ $\left|3 - \frac{x}{3}\right| < x - 5$

287.__________ $x^2 \leq 25$

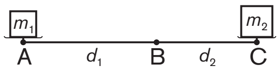
288.__________ $x^2 \geq 25$

289.__________ $x^2 + 4x - 4 > -8$

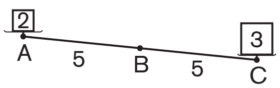
290.__________ $x^2 + 4x - 4 > -7$

Mass Point Geometry Stretch

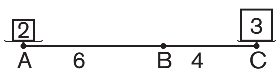
Center of Mass



Consider a seesaw, with a fulcrum at B, that has objects at A and C. As shown, the object at A has a mass of m_1, and its distance from B is d_1. The object at C has mass m_2, and its distance from B is d_2. These examples show how the position of the fulcrum determines whether the seesaw is balanced. The mass at B is $m_1 + m_2$, the sum of the masses at A and C.

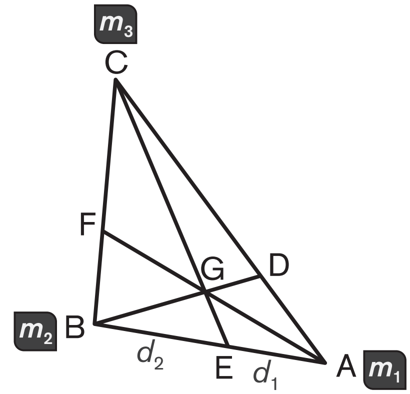


In this first example, B is positioned so that $d_1 = d_2 = 5$. Notice that $m_1 \times d_1 = 2 \times 5 = 10$ and $m_2 \times d_2 = 3 \times 5 = 15$. Although the objects at A and C are equidistant from B, the object at C is lower than the object at A because $m_1 \times d_1 < m_2 \times d_2$.



In this example, B is positioned so that $d_1 = 6$ and $d_2 = 4$. Here, $m_1 \times d_1 = 2 \times 6 = 12$ and $m_2 \times d_2 = 3 \times 4 = 12$. This time, the seesaw is balanced because $m_1 \times d_1 = m_2 \times d_2$. In this case, the position of B is known as the *center of mass*.

A *cevian* is a line segment that joins a vertex of a triangle with a point on the opposite side. Mass point geometry is a technique used to solve problems involving triangles and intersecting cevians by applying center of mass principles. Because triangle ABC, shown here, has cevians AF, BD and CE that intersect at point G, we can apply the center of mass principles presented. For example, side AB is balanced on point E when $m_1 \times d_1 = m_2 \times d_2$.



A *mass point*, denoted *m*P, consists of point P and its associated mass, *m*. Assume point G is the center of mass on which the entire triangle balances. Then the mass at G is the sum of the masses at the endpoints for each cevian and *m*G = *m*A + *m*F = *m*B + *m*D = *m*C + *m*E.

Suppose BF:CF = 3:4 and AD:CD = 2:5, and we are asked to determine the ratios AE:BE, AG:FG and BG:DG.

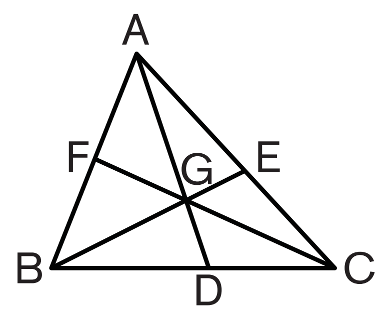
Start by finding *m*B and *m*C for side BC, which is balanced on point F. We know $m_2 \times 3 = m_3 \times 4$. We can let $m_2 = 4$ and $m_3 = 3$, so 4B + 3C = (4 + 3)F = 7F.

Next, find *m*A for side AC, which is balanced on point D. We know $m_1 \times 2 = m_3 \times 5$. Since $m_3 = 3$, it follows that $m_1 \times 2 = 3 \times 5$ and $m_1 = 15/2$. Rather than having mass point (15/2)A, we can multiply 4B, (15/2)A, 3C and 7F by 2 to get the following mass points: 8B, 15A, 6C and 14F. Now the mass at each point is of integer value.

Now, there is enough information to find *m*D and *m*E, since 15A + 6C = (15+ 6)D = 21D and 15A + 8B = (8 + 15)E = 23E. Therefore, given mass points 15A and 8B, it follows that side AB is balanced on point E when AE:BE = 8:15. In addition, given mass points 21D and 14F, we see that cevians AF and BD both are balanced on point G when AG:FG = 14:15 and BG:DG =21:8.

Solve the following problems by using mass point geometry. Express ratio answers as common fractions.

Triangle ABC, shown here, has cevians AD, BE and CF intersecting at point G, with AF:BF = 3:2 and BD:CD = 5:3.

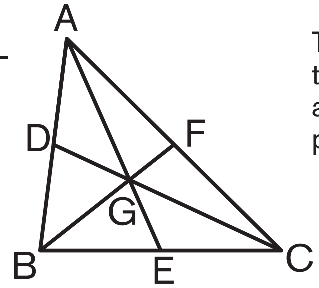


291.__________ What is the ratio of AE to CE?

292.__________ What is the ratio of BG to EG?

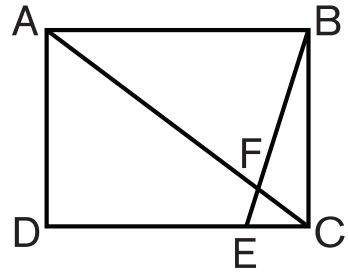
293.__________ What is the ratio of DG to AG?

294.__________

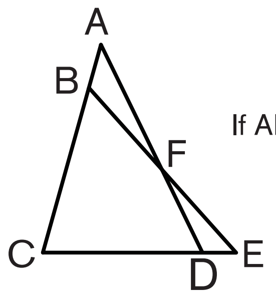


The medians of a triangle intersect at a point in the interior of the triangle as shown. What is the ratio of the lengths of the shorter and longer segments into which each median is divided at the point of intersection?

295.__________ In rectangle ABCD, point E is on side DC such that BC = 8, BE = 10 and AC = 17. If segments AC and BE intersect at F, what is the ratio of the area of triangle CFE to the area of triangle AFB?

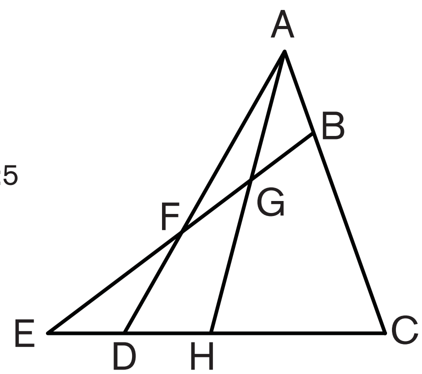


296.__________

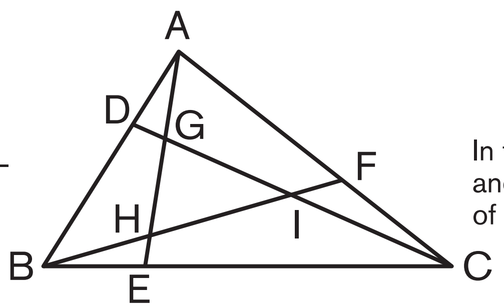


If AB:BC = 1:4 and AF:DF = 5:4, what is the ratio of DE to CD?

297.__________ For integers x, y and z, if AB:BC = 1:4, AG:GH = 3:5 and AF:DF = 5:4, then CH:DH:DE = x:y:z. What is the value of $x + y + z$?



298.__________

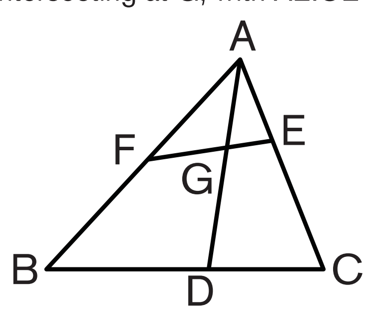


In triangle ABC, AD:BD = 1:2, BE:EC = 1:3 and AF:CF = 3:2. What is the ratio of the area of triangle GHI to the area of triangle ABC?

Triangle ABC, shown here, has cevian AD and transversal EF intersecting at G, with AE:CE = 1:2, AF:BF = 5:4 and BD:CD = 3:2.

299.__________ What is the ratio of AG to DG?

300.__________ What is the ratio of EG to FG?



PROGRAM DETAILS

The MATHCOUNTS Foundation administers its math enrichment, coaching and competition program with a grassroots network of more than 17,000 volunteers who organize MATHCOUNTS competitions nationwide. Each year more than 500 local competitions and 56 "state" competitions are conducted, primarily by chapter and state societies of the National Society of Professional Engineers. All 50 states, the District of Columbia, Puerto Rico, Guam, Virgin Islands and schools worldwide that are affiliated with the U.S. Departments of Defense and State participate in MATHCOUNTS.

This section of the handbook should serve as your main guide for making the most of your time when coaching for competitions. From program rules and procedures to tips from veteran coaches about growing your school's program, this resource was designed with teachers and coaches in mind.

GETTING STARTED AS A COACH

Thank you so much for serving as a coach in the MATHCOUNTS Competition Series!
Your work truly does make a difference in the lives of the students you mentor.

All MATHCOUNTS materials, including this handbook, can be incorporated into regular classroom instruction so that all students learn problem-solving techniques and develop critical thinking skills. If your school's MATHCOUNTS team meetings are limited to extracurricular sessions, all interested students should be invited to participate—regardless of their academic standing. Because the greatest amount of time in this program will be invested at the school level, having lots of Mathletes participate helps ensure that more students benefit from an experience in MATHCOUNTS.

Generating Student Interest: Here are some tips given to us from successful competition coaches and club leaders for getting students involved in the program at the beginning of the year.

- Ask Mathletes who have participated in the past to talk to other students about participating.
- Ask teachers, parent volunteers and counselors to help you recruit.
- Reach parents through school newsletters, PTA meetings or Back-to-School-Night presentations.
- Advertise around your school by.
  1) posting intriguing math questions (specific to your school) and referring students to the first meeting for answers.
  2) designing a bulletin board or display case with your MATHCOUNTS poster, photos and awards from past years.
  3) attending meetings of other extracurricular clubs (such as honor society) so you can invite their members to participate.
  4) adding information about the MATHCOUNTS team to your school's website.
  5) making a presentation at the first pep rally or student assembly.

Maintaining a Strong School Program: Having a great start is important, but here are some tips from coaches and club leaders about keeping your program going strong for the entire year.

- Publicize meetings on your school's website, in a school newsletter and in morning announcements.
- Celebrate your team's success by
  1) scheduling a special pep rally or awards ceremony for the Mathletes at your school.

2) planning a special field trip, such as to a local college campus or museum, as a reward.
3) planning an end-of-year event, such as a "students vs. teachers" Countdown Round, that gets the entire school involved and recognizes the Mathletes.
4) giving prizes, such as T-shirts, to the students who participate. (You can ask parents and local businesses to help with donations for these.)
5) posting event information, awards won and photos of your team on your school's website.

USING THIS HANDBOOK

Beginning your team meetings in the fall will help you maximize your coaching time and expose your students to more handbook problems before the Chapter Competition.

The *MATHCOUNTS School Handbook* is released every fall and provides the foundation for coaches to prepare their Mathletes to compete in the MATHCOUNTS Competition Series. This resource contains 300 challenging and creative problems that are written to meet the National Council of Teachers of Mathematics Standards for Grades 6-8. The handbook is made available electronically to all U.S. schools through the MATHCOUNTS website, and a hard copy is available upon request to all schools—free of charge. Coaches who register for the MATHCOUNTS Competition Series will receive the handbook in their School Competition Kit.

Handbook Contents and Structure: This handbook consists primarily of three types of math problems—Warm-Ups, Workouts and Stretches—that are designed to complement the Competition Series. All of the problems provide students with practice in a variety of problem-solving situations and may be used to diagnose skill levels, to practice and apply skills or to evaluate growth in skills. The handbook also contains additional resources for coaches to use, as described in the list.

- *Warm-Ups (pp. 9-34)* serve as excellent practice for the Sprint Round and assume that students will not be using calculators. These problems increase in difficulty as students go through the handbook.
- *Workouts (pp. 11-35)* serve as practice for the Target and Team Rounds and assume the use of a calculator. These problems also increase in difficulty as students go through the handbook.
- *Stretches (pp. 36-40)* focus on specific standards and topics and cover a variety of difficulty levels. These problems can be incorporated into your students' practice at any time.
- *Vocabulary and Formulas (pp. 52-53)*
- *Solutions (pp. 54-76)* provide complete explanations for how to solve all problems in this handbook. These are only possible solutions; you or your students may come up with more elegant solutions.
- *Difficulty Rating (p. 77)* explains ratings on a scale of 1-7, with 7 being the most difficult.
- *Answer Key (pp. 77-81)* provides answers for all handbook problems.
- *Common Core State Standards (p. 82)* explain how problems are aligned to the Common Core, as shown in the Problem Index.
- *Problem Index (pp. 83-84)* helps you incorporate handbook problems into your curriculum. This index organizes all 300 problems by topic, difficulty rating and mapping to the Common Core.

Tips for Productive Practices: Here are some suggestions for getting the most out of team meetings.

- Encourage discussion of the problems so that students learn from one another.
- Encourage a variety of methods for solving problems.
- Have students write problems for each other.
- Use the Problem of the Week, posted on www.mathcounts.org/potw every Monday.
- Practice working in groups to develop teamwork (and to prepare for the Team Round).
- Practice oral presentations to reinforce understanding.
- Use the Interactive MATHCOUNTS Platform, which includes current and past materials, as well as features that make it easy to collaborate.
- Provide refreshments and vary the location of your meetings to create an, enjoyable atmosphere.
- Recruit volunteers (such as MATHCOUNTS alumni, high school students, parents, communtiy professionals and reitrees) to serve as assistant coaches.

Suggested Practice Schedule: On average, coaches meet with their students for an hour once a week at the beginning of the year, and more often as the competitions approach. Practice sessions may be held before school, during lunch, after school, on weekends or at other times, coordinating with your school's schedule and avoiding conflicts with other activities.

Designing a schedule for your practices will help ensure you are able to cover more problems and prepare your students for competitions. Handbook Stretches can be used at any time during the fall, but below is the recommended schedule for using Warm-Ups and Workouts if you are participating in the Competition Series.

September 2014	Warm-Ups 1-2	Workout 1
October	Warm-Ups 3-6	Workouts 2-3
November	Warm-Ups 7-10	Workouts 4-5
December	Warm-Ups 11-14	Workouts 6-7
January 2015	Warm-Ups 15-16	Workout 8
	MATHCOUNTS School Competition	
February	Warm-Ups 17-18	Workout 9
	Selection of competitors for Chapter Competition	
	MATHCOUNTS Chapter Competition	

To encourage participation by the greatest number of students, postpone selection of your school's official competitors until just before the Chapter Competition.

ADDITIONAL RESOURCES FOR COACHES

Take advantage of other free resources available to coaches. The more MATHCOUNTS problems your students can work on in the fall, the better they will do in competitions.

Free Web Resources: These resources are available to coaches at no cost.

- *Past MATHCOUNTS Competitions:* the 2014 School, Chapter and State MATHCOUNTS Competitions and Answer Keys. Visit www.mathcounts.org/pastcompetition.
- *MATHCOUNTS Problem of the Week:* weekly set of 3-4 theme-based problems focusing on critical thinking and problem-solving skills. You can access the Problem of the Week Archive with problems starting from 2012. Visit www.mathcounts.org/potw.
- *MATHCOUNTS Minis:* monthly instructional math videos featuring Richard Rusczyk from Art of Problem Solving, explaining how to solve many types of problems. You can access the Minis Archive and choose from over 35 past videos. Visit www.mathcounts.org/minis.
- *Interactive MATHCOUNTS Platform:* forum that contains current and past resources, and lets users discuss problems and get instant feedback on their progress. Visit mathcounts.nextthought.com.

Additional Resources: These resources are available for purchase.

- *MATHCOUNTS OPLET:* The Online Problem Library and Extraction Tool (OPLET) is a database of over 13,000 problems organized by topic and difficulty level. Visit www.mathcounts.org/oplet.
- *MATHCOUNTS Online Store:* Purchase practice books, supplies, prizes and awards for your team. Visit www.mathcounts.org/store. Below is a list of the most popular practice books available.

  2014 Competitions (previous years available while supplies last)
  The Most Challenging MATHCOUNTS Problems Solved (step-by-step solutions to National Competition Sprint and Target Round problems from 2001 to 2010)
  Practice Problems for MATHCOUNTS, Vol. 1
  Practice Problems for MATHCOUNTS, Vol. 2
  The All-Time Greatest MATHCOUNTS Problems

OFFICIAL RULES AND PROCEDURES

The following rules and procedures govern all MATHCOUNTS competitions. The MATHCOUNTS Foundation reserves the right to alter these rules and procedures at any time. **Coaches are responsible for being familiar with the rules and procedures outlined in this handbook.** Coaches should bring any difficulty in procedures or in student conduct to the immediate attention of the appropriate chapter, state or national official. Students violating any rules may be subject to immediate disqualification.

REGISTRATION

The fastest and easiest way to register for the MATHCOUNTS Competition Series is online at www.mathcounts.org/compreg.

For your school to participate in the MATHCOUNTS Competition Series, a school representative is required to complete a registration form and pay the registration fees. A school representative can be a teacher, an administrator or a parent volunteer who has received express permission from his or her child's school administration to register. By completing the Competition Series Registration Form, the coach attests to the school administration's permission to register students for MATHCOUNTS.

School representatives can register online at www.mathcounts.org/compreg or download the Competition Series Registration Form and mail, fax or e-mail a scanned copy of it to the MATHCOUNTS Registration Office. Refer to the Critical 2014-2015 Dates on page 4 of this handbook for registration contact information.

What Registration Covers: Registration in the Competition Series entitles a school to

1) send 1-10 students (depending on number registered) to the Chapter Competition. *Students can advance beyond the chapter level, but this is determined by their performance at the competition.*
2) receive the School Competition Kit, which includes the *2014-2015 MATHCOUNTS School Handbook*, a recognition ribbon for each registered student, 10 student participation certificates and a catalog of additional coaching materials. *Mailings of School Competition Kits will occur on a rolling basis through December 31, 2014.*
3) receive online access to the 2015 School Competition, along with electronic versions of other competition materials, at www.mathcounts.org/competitioncoaches. *Coaches will receive an e-mail notification no later than November 3, 2014 when the 2015 School Competition is available online.*

Your state or chapter coordinator will be notified of your registration, and you then will be informed of the date and location of your Chapter Competition. **If you have not been contacted by mid-January with competition details, it is your responsibility to contact your local coordinator** to confirm that your registration has been properly routed and that your school's participation is expected. Coordinator contact information is available at www.mathcounts.org/competition.

Deadlines: The sooner your Registration Form is received, the sooner you will receive your preparation materials. To guarantee your school's registration, submit your registration by one of the following deadlines:

Early Bird Discount Deadline: November 14, 2014	Online registrations: submitted by 11:59 p.m. PST E-mailed or faxed forms: received by 11:59 p.m. PST Mailed forms: postmarked by November 14, 2014
Regular Registration Deadline: December 12, 2014*	Online registrations: submitted by 11:59 p.m. PST E-mailed or faxed forms: received by 11:59 p.m. PST Mailed forms: postmarked by December 12, 2014

*Late registrations may be accepted at the discretion of the MATHCOUNTS national office and your local coordinators, but acceptance is not guaranteed. If a school's late registration is accepted, an additional $20 processing fee will be assessed.

Registration Fees: The cost of your school's registration depends on when your registration is postmarked/e-mailed/faxed/submitted online. The cost of your school's registration covers the students for the entire Competition Series; there are no additional registration fees for competing at the state or national level. Title I schools (as affirmed by a school's administration) receive a 50% discount off the total cost of their registration.

Number of Registered Students	Registration Postmarked by 11/14/2014	Postmarked between 11/14/2014 and 12/12/2014	Postmarked after 12/12/2014 (with Late Fee)
1 individual	\$25	\$30	\$50
2 ind.	\$50	\$60	\$80
3 ind.	\$75	\$90	\$110
1 team of 4	\$90	\$100	\$120
1 tm. + 1 ind.	\$115	\$130	\$150
1 tm. + 2 ind.	\$140	\$160	\$180
1 tm. + 3 ind.	\$165	\$190	\$210
1 tm. + 4 ind.	\$190	\$220	\$240
1 tm. + 5 ind.	\$215	\$250	\$270
1 tm. + 6 ind.	\$240	\$280	\$300

ELIGIBILITY REQUIREMENTS

Eligibility requirements for the MATHCOUNTS Competition Series are different from requirements for other MATHCOUNTS programs. Eligibility for The National Math Club or the Math Video Challenge does not guarantee eligibility for the Competition Series.

Who IS Eligible:

- U.S. students enrolled in the 6th, 7th or 8th grade can participate in MATHCOUNTS competitions.
- Schools that are the students' official school of record can register.
- Any type of school, of any size, can register—public, private, religious, charter, virtual or homeschools—but virtual and homeschools must fill out additional forms to participate (see p. 46).
- Schools in the 50 U.S. states, District of Columbia, Guam, Puerto Rico and Virgin Islands can register.
- Overseas schools that are affiliated with the U.S. Department of Defense or State can register.

Who IS NOT Eligible:

- Students who are not full-time 6th, 7th or 8th graders cannot participate, even if they are taking middle school math classes.
- Academic centers, tutoring centers or enrichment programs that do not function as students' official school of record cannot register. *If it is unclear whether your educational institution is considered a school, please contact your local Department of Education for specific criteria governing your state.*
- Schools located outside of the U.S. states and not in the territories listed above cannot register.
- Overseas schools not affiliated with the U.S. Department of Defense or State cannot register.

Number of Students Allowed: A school can register a maximum of one team of four students and six individuals; these 1-10 students will represent the school at the Chapter Competition. Any number of students can participate at the school level. Prior to the Chapter Competition, coaches must notify their chapter coordinator to identify which students will be team members and which students will compete as individuals.

Number of Years Allowed: Participation in MATHCOUNTS competitions is limited to three years for each student, but there is no limit to the number of years a student may participate in school-based coaching.

What Team Registration Means: Members of a school team will participate in the Target, Sprint and Team Rounds. Members of a school team also will be eligible to qualify for the Countdown Round (where conducted). Team members will be eligible for team awards, individual awards and progression to the state and national levels based on their individual and/or team performance. It is recommended that your strongest four Mathletes form your school team. Teams of fewer than four will be allowed to compete; however, the Team Score will be computed by dividing the sum of the team members' scores by 4 (see p. 49), meaning teams of fewer than four students will be at a disadvantage. *Only one team (of up to four students) per school is eligible to compete.*

What Individual Registration Means: Students registered as individuals will participate in the Target and Sprint Rounds but not the Team Round. Individuals will be eligible to qualify for the Countdown Round (where conducted). Individuals also will be eligible for individual awards and progression to the state and national levels. A student registered as an individual may not help his or her school's team advance to the next level of competition. *Up to six students may be registered in addition to or in lieu of a school team.*

How Students Enrolled Part-Time at Two Schools Participate: *A student may compete only for his or her official school of record.* A student's school of record is the student's base or main school. A student taking limited course work at a second school or educational center may not register or compete for that second school or center, even if the student is not competing for his or her school of record. MATHCOUNTS registration is not determined by where a student takes his or her math course. If there is any doubt about a student's school of record, the chapter or state coordinator must be contacted for a decision before registering.

How Small Schools Participate: MATHCOUNTS does not distinguish between the sizes of schools for Competition Series registration and competition purposes. Every "brick-and-mortar" school will have the same registration allowance of up to one team of four students and/or up to six individuals. A school's participants may not combine with any other school's participants to form a team when registering or competing.

How Homeschools Participate: Homeschools and/or homeschool groups in compliance with the homeschool laws of the state in which they are located are eligible to participate in MATHCOUNTS competitions in accordance with all other rules. Homeschool coaches must complete a Homeschool Participation Form, verifying that students from the homeschool or homeschool group are in the 6th, 7th or 8th grade and that each homeschool complies with applicable state laws. Forms can be downloaded at www.mathcounts.org/competition and must be submitted to the MATHCOUNTS national office in order for registrations to be processed.

How Virtual Schools Participate: Virtual schools that want to register must contact the MATHCOUNTS national office by December 1, 2014 for specific registration details. Any student registering as a virtual-school student must compete in the MATHCOUNTS Chapter Competition assigned according to the student's home address. Additionally, virtual-school coaches must complete a Homeschool Participation Form verifying that the students from the virtual school are in the 6th, 7th or 8th grade and that the virtual school complies with applicable state laws. Forms can be downloaded at www.mathcounts.org/competition and must be submitted to the national office in order for registrations to be processed.

What Is Done for Substitutions of Students: Coaches determine which students will represent the school at the Chapter Competition. Coaches cannot substitute team members for the State Competition unless a student voluntarily releases his or her position on the school team. Additional requirements and documentation for substitutions (such as requiring parental release or requiring that the substitution request be submitted in writing) are at the discretion of the state coordinator. A student being added to a team need not be a student who was registered for the Chapter Competition as an individual. Coaches cannot make substitutions for students progressing to the State Competition as individuals. At all levels of competition, student substitutions are not permitted after on-site competition registration has been completed.

What Is Done for Religious Observances: A student who is unable to attend a competition due to religious observances may take the written portion of the competition up to one week in advance of the scheduled competition. In addition, all competitors from that student's school must take the Sprint and Target Rounds at the same earlier time. If the student who is unable to attend the competition due to a religious observance (1) is a member of the school team, then the team must take the Team Round at the same earlier time; (2) is not part of the school team, then the team has the option of taking the Team Round during this advance testing or on the regularly scheduled day of the competition with the other school teams. The coordinator must be made aware of the team's decision before the advance testing takes place. *Advance testing will be done at the discretion of the chapter and state coordinators. If advance testing is deemed possible, it will be conducted under proctored conditions.* Students who qualify for an official Countdown Round but are unable to attend will automatically forfeit one place standing.

What Is Done for Students with Special Needs: Reasonable accommodations may be made to allow students with special needs to participate. However, many accommodations that are employed in a classroom or teaching environment cannot be implemented in the competition setting. Accommodations that are not permissible include, but are not limited to, granting a student extra time during any of the competition rounds or allowing a student to use a calculator for the Sprint or Countdown Rounds. *A request for accommodation of special needs must be directed to chapter or state coordinators in writing at least three weeks in advance of the Chapter or State Competition.* This written request should thoroughly explain a student's special needs, as well as what the desired accommodation would entail. In conjunction with the MATHCOUNTS Foundation, coordinators will review the needs of the student and determine if any accommodations will be made. In making final determinations, the feasibility of accommodating these needs at the National Competition will be taken into consideration.

LEVELS OF COMPETITION

There are four levels in the MATHCOUNTS Competition Series: school, chapter (local), state and national. Competition questions are written for 6th, 7th and 8th graders. The competitions can be quite challenging, particularly for students who have not been coached using MATHCOUNTS materials. All competition materials are prepared by the national office.

School Competitions (Ideally Held in January 2015): After several months of coaching, schools registered for the Competition Series should administer the 2015 School Competition to all interested students. The School Competition should be an aid to the coach in determining competitors for the Chapter Competition. *Selection of team and individual competitors is entirely at the discretion of the coach and does not need to be based solely on School Competition scores.* School Competition materials are sent to the coach of a school, and it may be used by the teachers and students only in association with that school's programs and activities. The current year's School Competition questions must remain confidential and may not be used in outside activities, such as tutoring sessions or enrichment programs with students from other schools. For updates or edits, please check www.mathcounts.org/competitioncoaches before administering the School Competition.

It is important that the coach look upon coaching sessions during the academic year as opportunities to develop better math skills in all students, not just in those students who will be competing. Therefore, it is suggested that the coach postpone selection of competitors until just prior to the Chapter Competition.

Chapter Competitions (Held from January 31 to February 28, 2015): The Chapter Competition consists of the Sprint, Target and Team Rounds. The Countdown Round (official or just for fun) may or may not be conducted. The chapter and state coordinators determine the date and location of the Chapter Competition in accordance with established national procedures and rules. Winning teams and students will receive recognition. The winning team will advance to the State Competition. Additionally, the two highest-ranking competitors not on the winning team (who may be registered as individuals or as members of a team) will advance to the State Competition. This is a minimum of six advancing Mathletes (assuming the winning team has four members). Additional teams and/or individuals also may progress at the discretion of the state coordinator, but the policy for progression must be consistent for all chapters within a state.

State Competitions (Held from March 1 to March 31, 2015): The State Competition consists of the Sprint, Target and Team Rounds. The Countdown Round (official or just for fun) may or may not be included. The state coordinator determines the date and location of the State Competition in accordance with established national procedures and rules. Winning teams and students will receive recognition. The four highest-ranked Mathletes and the coach of the winning team from each State Competition will receive an all-expenses-paid trip to the National Competition.

2015 Raytheon MATHCOUNTS National Competition (Held Friday, May 8, 2015 in Boston, MA): The National Competition consists of the Sprint, Target, Team and Countdown Rounds (conducted officially). Expenses of the state team and coach to travel to the National Competition will be paid by MATHCOUNTS. The national program does not make provisions for the attendance of additional students or coaches. All national competitors will receive a plaque and other items in recognition of their achievements. Winning teams and individuals also will receive medals, trophies and college scholarships.

COMPETITION COMPONENTS

The following four rounds of a MATHCOUNTS competition are designed to be completed in approximately three hours.

Target Round (approximately 30 minutes): In this round 8 problems are presented to competitors in four pairs (6 minutes per pair). The multistep problems featured in this round engage Mathletes in mathematical reasoning and problem-solving processes. *Problems assume the use of calculators.*

Sprint Round (40 minutes): Consisting of 30 problems, this round tests accuracy, with the time period allowing only the most capable students to complete all of the problems. *Calculators are not permitted.*

Team Round (20 minutes): In this round, interaction among team members is permitted and encouraged as they work together to solve 10 problems. *Problems assume the use of calculators.*

Note: The order in which the written rounds (Target, Sprint and Team) are administered is at the discretion of the competition coordinator.

Countdown Round: A fast-paced oral competition for top-scoring individuals (based on scores in the Target and Sprint Rounds), this round allows pairs of Mathletes to compete against each other and the clock to solve problems. *Calculators are not permitted.*

At Chapter and State Competitions, a Countdown Round (1) may be conducted officially, (2) may be conducted unofficially (for fun) or (3) may be omitted. However, the use of an official Countdown Round must be consistent for all chapters within a state. In other words, *all* chapters within a state must use the round officially in order for *any* chapter within a state to use it officially. All students, whether registered as part of a school team or as individual competitors, are eligible to qualify for the Countdown Round.

An official Countdown Round determines an individual's final overall rank in the competition. If a Countdown Round is used officially, the official procedures as established by the MATHCOUNTS Foundation must be followed, as described below.*

- The top 25% of students, up to a maximum of 10, are selected to compete. These students are chosen based on their Individual Scores.
- The two lowest-ranked students are paired; a question is read and projected, and students are given 45 seconds to solve the problem. A student may buzz in at any time, and if he or she answers correctly, a point is scored. If a student answers incorrectly, the other student has the remainder of the 45 seconds to answer.
- A total of three total questions are read to the pair of students, one question at a time, and the student who scores the higher number of points (not necessarily 2 out of 3) progresses to the next round and challenges the next-higher-ranked student.
- If students are tied in their matchup after three questions (at 1-1 or 0-0), questions should continue to be read until one is successfully answered. The first student who answers an additional question correctly progresses to the next round.
- This procedure continues until the fourth-ranked Mathlete and hisor her opponent compete. For the final four matchups, the first student to correctly answer three questions advances.
- The Countdown Round proceeds until a first place individual is identified. More details about Countdown Round procedures are included in the 2015 School Competition.

**Rules for the Countdown Round change for the National Competition.*

An unofficial Countdown Round does not determine an individual's final overall rank in the competition but is done for practice or for fun. The official procedures do not have to be followed. Chapters and states choosing not to conduct the round officially must determine individual winners solely on the basis of students' scores in the Target and Sprint Rounds of the competition.

SCORING

MATHCOUNTS Competition Series scores do not conform to traditional grading scales. Coaches and students should view an Individual Score of 23 (out of a possible 46) as highly commendable.

Individual Score: calculated by taking the sum of the number of Sprint Round questions answered correctly and twice the number of Target Round questions answered correctly. There are 30 questions in the Sprint Round and 8 questions in the Target Round, so the maximum possible Individual Score is $30 + 2(8) = 46$. If used officially, the Countdown Round yields final individual standings.

Team Score: calculated by dividing the sum of the team members' Individual Scores by 4 (even if the team has fewer than four members) and adding twice the number of Team Round questions answered correctly. The highest possible Individual Score is 46. Four students may compete on a team, and there are 10 questions in the Team Round. Therefore, the maximum possible Team Score is $((46 + 46 + 46 + 46) \div 4) + 2(10) = 66$.

Tiebreaking Algorithm: used to determine team and individual ranks and to determine which individuals qualify for the Countdown Round. In general, questions in the Target, Sprint and Team Rounds increase in difficulty so that the most difficult questions occur near the end of each round. In a comparison of questions to break ties, generally those who correctly answer the more difficult questions receive the higher rank. The guidelines provided below are very general; competition officials receive more detailed procedures.

- *Ties between individuals:* The student with the higher Sprint Round score will receive the higher rank. If a tie remains after this comparison, specific groups of questions from the Target and Sprint Rounds are compared.
- *Ties between teams:* The team with the higher Team Round score, and then the higher sum of the team members' Sprint Round scores, receives the higher rank. If a tie remains after these comparisons, specific questions from the Team Round will be compared.

RESULTS DISTRIBUTION

Coaches should expect to receive the scores of their students, as well as a list of the top 25% of students and top 40% of teams, from their competition coordinators. In addition, single copies of the blank competition materials and answer keys may be distributed to coaches after all competitions at that level nationwide have been completed. *Before distributing blank competition materials and answer keys, coordinators must wait for verification from the national office that all such competitions have been completed.* Both the problems and answers from Chapter and State Competitions will be posted on the MATHCOUNTS website following the completion of all competitions at that level nationwide, replacing the previous year's posted tests.

Student competition papers and answers will not be viewed by or distributed to coaches, parents, students or other individuals. Students' competition papers become the confidential property of MATHCOUNTS.

ADDITIONAL RULES

All answers must be legible.

Pencils and paper will be provided for Mathletes by competition organizers. However, students may bring their own pencils, pens and erasers if they wish. They may not use their own scratch paper or graph paper.

Use of notes or other reference materials (including dictionaries and translation dictionaries) is prohibited.

Specific instructions stated in a given problem take precedence over any general rule or procedure.

Communication with coaches is prohibited during rounds but is permitted during breaks. All communication between guests and Mathletes is prohibited during competition rounds. Communication between teammates is permitted only during the Team Round.

Calculators are not permitted in the Sprint and Countdown Rounds, but they are permitted in the Target, Team and Tiebreaker (if needed) Rounds. When calculators are permitted, students may use any calculator (including programmable and graphing calculators) that does not contain a QWERTY (typewriter-like) keypad. Calculators that have the ability to enter letters of the alphabet but do not have a keypad in a standard typewriter arrangement are acceptable. Smart phones, laptops, iPads®, iPods®, personal digital assistants (PDAs) and any other "smart" devices are not considered to be calculators and may not be used during competitions. Students may not use calculators to exchange information with another person or device during the competition.

Coaches are responsible for ensuring their students use acceptable calculators, and students are responsible for providing their own calculators. Coordinators are not responsible for providing Mathletes with calculators or batteries before or during MATHCOUNTS competitions. Coaches are strongly advised to bring backup calculators and spare batteries to the competition for their team members in case of a malfunctioning calculator or weak or dead batteries. Neither the MATHCOUNTS Foundation nor coordinators shall be responsible for the consequences of a calculator's malfunctioning.

Pagers, cell phones, iPods® and other MP3 players should not be brought into the competition room. Failure to comply could result in dismissal from the competition.

Should there be a rule violation or suspicion of irregularities, the MATHCOUNTS coordinator or competition official has the obligation and authority to exercise his or her judgment regarding the situation and take appropriate action, which might include disqualification of the suspected student(s) from the competition.

FORMS OF ANSWERS

The following rules explain acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lb 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, \$0.25 will not be accepted.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and GCF(a, b) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not have a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and GCF(a, b) = 1. Examples:

Problem: What is $8 \div 12$ expressed as a common fraction? *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: What is $12 \div 8$ expressed as a common fraction? *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: What is the sum of the lengths of the radius and the circumference of a circle of diameter $\frac{1}{4}$ unit, expressed as a common fraction in terms of π? *Answer:* $\frac{1+2\pi}{8}$

Problem: What is $20 \div 12$ expressed as a mixed number? *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Acceptable Simplified Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:
Problem: What is $\sqrt{15} \times \sqrt{5}$ expressed in simplest radical form? *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form (\$)*a.bc*, where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when *a* is zero, in which case it may be omitted, or when *b* and *c* are both zero, in which case they may both be omitted. Answers in the form (\$)*a.bc* should be rounded to the nearest cent, unless otherwise specified. Examples:
Acceptable Forms: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not do any intermediate rounding (other than the "rounding" a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \leq |a| < 10$, and n is an integer. Examples:
Problem: What is 6895 expressed in scientific notation? *Answer:* 6.895×10^3
Problem: What is 40,000 expressed in scientific notation? *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole-number answers should be expressed in their whole-number form. Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

VOCABULARY AND FORMULAS

The following list is representative of terminology used in the problems but should not be viewed as all-inclusive. It is recommended that coaches review this list with their Mathletes.

absolute difference
absolute value
acute angle
additive inverse *(opposite)*
adjacent angles
algorithm
alternate exterior angles
alternate interior angles
altitude *(height)*
apex
area
arithmetic mean
arithmetic sequence
base 10
binary
bisect
box-and-whisker plot
center
chord
circle
circumference
circumscribe
coefficient
collinear
combination
common denominator
common divisor
common factor
common fraction
common multiple
complementary angles
composite number
compound interest
concentric
cone
congruent
convex
coordinate plane/system
coordinates of a point
coplanar
corresponding angles
counting numbers
counting principle
cube
cylinder
decagon
decimal
degree measure
denominator
diagonal of a polygon
diagonal of a polyhedron
diameter
difference
digit
digit-sum
direct variation
dividend
divisible
divisor
dodecagon
dodecahedron
domain of a function
edge
endpoint
equation
equiangular
equidistant
equilateral
evaluate
expected value
exponent
expression
exterior angle of a polygon
factor
factorial
finite
formula
frequency distribution
frustum
function
GCF
geometric mean
geometric sequence
height *(altitude)*
hemisphere
heptagon
hexagon
hypotenuse
imageof a point
(under a transformation)
improper fraction
inequality
infinite series
inscribe
integer
interior angle of a polygon
interquartile range
intersection
inverse variation
irrational number
isosceles
kite
lateral edge
lateral surface area
lattice point(s)
LCM
linear equation
mean
median of a set of data
median of a triangle
midpoint
mixed number
mode(s) of a set of data
multiple
multiplicative inverse *(reciprocal)*
natural number
nonagon
numerator
obtuse angle
octagon
octahedron
odds (*probability)*
opposite of a number *(additive inverse)*
ordered pair
origin
palindrome
parallel
parallelogram
Pascal's Triangle
pentagon
percent increase/decrease
perimeter
permutation
perpendicular
planar
polygon
polyhedron
prime factorization

prime number
principal square root
prism
probability
product
proper divisor
proper factor
proper fraction
proportion
pyramid
Pythagorean Triple
quadrant
quadrilateral
quotient
radius
random
range of a data set
range of a function
rate
ratio
rational number
ray
real number
reciprocal *(multiplicative inverse)*
rectangle
reflection
regular polygon
relatively prime
remainder
repeating decimal
revolution
rhombus
right angle
right circular cone
right circular cylinder
right polyhedron
right triangle
rotation
scalene triangle
scientific notation
sector
segment of a circle
segment of a line
semicircle
semiperimeter
sequence
set
significant digits
similar figures
simple interest
slope
slope-intercept form
solution set
sphere
square
square root
stem-and-leaf plot
sum
supplementary angles
system of equations/inequalities
tangent figures
tangent line
term
terminating decimal
tetrahedron
total surface area
transformation
translation
trapezoid
triangle
triangular numbers
trisect
twin primes
union
unit fraction
variable
vertex
vertical angles
volume
whole number
x-axis
x-coordinate
x-intercept
y-axis
y-coordinate
y-intercept

The list of formulas below is representative of those needed to solve MATHCOUNTS problems but should not be viewed as the only formulas that may be used. Many other formulas that are useful in problem solving should be discovered and derived by Mathletes.

CIRCUMFERENCE

Circle	$C = 2 \times \pi \times r = \pi \times d$

AREA

Circle	$A = \pi \times r^2$
Square	$A = s^2$
Rectangle	$A = l \times w = b \times h$
Parallelogram	$A = b \times h$
Trapezoid	$A = \frac{1}{2}(b_1 + b_2) \times h$
Rhombus	$A = \frac{1}{2} \times d_1 \times d_2$
Triangle	$A = \frac{1}{2} \times b \times h$
Triangle	$A = \sqrt{s(s-a)(s-b)(s-c)}$
Equilateral triangle	$A = \frac{s^2\sqrt{3}}{4}$

SURFACE AREA AND VOLUME

Sphere	$SA = 4 \times \pi \times r^2$
Sphere	$V = \frac{4}{3} \times \pi \times r^3$
Rectangular prism	$V = l \times w \times h$
Circular cylinder	$V = \pi \times r^2 \times h$
Circular cone	$V = \frac{1}{3} \times \pi \times r^2 \times h$
Pyramid	$V = \frac{1}{3} \times B \times h$

Pythagorean Theorem	$c^2 = a^2 + b^2$
Counting/ Combinations	${}_nC_r = \frac{n!}{r!(n-r)!}$

SOLUTIONS

The solutions provided here are only *possible* solutions. It is very likely that you or your students will come up with additional—and perhaps more elegant—solutions. Happy solving!

Warm-Up 1

1. The two-digit multiples of 11 are 11, 22, 33, 44, 55, 66, 77, 88 and 99. The sum of these integers is the sum of the units digits plus 10 times the sum of the tens digits: $1 + 2 + 3 + \cdots + 9 + 10(1 + 2 + 3 + \cdots + 9) = 11(1 + 2 + 3 + \cdots + 9) = 11 \times 45 =$ **495**.

2. According to the rule, 3 # 4 = 3 + 2 × 4 = 3 + 8 = **11**.

3. One hundred seconds is 100/60 = 5/3 of a minute, so Kimba would chew her gum 42 × 5/3 = 14 × 5 = **70** times.

4. To find Carver's average distance, we add the four distances and divide the result by 4. To add these mixed numbers, we need a common denominator. Since 1/3 = 2/6 and 1/2 = 3/6, we have $4\frac{2}{6} + 3\frac{3}{6} + 3\frac{5}{6} + 4\frac{1}{6} = 4 + 3 + 3 + 4 + \frac{2+3+5+1}{6} = 14\frac{11}{6} = \frac{95}{6}$. Dividing by 4 yields $\frac{95}{6} \div 4 = \frac{95}{6} \times \frac{1}{4} = \frac{95}{24} = \mathbf{3\frac{23}{24}}$ miles.

5. In the worst-case scenario, Rob might select all 8 red and all 6 blue socks before he gets 2 white socks. So, in order to guarantee that he retrieves a pair of white socks, he must remove 8 + 6 + 2 = **16** socks from the drawer.

6. To maximize the number of hamburgers, buy as many groups of 5 hamburgers for $9 as possible, since that's the best deal. Since 48 ÷ 9 = 5 r3, buy 5 groups of 5, or 25 hamburgers, totalling $45, plus 1 more with the remaining $3. A maximum of **26** hamburgers can be purchased for $48.

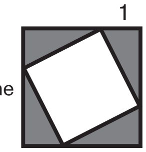
7. If we assume that Statement A is true, we have a contradiction, since Statement A asserts that Statement C is true and Statement C asserts that Statement A is false. Therefore, Statement A must be false. If we assume that Statement B is true, then Statement C is true and Statement D is false. There is no contradiction. If Statement C is true, then Statement A is false and Statement D is false. Again, there is no contradiction. If Statement D is true, then Statement A is also true, but we already know that Statement A leads to a contradiction. Therefore, the true statements are Statement **B** and Statement **C**.

Summary of Statements
A: B false, C true
B: C true, D false
C: D false, A false
D: A true, B true

Summary of Implications
A true ⇒ C true ⇒ A false
B true ⇒ C true ⇒ A false ⇒ D false
C true ⇒ A false ⇒ D false
D true ⇒ A true

8. There are 60 × 60 = 3600 seconds in one hour. We can calculate 3.14 × 3600 more simply as 314 × 36, which is **11,304** seconds.

9. Since the question is about a ratio, we have the freedom to assign a side length of 3 units to the larger square, resulting in an area of 9 units^2. As the figure shows, each of the 4 right triangles, shaded here, would have legs of length 1 and 2 units and an area of (1/2) × 1 × 2 = 1 unit^2. The 4 triangles, then, would have a combined area of 1 × 4 = 4 units^2. It follows that the area of the smaller square would be 9 − 4 = 5 units^2, making the ratio of the areas of the smaller and larger squares **5/9**.



10. Since 1/3 of the students wanted more fresh fruits and vegetables, 2/3 of the students did not want more fresh fruits and vegetables. Since 1/8 of this 2/3 of the students wanted more seafood, that's 1/8 × 2/3 = 1/12 of the students surveyed. In order for 1/12 of the students surveyed to be a whole number of students, the minimum number of students surveyed must be **12** students.

Warm-Up 2

11. The volume of a cylinder with base radius r and height h is $\pi r^2 h$. So, initially, the volume of the first cylinder, $\pi(r_1)^2h_1$, is equivalent to the volume of the second cylinder, $\pi(r_2)^2h_2$. When the height of the second cylinder is increased, its new volume must be equivalent to the new volume of the first cylinder after its radius is doubled, which is $\pi(2r_1)^2h_1 = 4\pi(r_1)^2h_1$. So $k[\pi(r_2)^2h_2] = 4[\pi(r_1)^2h_1]$, and $k =$ **4**.

12. When another positive integer, let's call it n, is included in the given set, the median will be 2, 4 or n. We're looking for the value of n that also gives us a mean, which is $(15 + n)/5$, of 2, 4 or n. If the mean is 2, we have $(15 + n)/5 = 2 \rightarrow 15 + n = 10 \rightarrow n = -5$, which is not the solution since we're looking for a positive integer n. If the mean is 4, we have $(15 + n)/5 = 4 \rightarrow 15 + n = 20 \rightarrow n = 5$. This works, but let's also check the final possibility. If the mean is n, we have $(15 + n)/5 = n \rightarrow 15 + n = 5n \rightarrow 15 = 4n \rightarrow n = 15/4$, which is not the solution since we are told n is an integer. It is when we include **5** in the set that we get a mean of 5 and a median of 5.

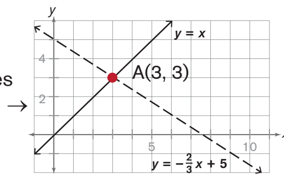
13. Since the numbers of dimes and quarters are known to be equal, we can imagine pairing all the dimes and quarters into groups of 35 cents. In $1.40, there would be 140 ÷ 35 = 4 of these groups. That's 4 dimes and 4 quarters, for a total of **8** coins.

14. The mean of an odd number of consecutive numbers is the middle number. The middle number of the list of the first 99 counting numbers is **50**.

15. Driving 35 miles at 30 mi/h would take 35/30 = 7/6 = 1 1/6 hours, or 1 hour 10 minutes. Driving 35 miles at 25 mi/h would take 35/25 = 7/5 = 1 2/5 hours, or 1 hour 24 minutes. To drive 35 miles at the faster speed takes 24 − 10 = **14** fewer minutes.

16. Since the price of the computer was reduced by 20%, the $800 must be 80% of the price before the reduction. We can set up the proportion $800/x = 80/100$. Solving for x, we see that $80x = 80{,}000 \rightarrow x =$ **$1000** was the price of the computer before the reduction.

17. The package of Koka-Kola contains a total of $8 \times 12 = 96$ ounces. The package of Pepsy-Kola contains $6 \times 16 = 96$ ounces. The absolute difference in the number of ounces in these two packages is **0** ounces.



18. The graph shows that the intersection of $y = -(2/3)x + 5$ and $y = x$ is A**(3, 3)**. We also can determine the coordinates of A algebraically by setting the equations equal to each other and solving for x as follows: $-(2/3)x + 5 = x \rightarrow (5/3)x = 5 \rightarrow x = 3$. Substituting, we see that $y = 3$ and the coordinates of A are **(3, 3)**.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

19. There are $6 \times 6 = 36$ possible outcomes when rolling two fair dice. Of those, 20 are multiples of 3 or 4, as shown. Thus, the probability that the sum of the numbers rolled is a multiple of 3 or 4 is $20/36 =$ **5/9**.

20. Consider the right triangle formed by the ground, the pole and one wire. The lengths of one leg and the hypotenuse are 10 feet and 11 feet, respectively. Using the Pythagorean Theorem, we see that the other leg, which is the pole, has a height of $\sqrt{(11^2 - 10^2)} = \sqrt{(121 - 100)} =$ $\mathbf{\sqrt{21}}$ feet.

Workout 1

21. The prime factorization of 2015 is $5 \times 13 \times 31$. The sum of the prime factors is $5 + 13 + 31 =$ **49**.

22. Using the properties of exponents, $(3.5 \times 10^4)^2$ can be expressed as $(3.5)^2 \times (10^4)^2 = 12.25 \times 10^8$. This can be rewritten in scientific notation as $\mathbf{1.225 \times 10^9}$.

23. Since the measure of $\angle AOB$ is 22.5 degrees, we know that the area of the shaded sector is $22.5/360 = 1/16$ of the total area of the circle. The circle must have a total area of $16 \times 9\pi = 144\pi$ m^2. So $\pi r^2 = 144\pi$, and it follows that $r^2 = 144$ and circle O has radius $\sqrt{144} =$ **12** meters.

24. After a rise of 6%, the new price would be 106% = 1.06 times the initial price. After a drop of 10%, the new price would be 90% = 0.9 times the price after the increase. Combining the two steps, we can multiply the initial price by $1.06 \times 0.9 = 0.954$ to get $3.50 \times 0.954 \approx$ **$3.34** as the cost of a gallon of gas at the end of May.

25. Since 1% of 100 is 1, there is a possible error in measurement of ±1 foot. That means the smallest the lot could be is $99 \times 99 = 9801$ ft^2, and the largest it could be is $101 \times 101 = 10{,}201$ ft^2. The absolute difference in these two areas is $10{,}201 - 9801 =$ **400** ft^2.

26. Let Y, R and V be Yumi's, Rana's and Victoria's current ages, respectively. Then we can write Y + R + V = 42. Four years ago, Rana and Victoria were each 4 years younger, so we also can write (R – 4) + (V – 4) = Y. We can substitute this expression for Y in the first equation and simplify to get (R – 4) + (V – 4) + R + V = 42 → 2R + 2V – 8 = 42 → 2R + 2V = 50 → R + V = 25. Now that we know the sum of Rana's and Victoria's ages, we can conclude that Yumi's age must be 42 – 25 = **17** years old.

27. Eighty-five percent of the 600 students at Goodnight Middle School, or $0.85 \times 600 = 510$ students are not native Texans. Sixty percent of these students, or $0.6 \times 510 = 306$ students have lived in Texas for more than 10 years. That means $510 - 306 = 204$ students have been in Texas less than 10 years. Deducting the 30 students who have been in Texas for less than a year, we see that $204 - 30 =$ **174** students have been in Texas for at least 1 year but not more than 10 years.

28. Since each of the semicircular ends of the track is 1/4 of the total distance, together they make up 1/2 of the 1/4 mile, which is 1/8 mile. So the distance between the straight segments of the track is the diameter, d, of a circle with a circumference of 1/8 mile. Solving $\pi d = 1/8$, we see that $d = 1/(8\pi)$, so the distance between the two parallel sides of the track is about **0.04** mile.

29. At the point of intersection of the two lines, the y-coordinates are the same, so we can set the two equations equal to each other and solve for x to get $x - 3 = -2x + 9 \rightarrow 3x = 12 \rightarrow x = 4$. Now, we can determine that the y-coordinate is $y = 4 - 3 = 1$. The point of intersection of the two lines is (4, 1), and the sum of the coordinates is $4 + 1 =$ **5**.

30. The probability that the pointer will land within the blue section is 1/3, and the probability that an even number is rolled is $3/6 = 1/2$. Therefore, the probability that the two events will happen together is the product of these probabilities, $1/3 \times 1/2 =$ **1/6**.

Warm-Up 3

31. The factorial of a natural number n, written $n!$, is the product of the natural numbers from 1 to n. For example, $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$. The value of $\frac{6!}{5! + 4!}$, then, is $\frac{6 \times 5 \times (4 \times 3 \times 2 \times 1)}{5 \times (4 \times 3 \times 2 \times 1) + (4 \times 3 \times 2 \times 1)} = \frac{6 \times 5}{5 + 1} = \frac{30}{6} = \mathbf{5}$.

32. A number is divisible by 9 if the sum of its digits is a multiple of 9. The sum of the digits of 1A,2A2 is 5 + 2A. If 5 + 2A = 9, then 2A = 4 and A = **2**. (When we let 5 + 2A = 18, A is not an integer and 5 + 2A = 27 yields a two-digit value of A, so there are no other solutions.)

33. Let a and b represent the weights of Tweedledum and Tweedledee, respectively. Tweedledum's and Tweedledee's statements yield the following equations, respectively: $2a + b = 361$ and $a + 2b = 362$. Combining these two equations, we get $3a + 3b = 723$, and dividing by 3, we see that $a + b = 241$. Since $2a + b = a + a + b$, it follows that $a + 241 = 361$, so $a = 120$ pounds and $b = 241 - 120 = 121$ pounds. Therefore, the absolute difference in the weights of Tweedledee and Tweedledum is $121 - 120 =$ **1** pound.

34. Beginning with a 63-inch long string, first cut off a 1-inch piece leaving, a 62-inch piece of string. Then cut off a 2-inch piece, which leaves a 60-inch piece of string. Next cut off a 4-inch piece, leaving a 56-inch piece of string, followed by an 8-inch piece, leaving a 48-inch piece of string. Then cut off a 16-inch piece, leaving a 32-inch piece of string, which happens to be twice as long as the last piece cut. Therefore, **6** pieces of string can be cut from the string in this manner. It is also worth noting that the lengths of the pieces of string make the sequence 1, 2, 4, 8, 16, 32. This is equivalent to the sequence $2^0, 2^1, 2^2, 2^3, 2^4, 2^5$, consisting of powers of 2. The sum of consecutive powers of 2 from 1 to some power of 2 is always 1 less than the next power of 2. In other words, $1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1$. Thus, a sum of 63, which is $64 - 1$, would be the result of summing the 6 powers of 2 from 1 to 32, representing the lengths of the **6** pieces of string.

35. The table shows all 36 possible sums. Exactly 7 of these are multiples of 5, so the probability that the sum of the numbers rolled is divisible by 5 is **7/36**.

+	1	3	5	7	9	11
2	3	5	7	9	11	13
4	5	7	9	11	13	15
6	7	9	11	13	15	17
8	9	11	13	15	17	19
10	11	13	15	17	19	21
12	13	15	17	19	21	23

36. An order of 10 shirts costs \$84, while an order of 20 shirts costs \$159. The 10 additional shirts must account for the 159 – 84 = \$75 difference in cost. Since an order of 30 shirts contains 10 more shirts than the order of 20 shirts contains, and since we know the cost of 10 shirts is \$75, it follows that the cost for 30 shirts would be 159 + 75 = **\$234**.

37. Since π is approximately 3.14, it follows that the value of π^2 is between $3.1^2 = 9.61$ and $3.2^2 = 10.24$. Therefore, the nearest integer to π^2 is **10**.

38. It would take Cara 200/50 = 4 hours to drive along the 200-mile highway at 50 mi/h. The alternate route would take 150/60 + 50/40 = 5/2 + 5/4 = 15/4 = 3 3/4 hours. Taking the alternate route would save Cara 4 – 3 3/4 = 1/4 hour, which is **15** minutes.

39. A 4-foot by 6-foot floor measures 48 inches by 72 inches. Using 4-inch tiles it would take $48 \div 4 = 12$ rows with $72 \div 4 = 18$ tiles each. That's a total of $12 \times 18 = 216$ tiles. The cost for 216 4-inch tiles would be $216 \times 0.20 =$ \$43.20. Using 6-inch tiles it would take $48 \div 6 = 8$ rows with $72 \div 6 = 12$ tiles each. That's a total of $8 \times 12 = 96$ tiles. The cost for 96 6-inch tiles would be $96 \times 0.40 =$ \$38.40. The savings would be 43.20 – 38.40 = **\$4.80**.

40. The shaded region is 1/4 of one unit square, so it has an area of **1/4** unit^2.

Warm-Up 4

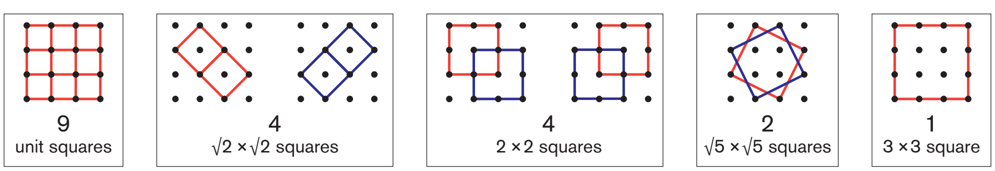
41. If $a - b = 0$, then $a = b$. Since the two numbers are equal, the product $a \times b$ is equal to the square of a or the square of b. We are asked to express our answer in terms of a, so the answer is $\boldsymbol{a^2}$.

42. The table shows the 4 possible number pairs that have a sum of 9. Each of these pairs also can occur with the numbers reversed, so there are a total of 8 ways to get a sum of 9. Since there are 8 possible outcomes for each die, there are $8 \times 8 = 64$ possible sums. Thus, the probability that the two numbers rolled have a sum of 9 is 8/64 = **1/8**.

Die 1	Die 2
1	8
2	7
3	6
4	5

43. Since $2015 = 101a + 19b$, it follows that $2015 - 101a = 19b$. After subtracting this multiple of 101 from 2015, the result must be a multiple of 19. Note that $2015 \div 101 = 19$ r96, but 96 is not a multiple of 19, so $a \neq 19$. If $a = 18$, then $2015 - (101 \times 18) = 2015 - 1818 = 197$. This is 7 more than a multiple of 19, so $a \neq 18$. If $a = 17$, then $2015 - (101 \times 17) = 2015 - 1717 = 298$. This is 13 more than a multiple of 19, so $a \neq 17$. If $a = 16$, then $2015 - (101 \times 16) = 2015 - 1616 = 399$, which is 19×21. So $101 \times 16 + 19 \times 21 = 2015$ and $a + b = 16 + 21 =$ **37**.

44. The 9 + 4 + 4 + 2 + 1 = **20** squares of various sizes are shown here.



45. The mixture contains 6 ounces of milk and 2 ounces of chocolate syrup, which is 8 ounces in all. The mixture is 3/4 milk and 1/4 chocolate syrup. After 2 ounces of the mixture are poured out, the remaining 6 ounces contain $6 \times 1/4 = 6/4 =$ $\mathbf{1\frac{1}{2}}$ ounces of chocolate syrup.

46. First, solve the equation $\sqrt{(x + 4)} = 3$ by squaring both sides of the equation. We get $x + 4 = 9$, so $x =$ **5**.

47. Since $48 = 2^4 \times 3$ and $72 = 2^3 \times 3^2$, the greatest common factor of 48 and 72 is $2^3 \times 3 = 24$ and their least common multiple is $2^4 \times 3^2 = 144$. The product of these two values is $24 \times 144 =$ **3456**. In general, the product of the GCF and the LCM of two numbers always is equal to the product of the two numbers. In this case, $48 \times 72 =$ **3456**.

48. A trip is defined by 2 of the 13 stations – its starting station and its ending station. We are looking for *combinations* of 2 stations chosen from a group of 13 stations. The formula used to determine the number of combinations of n objects chosen from a group of m objects is ${}_mC_n = \frac{n!}{n!(m-n)!}$. We can use this formula to find ${}_{13}C_2$, read "13 choose 2." So ${}_{13}C_2 = \frac{13!}{2!(13-2)!} = \frac{13\times12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{(2\times1)(11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1)}$. Simplifying, we see that among the 13 stations there are (13 × 12)/(2 × 1) = 13 × 6 = **78** trips.

49. Notice that side AC is a horizontal line and side BC is a vertical line. These two sides are perpendicular and have an included angle that is 90 degrees, which means △ABC is a right triangle. Side AC has length 17 − 2 = 15 units, and side BC has length 11 − 3 = 8 units. It follows, then, that the area of △ABC is 1/2 × 15 × 8 = **60** units^2.

50. When any quantity q is doubled and then divided by half the original quantity, the result is **4**. This can be proven algebraically since $2q \div (1/2)q = (2 \times 2q)/q = 4q/q =$ **4**.

Workout 2

51. The information provided yields the equations A + B = 23, A + C = 21 and B + C = 20. Adding all three equations, we get 2A + 2B + 2C = 23 + 21 + 20 → 2A + 2B + 2C = 64. Since we want A + B + C, we divide by 2 and see that altogether, the three girls scored **32** points.

52. The portion of Mauna Kea's altitude that is below sea level is 33,100 − 13,803 = 19,297 feet, which represents 19,297 ÷ 33,100 ≈ 0.58 = **58**% of Mauna Kea's altitude.

53. To serve 12 people instead of 4 people, we need a pizza that has 3 times the area. The area of the pizza is measured in square inches, and its diameter, which is linear, is measured in inches. It follows, then, that the diameter of a pizza that serves 12 people must be $\sqrt{3}$ times the diameter of one that serves 4. Therefore, the pizza with an area large enough to serve 12 people is $16 \times \sqrt{3} \approx$ **27.7** inches.

54. Square ABCD has an area of 12 × 12 = 144 cm^2, so each of the three regions has an area of 144 ÷ 3 = 48 cm^2. Let's focus on isosceles triangle DCF. With an area of 48 cm^2, base DC of length 12 cm and height h, we have the equation $48 = (1/2) \times 12 \times h$. Solving this equation, we see that $6h = 48 \rightarrow h = 8$ cm is the height of △DCF. This altitude of length 8 cm, drawn from F perpendicular to base DC, divides △DCF into two right triangles, each with legs 6 cm and 8 cm in length. Notice that the side lengths of each right triangle are a multiple of the 3-4-5 Pythagorean Triple, namely 6-8-10. So the length of segment CF is **10** cm.

55. We are told that a is an odd integer between 0 and 60, inclusive. So the possible values for a are 1, 3, 5, 7, ..., 59. The possible even integer values for $b = (a - 1)/2$ are 0, 2, 4, 6, ..., 28. The possible even integer values for $c = b/2$ are 0, 2, 4, ..., 14. The possible odd integer values for $d = c/2$ are 1, 3, 5 and 7. The possible odd integer values for $e = (d - 1)/2$ are 1 and 3. If $e = 1$, then $f = 0/2 = 0$. If $e = 3$, then $f = 2/2 = 1$. We conclude that $e = 1$ since we are told that f is an even integer. Therefore, $1 = (d - 1)/2 \rightarrow d - 1 = 2 \rightarrow d = 3$; and $3 = c/2 \rightarrow c = 6$; and $6 = b/2 \rightarrow b = 12$; and $12 = (a - 1)/2 \rightarrow 24 = a - 1 \rightarrow a =$ **25**.

56. With sales of $7200 for selling 240 tickets, it follows that the price per ticket is 7200 ÷ 240 = $30. To take in $7200 for selling 15 fewer tickets, the theater would need to charge 7200 ÷ 225 = $32 per ticket. That's an increase of 32 − 30 = $**2**.

57. Since the web interface deletes a batch of 50 messages in 0.5 second, it deletes 2 batches, or 100 messages, in 1 second. It would take 34,000 ÷ 100 = 340 seconds, almost **6** minutes, to delete all 34,000 messages.

58. The two squares closest in value to 2015 are 2025 and 1936. The value of $n + m$ is the same as the difference of these two squares, which is 2025 − 1936 = **89**.

Year	Salary
2000	$40,000.00
2001	$44,000.00
2002	$48,400.00
2003	$53,240.00
2004	$58,564.00
2005	$64,420.40
2006	$70,862.44
2007	$77,948.68
2008	$85,743.55

59. A yearly increase of 10% means each year Emilio's new salary was 110% = 1.1 times his salary the previous year. The table shows that Emilio's salary first exceeded $80,000 in **2008**.

60. If the distance of Anne's third attempt equals that of her first attempt, the average of the three triple jump attempts would be (36' 4" + 38' 4" + 36' 4") ÷ 3 = 111 ÷ 3 = 37'. If her third attempt is equal in distance to her second attempt, the average of the three triple jump attempts would be (36' 4" + 38' 4" + 38' 4") ÷ 3 = 113 ÷ 3 = 37' 8". The difference in these two averages is **8** inches.

Warm-Up 5

61. Altogether, the four women contributed a total of 12 + 15 + 20 + 8 = $55. Siriana's $15 contribution represents 15/55 = 3/11 of this total. So Siriana should receive 3/11 of the $1100 winnings, which is (3/11) × 1100 = $**300**. Alternatively, you may recognize that 1100 = 20 × 55, and 20 × 15 = $**300**, which is Siriana's share of the winnings.

62. We are looking for ways to select 5 of the 8 players, giving no consideration to the positions they will play. In other words, we need to determine the value of 8 choose 5. This is calculated as follows: ${}_8C_5 = \frac{8!}{5!(8-5)!} = \frac{8\times7\times6\times5\times4\times3\times2\times1}{(5\times4\times3\times2\times1)(3\times2\times1)} = 8 \times 7 = 56$. Therefore, 5 starting players can be selected from the 8 players in **56** ways.

63. The prime factorization of 144 is $2^4 \times 3^2$. So any factor of 144 must be the product of a power of 2 and a power 3, with the exponent of 2 being 0, 1, 2, 3 or 4 and the exponent of 3 being 0, 1 or 2. That's five possible exponents for the 2 and three possible exponents for the 3, for a total of 5 × 3 = 15 factors. Since these 15 factors are the products of the 15 combinations of a power of 2 chosen from 2^0, 2^1, 2^2, 2^3 and 2^4 and a power of 3 chosen from 3^0, 3^1 and 3^2, it follows that the sum of these factors is $(2^0 + 2^1 + 2^2 + 2^3 + 2^4) \times (3^0 + 3^1 + 3^2) = (1 + 2 + 4 + 8 + 16) \times (1 + 3 + 9) = 31 \times 13 = 403$. And 5 times this sum is 5 × 403 = **2015**.

64. If we let n represent the numerator, then the denominator is $n + 2$. Then we have $n/(n + 2) = (n + 2)/n$. Cross-multiplying, we get $n^2 = (n + 2)^2$. Now, solving for n yields $n^2 = n^2 + 4n + 4 \rightarrow 4n + 4 = 0 \rightarrow 4n = -4 \rightarrow n = -1$. The fraction must be −1/1, with a reciprocal of 1/−1. The sum of the numerator and denominator, then, is −1 + 1 = **0**.

65. The quarters to dimes ratio of 3:4 is equivalent to 12:16, and the quarters to nickels ratio of 4:5 is equivalent to 12:15. So, for every 12 quarters, Julius has 16 dimes and 15 nickels. Therefore, the dimes to nickels ratio must be 16:15 or **16/15**. Alternatively, this problem can be solved algebraically. Let q, d and n represent the numbers of quarters, dimes and nickels, respectively. Since $q/d = 3/4 = 12/16$ and $q/n = 4/5 = 12/15$, $d/q \times q/n = 16/12 \times 12/15 =$ **16/15**.

66. Rate and time are inversely proportional, so when Mr. Tesla drives 60 mi/h instead of 40 mi/h, he is traveling 3/2 as fast, and it will take him 2/3 as much time to get there. The reduction of 30 minutes, or 1/2 hour, must account for the remaining 1/3 of the time. Therefore, it would take him 3 × 30 = 90 minutes = 1.5 hours, to get to work when he drives 40 mi/h. Traveling 40 mi/h for 1.5 hours, Mr. Tesla will travel 1.5 × 40 = **60** miles. To check our reasoning, we note that it would take him 1 hour to drive 60 miles at 60 mi/h, which, indeed, is 30 minutes less than 1.5 hours.

67. Jack Frost seems to net 18 – 8 = 10 snowballs every hour. To get 2 dozen, or 24 snowballs, it will take him **2.4** hours.

68. From the first sentence, we can write the equation $m = d + 26$, where m is the mother's current age and d is the daughter's current age. From the second sentence, we can write $m = 3d - 6$. Setting the two equations equal to each other, we get $d + 26 = 3d - 6$. Solving for d yields $2d = 32 \rightarrow d = 16$. Therefore, the daughter is **16** years old now.

69. The information provided can be represented algebraically with the equations $r + b = 36$ and $2r = b - 6$. The second equation can be rewritten as $2r - b = -6$. Adding the first equation to the rewritten second equation and solving yields $3r = 30 \rightarrow r = 10$. So, **10** red marbles are in the bag.

70. We need to evaluate the expression, beginning with the innermost set of parentheses, working outward. Since 2 < 5, to find 2 @ 6, we apply the second rule, which yields (6 + 2)(6 – 2) = 8 × 4 = 32. Now, since 32 > 5, we apply the first rule to 32 @ −4, which yields (32 + −4)(32 – (−4)) = (32 – 4)(32 + 4) = 28 × 36 = 1008. Finally, we have 2 × 1008 – 1 = 2016 – 1 = **2015**.

Warm-Up 6

71. The common denominator of the three fractions is $6n$, so we have $6/(6n) + 3/(6n) + 2/(6n) = k \rightarrow 11/6n = k \rightarrow$ **11/6** $= nk$.

72. Since Ellie can paint the room in 2 hours, she can paint 1/2 of the room each hour. Likewise, since Diedre can paint the room in 4 hours, she can paint 1/4 of the room per hour. Working together, they can paint 1/2 + 1/4 = 3/4 of the room each hour. This is equivalent to painting 1/4 of the room every 20 minutes. Thus, Ellie and Diedre working together to paint the entire room will take 4 × 20 = **80** minutes.

73. Recall that the formula for the area of an equilateral triangle with side-length s is $(\sqrt{3}/4)s^2$. The fraction of the triangle that is shaded is proportional to the fraction of the base that corresponds to the shaded regions. Since the five congruent segments correspond to the shaded regions, it follows that the shaded regions have a total area of $3/5 \times \sqrt{3}/4 \times 10^2 = 300\sqrt{3}/20 =$ **15√3** cm².

74. In the 48 hours from 6 a.m. on May 8th to 6 a.m. on May 10th, the center of home plate rotates 360 × 2 = 720 degrees about the Earth's axis. In the 6 hours from 6 a.m. to noon on May 10th, it completes 1/4 of a full rotation, or 90 degrees, for a total of 720 + 90 = **810** degrees.

75. Since the terminal sides of the interior and exterior angles of a regular polygon form a straight line, the angles are supplementary, meaning they have a sum of 180 degrees. If we call the interior angle x, then the exterior angle is $180 - x$. We are told that they differ by 100°, so we can write $x - (180 - x) = 100$. Solving for x, we get $x - 180 + x = 100 \rightarrow 2x - 180 = 100 \rightarrow 2x = 280 \rightarrow x = 140°$. A property of all polygons is that the sum of the exterior angles always is 360 degrees. Since the polygon in question is regular, its exterior angles are congruent So this polygon has 360 ÷ 40 = 9 exterior angles, each measuring 40 degrees. Therefore, this polygon has **9** sides.

76. Consider the four-digit number ABCD, where each letter represents a digit. We have A + B = C, B + C = D and C + D = 10A + B. If we rewrite the last equation as D = 10A + B – C, then we can substitute this for D in the second equation. We get B + C = 10A + B – C → 2C = 10A → C = 5A. Since 1 is the only nonzero one-digit number that yields another one-digit number when multiplied by 5, it follows that A = 1 and C = 5. So 1 + B = 5 and B = 4. Also, we have 4 + 5 = D, so D = 9. Thus, the four-digit number is **1459**.

77. For a sphere with radius r, the surface area equals $4\pi r^2$, and the volume equals $(4/3)\pi r^3$. Since we are told that the surface area and volume are numerically equal, we have $4\pi r^2 = (4/3)\pi r^3 \rightarrow 4\pi r^2 = (1/3)r \times (4\pi r^2) \rightarrow 1 = (1/3)r \rightarrow r =$ **3** meters.

78. If Ben selects 3 slips for each of the 4 different prizes, he will have selected 12 slips and still not know his prize. But when he selects the 13th slip, it will name the same prize as 3 of his previously selected slips. So Ben can draw, at most, **12** slips without knowing his prize.

79. The longer loop of ribbon runs down the middle of two 12-inch faces and two 4-inch faces. The shorter loop of ribbon runs down the middle of two 3-inch faces and two 4-inch faces. That's a total of $2 \times 12 + 2 \times 3 + 4 \times 4 = 24 + 6 + 16 =$ **46** inches of ribbon.

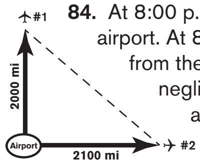
80. There are 6 possible displays in which all the digits are the same. They are 1:11, 2:22, 3:33, 4:44, 5:55 and 11:11. These occur twice a day (a.m. and p.m.), for a total of **12** times.

Workout 3

81. If we expand $(x + y)^2$, we get $x^2 + 2xy + y^2$. For some values of x and y, if it is true that $x^2 + 2xy + y^2 = x^2 + y^2$, then $2xy = 0$, and $xy =$ **0**.

82. Kiera received 44% of the 325 7th-grade votes, which is $0.44 \times 325 = 143$ votes. Aubrey must have received the other $325 - 143 = 182$ 7th-grade votes. Aubrey received 42% of the 350 8th-grade votes, which is $0.42 \times 350 = 147$ votes. Kiera must have received the other $350 - 147 = 203$ 8th-grade votes. Kiera received a total of $143 + 203 = 346$ votes, and Aubrey received a total of $182 + 147 = 329$ votes. Kiera was the winner with 346 out of the $325 + 350 = 675$ votes. That is $346/675 \approx 0.51 =$ **51**%.

83. When enlarged, the photo dimensions will be 125% of its original dimensions. The height will increase from 8 inches to $8 \times 1.25 = 10$ inches, and the width will increase from 6 inches to $6 \times 1.25 = 7.5$ inches. Enoch will have to crop the width of the picture **0.5** inch.



84. At 8:00 p.m., the first plane would have been flying east for 7 hours at 300 mi/h and would be $7 \times 300 = 2100$ miles from the airport. At 8:00 p.m., the second plane would have been flying north for 5 hours at 400 mi/h and would be $5 \times 400 = 2000$ miles from the airport. Though these distances are great enough that the Earth's curvature may be a consideration, if we assume it is negligible, the distance between the two planes is the length of the hypotenuse of a right triangle with legs of length 2000 miles and 2100 miles, as shown. We can use the Pythagorean Theorem to determine that the planes are $\sqrt{(2100^2 + 2000^2)} = \sqrt{(4{,}410{,}000 + 4{,}000{,}000)} = \sqrt{8{,}410{,}000} =$ **2900** miles.

85. Since there is no mode among the five known integers, it follows that x has to be equal in value to one of the known integers, thus, making x the mode. The known integers have a sum of 39, so the mean of the six integers is $(39 + x)/6$. Since the next integer greater than 39 that is divisible by 6 is 42, we should consider $x = 3$. This results in a mode of 3 and a mean of 7, which satisfies the assertion that the mean is 4 more than the mode. So, $x =$ **3**.

86. Let s be the initial number of students in the Happy Hearts Childcare Center. From the information provided, we have $s + 6 - 2 = 3s \rightarrow s + 4 = 3s \rightarrow 4 = 2s \rightarrow s = 2$. There were **2** students in the center before students entered and exited.

87. Since the prime numbers are consecutive, their product should be close to the cube of the middle prime. The cube root of 2431 is about 13.4. Considering the consecutive primes 11, 13 and 17, we see that $11 \times 13 \times 17 = 2431$. The sum of the three primes is $11 + 13 + 17 =$ **41**.

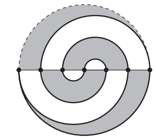
88. Since the boards are 2 inches thick and overlap, the interior width of the square garden bed is 2 inches less than the length of the boards, which is 8 feet = 96 inches. It takes $94 \times 94 \times 6 = 53{,}016$ in^3 of soil to completely fill the bed. Since 1 ft^3 is $12 \times 12 \times 12 = 1728$ in^3, the amount of soil needed to completely fill the bed is $53{,}016/1728 \approx$ **31** ft^3.

89. Let p represent the maximum price Bennie can afford to pay for the computer. Then we have $1.0625p + 11 \le 500 \rightarrow 1.0625p \le 489 \rightarrow p \le 460.24$. So Bennie can afford to order a computer that costs at most **460** dollars.

90. Let's suppose the league consists of n teams. If each team played each other team exactly once, the total number of games played would be ${}_nC_2 = n(n - 1)/2$ games. Since 21 games were played, we have $n(n - 1)/2 = 21$, so $n(n - 1) = 42$. When $n = 7$, we have $7 \times 6 = 42$. So there are **7** teams in the league. You may recognize that the number of games played is the triangular number 21, which equals $6 + 5 + 4 + 3 + 2 + 1$.

Warm-Up 7

91. In the three classes, there are $28 + 24 + 32 = 84$ students. To have a combined average for all three classes of 80%, there must be a combined total of $84 \times 80 = 6720$ percentage points. The 1st and 2nd period classes have already earned $28 \times 84 + 24 \times 86 = 2352 + 2064 = 4416$ percentage points. The 3rd period class must earn the remaining $6720 - 4416 = 2304$ percentage points. Dividing this among the 32 students in the 3rd period class, we see the class must earn a test average of $2304 \div 32 =$ **72**%.



92. The shaded region is a 180-degree rotation of the white region, so the each of the two regions must have an area half that of a circle with diameter 12 units. That would be $(1/2) \times \pi \times 6^2 =$ **18π** units^2.

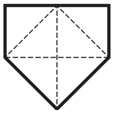
93. Since $b = c$, we can rewrite the first equation as $5a - 2c = 36$. The second equation can be rewritten as $2c = a$. Substituting a for $2c$ in the first equation, we get $5a - a = 36$. So $4a = 36$ and $a =$ **9**.

94. Since the bus was 50% full and after a net change of $7 - 2 = 5$ additional passengers the bus was 60% full, the additional 5 passengers must have accounted for the 10% increase in occupied seats. Since 5 is 10% of 50, there must be **50** seats on the bus.

95. When three fractions are inserted between 1/4 and 1/2 to form an arithmetic sequence of five fractions, the common difference is added four times. So the difference 1/2 − 1/4 = 1/4 must be divided by 4. Doing so, we see the common difference is 1/4 ÷ 4 = 1/16. The three new fractions are 5/16, 6/16 and 7/16, and they have a sum of 18/16 = **9/8**.

96. If we set up the problem vertically, we can multiply 123 by the 6 and the 4 in 4A6, as shown. Notice that in order for 3 + ★ + 0 = 4, ★ = 1. Also notice that ★ is the units digit of the product A × 3, which can only be 1 if we have 7 × 3. Thus, A = 7. Now, to determine the value of B, we multiply and get 476 × 123 = 58548. So B = 8, and A × B = 7 × 8 = **56**.

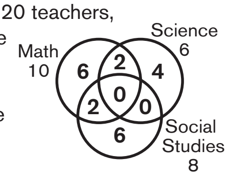
$$\begin{array}{r} 123 \\ \times\, 4A6 \\ \hline 738 \\ \_\,\_\,\_\,\bigstar 0 \\ +\ 49200 \\ \hline 5B548 \end{array}$$



97. If we introduce the dashed lines shown in the figure, we can see that each of the six right isosceles triangles is 1/4 of the area of the square of area 1 ft^2. It follows, then, that the home plate has an area of 6/4 = **3/2** ft^2.

98. Working backward, Rocky gave Aster 10 goldfish but kept 8. So Rocky received 10 + 8 = 18 goldfish from Elisa. It follows, then, that Elisa received 2 × 18 = 36 goldfish from Kendell. That means Kendell had 2 × 36 = 72 goldfish before giving any away. So Elisa received 36 fish but put only 18 in her bowl. The difference between Kendell's initial 72 goldfish and the 18 goldfish Elisa put in her new fishbowl is 72 − 18 = **54** goldfish.

99. If we make a Venn diagram for the three subjects, it may be easier to determine the quantity for each of the regions. Of the 20 teachers, none teach both science and social studies, so we can place a 0 in the intersection of science and social studies, as well as the intersection of all three subjects. It then becomes clear that a 2 should be placed in the intersection of math and social studies. Adding the 10 math teachers, 8 social studies teachers and 6 science teachers, we get a total of 24 teachers. However, we know that the school has only 20 teachers, but we've counted the 2 math and social studies teachers twice. We also must have double counted the 22 − 2 = **2** teachers who teach math and science. The figure shows the completed Venn diagram.



100. Both the *x*- and the *y*-coordinates of a midpoint of a segment must be halfway between the *x*- and the *y*-coordinates of the endpoints of the segment. If $x = 13$ is halfway between $x = -5$ and $x = a$, then $(-5 + a)/2 = 13 \rightarrow -5 + a = 26 \rightarrow a = 31$. If $y = -2$ is halfway between $y = 10$ and $y = b$, then $(10 + b)/2 = -2 \rightarrow 10 + b = -4 \rightarrow b = -14$. So $|a - b| = |31 - (-14)| = $ **45**.

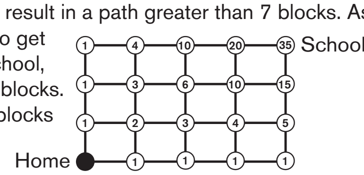
Warm-Up 8

101. By the Triangle Inequality Theorem, each side of the triangle must be shorter than the sum of the other two sides. This means *n* must be greater than 12 – 5 = 7 and less than 12 + 5 = 17. The **9** integers that are possible values of *n* are 8, 9, 10, 11, 12, 13, 14, 15 and 16.

102. Since 3/8 of one lap was 1/32 of the race, the race must have consisted of 32 of those 3/8 laps, which is a total of 32 × 3/8 =**12** laps.

103. The first 14 rows use up the first 14 × 6 = 84 counting numbers. Since the numbers in the odd rows are increasing from left to right, the first number in the 15th row is **85**.

104. One way to solve this type of problem is to indicate the number of ways to get to each intersection on the grid, as shown. To go only 7 blocks, he must always go north or east in the direction of the school. Walking south or west or backtracking will result in a path greater than 7 blocks. As we move north and east, the number of ways to get to an intersection is the sum of the number of ways to get to the intersections that lead to the new intersection. Since there are 35 different routes from home to school, Alvin can take different routes for **35** days. Alternatively, we might reason that Alvin has to go a total of 7 blocks. There are $_7C_3$ = 35 ways to pick the 3 blocks that are north, or likewise $_7C_4$ = 35 ways to choose the 4 blocks that are east. Either way, the result still is a different route each day for **35** days.



105. From the information given, we can write the equation $(4 + 20 + x)/3 = (y + 16)/2$. Cross-multiplying, we get $2 \times (24 + x) = 3 \times (y + 16) \rightarrow 48 + 2x = 3y + 48 \rightarrow 2x = 3y$. Since we want the smallest possible value of the positive integers *x* and *y*, we can make $x = 3$ and $y = 2$. The value of $x + y$, then, is 3 + 2 = **5**.

106. The sum of the areas of the five rectangles is (32 × 16) + (16 × 8) + (8 × 4) + (4 × 2) + (2 × 1) = 512 + 128 + 32 + 8 + 2 = **682** cm^2.

107. The information given can be written algebraically as $(1/2)x = (2/3)x - 8 \rightarrow (1/6)x = 8 \rightarrow x = $ **48**.

108. Let *n*, *d* and *q* represent the numbers of nickels, dimes and quarters, respectively. The following three equations can be written from the given information: $5n + 10d + 25q = 195$, $2n + 2 = d$ and $q = n + d + 1$. Substitute the expression in the second equation for *d* in the third equation to derive $q = n + 2n + 2 + 1 \rightarrow q = 3n + 3$. Next, divide the first equation by 5 to derive $n + 2d + 5q = 39$. Then replace *d* and *q* in this equation with the expression for *d* in the second equation and with the expression for *q* as derived. We have $n + 2(2n + 2) + 5(3n + 3) = 39$. Solving for *n*, we get $n + 4n + 4 + 15n + 15 = 39 \rightarrow 20n + 19 = 39 \rightarrow 20n = 20 \rightarrow n = 1$. Substituting for *n* in the second equation, we can solve for *d* as follows: $d = 2(1) + 2 = 4$. Finally, substituting for *n* in the derived equation for *q* yields $q = 3(1) + 3 = 6$. Colton paid for his soda using 1 nickel, 4 dimes and 6 quarters, or 1 + 4 + 6 = **11** coins in all.

109. At the end of the first year, Hiro's salary was 50,000(1.1) = \$55,000. At the end of his second year, his salary was 55,000(1.2) = \$66,000. At the end of Hiro's third year, his salary was 66,000(1.3) = \$85,800. After three years, Hiro's salary had increased by 85,800 − 50,000 = \$**35,800.**

110. Let's call the whole number n. The number plus 4 times its reciprocal can be written in algebra as $n + 4/n$. The product of the number and 4 times its reciprocal can be written as $n \times 4/n$, which simplifies to 4. These two expressions are said to be equal, so we have $n + 4/n = 4$. To solve for n, we can multiply both sides of the equation by n, which gives us the quadratic equation $n^2 + 4 = 4n$. Subtracting $4n$ from both sides, we rewrite the equation as $n^2 - 4n + 4 = 0$. At this point, you may recognize that the expression on the left is the square of the quantity $(n - 2)$, which means we have $(n - 2)^2 = 0$. This equation is true when $n =$ **2**. Let's check that this satisfies the original statement of the problem: $2 + 4/2 = 2 \times 4/2 \rightarrow 2 + 2 = 2 \times 2$. And it's a well-known fact that 2 plus 2 equals 2 times 2.

Workout 4

111. You may recognize that the side lengths of the right triangle formed by the rectangle's diagonal and two of its adjacent sides are a multiple of the 3-4-5 Pythagorean Triple. Thus, the diameter of the circumscribing circle, also the diagonal of a 6-cm by 8-cm rectangle, is 10 cm. The circle, then, has area equal to $\pi \times 5^2 = 25\pi \approx$ **79** cm^2.

112. Using the given rules, we have $4\# \cdot 3\# \cdot 2\# \cdot 1\# = (4? \cdot 3? \cdot 2? \cdot 1?)(3? \cdot 2? \cdot 1?)(2? \cdot 1?)(1?)$

$$= [(4! \cdot 3! \cdot 2! \cdot 1!)(3! \cdot 2! \cdot 1!)(2! \cdot 1!)(1!)][(3! \cdot 2! \cdot 1!)(2! \cdot 1!)(1!)][(2! \cdot 1!)(1!)](1!)$$
$$= [(24 \cdot 6 \cdot 2 \cdot 1)(6 \cdot 2 \cdot 1)(2 \cdot 1)(1)][(6 \cdot 2 \cdot 1)(2 \cdot 1)(1)][(2 \cdot 1)(1)](1)$$
$$= 24 \cdot 6^3 \cdot 2^6$$
$$= 24 \cdot 216 \cdot 64$$
$$= \mathbf{331{,}776}$$

113. Recall that the height of an equilateral triangle with a side length of 6 cm is $\sqrt{3}/2 \times 6 = 3\sqrt{3}$ cm. The circle has a diameter of $3\sqrt{3} + 6 + 3\sqrt{3} = 6\sqrt{3} + 6$ cm and a radius of $3\sqrt{3} + 3$ cm. The area of the circle, then, is $\pi(3 + 3\sqrt{3})^2 = \pi(9 + 18\sqrt{3} + 27) = \pi(36 + 18\sqrt{3})$ cm^2. Each triangle has an area of $(1/2) \times 6 \times 3\sqrt{3} = 9\sqrt{3}$ cm^2, and the square has an area of $6^2 = 36$ cm^2. So the combined area of the four triangles and the square is $4 \times 9\sqrt{3} + 36 = 36\sqrt{3} + 36$ cm^2. The area of the unshaded region is $(36\sqrt{3} + 36)/(\pi(36 + 18\sqrt{3})) \approx 0.47 =$ **47**% of the circle's area.

114. Twenty-one is the sum of the first 6 positive integers: $21 = 1 + 2 + 3 + 4 + 5 + 6$. The greatest possible value of n is **6**.

115. Since the hose allows 1/3 of the water flowing through it to spill, only 2/3 of the water is filling the pool. At 2/3 of the flow rate, it will take 3/2 as long to fill the pool. Thus, it will take $15 \times 3/2 = 45/2 =$ **22.5** minutes to fill the pool.

116. There are 26 letters in the alphabet, so there are $26 \times 26 = 676$ possible pairs of first and last initials. In the "worst case scenario," we could have 2 sets of people with all 676 possible pairs, which would be $2 \times 676 = 1352$ people, and still not have 3 people with the same initials. In a group of $1352 + 1 =$ **1353** people, there are guaranteed to be at least 3 people with the same first and last initials.

117. The ratio 16:9 is without units. Suppose we have a TV screen that measures 16 inches by 9 inches. Then, by the Pythagorean Theorem, the diagonal of the TV screen would be $\sqrt{(16^2 + 9^2)} = \sqrt{(256 + 81)} = \sqrt{337}$ inches. The ratio of the width to the diagonal of our 16-inch by 9-inch TV screen would be $16:\sqrt{337}$. The TV screen with a 37-inch diagonal would have the same ratio of width to diagonal, so with unknown width w, we can set up the proportion $16/\sqrt{337} = w/37$. Cross-multiplying and solving, we see that the TV screen has width $w\sqrt{337} = 16 \times 37 \rightarrow w = 592/\sqrt{337} \approx$ **32.2** inches.

118. Since the base of the pyramid is a square, we know the length of the diagonal is $6\sqrt{2}$ m, which is also the height of the pyramid. The volume of a pyramid is 1/3 times the product of the height and the area of the base. Thus, the volume of this pyramid is $(1/3) \times (6\sqrt{2}) \times 6^2 = \mathbf{72\sqrt{2}}$ m^3.

119. The $3/4 = 0.75$ inch of frosting on the top and on each side requires us to subtract 0.75 inch from the height of the cake and to subtract $2 \times 0.75 = 1.5$ inches from the length and width of the cake. The dimensions of the cake without the frosting, then, would be $16.5 \times 22.5 \times 3.25$ inches. So the volume of the cake without the frosting is $16.5 \times 22.5 \times 3.25 = 1206.5625 \approx$ **1207** in^3.

120. A property of every perfect square number is that each prime factor in its prime factorization is raised to an even power. The prime factorization of $54x$ is $2 \times 3^3 \times x$. Both 2 and 3 have an odd exponent. If x introduces one more factor of 2 and one more factor of 3, each prime factor then will be raised to an even power. The least value of x that does this is $x = 2 \times 3 =$ **6**. That results in the perfect square $54 \times 6 = 324$, which is 18^2.

Warm-Up 9

121. The angles of a quadrilateral must add to 360 degrees. Since the angles are a multiple of 3, 4, 5 and 6, it follows that for some integer a, $3a + 4a + 5a + 6a = 360$. Solving for a, we get $18a = 360$, so $a = 20$. Therefore, the largest angle in the quadrilateral has measure $6a = 6 \times 20 =$ **120** degrees.

122. Let n represent the largest of the six numbers arranged in ascending order. To maximize n, the other five numbers must be as small as possible. The smallest possible value for the first two numbers is 1. Since the median, in this case 8, of the ordered list is the average of the 3rd and 4th numbers, and since the 5th number can be no less than the 4th number, it follows that 8 is the smallest possible value for these three numbers. Finally, we know that the mean of 1, 1, 8, 8, 8 and n is 6, so it follows that $(1 \times 2 + 8 \times 3 + n) = 36 \rightarrow 26 + n = 36 \rightarrow n =$ **10**.

123. If the 3rd place winner's prize was d, then the 2nd place winner got $2d$ and the 1st place winner got $3d$. The combined total for all three prizes, then, would be $6d$. So we have $6d = 2400$ and $d = 400$. Therefore, the 3rd place winner received **$400.**

124. First, let's find $f(2)$. Since $2 \geq -1$, we use the second rule and get $f(2) = 2^2 - 6 = 4 - 6 = -2$. Now we'll evaluate $f(x)$ for $x = -2$ to determine $f(-2)$. Since $-2 < -1$, we use the first rule and get $f(-2) = -2 + 4 = 2$. Thus, the value of $f(f(-2))$ is **2**.

125. Although squaring these integers wouldn't be too bad without a calculator, let's see if we can find the value without having to square them.

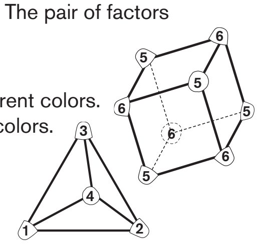
The numerator and denominator of the expression are both the difference of squares. Recall that $x^2 - y^2 = (x + y)(x - y)$. So we have $\frac{20^2 - 15^2}{18^2 - 17^2} = \frac{(20+15)(20-15)}{(18+17)(18-17)}$. Since $20 + 15 = 18 + 17$, we can cancel those terms from the numerator and denominator. That leaves us with $\frac{(20-15)}{(18-17)} = $ **5**.

126. The hypotenuse of the first triangle has length $1 \times \sqrt{2} = \sqrt{2}$ units. Since this is the length of the leg of the next triangle, the hypotenuse of that triangle would have length $\sqrt{2} \times \sqrt{2} = 2$ units. Notice that each subsequent hypotenuse is $\sqrt{2}$ times as long as the previous hypotenuse. Therefore, the hypotenuse of the 20th triangle would be $1 \times (\sqrt{2})^{20} = 1 \times 2^{10} =$ **1024** units.

127. If we subtract from the total the \$30 charged to hook the car to the tow truck, we see that $59.75 - 30 = \$29.75$ was the charge for the mileage. So from the school, Mr. Alman's car was towed $29.75 \div 1.75 =$ **17** miles to his house.

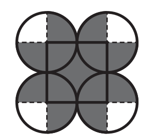
128. When the car's value went from \$20,000 to \$18,000, it retained 90% = 9/10 of its value. Multiplying \$18,000 by 9/10, we see that the car's value will be \$16,200 after 2 years. Multiplying \$16,200 by 9/10, we see that the car's value will be **\$14,580** after 3 years. Alternatively, if you recall $9^3 = 729$, and note that $(9/10)^3 = 729/1000$, you can multiply to get $20{,}000 \times 729/1000 = 20 \times 729 =$ **\$14,580**.

129. The prime factorization of 182 is $2 \times 7 \times 13$, so it has $2 \times 2 \times 2 = 8$ factors. They are 1, 2 ,7, 13, 14, 26, 91 and 182. The pair of factors closest to one another in value, 13 and 14, yields the least possible sum, which is $13 + 14 =$ **27**.



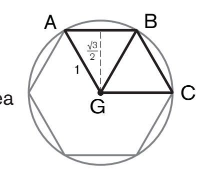
130. As shown, the tetrahedron will require gumdrops in 4 different colors, but the cube will require gumdrops in only 2 different colors. Since none of the gumdrops on the cube will be the same color as the gumdrops on the tetrahedron, Chin-Chin will need **6** colors.

Warm-Up 10



131. There are 4 circles, each with 3/4 of its area shaded. The 4 shaded regions of the circles have a combined area equivalent to that of 12 quarter circles, or 3 whole circles. If we use 4 of the quarter circles to shade the unshaded regions of the square, as shown, we will have shaded regions with a combined area equivalent to that of 1 whole square, $8 \times 8 = 64$ mm^2, plus 2 whole circles, $2 \times (\pi \times 4^2) = 32\pi$ mm^2. The total area of the shaded regions, then, is **$64 + 32\pi$** mm^2.

132. Since we want to maximize the difference, we should try the difference of a number in the nine-hundreds and a number in the one-hundreds. The greatest difference is $921 - 129 = 931 - 139 = 941 - 149 = 951 - 159 = 961 - 169 = 971 - 179 = 981 - 189 =$ **792**.



133. If we draw radius GB, as shown, quadrilateral ABCG is subdivided into two equilateral triangles with edge length 1 unit. In general, the altitude of an equilateral triangle is $\sqrt{3}/2$ times the side length. So in this case, we have an altitude of $\sqrt{3}/2$ unit. The area of each triangle is $(1/2) \times 1 \times (\sqrt{3}/2) = \sqrt{3}/4$ unit^2, making the area of quadrilateral ABCG $2 \times (\sqrt{3}/4) =$ **$\sqrt{3}/2$** unit^2.

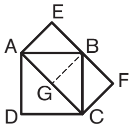
134. The surface area of a sphere with radius r is 4 times the area of its "great circle," or $4 \times \pi \times r^2$. So the total surface of the five spheres of radius 10 inches is $5 \times 4 \times \pi \times 10^2 = 2000\pi$ in^2. The surface area of a cylinder is the sum of the areas of the two circular bases and the lateral surface area, which is the area of the rectangle between the bases that wraps around the circumference of the bases. The area of each base of the cylinder with radius 20 inches is $\pi \times 20^2 = 400\pi$ in^2. The lateral surface area, which is the product of the circumference of a base and the cylinder's height h, is equal to $\pi \times 40 \times h = 40\pi h$ in^2. Therefore, the surface area of the cylinder is $2 \times 400\pi + 40\pi h = 800\pi + 40\pi h = \pi(800 + 40h)$. The surface area of the cylinder must be equal to the surface area of the five spheres, so we have $2000\pi = \pi(800 + 40h) \rightarrow 2000 = 800 + 40h \rightarrow 40h = 1200 \rightarrow h = 30$. Thus, the height of the cylinder is **30** inches.

135. Adding the equations $x + 2y + 3z = 6$, $2x + 3y + z = 8$ and $3x + y + 2z = 10$, we get $6x + 6y + 6z = 24$. Dividing both sides of the equation by 6 yields $x + y + z =$ **4**.

136. From the figure, we see that BC = BD + DC. According to the Angle Bisector Theorem, AB/AC = BD/DC. Substituting, we get the proportion 30/51 = 10/DC. Cross-multiplying yields 30 × DC = 510, so DC = 17. That means BC = 10 + 17 = **27** units.

137. Recall that the sum of the arithmetic sequence of consecutive numbers $a_1, a_2, a_3, \ldots a_n$ is $n \times (a_1 + a_n) \div 2$. So the sum of the integers in the 4th row is $7 \times (5 + 11) \div 2 = 56$, and the 5th row will have the 56 integers from 12 to $12 + 55 = 67$. The sum of the integers in the 5th row is $(12 + 67) \times 56 \div 2 = 2212$, so the 6th row will have the 2212 integers from 68 to $68 + 2211 =$ **2279**.

138. Since x and y are both negative integers, the product xy will be positive. The least possible value of xy occurs when $x = -1$ and $y = -2$. The condition $x - y = 1$ is met since $-1 - (-2) = -1 + 2 = 1$, and $xy = (-1)(-2) =$ **2**.

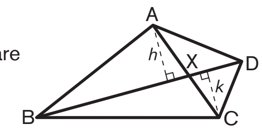


139. The figure shows that rectangle AEFC can be subdivided into four triangles, each of which is 1/4 of the area of square ABCD. The area of AEFC, therefore, is equivalent to that of square ABCD, which is $4 \times 4 =$ **16** units^2. Alternatively, you might use the properties of 45-45-90 right triangles to determine that the length and width of rectangle AEFC are $4\sqrt{2}$ and $2\sqrt{2}$, respectively, making the area of AEFC equal to $4\sqrt{2} \times 2\sqrt{2} =$ **16** units^2.

140. Since the original 100 liters of salt and water contained 1% salt, the composition of the solution was 1 liter of salt and 99 liters of water. After some of the water evaporated, the salt did not evaporate, so subsequently the 1 liter of salt would account for 5% of the solution. Since 1 liter is 5% of the solution, there must be $1 \div 0.05 = 20$ liters of solution. That means $100 - 20 =$ **80** liters of water evaporated.

Workout 5

141. The sum of the set of n numbers is $12 \times n = 12n$, and the sum of the set of $3n$ numbers is $6 \times 3n = 18n$. It follows, then, that the sum of the combined set of $n + 3n = 4n$ numbers is $12n + 18n = 30n$. Thus, the mean of the two sets combined is $30n/4n = 30/4 =$ **7.5**.



142. The figure shows convex quadrilateral ABCD with diagonals that intersect at X. Notice that $\triangle$BXC and $\triangle$DXC share an altitude of height k from C to diagonal BD. The area of $\triangle$BXC, denoted [BXC], is $1/2 \times \text{BX} \times k$, and the area of $\triangle$DXC, denoted [DXC], is $1/2 \times \text{DX} \times k$. The ratio of these areas is $\frac{[\text{BXC}]}{[\text{DXC}]} = \frac{1/2 \times \text{BX} \times k}{1/2 \times \text{DX} \times k} = \frac{\text{BX}}{\text{DX}}$. Similarly, $\triangle$BXA and $\triangle$DXA share an altitude of height h from A to diagonal BD. The area of $\triangle$BXA, denoted [BXA], is $1/2 \times \text{BX} \times h$, and the area of $\triangle$DXA, denoted [DXA], is $1/2 \times \text{DX} \times h$. The ratio of these areas is $\frac{[\text{BXA}]}{[\text{DXA}]} = \frac{1/2 \times \text{BX} \times h}{1/2 \times \text{DX} \times h} = \frac{\text{BX}}{\text{DX}}$. So $\frac{[\text{BXC}]}{[\text{DXC}]} = \frac{\text{BX}}{\text{DX}} = \frac{[\text{BXA}]}{[\text{DXA}]}$, and therefore, $\frac{[\text{BXC}]}{[\text{DXC}]} = \frac{[\text{BXA}]}{[\text{DXA}]}$. Substituting the areas of triangles BXC, DXC and BXA, we have 15/5 = 8/[DXA]. Cross-multiplying yields 15 × [DXA] = 40 and [DXA] = 40/15 ≈ **2.7** units^2.

143. There are $4! = 4 \times 3 \times 2 \times 1 = 24$ ways to permute the digits 1, 2, 3 and 4. Each digit will occur in each of the units, tens, hundreds and thousands places 6 times. Since $1 + 2 + 3 + 4 = 10$, the total for each place value will be $10 \times 6 = 60$. Therefore, the sum of the permutations of 1234 is 60 + 600 + 6000 + 60,000 = **66,660**.

144. Let D represent Duncan's starting number and T represent Taz's starting number. We know that D – (6 × 14) = 25 → D – 84 = 25 → D = 109. Also, we know that T + (8 × 14) = 25 → T + 112 = 25 → T = –87. The sum of their starting numbers, then, is 109 + (–87) = **22**.

145. Since the radius of the original circle is 2 units, its diameter, which also is the diagonal of the square, is 4 units. A property of every square is that the length of its diagonal is $\sqrt{2}$ times the side length. That means this square has side length $4 \div \sqrt{2} = 2\sqrt{2}$ units. The distance from the center of the circle to a midpoint of the side of the square is half this amount, or $\sqrt{2}$ units. The diameter of the inscribed circle, then, is $2 - \sqrt{2}$ units, making its radius $(2 - \sqrt{2})/2 \approx$ **0.3** unit.

146. Triangles ADE and BCE are right isosceles triangles, each with legs of length 2 cm and a hypotenuse of length $2\sqrt{2}$ cm. That means that angles CBE, CEB, DAE and DEA all measure 45 degrees. It follows, then, that angle AEB is 90 degrees. That means that arc AB also is 90 degrees and sector AEB is 1/4 of a circle of radius $2\sqrt{2}$ cm. The area of the shaded segment equals the area of sector AEB less the area of triangle AEB. Since the area of sector AEB is $1/4 \times \pi \times (2\sqrt{2})^2 = 2\pi$ cm^2, and the area of triangle ABE is $1/2 \times 4 \times 2 = 4$ cm^2, it follows that the area of the shaded segment is $2\pi - 4 \approx$ **2.3** cm^2.

147. The two girls are approaching each other at a rate of 2.4 + 2.8 = 5.2 mi/h. Since distance divided by rate equals time, the girls will meet 1.04 ÷ 5.2 = 0.2 hour = 12 minutes after leaving their respective homes. The time will be **8:12** a.m.

148. To get the smallest difference, we need the smallest possible squares for two four-digit numbers. To get the smallest squares, we need the smallest possible four-digit integers, which are 1000 and 1001. So, the smallest absolute difference between the squares of two distinct four-digit integers is $1001^2 - 1000^2 = 1{,}002{,}001 - 1{,}000{,}000 =$ **2001**.

149. A total of 40 × 2.15 = $86 would have been paid for the 40 bowls of chocolate ice cream. The remaining 158.20 – 86 = $72.20 would have been paid for bowls of vanilla ice cream. At $1.90 per bowl, that would mean 72.20 ÷ 1.90 = 38 bowls of vanilla ice cream were sold. Thus, a total of 40 + 38 = **78** bowls of ice cream were sold.

150. Dusty was traveling at 66 mi/h, which is equivalent to 66/60 = 11/10 miles each minute. At this rate, Dusty would have traveled 11 miles in **10** minutes.

Warm-Up 11

151. The least common multiple of 4 and 7 is 28. So the two lights will blink together every 28 seconds. In the 15 minutes from 10:00 p.m. to 10:15 p.m. there are 15 × 60 = 900 seconds. Since 900/28 = 225/7 = 32 1/7, the lights will blink together **32** more times before 10:15 p.m.

152. The largest three-digit base 5 number is 444_5, which equals $4 \times 25 + 4 \times 5 + 4 \times 1 = 100 + 20 + 4 = 124$ in base 10. The smallest four-digit base 4 number is 1000_4, which equals $1 \times 64 + 0 \times 16 + 0 \times 4 + 0 \times 1 = 64$ in base 10. These two values have an absolute difference of 124 – 64 = **60** in base 10.

153. We are looking for the combination of 5s and 7s that equals 53. One approach is to subtract 7s from 53 until the result is a multiple of 5. We have $53 - 7 = 46$; $46 - 7 = 39$; $39 - 7 = 32$; $32 - 7 = 25$. Since $25/5 = 5$, the store currently has in stock five boxes, each containing 5 trains, and four boxes, each containing 7 trains. The probability that the box randomly selected by Charles contains 7 trains is **4/9**.

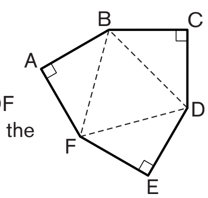
154. Since $ab = 1$, it follows that $a = 1/b$ and $b = 1/a$. That means a and b are reciprocals of each other. The fourth equation, $b = -c$, tells us that b and c are opposites. If we replace b with $-c$ in the second equation, we get $-c \times c = -9 \rightarrow c^2 = 9$. So $c = 3$ and $b = -3$, or $c = -3$ and $b = 3$. If $b = 3$, then $a = 1/3$; and if $b = -3$, then $a = -1/3$. When $a = -1/3$ and $b = -3$, then $c = 3$. This contradicts the final fact that $c < -a$, since $3 > 1/3$. Therefore, we conclude that $a = 1/3$, $b = 3$ and $c = -3$, so $a + b = 1/3 + 3 = \mathbf{3\frac{1}{3}}$.

155. The surface area of the original $3 \times 3 \times 3$ cube is $6 \times 3^2 = 6 \times 9 = 54$ units^2. Each unit cube removed from the center of a face, creates an additional 4 units^2 of surface area. That's another $6 \times 4 = 24$ units^2 of surface area. The surface area of the resulting solid is $54 + 24 =$ **78** units^2.

156. Let's suppose the area of the smallest triangle is 1 unit^2. Since each successive triangle has a base length that is 3/2 times the base length of the triangle above it, it follows that the areas of the next three triangles are $1 \times 3/2 = 3/2$ units^2, $1 \times (3/2)^2 = 9/4$ units^2 and $1 \times (3/2)^3 = 27/8$ units^2. The ratio of the area of the smallest triangle to the area of the largest is $1/(27/8) =$ **8/27**.

157. We'll use the slope-intercept form $y = mx + b$, where m is the slope and b is the y-intercept, to write equations for these two line segments. For the first segment, the slope is $(9 - 6)/(5 - 2) = 3/3 = 1$. Substituting $x = 2$, $y = 6$ and $m = 1$, we get $6 = 1(2) + b$ and $b = 4$. So the equation for the first segment is $y = x + 4$. The second segment has slope $(-1 - (-7))/(-1 - 5) = 6/-6 = -1$. Substituting $x = -1$, $y = -1$ and $m = -1$, we get $-1 = (-1)(-1) + b$ and $b = -2$. So the equation for the second segment is $y = -x - 2$. We are looking for the ordered pair (x, y) that satisfies both of these equations—in other words, when $x + 4 = -x - 2$. Solving for x yields $2x = -6$ and $x = -3$. When $x = -3$, $y = -3 + 4 = 1$. Therefore, the intersection has coordinates **(–3, 1)**.

158. The factor pairs (a, b) that satisfy $a \times b = 48$ are (48, 1), (24, 2), (16, 3), (12, 4) and (8, 6). Of these, the pair that satisfies $a - b = 8$ is (12, 4). Therefore, $a + b = 12 + 4 =$ **16**.



159. Drawing segments FB, BD and FD, as shown, creates three isosceles right triangles ABF, BCD and DEF and equilateral triangle BDF. The area of each of the right triangles is $1/2 \times 2 \times 2 = 2$ in^2. Based on the properties of 45-45-90 right triangles, each right triangle has a hypotenuse of length $2\sqrt{2}$ inches, which is also the side length of triangle BDF. The height of triangle BDF is $\sqrt{3}/2$ times its side length, or $\sqrt{3}/2 \times 2\sqrt{2} = \sqrt{6}$ inches. The area of triangle BDF, then, is $1/2 \times 2\sqrt{2} \times \sqrt{6} = 2\sqrt{3}$ in^2. Therefore, the area of the hexagon, in simplest radical form, is $3 \times 2 + 2\sqrt{3} = 6 + 2\sqrt{3}$ in^2. So $p = 6$, $q = 2$ and $pq = 6 \times 2 =$ **12**.

160. Since the first digit of a three-digit number cannot be zero, there are 9 choices for the first digit. If all three digits are to be different, there are only 9 choices for the second digit, and there are 8 choices for the third digit. In all, there are $9 \times 9 \times 8 =$ **648** integers from 100 to 999 that have three different digits.

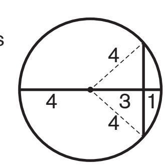
Warm-Up 12

161. The numerator is $2 \times 4 \times 6 \times 8 \times 10 \times \cdots \times 20$, and the denominator is 10!, which equals $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$. Each of the ten factors in the numerator is two times one of the ten factors in the denominator. So the expression simplifies to $2^{10} =$ **1024**.

162. The table shows how each positive integer less than 20 can be written as the sum of two primes except the integers 1, 2, 3, 11 and 17. The sum of these five integers is $1 + 2 + 3 + 11 + 17 =$ **34**. Notice that 2 is the only positive even integer less than 20 that cannot be written as the sum of two primes. In fact, the strong Goldbach conjecture states that every even integer greater than 2 can be expressed as a sum of two primes. Although this was never proven definitively, a counter-example has never been found that disproves it.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
			2 + 2	2 + 3	3 + 3	2 + 5	3 + 5	2 + 7	5 + 5		5 + 7	2 + 11	3 + 11	2 + 13	3 + 13		7 + 11	2 + 17

163. Since the chord divides the diameter into segments of lengths 1 cm and 7 cm, the chord divides a radius into segments of lengths 1 cm and 3 cm, as shown. Since the chord is perpendicular to the diameter, and hence the radius, the right triangle is formed with a hypotenuse that is a radius of length 4 cm and a leg of length 3 cm. By the Pythagorean Theorem, the other leg has length $\sqrt{(4^2 - 3^2)} = \sqrt{(16 - 9)} = \sqrt{7}$ cm. Since this leg is half of the chord, it follows that the chord has length $\mathbf{2\sqrt{7}}$ cm.



164. Base XY in triangle AXY is 2/3 the length of base BC in triangle ABC. In addition, the height of triangle AXY is 2/3 the height of triangle ABC. The area of triangle AXY is therefore $(2/3)^2 = 4/9$ of the area of triangle ABC. Trapezoid XYCB must account for the other 5/9 of the area of triangle ABC. Therefore, the ratio of the area of triangle AXY to that of trapezoid XYCB is $(4/9)/(5/9) = 4/9 \times 9/5 =$ **4/5**.

165. There are five groups of three digits that have a sum of 5: $1 + 1 + 3$, $1 + 2 + 2$, $1 + 4 + 0$, $2 + 3 + 0$ and $5 + 0 + 0$. We can make 3 different three-digit numbers using 1, 1, 3. Likewise, we can make 3 different numbers using 1, 2, 2. We can make 4 different three-digit numbers using 1, 4, 0 and another 4 different numbers using 2, 3, 0. There is 1 three-digit number using 5, 0, 0. That's a total of $3 + 3 + 4 + 4 + 1 =$ **15** numbers that have three digits with a digital sum of 5.

166. Let's say that the courier's speed *to* the delivery location was r mi/h. Then her speed *from* the delivery location was $r - 20$ mi/h. Time equals distance divided by rate, so the time going to the delivery location can be expressed as $180/r$, and the time to get from the delivery location back to her point of origin was $180/(r - 20)$. The entire trip took 7.5 hours, so we have $180/r + 180/(r - 20) = 7.5 \rightarrow 180/(r - 20) = 7.5 - 180/r \rightarrow 180/(r - 20) = (7.5r - 180)/r$. Cross-multiplying yields $180r = (r - 20)(7.5r - 180) \rightarrow 180r = 7.5r^2 - 180r - 150r + 3600 \rightarrow 7.5r^2 - 510r + 3600 = 0 \rightarrow r^2 - 68r = -480$. Let's solve this quadratic equation by completing the square. We need to add $(-68/2)^2 = (-34)^2 = 1156$ to both sides to get $r^2 - 68r + 1156 = -480 + 1156 \rightarrow (r - 34)^2 = 676 \rightarrow r - 34 = \pm\sqrt{676} \rightarrow r = 34 \pm 26$. So the solutions to this quadratic equation are $r = 34 + 26 = 60$ mi/h and $r = 34 - 26 = 8$ mi/h. The rate to the delivery location could not have been 8 mi/h because that leads to a negative rate, which is not possible. Therefore, we can conclude that the courier's average speed driving to the delivery location was **60** mi/h.

167. All four unshaded triangles are 30-60-90 right triangles. In a 30-60-90 right triangle, the length of the shorter leg is half the length of the hypotenuse, while the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg. Since each of the 4-unit sides of the inner square is the hypotenuse of one of these right triangles, it follows that each triangle has sides of length 4, 2 and $2\sqrt{3}$ units. The outer square has sides of length $2 + 2\sqrt{3}$ units. Its area, in simplest radical form, is $(2 + 2\sqrt{3})^2 = 4 + 8\sqrt{3} + 12 = 16 + 8\sqrt{3}$ units^2. Thus, $a = 16$, $b = 8$ and $a + b = 16 + 8 =$ **24**.

168. For $0 \times 0 = 0$, $0 \times (-1) = 0$ and $0 \times k = 0$, the result is 0, which is in the set. Since the product $(-1) \times (-1) = 1$ also must be in the set, it follows that $k =$ **1**. This is confirmed when we multiply $(-1) \times 1$ to get -1, an element in the set.

169. The width of the box is w, the length is $2w$ and the height is $3w$. The volume of one such box is $w \times 2w \times 3w = 6w^3$ units^3, and the volume of 24 of these boxes is $24 \times 6w^3 = \mathbf{144w^3}$ units^3.

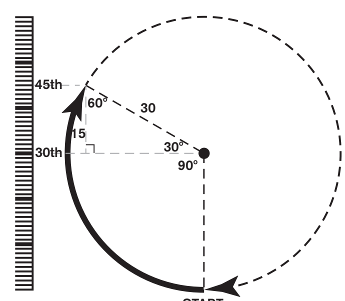
170. The difference between $a_1 = 13$ and $a_n = 77$ is $77 - 13 = 64$. The common difference, d, of this arithmetic sequence must be a factor of 64. The 7 factors of 64 are 1, 2, 4, 8, 16, 32 and 64. We need to determine the number of terms in the sequence that result when each of these factors is the common difference.
The common difference $d = 1$ can be added 64 times, resulting in the sequence from $a_1 = 13$ to $a_{65} = 77$.
The common difference $d = 2$ can be added 32 times, resulting in the sequence from $a_1 = 13$ to $a_{33} = 77$.
The common difference $d = 4$ can be added 16 times, resulting in the sequence from $a_1 = 13$ to $a_{17} = 77$.
The common difference $d = 8$ can be added 8 times, resulting in the sequence from $a_1 = 13$ to $a_9 = 77$.
The common difference $d = 16$ can be added 4 times, resulting in the sequence from $a_1 = 13$ to $a_5 = 77$.
The common difference $d = 32$ can be added 2 times, resulting in the sequence from $a_1 = 13$ to $a_3 = 77$.
The common difference $d = 64$ can be added 1 time, resulting in the sequence from $a_1 = 13$ to $a_2 = 77$.
The possible values of n are 65, 33, 17, 9, 5, 3, 2. The median of these values is **9**.

Workout 6

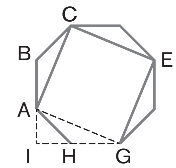
171. Using function notation, let $h(x)$ be the function that returns the number of 1s that result when Becky's rule is applied to the number x. The proper factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24 and 36. Right away, we see that $h(1) = h(2) = h(3) = 1$. Now we'll use this to determine the number of 1s for each of the other factors.
$h(4) = h(1) + h(2) = 1 + 1 = 2$
$h(6) = h(1) + h(2) + h(3) = 1 + 1 + 1 = 3$
$h(8) = h(1) + h(2) + h(4) = 1 + 1 + 2 = 4$
$h(9) = h(1) + h(3) = 1 + 1 = 2$
$h(12) = h(1) + h(2) + h(3) + h(4) + h(6) = 1 + 1 + 1 + 2 + 3 = 8$
$h(18) = h(1) + h(2) + h(3) + h(6) + h(9) = 1 + 1 + 1 + 3 + 2 = 8$
$h(24) = h(1) + h(2) + h(3) + h(4) + h(6) + h(8) + h(12) = 1 + 1 + 1 + 2 + 3 + 4 + 8 = 20$
$h(36) = h(1) + h(2) + h(3) + h(4) + h(6) + h(9) + h(12) + h(18) = 1 + 1 + 1 + 2 + 3 + 2 + 8 + 8 = 26$
So, $h(72) = h(1) + h(2) + h(3) + h(4) + h(6) + h(8) + h(9) + h(12) + h(18) + h(24) + h(36) = 1 + 1 + 1 + 2 + 3 + 4 + 2 + 8 + 8 + 20 + 26 = 76$. There will be **76** 1s on the board when Becky is finished.

172. Since the bug walks 1 mm every second, after 60 seconds the bug will have walked 60 mm along the second hand. The tricky part is determining the distance the bug will travel as a result of the rotation of the second hand. The second hand rotates exactly $360 \div 60 = 6$ degrees every second. The bug travels $6/360 = 1/60$ of the circumference of a circle with a radius equal to the distance from the center of the clock to the bug's current position along the second hand. After 1 second he will be 1 mm from the center of the clock, and when the second hand rotates 6 degrees, he travels 1/60 of the circumference of a circle of radius 1 mm, or $1/60 \times 2\pi \times 1$ mm. The total distance the bug will travel as a result of the second hand's rotation is $1/60 \times 2\pi \times (1 + 2 + 3 + \ldots + 60) = (\pi/30) \times (1 + 2 + 3 + \ldots + 60) = (\pi/30) \times (60 \times 61/2) = 61\pi$ mm. Adding this to the 60 mm he walks along the second hand, we get a total distance of $60 + 61\pi \approx$ **251.6** mm.

173. Think of the Ferris wheel as a circle with a diameter equivalent to the height of 60 floors. We need to determine how many degrees the Ferris wheel has already rotated to determine how long it will take to bring Courtney back to the starting place. It is important to note that as the Ferris wheel rotates at a constant rate, Courtney's vertical distance from the ground is not increasing at a constant rate. When Courtney has risen to the height of the 30th floor, halfway up the building, she has rotated 90 degrees on the Ferris wheel, only a quarter of the way around. From there to the 45th floor, Courtney rises another 15 floors and rotates another 30 degrees on the Ferris wheel, as shown. That means when Courtney has risen to the height of the 45th floor, she has rotated on the Ferris wheel a total of $90 + 30 = 120$ degrees. So in 8 minutes 26 seconds, or 506 s, Courtney has done $120/360 = 1/3$ of a complete rotation on the Ferris wheel. To rotate the remaining $360 - 120 = 240$ degrees back to her starting place, it will take twice as long, or $506 \times 2 =$ **1012** seconds.



174. We will extend line segments AB and GH until they intersect at I, as shown. For isosceles right triangle AIH, AH = 1 unit and AI = IH = $\sqrt{2}/2$ unit. For right triangle AIG, IG = $(\sqrt{2}/2) + 1$ units. We are asked to determine the area of square ACEG, which is equal to AG^2. Based on the Pythagorean Theorem, $AG^2 = AI^2 + IG^2 \rightarrow AG^2 = (\sqrt{2}/2)^2 + ((\sqrt{2}/2) + 1)^2$. Therefore, square ACEG has area $AG^2 = 1/2 + 1/2 + \sqrt{2} + 1 = 2 + \sqrt{2} =$ **3.4** units^2.



175. If we set aside one of the *L*s, using just the letters *SILY*, 4 × 3 × 2 = 24 three-letter strings can be formed. Now let's determine the number of three-letter strings that can be formed using both *L*s with each of the other three letters. Three strings can be formed with each of *LLS*, *LLI* and *LLY*, for a total of 3 + 3 + 3 = 9 additional three-letter strings. Thus, the total number of three-letter strings that can be formed is 24 + 9 = **33** strings.

176. If all 27 of the animals in the area were chicks, there would be 2 × 27 = 54 legs, which leaves 78 – 54 = 24 legs unaccounted for. That means there were 24 ÷ 2 = **12** bunnies.

177. The greatest percent reduction would be on a square field. The distance along two sides of a square field of side length s is $2s$, and the distance along the diagonal is $s\sqrt{2}$. The percent reduction in distance by using the diagonal path is $(2s - s\sqrt{2})/2s = (2 - \sqrt{2})/2 \approx 0.29 =$ **29**%.

178. In this case, since 50 ÷ 3 = 16 2/3, the rectangular solid with the greatest possible volume would have edges with lengths 16 cm, 17 cm and 17 cm and a volume of 16 × 17 × 17 = 4624 cm^3. The rectangular solid with the least possible volume would have edges with lengths 1 cm, 1 cm and 48 cm and a volume of 1 × 1 × 48 = 48 cm^3. The absolute difference in these two measures of volume is 4624 – 48 = **4576** cm^3.

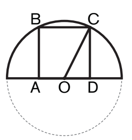
179. The probability that two jelly beans selected at random are the same color is the sum of the probabilities that they are both red, both black or both green, which is $\frac{28}{48} \times \frac{27}{47} + \frac{14}{48} \times \frac{13}{47} + \frac{6}{48} \times \frac{5}{47} = \frac{756+182+30}{2256} = \mathbf{\frac{121}{282}}$.

180. The units digits of powers of 2 form the repeating pattern 2, 4, 8, 6, then back to 2. Since 2015 is three more than a multiple of 4, the units digit of 2^{2015} is 8, the third number in the pattern. Similarly, the units digits of powers of 7 form the repeating pattern 7, 9, 3, 1, then back to 7. This also is a pattern of four digits, so the units digit of 7^{2015} is 3, the third number in the pattern. Since 8 × 3 = 24, the units digit of $2^{2015} \times 7^{2015}$ is **4**.

Warm-Up 13

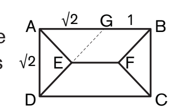
181. Recall that $12! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 = 2^{10} \times 3^5 \times 5^2 \times 7 \times 11$. Any perfect square factor will have an even number of each prime in its prime factorization. We can rewrite part of the shown prime factorization as a kind of "square factorization" as follows: $2^{10} \times 3^4 \times 5^2 = 4^5 \times 9^2 \times 25^1$. If we were to multiply a number from each of the three columns shown, the result would be a perfect square factor of 12!. In this way, we can get the 6 × 3 × 2 = **36** perfect squares that are factors of 12!.

4^0	9^0	25^0
4^1	9^1	25^1
4^2	9^2	
4^3		
4^4		
4^5		



182. Point O is the midpoint of side AD of square ABCD, shown here, inscribed in a semicircle. If we suppose ABCD has side length 2 units, then OD = 1 unit. Using the Pythagorean Theorem to determine the length of radius OC, we have $OC = \sqrt{(1^2 + 2^2)} = \sqrt{5}$ units. Therefore, the ratio of the radius of the semicircle to the side length of the square is $\mathbf{\sqrt{5}/2}$.

183. Triangle AED is a right isosceles triangle. Based on the properties of 45-45-90 right triangles, since each leg has length 1 unit, the hypotenuse has length $\sqrt{2}$ units. This is also the width of the rectangle. If we extend segment DE until it intersects side AB at point G, as shown, we can see that the length of the rectangle is $\sqrt{2} + 1$ units. The area of the rectangle, then, is $\sqrt{2} \times (1 + \sqrt{2}) = \mathbf{\sqrt{2} + 2}$ units^2.



184. The number 1111_3 is a four-digit number in base 3, in which the place values of the digits from left to right are 3^3, 3^2, 3^1 and 3^0. Likewise, 1111_2 is a four-digit number in base 2, in which the place values of the digits from left to right are 2^3, 2^2, 2^1 and 2^0. Written as decimals, 1111_3 = 27 + 9 + 3 + 1 = 40 and 1111_2 = 8 + 4 + 2 + 1 = 15. The difference between these two values is 40 – 15 = **25**.

185. The candle lost 11 – 8 = 3 inches in 6 – 4 = 2 hours. Since we are told it burned for a total of 6 hours, its height must have been reduced by 2 × 3 = 6 inches during the first 4 hours. That means the candle had an initial height of 11 + 6 = **17** inches.

186. The express train takes 18 ÷ 72 = 1/4 hour = 15 minutes to travel from Addington to Summit. The local train takes 18/54 = 1/3 hour = 20 minutes, plus 6 × 1.5 = 9 minutes for stops. That's a total of 29 minutes. The local train takes 29 – 15 = **14** minutes more than the express train.

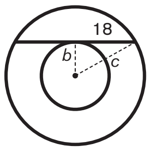
187. The positive integer divisors of 20 are 1, 2, 4, 5, 10, 20. The positive integer divisors of 30 are 1, 2, 3, 5, 6, 10, 15, 30. The four divisors in common are 1, 2, 5, 10. Thus, of the 6 × 8 = 48 possible pairs of integer divisors that Jack and Jill could choose, there are 4 ways in which they can choose the same number. The probability that Jack and Jill will pick the same number, then, is 4/48 = **1/12**.

188. Since 45 minutes is 45/60 = 3/4 hour, the first dragonfly flew 36 × 3/4 = 27 miles, and the second dragonfly rode 60 × 3/4 = 45 miles. Therefore, the second dragonfly traveled 45 – 27 = **18** more miles than the first dragonfly.

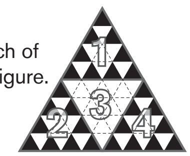
189. Let u and v represent the two numbers. We are told that $u + v = 1$ and $u - v = 2$. Adding these two equations yields $2u = 3$, so $u = 3/2$. It follows, then, that $v = 1 - 3/2 = -1/2$ and $(3/2) \times (-1/2) =$ **–3/4**.

190. The first term on the right side of the equation is the result when $2a$ is raised to the $\sqrt{x}$ power. Solving $(2a)^{\sqrt{x}} = 16a^4 \rightarrow (2a)^{\sqrt{x}} = (2a)^4$, we see that $\sqrt{x} = 4$ and $x =$ **16**.

Warm-Up 14



191. The smaller circle, of radius b, has area $\pi \times b^2$, and the larger circle, of radius c, has area $\pi \times c^2$. The area of the region between the circles, then, is $\pi \times c^2 - \pi \times b^2 = \pi(c^2 - b^2)$. Since a tangent line is always perpendicular to a radius drawn to the point of tangency, the smaller circle's radius drawn perpendicular to the segment also bisects the segment. Then the larger circle's radius drawn to its point of intersection with the segment, as shown, creates a right triangle. According to the Pythagorean Theorem, $18^2 + b^2 = c^2$, which can be rewritten as $324 = c^2 - b^2$. Substituting for $c^2 - b^2$ in $\pi(c^2 - b^2)$, we see that the area of the region between the circles is **324π** in^2.



192. The largest triangle can be divided into the 4 congruent equilateral triangles, which are numbered 1 through 4 in the figure. Each of these triangles can be further subdivided into 16 congruent triangles. That's a total of $4 \times 16 = 64$ congruent triangles in the entire figure. The triangles numbered 1, 2 and 4 have 7 white triangles each. The triangle numbered 3 has 16 white triangles. That's $3 \times 7 + 16 = 21 + 16 = 37$ white triangles in all. Therefore, **37/64** of the entire figure is white.

193. Simplifying the equation, we get $(m + n)/(m - n) = (25/4)(m - n)/(m + n) \rightarrow [(m + n)/(m - n)] \times [(m + n)/(m - n)] = 25/4 \rightarrow (m + n)^2/(m - n)^2 = 25/4$. Then taking the square root of both sides and cross-multiplying yields $(m + n)/(m - n) = 5/2 \rightarrow 2(m + n) = 5(m - n) \rightarrow 2m + 2n = 5m - 5n \rightarrow 7n = 3m \rightarrow$ **7/3** $= m/n$. Note: m and n are non-negative integers and $m > n$, so we need not consider the case where $m + n < 0$ or $m - n < 0$.

194. Each of the cube's six faces has an area of $6 \times 6 = 36$ cm^2. That's a total surface area of $6 \times 36 = 216$ cm^2. To cover the entire surface area with a silver layer 1 mm = 0.1 cm thick, the amount of silver needed is $216 \times 0.1 = 21.6 \approx$ **22** cm^3.

195. Consider the scenario at stage 2015 on a much smaller scale. Figure 1 shows three squares, placed one on top of another and positioned so that the upper left vertices of the squares coincide. The two black squares have side lengths 1 unit and 3 units, and the white square has side length 2 units. Notice that the figure is symmetrical about the dashed line. Let's cut the figure in half along the dashed line and remove the top half to obtain Figure 2. Since the two halves are identical reflections of each other, let's duplicate the half that remains and rotate it to replace the half that was removed. The result is the striped square shown in Figure 3. The stripes are congruent, each 1 unit by 3 units. There are 2 black stripes and 1 white stripe, so the figure is 2/3 black. Doing the same to the whole figure at stage 2015 of the original problem would result in a square with 2015 stripes, each 1 unit by 2015 units. There would be 1008 black stripes and 1007 white stripes, and **1008/2015** of the figure would be black.

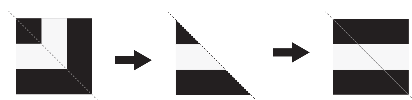


Figure 1 Figure 2 Figure 3

x	$f(x)$
−4	13
−3	6
−2	1
−1	−2
0	−3
1	−2
2	1
3	6
4	13

196. The table at right shows the value of $f(x)$ for each value of x in the given domain. The range is the set of possible outputs, which in this case is $\{-2, -3, 1, 6, 13\}$. The **3** integers that are in both the range and the domain of f are −3, −2 and 1.

197. Using properties of similar triangles, we can set up the proportion $5/x = 12/(x + 6)$. Cross-multiplying and solving for x, we get $5(x + 6) = 12x \rightarrow 5x + 30 = 12x \rightarrow 7x = 30 \rightarrow x = 30/7$. Converting this to a mixed number, we see that her shadow is $\mathbf{4\frac{2}{7}}$ feet long.

198. The only positive integers that are the same when expressed in base 7 and in base 13 are one-digit numbers. Six is the greatest one-digit integer in base 7. Therefore, 1, 2, 3, 4, 5 and 6 are the **6** integers that are the same when expressed in base 7 and in base 13.

199. The number of ways to randomly select three slips from the hat, all at the same time, is ${}_{10}C_3 = 10!/(7! \times 3!) = (10 \times 9 \times 8)/(3 \times 2 \times 1) = 120$ ways. For the mean of the three values to be one of the numbers, the selected values must form an arithmetic sequence with common difference d. The three-number sets for which the mean is one of the selected values are 1-2-3, 2-3-4, 3-4-5, 4-5-6, 5-6-7, 6-7-8, 7-8-9, 8-9-10 when $d = 1$; 1-3-5, 2-4-6, 3-5-7, 4-6-8, 5-7-9, 6-8-10 when $d = 2$; 1-4-7, 2-5-8, 3-6-9, 4-7-10 when $d = 3$; and 1-5-9, 2-6-10 when $d = 4$. These are the 20 scenarios in which the mean of the three numbers is one of the selected values. Therefore, the probability that the mean of the three randomly selected values is a number written on one of the slips is 20/120 = **1/6**.

200. If $2015 + a = b$, then $a = b - 2015$. Since a and b are both positive integers, we start subtracting 2015 from each of the possible b palindromes, beginning with 2112, which is the smallest palindrome greater than 2015. As the table shows, the first difference we obtain that also is a palindrome is 2772 − 2015 = **757**.

b	2112	2222	2332	2442	2552	2662	**2772**
a	97	207	317	427	537	647	**757**

Workout 7

201. If $a = 2$, $b = 10$, $c = 7$ and $d = 9$, then $a + b + c + d = 2 + 10 + 7 + 9 = 28$. Since $2^{10} \times 7^9 = (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2) \times (7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7)$, $2^{10} \times 7^9$ also can be written as $2^1 \times 14^9$. In this case, $a + b + c + d = 2 + 1 + 14 + 9 = 26$. However, the closer in value a, b, c and d are, the smaller their sum will be. We actually obtain the smallest value of $a + b + c + d$ when we, instead, rewrite $2^{10} \times 7^9$ as $4^5 \times 7^9$. We have $a + b + c + d = 4 + 5 + 7 + 9 =$ **25**.

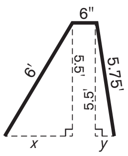
202. If we let $x = 0.2\overline{36}$, then $100x = 23.6\overline{36}$. Subtracting these two equations, we get $99x = 23.4$, so $x = 23.4/99 = 234/990 =$ **13/55**, which is the common fraction equivalent of $0.2\overline{36}$.

203. The original 3-4-5 triangle has an area of $1/2 \times 3 \times 4 = 6$ units^2. If we use the hypotenuse of length 5 as a base, then the altitude must be $6 \times 2 \div 5 = 12/5$ units. This is the length of the first cut. The perimeter of the original triangle was $3 + 4 + 5 = 12$ units. The sum of the perimeters of the 2 triangles is now longer by exactly twice the length of this first cut, so it's $12 + 2 \times 12/5 = 60/5 + 24/5 = 84/5$ units. If we think of this change from 60/5 to 84/5 as growth by a factor of $84/5 \div 60/5 = 84/5 \times 5/60 = 84/60 = 7/5$, we can multiply by this growth factor three more times to get the sum of the perimeters of the 16 triangles. We get $84/5 \times (7/5)^3 = 84/5 \times 343/125 = 28{,}812/625 \approx$ **46.1** units.

204. Since the area of $\triangle ABC$ is $(1/2)bh$, it follows that the shaded region has area $(2/3)bh - (1/2)bh = (1/6)bh$. Solving $-(1/4)x^2 + 16 = 0$, we get $x^2 - 64 = 0 \rightarrow x^2 = 64 \rightarrow x = -8$ and $x = 8$. So, $b = 8 - (-8) = 16$ units. Solving $y = -(1/4) \times 0 + 16$, we get $y = 16$. So, $h = 16 - 0 = 16$ units. The area of the shaded region is $(1/6) \times 16 \times 16 = 256/6 \approx$ **42.7** units^2.

205. Given $S_{100} = 1^2 - 2^2 + 3^2 - 4^2 + \ldots + 99^2 - 100^2$, the table shows the first few sums. Notice that these sums appear to be the triangular numbers with alternating signs, such that sums of an odd number of terms are positive while sums of an even number of terms are negative. The 100th triangular number is $100 \times 101 \div 2 = 5050$. Therefore, the value of S_{100}, which is the sum of an even number of terms, is **−5050**.

n	1	2	3	4	5	6
S_n	1	−3	6	−10	15	−21



206. If we draw perpendicular lines from the intersection of each support with the platform, as shown, two right triangles are formed. The distance between the two supports on the floor is $x + 0.5 + y$. We can determine x and y using the Pythagorean Theorem. We have $5.5^2 + x^2 = 6^2 \rightarrow x^2 = 36 - 30.25 \rightarrow x = \sqrt{5.75}$, and $5.5^2 + y^2 = 5.75^2 \rightarrow y^2 = 33.0625 - 30.25 \rightarrow y = \sqrt{2.8125}$. So, on the floor, the supports of the ladder are $\sqrt{5.75} + 0.5 + \sqrt{2.8125} \approx$ **4.57** feet apart.

207. We can get an idea of the value of the middle term in this sequence of consecutive odd integers by taking the cube root of 64,000,000. Since $\sqrt[3]{64{,}000{,}000} = 400$, and since we're looking for a product greater than 64,000,000, but less than 65,000,000, we'll try making the middle term 401. This results in the product $399 \times 401 \times 403 = 64{,}479{,}597$. The greatest of the three consecutive odd integers is **403**.

208. Dividing 1 by 13 gives us the repeating decimal 0.076923076..., in which the six-digit sequence 076923 repeats. Since 2015 is five more than a multiple of six, the 2015th digit after the decimal point will be the 5th digit in the pattern, which is **2**.

209. One tenth of an acre is $43{,}560 \div 10 = 4356$ ft^2. Since $\sqrt{4356} = 66$, Jason's square garden is 66 feet by 66 feet. The diameter of each watered area is 22 feet, and $66 \div 22 = 3$. So Jason can install sprinklers arranged in 3 rows with 3 sprinklers per row, using **9** sprinklers.

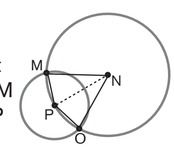
210. As shown in the table, the least amount of time is it will take to finish running the program is about **12** years. The optimal strategy is to wait ten years and then run the program using technology that will allow the program to finish in 2 years. Waiting 11 years for technological advances that will allow the program to finish in 1 year also takes about 12 years. After that, the increase in computing speed does not make up for the increased wait time, and the total time starts to increase.

Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Total Years *(nearest tenth)*	2000.0 + 0 = 2000.0	1000.0 + 1 = 1001.0	500.0 + 2 = 502.0	250.0 + 3 = 253.0	125.0 + 4 = 129.0	62.5 + 5 = 67.5	31.3 + 6 = 37.3	15.6 + 7 = 22.6	7.8 + 8 = 15.8	3.9 + 9 = 12.9	2.0 + 10 = 12.0	1.0 + 11 = 12.0	0.5 + 12 = 12.5	0.2 + 13 = 13.2

Warm-Up 15

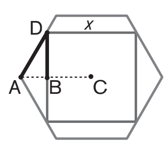
BBB	BGG	BRR	BYY	GRY
GGG	GBB	GRR	GYY	BRY
RRR	RBB	RGG	RYY	BGY
YYY	YBB	YGG	YRR	BGR

211. Let's say the colors are Blue, Green, Red and Yellow. There are 4 ways the vertices all can be the same color and 4 ways they all can be different colors. For any one of the 4 colors that might be repeated, there are 3 choices for the other vertex, so there are $4 \times 3 = 12$ ways to have two vertices the same color. That's a total of $4 + 4 + 12 =$ **20** ways to color the vertices, as summarized here.



212. We are told that the measure of angle MNO is 64 degrees. As shown, this angle is bisected by segment NP, so it follows that the measure of angle ONP is 32 degrees. Triangles NOP and NMP are congruent isosceles triangles, so the measure of angle OPM is equivalent to the sum of the base angles of one of these triangles. The measure of the central angle formed by the radii of circle P that intersect points M and N is $180 - 32 =$ **148** degrees.

213. First, evaluate $f(3) = 3 \times 3 - 7 = 9 - 7 = 2$. Next, evaluate $g(2) = 2^2 - 4 = 4 - 4 = 0$. Finally, evaluate $f(0) = 3 \times 0 - 7 = 0 - 7 =$ **−7**.



214. Let x represent the side length of the square. In the figure shown, then, $BC = BD = x/2$. Since each interior angle of a regular hexagon measures 120 degrees, it follows that the measure of angle DAB is $120 \div 2 = 60$ degrees, making triangle ABD a 30-60-90 right triangle. By the properties of 30-60-90 right triangles, if $x/2$ is the length of the longer leg, then the length of the shorter leg AB is $x/2 \div \sqrt{3} = (x\sqrt{3})/6$. Since $AB + BC = 1$, we have $(x\sqrt{3})/6 + x/2 = 1$. Solving for x yields $x\sqrt{3} + 3x = 6 \rightarrow x(3 + \sqrt{3}) = 6 \rightarrow x = 6/(3 + \sqrt{3})$. To write this answer in simplest radical form, we need to eliminate all radicals from the denominator. This process is called rationalizing the denominator. We do this by multiplying the numerator and denominator of $6/(3 + \sqrt{3})$ by the conjugate $(3 - \sqrt{3})$. We get $(6/(3 + \sqrt{3})) \times ((3 - \sqrt{3})/(3 - \sqrt{3})) \rightarrow 6(3 - \sqrt{3})/6 = 3 - \sqrt{3}$. So the inscribed square has sides of length $\mathbf{3 - \sqrt{3}}$ cm.

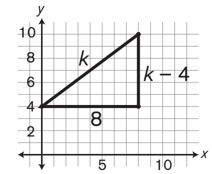
215. Increasing a price by 150% makes it $100 + 150 = 250\%$ of the previous price. Decreasing a price by 75% makes it $100 - 75 = 25\%$ of the previous price. After these two changes the current price is $2.5 \times 0.25 = 0.625 = 5/8$ of the original price. To return to the original price, the current price must be multiplied by the reciprocal of 5/8, which is 8/5. This corresponds to an increase of $8/5 - 1 = 8/5 - 5/5 = 3/5 = 0.6 =$ **60**%.

216. From 10 to 99, there are 90 numbers, and only one of them has the digits 42 adjacent and in that order. That means $P(2) = 1/90$. From 100 to 999, there are 900 numbers. There are 10 numbers of the form 42__ and 9 numbers of the form __42. So $P(3) = (10 + 9)/900 = 19/900$. Therefore, $P(3) - P(2) = 19/900 - 1/90 = 19/900 - 10/900 = 9/900 =$ **1/100**.

217. With an annual percent increase in weight of 10%, that's a total increase after five years of a $1.1^5 - 1 \approx 0.61 =$ **61**%.

218. We want the smallest value for the required $a + b + c$, so let's try splitting up a small square number into a and b and see how that works out. Suppose we try $a + b = 1 + 8 = 9$. Then the two other squares will be $8 - 1 = 7$ apart. The squares 9 and 16 are 7 apart, but that makes $c = 8$, which is a repeated number. If we try $a + b = 2 + 7 = 9$, we need two squares that are $7 - 2 = 5$ apart. Those squares would be 4 and 9, but that makes $c = 2$, which is again a repeat. Since starting with 9 doesn't work, we move on to 16. If we try $a + b = 1 + 15 = 16$, the other two squares would be $15 - 1 = 14$ apart, which doesn't happen. If we try $a + b = 2 + 14 = 16$, the other two squares would be $14 - 2 = 12$ apart. They would be 4 and 16, which lead to another repeat. In some cases, we would need a negative number. For example, if $a + b = 4 + 12 = 16$, then the other two squares would be 8 apart. They would be 1 and 9, but then c would have to be -3. Continuing the search, we eventually come to the solution $a + b = 6 + 19 = 25$. The other two squares must be $19 - 6 = 13$ apart. They would be 36 and 49, so c would have to be 30. Our solution is $a = 6$, $b = 19$ and $c = 30$. The three squares are $6 + 19 = 25$, $6 + 30 = 36$ and $19 + 30 = 49$. The sum $a + b + c$ is $6 + 19 + 30 =$ **55**.

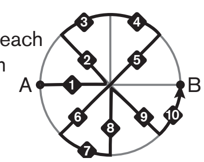
219. The three people had a total of 9 + 15 = 24 oranges. After sharing, they had 8 oranges each. Ronny had to give Lonny 1 orange, and Donny had to give Lonny 7 oranges, so Donny should receive $7 and Ronny should receive $**1**.



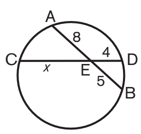
220. The distance from the point $(8, k)$ to the x-axis is k units. Since the distance from $(8, k)$ to $(0, 4)$ also is k units, k is the hypotenuse of a right triangle with legs of lengths 8 and $k - 4$ units, as shown. Using the Pythagorean Theorem, we have $8^2 + (k - 4)^2 = k^2 \rightarrow 64 + k^2 - 8k + 16 = k^2 \rightarrow 8k = 80 \rightarrow k =$ **10**.

Warm-Up 16

221. There are 9 one-digit counting numbers, 90 two-digit, 900 three-digit, etc. If we write the counting numbers in order, the number of digits written will be $1 \times 9 + 2 \times 90 + 3 \times 900 + \ldots$ After writing the first 99 numbers, we will have written $1 \times 9 + 2 \times 90 = 189$ digits, and $3 \times 900 = 2700$, so the 2015th digit written will be among the three-digit numbers. With the first 189 digits written, we need to write $2015 - 189 = 1826$ more digits to get to the 2015th digit. Since $1826 \div 3 = 608\ 2/3$, we need to write all of the first 608 three-digit numbers and the first two digits of the 609th three-digit number. The 609th three-digit number is $609 + 99 = 708$, so the 2015th digit written will be **0**.



222. There are a number of continuous paths, or circuits, that can be drawn from A to B, the longest of which all traverse four arcs, each 1/8 the circumference of the circle in length, and six radii. One such path is shown. The circumference of a circle with diameter 2 cm is 2π cm. The total length of the four arcs is $4/8 = 1/2$ of this length, or π cm. The total length of the six radii is 6 cm. Therefore, the length of the longest path from A to B drawn as a continuous pen stroke is **6 + π** cm.



223. A property of circles is that if two chords of a circle intersect, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord. That means in the circle with intersecting chords, shown here, that $AE \times EB = CE \times ED$. We have $8 \times 5 = 4x$, and $x = 10$. Therefore, the length of the entire chord is $4 + 10 =$ **14** cm.

224. Each of the 12 pentagons has 5 vertices and is completely surrounded by hexagons. There are exactly $12 \times 5 =$ **60** vertices. If we consider the 20 hexagons with 6 vertices each, we get $20 \times 6 = 120$ vertices, but every vertex of a hexagon touches one other hexagon vertex, so there are again $120 \div 2 =$ **60** vertices on a soccer ball.

225. Let r and h represent the radius and height, respectively, of the smallest cone. Then the largest cone has radius $4r$ and height $4h$. To find the volume of each frustum, we will subtract the volume of the smaller cone removed to make the frustum from the volume of the larger cone that contains both the smaller cone and that frustum. The volume of the smallest frustum, then, is $(1/3)\pi(2r)^2(2h) - (1/3)\pi r^2h = (8/3)\pi r^2h - (1/3)\pi r^2h = (7/3)\pi r^2h$. And the volume of the largest frustum is $(1/3)\pi(4r)^2(4h) - (1/3)\pi(3r)^2(3h) = (64/3)\pi r^2h - (27/3)\pi r^2h = (37/3)\pi r^2h$. Therefore, the ratio of the volumes of the smallest and largest frustums is $[(7/3)\pi r^2h]/[(37/3)\pi r^2h] = (7/3)/(37/3) = 7/3 \times 3/37 =$ **7/37**.

226. We are looking for an equation for the line of symmetry between the two points. By definition this line of symmetry is the perpendicular bisector of the segment with endpoints (1, 3) and (7, 11), and it passes through the segment's midpoint. This segment has midpoint $((1 + 7)/2, (3 + 11)/2) = (4, 7)$. This segment has slope $(11 - 3)/(7 - 1) = 8/6 = 4/3$. This is the opposite reciprocal of the slope of the line of symmetry. Thus, the slope of the line of symmetry is $-3/4$. We now have values for x, y and m, which we can use to determine the y-intercept, b. Substituting these values into the equation $y = mx + b$, we get $7 = (-3/4) \times 4 + b$. Solving for b yields $7 = -3 + b$, so $b =$ **10**.

227. The first few terms of the sequence are 1, 2, 3, 6, 12, 24, ... Notice that after the third term, each term is two times the previous term. As the table shows, the 15th term of the sequence is **12,288**. Alternatively, you may recognize that $a_4 = 3 \times 2$, $a_5 = 3 \times 2^2$, $a_6 = 3 \times 2^3$, and conclude that $a_n = 3 \times 2^{n-3}$. So $a_{15} = 3 \times 2^{12} = 3 \times 4096 =$ **12,288**.

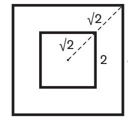
n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a_n	1	2	3	6	12	24	48	96	192	384	768	1536	3072	6144	**12,288**

228. The sum of the digits in each of Abe's four numbers is 12. Since he contributes all three of his digits as the hundreds digit X, he will contribute a total of 1200 to the final sum. Similarly, the sum of the digits in each of Bob's numbers is 15. He will contribute a total of 150 to the final sum, and Charlie will contribute 8. The final sum of the three numbers formed, then, is 1200 + 150 + 8 = **1358**.

229. Since 6^n is an addend six times, we can rewrite this equation as $6^{12} = 6 \times 6 \times 6^n$. Since we need 12 factors of 6, the value of n must be **10**.

230. There is one way Donald can use all 9 passes alone. If he takes a friend once, using all the passes will take 8 days, and there are 8 ways to choose on which day the friend goes. If the friend goes twice, it will take 7 days, and there are ${}_7C_2 = 21$ ways to choose on which 2 days the friend goes. If the friend goes three times, it will take 6 days, and there are ${}_6C_3 = 20$ ways to choose on which 3 days the friend goes. Finally, if the friend goes four times, it will take 5 days, and there are ${}_5C_4 = 5$ ways to choose on which 4 days the friend goes. Therefore, Donald can use the 9 passes in 1 + 8 + 21 + 20 + 5 = **55** ways.

Workout 8



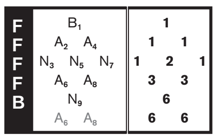
231. The distance from a top vertex of the smaller cube to the nearest vertex of the larger cube is the hypotenuse of a right triangle. One leg is an edge of the smaller cube and has length 2 inches. The figure, which gives the top view of the stacked cubes, shows that the distance from one of the smaller cubes bottom vertices to the closest of the larger cubes top vertices is 1/4 the length of a diagonal of the the larger cube's top face. So the other leg of the right triangle has length $\sqrt{2}$ inches. The distance we are looking for is $\sqrt{(2^2 + (\sqrt{2})^2)} = \sqrt{(4 + 2)} = \mathbf{\sqrt{6}}$ inches.

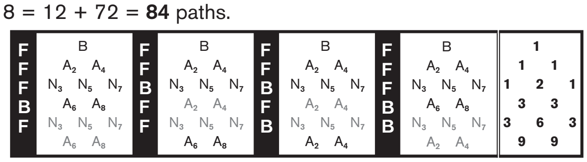
232. The greatest difference would occur if the digits in the thousands place and the hundreds place were transposed. The other two digits are arbitrary since they are the same in both four-digit numbers. So the greatest difference occurs when the 1 and 9 are transposed in \$9100 and \$1900, resulting in a difference of 9100 – 1900 = **\$7200**.

233. Since we are looking for a ratio, we'll consider this to be a unit square of area 1 unit^2. The ratio of the area of the rhombus to the area of the square is just the area of the rhombus over 1. The height of each equilateral triangle of side length 1 unit, then, is $(\sqrt{3})/2$ unit. The horizontal diagonal of the rhombus divides it into two smaller equilateral triangles. Each of these triangles has height $(\sqrt{3})/2 - 1/2 = (\sqrt{3} - 1)/2$ units, and the longer diagonal of the rhombus has length $2 \times (\sqrt{3} - 1)/2 = \sqrt{3} - 1$ units. The shorter diagonal of the rhombus, which is also the base of these smaller triangles, has length $(\sqrt{3} - 1)/2 \div \sqrt{3}/2 = (\sqrt{3} - 1)/\sqrt{3} = (3 - \sqrt{3})/3$ units. Since the area of a rhombus is 1/2 times the product of its diagonals, the rhombus has an area of $(\sqrt{3} - 1) \times ((3 - \sqrt{3})/3) \times (1/2) = ((4\sqrt{3} - 6)/3) \times (1/2) = (2\sqrt{3} - 3)/3$ units^2. Thus, the desired ratio is $(2\sqrt{3} - 3)/3 \approx$ **0.155**.

234. Suppose the nine cells are labeled as shown. All paths begin at B_1 and proceed by moving forward (right or down to a cell of higher number) or backward (left and up to a cell of lower number). Every path that spells *BANANA* has five moves and begins with two forward moves. If forward and backward moves are denoted by F and B, respectively, the possible orders of moves are FFFFB, FFFBF, FFBFF, FFBFB and FFFBB. To count the number of paths for each of these orders, we can create an arrangement of the letters of *BANANA* similar to Pascal's Triangle, as shown below. The arrangement on the left can be used to count the paths for the order of moves FFFFB. When each letter is replaced by the number of paths to that lettered cell, we can see that there are 6 + 6 = 12 paths that spell BANANA for this order of moves. The arrangements on the right can be used to count the number of paths for each of the remaining four orders: FFFBF, FFBFF, FFBFB and FFFBB. For each of these four orders of moves, there are 9 + 9 = 18 paths that spell *BANANA*. That's a total of 12 + 4 × 18 = 12 + 72 = **84** paths.

B_1	A_2	N_3
A_4	N_5	A_6
N_7	A_8	N_9



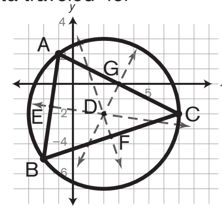


235. Let's suppose that the first three pitches are hits and the next six pitches are not hits. The probability of this happening is $(1/5)^3 \times (4/5)^6$ = 4096/1,953,125. But the hits could actually happen on any three of the nine pitches. So there are ${}_9C_3 = 9!/(6! \times 3!) = 84$ ways in which exactly three hits can occur in nine pitches. That's a probability of 4096/1,953,125 × 84 ≈ **0.18**.

236. A *great circle* of a sphere is the set of points on the sphere's surface that intersect a plane through the circle's center. Every great circle divides the sphere into two hemisphere. Since the three arcs drawn on this sphere are congruent and intersect at right angles, each arc must be part of a great circle of the sphere. In fact, the length of each arc is 1/4 of the circumference of a great circle, which is $2\pi r$. So, $A = 3 \times 2\pi r \div 4 = 3\pi r/2$. The surface area of the triangular region is 1/4 of the surface area of a hemisphere, which is 1/8 of the surface area of the entire sphere. Therefore, $B = 4\pi r^2 \div 8 = \pi r^2/2$. Substituting, we see that the value of $A^2/B = (3\pi r/2)^2/(\pi r^2/2) = (9(\pi)^2 r^2/4)/(\pi r^2/2) = (9(\pi)^2 r^2/4) \times 2/\pi r^2 = 9/2\pi$. So $k = 9/2 =$ **4.5**.

237. We'll use the quad ratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve this equation, given $a = 1$, $b = 7$ and $c = 5$. Substituting, we have $\frac{-7 \pm \sqrt{7^2 - 4 \times 1 \times 5}}{2 \times 1} = \frac{-7 \pm \sqrt{49 - 20}}{2} = \frac{-7 \pm \sqrt{29}}{2}$. This expression is in the form $\frac{l + u\sqrt{m}}{p}$, where $l = -7$, $u = 1$, $m = 29$ and $p = 2$, so $plum = -7 \times 1 \times 29 \times 2 =$ **−406**.

238. The two routes form an isosceles triangle. Drawing the altitude from the apex to the base, creates two 30-60-90 right triangles, each with a long leg of length 12 miles. The hypotenuse of each triangle would have length $12 \times 2/\sqrt{3} = 24/\sqrt{3}$ miles. So the total distance Anita traveled for the detour was $48/\sqrt{3}$ miles. By taking the detour, Anita traveled $48/\sqrt{3} - 24 \approx$ **3.7** miles more than if she'd taken the direct route.



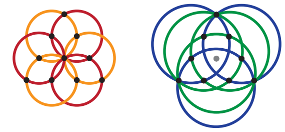
239. First, we graph A(−1, 2), B(−2, −5) and C(7, −2) and connect them to form triangle ABC, as shown. When we graph the perpendicular bisector of each side, they intersect at D(2, −2). This point is the center of the circumscribing circle. From the graph, we can see that the circle has radius CD = 5 units, and its area is $\pi r^2 = \pi \times 5^2 = 25\pi \approx$ **78.5** units^2.

240. Let's assign the variable n to the middle number. Then the three consecutive numbers are $n - 1$, n and $n + 1$. The pairwise products are $(n - 1) \times n$, $n \times (n + 1)$ and $(n - 1) \times (n + 1)$. We are told that the sum of these products is 8111. We have the equation $(n - 1) \times n + n \times (n + 1) + (n - 1) \times (n + 1) = 8111$. Solving for n, we get $n^2 - n + n^2 + n + n^2 - 1 = 8111 \rightarrow 3n^2 - 1 = 8111 \rightarrow 3n^2 = 8112 \rightarrow n^2 = 2704 \rightarrow n = 52$. Therefore, the largest of these consecutive numbers is $n + 1 = 52 + 1 =$ **53**.

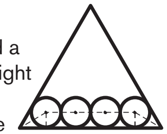
Warm-Up 17

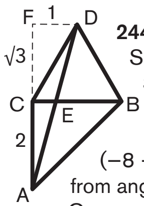
241. The common ratio of this geometric series,$1/\sqrt{2}$, has an absolute value less than 1, so the series converges. Let $S = 4 + 2\sqrt{2} + 2 + \sqrt{2} + \ldots$ It follows that $S/\sqrt{2} = 2\sqrt{2} + 2 + \sqrt{2} + \ldots$ Subtracting these two equations, we get $S - S/\sqrt{2} = (4 + 2\sqrt{2} + 2 + \sqrt{2} + \ldots) - (2\sqrt{2} + 2 + \sqrt{2} + \ldots) \rightarrow S(1 - 1/\sqrt{2}) = 4$. Solving for S, we have $S = 4/(1 - 1/\sqrt{2}) \times (1 + 1/\sqrt{2})/(1 + 1/\sqrt{2}) = 4(1 + 1/\sqrt{2})/(1/2) = 8(1 + 1/\sqrt{2}) = 8 + 8/\sqrt{2}$. In simplest radical form, $S = 8 + 8/\sqrt{2} \times \sqrt{2}/\sqrt{2} = 8 + 4\sqrt{2}$. So $m = 8$, $n = 4$ and $m + n = 8 + 4 =$ **12**. Alternatively, using the formula for the sum of a geometric series with common ratio r, $S = a_1/(1 - r)$, we can substitute to get the same result: $S = 4/(1 - 1/\sqrt{2}) = 8 + 4\sqrt{2}$, and $m + n =$ **12.**

242. There are **12** circles that each intersect exactly 4 of the 10 points. The circles, which occur in two different sizes, are shown here.

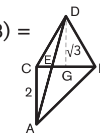


243. Let r be the radius of each circle. When a segment is drawn from the center of the leftmost circle to the closest base vertex, and a radius is drawn to the point of tangency with the base of the equilateral triangle, a 30-60-90 right triangle is created, as shown. This right triangle has sides of length r, $r\sqrt{3}$ and $2r$ cm. The figure also shows a congruent right triangle created with the rightmost circle. The distance between the center of the leftmost circle and the center of the rightmost circle is $6r$. That means the length of the base of the equilateral triangle is equivalent to $r\sqrt{3} + 6r + r\sqrt{3}$. So we can write the equation $r\sqrt{3} + 6r + r\sqrt{3} = 12$. Solving for r, we get $r(6 + 2\sqrt{3}) = 12 \rightarrow r = 12/(6 + 2\sqrt{3}) = 6/(3 + \sqrt{3}) \times (3 - \sqrt{3})/(3 - \sqrt{3}) = 6(3 - \sqrt{3})/6 = \mathbf{3 - \sqrt{3}}$ cm.





244. If we draw segment FD, as shown, parallel to side BC, and then extend segment AC to F, similar triangles ACE and AFD are created. Since triangle BCD is equilateral, it follows that angle DCF, the complement of angle BCD, measures 30 degrees, making triangle CDF a 30-60-90 right triangle. We know that AC = BC = CD = BD = 2 inches; therefore, DF = 1 inch and CF = $\sqrt{3}$ inches. Using proportional reasoning, we have $2/EC = (2 + \sqrt{3})/1 \rightarrow EC \times (2 + \sqrt{3}) = 2 \rightarrow EC = 2/(2 + \sqrt{3}) \times (2 - \sqrt{3})/(2 - \sqrt{3}) = (4 - 2\sqrt{3})/(4 - 3) = 4 - 2\sqrt{3}$ inches. Since $BE = 2 - EC = 2 - (4 - 2\sqrt{3}) = -2 + 2\sqrt{3}$ inches, $BE/EC = (-2 + 2\sqrt{3})/(4 - 2\sqrt{3}) \times (4 + 2\sqrt{3})/(4 + 2\sqrt{3}) = (-8 - 4\sqrt{3} + 8\sqrt{3} + 12)/(16 - 12) = (4 + 4\sqrt{3})/4 = \mathbf{1 + \sqrt{3}}$. Another option is to consider the altitude, of length $\sqrt{3}$ inches, drawn from angle BDC to G on side BC as shown. We have similar triangles DEG and AEC, so $EC/GE = AC/DG \rightarrow EC/(1 - EC) = 2/\sqrt{3}$. Cross-mulitiplying and solving leads to the values for EC, BE and BE/EC.

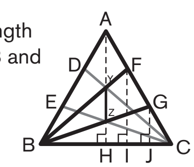


245. Since $(a + b + c) \times (a^2 + b^2 + c^2) = 6 \times 40$, we have $a^3 + ab^2 + ac^2 + a^2b + b^3 + bc^2 + a^2c + b^2c + c^3 = 240$. Rearranging the left-hand side yields $a^3 + b^3 + c^3 + a^2(b + c) + b^2(a + c) + c^2(a + b) = 240$. Since $a^3 + b^3 + c^3 = 200$, it follows that $a^2(b + c) + b^2(a + c) + c^2(a + b) = 240 - 200 = \mathbf{40}$.

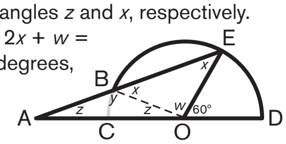
246. Let n be the number of days until the celebration. We want the least value of n such that $(520 + n)/(50 + n) = k$, where k is a positive integer. We note that when $n = 0$, we get $520/50 = 10.4$, and the larger n gets, the closer we get to $k = 1$. Since we want the least value of n, let's start with $k = 10$, then $k = 9$, etc. until we get an integer value for n. Solving $(520 + n)/(50 + n) = k$ for n, we get $520 + n = k(50 + n) \rightarrow 520 + n = 50k + kn \rightarrow kn - n = 520 - 50k \rightarrow n(k - 1) = 520 - 50k \rightarrow n = (520 - 50k)/(k - 1)$. As the table shows, $n = 44$ when $k = 6$. So in **44** days, when Kevin has made 520 + 44 = 564 hats and Devin has made 50 + 44 = 94 hats, they will celebrate since 564 ÷ 94 = 6.

k	***n***
10	(520 – 50 × 10)/(10 – 1) = 20/9
9	(520 – 50 × 9)/(9 – 1) = 70/8
8	(520 – 50 × 8)/(8 – 1) = 120/7
7	(520 – 50 × 7)/(7 – 1) = 170/6 = 85/3
6	(520 – 50 × 6)/(6 – 1) = 220/5 = 44

247. Draw segments AH, FI and GJ perpendicular to side BC, as shown. Since triangle ABC is an equilateral triangle of side length 6 inches, it follows that $AH = 3\sqrt{3}$ inches. FIC and GJC are 30-60-90 right triangles with side lengths, in inches, of 4, 2 and $2\sqrt{3}$ and 2, 1 and $\sqrt{3}$, respectively. Since ZHB and GJB are right triangles that share vertex angle HBZ, they are similar and, by definition, ZH/HB = GJ/JB. We know that HB = 3 inches, and JB = JI + IH + HB = 1 + 1 + 3 = 5 inches. That means $ZH/3 = \sqrt{3}/5 \rightarrow 5 \times ZH = 3 \times \sqrt{3} \rightarrow ZH = 3\sqrt{3}/5$ inches. Also YHB and FIB are right triangles that share vertex angle HBY, so they are similar and YH/HB = FI/IB. Since IB = IH + HB = 1 + 3 = 4 inches, we have $YH/3 = 2\sqrt{3}/4 = \sqrt{3}/2 \rightarrow 2 \times YH = 3 \times \sqrt{3} \rightarrow YH = 3\sqrt{3}/2$ inches. Finally, since YZ + ZH = YH, we have $YZ + 3\sqrt{3}/5 = 3\sqrt{3}/2$ and $YZ = 3\sqrt{3}/2 - 3\sqrt{3}/5 = (15\sqrt{3} - 6\sqrt{3})/10 = 9\sqrt{3}/10$ inches. Therefore, $r = 9$, $s = 10$ and $r + s = 9 + 10 = \mathbf{19}$.



248. In the figure, radius OB divides triangle AOE into isosceles triangles ABO and BOE. We have marked their base angles z and x, respectively. We know that $2z + y = 180$ degrees and $x + y = 180$ degrees. This implies that $x = 2z$ and $2x = 4z$. It also is true that $2x + w = 180$ degrees. Substituting $4z$ for $2x$, we get $4z + w = 180$ degrees. Since angle COE has measure 180 – 60 = 120 degrees, it follows that $z + w = 120$ degrees. Subtracting, we get $(4z + w) - (z + w) = 180 - 120$, so $3z = 60$ degrees and $z = 20$ degrees. Therefore, the measure of angle A is **20** degrees.

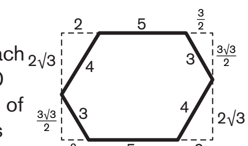


249. First, let's consider three players. If they are positioned so that no two of them stand the same distance apart, then we have a scalene triangle. The longest side of a triangle is opposite the greatest angle, so the player standing at the vertex of the angle with the greatest degree measure can get hit by both of the other players. Whatever the degree measure of the greatest angle, it must be greater than 60 degrees. We now can imagine fitting several triangles together so that one player stands at the vertex of the angle with the greatest measure in each of these non-overlapping triangles. Since 360 ÷ 60 = 6, we can divide 360 degrees into, at most, 5 angles of measure greater than 60 degrees. Thus, the maximum number of times a player can get hit is **5** times.

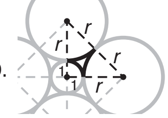
250. If, initially, n members voted for the bill, then $n + 6$ members voted against the bill initially. After the amendments, $n + 9$ members voted for the bill and $(n + 6) - 9 = n - 3$ members voted against it. Since $n + 9 + n - 3 = 2n + 6$ members voted in total and $n + 9$ represents 60% of that total, we can write $n + 9 = 0.6 \times (2n + 6) \rightarrow n + 9 = 1.2n + 3.6$. If we multiply through by 10 to eliminate the decimals, we get $10n + 90 = 12n + 36$ and $2n = 54$. Recall that $2n + 6$ members voted, so, in total, there are 54 + 6 = **60** members.

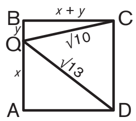
Warm-Up 18

251. Each exterior angle of this equiangular hexagon has measure 360 ÷ 6 = 60 degrees. The interior supplement to each exterior angle has measure 180 – 60 = 120 degrees. If we inscribe the hexagon in a rectangle, as shown, four 30-60-90 right triangles are created. The area of the hexagon is the difference between the area of the rectangle and the total area of the four right triangles. Each of the triangles with hypotenuse of length 4 units has legs with lengths 2 units and $2\sqrt{3}$ units and has an area of $1/2 \times 2 \times 2\sqrt{3} = 2\sqrt{3}$ units^2. Each of the triangles with hypotenuse of length 3 units has legs with lengths 3/2 units and $3\sqrt{3}/2$ units and has an area of $1/2 \times 3/2 \times 3\sqrt{3}/2 = 9\sqrt{3}/8$ units^2. The combined area of the four triangles is $(2 \times 2\sqrt{3}) + (2 \times 9\sqrt{3}/8) = 4\sqrt{3} + 9\sqrt{3}/4 = (16\sqrt{3} + 9\sqrt{3})/4 = 25\sqrt{3}/4$ units^2. The rectangle has side lengths $2\sqrt{3} + 3\sqrt{3}/2 = 7\sqrt{3}/2$ units and $2 + 5 + 3/2 = 17/2$ units and has an area of $7\sqrt{3}/2 \times 17/2 = 119\sqrt{3}/4$ units^2. Therefore, the area of the hexagon is $119\sqrt{3}/4 - 25\sqrt{3}/4 = 94\sqrt{3}/4 = \mathbf{47\sqrt{3}/2}$ units^2.



252. As shown, by drawing segments that connect the centers of two large circles, of radius r, and the center circle, of radius 1 mm, we create an isosceles right triangle. So, $(r + 1)^2 + (r + 1)^2 = (2r)^2 \rightarrow r^2 + 2r + 1 + r^2 + 2r + 1 = 4r^2 \rightarrow 2r^2 - 4r - 2 = 0 \rightarrow r^2 - 2r - 1 = 0$. Substituting $a = 1$, $b = -2$ and $c = -1$ into the quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ yields $r = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times (-1)}}{2 \times 1} = \frac{2 \pm \sqrt{4 + 4}}{2} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$. Since we are interested only in the positive soluton, the length of the radius of each large circle is **1 + √2** mm.

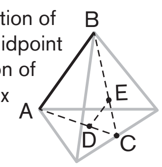




253. Let x = AQ and y = QB in square ABCD, shown here. So the square has side length $x + y$, and its area is $(x + y)^2$. Using the Pythagorean Theorem, we can write the following equations for triangles QAD and QBC, respectively: $x^2 + (x + y)^2 = 13$ and $y^2 + (x + y)^2 = 10$. Subtracting these two equations, we get $x^2 - y^2 = 3$ and $y^2 = x^2 - 3$. Substituting this back into the first equation and solving yields $x^2 + (x + \sqrt{(x^2 - 3)})^2 = 13 \rightarrow x^2 + x^2 + 2x\sqrt{(x^2 - 3)} + x^2 - 3 = 13 \rightarrow 3x^2 + 2x\sqrt{(x^2 - 3)} = 16 \rightarrow 2x\sqrt{(x^2 - 3)} = 16 - 3x^2$. Squaring both sides, we get $4x^2(x^2 - 3) = 256 - 96x^2 + 9x^4 \rightarrow 4x^4 - 12x^2 = 256 - 96x^2 + 9x^4 \rightarrow 5x^4 - 84x^2 + 256 = 0$. Factoring the quadratic expression, we get $(5x^2 - 64)(x^2 - 4) = 0$. So $5x^2 - 64 = 0$ and $x^2 = 64/5$, or $x^2 - 4 = 0$ and $x^2 = 4$. If $x^2 = 64/5$, then $64/5 + (x + y)^2 = 13$ and we get $(x + y)^2 = 13 - 64/5 = 1/5$ unit^2 as the area of the square. While this solution satisfies the equation, it leads to a negative value for y, which cannot be. If $x^2 = 4$, then $4 + (x + y)^2 = 13$ and the area of the square is $(x + y)^2 =$ **9** units^2.

254. If $45_a = 54_b$, then $4a + 5 = 5b + 4$, or $4a + 1 = 5b$. The greatest digit in 45_a and 54_b is 5, so we need $a > 5$ and $b > 5$, since the digits 0 through 5 are not used in base 2, base 3, base 4 or base 5. We should look for the least multiple of 5 that is greater than 5 × 5 = 25 and also is 1 more than a multiple of 4. The number 45 meets those criteria, in which case $b = 45 \div 5 = 9$ and $a = 44 \div 4 = 11$. In base 10, $45_{11} = 4 \times 11 + 5 = 49$ and $54_9 = 5 \times 9 + 4 = 49$. Therefore, the smallest possible value of $a + b$ is 11 + 9 = **20**.

255. We can use the laws of exponents to rewrite the expression. First, we rewrite $\sqrt[4]{2}$ as $2^{\frac{1}{4}}$. Then $2 \times 2^{\frac{1}{4}} = 2^{\frac{5}{4}}$. The cube root of this expression can be written as $(2^{\frac{5}{4}})^{\frac{1}{3}} = 2^{\frac{5}{12}}$ and $2 \times 2^{\frac{5}{12}} = 2^{\frac{17}{12}}$. Finally, the square root of $2^{\frac{17}{12}}$ can be written as $(2^{\frac{17}{12}})^{\frac{1}{2}} = 2^{\frac{17}{24}}$. Thus, $c =$ **17/24**.

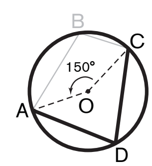


256. Each of the four faces of the original tetrahedron is an equilateral triangle. The center of an equilateral triangle is the point of intersection of its three medians. The distance from the midpoint of a side of the triangle to the the center of the triangle is 1/3 of the distance from the midpoint of a side to the opposite vertex, the distance we typically refer to as the altitude of the triangle. In the figure, triangle ABC is a cross-section of the original tetrahedron, and triangles ABC and DEC are similar isosceles triangles with EC/BC = 1/3. Since segment DE is one of the six edges of the smaller regular tetrahedron created when the centers of the four faces of the original regular tetrahedron are connected, the ratio of the volume of the smaller tetrahedron to the volume of the original one is $(1/3)^3 =$ **1/27**.

257. Let's use a "Daniel-hour" as our unit of work. Daniel does 1/3 of a Daniel-hour from 9:00 to 9:20 on his own. From 9:20 to 10:00, Daniel does 2/3 of a Daniel-hour and Yeong does 4/3 of a Daniel-hour. The whole room required 1/3 + 2/3 + 4/3 = 7/3, or 2 1/3 Daniel-hours of work. From 9:00 to 9:30, Daniel did 1/2 of a Daniel-hour, and from 9:20 to 9:30 Yeong did 1/3 of a Daniel-hour. By 9:30, 1/2 + 1/3 = 5/6 Daniel-hour of work had been done. So by 9:30, (5/6) ÷ (7/3) = 5/6 × 3/7 = **5/14** of the room had been painted.

258. There are ${}_{16}C_3 = 16!/(13! \times 3!) = (16 \times 15 \times 14) \div (3 \times 2 \times 1) = 560$ ways to choose three dots. There are 4 ways to choose 3 dots in any one of the 4 rows, 4 columns and 2 diagonals. That's 4 × (4 + 4 + 2) = 40 sets of three collinear dots. There are also 4 shorter diagonals that have just 3 dots, for a total of 44 possible sets of three collinear dots. The probability that three dots chosen at random are collinear is 44/560 = **11/140**.

259. On a balance scale, we can place some of the weights with the object being weighed. This has the effect of subtracting from the value of the weights placed on the other side of the scale. The result is that we can get all positive integer values by using powers of 3. To weigh any whole number of pounds up to 40, we need just four weights that are 1, 3, 9 and 27 pounds. The greatest weight is **27** pounds.



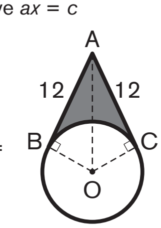
260. Together, arcs AB and BC form arc AC with measure 100 + 50 = 150 degrees. A property of circles is that an inscribed angle has a degree measure that is half that of the arc it intercepts. The measure of angle ADC, then, is 150 ÷ 2 = **75** degrees.

Workout 9

$a^2 + d^2$	$b^2 + c^2$
$1^2 + 2^2 = 5$	×
$2^2 + 3^2 = 13$	×
$3^2 + 4^2 = 25$	×
$4^2 + 5^2 = 41$	×
$5^2 + 6^2 = 61$	×
$6^2 + 7^2 = 85$	$9^2 + 2^2 = 85$

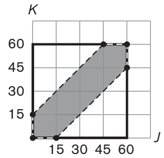
261. We are looking for a number that can be written as the sum of two squares in two different ways. For example, $7^2 + 4^2 = 8^2 + 1^2 = 65$, and $|7 - 3| = 4$. We are asked to find the least possible absolute difference between a and d. Since $a \neq d$, the smallest this difference can be is 1. We check the squares of consecutive integers, beginning with $1^2 + 2^2$ and continuing until we find a sum that can be expressed as the sum of the squares of two different integers. As the table shows, we find such a sum when we get to $6^2 + 7^2 = 9^2 + 2^2 = 85$ and $|d - a| = |7 - 6| =$ **1**.

262. The y-intercept occurs when $x = 0$, so we solve $by = c$ for y and get $y = c/b$. Similarly, the x-intercept occurs when $y = 0$, so we solve $ax = c$ for x and get $x = c/a$. The triangle has vertices (0, 0), (c/a, 0) and (0, c/b). The area of the triangle is 1/2 × c/a × c/b = **$c^2/(2ab)$**.



263. A property of circles is that tangent segments that share an endpoint not on the circle are congruent. So AB = AC = 12 inches. Furthermore, a tangent to a circle is perpendicular to a radius drawn to the point of tangency. In the figure, segment OA bisects angle BOC and forms 30-60-90 right triangles AOB and AOC. It follows, by properties of 30-60-90 right triangles, that OB = OC = 12 ÷ √3 = 4√3 inches. Triangles AOB and AOC, combined, have area 12 × 4√3 = 48√3 in^2. The area of sector BOC is 120/360 = 1/3 the area of the circle, or 1/3 × π × (4√3)^2 = 1/3 × π × 48 = 16π in^2. The area of the shaded region, then, is 48√3 − 16π ≈ **32.9** in^2.

264. In one extreme scenario, Khalid arrives at the dining hall at 8:00 and leaves at 8:15. Jack and Khalid will miss one another if Jack arrives at any time from 8:15 to 9:00, which is 3/4 of the possible arrival times. In another scenario, Khalid arrives just before 8:15. In this case, they will miss seeing each other unless Jack arrives any time between 8:00 and 8:15 since Jack would still be there when Khalid arrives just before 8:15. So, in this case, they will miss seeing each other if Jack arrives at a time from 8:30 to 9:00, which is 1/2 the available arrival times. At the other extreme is the scenario in which Khalid arrives at 9:00 and stays to 9:15. If Jack arrives any time from 8:00 to 8:45, he would not see Khalid. In the end, the probability that the two boys don't meet is 3/4 × 3/4 = 9/16, so the probability that they do meet is 1 − 9/16 = **7/16**.



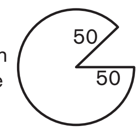
The figure shows another way to look at this kind of problem. (This is geometric probability.) The side lengths of the square are the possible arrival times, with the grid showing 15-minute intervals. The vertical axis could be Khalid's arrival time, and the horizontal axis could be Jack's arrival time. The shaded band represents the scenarios in which the two boys meet. The area of this shaded region is **7/16**.

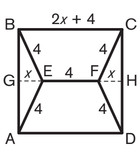
265. There are five ways for Colby to get 20 points or more. Colby could make all five 2-point and all five 3-point shots for 25 points. There is a $0.7^5 \times 0.4^5$ probability of this occurring. Colby could make four 2-point and all five 3-point shots for 23 points. There is a $5 \times 0.3 \times 0.7^4 \times 0.4^5$ probability of this occurring. Colby could make three 2-point and all five 3-point shots for 21 points. There is a $10 \times 0.3^2 \times 0.7^3 \times 0.4^5$ probability of this occurring. Colby could make all five 2-point and four 3-point shots for 22 points. There is a $0.7^5 \times 5 \times 0.6 \times 0.4^4$ probability of this occurring. Finally, Colby could make four 2-point and four 3-point shots for 20 points. There is a $5 \times 0.3 \times 0.7^4 \times 5 \times 0.6 \times 0.4^4$ probability of this occurring. The sum of these five probabilities is $(0.7^5 \times 0.4^5) + (5 \times 0.3 \times 0.7^4 \times 0.4^5) + (0.7^5 \times 5 \times 0.6 \times 0.4^4) + (0.7^5 \times 5 \times 0.6 \times 0.4^4) + (5 \times 0.3 \times 0.7^4 \times 5 \times 0.6 \times 0.4^4) \approx$ **0.049**.

266. When each cylinder is glued to a face of the cube, the area of the square face that is covered by the circular base of the cylinder is exactly replaced by the area of the other circular base of the cylinder. The only new area that is created is the rectangular band that wraps around each cylinder. Each of these bands is a rectangle with sides equal to the height of the cylinder and the circumference of the cylinder. The surface area of the resulting solid is thus $6 \times 4^2 + 6 \times 4 \times \pi \times 2^2 = 96 + 96\pi \approx$ **397.6** in^2.

267. Let S, T and P be the numbers of fish in Sam's, Taylor's and Pat's fish tanks. Then $S = (5/4)T$ and $P = (31/25)S$, so $P = 31/25 \times (5/4)T = (31/20)T$. Since the denominator of this fraction is 20, Taylor had to have a multiple of 20 fish. If Taylor had exactly 20 fish, then Sam had 25 fish and Pat had 31 fish. The minimum combined number of fish is 20 + 25 + 31 = **76** fish.

268. The radius of the lateral surface shown is $\sqrt{(48^2 + 14^2)} = \sqrt{(2304 + 196)} = \sqrt{2500} = 50$ feet. The circumference of the base with a radius of 48 feet is 96π feet. This is 96% of the 100π feet that a full circle with a radius of 50 feet would have. The missing 4% of the circumference corresponds to a central angle with a measure of 0.04 × 360 = **14.4** degrees.





269. Let's say the lengths GE and FH are x cm. Then the side length of the square is $2x + 4$ cm, and the long leg of right triangle BGE is $x + 2$ cm. To solve for x, we'll first use the Pythagorean Theorem to get $x^2 + (x + 2)^2 = 4^2 \rightarrow x^2 + x^2 + 4x + 4 = 16 \rightarrow 2x^2 + 4x - 12 = 0 \rightarrow x^2 + 2x - 6 = 0$. Then substituting $a = 1$, $b = 2$ and $c = -6$ into the quadratic formula, we get $x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-6)}}{2 \times 1} = \frac{-2 \pm \sqrt{4 + 24}}{2} = \frac{-2 \pm \sqrt{28}}{2} = \frac{-2 \pm 2\sqrt{7}}{2} = -1 \pm \sqrt{7}$. We choose the positive value $x = -1 + \sqrt{7}$ cm, since it is a measure of length. Then GB = $x + 2$, so it's $1 + \sqrt{7}$ cm. The area of triangle ABE is twice the area of triangle GBE, so it's equal to the product $(-1 + \sqrt{7}) \times (1 + \sqrt{7})$, which is 7 − 1 = **6** cm^2.

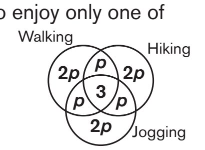
270. The table shows some possible pairs of factors, with the sum of digits column labeled S.O.D. The greatest product of two five-digit numbers whose digits do not repeat is 96420 × 87531 = 8,439,739,020. The sum of its digits is **45**.

a	b	Product	S.O.D
96420	87531	8,439,739,020	45
96421	87530	8,439,730,130	38
96430	87521	8,439,650,030	38
96431	87520	8,439,641,120	38
96520	87431	8,438,840,120	38
96521	87430	8,438,831,030	38
97531	86420	8,428,629,020	41
96543	87210	8,419,515,030	36

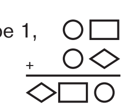
Logic Stretch

271. From Everett's statement, we know that Celia's and Desi's numbers could be 1 and 5, 2 and 4, 4 and 2, or 5 and 1. Then from Celia's statement, it is clear that Everett's number is either 2 or 5. So the sum of the numbers that Everett could have on his hat is 2 + 5 = **7**.

272. Of the 30 people who enjoy one or more of the activities walking, hiking and jogging, three people enjoy doing all three. To denote this, we've placed a "3" at the intersection of all three activities in the Venn diagram shown. Let p represent the number of people who enjoy two of the activities. This is denoted by the "p" at each of the three intersections of exactly two of the activities. The number of of people who enjoy only one of the activities, then, would be represented by $2p$ and denoted by the "$2p$" placed where walking, hiking and jogging intersect with no other activity. It follows that $p + p + p + 2p + 2p + 2p + 3 = 30 \rightarrow 3p + 6p + 3 = 30 \rightarrow 9p + 3 = 30$. Solving for p, we see that $9p = 27$, so $p = 3$. The total number of people who enjoy jogging is $p + p + 2p + 3 = 4p + 3$. Substituting and solving for p yields a total of 4 × 3 + 3 = 12 + 3 = **15** people who enjoy jogging.



273. The subtraction problem can be rewritten as the addition problem shown here. Notice that the hundreds digit of the sum has to be 1, so ◇ = 1. Therefore, the circle must be 5 or more so that the sum of two circles results in a 1 being carried to the hundreds place. Substituting 5, 6, 7 or 8 for each circle leads to a contradiction in the value of the rectangle. Only ○ = 9 works, which means that □ = 8, and □◇ ÷ ○ = 81 ÷ 9 = **9**.



274. Box 3 is incorrectly labeled “Tennis Balls & Baseballs.” Since a baseball was removed from this box, it must contain only baseballs. So the appropriate label for Box 3 is “Baseballs.” Box 1 contains only tennis balls or tennis balls and baseballs. The same is true for Box 2. Since the “Tennis Balls” label on Box 1 is incorrect, Box 1 must contain tennis balls and baseballs. Therefore, the box containing only tennis balls is Box **2**.

275. If no pages were missing, the page numbers would add up to $(50 \times 51)/2 = 1275$. Therefore, the numbers on the missing pages must have a sum of $1275 - 1242 = 33$. Let n and $n + 1$ be the page numbers on both sides of a single sheet. We know that n is odd and $n + 1$ is even. The two consecutive numbers with a sum of 33 are 16 and 17, and they cannot be on opposite sides of a single sheet. So no single missing sheet can account for the difference of 33. Perhaps two missing sheets could account for the difference of 33. But the sum of their page numbers would be an even number, so it is not possible for two missing sheets to account for the dfference of 33. However, it is possible for three missing sheets to account for this difference. If the sheet numbered 1 and 2 and the sheet numbered 3 and 4 were missing, their sum would be $1 + 2 + 3 + 4 = 10$. But we would still need to account for the remainder of the difference, which is $33 - 10 = 23$. Since $11 + 12 = 23$, if the sheet numbered 11 and 12 were missing, that would work. Therefore, the greatest page number that could be missing is **12**.

276. Suppose $T + E > 10$ and $T = 3$. Then $3 + E = 10 + H$ and $E + H + 1 = 3$ or $E + H + 1 = 13$. If $3 + E = 10 + H$, $E = 7 + H$, and if $E + H + 1 = 3$, we get $E + H = 2 \rightarrow 7 + H + H = 2 \rightarrow 7 + 2H = 2 \rightarrow 2H = -5$. If $E + H + 1 = 13$, we get $E + H = 12 \rightarrow 7 + H + H = 12 \rightarrow 7 + 2H = 12 \rightarrow 2H = 5$. There is no single-digit number H for which $2H = -5$ or $2H = 5$. Let’s suppose, then, that $T + E < 10$ and $T = 3$. Then $3 + E = H$ and $E + H = 3$ or $E + H = 13$. Substituting $E = H - 3$ in $E + H = 3$ or $E + H = 13$, we get $2H = 6$ or $2H = 16$. But if $T = 3$, then $H \neq 3$. Therefore, $2H = 16$ and $H = 8$. It follows that $E = H - 3 = 8 - 3 = 5$. Since $G + 4 = MA$, it must be true that $M = 1$ and G is 6, 7 or 9. If $G = 9$, that results in $A = 3$, but $T= 3$. So $G \neq 9$. If $G = 7$, that results in $A = 1$, but $M = 1$. So $G \neq 7$. Therefore, $G = 6$ and the value of the four-digit number MATH is **1038**.

277. Since we will be counting the number of moves Porscha and Micah make, let’s label the steps of this particular staircase beginning at the upper landing as steps A through K. After their first three moves, Porscha will be on step K and Micah will be on step A. After Porscha and Micah each make their respective three-move sequence three more times, they will be on step H and step D, respectively, separated by three steps. After Porscha makes her next two moves (up two steps), she will be on step F. After Micah makes his next two moves (down two steps), he also will be on step F. They each will have made a total of $4 \times 3 + 2 = 14$ moves when they both reach step F, $3 \times 13 =$ **39** seconds after making their first moves. The chart shows Porscha’s and Micah’s progress up and down the steps until they reach the same step.

Moves	Time	Porscha	Micah
		Lower Landing	Upper Landing
1	00:00	K	A
2	00:03	J	B
3	00:06	K	A
4	00:09	J	B
5	00:12	I	C
6	00:15	J	B
7	00:18	I	C
8	00:21	H	D
9	00:24	I	C
10	00:27	H	D
11	00:30	G	E
12	00:33	H	D
13	00:36	G	E
14	00:39	F	F

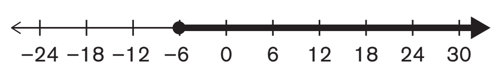
278. If 3D6,D92 is divisible by 11, then the absolute difference between $3 + 6 + 9 = 18$ and $D + D + 2 = 2D + 2$ must be divisible by 11. The least absolute difference that would be divisible by 11 is 0, in which case $18 - (2D+ 2) = 0 \rightarrow 16 - 2D = 0 \rightarrow 2D = 16 \rightarrow D =$ **8**.

279. If the middle card, showing the heart, does not have a 2 printed on the other side, there are $4 \times 3 = 12$ ways for the four suits to be distributed on the other side of the cards showing the number 2. In this scenario, the number printed on the other side of the middle card could be 1, 3 or 4. Therefore, there are $3 \times 12 = 36$ different ways for this scenario to occur, and for half of these ways, or 18 ways, there is a heart printed on the other side of one of the cards showing the number 2. If, on the other hand, the middle card does have the number 2 printed on the other side, there are $3 \times 2 = 6$ ways the other three suits can be distributed on the other side of the cards showing the number 2. But none of these scenarios has a heart printed on the other side of one of the cards showing the number 2. So of the $36 + 6 = 42$ possible scenarios, the probability that one of the cards showing the number 2 has a heart printed on the other side is 18/42 = **3/7**.

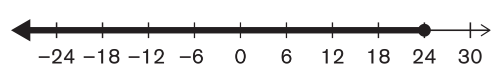
280. Let’s first determine if there is a solution that doesn’t involve borrowing. In that case, $C - A = 1$, $A - B = 6$ and $B - C = 2$. Rewriting the first equation as $C - 1 = A$ and substituting for A into the second equation yields $C - 1 - B = 6 \rightarrow C - B = 7$, which contradicts the third equation. So the solution must involve borrowing. Now, borrowing from the tens place, we get $C - A = 1$, $A - B - 1 = 6$ and $B - C + 10 = 2$. Eliminating A by adding the first two equations gives us $C - B = 8$, which agrees with the third equation. Rewriting the equations as $C = A + 1$, $A = B + 7$ and $C = B + 8$, we can see that A is smallest when B is as small as it can be. From the third equation, $B = 0$ and $C = 8$, which means $A = 7$ and ABC is 708. A is largest when B is as large as it can be. From the third equation, $B = 1$ and $C = 9$, which means $A = 8$ and ABC is 819. These are the only two solutions with borrowing in the tens place. Borrowing from the hundreds place gives the equations $C - A - 1 = 1$, $A - B + 10 = 6$ and $B - C = 2$. Solving as we did previously, we see that the the least and greatest values of ABC are 153 and 597, respectively. Borrowing from both the hundreds place and the tens place yields no additional solutions. Therefore, the sum of the least and greatest possible values of ABC is $153 + 819 =$ **972**.

Solving Inequalities Stretch

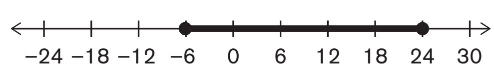
281. $3 - x/3 \le 5 \rightarrow 3 - 5 \le x/3 \rightarrow -2 \le x/3 \rightarrow \boldsymbol{x \ge -6}$.



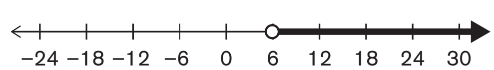
282. $3 - x/3 \ge -5 \rightarrow 3 + 5 \ge x/3 \rightarrow 8 \ge x/3 \rightarrow \boldsymbol{x \le 24}$.



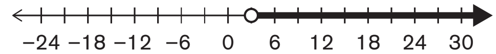
283. Since this is an absolute value inequality, we’re looking for all x such that $3 - x/3 \le 5$ and $3 - x/3 \ge -5$. Notice that we’ve already solved both of these inequalities in #281 and #282. Thus, our answer is $\boldsymbol{-6 \le x \le 24}$. Notice that the graph of this solution is the intersection of the graphed solutions for #281 and #282.



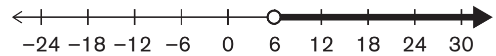
284. $3 - x/3 < x - 5 \rightarrow 3 + 5 < x + x/3 \rightarrow 8 < (4/3)x \rightarrow \boldsymbol{x > 6}$.



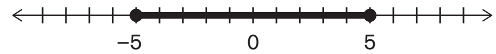
285. $3 - x/3 > 5 - x \rightarrow x - x/3 > 5 - 3 \rightarrow (2/3)x > 2 \rightarrow$ $\boldsymbol{x > 3}$.



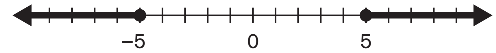
286. Since this is an absolute value inequality, we're looking for all x such that $3 - x/3 < x - 5$ and $3 - x/3 > 5 - x$. Again, we've already solved both of these inequalities, in #284 and #285. Thus, our answer is $\boldsymbol{x > 6}$ since this includes all x that are both greater than 3 and greater than 6. Notice that the graph of this solution is the intersection of the graphed solutions for #284 and #285.



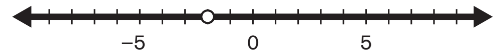
287. $x^2 \le 25 \rightarrow x \le \sqrt{25} \rightarrow x \le \sqrt{25}$ and $x \ge -\sqrt{25} \rightarrow$ $\boldsymbol{-5 \le x \le 5}$.



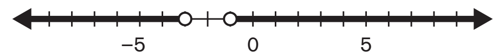
288. $x^2 \ge 25 \rightarrow x \ge \sqrt{25} \rightarrow x \ge \sqrt{25}$ or $x \le -\sqrt{25} \rightarrow$ $\boldsymbol{x \le -5}$ **or** $\boldsymbol{x \ge 5}$.



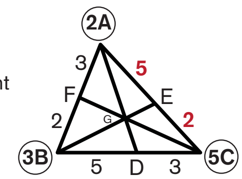
289. $x^2 + 4x - 4 > -8 \rightarrow x^2 + 4x + 4 > 0 \rightarrow (x + 2)^2 > 0$. When we take the square root of both sides, we see that the solution uses the positive root and $x + 2 > 0$, and the solution uses the negative root and $x + 2 < 0$. Thus, $\boldsymbol{x > -2}$ **or** $\boldsymbol{x < -2}$, meaning $\boldsymbol{x \ne -2}$.



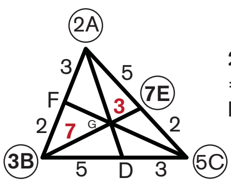
290. $x^2 + 4x - 4 > -7 \rightarrow x^2 + 4x + 3 > 0 \rightarrow (x + 3)(x + 1) > 0$. In order for this product to be positive (> 0), either both factors are positive, or both factors are negative. If $x < -3$, both factors will be negative. If $x > -1$, both factors will be positive. This quadratic inequality is satisfied for $\boldsymbol{x < -3}$ **or** $\boldsymbol{x > -1}$.



Mass Point Geometry Stretch

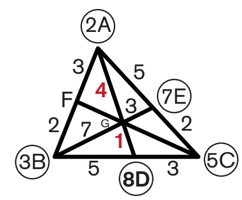


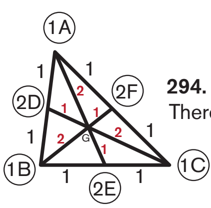
291. Since side AB balances on point F and AF:BF = 3:2, we need mA:mB = 2:3. Similarly, since side BC balances on point D and BD:CD = 5:3, we need mB:mC = 3:5. We can maintain balance with mass points 2A, 3B and 5C. The ratio of mass points 2A and 5C is 2:5, so to maintain balance, the ratio of AE to CE, as a common fraction, must be **5/2**.



292. From the previous problem, we have mass points 2A, 3B and 5C. Side AC balances on point E, so it follows that 2A + 5C = (2 + 5)E = 7E. With mass points 3B and 7E, the masses at point B and point E are in the ratio 3:7. Thus, the ratio of BG to EG, as a common fraction, is **7/3** .

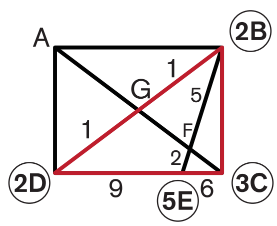
293. From the previous problems, we have mass points 2A, 3B and 5C. Side BC balances on point D, so 3B + 5C = (3 + 5)D = 8D. Cevian AD, which balances on point G, has mass points 8D and 2A, in the ratio 8:2 = 4:1. Therefore, the ratio of DG to AG, as a common fraction, must be **1/4**.

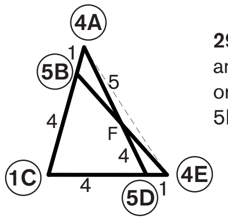




294. Since the medians divide the sides of the triangle in half, we can assign a mass of 1 at each vertex of the triangle as shown. Therefore, each median is divided into two segments with lengths in the ratio **1/2**.

295. Using the Pythagorean Theorem, we can determine that $BC^2 + DC^2 = BD^2 \rightarrow 8^2 + DC^2 = 17^2 \rightarrow DC^2 = 289 - 64 \rightarrow DC^2 = 225 \rightarrow DC = 15$. Similarly, we have $BC^2 + CE^2 = BE^2 \rightarrow 8^2 + CE^2 = 10^2 \rightarrow CE^2 = 100 - 64 \rightarrow CE^2 = 36 \rightarrow CE = 6$. Since DE + EC = DC, we have DE + 6 = 15 → DE = 15 − 6 = 9. Now we will apply mass point geometry to triangle BCD, shown here. Side CD balances on point E and CE:DE = 6:9 = 2:3. With mass points 3C and 2D, balance is maintained, and we add 3C + 2D to get mass point 5E. Since diagonal AC bisects hypotenuse BD at point G, we need mass point 2B to maintain the 1:1 ratio of BG:DG. So cevian BE, which balances on F, has mass points 2B and 5E in the ratio 2:5. For similar triangles CFE and AFB, the ratio, written as a common fraction, of the lengths of corresponding sides is 2/5. It follows, then, that their areas are in the ratio $2^2/5^2 =$ **4/25**.





296. If we draw segment AE, the result is triangle ACE with cevians AD and BE, as shown. Side AC balances on point B, and AB:BC = 1:4. With mass points 4A and 1C, this balance is maintained, and we add 4A + 1C to get mass point 5B. In order for cevian AD to remain balanced on point F, we need mass point 5D. Side CE balances on point D, and 1C + *m*E = 5D, so it takes mass point 4E to maintain this balance. Therefore, the ratio of DE to CD must be **1/4**, as a common fraction.

297. If we remove segment AH, we are left with Figure 1.1, shown here. It's the figure from the previous problem, reflected horizontally. So, from the previous problem, we have mass points 4A, 5B, 1C, 5D and 4E. Also, we know that DE:CD = 1:4. Now, if, instead, we remove segment AD, we are left with the Figure 1.2, shown here. Since AG:GH = 3:5 and we already have mass point 4A, we have 4 × 3 = *m*H × 5 → *m*H = (12/5)H. We can multiply by 5 to get all integer masses. Now we have mass points 20A, 25B, 5C and 12H. Since *m*E + 5C = 12H, it follows that *m*E = 7E and CH:EH = 7:5.So, DE is 1/5 of CE, CH is 7/12 of CE and DH = 1 − (1/5) − (7/12) = 13/60 of CE. So, CH:DH:DE = 7/12:13/60:1/5 = 35:13:12. Thus, *x* = 35, *y* = 13, *z* = 12 and *x* + *y* + *z* = 35 + 13 + 12 = **60**.

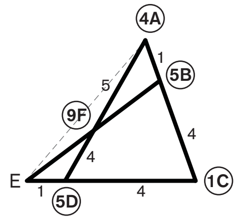


Figure 1.1

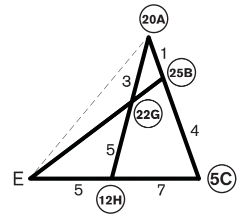


Figure 1.2

298. If we remove segment AE, the result is triangle ABC with cevians BF and CD that intersect at I (Figure 2.1). We have mass points 2A, 1B , 3C, 5F and 6I, and BI:FI = 5:1. Since BI is 5/6 of BF and CF is 2/5 of AC, the area of triangle BCI is (5/6) × (2/5) =1/3 the area of triangle ABC. Similarly, if we remove segment BF, the result is triangle ABC with cevians AE and CD that intersect at G (Figure 2.2). We have mass points 6A, 3B, 1C, 9D, 4E and 10G, and AG:EG = 2:3. Since AG is 2/5 of AE and CE is 3/4 of BC, the area of triangle ACG is (2/5) × (3/4) = 3/10 the area of triangle ABC. Finally, if we remove segment CD, the result is triangle ABC with cevians AE and BF that intersect at H (Figure 2.3). We have mass points 2A, 9B, 3C, 12E, 5F and 14H, and AH:EH = 6:1. Since AH is 6/7 of AE and BE is 1/4 of BC, the area of triangle ABH is (6/7) × (1/4) = 3/14 the area of triangle ABC. The area of triangle ABC is the sum of the areas of triangles GHI, BCI, ACG and ABH. Together, triangles BCI, ACG and ABH are 1/3 + 3/10 + 3/14 = (70 + 63 + 45)/210 = 178/210 = 89/105 the area of triangle ABC. Therefore, the ratio of the areas of triangles GHI and ABC is 1 − 89/105 = **16/105**.

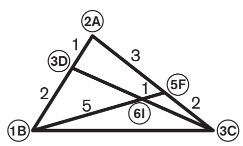


Figure 2.1

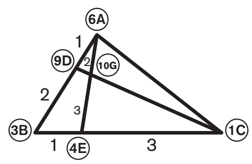


Figure 2.2

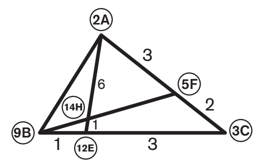


Figure 2.3

299. Since EF is not a cevian, first we will consider triangle ABC with cevians AD and CF (Figure 3.1), and then we'll consider triangle ABC with cevians AD and BE (Figure 3.2). For the triangle in Figure 3.1, *m*A:*m*B = 4:5 and *m*B:*m*C = 2:3, so in order to get integer masses, we need the mass at point B to be a multiple of both 2 and 5. We'll use mass point 10B, which leads to mass points 8A, 15C and 25D. For the triangle in Figure 3.2, we'll keep mass points 10B, 15C and 25D. To maintain the 2:1 ratio of *m*C to *m*A, we'll use mass point 30A. Now we can balance the figure on point G by assigning a mass of 30 + 8 = 38 to point A. It follows, then, that the ratio of AG to DG, as a common fraction, is **25/38**.

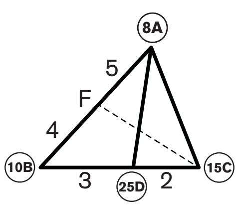


Figure 3.1

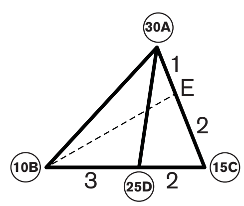


Figure 3.2

300. From the previous problem, the triangle, with cevian CF ,has mass points 8A and 10B. As shown in Figure 4.1, with mass point 18F, we can maintain the balance of side AB on point F. Similarly, the triangle, with cevian BE, has mass points 30A and 15C. As shown in Figure 4.2, with mass point 45E, we can maintain the balance of side AC on point E. Therefore, the ratio of EG to FG, as a common fraction, is 18/45 = **2/5**.

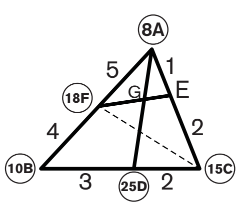


Figure 4.1

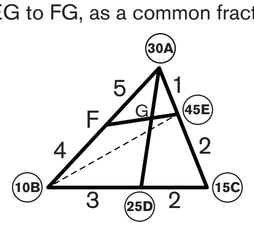


Figure 4.2

ANSWERS

In addition to the answer, we have provided a difficulty rating for each problem. Our scale is 1-7, with 7 being the most difficult. These are only approximations, and how difficult a problem is for a particular student will vary. Below is a general guide to the ratings:

Difficulty 1/2/3 - One concept; one- to two-step solution; appropriate for students just starting the middle school curriculum.
4/5 - One or two concepts; multistep solution; knowledge of some middle school topics is necessary.
6/7 - Multiple and/or advanced concepts; multistep solution; knowledge of advanced middle school topics and/or problem-solving strategies is necessary.

Warm-Up 1

Answer	Difficulty		
1. 495	(1)	6. 26	(3)
2. 11	(2)	7. B & C	(4)
3. 70	(2)	8. 11,304	(2)
4. $3\frac{23}{24}$	(3)	9. 5/9	(4)
5. 16	(3)	10. 12	(4)

Warm-Up 2

Answer	Difficulty		
11. 4	(3)	16. 1000 or 1000.00	(3)
12. 5	(4)	17. 0	(2)
13. 8	(3)	18. (3, 3)	(4)
14. 50	(3)	19. 5/9	(4)
15. 14	(4)	20. $\sqrt{21}$	(4)

Workout 1

Answer	Difficulty		
21. 49	(2)	26. 17	(4)
22. 1.225×10^9	(3)	27. 174	(3)
23. 12	(4)	28. 0.04	(5)
24. 3.34	(3)	29. 5	(3)
25. 400	(4)	30. 1/6	(3)

Warm-Up 3

Answer	Difficulty		
31. 5	(2)	36. 234 or 234.00	(2)
32. 2	(2)	37. 10	(3)
33. 1*	(3)	38. 15	(3)
34. 6	(3)	39. 4.80	(3)
35. 7/36	(3)	40. 1/4	(3)

Warm-Up 4

Answer	Difficulty		
41. a^2	(3)	46. 5	(3)
42. 1/8	(3)	47. 3456	(3)
43. 37	(4)	48. 78	(5)
44. 20	(3)	49. 60	(3)
45. $1\frac{1}{2}$	(3)	50. 4	(3)

Workout 2

Answer	Difficulty		
51. 32	(5)	56. 2 or 2.00	(3)
52. 58	(2)	57. 6	(2)
53. 27.7	(4)	58. 89	(3)
54. 10	(5)	59. 2008	(3)
55. 25	(5)	60. 8	(4)

** The plural form of the units is always provided in the answer blank, even if the answer appears to require the singular form of the units.*

Warm-Up 5

Answer	Difficulty		
61. 300 or 300.00	(3)	66. 60	(4)
62. 56	(4)	67. 2.4	(3)
63. 2015	(3)	68. 16	(4)
64. 0	(4)	69. 10	(4)
65. 16/15	(4)	70. 2015	(3)

Warm-Up 6

Answer	Difficulty		
71. 11/6	(3)	76. 1459	(3)
72. 80	(4)	77. 3	(4)
73. $15\sqrt{3}$	(4)	78. 12	(3)
74. 810	(3)	79. 46	(3)
75. 9	(4)	80. 12	(3)

Workout 3

Answer	Difficulty		
81. 0	(4)	86. 2	(3)
82. 51	(3)	87. 41	(2)
83. 0.5	(3)	88. 31	(4)
84. 2900	(4)	89. 460	(4)
85. 3	(4)	90. 7	(4)

Warm-Up 7

Answer	Difficulty		
91. 72	(3)	96. 56	(3)
92. 18π	(4)	97. 3/2	(3)
93. 9	(3)	98. 54	(2)
94. 50	(3)	99. 2	(3)
95. 9/8	(3)	100. 45	(3)

Warm-Up 8

Answer	Difficulty		
101. 9	(2)	106. 682	(3)
102. 12	(2)	107. 48	(3)
103. 85	(2)	108. 11	(3)
104. 35	(4)	109. 35,800 or 35,800.00	(3)
105. 5	(3)	110. 2	(4)

Workout 4

Answer	Difficulty		
111. 79	(4)	116. 1353	(4)
112. 331,776	(5)	117. 32.2	(4)
113. 47	(2)	118. $72\sqrt{2}$	(4)
114. 6	(2)	119. 1207	(4)
115. 22.5	(3)	120. 6	(5)

Warm-Up 9

Answer	Difficulty		
121. 120	(3)	126. 1024	(4)
122. 10	(3)	127. 17	(2)
123. 400 or 400.00	(2)	128. 14,580 or 14,580.00	(4)
124. 2	(2)	129. 27	(3)
125. 5	(3)	130. 6	(3)

Warm-Up 10

Answer	Difficulty		
131. $64 + 32\pi$ or $32\pi + 64$	(4)	136. 27	(5)
132. 792	(3)	137. 2279	(5)
133. $\sqrt{3}/2$	(3)	138. 2	(3)
134. 30	(4)	139. 16	(4)
135. 4	(3)	140. 80	(4)

Workout 5

Answer	Difficulty		
141. 7.5	(3)	146. 2.3	(4)
142. 2.7	(5)	147. 8:12	(3)
143. 66,660	(5)	148. 2001	(4)
144. 22	(3)	149. 78	(3)
145. 0.3	(5)	150. 10	(3)

Warm-Up 11

Answer	Difficulty		
151. 32	(3)	156. 8/27	(5)
152. 60	(4)	157. (−3, 1)	(4)
153. 4/9	(4)	158. 16	(3)
154. $3\frac{1}{3}$	(4)	159. 12	(5)
155. 78	(4)	160. 648	(4)

Warm-Up 12

Answer	Difficulty		
161. 1024	(3)	166. 60	(5)
162. 34	(3)	167. 24	(5)
163. $2\sqrt{7}$	(5)	168. 1	(4)
164. 4/5	(4)	169. $144w^3$	(3)
165. 15	(3)	170. 9	(5)

Workout 6

Answer	Difficulty		
171. 76	(4)	176. 12	(3)
172. 251.6	(5)	177. 29	(5)
173. 1012	(5)	178. 4576	(4)
174. 3.4	(4)	179. 121/282	(5)
175. 33	(4)	180. 4	(4)

Warm-Up 13

Answer	Difficulty		
181. 36	(4)	186. 14	(3)
182. $\sqrt{5}/2$	(4)	187. 1/12	(4)
183. $2 + \sqrt{2}$ or $\sqrt{2} + 2$	(4)	188. 18	(3)
184. 25	(3)	189. −3/4	(3)
185. 17	(2)	190. 16	(4)

Warm-Up 14

Answer	Difficulty		
191. 324π	(4)	196. 3	(4)
192. 37/64	(4)	197. $4\frac{2}{7}$	(4)
193. 7/3	(4)	198. 6	(4)
194. 22	(3)	199. 1/6	(5)
195. 1008/2015	(5)	200. 757	(4)

Workout 7

Answer	Difficulty		
201. 25	(4)	206. 4.57	(4)
202. 13/55	(3)	207. 403	(3)
203. 46.1	(6)	208. 2	(4)
204. 42.7	(4)	209. 9	(3)
205. −5050	(4)	210. 12	(4)

Warm-Up 15

Answer	Difficulty		
211. 20	(4)	216. 1/100	(4)
212. 148	(5)	217. 61	(2)
213. −7	(3)	218. 55	(4)
214. $3 - \sqrt{3}$ or $-\sqrt{3} + 3$	(6)	219. 1 or 1.00	(3)
215. 60	(4)	220. 10	(4)

Warm-Up 16

Answer	Difficulty		
221. 0	(3)	226. 10	(4)
222. $6 + \pi$ or $\pi + 6$	(5)	227. 12,288	(3)
223. 14	(4)	228. 1358	(3)
224. 60	(4)	229. 10	(4)
225. 7/37	(5)	230. 55	(5)

Workout 8

Answer	Difficulty		
231. $\sqrt{6}$	(4)	236. 4.5	(5)
232. 7200 or 7200.00	(3)	237. −406	(4)
233. 0.155	(5)	238. 3.7	(4)
234. 84	(5)	239. 78.5	(5)
235. 0.18	(5)	240. 53	(4)

Warm-Up 17

Answer	Difficulty	Answer	Difficulty
241. 12	(5)	246. 44	(4)
242. 12	(4)	247. 19	(6)
243. $3 - \sqrt{3}$ or $-\sqrt{3} + 3$	(5)	248. 20	(5)
244. $1 + \sqrt{3}$ or $\sqrt{3} + 1$	(5)	249. 5	(6)
245. 40	(4)	250. 60	(4)

Warm-Up 18

Answer	Difficulty	Answer	Difficulty
251. $(47\sqrt{3})/2$	(5)	256. 1/27	(5)
252. $1 + \sqrt{2}$ or $\sqrt{2} + 1$	(4)	257. 5/14	(5)
253. 9	(6)	258. 11/140	(5)
254. 20	(5)	259. 27	(5)
255. 17/24	(4)	260. 75	(4)

Workout 9

Answer	Difficulty	Answer	Difficulty
261. 1	(4)	266. 397.6	(4)
262. $c^2/(2ab)$	(4)	267. 76	(4)
263. 32.9	(5)	268. 14.4	(4)
264. 7/16	(6)	269. 6	(5)
265. 0.049	(6)	270. 45	(4)

Logic Stretch

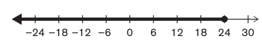
Answer	Difficulty	Answer	Difficulty
271. 7	(2)	276. 1038	(3)
272. 15	(3)	277. 39	(3)
273. 9	(3)	278. 8	(2)
274. 2	(1)	279. 3/7	(4)
275. 12	(3)	280. 972	(4)

Solving Inequalities Stretch

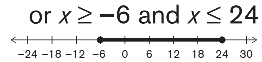
Answer **Difficulty**

281. $x \geq -6$ (3)

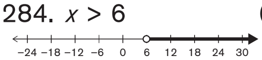
282. $x \leq 24$ (3)



283. $-6 \leq x \leq 24$ or $x \geq -6$ and $x \leq 24$ (4)

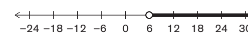


284. $x > 6$ (3)

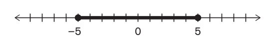


285. $x > 3$ (3)

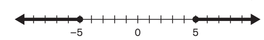
286. $x > 6$ (4)



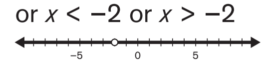
287. $-5 \leq x \leq 5$ or $x \geq -5$ and $x \leq 5$ (5)



288. $x \leq -5$ or $x \geq 5$ (5)



289. $x \neq -2$ or $x < -2$ or $x > -2$ (6)



290. $x < -3$ or $x > -1$ (6)

Mass Point Geometry Stretch

Answer	Difficulty	Answer	Difficulty
291. 5/2	(5)	296. 1/4	(6)
292. 7/3	(5)	297. 60	(6)
293. 1/4	(5)	298. 16/105	(7)
294. 1/2	(5)	299. 25/38	(7)
295. 4/25	(6)	300. 2/5	(7)

MATHCOUNTS Problems Mapped to Common Core State Standards (CCSS)

Forty-three states, the District of Columbia, four territories and the Department of Defense Education Activity (DoDEA) have adopted the Common Core State Standards (CCSS). As such, MATHCOUNTS considers it beneficial for teachers to see the connections between the *2014-2015 MATHCOUNTS School Handbook* problems and the CCSS. MATHCOUNTS not only has identified a general topic and assigned a difficulty level for each problem but also has provided a CCSS code in the Problem Index (pages 83-84). A complete list of the Common Core State Standards can be found at www.corestandards.org.

The CCSS for mathematics cover K-8 and high school courses. MATHCOUNTS problems are written to align with the NCTM Standards for Grades 6-8. As one would expect, there is great overlap between the two sets of standards. MATHCOUNTS also recognizes that in many school districts, algebra and geometry are taught in middle school, so some MATHCOUNTS problems also require skills taught in those courses.

In referring to the CCSS, the Problem Index code for each or the Standards for Mathematical Content for grades K-8 begins with the grade level. For the Standards for Mathematical Content for high school courses (such as algebra or geometry), each code begins with a letter to indicate the course name. The second part of each code indicates the domain within the grade level or course. Finally, the number of the individual standard within that domain follows. Here are two examples:

- *6.RP.3 → Standard #3 in the Ratios and Proportional Relationships domain of grade 6*
- *G-SRT.6 → Standard #6 in the Similarity, Right Triangles and Trigonometry domain of Geometry*

Some math concepts utilized in MATHCOUNTS problems are not specifically mentioned in the CCSS. Two examples are the Fundamental Counting Principle (FCP) and special right triangles. In cases like these, if a related standard could be identified, a code for that standard was used. For example, problems using the FCP were coded 7.SP.8, S-CP.8 or S-CP.9 depending on the context of the problem; SP → Statistics and Probability (the domain), S → Statistics and Probability (the course) and CP → Conditional Probability and the Rules of Probability. Problems based on special right triangles were given the code G-SRT.5 or G-SRT.6, explained above.

There are some MATHCOUNTS problems that either are based on math concepts outside the scope of the CCSS or based on concepts in the standards for grades K-5 but are obviously more difficult than a grade K-5 problem. When appropriate, these problems were given the code SMP for Standards for Mathematical Practice. The CCSS include the Standards for Mathematical Practice along with the Standards for Mathematical Content. The SMPs are (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (4) Model with mathematics; (5) Use appropriate tools strategically; (6) Attend to precision; (7) Look for and make use of structure and (8) Look for and express regularity in repeated reasoning.

PROBLEM INDEX

It is difficult to categorize many of the problems in the *MATHCOUNTS School Handbook*. It is very common for a MATHCOUNTS problem to straddle multiple categories and cover several concepts. This index is intended to be a helpful resource, but since each problem has been placed in exactly one category and mapped to exactly one Common Core State Standard (CCSS), the index is not perfect. In this index, the code **9 (3) 7.SP.3** refers to problem 9 with difficulty rating 3 mapped to CCSS 7.SP.3. For an explanation of the difficulty ratings refer to page 78. For an explanation of the CCSS codes refer to page 82.

Algebraic Expressions & Equations

2	(2)	SMP
13	(3)	7.EE.4
26	(4)	6.EE.7
33	(3)	8.EE.8
36	(2)	8.EE.8
41	(3)	6.EE.9
46	(3)	F-IF.2
50	(3)	6.EE.2
51	(5)	8.EE.8
64	(4)	SMP
68	(4)	8.EE.8
69	(4)	8.EE.8
70	(3)	SMP
81	(4)	8.EE.2
86	(3)	8.EE.8
93	(3)	8.EE.8
107	(3)	6.EE.2
108	(3)	8.EE.8
110	(4)	6.EE.2
112	(5)	SMP
123	(2)	6.EE.7
124	(2)	F-IF.2
125	(3)	A-SSE.2
127	(2)	6.EE.9
135	(3)	8.EE.8
144	(3)	6.EE.7
147	(3)	6.EE.9
149	(3)	8.EE.8
150	(3)	6.EE.9
154	(4)	8.EE.8
158	(3)	4.OA.4
176	(3)	7.NS.3
189	(3)	8.EE.8
193	(4)	F-IF.2
237	(4)	A-REI.4
240	(4)	A-CED.1
245	(4)	8.EE.2
281	(3)	7.EE.4
282	(3)	7.EE.4
283	(4)	A-CED.1
284	(3)	7.EE.4
285	(3)	7.EE.4
286	(4)	A-CED.1
287	(5)	A-CED.1
288	(5)	A-CED.1
289	(6)	A-CED.1
290	(6)	A-CED.1

General Math

1	(2)	5.MD.1
17	(2)	4.OA.3
161	(3)	7.NS.2
168	(4)	SMP
180	(4)	6.EE.2
196	(4)	F-IF.1
213	(3)	F-IF.2

Number Theory

8	(1)	4.OA.5
21	(2)	4.OA.4
31	(2)	SMP
32	(2)	SMP
37	(3)	SMP
47	(3)	6.NS.4
48	(5)	S-CP.9
57	(2)	7.NS.3
62	(4)	S-CP.9
63	(3)	4.OA.3
76	(3)	SMP
87	(2)	4.OA.4
96	(3)	SMP
120	(5)	SMP
129	(3)	4.OA.4
138	(3)	7.NS.1
148	(4)	SMP
152	(4)	SMP
162	(3)	SMP
165	(3)	SMP
175	(4)	S-CP.9
181	(4)	SMP
184	(3)	SMP
190	(4)	8.EE.2
198	(4)	SMP
200	(4)	SMP
201	(4)	N-RN.2
202	(3)	7.NS.3
207	(3)	SMP
208	(4)	7.NS.2
209	(3)	7.NS.3
210	(4)	SMP
221	(3)	SMP
231	(4)	N-RN.1
254	(5)	SMP
255	(4)	N-RN.2
261	(4)	8.EE.2
270	(4)	7.NS.3

Statistics

14	(3)	SMP
60	(4)	6.SP.5
199	(5)	7.SP.8
235	(5)	7.SP.8
4	(3)	6.SP.2
12	(4)	6.SP.5
85	(4)	6.SP.2
91	(3)	6.SP.2
105	(3)	6.SP.2
122	(3)	6.SP.5
141	(3)	6.SP.2

Logic

7	(4)	SMP
132	(3)	SMP
271	(2)	SMP
272	(3)	SMP
273	(3)	SMP
274	(1)	SMP
275	(3)	SMP
276	(3)	SMP
277	(3)	SMP
278	(2)	SMP
279	(4)	SMP
280	(4)	SMP

Problem Solving (Misc.)

6	(3)	SMP
43	(4)	SMP
44	(3)	SMP
55	(5)	SMP
58	(3)	SMP
90	(4)	SMP
99	(3)	SMP
104	(4)	SMP
116	(4)	SMP
166	(5)	6.EE.9
172	(5)	G-C.2
192	(4)	7.G.6
195	(5)	SMP
218	(4)	SMP
230	(5)	SMP
234	(5)	SMP
242	(4)	SMP
246	(4)	SMP
259	(5)	SMP

Measurement

3	(2)	6.RP.3
15	(4)	6.RP.3
22	(3)	8.EE.4
38	(3)	6.RP.3
39	(3)	6.RP.3
23	(4)	7.G.4
28	(5)	7.G.4
40	(3)	7.G.6
111	(4)	8.G.7
117	(4)	8.G.7
126	(4)	G-SRT.6
131	(4)	7.G.6
133	(3)	G-SRT.6
139	(4)	G-SRT.6
146	(4)	G-SRT.6
159	(5)	G-SRT.6
167	(5)	G-SRT.6
174	(4)	G-SRT.6
177	(5)	8.G.7
182	(4)	8.G.7
191	(4)	8.G.7
206	(4)	8.G.7
214	(6)	G-SRT.6
233	(5)	G-SRT.6
243	(5)	G-SRT.6
244	(5)	G-SRT.6
247	(6)	G-SRT.6
251	(5)	G-SRT.6
263	(5)	G-SRT.6
269	(5)	8.G.7

Coordinate Geometry

18	(4)	8.EE.8
29	(3)	8.EE.8
49	(3)	6.G.3
100	(3)	8.F.3
157	(4)	8.F.3
204	(4)	F-IF.2
220	(4)	8.G.8
228	(3)	8.F.3
238	(4)	G-SRT.6
239	(5)	SMP
262	(4)	8.F.3

Sequences, Series & Patterns

34	(3)	F-BF.2
71	(3)	F-BF.2
95	(3)	F-BF.2
103	(2)	F-BF.2
114	(2)	F-BF.2
137	(5)	F-BF.2
170	(5)	F-BF.2
171	(4)	SMP
205	(4)	F-BF.2
229	(4)	F-BF.2
241	(5)	F-BF.2

Plane Geometry

9	(4)	8.G.7
20	(4)	8.G.7
53	(4)	7.G.4
75	(4)	SMP
92	(4)	7.G.4
101	(2)	SMP
121	(3)	8.G.5
136	(5)	G-CO.10
145	(5)	G-C.2
163	(5)	G-C.2
212	(5)	G-C.2
222	(5)	7.G.4
223	(4)	G-C.2
225	(5)	G-GMD.3
248	(5)	G-C.2
249	(6)	G-C.2
252	(4)	8.G.7
253	(6)	8.G.7
260	(4)	G-C.2

Solid Geometry

11	(3)	G-GMD.3
77	(4)	G-GMD.3
79	(3)	7.G.6
88	(4)	7.G.6
106	(3)	7.G.6
118	(4)	8.G.9
119	(4)	7.G.6
130	(3)	SMP
134	(4)	G-GMD.3
155	(4)	7.G.6
169	(3)	6.G.2
178	(4)	7.G.6
194	(3)	7.G.6
224	(4)	SMP
226	(4)	SMP
227	(3)	G-GMD.3
236	(5)	G-GMD.3
256	(5)	SMP
266	(4)	G-GMD.3
268	(4)	7.G.4

Probability, Counting & Combinatorics

5	(3)	7.SP.8
19	(4)	7.SP.8
30	(3)	7.SP.8
35	(3)	7.SP.8
42	(3)	7.SP.8
78	(3)	7.SP.8
80	(3)	SMP
143	(5)	S-CP.9
151	(3)	SMP
153	(4)	S-CP.8
160	(4)	S-CP.8
179	(5)	7.SP.8
187	(4)	7.SP.8
211	(4)	S-CP.9
216	(4)	7.SP.8
232	(3)	S-CP.9
258	(5)	7.SP.8
264	(6)	7.SP.8
265	(6)	7.SP.8

Percents & Fractions

10	(4)	6.RP.3
16	(3)	7.RP.3
24	(3)	7.RP.3
25	(4)	6.RP.3
27	(3)	6.RP.3
52	(2)	6.RP.3
56	(3)	7.RP.3
59	(3)	6.RP.3
65	(4)	6.RP.3
66	(4)	6.RP.3
67	(3)	6.RP.3
72	(4)	7.RP.3
74	(3)	6.RP.3
82	(3)	7.RP.3
89	(4)	7.RP.3
94	(3)	6.RP.3
98	(2)	6.RP.3
102	(2)	6.NS.1
109	(3)	7.RP.3
128	(4)	7.RP.3
140	(4)	6.RP.3
188	(3)	6.RP.3
215	(4)	7.RP.3
217	(2)	7.RP.3
257	(5)	7.RP.3
267	(4)	6.RP.3

Proportional Reasoning

54	(5)	7.G.6
73	(4)	7.G.6
83	(3)	7.G.6
97	(3)	6.G.1
113	(2)	7.G.6
142	(5)	7.RP.2
156	(5)	7.G.6
164	(4)	G-SRT.5
183	(4)	6.G.1
197	(4)	G-SRT.5
203	(6)	7.G.6
291	(5)	6.RP.1
292	(5)	6.RP.1
293	(5)	6.RP.1
294	(5)	6.RP.1
295	(6)	6.RP.1
296	(6)	6.RP.1
297	(6)	6.RP.1
298	(7)	6.RP.1
299	(7)	6.RP.1
300	(7)	6.RP.1
45	(3)	6.RP.3
61	(3)	6.RP.3
84	(4)	6.RP.3
115	(3)	6.RP.3
173	(5)	G-C.2
185	(2)	6.RP.3
186	(3)	6.RP.3
219	(3)	6.RP.3
250	(4)	6.EE.7

MATHCOUNTS® COMPETITION SERIES

2014–2015 ADDITIONAL STUDENTS REGISTRATION FORM

Step 1: *Tell us about your school so we can find your original registration (please print legibly).*

Teacher/Coach Name ______________________ Teacher/Coach Email ______________________

School Name ______________________ Teacher/Coach Phone ______________________

School Mailing Address ______________________ City, State ZIP ______________________

Step 2: *Tell us how many students you are adding to your school's registration. Following the instructions below.*

Please circle *the number of additional students you will enter in the Chapter Competition and the associated cost below (depending on the date your registration is postmarked). The cost is $30 per student added, whether that student will be part of a team or will compete as an individual. The cost of adding students to a previous registration is not eligible for an Early Bird rate.*

# of Students You Are Adding	1	2	3	4	5	6	7	8	9
Regular Rate *(postmarked **by** Dec. 12, 2014)*	$30	$60	$90	$120	$150	$180	$210	$240	$270
Late Registration *(postmarked **after** Dec. 12, 2014)*	$50	$80	$110	$140	$170	$200	$230	$260	$290

☐ My school qualifies for the 50% Title I discount, so the Amount Due in Step 4 will be half the amount I circled above. Principal signature required to verify Title I eligibility.

X ______________________

Step 3: *Tell us what your school's FINAL registration should be (including all changes/additions).*

TOTAL # of Registered Students	1 (1 individual)	2 (2 ind)	3 (3 ind)	4 (1 team)	5 (1 tm, 1 ind)	6 (1 tm, 2 ind)	7 (1 tm, 3 ind)	8 (1 tm, 4 ind)	9 (1 tm, 5 ind)	10 (1 tm, 6 ind)

Step 4: *Almost done... just fill in payment information and turn in your form!*

Amount Due ____________ ☐ Check *(payable to MATHCOUNTS Foundation)* ☐ Money order ☐ Purchase order # ____________ *(must include P.O.)*

☐ Credit card *(include all information)* Name on card ______________________ ☐ Visa ☐ MasterCard

Signature ______________________ Card # ______________________ Exp. ____________

Mail, email a scanned copy or fax this completed form to:

MATHCOUNTS Registration | P.O. Box 441 | Annapolis Junction, MD 20701

Email: reg@mathcounts.org | Fax: 240-396-5602

Questions?
Please call the Registration Office at ***301-498-6141.***

Registration is fast and easy at www.mathcounts.org/clubreg!
Get an instant email confirmation without having to wait for processing.

THE NATIONAL **MATHCLUB** powered by MATHCOUNTS®

2014–2015 REGISTRATION FORM

Step 1: *Tell us about your group. Check and complete only 1 option.*

☐ ***U.S. school with students in 6th, 7th and/or 8th grade***

School Name: ______________________________

There can be multiple clubs at the same U.S. middle school, as long as each club has a different Club Leader.

☐ ***Chapter or member group of a larger organization.***
(Can be non-profit or for profit)

Organization: ______________________________

Chapter (or equivalent) Name: ______________________________

Examples of larger organizations: Girl Scouts, Boy Scouts, YMCA, Boys & Girls Club, nationwide tutoring/enrichment centers.

☐ ***A home school or group of students not affiliated with a larger organization.***

Club Name: ______________________________

Examples: home schools, neighborhood math groups, independent tutoring centers

Step 2: *Make sure your group is eligible to participate in The National Math Club.*

Please check off that all 3 of the following statements are true for your group, to the best of your knowledge:

☐ My group consists of at least 4 U.S. students.
☐ The students in my group are in 6th, 7th and/or 8th grade.
☐ My group has regular in-person meetings.

By signing below I, the Club Leader, affirm that all of the above statements are true, to the best of my knowledge, and that my group is therefore eligible to participate in The National Math Club. I understand that MATHCOUNTS can cancel my membership at any time if it is determined that my group is ineligible.

Club Leader Signature: ______________________________

Step 3: *Get signed up.*

Club Leader Name ______________________________ Club Leader Phone ______________

Club Leader Email ______________________________

Club Leader Alternate Email ______________________________

Club Mailing Address ______________________________

City, State ZIP ______________________________

Total # of participating students in club: ____________ ☐ Previously participated in MATHCOUNTS.

How did you hear about MATHCOUNTS? ☐ Mailing ☐ Word-of-mouth ☐ Conference ☐ Internet ☐ Email ☐ Prior Participant

For schools only: School Type: ☐ Public ☐ Charter ☐ Private ☐ Home school ☐ Virtual

If your school is overseas: My school is sponsored by ☐ DoDDS. OR ☐ State Department. Country ______________

Step 4: *Almost done... just turn in your form.*

Mail, email a scanned copy or fax this completed form to:
MATHCOUNTS Foundation, 1420 King Street, Alexandria, VA 22314
Email: info@mathcounts.org
Fax: 703–299–5009

Questions?
Please call the national office at
703-299-9006