2014 Raytheon MATHCOUNTS National Competition

Friday May 9, 2014 - Washington, D.C.

Rank	Student	State	Grade
1	Swapnil Garg	CA	8
2	Kevin Liu	IN	7
S	Daniel Zhu	MD	7
S	Alan Peng	MO	8
Q	Nicholas Sun	IL	8
Q	Hongyi Chen	CO	8
Q	Colin Tang	WA	7
Q	Vinjai Vale	TX	8
P	Freddie Zhao	MI	7
P	Jun-Hee Lee	IA	8
P	Graham O'Donnell	FL	8
P	Akshaj Kadaveru	VA	8
13	Elbert Du	IL	8
14	Andy Xu	SC	7
15	Daniel Kim	NJ	8
16	Franklyn Wang	VA	8
17	Kevin Feng	TX	8
18	Vincent Huang	TX	7
19	Alexander Gu	IN	7
20	Jason Lee	MD	7
21	Brian Reinhart	FL	8
22	Daniel Liu	WA	8
23	Zack Lee	NC	8
24	Raymond Feng	NY	7
25	Jeffrey Chang	MA	8
26	Chang Yu	TN	8
27	Allen Ryu	MS	7
28	Joseph Feffer	PA	8
29	Harry Wang	CA	8
30	Rajiv Movva	CA	8
31	William Sun	VA	8
32	Hannah Zhang	CO	8
33	William Wang	KS	7
34	Walker Kroubalkian	AZ	7
35	Christopher Lee	PA	7
36	Peter Rowley	MA	8
37	Jeffery Li	CA	7
38	Jae Hyun Lim	NE	8
39	Kaan Dokmeci	NM	8
40	Srivats Narayanan	KS	8
41	Joshua Lee	VA	8
42	Daniel Chu	GA	8
43	Spencer Liu	MI	8
44	Richard Xu	NY	8
45	Thomas Luo	MD	7
46	David Ma	MA	8
47	Matthew Dai	NC	8
48	Allen Chen	IL	7
49	Alan Tu	NY	7
50	Jeremy Chen	MA	8
51	Wanlin Li	PA	8
52	Samuel Merson	AZ	8
53	Michelle Shen	IN	8
54	Anders Olsen	OR	8
55	Richard Liu	FL	8
56	Peter Zhu	OH	8

Rank	Team
1	California
2	Maryland
3	Virginia
4	New York
5	Pennsylvania
6	Indiana
7	Texas
8	Massachusetts
9	Florida
10	Colorado
11	Illinois
12	Washington
13	Kansas
14	Michigan
15	Nevada
16	New Jersey
17	North Carolina
18	South Carolina
19	Iowa
20	Wisconsin
21	Oregon
22	Ohio
23	Arizona

Written Competition Champion - Kevin Liu, Indiana
Written Competition Runner-Up - Nicholas Sun, Illinois

S - Semifinalist; Q - Quarterfinalist; P - Countdown Round Participant

MATHCOUNTS

2014 Raytheon MATHCOUNTS National Competition

Orlando, FL

Friday, May 9, 2014

National Competition

Competition Statistics

Scoring Statistics

	Individual			Team	
	Indiv. Total	Sprint Score	Target Score	Team Total	Team Round
Minimum	4.00	4.00	0.00	14.75	6.00
Average	24.16	15.84	8.32	35.27	11.11
Maximum	43.00	28.00	16.00	52.75	18.00
Std. Dev.	9.37	5.64	4.23	9.97	2.88

Grade / Gender Distribution

	Gender:			
Grade:	**M**	**F**	**U**	**Total**
6	8	1		9
7	53	7		60
8	132	23		155
Total	193	31	0	224

MATHCOUNTS

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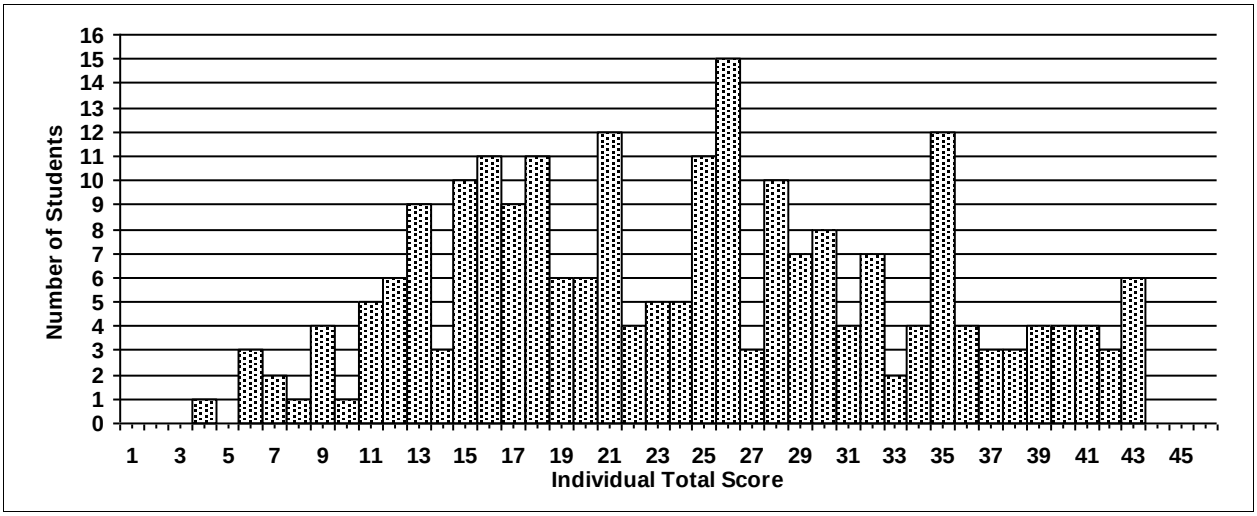
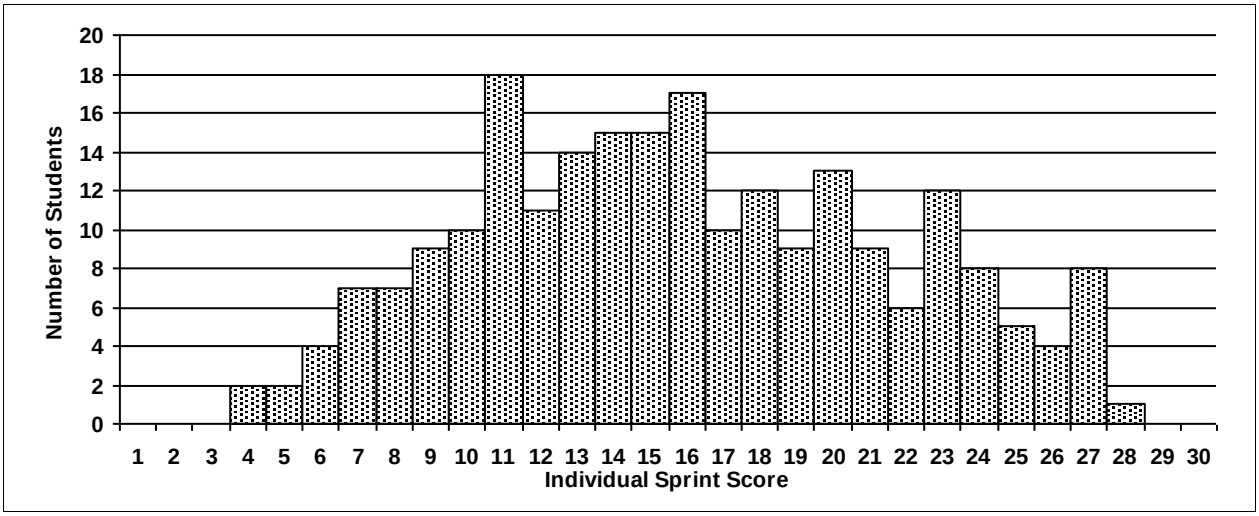
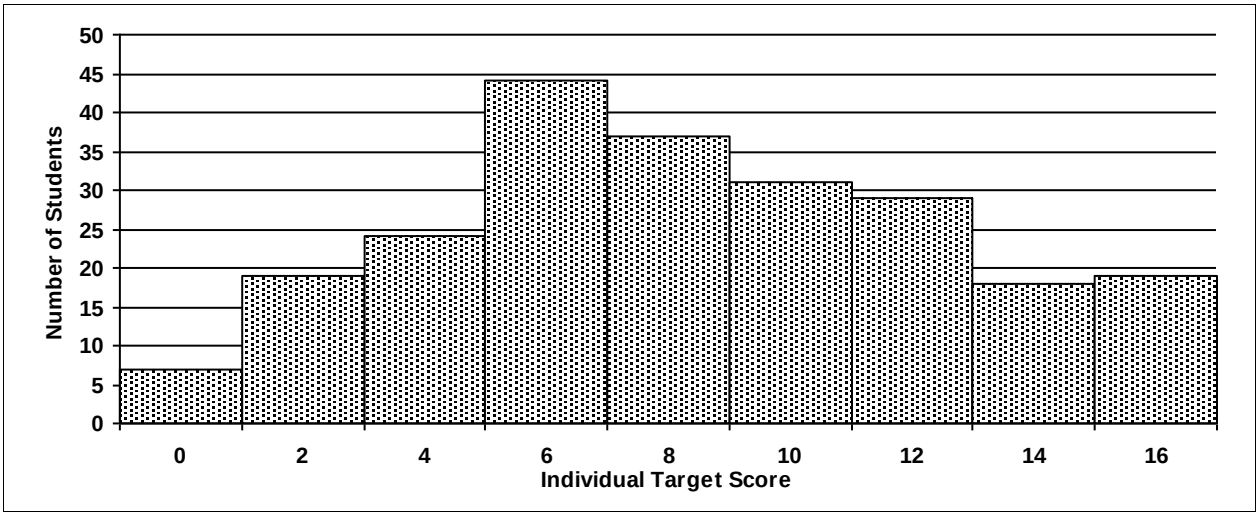
National Competition

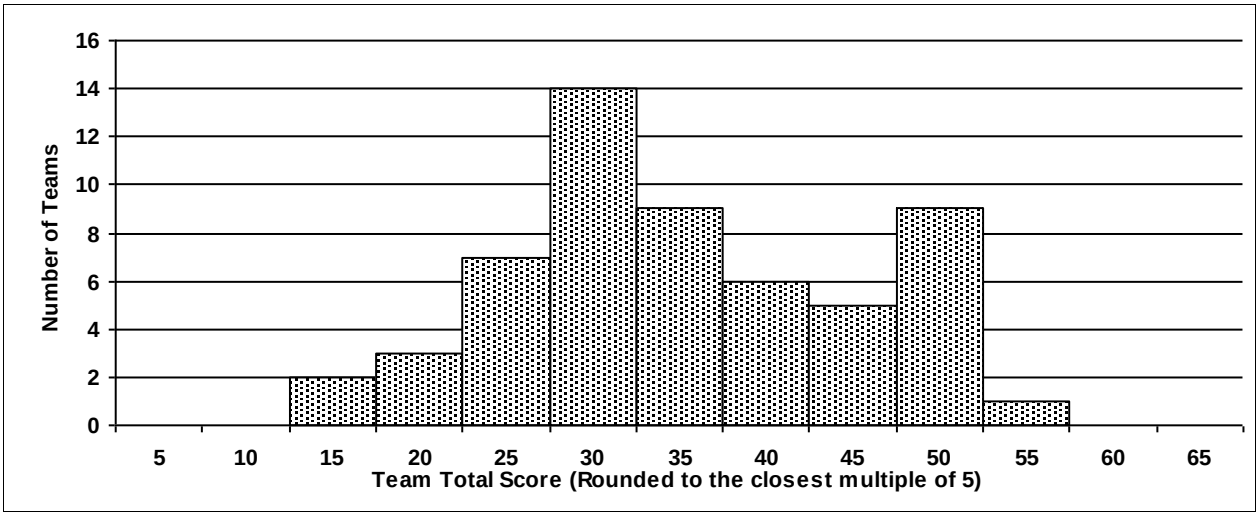
Score Distributions

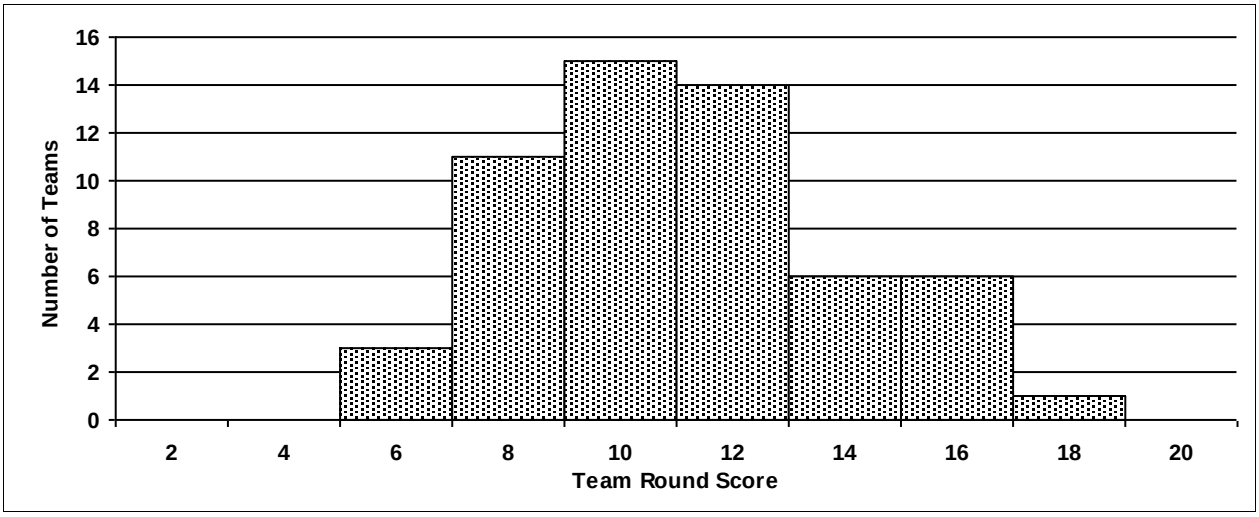






Score Distributions





MATHCOUNTS

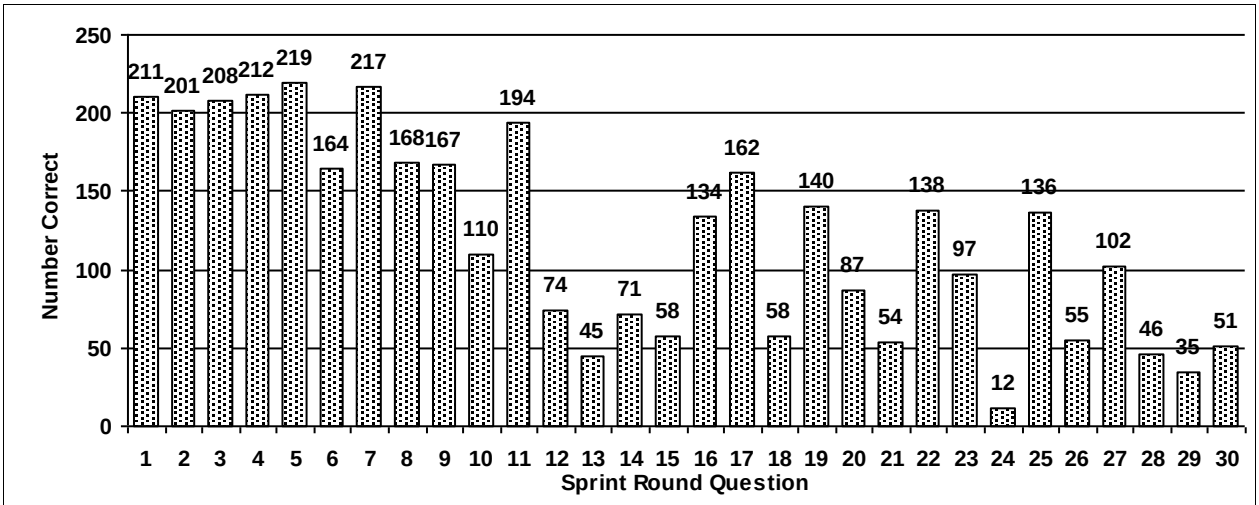
2014 Raytheon MATHCOUNTS National Competition

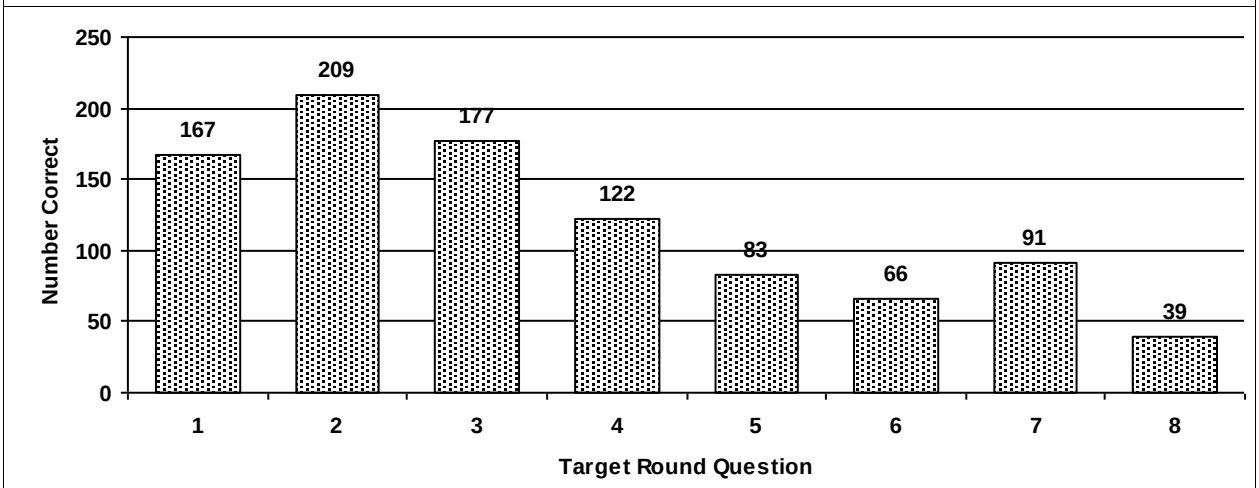
Orlando, FL

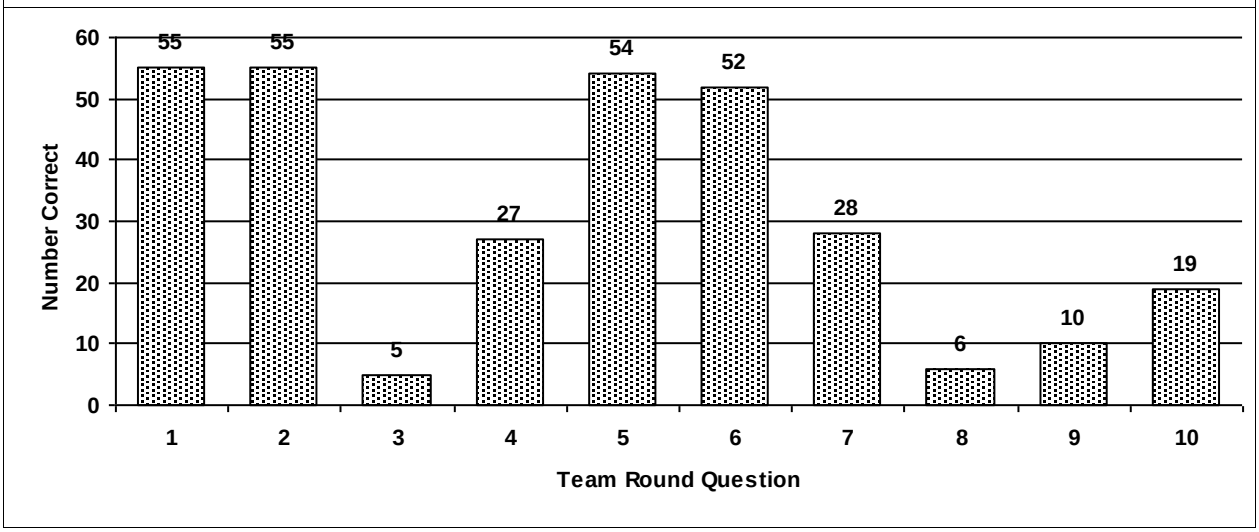
Friday, May 9, 2014

National Competition

Question Analysis







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2014
■ National Competition ■
Answer Key

The appropriate units (or their abbreviations) are provided in the answer blanks.

Note to coordinators: Answers to the Tiebreaker Round problems appear in the Tiebreaker Round Booklet.

Sprint Round Answers

1. 5000.5

2. 30 %

3. 8 years

4. \$6.50

5. 20

6. $(1, -1)$

7. 28

8. 24 cm

9. -11

10. $2 + \sqrt{3}$ cm or $\sqrt{3} + 2$

11. 90

12. 2013

13. 4 solutions

14. 9 pairs

15. $\frac{13}{1024}$

16. 11 m^2

17. 19

18. $\frac{35}{72}$

19. $\frac{26}{9}$

20. 5

21. $\frac{1}{4}$

22. 42

23. 9 base 10

24. 22

25. 225

26. \$3.75

27. $\frac{3}{5}$

28. 0

29. 32 pages

30. 8

Target Round Answers

1. 5

2. \$3.83

3. 65

4. 13 travelers

5. 56 in^2

6. $\frac{23}{2}$

7. $\frac{19}{118}$

8. $\frac{7}{10}$ in

Team Round Answers

1. 25 boxes

2. \$1150 or 1150.00

3. 2685

4. 65

5. $\frac{1}{54}$

6. 32 $units^2$

7. 46 in^2

8. $\frac{27}{15,625}$

9. 1020 ways

10. 80 degrees

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2014
■ National Competition ■
Sprint Round
Problems 1–30

HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

State _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

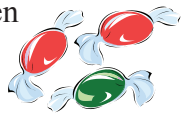
Total Correct	Scorer's Initials

1. _____ What is the mean of the 10,000 integers from 1 to 10,000, inclusive? Express your answer as a decimal to the nearest tenth.

2. _____ % What percent of the integers from 3 to 12, inclusive, are neither primes nor multiples of 4?

3. _____ years On the next full moon, Bob will celebrate being alive for 100 full moons. On average, the cycle of the moon has lasted 29.53 days since he was born. In years, how old will Bob be on his 100-moons birthday? Express your answer to the nearest whole number.

4. $ _____ Samantha bought 6 total pounds of red and green candies to share with her friends. The red candies cost $1.00 per pound and the green candies cost $1.25 per pound. She bought twice as many pounds of red candies as green candies. How much did Samantha pay for the 6 pounds of candies?

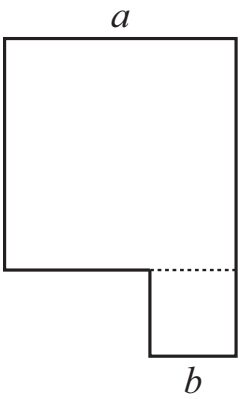


5. _____ The sum of nine consecutive integers is 216. What is the smallest of the nine integers?

6. (,) _____ A kite is a quadrilateral in which two pairs of adjacent sides are congruent. Points A(−2, 1), B(1, 5), C(4, 1) and D(x, y) are vertices of a convex kite with an area of 18 units^2. If x and y are integers, what are the coordinates of point D? Express your answer as an ordered pair.

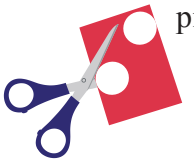
7. _____ What is the smallest value of x such that $3(x - 21) > 8$ and x is a multiple of 7?

8. _____ cm Two squares, with integer side lengths a and b, are arranged so that one entire side of the smaller square overlaps a part of a side of the larger square, and the two squares share a vertex, as shown. The perimeter of the entire figure is 86 cm, and the sum of the areas of the two squares is 386 cm^2. In centimeters, what is the value of $a + b$?

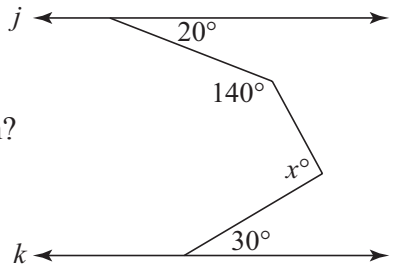


9. _____ The mean of 10, 4, 1, x and 1 is equal to the median. What is the smallest possible value of x?

10. _____ cm Lothario wants to cut out five circles, each 2 cm in diameter, from a rectangular piece of cardboard that is 6 cm long. What must be the minimum width of the rectangular cardboard? Express your answer in simplest radical form.

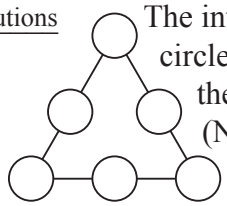


11. _____ If $j \parallel k$, what is the value of x, in the figure shown?



12. _____ What is the value of $\dfrac{2013^3 - 2 \cdot 2013^2 \cdot 2014 + 3 \cdot 2013 \cdot 2014^2 - 2014^3 + 1}{2013 \cdot 2014}$?

13. _____ solutions



The integers 1 through 6 are to be used, each exactly once to fill the six circles in the figure so that the sums of three integers along each side of the triangle are the same. How many different solutions are possible? (Note that two solutions are considered the same if one can be rotated or reflected to obtain the other.)

14. _____ pairs

How many pairs of consecutive, positive three-digit multiples of 9 contain the same three digits?

15. _____

7	5	3	1
5	5	3	1
3	3	3	1
1	1	1	1

A square dartboard is divided into 16 congruent regions, with a point value assigned to each region as shown. Assuming each dart thrown hits the dartboard in a region, what is the probability that the sum of the points earned from three randomly thrown darts will be greater than 15? Express your answer as a common fraction.

16. _____ m^2

A right triangle has a hypotenuse of 10 m and a perimeter of 22 m. In square meters, what is the area of the triangle?

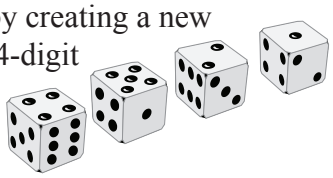
17. _____

For the following system of equations, what is the value of c?

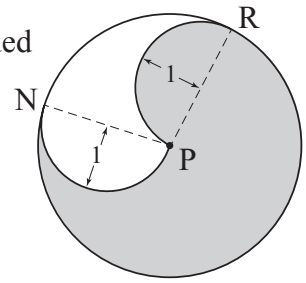
$$a + b + c + d = 88$$
$$a + b + c + e = 84$$
$$a + b + d + e = 82$$
$$a + c + d + e = 78$$
$$b + c + d + e = 72$$

18. _____

Alexi rolled four standard dice and lined them up to create a 4-digit number. He removed two dice from the line and rolled them again. Alexi then returned each re-rolled die to its original position in the line, thereby creating a new 4-digit number. What is the probability that the new 4-digit number is greater than the original 4-digit number? Express your answer as a common fraction.



19. _____ In circle P with radius 2 units, $m\angle NPR = 100°$. If the shaded region has area $k\pi$ units^2, what is the value of k? Express your answer as a common fraction.



20. _____ For integers a, b, c and d, $(x^2 + ax + b)(x^2 + cx + d) = x^4 + x^3 - 2x^2 + 17x - 5$. What is the value of $a + b + c + d$?

21. _____ Two random integers, a and b, are independently chosen, with replacement, from 1 to 1000, inclusive. What is the probability that both $2^a + 2^b$ and $3^a + 3^b$ are multiples of 5? Express your answer as a common fraction.

22. _____ What is the smallest integer greater than 38 that cannot be the length of the hypotenuse of a right triangle with integer side lengths?

23. _____ base 10 In base b, 441_b is equal to n^2 in base 10, and 351_b is equal to $(n-2)^2$ in base 10. What is the value of b, expressed in base 10?

24. _____ Larry tells Mary and Jerry that he is thinking of two consecutive integers from 1 to 10. He tells Mary one of the numbers, and he tells Jerry the other number. Then the following conversation occurs between Mary and Jerry:

Mary: I don't know your number.
Jerry: I don't know your number, either.
Mary: Ah, now I know your number.

Assuming both Mary and Jerry used correct logic, what is the sum of the possible numbers Mary could have?

25. _____ If the 4014th term of a geometric sequence of non-negative numbers is 135, and the 14th term is 375, what is the 2014th term?

26. $_____ A certain car drives 40 mi per gallon of gas at a rate of 60 mi/h. The same car drives 35 mi per gallon of gas at a rate of 75 mi/h. Thus, traveling at the higher speed saves time, but uses more gas. Gasoline costs $3.50 per gallon. What is the expense for each hour of time saved when traveling at the higher speed?

27. _____ An unfair coin has the property that when flipped four times, the probability of it landing twice heads up and twice tails up (in any order) is the same as the probability of it landing three times heads up and once tails up (in any order). What is the probability of this coin landing heads up in one flip? Express your answer as a common fraction.

28. _____ If $f(x) = \frac{ax+b}{cx+d}$, $abcd \neq 0$ and $f(f(x)) = x$ for all x in the domain of f, what is the value of $a + d$?

29. _____ pages Sam and Delilah are reading different books. Today, Sam and Delilah read one chapter in their respective books, and they each read more than one page. Interestingly, they read the same number of pages, but the sum of the page numbers for the chapter Sam read was 880, and the sum of the page numbers for the chapter Delilah read was 1008. How many pages did Sam read today?

30. _____ The area of the largest equilateral triangle that can be inscribed in a square of side length 1 unit can be expressed in the form $a\sqrt{b} - c$ units^2, where a, b and c are integers. What is the value of $a + b + c$?

MATHCOUNTS®

2014
■ National Competition ■
Target Round

Name _____

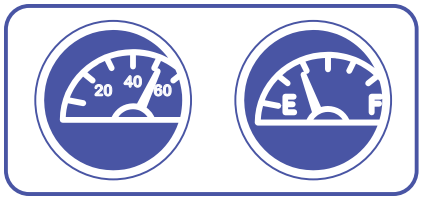
State _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

Total Correct	Scorer's Initials

1. _____ If $f(x) = ax^2 + bx + c, f(1) = 0, f(2) = 1$ and $f(3) = 8$, what is the value of c?

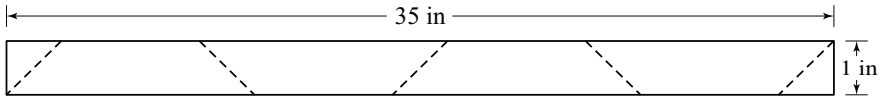
2. $_____ Ezra spent \$60 on gas each week for a period of three weeks. The first week gas was \$3 per gallon, the second week it was \$4 per gallon, and the third week it was \$5 per gallon. What was the average amount Ezra paid per gallon of gas over this three-week period? Express your answer as a decimal to the nearest hundredth.



3. _____ A two-digit, positive integer, b, is formed by reversing the digits of another two-digit, positive integer, a. If both $a + b$ and $a - b$ are perfect squares, what is the value of a?

4. _____travelers At a New York airport 135 international travelers were polled to see what language or languages each spoke. Of those polled, 87 spoke English; 86 spoke Spanish; 39 spoke French; 31 spoke English and Spanish, but not French; and 19 spoke English and French, but not Spanish. Only one person polled spoke Spanish and French but not English. If everyone spoke at least one of these three languages, how many travelers spoke all three languages?

5. _____ in^2 A 1-inch by 35-inch piece of wood was divided into six pieces by making five 45° cuts, as shown. The four trapezoidal pieces were kept, and the two triangular pieces were discarded. All four trapezoidal pieces were then used to form a rectangular picture frame. A picture measuring $17\frac{1}{2}$ inches by 20 inches was reduced proportionally so it could fit in the interior of the frame. If the cuts were made so that the re-sized picture fit exactly within the frame, what was the area of the interior of the frame?



6. _____ What is the sum of all real values of x that are solutions to the equation $\left(\frac{2}{3}x^2 - x - \frac{2}{3}\right)^{(x^2 - 9x + 20)} = 1$? Express your answer as a common fraction.

7. _____ If 1% of the planets in the universe contain water, and astronomers develop a test that is 95% accurate for determining whether or not a planet contains water, then what is the probability that a planet identified by the test as containing water really does contain water? Express your answer as a common fraction.

8. _____ in A circle passes through two diagonally opposite vertices of a 3-inch by 4-inch rectangle. What is the least possible distance between the center of the circle and a vertex of the rectangle? Express your answer as a common fraction.

MATHCOUNTS®

2014
■ National Competition ■
Team Round
Problems 1–10

State _____

Team Members _____ , Captain

_____

_____

_____

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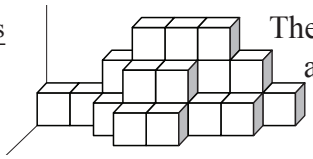
This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

1. _____ boxes



The figure shows some identical boxes in a room stacked flush against a wall and each other. How many boxes, including those not visible, are in this arrangement?

2. $_____

At a health food store, income from trail mix is $4.95 per pound. The expense, in dollars, of preparing n pounds of the trail mix is represented by $-0.005n^2 + 5n + 620$. If the store prepares and sells 600 pounds of trail mix, after expenses, what is the total profit?

3. _____

The prime factorization of 1995, which is $3 \times 5 \times 7 \times 19$, uses each odd digit exactly once and 1995 is the smallest positive integer with this property. What is the next smallest?

4. _____

A sequence is defined by $a_1 = 0$, $a_2 = 4$ and $a_n = 4(a_{n-1} - a_{n-2})$ for $n > 2$. What is the greatest value of n such that $n < 100$ and a_n is a power of 2?

5. _____

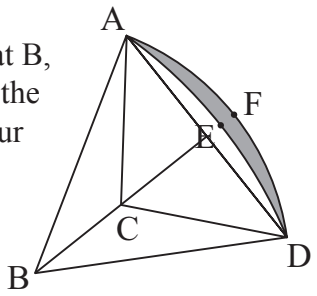
If three standard six-sided dice are rolled, what is the probability that the sum of the numbers on the top faces is 17 or 18? Express your answer as a common fraction.

6. _____ units^2

What is the area of the region defined by $|x| + |y| \leq 4$?

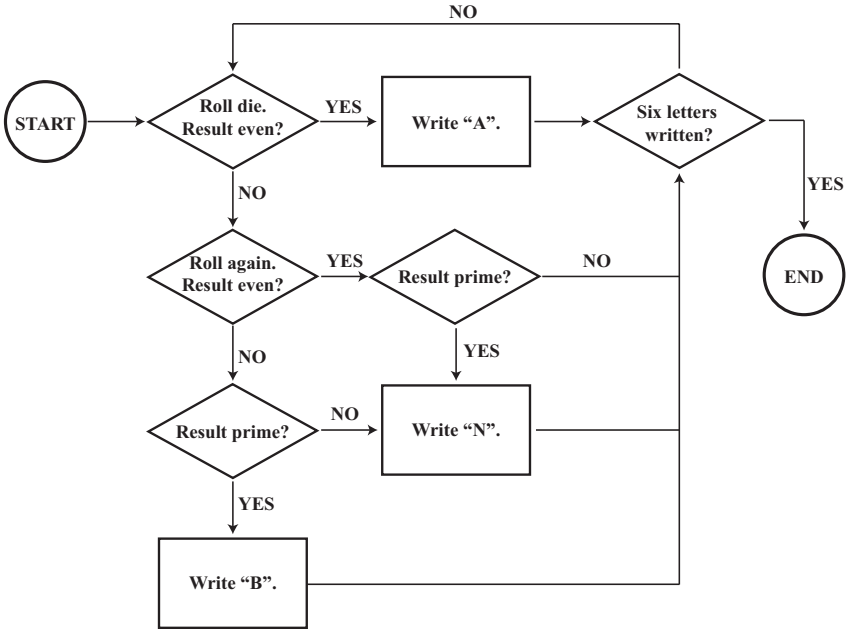
7. _____ in^2

In the figure, $\overset{\frown}{AFD}$ has its center at C, $\overset{\frown}{AED}$ has its center at B, AD = DB = 20 inches and $m\angle ACD = 2(m\angle ABD)$. What is the area of the shaded region between the two arcs? Express your answer to the nearest whole number.



8. _____

Using a standard six-sided die and the directions shown, a string of six letters is written in order. What is the probability that once 'END' is reached, the six letters that have been written are B, A, N, A, N and A, in that order? Express your answer as a common fraction.



9. _____ ways

How many ways are there to color the walls of a pentagonal room using five different colors, so that no two non-adjacent walls have the same color?

10. _____ degrees

In the triangle shown, what is the degree measure of $\angle ADB$?