2016–2017 MATHCOUNTS® School Handbook

Contains 250 creative math problems
that meet the NCTM Grades 6-8 Standards.

2017 MATHCOUNTS
National Competition Sponsor

National Sponsors:
Raytheon Company
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Change the Equation has recognized MATHCOUNTS as having one of the nation's most effective STEM learning programs, listing the Math Video Challenge as an Accomplished Program in STEMworks.

The National Association of Secondary School Principals has placed all three MATHCOUNTS programs on the NASSP Advisory List of National Contests and Activities for 2016-2017.

HOW TO USE THIS SCHOOL HANDBOOK

If You're a New Coach

Welcome! We're so glad you're a coach this year.
Check out the **Guide for New Coaches**
starting on the next page.

If You're a Returning Coach

Welcome back! Thank you for coaching again.
Get the **2016-2017 Handbook Materials**
starting on page 8.

GUIDE FOR NEW COACHES

Welcome to the MATHCOUNTS® Competition Series! Thank you so much for serving as a coach this year. Your work truly does make a difference in the lives of the students you mentor. We've created this Guide for New Coaches to help you get acquainted with the Competition Series and understand your role as a coach in this program.

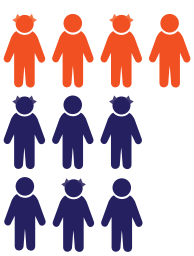
If you have questions at any point during the program year, please feel free to contact the MATHCOUNTS national office at (703) 299-9006 or info@mathcounts.org.

The MATHCOUNTS Competition Series in a Nutshell

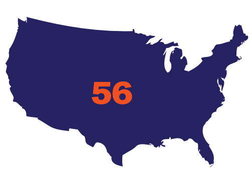
The **MATHCOUNTS Competition Series** is a national program that provides students the opportunity to compete in live, in-person math contests against and alongside their peers. Created in 1983, it is the longest-running MATHCOUNTS program and is open to all sixth-, seventh- and eighth-grade students.

HOW DOES IT WORK? The Competition Series has 4 levels of competition—school, chapter, state and national. Here's what a typical program year looks like.

Schools register in the fall and work with students during the year. Coaches administer the School Competition, usually in January. Any number of students from your school can participate in your team meetings and compete in the School Competition. MATHCOUNTS provides the School Competition to coaches in November. Many coaches use this to determine which student(s) will advance to the Chapter Competition.



Between 1 and 10 students from each school advance to the local Chapter Competition, which takes place in February. Each school can send a team of 4 students plus up to 6 individual competitors. All chapter competitors—whether they are team members or individuals—participate in the individual rounds of the competition; then just the 4 team members participate in the team round. Schools also can opt to send just a few individual competitors, rather than forming a full team. Over 500 Chapter Competitions take place across the country.



Top students from each Chapter Competition advance to their State Competition, which takes place in March. Your school's registration fees cover your students as far as they get in the Competition Series. If your students make it to one of the 56 State Competitions, no additional fees are required.

Top 4 individual competitors from each State Competition receive an all-expenses-paid trip to the National Competition, which takes place in May. These 224 students combine to form 4-person state teams, while also competing individually for the title of National Champion.

WHAT DOES THE TEST LOOK LIKE? Every MATHCOUNTS competition consists of 4 rounds—Sprint, Target, Team and Countdown Round. Altogether the rounds are designed to take about 3 hours to complete. Here's what each round looks like.

Sprint Round
40 minutes
30 problems total
no calculators used
focus on speed and accuracy

Target Round
Approx. 30 minutes
8 problems total
calculators used
focus on problem-solving and mathematical reasoning

The problems are given to students in 4 pairs. Students have 6 minutes to complete each pair.

Team Round
20 minutes
10 problems total
calculators used
focus on problem-solving and collaboration

Only the 4 students on a school's team can take this round officially.

VS

Countdown Round
Maximum of 45 seconds per problem
no calculators used
focus on speed and accuracy

Students with highest scores on Sprint and Target Rounds compete head-to-head. This round is optional at the school, chapter and state level.

HOW DO I GET MY STUDENTS READY FOR THESE COMPETITIONS? What specifically you do to prepare your students will depend on your schedule as well as your students' schedules and needs. But in general, working through lots of different MATHCOUNTS problems and completing practice competitions is the best way to prepare to compete. Each year MATHCOUNTS provides the *School Handbook* to all coaches, plus lots of additional free resources online.

The next sections of this Guide for New Coaches will explain the layout of the *MATHCOUNTS School Handbook* and other resources, plus give you tips on structuring your team meetings and preparation schedule.

The Role of the Competition Coach

Your role as the coach is such an important one, but that doesn't mean you need to know everything, be a math expert or treat coaching like a full-time job. Every MATHCOUNTS coach has a different coaching style and you'll find the style that works best for you and your students. But in general **every good MATHCOUNTS coach must do the following**.

- Schedule and run an adequate number of practices for participating students.
- Help motivate and encourage students throughout the program year.
- Select the 1-10 student(s) who will represent the school at the Chapter Competition in February.
- Take students to the Chapter Competition or make arrangements with parents and volunteers to get them there.

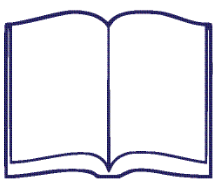
Looking for tools to help you become a top-notch coach? Check out our videos at the Coach Section of the MATHCOUNTS website!

You don't need to know how to solve every MATHCOUNTS problem to be an effective coach. In fact, many coaches have told us that they themselves improved in mathematics through coaching. Chances are, you'll learn with and alongside your students throughout the program year.

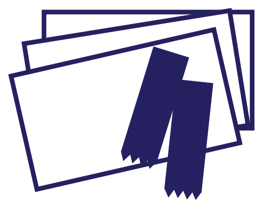
You don't need to spend your own money to be an effective coach. You can prepare your students using solely the free resources and this handbook. We give coaches numerous detailed resources and recognition materials so you can guide your Mathletes® to success even if you're new to teaching, coaching or competition math, and even if you use only the free resources MATHCOUNTS provides all competition coaches.

Making the Most of Your Resources

As the coach of a registered competition school, you already have received what we at MATHCOUNTS call the **School Competition Kit**. Your kit includes the following materials for coaches.



2016-2017 MATHCOUNTS School Handbook
The most important resource included in the School Competition Kit. Includes 250 problems.



Student Recognition Ribbons and Certificates
10 participation certificates and 1 ribbon for each registered chapter competitor.

You'll also get access to electronic resources. The following resources are available to coaches online at **www.mathcounts.org/coaches**. This section of the MATHCOUNTS website is restricted to coaches and you already should have received an email with login instructions. *If you have not received this email, please contact us at info@mathcounts.org to make sure we have your correct email address.*

Official 2017 MATHCOUNTS School Competition
Released in November 2016
Includes all 4 test rounds and the answer key

2016 MATHCOUNTS School, Chapter + State Competitions
Released in mid-April 2016
Each level includes all 4 test rounds and the answer key

MATHCOUNTS Problem of the Week
Released each Monday
Each multi-step problem relates to a timely event

You can use the **2017 MATHCOUNTS School Competition** to choose the students who will represent your school at the Chapter Competition. Sometimes coaches already know which students will attend the Chapter Competition. If you do not need the School Competition to determine your chapter competitors, then we recommend using it as an additional practice resource for your students.

The **2016-2017 MATHCOUNTS School Handbook** will be your primary resource for the Competition Series this year. It is designed to help your students prepare for each of the 4 rounds of the test, plus build critical thinking and problem-solving skills. This section of the Guide for New Coaches will focus on how to use this resource effectively for your team.

Check out our online coach resource videos:
Making the Most of Your Coaching Resources
How to Use the Handbook

WHAT'S IN THE HANDBOOK? There is a lot included in the *School Handbook*, and you can find a full table of contents on pg. 8 of this book, but below are the sections that you'll use the most when coaching your students.

- **Handbook Problems:** 250 math problems divided into Warm-Ups, Workouts and Stretches. These problems increase in difficulty as the students progress through the book. (pg. 13)

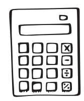
- **Solutions to Handbook Problems:** complete step-by-step explanations for how each problem can be solved. These detailed explanations are only available to registered coaches. (pg. 59)
- **Answers to Handbook Problems:** key available to the general public. Your students can access this key, but not the full solutions to the problems. (pg. 51)
- **Problem Index + Common Core State Standards Mapping:** catalog of all handbook problems organized by topic, difficulty rating and mapping to Common Core State Standards. (pg. 55)

There are 3 types of handbook problems to prepare students for each of the rounds of the competition. You'll want to have your students practice all of these types of problems.

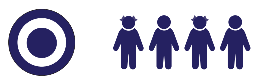
Warm-Ups
14 Warm-Ups in handbook
10 questions per Warm-Up
no calculators used

Warm-Ups prepare students particularly for the Sprint and Countdown Rounds.

Workouts
8 Workouts in handbook
10 questions per Workout
calculators used

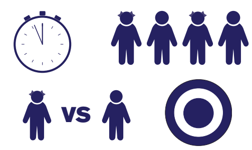


Workouts prepare students particularly for the Target and Team Rounds.



Stretches
3 Stretches in handbook
Number of questions and use of calculators vary by Stretch

Each Stretch covers a particular math topic that could be covered in any round. These help prepare students for all 4 rounds.



IS THERE A SCHEDULE I SHOULD FOLLOW FOR THE YEAR? On average coaches meet with their students for an hour once a week at the beginning of the year, and more often as the competitions approach. Practice sessions may be held before school, during lunch, after school, on weekends or at other times, coordinating with your school's schedule and avoiding conflicts with other activities.

Designing a schedule for your practices will help ensure you're able to cover more problems and prepare your students for competitions. We've designed the *School Handbook* with this in mind. Below is a suggested schedule for the program year that mixes in Warm-Ups, Workouts and Stretches from the *School Handbook*, plus free practice competitions from last year. This schedule allows your students to tackle more difficult problems as the School and Chapter Competition approach.

Mid-August – September 2016 Warm-Ups 1, 2 + 3 Workouts 1 + 2	**October 2016** Warm-Ups 4, 5 + 6 Workout 3 Fractions Stretch	**November 2016** Warm-Ups 7 + 8 Workouts 4 + 5 Angles and Arcs Stretch	**December 2016** Warm-Ups 9, 10 + 11 Workout 6 Bases Stretch
January 2017 Warm-Ups 12, 13 + 14 Workouts 7 + 8 *2017 MATHCOUNTS School Competition* *Select chapter competitors (optional at this time)*		**February 2017** Practice Competition: 2016 School Competition Practice Competition: 2016 Chapter Competition *Select chapter competitors (required by this time)* *2017 MATHCOUNTS Chapter Competition*	

You'll notice that in January or February you'll need to select the 1-10 student(s) who will represent your school at the Chapter Competition. This must be done before the start of your local Chapter Competition. You'll submit the names of your chapter competitors either online at **www.mathcounts.org/coaches** or directly to your local Chapter Coordinator.

It's possible you and your students will meet more frequently than once a week and need additional resources. If that happens, don't worry! You and your Mathletes can work together using the **Interactive MATHCOUNTS Platform**, powered by NextThought. This free online platform contains numerous *MATHCOUNTS School Handbooks* and past competitions, not to mention lots of features that make it easy for students to collaborate with each other and track their progress. You and your Mathletes can sign up for free at mathcounts.nextthought.com.

Check out the Interactive MATHCOUNTS Platform to get even more handbook problems and past competitions!

And remember, just because you and your students will meet once a week doesn't mean your students can only prepare for MATHCOUNTS one day per week. Many coaches assign "homework" during the week so they can keep their students engaged in problem solving outside of team practices. Here's one example of what a 2-week span of practices in the middle of the program year could look like.

Monday	Tuesday	Wednesday (Weekly Team Practice)	Thursday	Friday
-Students continue to work individually on Workout 4, due Wednesday	-Students continue to work on Workout 4 -Coach emails team to assign new Problem of the Week, due Wednesday	-Coach reviews solutions to Workout 4 -Coach gives Warm-Up 7 to students as timed practice and then reviews solutions -Students discuss solutions to Problem of the Week in groups	-Coach emails math team to assign Workout 5 as individual work, due Wednesday	-Students continue to work individually on Workout 5
-Students continue to work individually on Workout 5, due Wednesday	-Students continue to work on Workout 5 -Coach emails team to assign new Problem of the Week, due Wednesday	-Coach reviews solutions to Workout 5 -Coach gives Warm-Up 8 to students as timed practice and then reviews solutions -Students discuss solutions to Problem of the Week in groups	-Coach emails math team to assign Workout 6 as group work, due Wednesday	-Students work together on Workout 6 using online Interactive Platform

WHAT SHOULD MY TEAM PRACTICES LOOK LIKE? Obviously every school, coach and group of students is different, and after a few practices you'll likely find out what works and what doesn't for your students. Here are some suggestions from veteran coaches about what makes for a productive practice.

- Encourage discussion of the problems so that students learn from each other
- Encourage a variety of methods for solving problems
- Have students write math problems for each other to solve
- Use the **Problem of the Week** (posted online every Monday)
- Practice working in groups to develop teamwork (and to prepare for the Team Round)
- Practice oral presentations to reinforce understanding

On the following page is a sample agenda for a 1-hour practice session. There are many ways you can structure math team meetings and you will likely come up with an agenda that works better for you and your group. It also is probably a good idea to vary the structure of your meetings as the program year progresses.

MATHCOUNTS Team Practice Sample Agenda – 1 Hour

Review Problem of the Week (20 minutes)

- Have 1 student come to the board to show how s/he solved the first part of the problem.
- Discuss as a group other strategies to solve the problem (and help if student answers incorrectly).
- Have students divide into groups of 4 to discuss the solutions to the remaining parts of the problem.
- Have 2 groups share answers and explain their solutions.

Timed Practice with Warm-Up (15 minutes)

- Have students put away all calculators and have one student pass out Warm-Ups (face-down).
- Give students 12 minutes to complete as much of the Warm-Up as they can.
- After 12 minutes is up, have students hold up pencils and stop working.

Play Game to Review Warm-Up Answers (25 minutes)

- Have students divide into 5 groups (size will depend on number of students in meeting).
- Choose a group at random to start and then rotate clockwise to give each group a turn to answer a question. When it is a group's turn, ask the group one question from the Warm-Up.
- Have the group members consult their completed Warm-Ups and work with each other for a maximum of 45 seconds to choose the group's official answer.
- Award 2 points for a correct answer on questions 1-3, 3 points for questions 4-7 and 5 points for questions 8-10. The group gets 0 points if they answer incorrectly or do not answer in 45 seconds.
- Have all students check their Warm-Up answers as they play.
- Go over solutions to select Warm-Up problems that many students on the team got wrong.

Register for the National Math Club to get fun math games and activities... all for free! **Learn more on pg. 11.**

OK I'M READY TO START. HOW DO I GET STUDENTS TO JOIN? Here are some tips given to us from successful competition coaches and club leaders for getting students involved in the program at the beginning of the year.

- Ask Mathletes who have participated in the past to talk to other students about participating.
- Ask teachers, parent volunteers and counselors to help you recruit.
- Reach parents through school newsletters, PTA meetings or Back-to-School-Night presentations.
- Advertise around your school by:
    1. posting intriguing math questions (specific to your school) and referring students to the first meeting for answers.
    2. designing a bulletin board or display case with your MATHCOUNTS poster (included in your School Competition Kit) and/or photos and awards from past years.
    3. attending meetings of other extracurricular clubs (such as honor society) so you can invite their members to participate.
    4. adding information about the MATHCOUNTS team to your school's website.
    5. making a presentation at the first pep rally or student assembly.

Good luck in the competition! If you have any questions during the year, please contact the MATHCOUNTS national office at (703) 299-9006 or info@mathcounts.org.

Coach Resources: www.mathcounts.org/coaches

2016-2017 HANDBOOK MATERIALS

Thank you for being a coach in the MATHCOUNTS Competition Series this year!
We hope participating in the program is meaningful and enriching for you and your Mathletes.
Don't forget to log in at **www.mathcounts.org/coaches** for additional resources!

What's in This Year's Handbook

HIGHLIGHTED RESOURCES

Also access resources at www.mathcounts.org/coaches!

C Great for Coaches   M Great for Mathletes   A Advanced Level Book   Free Resource

OPLET

Online database of over 13,000 problems and over 5,000 step-by-step solutions. Create personalized quizzes, flash cards, worksheets and more!

Save $25 when you buy your subscription by Oct. 17, 2016
Renewers: use code RENEW17
First-Time Subscribers: use code NEW17

C

www.mathcounts.org/myoplet

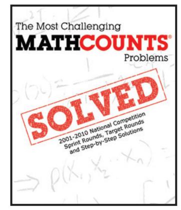
Practice Competitions for MATHCOUNTS, Vol. I & II

Practice books written by repeat national-level coach Josh Frost. Each volume includes 4 complete mock-competitions plus solutions.

C M

www.mathcounts.org/store

Most Challenging MATHCOUNTS Problems Solved

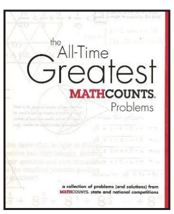


Advanced level practice book with 10 years of national-level Sprint Rounds, plus detailed step-by-step solutions to each problem.

C M A

www.mathcounts.org/store

All Time Greatest MATHCOUNTS Problems



A collection of some of the most creative, interesting and challenging MATHCOUNTS competition problems.

C M A

www.mathcounts.org/store

Interactive MATHCOUNTS Platform

Online platform of past and current handbook and competition problems. Interactive features make collaboration easy and fun!

C M

mathcounts.nextthought.com

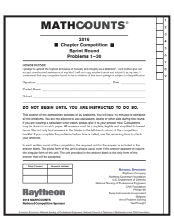
MATHCOUNTS Trainer App

Train your Mathletes with this fun app, featuring real-time leaderboards and lots of past MATHCOUNTS problems.

M

aops.com/mathcounts_trainer or download at the App Store

Past Competitions



Last year's School, Chapter and State competitions are free online! Other years' competitions can be purchased.

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www.mathcounts.org/pastcompetitions
www.mathcounts.org/store

Problem of the Week

A new, multi-step problem every week! Each problem focuses on a particular set of math skills and coincides with a timely event, holiday or season. Get the problem at the beginning of the week and the step-by-step solution the following week.

C M

www.mathcounts.org/potw

MATHCOUNTS Minis

A fun monthly video series featuring Richard Rusczyk from Art of Problem Solving. Each video looks at a particular math skill and walks through how to solve different MATHCOUNTS problems using creative problem-solving strategies.

C M

www.mathcounts.org/minis

Critical 2016-2017 Dates

2016

Aug. 15 – Dec. 16	Submit your school's registration to participate in the Competition Series and receive this year's School Competition Kit, which includes a hard copy of the *2016-2017 MATHCOUNTS School Handbook*. Kits are shipped on an ongoing basis between mid-August and December 31. The fastest way to register is online at **www.mathcounts.org/compreg**. You also can download the MATHCOUNTS Competition Series Registration form and mail or email it with payment to: MATHCOUNTS Foundation – Competition Series Registrations 1420 King Street, Alexandria, VA 22314 *Email*: reg@mathcounts.org To add students to your school's registration, log in at **www.mathcounts.org/coaches** to access the Dashboard. **Questions?** Call the MATHCOUNTS national office at (703) 299-9006 or email us at info@mathcounts.org.
Nov. 1	The 2017 School Competition will be available online. All registered coaches can log in at www.mathcounts.org/coaches to download the competition.
Nov. 18 (postmark)	**Deadline to register for the Competition Series at reduced registration rates** ($90 for a team and $25 for each individual). After November 18, registration rates will be $100 for a team and $30 for each individual.
Dec. 16 (postmark)	**Competition Series Registration Deadline** In some circumstances, late registrations might be accepted at the discretion of MATHCOUNTS and the local coordinator. *Late fees will apply. Register on-time to ensure your students' participation.*

2017

Early Jan.	If you have not been contacted with details about your upcoming competition, call your local or state coordinator. Coordinator contact information is available at www.mathcounts.org/findmycoordinator.
Late Jan.	If you have not received your School Competition Kit, contact the MATHCOUNTS national office at (703) 299-9006 or info@mathcounts.org.
Feb. 1-28	**Chapter Competitions**
March 1-31	**State Competitions**
May 14-15	**2017 Raytheon MATHCOUNTS National Competition in Orlando, FL**

Other MATHCOUNTS Programs

MATHCOUNTS was founded in 1983 as a way to provide new avenues of engagement in math for middle school students. MATHCOUNTS began solely as a competition, but has grown to include 3 unique but complementary programs: the **MATHCOUNTS Competition Series**, the **National Math Club** and the **Math Video Challenge**. Your school can participate in all 3 MATHCOUNTS programs!

THE NATIONAL MATHCLUB powered by MATHCOUNTS®

The **National Math Club** is a free enrichment program that provides teachers and club leaders with resources to run a math club. The materials provided through the National Math Club are designed to engage students of all ability levels—not just the top students—and are a great supplement for classroom teaching. This program emphasizes collaboration and provides students with an enjoyable, pressure-free atmosphere in which they can learn math at their own pace.

Active clubs also can earn rewards by having a minimum number of club members participate (based on school/organization/group size). **There is no cost to sign up for the National Math Club**, and registration is open to schools, organizations and groups that consist of at least 4 students in 6th, 7th and/or 8th grade and have regular in-person meetings. More information can be found at **www.mathcounts.org/club**, and the 2016-2017 School Registration Form is included on the next page.

MATHVIDEO CHALLENGE produced by MATHCOUNTS®

The **Math Video Challenge** is an innovative program that challenges students to work in teams of 4 to create a video explaining the solution to a MATHCOUNTS handbook problem and demonstrating its real-world application. This project-based activity builds math, communication and collaboration skills.

Students post their videos to the contest website, where the general public votes for the best videos. The 100 videos with the most votes advance to judging rounds, in which 20 semifinalists and, later, 4 finalists are selected. This year's finalists will present their videos to the students competing at the 2017 Raytheon MATHCOUNTS National Competition, and the 224 Mathletes will vote to determine the winner. Members of the winning team receive college scholarships. **Registration is completely free** and open to all 6th, 7th and 8th grade students. More information can be found at **videochallenge.mathcounts.org**.

! The fastest way to register your school is online at **www.mathcounts.org/clubreg**!

THE NATIONAL **MATHCLUB** powered by **MATHCOUNTS**®

2016-2017 **SCHOOL** REGISTRATION FORM

This registration form is for U.S. middle schools only. To register a non-school group (such as a Girl Scout Troop, Boys and Girls Club Chapter or math circle) for the National Math Club, please go to www.mathcounts.org/club to review eligibility requirements and register.

Step 1: Fill in your school's name and confirm eligibility to participate.

*required information

☒ ***U.S. school with students in 6th, 7th and/or 8th grade***

*School Name** ______________________________

***There can be multiple clubs at the same U.S. school**, as long as each club has a different club leader.*

By signing below I, the club leader, affirm that the school named above is a U.S. school with students in sixth-, seventh- and/or eighth-grade and is therefore eligible to participate in the National Math Club. I affirm that I have permission to register the school above for this program and I understand that MATHCOUNTS can cancel my membership at any time if it is determined that my group is ineligible.

Club Leader Signature* ______________________________

Step 2: Provide your information so we can send you materials and set up your online access.

*required information

Club Leader Name* ______________________________ Club Leader Phone ______________________________

Club Leader Email Address* ______________________________

Club Leader Alternate Email Address ______________________________

Club Address* ______________________________

City, State ZIP* ______________________________

Estimated total number of participating students in club (minimum 4 students)*: __________

My school previously participated in the National Math Club.

School Type (please check one)*: ☐ Public ☐ Charter ☐ Private ☐ Homeschool ☐ Virtual

Department of Defense or State Department schools, please provide additional information below.

Clubs located outside of the U.S. states or territories are not eligible to participate in the National Math Club unless they are in schools affiliated with the U.S. Department of Defense or State Department.

My school is sponsored by (please check one): ☐ U.S. Department of Defense (DoDDS)
☐ U.S. State Department

Country ______________________________

Step 3: Almost done... just turn in your form.

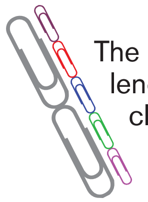
Mail or email a scanned copy of this completed form to:
MATHCOUNTS Foundation, 1420 King Street, Alexandria, VA 22314
Email: reg@mathcounts.org

Questions?
Please call the national office at
703-299-9006

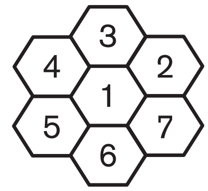
Warm-Up 1

1. ________ years A time capsule was sealed in 1940 and will be opened on the same date in 2017. How long will the capsule remain sealed?

2. ________ large paper clips



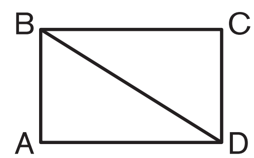
The length of 5 small paper clips is equal to the length of 2 large paper clips. The length of 8 small paper clips is equivalent to the length of how many large paper clips? Express your answer as a mixed number.



3. ________ paths In the figure shown, if no cell may be visited more than once and not every cell must be visited, how many paths start in cell 1 and end in cell 7?

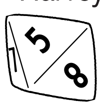
4. ________ What is the value of $4 \div \frac{2}{3} - 5$?

5. ________ What is the absolute difference between $\frac{1}{2}$ and $\frac{1}{3}$? Express your answer as a common fraction.



6. ________ cm If the perimeter of rectangle ABCD is 34 cm and AB = 5 cm, what is the perimeter of $\triangle ABD$?

7. ________ Harvey has a fair eight-sided die that has a different number from 1 to 8 on each side. If he rolls this die twice, what is the probability that the second number rolled is greater than or equal to the first number? Express your answer as a common fraction.



8. ________ In scientific notation, what is the product of 1.2×10^3 and 1.4×10^2? Express your answer to two significant figures.

9. ________ When the integer n is squared, the result is less than 150. What is the sum of all possible values of n?

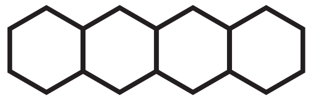
10. $________ Minnie paid a one-time registration fee of $30 for dance lessons. Additionally, she paid $20 per lesson. If she took seven lessons, how much did she pay altogether?

Warm-Up 2

11. ________ pieces How many pieces that are exactly 5 inches long can Sue cut from a string that is 7 feet long?

12. ________ units How many units away from the origin is the point $(-3, -4)$?

13. ________ tables A hexagonal table that is by itself seats 6 people, one person at each side. A row of hexagonal tables is created by pushing together a certain number of hexagonal tables so that a side of one table meets a side of the next table, in the way shown here. If 50 people can sit at the row of tables that was created, how many tables are in the row?



14. ________ coins In Oregon, which has no sales tax, Gloria bought three notebooks for \$1.57 each. If she paid \$5.00, what is the least number of U.S. coins that she could get in change?

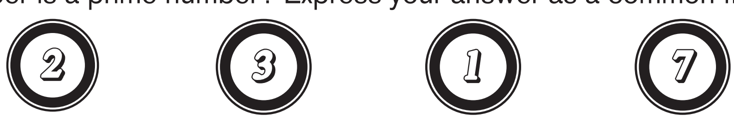
15. ________ g

The mass of a fluorine atom is 3.16×10^{-23} g. What is the mass of 1,000,000,000 fluorine atoms? Express your answer in scientific notation to three significant figures.

16. ________ meals A restaurant offers a dinner special in which diners can choose any one of 3 appetizers, any one of 4 entrées, any two different side dishes out of 5 and any one of 6 desserts. How many different meals are possible?

17. ________ minutes It takes 15 machines 15 minutes to make 500 raviolis. The machines produce raviolis at a steady rate. How long would it take 75 of these machines to make 6000 raviolis?

18. ________ Four chips are distinctly labeled with the digits 2, 3, 1 and 7, one chip for each digit. Two chips are drawn at random without replacement and placed in the order in which they are drawn, from left to right, to form a two-digit number. What is the probability that the two-digit number is a prime number? Express your answer as a common fraction.



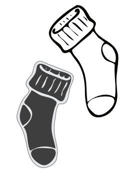
19. ________ The letters A, B, ... , Z are equally spaced in order on a number line, with A at 0 and Z at 25. What is the average of the two numbers that are 4 units from the letter M?

20. ________ units On a coordinate grid, point B is located 8 units below and 2 units to the left of A(0, 6). What is the length of segment AB? Express your answer in simplest radical form.

Warm-Up 3

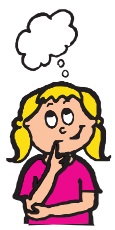
21.__________ What is the average of the integers from 13 to 31, inclusive?

22.__________ socks Kelly has 6 identical white socks and 5 identical black socks in a drawer. If she selects without looking, how many socks must she take from the drawer to be assured of a matching pair?



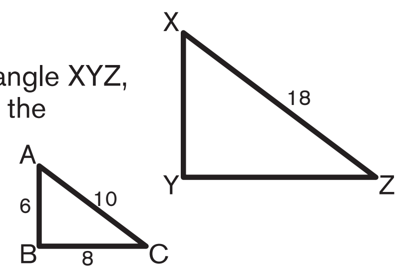
23.__________ For what value of m does $\frac{3}{m} = \frac{27}{72}$?

24.__________ chairs A room has 23 rows of 27 chairs each. How many chairs are in the room?



25.__________ Esme is thinking of two integers. One integer is 4 times the other, and their sum is 18 more than 3 times the smaller integer. What is the smaller of the integers Esme is thinking of?

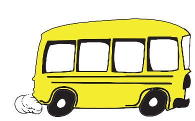
26.__________ Right triangle ABC has side lengths 6, 8 and 10 units. Right triangle XYZ, with hypotenuse of length 18 units, is similar to △ABC. What is the ratio of the area of △ABC to the area of △XYZ? Express your answer as a common fraction.



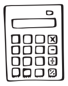
27.__________ A number is randomly selected from the integers 1 through 25, inclusive. What is the probability that the number chosen is divisible by 2, 3, 4 or 5? Express your answer as a common fraction.

28.__________ sequences Using only 1s and 2s, in how many different ordered sequences can Siddarth write a sum that equals 5? For example, a sum of 3 can be written 1 + 2, 2 + 1 or 1 + 1 + 1.

29.__________ miles Kevin takes a bus from home to school. The bus travels 8 miles west, then turns and travels 8 miles north, then turns and travels 7 miles west to the school. If the bus were able to travel directly from Kevin's house to the school, along a straight path, how much shorter would the trip be?



30.__________ What is the result when Ellen starts with the integer 123,456 and performs the following sequence of operations: subtract 6, divide by 10, subtract 5, divide by 10, subtract 4, divide by 10, subtract 3, divide by 10, subtract 2, divide by 10?



Workout 1

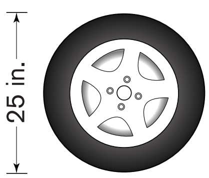
31. NZ$______ When one New Zealand dollar, NZ\$1.00, was worth US\$0.77, how much was US\$1.00 worth in New Zealand dollars?

32. ______ pennies A jar contains some number of pennies. When pennies are removed 2, 3, 4, 5, 6 or 8 at a time, one penny is left over. There are no pennies left over when they are removed 7 at a time. What is the least number of pennies that could be in the jar?

33. ______ lemmings The population of lemmings on an island in Norway varies drastically. In a base year, the population was n lemmings. The next year, it tripled. The third year, the number dropped by 3000 from the second year. The fourth year, the population was $\frac{1}{2}$ that of the third year. The fifth year, it increased by 1300 to a total of 1450 lemmings. What is the value of n?

34. ______ integers How many distinct positive four-digit integers can be formed using the digits 1, 2, 3 and 4 each once, such that no adjacent digits differ by more than 2?

35. ______ rotations

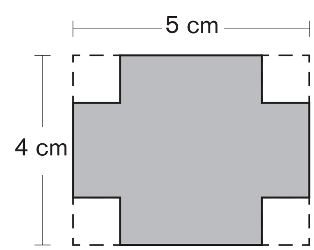


The tires on a certain car are 25 inches in diameter. If the car is moving at a constant speed of 65 miles per hour, how many rotations per second is the front left tire making? Express your answer as a decimal to the nearest tenth. (1 mile = 5280 feet)

36. week______ Nish trains to run a half marathon, a distance of 13.1 miles. Her training starts in week 1 with two 2-mile runs and one 4-mile run. Each week thereafter, the distances of her runs increase by 10% over the previous week's distances; therefore, she runs 8.8 miles in week 2. In which week of training does she first exceed 13.1 miles for the week?

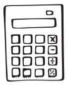
37. ______ Kylie writes a 1 after A, a 2 after B, and so on, writing a single digit in counting order after each of the first nine letters of the alphabet. When she reaches 10, she writes the 1 after J and the 0 after K. When she reaches the Z, the 1 from 18 follows it; then, she cycles back to the start of the alphabet, and the 8 follows the A. After what letter does Kylie write the 6 in 26?

38. ______ cm^2 A dodecagon is formed when a 1-cm by 1-cm square is removed from each corner of a 4-cm by 5-cm rectangle as shown. What is the area of the dodecagon?



39. ______ Jesse added all but one of the first ten positive integers together. The sum was a perfect square. Which one of the first ten positive integers did Jesse not include?

40. ______ purple marbles A bag contains 20 purple and 40 green marbles. How many purple marbles need to be added so that $\frac{5}{12}$ of the marbles will be green?



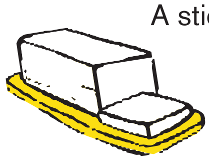
Workout 2

41. ________ feet A cube of cheese has edge length 1 inch and weighs 0.6 ounce. What is the edge length, in feet, of a cube of the same cheese that weighs 64.8 pounds? (1 pound = 16 ounces)

42. ________ integers How many positive integers from 1 to 100, inclusive have an even number of positive divisors?

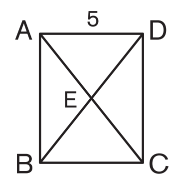
43. ________ What is the coordinate of the point on a number line that is $\frac{2}{3}$ of the way from −1.3 to $3\frac{1}{8}$? Express your answer as a decimal to the nearest hundredth.

44. ________ calories



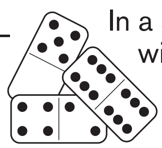
A stick of butter is a $1\frac{1}{2}$-inch by $1\frac{1}{2}$-inch by $3\frac{1}{4}$-inch rectangular prism and contains 800 calories. How many calories are in a pre-formed “pat” of butter measuring 1 inch by 1 inch by $\frac{3}{8}$ inch? Express your answer to the nearest whole number.

45. ________ degrees In rectangle ABCD, shown here, AD = BC = 5 units. Diagonals AC and BD, each of length 10 units, intersect at E. What is the degree measure of ∠AEB?



46. ________ What is the value of n if $4! + 5! = n!3!$?

47. ________



In a standard set of dominoes, a face of each domino has a line through the center, with 0 to 6 dots on each side of the line. Each possible combination of dots is used exactly once, one combination per domino. What is the probability that a randomly selected domino will have the same number of dots on both sides of the line? Express your answer as a common fraction.

48. ________ % Paige cuts a square out of a circular pizza. The corners of the square lie on the circumference of the pizza. To the nearest whole number, what percent of the pizza is left when Paige removes the square?

49. ________ zaggles In exchange for 5 ziggles and 4 zoggles, Jefferson gets 30 zaggles. In exchange for 2 ziggles and 3 zoggles, Monroe gets 19 zaggles. How many zaggles should Carter expect to get in exchange for 1 ziggle and 1 zoggle?

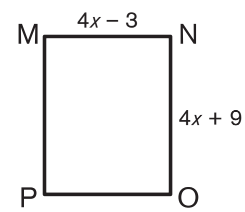
50. ________ feet When leaned against a vertical structure, a straight ladder can be used safely if its top is no more than 4 feet above the base of the structure for every foot that the bottom of the ladder is away from the base. How high can a 22-foot ladder safely reach up a vertical structure? Express your answer as a decimal to the nearest tenth.

Warm-Up 4

51.__________ Jamie's scores on the first five tests in his algebra class were 81, 75, 86, 98 and 92. After three more tests the median of his test scores was 88. What is the greatest possible value for the lowest score on these three tests?

52.__________ Consider the set of all possible two-digit numbers that can be created using an unlimited supply of 1s, 3s, 7s and 9s. What is the greatest absolute difference between any two primes in this set?

53.__________ The mean of a and b is 8. The mean of b and c is 16. The mean of a and c is 14. What is the value of $a + b + c$?



54.__________ units Rectangle MNOP has length $4x + 9$ and width $4x - 3$. What is the absolute difference between the length and width of rectangle MNOP?

55.__________ The faces of a cube are randomly and independently painted either red or blue with equal likelihood. What is the probability that the cube has all blue faces? Express your answer as a common fraction.

56.__________ If $n = n^2 - n$, what is the value of 5?

57.__________ Kavon has a fair eight-sided die with each side having a different one of the digits 1 through 8. He rolls the die twice and writes down, in order, the results to form a two-digit number. What is the probability that his two-digit number is prime? Express your answer as a common fraction.

58.__________ What is the value of $2x^2 + 3y^2 - 4x + 2y - 17$ when $x = 3$ and $y = -2$?

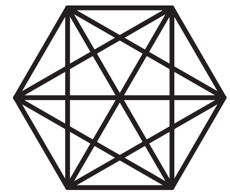
59.__________ Colorado used to issue license plates with the format of two letters (excluding Q) followed by four digits from 0 through 9. Later the state switched to a format of three letters (excluding Q) followed by three digits. What is the ratio of the number of possible old-style plates to the number of possible new-style ones? Express your answer as a common fraction.

60.__________ What is $40\% \times \frac{2}{3} \times 24 \div 0.8$?

Warm-Up 5

61. ________ What is the value of $(2 \times 6^3 + 6^2) - 7 \times 6^2$?

62. ________ triangles The figure shown consists of a regular hexagon and all of its diagonals. How many triangles in the figure have at least two congruent sides?



63. ________ ways Fido has to climb five stairs. If he steps on at least three of the five stairs, but never climbs more than three stairs in one step, in how many possible ways can Fido climb the stairs?

64. ________ The quotient $\dfrac{x^2(x^2)^3}{x^2}$ can be expressed as x^y. What is the integer value of y?

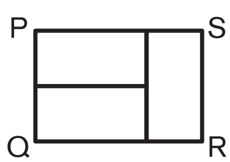
65. $ ________ A grocery store is required to charge customers an 8% sales tax on certain items. However, some purchases at the store, such as food products, are not subject to sales tax. During a certain month, the store sold $400,000 worth of groceries, not including the sales tax. If the store also collected $10,000 in sales tax that month, then what was the total amount (in dollars) of the store's sales that month that were not subject to sales tax?

66. ________ ways A school of 100 fish swims in the ocean and comes to a very wide horizontal pipe. The fish have three choices to get to the food on the other side: swim above the pipe, through the pipe or below the pipe. If we do not consider the fish individually, in how many ways can the entire school of fish be partitioned into three groups with each group choosing a different one of the three options and with at least one fish in each group?

67. ________ miles If Reid is traveling at a speed of 44 ft/s, how many miles will he travel in an hour given that 1 mile = 5280 feet?

68. ________ ways In how many different ways can the letters of CHAIRS be arranged?

69. ________ cm



The figure shows rectangle PQRS composed of three congruent rectangles. If the area of PQRS is 1536 cm^2, what is its perimeter?

70. ________ What is the value of the following expression?

$$2^{0^{12}} - 2^{2^{10}}$$

Warm-Up 6

71. $________ Amy's favorite lotion costs \$3.00 for 4 fluid ounces. At that same rate, what would she expect to pay for a quart of lotion? (1 quart = 32 fluid ounces)

72. ________

Multiply →, ↓

3	2	**6**
4	1	**4**
12	**2**	**24**

Multiply →, ↓

18	*a*	
b	4	
		5184

In the first grid, numbers were multiplied vertically and horizontally until a value was found for the shaded box. For instance, $3 \times 2 = 6$ and $4 \times 1 = 4$ were the results from the first two rows, and then $6 \times 4 = 24$ in the third column. The partially completed second grid follows the same rules, and both a and b are positive integers. What is the least possible value of $a + b$?

73. ________ The consecutive counting numbers are written in a triangular table, as shown, with one more number in each successive row. What is the sum of the numbers in the row that contains 25?

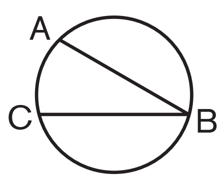
1
2 3
4 5 6
7 8 9 10

74. ________ In the arithmetic sequence 12, w, x, y, z, 47, what is the value of y?

75. ________ cm² The width of a rectangle is one-third of its length. If the perimeter of the rectangle is 136 cm, what is its area?

76. ________ permutations How many three-letter permutations can be made using letters from ALASKA?

77. ________ inches



The circle shown has a diameter of 12 inches, $m\angle ABC = 30$ degrees and AB = BC. What is the length of minor arc AC? Express your answer in terms of π.

78. ________ If A represents a digit in the equation $0.0A = \sqrt{0.0049}$, what is the value of A?

79. ________ The absolute difference between two numbers is 6, and the absolute difference between their squares is 24. What is the product of the two numbers?

80. ________ If a ☯ b is defined as $a^2 - 2b^2$. What is 5 ☯ (4 ☯ 3)?

Workout 3

81. ______ The sum of eleven consecutive integers is 11. What is the least of these eleven integers?

82. ______ The mean of x and y is 12 and the mean of y and 12 is $\frac{z}{2}$. What is the mean of x and z?

83. ______ cm Squares A and B have at least one point in common. The area of square A is 225 cm^2 and the area of square B is 16 cm^2. What is the maximum distance between the centers of the squares? Express your answer as a common fraction in simplest radical form.

84. ______ In the number sequence 3, 5, 2, ..., after the first two terms, the nth term is defined as $a_n = a_{n-1} - a_{n-2}$. For example, $a_3 = a_2 - a_1 = 5 - 3 = 2$. What is the sum of the first 200 terms of this sequence?

85. ______ What is the sum of the positive integer factors of 2017?

86. ______ feet Oberon and Lance sit directly opposite each other at a large round table. Arthur sits at the same table, 20 feet from Oberon and 21 feet from Lance. What is the diameter of the table?

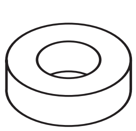
87. $ ______ The cost of 1 binder with photos of celebrities on the cover plus the cost of 8 regular binders is a total of $32.60. The cost of 1 binder with photos of celebrities on the cover plus the cost of 12 regular binders is a total of $46.00. How much more does it cost to buy a celebrity binder than a regular binder?

88. ______ widgets Mr. Jones makes 3% commission on his sales of widgets. At a different company, Mr. Smith makes 5% commission selling the same widgets at the same price. Mr. Smith sold 500 fewer widgets than Mr. Jones, and they both earned the same commission. How many widgets did Mr. Smith sell?

89. ______ The table of values shows the relationship between x and y, which can be modeled with the equation $y = ax^b$, for integers a and b. What is the value of $a + b$?

x	2	3	5	6
y	8	27	125	216

90. ______ in^2 Donny needs to find the surface area of a dozen donuts so he can make enough glaze. He doesn't know how to calculate the exact surface area of a donut, so he makes an approximation based on a roll of 1-inch-wide masking tape, which has the same volume as the donut. The outer radius of the tape is 2.5 inches, and the inner radius is 1.5 inches. What surface area did Donny estimate for a dozen donuts? Express your answer in terms of π.



Workout 4

91. ________ combinations On Saturday, three different football games are televised at noon and four different games are televised at 8 p.m. On Sunday, five different games are televised at noon. If Amanda watches one Saturday game at noon and another at 8 p.m. and one game at noon on Sunday, how many different combinations of games can she watch?

92. ________ If $a = 12$, $b = 4$, $c = 5$ and $x = \frac{1}{2}$, then what is the value of $\dfrac{\left(\frac{abc}{x}\right) - \left(6b^2 - 4\right)}{0.5}$?

93. ________ hours At noon, Randy's family left the Texas-Oklahoma border, traveling north on I-35. At noon, Marco's family left their home in Minnesota, 1029 miles from the Texas-Oklahoma border, traveling south on I-35. If Randy's family is traveling 45 mi/h and Marco's family is traveling 53 mi/h, how many hours will it take for the two families to pass each other? Express your answer as a decimal to the nearest tenth.

94. ________ What is the 41st digit after the decimal point in the decimal expansion of $\frac{1}{27}$?

95. ________ For the integers 15, 17, 11, 13, x, y, the mode, median and mean form an increasing arithmetic sequence, in that order. If $x \le y$, what is the greatest possible value of y?

96. ________ units^2

The hypotenuse of a 5-12-13 right triangle is the diameter of a semicircle containing the right angle vertex, as shown. What is the total area of the shaded regions? Express your answer to the nearest whole number.

97. ________ ways A group of 12 tourists will split up for two tours. A tour guide will lead one group on a hike. Another tour guide will lead the other group on a safari. If at least one tourist goes with each guide, in how many different ways can the tourists split up for the two tour groups?

98. ________ intersections What is the maximum number of distinct intersections of 30 different coplanar circles?

99. ________ % A sphere, a cone and a cylinder all have the same height and radius. The sphere and cone are completely filled with water. If the amount of water in the cylinder is the same as the total of the amounts in the sphere and cone, what percent of the cylinder's volume is filled?

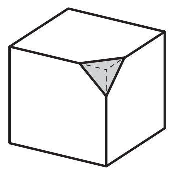
100. ________ What is the sum of all the prime numbers less than 500 with only 3s and 4s as digits?

Warm-Up 7

101. ________ marbles Mac has 25 marbles, of which 20% are red. Thayer has 20 marbles, of which 75% are not red. What is the absolute difference between the numbers of red marbles they have?

102. ________ If $f(x) = x^2 - 2$ and $g(x) = 2x + 4$, what is the value of $f(g(-3))$?

103. ________



In the figure shown, a triangular pyramid has been cut off the corner of the cube so that an equilateral triangle face is formed. If each corner of the cube is cut off in this manner, what is the maximum sum of the number of faces, edges and vertices on the new polyhedron?

104. $ ________ Four oranges cost a total of 90 cents. At this rate, what is the cost of 3 dozen oranges?

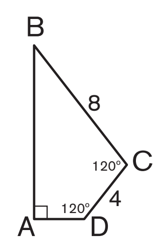
105. ________ When fully matured, a grape contains 80% water. After the drying process, called dehydration, the resulting raisin is only 20% water. What fraction of the original water in the grape remains after dehydration? Express your answer as a common fraction.

106. ________ The proper factors of a positive integer are all of the distinct positive integer factors of the number except the number itself. An abundant number is a positive integer whose proper factors sum to a value greater than the number. Which abundant number less than 50 has the greatest proper factor sum?

107. ________ If $(AAA)^3 = A6{,}926{,}0A7$, what digit does A represent?

108. ________ liters A Vermont syrup maker has 100 liters of a mixture that is $\frac{1}{4}$ maple syrup and $\frac{3}{4}$ base. She wants to add enough maple syrup to bring the ratio of maple syrup to base up to 1:1. If she has to evaporate 90% of the maple sap to get the maple syrup to add to that mixture, how many liters of maple sap does she need to start with?

109. ________ units² In quadrilateral ABCD, $m\angle C = m\angle D = 120$ degrees, $m\angle A = 90$ degrees, BC = 8, CD = 4. What is the area of ABCD? Express your answer in simplest radical form.



110. ________ arrange-ments Zzyzx Road is in California near Nevada. How many five-letter arrangements of the letters in the English alphabet follow Zzyzx alphabetically?

Warm-Up 8

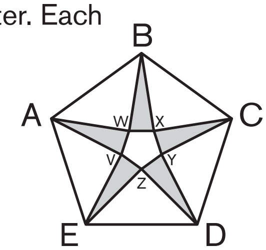
111.________ If $\frac{x}{y} = 10$, $x = 3z$ and $z = 20$, what is the value of y?

112.________ Cora has five balls—two red, two blue and one yellow—which are indistinguishable except for their color. She has two containers—one red and one green. If the balls are randomly distributed between the two containers, what is the probability that the two red balls will be alone in the red container? Express your answer as a common fraction.

113.______ units² What is the area of a circle that has diameter AB with endpoints A(−2, 4) and B(10, 2)? Express your answer in terms of π.

114.______ sets An arithmetic sequence of integers has 20 as the first term and 56 as the last term. How many different sets of integers form such a sequence?

115.______ degrees In the figure, regular pentagons ABCDE and VWXYZ have the same center. Each side of pentagon ABCDE is the hypotenuse of an isosceles right triangle. In each right triangle, the vertex opposite the hypotenuse is a vertex of pentagon VWXYZ. Each side of the smaller regular pentagon VWXYZ is also the base of one of the shaded acute isosceles triangles. What is the degree measure of the vertex angle of each shaded triangle?

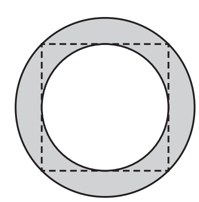


116.________ A 12-foot by 12-foot square bathroom needs to be tiled with 1-foot square tiles. Two of the tiles are the wrong color. If the tiles are placed randomly, what is the probability that the two wrong-colored tiles share an edge? Express your answer as a common fraction.

117.________ What digit is in the units place in the product $3^{17} \times 7^{23}$?

118.________ What is the geometric mean of 14 and 126?

119.______ in² The figure shows a square inscribed in a circle of radius 12 inches, and another circle inscribed within that square. What is the area of the shaded region between the two circles? Express your answer in terms of π.



120.______ base 3 What is the value of the sum $321_5 + 321_4$ when written in base 3?

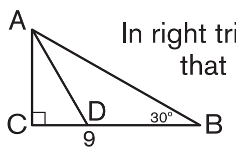
Workout 5

121. ________ A set of six different positive integers has a median and mean of 6. If the largest number in the set is 12, what is the largest possible sum for the three largest numbers?

122. ________ minutes per mile Nathan ran 2.5 miles at a pace of 7 minutes 36 seconds per mile. If he wishes to complete the entire 5-mile run at an average pace of 7 minutes 24 seconds per mile, what should his pace be for the next 2.5 miles? Express your answer as a decimal to the nearest tenth.

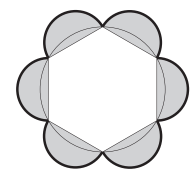
123. ________ The graphs of $y = x^2 - 3x + 3$ and $4x - 12y = -19$ intersect in two points. What is the sum of the x-coordinates of those points? Express your answer as a common fraction.

124. ________ cm



In right triangle ABC, $m\angle B$ = 30 degrees and BC = 9 cm. If D is on side BC so that segment AD bisects acute $\angle A$, what is DC?

125. ________ units A regular hexagon is inscribed in a circle of radius 1 unit. On each side of the hexagon, a semicircle is constructed with the side of the hexagon as a diameter, as shown. What is the perimeter of the figure formed by these semicircles? Express your answer as a decimal to the nearest tenth.

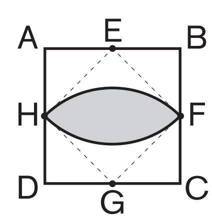


126. ________ integers For how many three-digit positive integers is the sum of the digits of the integer equal to 9?

127. ________ A toy manufacturer produces blue yo-yos and red yo-yos simultaneously at the same rate. During production, yo-yos of each color exit the assembly line in random order. What is the probability that the next four yo-yos that exit are all the same color? Express your answer as a common fraction.

128. ________ cm Ginger wants to make bubble tea in a cylinder-shaped cup with inside measurements of diameter 6 cm and height 12 cm. After she places 48 identical spherical tapioca bubbles into her empty cup, exactly 100π mL of liquid will fill the cup right to the top. Given that 1 mL = 1 cm^3, what is the radius of each tapioca bubble? Express your answer as a decimal to the nearest tenth.

129. ________ m^2 Square ABCD, shown here, has side length 2 meters, and E, F, G and H are midpoints of the sides. The curved lines are arcs of circles with centers at E and G. What is the area of the shaded region? Express your answer as a decimal to the nearest hundredth.

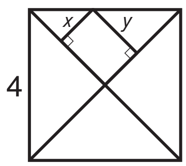


130. ________ Tony chooses a positive integer k. After his friends make the following five statements, Tony says that exactly two of them are correct. What is the least possible value of k?

a. Bruce guesses that k is a multiple of 15.
b. Steven guesses that k is a multiple of 18.
c. Thor guesses that k is a multiple of 20.
d. Clint guesses that k is a multiple of 28.
e. Natasha guesses that k is a multiple of 60.

Workout 6

131. ________ Caynan wrote a sequence of consecutive integers beginning with −37. If the sum of the integers he wrote is 200, what is the greatest integer in the sequence Caynan wrote?

132. ________ 

In the figure, the segments of lengths x and y lie on perpendiculars to the diagonals of a square of side length 4. The sum $x + y$ can be written in the form $\sqrt{z}$. What is the value of z?

133. ________ A box contains 26 slips of paper, each showing a different letter of the alphabet. If two slips of paper are drawn from the box at the same time, what is the probability that both letters appear in the word ALGEBRA? Express your answer as a common fraction.

134. $________ In Swimmington, where Maxwell lives, the charge for water usage is 0.15 cent per gallon. Maxwell has a cylindrical pool of height 4.5 feet and diameter 24 feet. What is the cost for Maxwell to fill his pool so that the water surface is 3 inches below the top of the pool, given that 1 gallon = 231 in^3? Express your answer to the nearest whole number.

135. ________ base 8 What is 10111010_2 when written in base 8?

136. ________ spheres Solid metal spheres with diameter $\frac{1}{6}$ inch are dropped into a rectangular prism tank, where they sink to the bottom. The tank is 10 inches wide by 15 inches long by 8 inches deep, and the water level is currently 3 inches. How many spheres does it take to raise the water level 1 inch? Express your answer to the nearest hundred.

137. ________ % Recently, the manufacturer changed how Leon's favorite pens are sold. The price of a box of pens has been reduced by 10%, and there are now 25% fewer pens per box. What is the percent change in the cost per pen?

138. ________ What is the greatest integer n such that n! has n digits?

139. ________ Adam has a triangle with vertices labeled 1 through 3. Jayvon has an octagon with vertices labeled 1 through 8. Each boy starts at position 1 and counts consecutive vertices on his polygon, continuing in the same direction, until he has reached 120 and is back at the vertex labeled 1. Percy did the same activity with his polygon, and he also finished at the vertex labeled 1. If Percy's polygon is not a triangle or an octagon, what is the sum of all the possible numbers of sides his polygon might have?

140. ________ On Mars a day is called a sol. Mars has a 668-sol year with a 7-sol week. If a regular Martian year has 95 weeks, and a leap year is one week longer, what fraction of the years are leap years? Express your answer as a common fraction.

Warm-Up 9

141. ________ Two positive integers have a sum of 11 and a product of 24. What is the absolute difference between those two numbers?

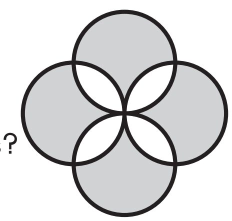
142. ________ Becca and Varun are walking side by side at the same constant speed. Becca steps onto a moving walkway and continues to walk at the same speed, while Varun walks alongside, maintaining his speed. When Becca reaches the end, Varun has covered only two-fifths of the length of the walkway. What is the ratio of the walkway's rate to Becca and Varun's walking speed? Express your answer as a common fraction.

143. ________ % Two sides of a regular pentagon are doubled and a new pentagon is formed. By what percent is the perimeter increased?

144. ________ quarters Joe has some nickels, dimes and quarters. He has 37 coins in all, with 4 more nickels than dimes and 2 more quarters than nickels. How many quarters does Joe have?

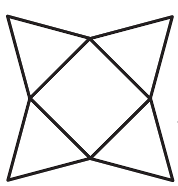
145. ________ numbers A *repeating* integer is one in which a sequence of digits occurs two or more times to make the entire number. The 4-digit number 4242 is a repeating integer. How many numbers are six-digit repeating integers?

146. ________ cm^2 Lucky draws a four-leaf clover by shading portions of four overlapping circles of radius 2 cm as shown. What is the area of the shaded regions?



147. ________ dates Eve's cousin, Fin, lives in a different country. According to a postcard Eve got, Fin plans to visit the U.S., but Eve can't tell the exact date of Fin's visit, because of the way the date is written. Eve doesn't know if the date format used in Fin's country is M/D or D/M, where M and D are different and represent the two-digit month and two-digit day, respectively. For how many dates in the year would both interpretations of the date written on the postcard result in a valid date?

148. ________



The net of a square pyramid, shown here, is a square with an equilateral triangle on each of its sides. The side length of the square can be expressed as $6x - 6$ or $2x + 14$, for the same value of x. When the net is folded to form a square pyramid, its surface area can be expressed in simplest radical form as $a^2(\sqrt{b} + c)$. What is the value of $a + b + c$?

149. ________ What single digit does D represent when $2D \times D51 = 807D$?

150. ________ extensions

The numbers 2 through 9 on a telephone keypad, like the one shown, are associated with the letters of the alphabet. Each person in a particular office is assigned a phone extension based on the first three letters of his or her last name. For instance, John DOE has the extension 363, and Marvella JOHnson has the extension 564. How many unique three-digit extensions can be assigned using the digits 2 through 9?

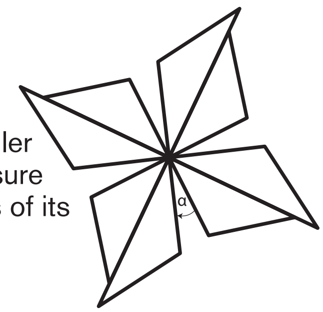
Warm-Up 10

151. ________ inches Keaton wants to build a rectangular prism with volume 2016 in^3 so that the length of each edge is a whole number of inches. What is the least possible sum of the three dimensions of the prism he builds?

152. ________ Cody's ZIP code is a five-digit number whose digits are all different. In this number, there are two pairs of adjacent digits in which the digits differ by 1. There is a pair of adjacent digits in which one digit is 4 times the other. There is a pair of adjacent digits whose sum is 10, as well as a pair of non-adjacent digits whose sum is 10. The sum of all five digits is a multiple of 10. If the leftmost digit of the number is 7, what is Cody's ZIP code?

153. ________ The product of the 3-digit number ABC and its reverse, CBA, is 140,209. If A, B and C each represent a different digit, what is the value of A + B + C?

154. ________ degrees A fan design has four pairs of similar isosceles triangles that create four blades, as shown. In each pair of triangles, the base of the smaller triangle is a segment of the base of the larger triangle, and the measure of the vertex angle of each triangle is twice the sum of the measures of its base angles. What is the degree measure of the angle labeled α?



155. ________ teams  Twelve couples participate in a fitness retreat. One strength-building exercise requires participants to form teams of three so that the two people who make up a couple are not on the same team. How many different teams of three can be formed in this manner?

156. ________ What common fraction is equivalent to $0.3\overline{27}$?

157. ________ Parallel lines *l*, *m* and *n* are in a plane with line *m* a distance of 1 cm from each of the lines *l* and *n*. Line *l* is tangent to a circle that has radius 3 cm. Lines *m* and *n* intersect the circle, and the four points of intersection are connected to form a trapezoid. If the area of the trapezoid is expressed in the form $\sqrt{a}+\sqrt{b}$ cm^2, what is the value of the product *ab*?

158. ________ What is the value of the expression $\dfrac{2017^2+11(2017)-42}{2014}$?

159. ________ What is the greatest integral value of *n* for which 32! has 2^n as a the factor?

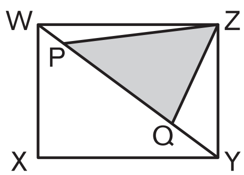
160. ________ combinations  Storm places coins, having a total value of at least \$1.00, in a bag. The coins may include pennies, nickels, dimes and quarters, but no more than three of any single denomination. How many different combinations of coins can Storm place in the bag?

Warm-Up 11

161.________ Three students each flip three fair coins. What is the probability that all three students get the same number of tails? Express your answer as a common fraction.

162.________ years Twins Taylor and Tyler were born on 05/02/07. This date is referred to as a *sum date* because the sum of the month and day is equal to the two-digit year: 05 + 02 = 07. How many years in the 21st century will have a sum date in each month during that year?

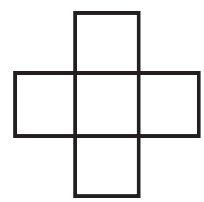
163.________ cm^2



The area of rectangle WXYZ is 90 cm^2. P and Q are points on diagonal WY such that 3(WP + QY) = 2PQ. What is the area of triangle PQZ?

164.________ There are 50 equally spaced points marked on a circle. Sara numbers them clockwise from 1 to 50. Starting at 1, she then draws congruent, connected segments between points that are 8 spaces apart, moving clockwise from the end of the previous segment. For example, the first segment is drawn from 1 to 9, and the second from 9 to 17. What is the sum of the numbers on the endpoints of the 23rd segment drawn?

165.________ colorings The five squares of the diagram shown are to be colored orange, yellow, green, blue and indigo, with exactly one color per square. Two colorings are the same if one is just a rotation of the other (but not if the diagram must be flipped over). How many distinct colorings are there?



166.________ What is the greatest four-digit palindrome that is divisible by 7 and 8?

167.________ players Tryouts were held for three positions on the school basketball team: center, guard and forward. There were 4 players who tried out for center, 10 for guard and 10 for forward. These numbers include 1 player who tried out for center and guard, 3 who tried out for center and forward, and 4 who tried out for guard and forward. All six of these counts include 1 player who tried out for all three positions. If 17 players, in all, tried out for the team, how many players tried out only for guard?

168.________ cm When ice melts and becomes water, its volume decreases by 8%. A cylindrical block of ice completely fills a container with a height of 10 cm and a radius of 4 cm. When all of the ice melts, what will be the height of the water in the cylinder? Express your answer as a decimal to the nearest tenth.

169.________ values A triangle exists with side lengths $2x$, $3x + 7$ and $6x - 5$ for how many integer values of x?

170.________ If x and y are integers, such that $x > y$, $(x + y)^2 = 9$ and $x^2 + y^2 = 29$, what is the smallest possible value for x?

Workout 7

171. ________ in^2 A circular pizza is cut along four diameters into eight identical sectors. If the total perimeter of each sector is 10 inches, what is the area of the whole pizza? Express your answer as a decimal to the nearest tenth.

172. ________ minutes Because of a traffic jam, Alana's 18-mile commute to work took 4 minutes longer than usual, and her average speed was decreased by 9 mi/h. How many minutes did it take her to get to work that day?

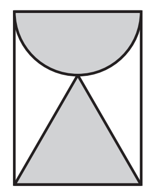
173. ________ g_n The standard gravitational acceleration of an object near Earth's surface is $g_n \approx 32$ ft/s^2. Kingda Ka is a roller coaster at the Six Flags amusement park in Jackson, New Jersey. It accelerates from a stop to 128 mi/h in 3.5 seconds. What is the acceleration of the roller coaster as a multiple of g_n? Express your answer as a decimal to the nearest tenth.

174. ________ % To the nearest whole percent, what percent of all positive integers are not multiples of 2, 3, 4, 5 or 6?

175. ________ Seventy-five bingo balls, each with a different positive integer from 1 through 75, are placed in a cage. A random ball is selected, its number is announced, and the ball is returned to the cage. This process occurs a total of 20 times. What is the probability that at least one ball is selected more than once? Express your answer as a decimal to the nearest hundredth.

176. ________ Tina and Tricia play on a softball team. Tina has 8 hits out of 20 times at bat, and Tricia has 6 hits out of 16 times at bat. Based on their past performance, what is the probability that both girls will get a hit the next time they bat? Express your answer as a common fraction.

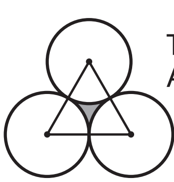
177. ________ % To create a new flag design, Howard paints a semicircle and an equilateral triangle inscribed in a rectangle as shown. What percent of the flag does the painted area cover? Express your answer to the nearest whole number.



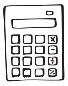
178. ________ The inhabitants of the planet Rundia run footraces similar to those run on Earth. However, Rundians measure the distances that they run in *bars*, where one bar measures four-fifths of a meter, and they measure time in *ticks*, where there are 100 ticks in one minute. Rundian sprinter Sejes Wesno can run a 100-bar race in 11.53 ticks. The great Earth sprinter Usain Bolt ran a 100-meter race in 9.58 seconds. If Wesno is k times as fast as Bolt, what is the value of k? Express your answer as a decimal to the nearest hundredth.

179. ________ Jason's line has a slope of $-\frac{1}{3}$ and contains the point $(5, -2)$. Amisha's line is perpendicular to Jason's and passes through the point $(4, 1)$. If the intersection of these lines is (x, y), what is the value of $x + y$? Express the answer as a common fraction.

180. ________ units^2



Three circles, each having radius 4 units, are externally tangent to each other. A triangle joins the centers of the circles. What is the area of the shaded region within the triangle but outside the circles? Express your answer as a decimal to the nearest tenth.

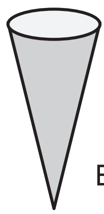
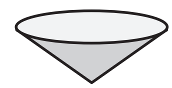


Workout 8

181. ________ units² In square WXYZ, point V is the midpoint of side YZ, and the area of $\triangle XYV$ is $\frac{4}{5}$ unit². What is the area of square WXYZ? Express your answer as a common fraction.

182. ________ tokens Ali gives Stan a closed box that contains at least one of each token worth 5, 11 or 19 points. Ali says that the tokens have a combined value of 56 points. How many tokens are in the box?

183. ________ Xera and Yeta use this method to decide who will sit in the front passenger seat of the car. Xera throws a standard six-sided die, after which Yeta picks a card from a standard deck of 52 cards, with replacement. They continue to take turns die-throwing and card-picking until either Xera wins by rolling a four or Yeta wins by picking a card with a four. What is the probability that Yeta wins? Express your answer as a common fraction.

184. ________ %  When a cone's height is decreased by a factor of four, to maintain the same volume, the radius must be increased by a factor of two, or 100%. When the cone's height is decreased by a factor of three, by what percent must the radius be increased to maintain the same volume? Express your answer to the nearest whole number. 

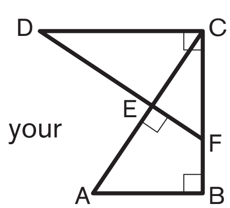
185. ________ integers Given the expression $\frac{n^2-9}{n^2-4}$, for how many positive integers n from 1 to 2016, inclusive, is the GCF of the numerator and denominator greater than 1?

186. ________ values The mean, the median and the mean of all modes of the integers 7, 4, 5, 6, 5, x are equal. How many possible values are there for x?

187. ________ A box contains 15 slips of paper, each bearing a different natural number from 1 to 15, inclusive. If three of these slips are randomly drawn, one at a time, without replacement, what is the probability that three consecutive numbers are drawn in increasing order? Express your answer as a common fraction.

188. ________ inches A rectangle has a diagonal of length 8 inches and an area of 26 square inches. What is its perimeter? Express your answer in simplest radical form.

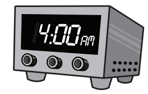
189. ________ units In pentagon ABCDE, with right angles ABC, BCD and AEF, as shown, AE = 14, DE = 18 and EF = 8. What is the length of side BC? Express your answer in simplest radical form.



190. ________ hours Pump P can fill a water tank in 12 hours, and Pump Q can fill the same tank in 15 hours. The two pumps started filling the tank at the same time and worked together until the tank was 60% full. At that point, Pump P was turned off, and Pump Q continued to fill the tank until it was completely full. How many hours did it take to completely fill the tank?

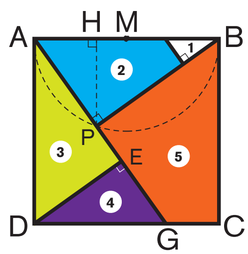
Warm-Up 12

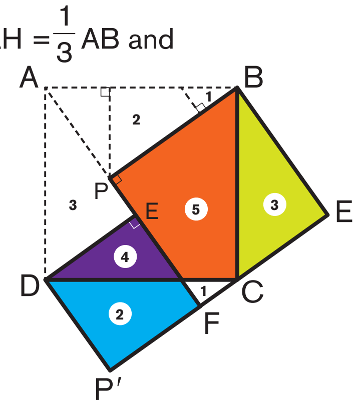
191.______ minutes The digital clock shown can display up to four digits to represent the hour and the minute. For how many minutes in a 12-hour period does the digit 0 appear on the clock?



192.______ What is the absolute difference between $1.\overline{18}$ and $2.3\overline{6}$? Express your answer as a common fraction.

193.______ inches A square is dissected into five pieces as shown on the left, with AH = $\frac{1}{3}$ AB and AM = MP = MB. The pieces are shaded and numbered. The pieces are rearranged so as to form two squares, one with twice the area of the other. The rearrangement of the pieces is shown on the right, overlaid on top of the original square. If square DEFP' in the second picture has a side length of 1 inch, then what is the side length of the original square? Express your answer in simplest radical form.



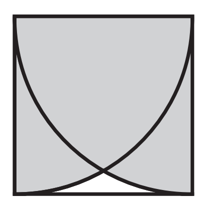


194.______ arrangements How many arrangements of all eight letters in TRESPASS do not have S as the final letter?

195.______ Four vertices of a regular octagon are chosen at random. What is the probability that a square can be made by connecting the vertices? Express your answer as a common fraction.

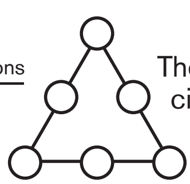
196.______ mi/h A private jet made a trip from Denver to Los Angeles in 3 hours, flying against a steady headwind. On the return trip the wind speed doubled and became a tailwind. The return trip took only 2.5 hours. If the plane's speed on the return trip was 450 mi/h, what was the speed of the original headwind?

197.______ In the square shown, the side lengths are 6 cm, and the intersecting arcs are quarter-circles. The area of the shaded region, expressed in simplest radical form in terms of π, is $a\pi + b\sqrt{c}$ cm^2. What is the value of $a + b + c$?



198.______ prisms A rectangular prism with length ≥ width ≥ height has positive integer dimensions and a volume of 60 units^3. How many different prisms are there that meet these conditions?

199.______ solutions



The numbers 1, 2, 3, 4, 5 and 6 are to be placed in this figure, one number per circle, so that the sums of the numbers on each side of the triangle are the same. How many distinct solutions are there, not including rotations and reflections?

200.______ minutes Xena runs halfway across a field in 1 minute. The next fourth of the field takes her $\frac{2}{3}$ of a minute to cross, the next eighth takes 4/9 of a minute, and so on, with each half of the previous distance taking $\frac{2}{3}$ of the previous time. How many minutes does it take Xena to cross the field?

Warm-Up 13

201. ________ Given $a - b = 3$ and $a^2 + b^2 = 65$, what is the value of $a^3 - b^3$?

202. ________ integers How many positive three-digit integers have one digit equal to the average of the other two digits?

203. ________ marbles A bag contains only red marbles and green marbles, two of which are to be drawn without replacement. There are at least two marbles of each color in the bag. If the probability of both marbles being red is half the probability of both marbles being green, then what is the minimum possible number of marbles in the bag?

204. ________ M(a, b) is the midpoint of the longest side of the triangle bounded by the lines $x + 2y = 8$, $5x + 2y = 48$ and $x - 2y = 0$. What is the value of $a + b$? Express the answer as a common fraction.

205. ________ A math club has 16 members. The coach wants to select three boys and three girls to represent their school at a tournament. There are six times as many ways to choose the girls as there are ways to choose the boys. What is the ratio of girls to boys in the club? Express your answer as a common fraction.

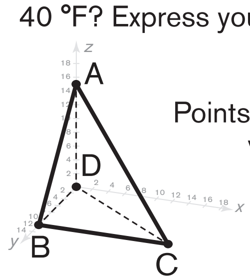
206. ________ The number 3638 has a digital sum of $3 + 6 + 3 + 8 = 20$ and a digital product of $3 \times 6 \times 3 \times 8 = 432$. What is the absolute difference between the least and greatest four-digit numbers that each have a digital sum of 20 and a digital product of 432?

207. ________ Jebediah has two coins in his pocket. One is a fair coin, while the other has heads on both sides. He pulls one coin out at random and flips it three times. If the coin lands heads all three times, what is the probability that it is the fair coin? Express your answer as a common fraction.

208. ________ people What is the minimum number of people that must be in a room to ensure that there are three people who, when considered pairwise, all know each other or three people who, when considered pairwise, all do not know each other?

209. ________ % Tire pressure is directly proportional to temperature on a temperature scale where zero degrees is absolute zero. Given that temperatures in degrees Celsius (C) and degrees Fahrenheit (F) are related by the formula $F = \frac{9}{5}C + 32$, and absolute zero is -273.15 °C, by what percent does tire pressure decrease when the temperature drops from 80 °F to 40 °F? Express your answer to the nearest whole number.

210. ________ units



Points A(0, 0, 15), B(0, 12, 0), C(16, 12, 0) and D(0, 0, 0) determine the vertices of a tetrahedron as shown. What is the shortest distance from B to the face ADC? Express your answer as a common fraction.

Warm-Up 14

211. ______ chords Eight points on a circle are labeled. How many chords can be drawn connecting any two of these points?

212. ______ Tad draws three cards at random, without replacement, from a deck of ten cards numbered 1 through 10. What is the probability that no two of the cards drawn have numbers that differ by 1? Express your answer as a common fraction.

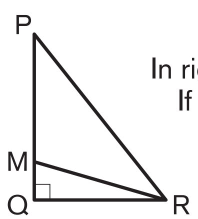
213. ______ people Twelve people have sheared $\frac{1}{3}$ of a field of pine trees in 7 days. How many more people need to be added to the crew to shear the rest of the trees in the field in the next 6 days?

214. ______ If three distinct integer lattice points are randomly selected from the interior of the circle defined by $x^2 + y^2 = 8$, what is the probability that they are the vertices of a triangle? Express your answer as a common fraction.

215. ______ numbers For how many four-digit numbers is the sum of the digits equal to the product of the digits?

216. ______ cubes A cube has one red, one green, one yellow and three blue faces. How many distinct cubes satisfying this description are possible?

217. ______ units



In right triangle PQR, shown here, M is on PQ such that PM = MR. If PQ = 12 units and QR = 9 units, what is the value of MQ? Express your answer as a common fraction.

218. ______ Karla writes down six different prime numbers in increasing order. She notices that the product of the first three prime numbers she has written is equal to the sum of the last three prime numbers she has written. What is the least possible value of the last prime number Karla wrote?

219. ______ Circle A has center (0, 0) with radius 4. Circle B has center (40, 40) with radius 6. The radius of circle B increases at twice the rate as the radius of circle A increases. When the circles are externally tangent at P(x, y), which is located between the centers of circles A and B, what is the value of x? Express your answer as a fraction in simplest radical form.

220. ______ ways

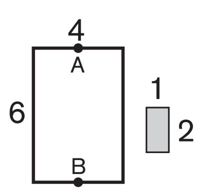


Figure 1

Figure 1 shows a 2 × 1 gray rectangle and a 6 × 4 white rectangle with the midpoints of two sides labeled A and B. If the larger rectangle cannot be rotated, in how many ways can 12 of the gray rectangles be arranged inside the white rectangle so that none crosses a segment from A to B? One such arrangement is shown in Figure 2, but not in Figure 3.

YES

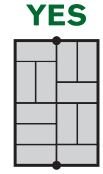


Figure 2

NO

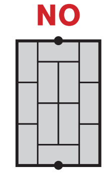


Figure 3

Fractions Stretch

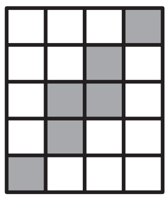
Solve the following problems. Express any non-integer answer as a common fraction.

221.________ What fraction of 100 is 25?

222.________ What fraction of $\frac{3}{8}$ is $\frac{9}{16}$?

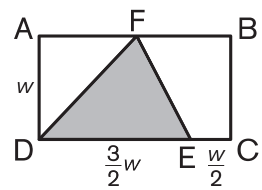
223.________ What is the value of $\sqrt{\frac{3}{11} \div \frac{11}{12}}$?

224.________



What fractional part of this grid of 20 unit squares is shaded?

225.________ What fraction of the area of rectangle ABCD is the area of inscribed triangle DEF?



226.________ On a number line, what common fraction is $\frac{3}{4}$ of the way from $\frac{1}{2}$ to $\frac{3}{4}$?

227.________ What is the reciprocal of $\frac{1}{2+\frac{1}{3}}$?

228.________ What common fraction is equal to $0.7\overline{5}$?

229.________ If $\dfrac{1}{\dfrac{1}{\frac{1}{n}+\frac{1}{3}}+\dfrac{1}{\frac{1}{3}+\frac{1}{n}}} = \dfrac{5}{12}$, what is the value of n?

230.________ If $\dfrac{2x}{x-3} - 2 = \dfrac{4}{x+2}$, what is the value of x?

Angles and Arcs Stretch

Secant	a line that intersects the circle at two points
Chord	a line segment whose endpoints are two points on the circle
Tangent	a coplanar line that intersects the circle at a single point of tangency
Central Angle	an angle with its vertex at the center of the circle
Inscribed Angle	an angle with its vertex on the circle and whose sides are chords of the circle
Major Arc	an arc of the circle with measure greater than or equal to 180°
Minor Arc	an arc of the circle with measure less than 180°

Angle and Arc Measures

In the figures below, observe how the degree measure of ∠AXB decreases as the distance between the vertex of the angle and the center of the circle increases.

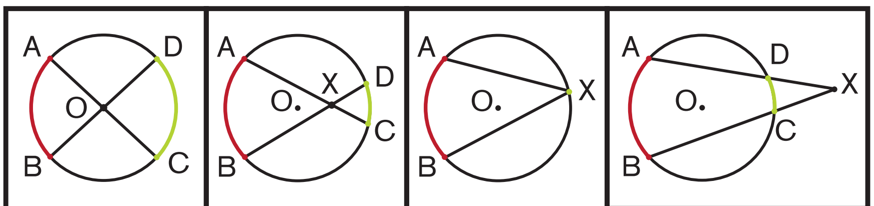


Figure I	Figure II	Figure III	Figure IV
$m\overset{\frown}{AB} = 80°$	$m\overset{\frown}{AB} = 80°$	$m\overset{\frown}{AB} = 80°$	$m\overset{\frown}{AB} = 80°$
$m\overset{\frown}{CD} = 80°$	$m\overset{\frown}{CD} = 40°$	$m\overset{\frown}{CD} = 0°$	$m\overset{\frown}{CD} = 40°$
$m\angle AOB = 80°$	$m\angle AXB = 60°$	$m\angle AXB = 40°$	$m\angle AXB = 20°$

- In Figure I, angles AOB and COD are central angles of circle O that intercept arcs AB and CD, respectively. The degree measure of a central angle and the arc it intercepts are equal.

$$m\angle AOB = m\overset{\frown}{AB} \text{ and } m\angle COD = m\overset{\frown}{CD}$$

- In Figure II, vertical angles AXB and CXD, formed by the intersection of chords AC and BD inside circle O, intercept arcs AB and CD, respectively. The degree measure of vertical angles formed by two chords intersecting inside a circle is half the sum of the measures of their intercepted arcs.

$$m\angle AXB = m\angle CXD = \tfrac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{CD})$$

- In Figure III, ∠AXB is inscribed in circle O. The degree measure of an inscribed angle is half the measure of the intercepted arc.

$$m\angle AXB = \tfrac{1}{2}m\overset{\frown}{AB}$$

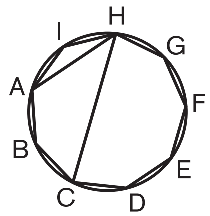
- In Figure IV, ∠AXB, formed by the intersection of two secants at point X outside of circle O, intercepts arcs AB and CD. The degree measure of an angle formed by two secants, two tangents or a secant and a tanget is half the difference of the measures of its intercepted arcs.

$$m\angle AXB = \tfrac{1}{2}(m\overset{\frown}{AB} - m\overset{\frown}{CD})$$

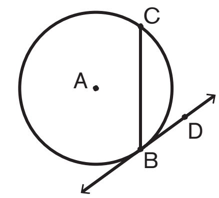
It may appear that there are four different formulas for calculating the four types of angles. But in each case, the measure of the angle in question is, essentially, the average of the measures of the intercepted arcs. In Figure IV, note that, with respect to ∠AXB, $\overset{\frown}{AB}$ appears concave, while $\overset{\frown}{CD}$ appears convex. So the measure of ∠AXB can be thought of as the average of 80° and −40°.

Solve the following problems by using what you've learned about angles and arcs. Express any non-integer value as a decimal to the nearest tenth.

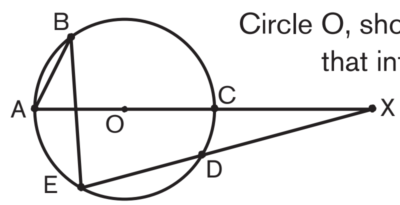
231.________° Regular nonagon ABCDEFGHI is inscribed in a circle, as shown. What is $m\angle AHC$?



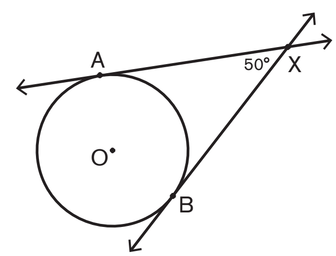
232.________° In circle A, shown here, $\overleftrightarrow{BD}$ is tangent to the circle at B, and major $\overset{\frown}{BC}$ has measure 230°. What is $m\angle CBD$?



233.________° Circle O, shown here with chords AB and BE, has secants AC and DE that intersect at X. If $m\angle ABE = 35°$ and $m\angle AXE = 15°$, what is the measure of $\overset{\frown}{CD}$?



234.________° In this figure, lines AX and BX are tangent to circle O at A and B, respectively. If $m\angle AXB = 50°$, what is the measure of major $\overset{\frown}{AB}$?



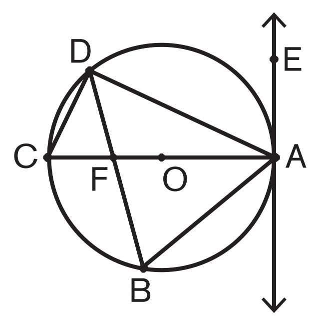
Use the figure at the right for questions 235 through 238.

235.________° What is $m\angle ABD$?

236.________° What is $m\overset{\frown}{AB}$?

237.________° What is $m\angle BAE$?

238.________° What is $m\angle CFD$?



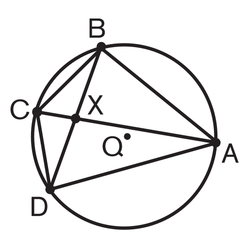
$\overleftrightarrow{AE}$ is tangent to circle O

$\overleftrightarrow{AE} \perp \overline{AC}$

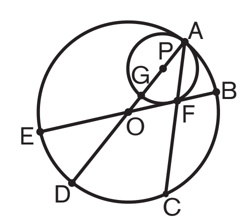
$m\angle BDC = 40°$

$m\overset{\frown}{AD} = 125°$

239.________° Quadrilateral ABCD is inscribed in circle Q, as shown, with diagonals intersecting at X. If $m\overset{\frown}{AB} = 110°$, $m\overset{\frown}{BC} = 60°$ and AB = BD, what is $m\angle CXD$?



240.________° Circle P is internally tangent to circle O at A, as shown. $\overline{AC}$ and $\overline{BE}$ intersect at F, which is also the point of tangency between $\overline{BE}$ and circle P. $\overline{AD}$ and $\overline{BE}$ are diameters of circle O, and $\overline{AG}$ is a diameter of circle P. If $m\overset{\frown}{CD} = 50°$, what is the measure of minor $\overset{\frown}{BC}$?



Bases Stretch

The **base 10** number system, the number system we are most familiar with, uses the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Numerals with these digits in the ones, tens, hundreds and higher places express specific numerical quantities. In base 10, the number 245, for example, is composed of 2 hundreds, 4 tens and 5 ones. That is, $2(10^2) + 4(10^1) + 5(10^0) = 200 + 40 + 5 = 245$.

A **base *b*** number system uses the digits 0, 1, ..., $b - 1$. Numerical quantities are expressed with these digits in the b^0, b^1, b^2 and higher places. In base b, if $b \geq 6$, the numeral 245_b represents the number $2(b^2) + 4(b^1) + 5(b^0)$. In base 8, for example, $245_8 = 2(8^2) + 4(8^1) + 5(8^0) = 2(64) + 4(8) + 5(1) = 128 + 32 + 5 = 165$.

Bases greater than 10 use letters to represent the digits greater than 9. For example, the 12 digits used in base 12 are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A and B. The numeral 10 in base 12 has 1 twelve and 0 ones. That is, $10_{12} = 1(12^1) + 0(12^0) = 1(12) + 0(1) = 12 + 0 = 12$.

Practice Problems

What is the representation of each of the following in base 10?

241.________ 24_9

242.________ 24_8

243.________ 24_7

What is the representation of 24 in each of the following bases?

244.________ base 9

245.________ base 8

246.________ base 7

Now try these.

247.________ What is the representation of 4991 in base 12?

248.________ What is the representation of $3BB_{12}$ in base 6?

249.________ If $523_b = 262$, what is the value of b?

250.________ If $441_b = n^2$ and $351_b = (n - 2)^2$, for some $b < 10$, what is the value of n?

OFFICIAL RULES + PROCEDURES

The following rules and procedures govern all MATHCOUNTS competitions. The MATHCOUNTS Foundation reserves the right to alter these rules and procedures at any time. **Coaches are responsible for being familiar with the rules and procedures outlined in this handbook.** Coaches should bring any difficulty in procedures or in student conduct to the immediate attention of the appropriate chapter, state or national official. Students violating any rules may be subject to immediate disqualification.

Any questions regarding the MATHCOUNTS Competition Series Official Rules + Procedures articulated in this handbook should be addressed to the MATHCOUNTS national office at (703) 299-9006 or info@mathcounts.org.

REGISTRATION

The fastest and easiest way to register for the MATHCOUNTS Competition Series is online at www.mathcounts.org/compreg.

For your school to participate in the MATHCOUNTS Competition Series, a school representative is required to complete a registration form and pay the registration fees. A school representative can be a teacher, administrator or parent volunteer who has received expressed permission from his/her child's school administration to register. By completing the Competition Series Registration Form, the coach attests to the school administration's permission to register students for MATHCOUNTS.

School representatives can register online at www.mathcounts.org/compreg or download the Competition Series Registration Form and mail or email a scanned copy of it to the MATHCOUNTS national office. Refer to the Critical 2016-2017 Dates on pg. 10 of this handbook for contact information.

WHAT REGISTRATION COVERS: Registration in the Competition Series entitles a school to:

1) send 1-10 student(s)—depending on number registered—to the Chapter Competition. *Students can advance beyond the chapter level, but this is determined by their performance at the competition.*
2) receive the School Competition Kit, which includes the *2016-2017 MATHCOUNTS School Handbook*, one recognition ribbon for each registered student, 10 student participation certificates and a catalog of additional coaching materials. *Mailings of School Competition Kits will occur on a rolling basis through December 31, 2016.*
3) receive online access to the 2017 School Competition, along with electronic versions of other competition materials at www.mathcounts.org/coaches. *Coaches will receive an email notification no later than November 1, 2016 when the 2017 School Competition is available online.*

Your state or chapter coordinator will be notified of your registration, and then you will be informed of the date and location of your Chapter Competition. **If you have not been contacted by mid-January with competition details, it is your responsibility to contact your local coordinator** to confirm that your registration has been properly routed and that your school's participation is expected. Coordinator contact information is available at www.mathcounts.org/findmycoordinator.

DEADLINES: The sooner your Registration Form is received, the sooner you will receive your preparation materials. To guarantee your school's participation, submit your registration by one of the following deadlines:

Early Bird Discount Deadline: November 18, 2016	Online registrations: submitted by 11:59 PST Emailed forms: received by 11:59 PST Mailed forms: postmarked by November 18, 2016
Regular Registration Deadline: December 16, 2016*	Online registrations: submitted by 11:59 PST Emailed forms: received by 11:59 PST Mailed forms: postmarked by December 16, 2016

*Late Registrations may be accepted at the discretion of the MATHCOUNTS national office and your local coordinators, but are not guaranteed. If a school's late registration is accepted, an additional $20 processing fee will be assessed.

REGISTRATION FEES: The cost of your school's registration depends on when your registration is postmarked/emailed/submitted online. The cost of your school's registration covers the students for the entire Competition Series; there are no additional registration fees to compete at the state or national level. Title I schools (as affirmed by a school's administration) receive a 50% discount off the total cost of their registration.

Number of Registered Students	Registration Postmarked by 11/18/2016	Postmarked between 11/18/2016 and 12/16/2016	Postmarked after 12/16/2016 (+ Late Fee)
1 individual	$25	$30	$50
2 ind.	$50	$60	$80
3 ind.	$75	$90	$110
1 team of 4	$90	$100	$120
1 tm. + 1 ind.	$115	$130	$150
1 tm. + 2 ind.	$140	$160	$180
1 tm. + 3 ind.	$165	$190	$210
1 tm. + 4 ind.	$190	$220	$240
1 tm. + 5 ind.	$215	$250	$270
1 tm. + 6 ind.	$240	$280	$300

ELIGIBILITY REQUIREMENTS

Eligibility requirements for the MATHCOUNTS Competition Series are different from other MATHCOUNTS programs. Eligibility for the National Math Club or the Math Video Challenge does not guarantee eligibility for the Competition Series.

WHO IS ELIGIBLE:

- U.S. students enrolled in the 6th, 7th or 8th grade can participate in MATHCOUNTS competitions.
- Schools that are the students' official school of record can register.
- Any type of school, of any size, can register—public, private, religious, charter, virtual or homeschools—but virtual and homeschools must fill out additional forms to participate (see pgs. 41-42).
- Schools in 50 U.S. states, District of Columbia, Guam, Puerto Rico and Virgin Islands can register.
- Overseas schools that are affiliated with the U.S. Departments of Defense and State can register.

WHO IS NOT ELIGIBLE:

- Students who are not full-time 6th, 7th or 8th graders cannot participate, even if they are taking middle school math classes.
- Academic centers, tutoring centers or enrichment programs that do not function as students' official school of record cannot register. *If it is unclear whether your educational institution is considered a school, please contact your local Department of Education for specific criteria governing your state.*
- Schools located outside of the U.S. states and territories listed on the previous page cannot register.
- Overseas schools not affiliated with the U.S. Departments of Defense or State cannot register.

NUMBER OF STUDENTS ALLOWED: A school can register a maximum of one team of four students and six individuals; these 1-10 student(s) will represent the school at the Chapter Competition. Any number of students can participate at the school level. Prior to the Chapter Competition, coaches must notify their chapter coordinator of which students will be team members and which students will compete as individuals.

NUMBER OF YEARS ALLOWED: Participation in MATHCOUNTS competitions is limited to 3 years for each student, but there is no limit to the number of years a student may participate in school-based coaching.

WHAT TEAM REGISTRATION MEANS: Members of a school team will participate in the Target, Sprint and Team Rounds. Members of a school team also will be eligible to qualify for the Countdown Round (where conducted). Team members will be eligible for team awards, individual awards and progression to the state and national levels based on their individual and/or team performance. It is recommended that your strongest four Mathletes form your school team. Teams of fewer than four will be allowed to compete; however, the team score will be computed by dividing the sum of the team members' scores by four (see pg. 45), meaning, teams of fewer than four students will be at a disadvantage. *Only one team (of up to four students) per school is eligible to compete.*

WHAT INDIVIDUAL REGISTRATION MEANS: Students registered as individuals will participate in the Target and Sprint Rounds, but not the Team Round. Individuals will be eligible to qualify for the Countdown Round (where conducted). Individuals also will be eligible for individual awards and progression to the state and national levels. A student registered as an "individual" may not help his/her school's team advance to the next level of competition. *Up to six students may be registered in addition to or in lieu of a school team.*

HOW STUDENTS ENROLLED PART-TIME AT TWO SCHOOLS PARTICIPATE: *A student may compete only for his/her official school of record.* A student's school of record is the student's base or main school. A student taking limited course work at a second school or educational center may not register or compete for that second school or center, even if the student is not competing for his/her school of record. MATHCOUNTS registration is not determined by where a student takes his or her math course. If there is any doubt about a student's school of record, the chapter or state coordinator must be contacted for a decision before registering.

HOW SMALL SCHOOLS PARTICIPATE: MATHCOUNTS does not distinguish between the sizes of schools for Competition Series registration and competition purposes. Every "brick-and-mortar" school will have the same registration allowance of up to one team of four students and/or up to six individuals. A school's participants may not combine with any other school's participants to form a team when registering or competing.

HOW HOMESCHOOLS PARTICIPATE: Homeschools and/or homeschool groups in compliance with the homeschool laws of the state in which they are located are eligible to participate in MATHCOUNTS competitions in accordance with all other rules. Homeschool coaches must complete the 2016-2017 Homeschool + Virtual School Participation Form, verifying that students from the homeschool or homeschool group are in the 6th, 7th or 8th grade and that each homeschool complies with applicable state laws. Forms can be downloaded at www.mathcounts.org/competition and must be submitted to the MATHCOUNTS national office in order for registrations to be processed.

HOW VIRTUAL SCHOOLS PARTICIPATE: Virtual schools that want to register must contact the MATHCOUNTS national office by December 9, 2016 for specific registration details. Any student registering as a virtual school student must compete in the MATHCOUNTS Chapter Competition assigned according to the student's home address. Additionally, virtual school coaches must complete the 2016-2017 Homeschool + Virtual School Participation Form, verifying that the students from the virtual school are in the 6th, 7th or 8th grade and that the virtual school complies with applicable state laws. Forms must be submitted to the national office in order for registrations to be processed; forms can be downloaded at www.mathcounts.org/competition.

WHAT IS DONE FOR SUBSTITUTIONS OF STUDENTS: Coaches determine which students will represent the school at the Chapter Competition. Coaches cannot substitute team members for the State Competition unless a student voluntarily releases his/her position on the school team. Additional requirements and documentation for substitutions (such as requiring parental release or requiring the substitution request be submitted in writing) are at the discretion of the State Coordinator. A student being added to a team need not be a student who was registered for the Chapter Competition as an individual. Coaches cannot make substitutions for students progressing to the State Competition as individuals. At all levels of competition, student substitutions are not permitted after on-site competition registration has been completed.

WHAT IS DONE FOR RELIGIOUS OBSERVANCES: A student who is unable to attend a competition due to religious observances may take the written portion of the competition up to one week in advance of the scheduled competition. In addition, all competitors from that student's school must take the Sprint and Target Rounds at the same earlier time. If the student who is unable to attend the competition due to a religious observance: (1) is a member of the school team, then the team must take the Team Round at the same earlier time; (2) is not part of the school team, then the team has the option of taking the Team Round during this advance testing or on the regularly scheduled day of the competition with the other school teams. The coordinator must be made aware of the team's decision before the advance testing takes place. *Advance testing will be done at the discretion of the chapter and state coordinators. If advance testing is deemed possible, it will be conducted under proctored conditions.* Students who qualify for an official Countdown Round but are unable to attend will automatically forfeit one place standing.

WHAT IS DONE FOR STUDENTS WITH SPECIAL NEEDS: Reasonable accommodations may be made to allow students with special needs to participate. However, many accommodations that are employed in a classroom or teaching environment cannot be implemented in the competition setting. Accommodations that are not permissible include, but are not limited to: granting a student extra time during any of the competition rounds or allowing a student to use a calculator for the Sprint or Countdown Rounds. *A request for accommodation of special needs must be directed to chapter or state coordinators in writing at least three weeks in advance of the Chapter or State Competition.* This written request should thoroughly explain a student's special need, as well as what the desired accommodation would entail. In conjunction with the MATHCOUNTS Foundation, coordinators will review the needs of the student and determine if any accommodations will be made. In making final determinations, the feasibility of accommodating these needs at the National Competition will be taken into consideration.

LEVELS OF COMPETITION

There are four levels in the MATHCOUNTS Competition Series: school, chapter (local), state and national. Competition questions are written for 6th, 7th and 8th graders. The competitions can be quite challenging, particularly for students who have not been coached using MATHCOUNTS materials. All competition materials are prepared by the national office.

SCHOOL COMPETITIONS (TYPICALLY HELD IN JANUARY 2017): After several months of coaching, schools registered for the Competition Series should administer the 2017 School Competition to all interested students. The School Competition should be an aid to the coach in determining competitors for the Chapter Competition. *Selection of team and individual competitors is entirely at the discretion of the coach and does not need to be based solely on School Competition scores.* School Competition materials are sent to the coach of a school, and it may be used by the teachers and students only in association with that school's programs and activities. The current year's School Competition questions must remain confidential and may not be used in outside activities, such as tutoring sessions or enrichment programs with students from other schools. For updates or edits, please check www.mathcounts.org/coaches before administering the School Competition.

It is important that the coach look upon coaching sessions during the academic year as opportunities to develop better math skills in all students, not just in those students who will be competing. Therefore, it is suggested that the coach postpone selection of competitors until just prior to the Chapter Competition.

CHAPTER COMPETITIONS (HELD FROM FEB. 1–28, 2017): The Chapter Competition consists of the Sprint, Target and Team Rounds. The Countdown Round (official or just for fun) may or may not be conducted. The chapter and state coordinators determine the date and location of the Chapter Competition in accordance with established national procedures and rules. Winning teams and students will receive recognition. The winning team will advance to the State Competition. Additionally, the two highest-ranking competitors not on the winning team (who may be registered as individuals or as members of a team) will advance to the State Competition. This is a minimum of six advancing Mathletes (assuming the winning team has four members). Additional teams and/or individuals also may progress at the discretion of the state coordinator, but the policy for progression must be consistent for all chapters within a state.

STATE COMPETITIONS (HELD FROM MAR. 1–31, 2017): The State Competition consists of the Sprint, Target and Team Rounds. The Countdown Round (official or just for fun) may or may not be included. The state coordinator determines the date and location of the State Competition in accordance with established national procedures and rules. Winning teams and students will receive recognition. The four highest-ranked Mathletes and the coach of the winning team from each State Competition will receive an all-expenses-paid trip to the National Competition.

2017 RAYTHEON MATHCOUNTS NATIONAL COMPETITION (HELD MAY 14–15 IN ORLANDO, FL): The National Competition consists of the Sprint, Target, Team and Countdown Rounds (conducted officially). Expenses of the state team and coach to travel to the National Competition will be paid by MATHCOUNTS. The national program does not make provisions for the attendance of additional students or coaches. All national competitors will receive a plaque and other items in recognition of their achievements. Winning teams and individuals also will receive medals, trophies and college scholarships.

COMPETITION COMPONENTS

The four rounds of a MATHCOUNTS competition, each described below, are designed to be completed in approximately three hours:

TARGET ROUND (approximately 30 minutes): In this round eight problems are presented to competitors in four pairs (six minutes per pair). The multi-step problems featured in this round engage Mathletes in mathematical reasoning and problem-solving processes. *Problems assume the use of calculators.*

SPRINT ROUND (40 minutes): Consisting of 30 problems, this round tests accuracy, with the time period allowing only the most capable students to complete all of the problems. *Calculators are not permitted.*

TEAM ROUND (20 minutes): In this round, interaction among team members is permitted and encouraged as they work together to solve 10 problems. *Problems assume the use of calculators.*

Note: The order in which the written rounds (Target, Sprint and Team) are administered is at the discretion of the competition coordinator.

COUNTDOWN ROUND: A fast-paced oral competition for top-scoring individuals (based on scores on the Target and Sprint Rounds), this round allows pairs of Mathletes to compete against each other and the clock to solve problems. *Calculators are not permitted.*

At Chapter and State Competitions, a Countdown Round (1) may be conducted officially, (2) may be conducted unofficially (for fun) or (3) may be omitted. However, the use of an official Countdown Round must be consistent for all chapters within a state. In other words, *all* chapters within a state must use the round officially in order for *any* chapter within a state to use it officially. All students, whether registered as part of a school team or as individual competitors, are eligible to qualify for the Countdown Round.

An official Countdown Round determines an individual's final overall rank in the competition. If a Countdown Round is used officially, the official procedures as established by the MATHCOUNTS Foundation must be followed, as described below.*

- The top 25% of students, up to a maximum of 10, are selected to compete. These students are chosen based on their Individual Scores.
- The two lowest-ranked students are paired; a question is read and projected, and students are given 45 seconds to solve the problem. A student may buzz in at any time, and if s/he answers correctly, a point is scored. If a student answers incorrectly, the other student has the remainder of the 45 seconds to answer.
- Three total questions are read to the pair of students, one question at a time, and the student who scores the higher number of points (not necessarily 2 out of 3) progresses to the next round and challenges the next-higher-ranked student.
- If students are tied in their matchup after three questions (at 1-1 or 0-0), questions should continue to be read until one is successfully answered. The first student who answers an additional question correctly progresses to the next round.
- This procedure continues until the 4th-ranked Mathlete and his/her opponent compete. For the final four matchups, the first student to correctly answer three questions advances.
- The Countdown Round proceeds until a 1st place individual is identified. More details about Countdown Round procedures are included in the 2017 School Competition.

**Rules for the Countdown Round change for the National Competition.*

An unofficial Countdown Round does not determine an individual's final overall rank in the competition, but is done for practice or for fun. The official procedures do not have to be followed. Chapters and states choosing not to conduct the round officially must determine individual winners solely on the basis of students' scores in the Target and Sprint Rounds of the competition.

SCORING

MATHCOUNTS Competition Series scores do not conform to traditional grading scales. Coaches and students should view an Individual Score of 23 (out of a possible 46) as highly commendable.

INDIVIDUAL SCORE: calculated by taking the sum of the number of Sprint Round questions answered correctly and twice the number of Target Round questions answered correctly. There are 30 questions in the Sprint Round and eight questions in the Target Round, so the maximum possible Individual Score is $30 + 2(8) = 46$. If used officially, the Countdown Round yields final individual standings.

TEAM SCORE: calculated by dividing the sum of the team members' Individual Scores by four (even if the team has fewer than four members) and adding twice the number of Team Round questions answered correctly. The highest possible Individual Score is 46. Four students may compete on a team, and there are 10 questions in the Team Round. Therefore, the maximum possible Team Score is $((46 + 46 + 46 + 46) \div 4) + 2(10) = 66$.

TIEBREAKING ALGORITHM: used to determine team and individual ranks and to determine which individuals qualify for the Countdown Round. In general, questions in the Target, Sprint and Team Rounds increase in difficulty so that the most difficult questions occur near the end of each round. In a comparison of questions to break ties, generally those who correctly answer the more difficult questions receive the higher rank. The guidelines provided below are very general; competition officials receive more detailed procedures.

- *Ties between individuals:* the student with the higher Sprint Round score will receive the higher rank. If a tie remains after this comparison, specific groups of questions from the Target and Sprint Rounds are compared.
- *Ties between teams:* the team with the higher Team Round score, and then the higher sum of the team members' Sprint Round scores, receives the higher rank. If a tie remains after these comparisons, specific questions from the Team Round will be compared.

RESULTS DISTRIBUTION

Coaches should expect to receive the scores of their students, as well as a list of the top 25% of students and top 40% of teams, from their competition coordinators. In addition, single copies of the blank competition materials and answer keys may be distributed to coaches after all competitions at that level nationwide have been completed. *Before distributing blank competition materials and answer keys, coordinators must wait for verification from the national office that all such competitions have been completed.* Both the problems and answers from Chapter and State Competitions will be posted on the MATHCOUNTS website following the completion of all competitions at that level nationwide, replacing the previous year's posted tests.

Student competition papers and answers will not be viewed by or distributed to coaches, parents, students or other individuals. Students' competition papers become the confidential property of MATHCOUNTS.

ADDITIONAL RULES

All answers must be legible.

Pencils and paper will be provided for Mathletes by competition organizers. However, students may bring their own pencils, pens and erasers if they wish. They may not use their own scratch paper or graph paper.

Use of notes or other reference materials (including dictionaries and translation dictionaries) is prohibited.

Specific instructions stated in a given problem take precedence over any general rule or procedure.

Communication with coaches is prohibited during rounds but is permitted during breaks. All communication between guests and Mathletes is prohibited during competition rounds. Communication between teammates is permitted only during the Team Round.

Calculators are not permitted in the Sprint and Countdown Rounds, but they are permitted in the Target, Team and Tiebreaker (if needed) Rounds. When calculators are permitted, students may use any calculator (including programmable and graphing calculators) that does not contain a QWERTY (typewriter-like) keypad. Calculators that have the ability to enter letters of the alphabet but do not have a keypad in a standard typewriter arrangement are acceptable. Smart phones, laptops, tablets, iPods®, personal digital assistants (PDAs) and any other "smart" devices are not considered to be calculators and may not be used during competitions. Students may not use calculators to exchange information with another person or device during the competition.

Coaches are responsible for ensuring their students use acceptable calculators, and students are responsible for providing their own calculators. Coordinators are not responsible for providing Mathletes with calculators or batteries before or during MATHCOUNTS competitions. Coaches are strongly advised to bring backup calculators and spare batteries to the competition for their team members in case of a malfunctioning calculator or weak or dead batteries. Neither the MATHCOUNTS Foundation nor coordinators shall be responsible for the consequences of a calculator's malfunctioning.

Pagers, cell phones, tablets, iPods® and other MP3 players should not be brought into the competition room. Failure to comply could result in dismissal from the competition.

Should there be a rule violation or suspicion of irregularities, the MATHCOUNTS coordinator or competition official has the obligation and authority to exercise his/her judgment regarding the situation and take appropriate action, which might include disqualification of the suspected student(s) from the competition.

FORMS OF ANSWERS

The following rules explain acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Competition answers will be scored in compliance with these rules for forms of answers.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lb 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, $0.25 will not be accepted.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and GCF(a, b) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not have a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and GCF(a, b) = 1. Examples:

Problem: What is 8 ÷ 12 expressed as a common fraction? *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: What is 12 ÷ 8 expressed as a common fraction? *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: What is the sum of the lengths of the radius and the circumference of a circle of diameter $\frac{1}{4}$ unit expressed as a common fraction in terms of π? *Answer:* $\frac{1+2\pi}{8}$

Problem: What is 20 ÷ 12 expressed as a mixed number? *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Acceptable Simplified Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:
Problem: What is $\sqrt{15} \times \sqrt{5}$ expressed in simplest radical form? *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form ($) *a.bc*, where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when a is zero, in which case it may be omitted, or when b and c are both zero, in which case they both may be omitted. Answers in the form ($) *a.bc* should be rounded to the nearest cent, unless otherwise specified. Examples:
Acceptable Forms: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not do any intermediate rounding (other than the "rounding" a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \leq |a| < 10$, and n is an integer. Examples:
Problem: What is 6895 expressed in scientific notation? *Answer:* 6.895×10^3
Problem: What is 40,000 expressed in scientific notation? *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole-number answers should be expressed in their whole-number form. Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

VOCABULARY AND FORMULAS

The following list is representative of terminology used in the problems but should not be viewed as all-inclusive. It is recommended that coaches review this list with their Mathletes.

absolute difference
absolute value
acute angle
additive inverse *(opposite)*
adjacent angles
algorithm
alternate exterior angles
alternate interior angles
altitude *(height)*
apex
area
arithmetic mean
arithmetic sequence
base 10
binary
bisect
box-and-whisker plot
center
chord
circle
circumference
circumscribe
coefficient
collinear
combination
common denominator
common divisor
common factor
common fraction
common multiple
complementary angles
composite number
compound interest
concentric
cone
congruent
convex
coordinate plane/system
coordinates of a point
coplanar
corresponding angles
counting numbers
counting principle
cube
cylinder
decagon

decimal
degree measure
denominator
diagonal of a polygon
diagonal of a polyhedron
diameter
difference
digit
digit-sum
direct variation
dividend
divisible
divisor
dodecagon
dodecahedron
domain of a function
edge
endpoint
equation
equiangular
equidistant
equilateral
evaluate
expected value
exponent
expression
exterior angle of a polygon
factor
factorial
finite
formula
frequency distribution
frustum
function
GCF
geometric mean
geometric sequence
height *(altitude)*
hemisphere
heptagon
hexagon
hypotenuse
image(s) of a point(s)
(under a transformation)
improper fraction
inequality

infinite series
inscribe
integer
interior angle of a polygon
interquartile range
intersection
inverse variation
irrational number
isosceles
kite
lateral edge
lateral surface area
lattice point(s)
LCM
linear equation
mean
median of a set of data
median of a triangle
midpoint
mixed number
mode(s) of a set of data
multiple
multiplicative inverse *(reciprocal)*
natural number
nonagon
numerator
obtuse angle
octagon
octahedron
ordered pair
origin
palindrome
parallel
parallelogram
Pascal's Triangle
pentagon
percent increase/decrease
perimeter
permutation
perpendicular
planar
polygon
polyhedron
polynomial
prime factorization
prime number

principal square root
prism
probability
product
proper divisor
proper factor
proper fraction
proportion
pyramid
Pythagorean Triple
quadrant
quadrilateral
quotient
radius
random
range of a data set
range of a function
rate
ratio
rational number
ray
real number
reciprocal *(multiplicative inverse)*
rectangle
reflection
regular polygon
relatively prime
remainder
repeating decimal
revolution
rhombus
right angle
right circular cone
right circular cylinder
right polyhedron
right triangle
rotation
scalene triangle
scientific notation
sector
segment of a circle
segment of a line
semicircle
semiperimeter
sequence
set
significant digits
similar figures
simple interest
slope
slope-intercept form
solution set
space diagonal
sphere
square
square root
stem-and-leaf plot
sum
supplementary angles
system of equations/inequalities
tangent figures
tangent line
term
terminating decimal
tetrahedron
total surface area
transformation
translation
trapezoid
triangle
triangular numbers
trisect
twin primes
union
unit fraction
variable
vertex
vertical angles
volume
whole number
x-axis
x-coordinate
x-intercept
y-axis
y-coordinate
y-intercept

The list of formulas below is representative of those needed to solve MATHCOUNTS problems but should not be viewed as the only formulas that may be used. Many other formulas that are useful in problem solving should be discovered and derived by Mathletes.

CIRCUMFERENCE

Circle	$C = 2 \times \pi \times r = \pi \times d$

AREA

Circle	$A = \pi \times r^2$
Square	$A = s^2$
Rectangle	$A = l \times w = b \times h$
Parallelogram	$A = b \times h$
Trapezoid	$A = \frac{1}{2}(b_1 + b_2) \times h$
Rhombus	$A = \frac{1}{2} \times d_1 \times d_2$
Triangle	$A = \frac{1}{2} \times b \times h$
Triangle	$A = \sqrt{s(s-a)(s-b)(s-c)}$
Equilateral triangle	$A = \frac{s^2\sqrt{3}}{4}$

SURFACE AREA AND VOLUME

Sphere	$SA = 4 \times \pi \times r^2$
Sphere	$V = \frac{4}{3} \times \pi \times r^3$
Rectangular prism	$V = l \times w \times h$
Circular cylinder	$V = \pi \times r^2 \times h$
Circular cone	$V = \frac{1}{3} \times \pi \times r^2 \times h$
Pyramid	$V = \frac{1}{3} \times B \times h$

Pythagorean Theorem	$c^2 = a^2 + b^2$
Counting/ Combinations	${}_nC_r = \frac{n!}{r!(n-r)!}$

ANSWERS

In addition to the answer, we have provided a difficulty rating for each problem. Our scale is 1-7, with 7 being the most difficult. These are only approximations, and how difficult a problem is for a particular student will vary. Below is a general guide to the ratings:

Difficulty 1/2/3 - One concept; one- to two-step solution; appropriate for students just starting the middle school curriculum.

4/5 - One or two concepts; multistep solution; knowledge of some middle school topics is necessary.

6/7 - Multiple and/or advanced concepts; multistep solution; knowledge of advanced middle school topics and/or problem-solving strategies is necessary.

Warm-Up 1

Answer		Difficulty			
1.	77	(1)	6.	30	(4)
2.	$3\frac{1}{5}$	(2)	7.	9/16	(4)
3.	11	(2)	8.	1.7×10^5	(2)
4.	1	(2)	9.	0	(3)
5.	1/6	(2)	10.	170 or 170.00	(2)

Warm-Up 2

Answer		Difficulty			
11.	16	(2)	16.	720	(3)
12.	5	(3)	17.	36	(5)
13.	12	(3)	18.	7/12	(4)
14.	5	(3)	19.	12	(3)
15.	3.16×10^{-14}	(4)	20.	$2\sqrt{17}$	(4)

Warm-Up 3

Answer		Difficulty			
21.	22	(2)	26.	25/81	(4)
22.	3	(2)	27.	18/25	(4)
23.	8	(2)	28.	8	(4)
24.	621	(1)	29.	6	(4)
25.	9	(4)	30.	1	(2)

Workout 1

Answer		Difficulty			
31.	1.30	(3)	36.	7	(4)
32.	721	(4)	37.	Q	(3)
33.	1100	(4)	38.	16	(2)
34.	12	(4)	39.	6	(3)
35.	14.6	(4)	40.	36	(4)

Workout 2

Answer		Difficulty			
41.	1*	(3)	46.	4	(4)
42.	90	(3)	47.	1/4	(4)
43.	1.65	(3)	48.	36	(4)
44.	41	(1)	49.	7	(4)
45.	120	(4)	50.	21.3	(4)

Warm-Up 4

Answer		Difficulty			
51.	88	(4)	56.	380	(3)
52.	86	(3)	57.	15/64	(4)
53.	38	(4)	58.	−3	(2)
54.	12	(2)	59.	2/5	(4)
55.	1/64	(3)	60.	8	(3)

** The plural form of the units is always provided in the answer blank, even if the answer appears to require the singular form of the units.*

Warm-Up 5

Answer	Difficulty			
61.	216	(2)	66. 4851	(5)
62.	38	(4)	67. 30	(3)
63.	11	(4)	68. 720	(3)
64.	6	(3)	69. 160	(5)
65.	275,000 or 275,000.00	(4)	70. −3	(4)

Workout 4

Answer	Difficulty			
91.	60	(3)	96. 36	(4)
92.	776	(3)	97. 4094	(4)
93.	10.5	(4)	98. 870	(4)
94.	3	(2)	99. 100	(4)
95.	35	(5)	100. 922	(3)

Warm-Up 6

Answer	Difficulty			
71.	24 or 24.00	(2)	76. 34	(4)
72.	17	(3)	77. 2π	(5)
73.	175	(3)	78. 7	(3)
74.	33	(4)	79. −5	(4)
75.	867	(4)	80. 17	(4)

Warm-Up 7

Answer	Difficulty			
101.	0	(2)	106. 48	(4)
102.	2	(3)	107. 3	(4)
103.	74	(5)	108. 500	(4)
104.	8.10	(2)	109. $14\sqrt{3}$	(5)
105.	1/16	(4)	110. 678	(4)

Workout 3

Answer	Difficulty			
81.	−4	(3)	86. 29	(4)
82.	18	(3)	87. 2.45	(4)
83.	$\frac{19\sqrt{2}}{2}$	(4)	88. 750	(4)
84.	8	(2)	89. 4	(4)
85.	2018	(2)	90. 192π	(4)

Warm-Up 8

Answer	Difficulty			
111.	6	(2)	116. 1/39	(5)
112.	1/32	(4)	117. 9	(4)
113.	37π	(4)	118. 42	(4)
114.	9	(4)	119. 72π	(5)
115.	18	(5)	120. 12022	(4)

Workout 5

Answer	Difficulty		
121. 29	(4)	126. 45	(3)
122. 7.2	(4)	127. 1/8	(3)
123. 10/3	(5)	128. 0.5	(5)
124. 3	(4)	129. 1.14	(6)
125. 9.4	(5)	130. 90	(3)

Workout 6

Answer	Difficulty		
131. 42	(3)	136. 61,900	(4)
132. 8	(5)	137. 20	(5)
133. 3/65	(4)	138. 24	(4)
134. 22 or 22.00	(5)	139. 346	(3)
135. 272	(3)	140. 3/7	(3)

Warm-Up 9

Answer	Difficulty		
141. 5	(2)	146. 32	(6)
142. 3/2	(4)	147. 132	(3)
143. 40	(3)	148. 28	(5)
144. 15	(3)	149. 3	(3)
145. 981	(4)	150. 512	(3)

Warm-Up 10

Answer	Difficulty		
151. 38	(4)	156. 18/55	(3)
152. 78230	(4)	157. 40	(6)
153. 14	(4)	158. 2031	(4)
154. 30	(5)	159. 31	(4)
155. 1760	(5)	160. 32	(4)

Warm-Up 11

Answer	Difficulty		
161. 7/64	(4)	166. 8008	(4)
162. 18	(3)	167. 6	(3)
163. 27	(5)	168. 9.2	(4)
164. 62	(3)	169. 9	(5)
165. 30	(4)	170. 2	(5)

Workout 7

Answer	Difficulty		
171. 40.5	(4)	176. 3/20	(3)
172. 24	(5)	177. 60	(5)
173. 1.7	(3)	178. 1.11	(4)
174. 27	(5)	179. 9/5	(5)
175. 0.94	(4)	180. 2.6	(4)

Workout 8

Answer	Difficulty		
181. 16/5	(2)	186. 2	(4)
182. 6	(2)	187. 1/210	(4)
183. 5/18	(5)	188. $4\sqrt{29}$	(6)
184. 73	(5)	189. $6\sqrt{13}$	(6)
185. 806	(5)	190. 10	(4)

Warm-Up 12

Answer	Difficulty		
191. 225	(3)	196. 25	(5)
192. 391/330	(4)	197. 18	(7)
193. $\sqrt{3}$	(5)	198. 10	(3)
194. 4200	(4)	199. 4	(4)
195. 1/35	(4)	200. 3	(5)

Warm-Up 13

Answer	Difficulty		
201. 279	(5)	206. 6777	(5)
202. 121	(5)	207. 1/9	(5)
203. 7	(4)	208. 6	(6)
204. 15/2	(5)	209. 7	(4)
205. 5/3	(5)	210. 48/5	(6)

Warm-Up 14

Answer	Difficulty		
211. 28	(3)	216. 5	(5)
212. 7/15	(5)	217. 21/8	(5)
213. 16	(4)	218. 47	(5)
214. 124/133	(7)	219. $\frac{\sqrt{2}+40}{3}$ or $\frac{40+\sqrt{2}}{3}$	(5)
215. 12	(5)		
		220. 169	(5)

Fractions Stretch

Answer	Difficulty		
221. 1/4	(1)	226. 11/16	(4)
222. 3/2	(2)	227. 7/3	(3)
223. 6/11	(3)	228. 34/45	(4)
224. 3/10	(1)	229. 2	(5)
225. 3/8	(4)	230. −12	(4)

Angles and Arcs Stretch

Answer	Difficulty		
231. 40	(3)	236. 100	(3)
232. 65	(3)	237. 130	(4)
233. 40	(4)	238. 77.5	(4)
234. 230	(4)	239. 80	(4)
235. 62.5	(3)	240. 90	(5)

Bases Stretch

Answer	Difficulty		
241. 22	(2)	246. 33	(3)
242. 20	(2)	247. 2A7B	(4)
243. 18	(2)	248. 2355	(5)
244. 26	(3)	249. 7	(5)
245. 30	(3)	250. 19	(6)

MATHCOUNTS Problems Mapped to Common Core State Standards (CCSS)

Forty-three states, the District of Columbia, four territories and the Department of Defense Education Activity (DoDEA) have adopted the Common Core State Standards (CCSS). As such, MATHCOUNTS considers it beneficial for teachers to see the connections between the *2016-2017 MATHCOUNTS School Handbook* problems and the CCSS. MATHCOUNTS not only has identified a general topic and assigned a difficulty level for each problem but also has provided a CCSS code in the Problem Index (pages 56-57). A complete list of the Common Core State Standards can be found at www.corestandards.org.

The CCSS for mathematics cover K-8 and high school courses. MATHCOUNTS problems are written to align with the NCTM Standards for Grades 6-8. As one would expect, there is great overlap between the two sets of standards. MATHCOUNTS also recognizes that in many school districts, algebra and geometry are taught in middle school, so some MATHCOUNTS problems also require skills taught in those courses.

In referring to the CCSS, the Problem Index code for each of the Standards for Mathematical Content for grades K-8 begins with the grade level. For the Standards for Mathematical Content for high school courses (such as algebra or geometry), each code begins with a letter to indicate the course name. The second part of each code indicates the domain within the grade level or course. Finally, the number of the individual standard within that domain follows. Here are two examples:

- *6.RP.3 → Standard #3 in the Ratios and Proportional Relationships domain of grade 6*
- *G-SRT.6 → Standard #6 in the Similarity, Right Triangles and Trigonometry domain of Geometry*

Some math concepts utilized in MATHCOUNTS problems are not specifically mentioned in the CCSS. Two examples are the Fundamental Counting Principle (FCP) and special right triangles. In cases like these, if a related standard could be identified, a code for that standard was used. For example, problems using the FCP were coded 7.SP.8, S-CP.8 or S-CP.9 depending on the context of the problem; SP → Statistics and Probability (the domain), S → Statistics and Probability (the course) and CP → Conditional Probability and the Rules of Probability. Problems based on special right triangles were given the code G-SRT.5 or G-SRT.6, explained above.

There are some MATHCOUNTS problems that either are based on math concepts outside the scope of the CCSS or based on concepts in the standards for grades K-5 but are obviously more difficult than a grade K-5 problem. When appropriate, these problems were given the code SMP for Standards for Mathematical Practice. The CCSS include the Standards for Mathematical Practice along with the Standards for Mathematical Content. The SMPs are (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (4) Model with mathematics; (5) Use appropriate tools strategically; (6) Attend to precision; (7) Look for and make use of structure and (8) Look for and express regularity in repeated reasoning.

PROBLEM INDEX

It is difficult to categorize many of the problems in the *MATHCOUNTS School Handbook*. It is very common for a MATHCOUNTS problem to straddle multiple categories and cover several concepts. This index is intended to be a helpful resource, but since each problem has been placed in exactly one category and mapped to exactly one Common Core State Standard (CCSS), the index is not perfect. In this index, the code **9 (3) 7.SP.3** refers to problem 9 with difficulty rating 3 mapped to CCSS 7.SP.3. For an explanation of the difficulty ratings refer to page 51. For an explanation of the CCSS codes refer to page 55.

[1] CCSS 7.G.5

[2] CCSS 8.EE.1

Algebraic Expressions & Equations

25	(4)	6.EE.2
33	(4)	A-CED.1
49	(4)	A-CED.2
53	(4)	8.EE.8
56	(3)	6.EE.2
58	(2)	6.EE.2
64	(3)	8.EE.1
79	(4)	A-REI.5
80	(4)	6.EE.2
87	(4)	8.EE.8
88	(4)	8.EE.8
89	(4)	8.EE.1
92	(3)	6.EE.2
102	(3)	F-IF.2
111	(2)	8.EE.8
123	(5)	A-REI.7
141	(2)	8.EE.8
144	(3)	8.EE.8
158	(4)	A-SSE.2
170	(5)	A-SSE.2
172	(5)	A-CED.2
188	(6)	G-SRT.3
196	(5)	G-C.5
201	(5)	S-CP.9
204	(5)	S-CP.9

Statistics

21	(2)	6.SP.5
51	(4)	6.SP.5
82	(3)	ACED.2
95	(5)	6.SP.5
118	(4)	SMP
121	(4)	6.SP.5
186	(4)	6.SP.5

Problem Solving (Misc.)

9	(3)	6.EE.1
19	(3)	SMP
39	(3)	6.EE.1
62	(4)	SMP
152	(4)	SMP
182	(2)	SMP
191	(3)	SMP
208	(6)	SMP
216	(5)	SMP

Probability, Counting & Combinatorics

7	(4)	SCP.9
16	(3)	SCP.9
18	(4)	SCP.9
27	(4)	SCP.6
28	(4)	SMP
34	(4)	SMP
47	(4)	SCP.9
55	(3)	7.SP.8
57	(4)	SCP.9
59	(4)	SCP.9
66	(5)	SCP.9
68	(3)	SCP.9
76	(4)	SCP.9
91	(3)	SCP.9
97	(4)	SCP.9
110	(4)	SCP.9
112	(4)	SCP.9
116	(5)	SCP.9
126	(3)	SMP
127	(3)	7.SP.8
133	(4)	7.SP.8
145	(4)	SMP
147	(3)	SMP
150	(3)	SCP.9
155	(5)	SMP
160	(4)	SMP
161	(4)	SCP.9
162	(3)	SMP
165	(4)	SMP
175	(4)	7.SP.8
176	(3)	7.SP.8
183	(5)	7.SP.8
187	(4)	SCP.9
194	(4)	SCP.9
195	(4)	SCP.9
202	(5)	SCP.9
203	(4)	7.SP.8
205	(5)	SCP.9
207	(5)	7.SP.8
211	(3)	SMP
212	(5)	7.SP.8
214	(7)	SMP
220	(5)	SMP

Sequences, Series & Patterns

13	(3)	SMP
73	(3)	SMP
74	(4)	FIF.3
84	(2)	FIF.3
94	(2)	8.NS.1
114	(4)	FLE.2
131	(3)	FBF.2
164	(3)	SMP
200	(5)	FBF.2

Proportional Reasoning

2	(2)	6.RP.3
17	(5)	6.RP.3
23	(2)	4.NF.2
26	(4)	8.G.4
31	(3)	4.MD.2
40	(4)	6.RP.3
41	(3)	6.RP.3
67	(3)	6.RP.3
71	(2)	6.RP.3
93	(4)	6.RP.3
104	(2)	6.RP.3
105	(4)	7.RP.2
108	(4)	7.RP.2
122	(4)	S.RP.3
140	(3)	7.RP.1
142	(4)	7.RP.1
178	(4)	6.RP.3
190	(4)	7.RP.1
209	(4)	6.EE.2
213	(4)	6.RP.3

Percents & Fractions

5	(2)	5.NF.1
36	(4)	7.EE.3
60	(3)	6.RP.3
65	(4)	7.RP.3
101	(2)	6.RP.3
137	(5)	7.RP.3
177	(5)	7.G.4
192	(4)	6.RP.3
Fractions Stretch[3]		

[3] CCSS 6.RP.3

SOLUTIONS

The solutions provided here are only *possible* solutions. It is very likely that you or your students will come up with additional—and perhaps more elegant—solutions. Happy solving!

Warm-Up 1

1. Subtract the year the capsule was sealed, 1940, from the year it will be opened, 2017. This difference is the amount of time in between, or 2017 − 1940 = **77** years.

2. This can be done using the proportion 5/2 = 8/x, x being the number of large paper clips equivalent in length to 8 small paper clips. To find x, multiply each side of the equation by x and then by 2/5. The result is x = 16/5 large clips, which as a mixed number is $\mathbf{3\frac{1}{5}}$ large paper clips.

3. One path goes directly from cell 1 to cell 7. To find all the other paths, we will systematically list five clockwise paths and then five counterclockwise paths that end in cell 7. The **11** paths are 1-7, 1-2-7, 1-3-2-7, 1-4-3-2-7, 1-5-4-3-2-7, 1-6-5-4-3-2-7, 1-6-7, 1-5-6-7, 1-4-5-6-7, 1-3-4-5-6-7, 1-2-3-4-5-6-7.

4. Dividing by 2/3 is the same as multiplying by 3/2, so 4 ÷ (2/3) − 5 = 4 × (3/2) − 5 = 6 − 5 = **1**.

5. The absolute difference, or the absolute value of the difference, can be thought of as the positive difference. To find the positive difference between the two values, subtract the smaller fraction from the larger fraction as follows: (1/2) – (1/3) = (3/6) – (2/6) = **1/6**.

6. If the perimeter of rectangle ABCD is 34 cm, then the semiperimeter is half that measure, or 17 cm, and side AD must measure 17 − 5 = 12 cm. To find the length of diagonal BD, we use the Pythagorean Theorem as follows: $5^2 + 12^2 = x^2 \rightarrow 25 + 144 = x^2 \rightarrow 169 = x^2 \rightarrow x = 13$. (5-12-13 is one of several Pythagorean Triples memorized by competitive Mathletes.) The perimeter of △ABD is 5 + 12 + 13 = **30** cm.

7. There are 8 × 8 = 64 ways the two rolls of the eight-sided die can occur. This will be the denominator of our probability fraction. If the first roll of the die is a 1, then the second roll can be any of the 8 numbers for it to be greater than or equal to the first roll. If the first roll is a 2, then the second roll can be any of 7 possibilities, excluding 1. With a 3, we can have 6 possibilities, etc. The total number of acceptable rolls for the second die is, thus, 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36. The probability is 36/64 = **9/16**.

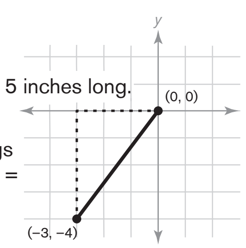
8. The product $1.2 \times 10^3 \times 1.4 \times 10^2$ can be rewritten as $1.2 \times 1.4 \times 10^3 \times 10^2$, which is $1.68 \times 10^{3+2} = 1.68 \times 10^5 \approx \mathbf{1.7 \times 10^5}$.

9. The square of a positive integer is equal to the square of its opposite. For example $5^2 = (-5)^2$. So all the integers n that satisfy the requirement come in pairs of opposites. Therefore, their sum is **0**.

10. If Minnie took 7 lessons, and lessons cost $20 each, she paid 7 × 20 = $140, plus the $30 registration fee, for a total of 140 + 30 = **$170**.

Warm-Up 2

11. A string that is 7 feet long is 7 × 12 = 84 inches long, which is 16 × 5 + 4. So, Sue can cut **16** pieces of string that are 5 inches long.



12. As the figure shows, the segment from the point (−3, −4) to the origin (0, 0) is the hypotenuse of a right triangle with legs of length 3 and 4 units. This is the well-known 3-4-5 right triangle that satisfies the Pythagorean Theorem as follows: $3^2 + 4^2 = 9 + 16 = 25 = 5^2$. The distance from (−3, −4) to the origin is **5** units.

13. The hexagonal tables that are on the ends seat 5 people each. The other 50 − 2 × 5 = 40 people are seated at hexagonal tables that are in the middle of the row and seat only 4 people each. Thus, 40 ÷ 4 = 10 more tables are needed in between the two end tables. That's a total of 2 + 10 = **12** tables.

14. The three notebooks cost 3 × 1.57 = $4.71. The change from $5.00 would be 29¢. The fewest coins Gloria could get in change would be a quarter and four pennies, which is **5** coins.

15. The number 1,000,000,000 is 10^9 as a power of 10. When we multiply numbers in scientific notation, we can add the exponents that have the same base. The mass of 1,000,000,000 fluorine atoms is $10^9 \times 3.16 \times 10^{-23} = 3.16 \times 10^{9-23} = \mathbf{3.16 \times 10^{-14}}$ g.

16. Each of the 3 possible appetizers can be combined with each of the 4 entrées for 3 × 4 = 12 meals so far. The two side dishes can be selected in 5 × 4 ÷ 2 = 10 ways. We divide by 2 because the 5 × 4 = 20 ways would include duplicates of every pair of side dishes chosen the other way around. We now have 12 × 10 = 120 meals with an appetizer, an entrée and two side dishes. Each of these 120 meals can be combined with any of the 6 desserts for 120 × 6 = **720** meals.

17. If 15 machines make 500 raviolis in 15 minutes, then 15 × 5 = 75 machines would make 5 × 500 = 2500 raviolis in the same amount of time, 15 minutes. To get from 2500 raviolis to 6000 raviolis, those 75 machines would need to run for 6000/2500 or 12/5 times as long and 12/5 × 15 = **36** minutes.

18. Let's list all 4 × 3 = 12 possible two-digit numbers that can be made from the four digits and count those that are prime: 12, $\underline{13}$, $\underline{17}$, 21, $\underline{23}$, 27, $\underline{31}$, 32, $\underline{37}$, $\underline{71}$, 72, $\underline{73}$. Seven of these numbers are prime, so the probability is **7/12**.

19. The two letters that are 4 units from the letter M are I and Q. These are the 9th and 17th letters in the alphabet, so they would be placed at 8 and 16 on the number line. Not surprisingly, the average of these two numbers is exactly the number of M, which is **12**.

20. The location of point B is (−2, −2), but we don't really need to know this. The more important information is the lengths of the legs of the right triangle that can be formed on the grid with points A and B at opposite ends of the hypotenuse, and these lengths were given. Using the Pythagorean Theorem, we can find the length of segment AB as follows: $2^2 + 8^2 = x^2 \rightarrow 4 + 64 = x^2 \rightarrow 68 = x^2 \rightarrow x = \sqrt{68} = \sqrt{(4 \times 17)} = \sqrt{4} \times \sqrt{17} = \mathbf{2\sqrt{17}}$ units.

Warm-Up 3

21. The average of an arithmetic sequence is the average of its first and last terms. In this case, we have (13 + 31) ÷ 2 = 44 ÷ 2 = **22**.

22. Suppose Kelly first selects a white sock and then selects a black sock. The third sock she selects must match either the white or the black, forming a pair either way. Thus, she must select **3** socks to be assured of a matching pair.

23. The fraction 27/72 expressed as a common fraction is 3/8. So m must be **8**.

24. The total number of chairs in the room must be 23 × 27 = **621**. Most students will use the standard algorithm for multiplying two-digit integers. But to quickly multiply these two numbers, we can write the product as the difference of two squares: $(25 - 2)(25 + 2) = 25^2 - 2^2 = 625 - 4 = \mathbf{621}$.

25. Without equations, we might think about this problem as follows: Since the larger number is 4 times the smaller number, then the sum of the two numbers must be 5 times the smaller number. This same sum is 3 times the smaller number plus 18, so 18 must be 2 times the smaller number and the smaller number is **9**. If we write equations, we have $y = 4x$ and $x + y = 3x + 18$. Replacing the y in the second equation with $4x$, we get $x + 4x = 3x + 18 \rightarrow 5x = 3x + 18 \rightarrow 2x = 18 \rightarrow x = \mathbf{9}$. This matches our earlier thinking exactly.

26. The ratio of the hypotenuse of △ABC to the hypotenuse of △XYZ is 10/18, or 5/9. This is a one-dimensional scale factor. The area of the triangles is two-dimensional, so we have to use this scale factor twice. The ratio we want is 5/9 × 5/9 = **25/81**.

27. There are seven numbers from 1 through 25, inclusive, that are *not* divisible by 2, 3, 4 or 5. They are 1, 7, 11, 13, 17, 19 and 23. The other 25 − 7 = 18 numbers *are* divisible by one or more of 2, 3, 4 or 5, so the probability is **18/25**.

28. The **8** ordered sequences that Siddarth can use to write 5 as a sum of 1s and 2s are 1 + 1 + 1 + 1 + 1, 2 + 1 + 1 + 1, 1 + 2 + 1 + 1, 1 + 1 + 2 + 1, 1 + 1 + 1 + 2, 2 + 2 + 1, 2 + 1 + 2 and 1 + 2 + 2. In general, the number of distinct ordered sequences containing only 1s and 2s and having a sum of n is the nth Fibonacci number, where the first and second terms are 1 and 2, respectively.

29. Kevin's bus traveled 15 miles west and 8 miles north for a total of 23 miles. The direct path would be the hypotenuse of a triangle with legs of length 8 miles and 15 miles. Using the Pythagorean Theorem, we can find the length of the hypotenuse as follows: $8^2 + 15^2 = x^2 \rightarrow 64 + 225 = x^2 \rightarrow 289 = x^2 \rightarrow x = 17$. (This is another Pythagorean Triple that serious Mathletes should memorize.) Kevin's bus would travel 17 miles on the direct route, which is shorter by 23 − 17 = **6** miles.

30. Ellen would get the following sequence of calculations: 123,456 − 6 = 123,450 → 123,450 ÷ 10 = 12,345 → 12,345 − 5 = 12,340 → 12,340 ÷ 10 = 1234 → 1234 − 4 = 1230 → 1230 ÷ 10 = 123 → 123 − 3 = 120 → 120 ÷ 10 = 12 → 12 − 2 = 10 → 10 ÷ 10 = **1.**

Workout 1

31. The exchange rate in the other direction is the reciprocal of 0.77, or 1 ÷ 0.77, which is 1.2987012.... So US$1.00 is about NZ$**1.30**.

32. We are looking for a multiple of 7 that is one more than a multiple of 2, 3, 4, 5, 6 and 8. The LCM of 2, 3, 4, 5, 6 and 8 is $3 \times 5 \times 8 = 120$, so 121 will leave a remainder of 1 if we divide by 2, 3, 4, 5, 6 or 8. Unfortunately 121 is not a multiple of 7; it's 2 more than a multiple of 7. Let's try $2 \times 120 + 1 = 241$. This is 3 more than a multiple of 7 or 3 (mod 7). The pattern continues with 361 = 4 (mod 7), 481 = 5 (mod 7) and 601 = 6 (mod 7), but finally 721 = 0 (mod 7). The least number of pennies that could be in the jar is **721** pennies.

33. For the first through fifth years, we have populations of n, then $3n$, then $3n - 3000$, then $(3n - 3000)/2$, then finally $(3n - 3000)/2 + 1300 = 1450$ lemmings. Now, starting at the fifth year and working backwards, we have $(3n - 3000)/2 = 150$, then $3n - 3000 = 300$, then $3n = 3300$, then at the start n = **1100** lemmings.

34. With no restrictions, $4 \times 3 \times 2 \times 1 = 24$ positive integers can be formed from the four given digits. The restriction means that we cannot have a 1 next to a 4. There are 3 ways to place a 4 and a 1 next to each other; as the first two digits, the middle two digits and the last two digits. There are 2 ways to order the 4 and the 1 and 2 ways to order the 3 and the 2, for $3 \times 2 \times 2 = 12$ integers. This eliminates 12 of the 24 integers and leaves **12** integers.

35. A tire with a diameter of 25 inches has a circumference of 25π inches, so it's covering 25π inches per rotation. Here is a single expression that uses units cancellation to convert 65 miles per hour to the number of tire rotations per second.

$$\frac{1 \text{ rotation}}{25\pi \cancel{\text{ inches}}} \times \frac{12 \cancel{\text{ inches}}}{1 \cancel{\text{ foot}}} \times \frac{5280 \cancel{\text{ feet}}}{1 \cancel{\text{ mile}}} \times \frac{65 \cancel{\text{ miles}}}{1 \cancel{\text{ hour}}} \times \frac{1 \cancel{\text{ hour}}}{60 \cancel{\text{ minutes}}} \times \frac{1 \cancel{\text{ minute}}}{60 \text{ seconds}} = \frac{12 \times 5280 \times 65 \text{ rotations}}{25\pi \times 60 \times 60 \text{ seconds}} \approx \mathbf{14.6} \text{ rotations per second}$$

36. To increase a number by 10%, multiply by 1.1. In week 2, Nish runs $8 \times 1.1 = 8.8$ miles, as stated. In week 3, she runs $8.8 \times 1.1 = 9.68$ miles. In week 4, she runs $9.68 \times 1.1 = 10.648$ miles. In week 5, she runs $10.648 \times 1.1 = 11.7128$ miles. In week 6, she runs $11.7128 \times 1.1 = 12.88408$ miles. In week 7, she runs $12.88408 \times 1.1 = 14.172488$ miles. The first week of training in which Nish exceeds 13.1 miles is week **7**.

37. We are told that the 8 from 18 follows the A in the second time through the alphabet. There are 8 more two-digit numbers from 19 to 26, inclusive. This is 16 more digits that must follow 16 more letters after the A. We want the 17th letter of the alphabet, which is **Q**.

38. The area of the original rectangle is $4 \times 5 = 20$ cm^2. The dodecagon is 4 cm^2 less, so it's **16** cm^2.

39. The sum of the first ten positive integers is $10 \times 11 \div 2 = 55$. The nearest perfect square less than 55 is 49, sothe **6** must not have been included in the sum.

40. If we want the green marbles to be 5/12 of the marbles, then the 40 green marbles must be 5 parts, so each part is $40 \div 5 = 8$. The total number of marbles needs to be $8 \times 12 = 96$. There are $20 + 40 = 60$ marbles so far, so $96 - 60$ = **36** purple marbles must be added.

Workout 2

41. The second cube of cheese weighs $64.8 \times 16 = 1036.8$ ounces. This type of cheese weighs 0.6 ounce per cubic inch. So the cube of cheese weighing 1036.8 ounces would have volume $1036.8 \div 0.6 = 1728$ in^3 and edge length $\sqrt[3]{1728} = 12$ inches = **1** foot.

42. Only the perfect squares have an odd number of divisors. There are $\sqrt{100} = 10$ perfect squares from 1 to 100, inclusive. That means there are $100 - 10$ = **90** integers with an even number of factors.

43. The distance from −1.3 to 3 1/8, or 3.125, is $1.3 + 3.125 = 4.425$ units. One-third of this distance is 1.475 and 2/3 is 2.95. If we add 2.95 to −1.3 or subtract 1.475 from 3.125, we get to the same point that is 2/3 of the way between the endpoints: $-1.3 + 2.95$ = **1.65**.

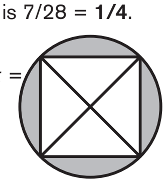
44. The stick of butter has a volume of $1.5 \times 1.5 \times 3.25 = 7.3125$ in^3. The pat of butter has a volume of $1 \times 1 \times 0.375 = 0.375$ in^3. The number of calories in a pat of butter is $\frac{800 \text{ calories}}{1.5 \times 1.5 \times 3.25 \cancel{\text{ in}^3}} \times \frac{1 \times 1 \times 0.375 \cancel{\text{ in}^3}}{1 \text{ pat}} \approx$ **41** calories.

45. The two diagonals bisect each other, so triangles AED and BEC must be equilateral, with side lengths of 5 and angles of 60 degrees. The measure of ∠AEB is the supplement of 60, which is 180 − 60 = **120** degrees.

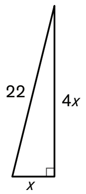
46. The expression 4! + 5! can be rewritten as $4!(1 + 5) = 6 \times 4 \times 3! = 24 \times 3!$. This means that $n! = 24$, so n = **4**.

47. There are 28 dominoes in a full set. There are $7 \times 6 \div 2 = 21$ dominoes with different numbers of dots on the two sides of the line and 7 with the same number of dots on both sides of the line, called doubles. The probability that one of the doubles is selected at random is 7/28 = **1/4**.

48. Since we are looking for a percent, we can assign a radius of 1 unit to our circular pizza. The area of the pizza is then $1^2 \times \pi = \pi$ units^2. The inscribed square can be cut into four isosceles right triangles that have legs of 1 unit. Each triangle has an area of 1/2 units^2 and the four of them can be rearranged to form 2 unit squares. The difference $\pi - 2$ is the area of the four segments that remain when Paige removes the square. As a percent, this is $(\pi - 2)/\pi \times 100\%$ ≈ **36**%.



49. If we combine what Jefferson and Monroe exchange, we get a total of 7 ziggles and 7 zoggles exchanged for a total of 49 zaggles. Dividing these totals by 7, we find that Carter can expect to exchange 1 ziggle and 1 zoggle for **7** zaggles.



50. Assuming that the ground is perpendicular to the vertical structure, we can draw a right triangle with legs of x and $4x$ feet and a hypotenuse of 22 feet. We can then use the Pythagorean Theorem and solve for x as follows: $x^2 + (4x)^2 = 22^2 \rightarrow 1x^2 + 16x^2 = 484 \rightarrow 17x^2 = 484 \rightarrow x^2 = 484/17 \rightarrow x = \sqrt{(484/17)} \rightarrow x \approx 5.336$. This is the value of x, which tells us how far the base of the ladder can be from the wall. We want to know how high the ladder can safely reach up the vertical structure, which is $4x$, or about $4 \times 5.336 \approx$ **21.3** feet.

Warm-Up 4

51. From least to greatest, Jamie's first five tests scores were 75, 81, 86, 92, 98. We want to maximize the value of the lowest of the next three test scores so that the median of all eight scores is 88. The median will be the average of the 4th and 5th scores in an ordered list of the eight scores. There is no way to get a median of 88 with two of the five known scores being the 4th and 5th scores. We could let two of the unknown scores be 87 and 89 to get a median of 88. In this case the third unknown score just has to be greater than or equal to 89, and the lowest of these three scores is 87. We could let 86 be the 4th score and make the 5th score 90 to get a median of 88. Then the lowest of the three unknown scores would be 86. If we let the 4th and 5th scores both be 88, we get a median of 88, and the final unknown score can have any value greater than or equal to 88. Further analysis shows that if the lowest of the unknown scores is greater than 88, the median would be 89 or more. Therefore, the greatest possible value of the lowest of the final three scores is **88**.

52. The least and greatest possible two-digit primes containing only 1s, 3s, 7s and 9s as digits are 11 and 97, respectively. The absolute difference between these two values is $97 - 11 =$ **86**.

53. The sum of each pair of numbers is twice the mean of the pair. From the information given, we can write the following system of equations: $a + b = 16$, $b + c = 32$ and $a + c = 28$. Adding these three equations, we get $2a + 2b + 2c = 76$. So $a + b + c$ will be half of 76, or **38**.

54. The difference between the length and the width of the rectangle is $(4x + 9) - (4x - 3) = 4x + 9 - 4x + 3 =$ **12** units.

55. Each of the six faces of the cube can be painted one of two colors, so there are $2^6 = 64$ equally likely possible ways to color the six faces. Some of these ways will look the same under rotation, but only one of them is all blue, so the probability is **1/64**.

56. We first evaluate $n^2 - n$ for $n = 5$ to get $25 - 5 = 20$. Now we evaluate $n^2 - n$ for $n = 20$ to get $400 - 20 =$ **380**.

57. The possible two-digit primes that use only the digits 1 through 8 are 11, 13, 17, 23, 31, 37, 41, 43, 47, 53, 61, 67, 71, 73 and 83. That's 15 out of $8 \times 8 = 64$ possible outcomes when an eight-sided die is rolled twice. So the probability is **15/64**.

58. Substituting the given values of x and y, we get $2 \times 3^2 + 3 \times (-2)^2 - 4 \times 3 + 2 \times (-2) - 17 = 2 \times 9 + 3 \times 4 - 12 - 4 - 17 = 18 + 12 - 12 - 21 = 18 - 21 =$ **−3**.

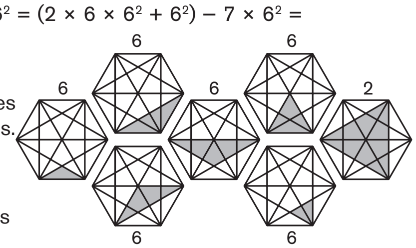
59. The number of possible old-style plates is $25 \times 25 \times 10 \times 10 \times 10 \times 10$. The number of possible new-style plates is $25 \times 25 \times 25 \times 10 \times 10 \times 10$. The ratio is $(25 \times 25 \times 10 \times 10 \times 10 \times 10)/(25 \times 25 \times 25 \times 10 \times 10 \times 10) = 10/25 =$ **2/5**.

60. Since 40% = 2/5 and 0.8 = 4/5, we have $2/5 \times 2/3 \times 24 \div 4/5 = (2 \times 2 \times 24 \times 5)/(5 \times 3 \times 4) =$ **8**.

Warm-Up 5

61. We will factor out a common factor of 6^2 to make the calculations simpler. We have $(2 \times 6^3 + 6^2) - 7 \times 6^2 = (2 \times 6 \times 6^2 + 6^2) - 7 \times 6^2 = 6^2 \times (12 + 1 - 7) = 6^2 \times 6 = 6^3 =$ **216**.

62. There are seven types of triangles in the figure that have at least two sides congruent. As shown, six types occur in six different locations and one type occurs in just two locations. That's a total of $6 \times 6 + 2 =$ **38** triangles.



63. This problem is similar to problem 28, which asked in how many ways Sidarth could write 5 as a sum of 1s and 2s. Since Fido never climbs more than three stairs in one step, instead of just partitioning the stairs into groups of 1 and 2, we can also include groups of 3.
There is <u>1</u> way for Fido to climb the stairs if he steps on each stair (1 + 1 + 1 + 1 + 1).
There are <u>4</u> ways for Fido to climb the stairs if he steps on exactly four stairs (2 + 1 + 1 + 1; 1 + 2 + 1 + 1; 1 + 1 + 2 + 1; 1 + 1 + 1 + 2).
There are <u>6</u> ways for Fido to climb the stairs if he steps on exactly three stairs (2 + 2 + 1; 2 + 1 + 2; 1 + 2 + 2; 3 + 1 + 1; 1 + 3 + 1; 1 + 1 + 3).
These are the 1 + 4 + 6 = **11** ways Fido can climb the five stairs.

64. The x^2 term in the numerator "cancels" the x^2 in the denominator since x^2/x^2 is 1. That leaves $(x^2)^3 = x^2 \cdot x^2 \cdot x^2 = x^6$. So $y =$ **6**.

65. The \$10,000 collected in tax is 8% of some unknown amount, say x dollars, in sales, and we can write the equation $10{,}000 = 0.08x$. Solving, we get $x = 125{,}000$, which means that \$125,000 in sales was subject to the 8% sales tax, and the remaining \$400,000 − \$125,000 = **\$275,000** in sales was not.

66. To ensure that at least one fish travels through each route to the food, let's assign one fish to each of these three routes: above the pipe, through the pipe and below the pipe. Now we need only consider the possibilities for the remaining 97 fish to choose one of the three routes. If none of the 97 fish go above the pipe, then there are 98 ways to divide the 97 fish between the other two routes. If one of the 97 fish goes above the pipe, then there are 97 ways to divide the remaining 96 fish between the other two routes. We can continue counting in this manner until we reach the cases in which all 97 fish go above the pipe and none go through or below it. We get $98 + 97 + 96 + \cdots + 3 + 2 + 1 = (98 \times 99)/2 = 49 \times 99$. At this point, instead of multiplying this out by hand, it would be simpler to calculate $49 \times (100 - 1)$. Doing so, we see that there are $49 \times (100 - 1) = 4900 - 49 =$ **4851** ways for the school of 100 fish to get to the food.

67. Some Mathletes may have already learned that 60 mi/h is 88 ft/s. These students will quickly realize that 44 ft/s must be $(1/2) \times 60 =$ **30** mi/h. Others might convert the speed as follows: $\dfrac{44\ \cancel{\text{feet}}}{1\ \cancel{\text{second}}} \times \dfrac{60\ \cancel{\text{seconds}}}{1\ \cancel{\text{minute}}} \times \dfrac{60\ \cancel{\text{minutes}}}{1\text{ hour}} \times \dfrac{1\text{ mile}}{5280\ \cancel{\text{feet}}} = (44 \times 60 \times 60)/5280 = (44 \times 60)/88 = (22 \times 30)/22 =$ **30** mi/h.

68. The word CHAIRS has six letters with no letters that repeat. So there are $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 =$ **720** ways the letters can be arranged.

69. Let PQ = $2a$ cm. Then PS is $3a$ cm, so PQRS has perimeter $2(2a + 3a) = 10a$ cm and area $(2a)(3a) = 6a^2$ cm^2. We are told that the area of PQRS is 1536 cm^2, so $6a^2 = 1536$. Solving for a, we get $a^2 = 1536/6 \rightarrow a^2 = 256 \rightarrow a = 16$. Then PQRS has perimeter $10a = 10 \times 16 =$ **160** cm.

70. To evaluate a tower of exponents, we start at the top and work our way down. For the first term, $2^{0^{1^2}}$, since $1^2 = 1$ and $0^1 = 0$, we evaluate $2^0 = 1$. For the second term, $2^{2^{1^0}}$, since $1^0 = 1$ and $2^1 = 2$, we evaluate $2^2 = 4$. Therefore, the final result is $1 - 4 =$ **−3**.

Warm-Up 6

71. Since 1 quart = 32 fluid ounces, a quart is 8 times the 4 fluid ounces of lotion that costs \$3, so the cost of a quart will be 8 times \$3, or **\$24**.

72. The number in the bottom right box is the product of the original four numbers. This means that $18 \times a \times 4 \times b = 5184$, so $72ab = 5184$ and $ab = 5184/72 = 72$. We are looking for the least possible value of $a + b$, which occurs when a and b are 8 and 9. We have $8 + 9 =$ **17**.

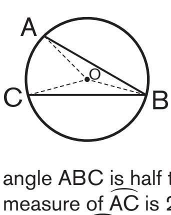
73. As shown, the number 25 is at the center, or the median, of the seventh row. Since the numbers are consecutive, we know that 25 is the average of the seven numbers in that row, so the total of the seven numbers must be $7 \times 25 =$ **175**.

1
2 3
4 5 6
7 8 9 10
11 12 13 14 15
16 17 18 19 20 21
22 23 24 25 26 27 28

74. The terms of an arithmetic sequence must increase or decrease by a constant amount, called the common difference. In this sequence, there must be 5 equal increases from the first term, 12, to the sixth term, 47. Therefore, the common difference must be $(47 - 12) \div 5 = 35 \div 5 = 7$. From the first term, 12, to the fourth term, y, this common difference would be applied 3 times to get $y = 12 + 3(7) = 12 + 21 =$ **33**.

75. Since the width of the rectangle is one-third of its length, then the perimeter must be $1 + 3 + 1 + 3 = 8$ widths. Dividing 136 cm by 8, we see that the width is 17 cm, which means the length is 3×17 cm = 51 cm. Thus, the area of the rectangle is $17 \times 51 =$ **867** cm^2.

76. This problem is a little tricky because of the three occurrences of the letter A in the word ALASKA. We will consider different cases based on the number of As present and create organized lists. First, let's consider three-letter permutations with three As. There is only 1 such permutation (AAA). Next, let's determine the number of three-letter permutations with two As. There are 3 choices for the third letter, and each three-letter combination can be arranged 3 different ways for a total of $3 \times 3 = 9$ permutations (AAL, ALA, LAA, AAS, ASA, SAA, AAK, AKA, KAA). If we consider three-letter permutations with one A, there are 3 choices for the other two letters, and there are 3 ways to choose two of these three letters. Each three-letter combination can be arranged in 6 different orders for a total of $3 \times 6 = 18$ permutations (ALS, ASL, LAS, LSA, SAL, SLA, AKS, ASK, KAS, KSA, SAK, SKA, AKL, ALK, KAL, KLA, LAK, LKA). Finally, we need to count the three-letter permutations with no As. There are 3 choices for the three letters and $3 \times 2 = 6$ arrangements (KLS, KSL, LKS, LSK, SKL, SLK). That's a total of $1 + 9 + 18 + 6 =$ **34** permutations.



77. If we draw three radii from center O to A, B and C, as shown, we create congruent isosceles triangles AOB and COB. Knowing $m\angle ABC = 30$ degrees, it follows that $m\angle OBC = m\angle OCB = m\angle OAB = m\angle OBA = 15$ degrees. That also means that $m\angle AOB = m\angle COB = 180 - 2(15) = 150$ degrees and $m\angle AOC = 360 - 2(150) = 60$ degrees. Angle AOC is the central angle that intercepts minor $\overset{\frown}{AC}$. Since it is $60/360 = 1/6$ of the circle, it follows that the length of $\overset{\frown}{AC}$ is 1/6 of the circle's circumference. The circumference of a circle of diameter 12 inches is $C = d \times \pi = 12\pi$ inches. Therefore, minor $\overset{\frown}{AC}$ has length $(1/6) \times 12\pi =$ **2π** inches. This problem can be solved much faster if you know that the degree measure of inscribed angle ABC is half the measure of its intercepted $\overset{\frown}{AC}$ (see "Angles and Arcs Stretch" on p. 36). So if $m\angle ABC = 30$ degrees, then the degree measure of $\overset{\frown}{AC}$ is $2 \times 30 = 60$ degrees. Since the degree measure of $\overset{\frown}{AC}$ is $60/360 = 1/6$ of the circle, this again leads us to conclude that the length of $\overset{\frown}{AC}$ is 1/6 of the circle's circumference, which we calculated to be **2π** inches.

78. For those who do not recognize that A must be **7** since $0.0049 = (0.07)^2$, one way to solve this problem is by converting 0.0049 to a fraction. We have $\sqrt{(0.0049)} = \sqrt{(49/10{,}000)} = \sqrt{(49)}/\sqrt{(10{,}000)} = 7/100 = 0.07$. Therefore, A is **7**.

79. Let's call the two numbers p and q. From the information given, we can write two equations: $|p-q| = 6$ and $|p^2 - q^2| = 24$, which means that $p - q = 6$ or $p - q = -6$ and $p^2 - q^2 = 24$ or $p^2 - q^2 = -24$. Factoring $p^2 - q^2$, we get $(p - q)(p + q) = 24$ or $(p - q)(p + q) = -24$. We can substitute 6 for $p - q$ in these two equations to get $6(p + q) = 24$ and $6(p + q) = -24$, which means that $p + q = 4$ or $p + q = -4$. (Note: Substituting -6 for $p - q$ in the two equations yields the same result.) Now we know that the difference of the two numbers is 6 or -6 and their sum is 4 or -4. Now we can solve the four systems of equations, as shown, to see that the product is $5 \times (-1) = 1 \times (-5) = \mathbf{-5}$.

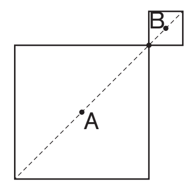
$p+q=4$ $\underline{p-q=6}$ $2p=10$ $p=5 \quad q=-1$	$p+q=4$ $\underline{p-q=-6}$ $2p=-2$ $p=-1 \quad q=5$	$p+q=-4$ $\underline{p-q=-6}$ $2p=-10$ $p=-5 \quad q=1$	$p+q=-4$ $\underline{p-q=6}$ $2p=2$ $p=1 \quad q=-5$

80. We start inside the parentheses and evaluate as follows: 4 ☯ 3 = $4^2 - 2 \times 3^2 = 16 - 18 = -2$. Now we evaluate 5 ☯ −2 = $5^2 - 2 \times (-2)^2 = 25 - 8 = \mathbf{17}$.

Workout 3

81. The average of eleven integers with a sum of 11 must be 11/11 = 1. Since the integers are consecutive, 1 is also the median. In this case the median is the sixth integer in the ordered list. Therefore, the least of the eleven integers must be $1 - 5 = \mathbf{-4}$. A student who doesn't know these properties might approach solving the problem differently. Since the sum of eleven *positive* integers would be greater than 11, we deduce that the consecutive integers in question must include negative values, and when summing these integers, opposites will add to zero, thus leaving only a few consecutive positive integers to add to 11. We can start by considering the eleven integers from −5 to 5, but these integers have a sum of 0. If, however, we consider the integers from −4 to 6, we see that the consecutive integers from −4 to 4 sum to 0, and the final two integers, 5 and 6, sum to 11. Once again we conclude that the least integer is **−4**.

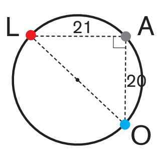
82. The mean of x and y is 12. So the sum of x and y must be $2 \times 12 = 24$, and $x = 24 - y$. The mean of y and 12 is $z/2$. So $(y + 12)/2 = z/2$, and $z = y + 12$. Using these two expressions for x and z, we can write their mean as $(x + z)/2 = (24 - y + y + 12)/2 = 36/2 = \mathbf{18}$.



83. Square A has area 225 cm^2, so it must have side length $\sqrt{225} = 15$ cm and diagonal length $15\sqrt{2}$ cm. Square B has area 16 cm^2, so it must have side length $\sqrt{16} = 4$ cm and diagonal length $4\sqrt{2}$ cm. These squares, with centers labeled A and B, are positioned so that A, B and the common vertex are collinear, as shown. This arrangement gives us the greatest distance between A and B, half the sum of the diagonal lengths of the two squares. That distance is $\frac{15\sqrt{2} + 4\sqrt{2}}{2} = \mathbf{\frac{19\sqrt{2}}{2}}$ cm.

84. If we continue the sequence, we get a repeating pattern: 3, 5, 2, −3, −5, −2, 3, 5, 2, −3, −5, −2… We note that the sum of the six repeating terms is 0. So for any integer n that is a multiple of 6, the sum of the first n terms of this sequence will be 0. The greatest multiple of 6 that is less than 200 is 198. So the sum of the first 198 terms will be 0, making the sum of the first 200 terms $3 + 5 = \mathbf{8}$.

85. Using divisibility rules, we can quickly determine that 2017 is not divisible by 2, 3 or 5. We might then try dividing by 7, 11, 13, etc., and begin to suspect that 2017 is a prime number. To be sure, we would have to try dividing by all the prime numbers less than or equal to the square root of 2017. Since $45 \times 45 = 2025$, we can be sure that 2017 is prime if it has none of the following primes as factors: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41 and 43. Since none of them is a factor, we conclude that 2017 is prime, and the sum of its factors is $1 + 2017 = \mathbf{2018}$.



86. Because of the way that Oberon, Lance and Arthur are seated, a right triangle is formed with legs of length 20 feet and 21 feet. The hypotenuse is a diameter of the circle. You may recognize these two values as being part of the 20-21-29 Pythagorean Triple. Those who do not can use the Pythagorean Theorem to calculate the diameter as $\sqrt{(20^2 + 21^2)} = \sqrt{(400 + 441)} = \sqrt{841} = \mathbf{29}$ feet.

87. Comparing the two transactions, we see that the extra $12 - 8 = 4$ regular binders in the second transaction have to account for the $46.00 - 32.60 = \$13.40$ difference in cost. Therefore, a regular binder must cost $13.40 \div 4 = \$3.35$. Using this and the information from the first transaction, we calculate the cost of the celebrity binder to be $32.60 - 8 \times 3.35 = 32.60 - 26.80 = 5.80$. The difference between the cost of a celebrity binder and that of a regular binder is $5.80 - 3.35 = \mathbf{\$2.45}$.

88. Let p represent the price of a widget. Since Mr. Jones sold n widgets, he earned $0.03 \times p \times n$, and Mr. Smith earned $0.05 \times p \times (n - 500)$. These two commission amounts are equal, so we can write the equation $0.03pn = 0.05p(n - 500) \rightarrow 0.03n = 0.05n - 25$. Solving for n, we get $0.03n = 0.05n - 25 \rightarrow 0.02n = 25 \rightarrow n = 1250$. So Mr. Jones sold 1250 widgets, and Mr. Smith sold $1250 - 500 = \mathbf{750}$ widgets.

89. The y values in the table appear to be the cubes of the corresponding x values. This would suggest that $a = 1$ and $b = 3$, which results in the sum $a + b = 1 + 3 = \mathbf{4}$. Alternatively, from the table of values we have the following equations: $27 = a \times 3^b$ and $8 = a \times 2^b$. Dividing these two equations yields $27/8 = 3^b/2^b$, so $b = 3$, $a = 1$ and $a + b = 1 + 3 = \mathbf{4}$.

90. We need to consider the top and bottom surfaces, which are both rings, and the inner and outer lateral surfaces. The top and bottom surfaces each have area $(2.5^2 - 1.5^2)\pi = (6.25 - 2.25)\pi = 4\pi$ in^2. The inner lateral surface has area $2 \times 1.5 \times \pi \times 1 = 3\pi$ in^2. The outer lateral surface has area $2 \times 2.5 \times \pi \times 1 = 5\pi$ in^2. The total surface area Donny calculated for each roll of tape was $4\pi + 4\pi + 3\pi + 5\pi = 16\pi$ in^2. That means the estimated surface area for a dozen donuts is $16\pi \times 12 = \mathbf{192\pi}$ in^2.

Workout 4

91. For each of the 3 games Amanda could watch at noon on Saturday, there are 4 games she could watch at 8 p.m. and then 5 games the next day. So there are 3 × 4 × 5 = **60** combinations of games she can watch.

92. When the values of a, b and c are substituted into the expression, we get $\dfrac{\frac{12\times4\times5}{1/2}-(6\times4^2-4)}{0.5}=\dfrac{\frac{240}{1/2}-(96-4)}{0.5}=\dfrac{480-92}{0.5}=\dfrac{388}{0.5}=\mathbf{776}$.

93. The two families are approaching each other at 45 + 53 = 98 mi/h. Since time = distance ÷ rate and the two families start out 1029 miles apart, it will take 1029 ÷ 98 = **10.5** hours for them to pass each other.

94. Using a calculator to divide 1 by 27, we see that this is a non-terminating decimal with three digits repeating: 0.037037…. The third digit of the repeating pattern, the 7, will be in the 3rd, 6th, 9th, ..., 36th, 39th and 42nd places after the decimal point. Because the 42nd place contains the digit 7, the place before that, the 41st place, will contain the digit **3**.

95. Since the mode, median and mean form an increasing sequence and we are trying to maximize the value of y, let's start by assuming $x = 11$ and $y \geq 17$. Our list of integers is now, from least to greatest, 11, 11, 13, 15, 17, y. Now these integers have a mode of 11, a median of 14 and a sum of $67 + y$. The difference between the median and the mode is 14 − 11 = 3. So in order for the mode, median and mean to form an increasing sequence, the mean must be 14 + 3 = 17. We can write the following equation for the mean: $(67 + y) \div 6 = 17$. Solving for y, we see that $67 + y = 102$, so $y =$ **35**, which is the greatest possible value of y.

96. The semicircle has radius 13/2 = 6.5 and area $(\pi \times 6.5^2) \div 2 = 21.125\pi$ units^2. The area of the triangle is (1/2) × 5 × 12 = 30 units^2. The difference between these two areas, $21.125\pi - 30 \approx$ **36** units^2, is the total area of the shaded regions.

97. Each of the 12 tourists has two choices for a tour. That would be $2^{12} = 4096$ different ways to split into two groups. But we cannot allow all the tourists to go on either one of the tours, so our answer is 4096 − 2 = **4094** ways.

98. It's virtually impossible to draw 30 circles carefully enough to count the number of distinct intersections. You might try drawing one circle and then adding more circles, one at a time, and counting intersections to see if a pattern emerges. Doing so reveals that with 1 circle, there are 0 intersections; with 2 circles, there are, at most, 2 intersections; with 3 circles, there are, at most, 6 intersections; with 4 circles, there are, at most, 12 intersections. Organizing these findings in a table like the one shown, we see that for n circles, there appear to be $n(n - 1)$ distinct intersections. Therefore, 30 circles will have, at most, 30 × 29 = **870** intersections. Alternatively, since every pair of circles intersects in, at most, 2 points, and there are ${}_{30}C_2$ ways to select a pair from a collection of 30 circles, it follows that there are 2 × (30 ×29)/2 = 30 × 29 = **870** intersections.

Circles	1	2	3	4	⋯	n
Intersections	0	2	6	12	⋯	$n(n-1)$

99. The sphere has radius r, so its "height" would be $2r$. This means the height of both the cone and the cylinder is $2r$. The volume of the cylinder is $\pi \times r^2 \times h = \pi \times r^2 \times 2r = 2 \times \pi \times r^3$. The volume of the cone is $(1/3) \times \pi \times r^2 \times h = (1/3) \times \pi \times r^2 \times 2r = (2/3) \times \pi \times r^3$. The volume of the sphere is $(4/3) \times \pi \times r^3$. The volumes of the cone and sphere, combined, are $(2/3) \times \pi \times r^3 + (4/3) \times \pi \times r^3 = 2 \times \pi \times r^3$, which is exactly the volume of the cylinder. So **100**% of the cylinder is filled.

100. Let's list all the odd numbers less than 500 that we can create with 3s and 4s and strike through those that are not prime: 3, ~~33~~, 43, ~~333~~, ~~343~~, 433, 443. The sum of the primes among them is 3 + 43 + 433 + 443 = **922**.

Warm-Up 7

101. Mac has 25 × 0.2 = 5 red marbles, and Thayer also has 20 × 0.25 = 5 red marbles. Thus, the absolute difference between the numbers of red marbles they have is **0** marbles.

102. First we evaluate $g(-3) = 2(-3) + 4 = -6 + 4 = -2$. Next we evaluate $f(-2) = (-2)^2 - 2 = 4 - 2 =$ **2**.

103. A cube has 6 faces, 12 edges and 8 vertices. When each vertex is cut off, 1 new equilateral triangle face is created, though the size of the triangle can vary. There is a net gain of, at most, 3 edges for a maximum of 8 × 3 = 24 additional edges. There is also a net gain of, at most, 2 vertices, for a maximum of 8 × 2 = 16 additional vertices. At most, the sum of the number of faces, edges and vertices will be (6 + 8) + (12 + 24) + (8 + 16) = 14 + 36 + 24 = **74** . Note that this maximum can be achieved if the cube is cut so that for each new triangle face none of its vertices reaches the midpoint of an edge of the cube (Fig. 1). Otherwise, the resulting solid would have fewer edges and vertices (Fig. 2).

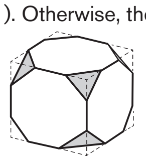


Figure 1

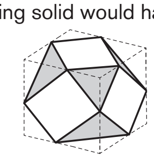


Figure 2

104. Since 4 oranges cost 90 cents, 1 dozen oranges cost 3 × 90 = 270 cents. Therefore, 3 dozen oranges cost 3 × 270 = 810 cents, or $**8.10**.

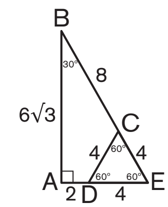
105. Let's imagine that we have 100 pounds of fully matured grapes, consisting of 80 pounds of water and 20 pounds of what we will call pulp. After the drying process, the amount of pulp is still 20 pounds, but the raisins are now only 20% water. The pulp accounts for the other 80%, and the raisins must weigh 25 pounds since 20 is 80% of 25. So, 5 out of 80 pounds of water remain, which is 5/80 = **1/16** of the original water.

106. We are looking for a number with lots of factors, probably a number that is a multiple of both 2 and 3, which makes it a multiple of 6. It turns out that **48** has the greatest sum of proper factors. That sum is 1 + 2 + 3 + 4 + 6 + 8 + 12 + 16 + 24 = 76.

107. We see that $(AAA)^3$ has a units digit of 7, which can only occur if the number being cubed has a units digit of 3, so A = **3**.

108. First, we note that the syrup maker initially has 25 liters of maple syrup and 75 liters of base. She will need to add 50 liters of maple syrup to make the ratio 1:1. Since 90%, or 9/10 of the amount of sap will evaporate, she can obtain those 50 liters of maple syrup from 50 × 10 = **500** liters of maple sap.

109. Suppose we extend sides AD and BC to intersect at E, as shown. Since the $m\angle B = 360 - (90 + 120 + 120) = 360 - 330 = 30$ degrees, it follows that △ABE is a 30-60-90 right triangle. The area of ABCD is the area of △ABE minus the area of △CDE. Within the small triangle, since $m\angle C = m\angle D = 180 - 120 = 60$ degrees, we know that △CDE is equilateral, so CD = DE = CE = 4 units. The area of an equilateral triangle with side length s is $(1/4) \times s^2\sqrt{3}$. So △CDE has area $(1/4) \times 16\sqrt{3} = 4\sqrt{3}$ units². For △ABE, BE = 8 + 4 = 12 units. In a 30-60-90 right triangle, the length of the short leg is half that of the hypotenuse, and the length of the long leg is $\sqrt{3}$ times the length of the short leg. So AE = 12 ÷ 2 = 6 units, and AB = $6\sqrt{3}$ units. Triangle ABE, then, has area $(1/2) \times 6 \times 6\sqrt{3} = 18\sqrt{3}$ units². Therefore, the area of ABCD is $18\sqrt{3} - 4\sqrt{3} = \mathbf{14\sqrt{3}}$ units².



110. The next 2 five-letter arrangements that follow Zzyzx alphabetically are Zzyzy and Zzyzz. The remaining arrangements are of the form Zzz _ _. There are 26 × 26 = 676 ways to fill in the last two letters, so following Zzyzx alphabetically, there are 2 + 676 = **678** arrangements.

Warm-Up 8

111. We know that $z = 20$. So, $x = 3 \times 20 = 60$. Thus, $60/y = 10$ and $y = 60/10 =$ **6**.

112. There are $2^5 = 32$ different ways the balls can be distributed between the two containers. Having the two red balls alone in the red container is just one of these 32 possibilities. The probability of this occurring, therefore, is **1/32**.

113. The diameter of the circle, which is the distance from A to B, is $d = \sqrt{[(10 - (-2))^2 + (2 - 4)^2]} = \sqrt{(12^2 + (-2)^2)} = \sqrt{(144 + 4)} = \sqrt{148} = 2\sqrt{37}$ units. The radius of the circle is half this amount, which is $\sqrt{37}$ units, so the area of the circle is $\pi \times r^2 = \pi \times (\sqrt{37})^2 = \mathbf{37\pi}$ units².

114. Since the sequence is arithmetic, the difference of 56 − 20 = 36 must be divided into equal intervals. We can do this using the factors of 36. If we list the factors of 36 in pairs, we can think of the first factor as the number of intervals and the second factor as the size of each interval: 1 × 36, 2 × 18, 3 × 12, 4 × 9, 6 × 6, 9 × 4, 12 × 3, 18 × 2 and 36 × 1. That makes **9** possible sets of integers that form arithmetic sequences.

115. If you don't recall that the interior angle measures of a regular pentagon are 108 degrees, you can subdivide the pentagon into three triangles. Since the sum of the interior angles of a triangle is 180 degrees, the sum of the interior angles of the pentagon must be 3 × 180 = 540 degrees. Divide by 5, and we find that each interior angle measures 108 degrees. The isosceles right triangles have angle measures of 45, 45 and 90. The vertex angle of each shaded triangle is what is left when two 45-degree angles are subtracted from the 108-degree angle, which is **18** degrees.

116. There are 12 × 12 = 144 tiles on the bathroom floor. We can group the tiles into three categories: 4 corner tiles, 40 edge tiles and 100 interior tiles. There is a 100/144 chance that the first of the wrong-colored tiles is placed among the 100 interior tiles. If this happens, there is a 4/143 chance that the other wrong-colored tile shares an edge with it. There is a 40/144 chance that the first wrong-colored tile is one of the 40 tiles around the edge of the room. If this happens, there is a 3/143 chance that the other wrong-colored tile shares an edge with it. Finally, there is a 4/144 chance that the first wrong-colored tile is placed in one of the 4 corners. If this happens, there is a 2/143 chance that the other wrong-colored tile shares an edge with it. The total probability, then, that the two wrong-colored tiles share an edge is (100/144) × (4/143) + (40/144) × (3/143) + (4/144) × (2/143) = 528/(144 × 143) = (11 × 48)/(144 × 143) = **1/39**. Alternatively, there are 11 ways for tiles to share an edge in each row and 11 ways in each column, for a total of 2 × 12 × 11 ways. Dividing this by the number of ways to place the two tiles, which is (144 × 143)/2, we get (2 × 12 × 11) ÷ (144 × 143)/2 = (2 × 12 × 11 × 2)/(144 × 143) = **1/39**.

117. The units digits of the powers of 3 repeat in a cycle of four numbers: 3, 9, 7, 1. Since $17 \equiv 1 \bmod 4$, the units digit of 3^{17} will be the first number in this pattern, which is 3. Similarly, the units digits of the powers of 7 repeat in the pattern 7, 9, 3, 1. Since $23 \equiv 3 \bmod 4$, the units digit of 7^{23} will be the third number in this pattern, which is 3. The units digit of the product $3^{17} \times 7^{23}$ will be the product of the units digits of 3^{17} and 7^{23}, which is 3 × 3 = **9**.

118. The geometric mean of two numbers is the square root of the product of the two numbers. For 14 and 126, the geometric mean is $\sqrt{(14 \times 126)} = \sqrt{(14 \times 14 \times 9)} = \sqrt{(14^2 \times 3^2)} = 14 \times 3 =$ **42**.

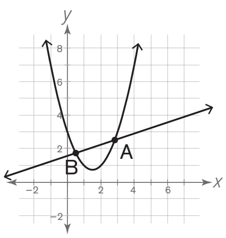
119. The area of the larger circle is $\pi \times 12^2 = 144\pi$ inches^2. The diagonal of the square is the same as the diameter of the circle, which is $2 \times 12 =$ 24 inches. By the properties of 45-45-90 right triangles, we know that the side length of the square is $24/\sqrt{2} = 12\sqrt{2}$, which is also the diameter of the smaller circle. So the radius of the smaller circle is $12\sqrt{2} \div 2 = 6\sqrt{2}$. The area of the smaller circle, then, is $\pi \times (6\sqrt{2})^2 = 72\pi$. The shaded region has area $144\pi - 72\pi =$ **72π** in^2.

120. The base-10 value of 321_5 is $3 \times 5^2 + 2 \times 5^1 + 1 \times 5^0 = 75 + 10 + 1 = 86$. The base-10 value of 321_4 is $3 \times 4^2 + 2 \times 4^1 + 1 \times 4^0 =$ $48 + 8 + 1 = 57$. Their base-10 sum is $86 + 57 = 143$. We can write 143 as a sum of powers of 3 as follows: $143 = \underline{1} \times 3^4 + \underline{2} \times 3^3 + \underline{0} \times 3^2 + \underline{2} \times 3^1 + \underline{2} \times 3^0$. Therefore, the sum in base 3 is **12022**.

Workout 5

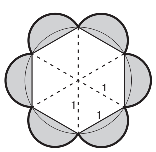
121. If six numbers have a mean of 6, then their sum must be $6 \times 6 = 36$. Since we want to maximize the sum of the largest three numbers, we should minimize the sum of the smallest three numbers. The least possible sum would be $1 + 2 + 3 = 6$. We need to see if this works with the given conditions. With 6 as the median and 12 as the greatest number in the set, we could have 1, 2, 3, 9, 9, 12. But this won't work because the set does not contain six different integers. If we change the third and fourth numbers to 4 and 8, respectively, we have 1, 2, 4, 8, 9, 12. The median is still 6, and there are no duplicates in the set. The sum of the three largest numbers is $8 + 9 + 12 =$ **29**.

122. Because Nathan ran 12 seconds per mile slower than his goal in the first half of the run, it would seem that he should run 12 seconds per mile faster than his goal in the second half. That would be a pace of 7 minutes 12 seconds per mile, or **7.2** minutes per mile. We should confirm this using a different approach. To complete the 5-mile run at an average pace of 7 minutes 24 seconds per mile, or 7.4 minutes per mile, Nathan must finish in $7.4 \times 5 = 37$ minutes. Running at a pace of 7 minutes 36 seconds per mile, or 7.6 minutes per mile, for the first 2.5 miles took $7.6 \times 2.5 =$ 19 minutes. Nathan would then need to complete the final 2.5 miles in $37 - 19 = 18$ minutes. This confirms that his pace should be $18 \div 2.5 =$ **7.2** minutes per mile.



123. Let's get an idea of what the functions $y = x^2 - 3x + 3$ and $4x - 12y = -19$ look like and how they intersect each other by graphing them on a coordinate grid as shown. We have a line that intersects a parabola at two points. Solving the second equation for y yields $4x - 12y = -19 \rightarrow 12y = 4x + 19 \rightarrow y = (4x + 19)/12$. Setting the expressions for y equal to each other and solving for x, we get $x^2 - 3x + 3 = (4x + 19)/12 \rightarrow 12(x^2 - 3x + 3) = 4x + 19 \rightarrow 12x^2 - 36x + 36 =$ $4x + 19 \rightarrow 12x^2 - 40x + 17 = 0$. Factoring the quadratic equation gives us $(2x - 1)(6x - 17) = 0$. So $2x - 1 = 0$ and $x = 1/2$ or $6x - 17 = 0$ and $x = 17/6$. The sum of these x-coordinates is $1/2 + 17/6 = (3 + 17)/6 = 20/6 =$ **10/3**. Alternatively, the sum of the roots of a quadratic equation is always the opposite of the coefficient of the linear term when the quadratic is written with leading coefficient 1. The equation $12x^2 - 40x + 17 = 0$ written in the required form is $x^2 - (10/3)x + (17/12)$. The sum of the roots, then, is the opposite of $-10/3$, which is **10/3**.

124. ABC is a 30-60-90 right triangle. The ratio of the short leg to the long leg of a 30-60-90 triangle is 1 to $\sqrt{3}$, so the length of AC must be $9 \div \sqrt{3} = 3\sqrt{3}$ cm. Triangle ACD is also a 30-60-90 triangle, so the same ratio applies and length DC must be $3\sqrt{3} \div \sqrt{3} =$ **3** cm.



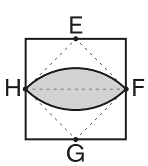
125. The diameter of the circle is $1 \times 2 = 2$ units. If we use the vertices of the hexagon to draw three diameters, we divide the hexagon into 6 equilateral triangles, as shown. The side length of the regular hexagon, also the diameter of each semicircle, then, is 1 unit. When paired, the six semicircles together make three circles of diameter 1 unit. The sum of their circumferences, which is also the perimeter we seek, is $3 \times \pi \times 1 \approx$ **9.4** units.

126. This organized list shows all the three-digit positive integers whose digits have a sum of 9. There are $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 =$ **45** such integers.

108	207	306	405	504	603	702	801	900
117	216	315	414	513	612	711	810	
126	225	324	423	522	621	720		
135	234	333	432	531	630			
144	243	342	441	540				
153	252	351	450					
162	261	360						
171	270							
180								

127. There are $2 \times 2 \times 2 \times 2 = 16$ ways the next four yo-yos can exit the assembly line. In only 2 of these 16 ways do the yo-yos all have the same color (all blue or all red), so the probability is 2/16, or **1/8**.

128. The diameter of the cup is 6 cm, so the radius is 3 cm. The volume of the cup is $\pi \times r^2 \times h = \pi \times 3^2 \times 12 = 108\pi$ cm^3. The 100π mL of tea will occupy 100π cm^3 of space, so the spherical bubbles need to occupy the remaining $108\pi - 100\pi = 8\pi$ cm^3. Each spherical bubble has volume $8\pi \div 48 = \pi/6$ cm^3. So $4/3 \times \pi \times r^3 = \pi/6$. Solving for r, we get $r^3 = (1/6) \times (3/4) \rightarrow r^3 = 1/8 \rightarrow r = \sqrt[3]{(1/8)} =$ **0.5** cm.



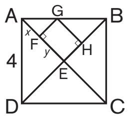
129. Square ABCD has side length 2 m, so the right triangle at each corner has side length 1 m and hypotenuse length $\sqrt{2}$ m. The hypotenuse length is also the radius of the arcs of the circles. Since triangles EFH and GFH are right triangles, it follows that the area of each sector is 1/4 of the area of a circle with radius $\sqrt{2}$ m. As the figure shows, the area of the shaded region is the difference between the combined areas of the sectors, $2(1/4 \times \pi \times (\sqrt{2})^2) = \pi$ m^2, and the combined area of triangles EFH and GFH, $(\sqrt{2})^2 = 2$ m^2. Therefore, the shaded region has area $\pi - 2 \approx$ **1.14** m^2.

130. We should consider the LCMs of all pairs of guesses, as shown in the table. The value of k cannot be 60, because that would make three statements correct. The next possibility is 90, which would make only Bruce and Steven correct. The least possible value of k, then, is **90**.

LCM(15, 18) = 90
LCM(15, 20) = 60
LCM(15, 28) = 420
LCM(15, 60) = 60
LCM(18, 20) = 180
LCM(18, 28) = 252
LCM(18, 60) = 180
LCM(20, 28) = 140
LCM(20, 60) = 60
LCM(28, 60) = 420

Workout 6

131. The consecutive integers from –37 to 37 have a sum of zero, so we should start with 38 and add consecutive integers until we get a sum of 200. Five times 40 is 200, so let's try 38 + 39 + 40 + 41 + 42. This works. The greatest integer in Caynan's sequence is **42**.



132. In the figure, right triangle AFG is isosceles with legs of length x, and rectangle EFGH has sides of lengths x and y. That means that AE = $x + y$. Square ABCD, with side length 4 units, has diagonal length $4\sqrt{2}$ units. Notice that AE is half the diagonal length, so we can write $x + y = 2\sqrt{2} = \sqrt{(2^2 \times 2)} = \sqrt{8}$. Therefore, z = **8**.

133. Two letters can be chosen from the 26 slips of paper in 26 × 25 ÷ 2 = 325 different ways. The 7-letter word ALGEBRA contains 6 different letters, which can be paired in 6 × 5 ÷ 2 = 15 different ways. The probability that two randomly drawn slips are one of those pairs is 15/325 = **3/65**.

134. The radius of the pool is 12 feet = 12 × 12 = 144 inches. Since 4.5 feet = 12 × 4 + 6 = 54 inches, we know that Maxwell will fill the pool to a height of 54 – 3 = 51 inches. So the volume of water used is $\pi \times 144^2 \times 51 = 1{,}057{,}536\pi$ in^3. That's 1,057,536π/231 gallons of water. Since every gallon costs 0.15 cent, this will cost 1,057,536π/231 × 0.0015 ≈ $**22**.

135. The place values for this binary number, from left to right, are 2^7, 2^6, 2^5, 2^4, 2^3, 2^2, 2^1 and 2^0. So $10111010_2 = 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 128 + 32 + 16 + 8 + 2$. Since 8 is a power of 2, there is no need to find the base 10 value of this number to convert it to base 8. We know that $128 = 2 \times 8^2$, $32 + 16 + 8 = (4 + 2 + 1) \times 8^1 = 7 \times 8^1$ and $2 = 2 \times 8^0$. Therefore, $10111010_2 = 2 \times 8^2 + 7 \times 8^1 + 2 \times 8^0$ = **272** base 8.

136. To raise the water in the tank by 1 inch, the metal spheres will need to occupy 10 × 15 × 1 = 150 in^3 of space. Each sphere has a radius of 1/6 ÷ 2 = 1/12 inch and a volume of $4/3 \times \pi \times (1/12)^3 = \pi/1296$ in^3. To raise the water level 1 inch, it takes 150 ÷ (π/1296) ≈ **61,900** spheres.

137. If Leon's favorite pens were previously x dollars for a box of n pens, then the cost per pen was x/n dollars per pen. A reduction of 10% in the price means that each box now costs $0.9x$. With 25% fewer pens, each box now contains $0.75n$ pens. So the cost per pen is now $0.9x/0.75n = 1.2(x/n)$. That's an increase in cost of 1.2 – 1 = 0.2 = **20**% per pen.

138. This problem is a calculator exploration. When the numbers begin displaying in scientific notation, it helps to recognize that the number of digits is one more than the exponent. Using the calculator, we see that 10! has 7 digits; 15! has 13 digits; 20! has 19 digits; 21! has 20 digits; 22! has 22 digits. This means that 22 is the *least* integer for which n! has n digits. Continuing on, we see that 23! has 23 digits; 24! has 24 digits; 25! has 26 digits. Since the number of digits keeps increasing at least as fast as the number does when we pass 24, it follows that **24** is the greatest integer for which n! has n digits.

1	3	5	15
2	6	10	30
4	12	20	60
8	24	40	120

139. The number of sides on Percy's polygon must be a factor of 120. Once we've listed the factors, it's not difficult to find the sum by using a calculator. If, however, we had to find the sum without the aid of a calculator, we could arrange the factors in an array such as the one shown to find the sum more directly. The sum of the numbers in the first column, 1 + 2 + 4 + 8 = 15, times the sum of the numbers in the first row, 1 + 3 + 5 + 15 = 24, is 15 × 24 = 360, which is the sum of all the factors of 120. Since we cannot have a polygon with fewer than 3 sides, and since Percy's polygon is not a triangle or an octagon, we need to subtract 1 + 2 + 3 + 8 = 14 from that sum. Our answer is 360 – 14 = **346**.

140. On Mars a standard year has 95 weeks and each week has 7 sols, for a total of 95 × 7 = 665 sols. During a leap year, there is one more week, or an additional 7 sols, for a total of 672 sols. Since Mars has a 668-sol year, there are an average of 668 sols per year. If we let n and m represent the number of standard years and leap years, respectively, we have $(665n + 672m) \div (n + m) = 668 \rightarrow 665n + 672m = 668n + 668m \rightarrow 4m = 3n \rightarrow m/n = 3/4$. The ratio of leap years to standard years is 3 to 4. So 3 out of every 7 years, or **3/7** of the years, are leap years.

Warm-Up 9

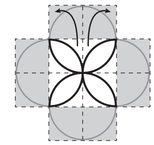
141. There are many pairs of numbers to consider with a sum of 11. However, there are only four factor pairs of 24 to consider: 1 × 24, 2 × 12, 3 × 8 and 4 × 6. The pair 3 and 8 have a sum of 11 and an absolute difference of 8 – 3 = **5**.

142. Varun walked 2/5 of the length of the walkway, so Becca must have also walked 2/5 of the length of the walkway. In the same amount of time, the walkway took her the other 3/5 of the length of the walkway. The ratio of the walkway's speed to Becca and Varun's speed is 3/5 to 2/5 = **3/2**.

143. Suppose the original pentagon has side length 1 unit and perimeter 1 × 5 = 5 units. When two sides of this pentagon are doubled, the perimeter of the new pentagon is 1 × 3 + 2 × 2 = 3 + 4 = 7 units. The increase in the perimeter is (7 – 5)/5 = 2/5 = **40**%.

144. Let d represent the number of dimes Joe has. Joe has 4 more nickels than dimes, so he has $d + 4$ nickels. He has 2 more quarters than nickels, so he has $d + 4 + 2 = d + 6$ quarters. Joe has a total of 37 coins, so we can write the equation $d + d + 4 + d + 6 = 37$. This simplifies to $3d + 10 = 37$. So $3d = 27$, and $d = 9$. If follows, then, that Joe has $d + 6 = 9 + 6 =$ **15** quarters.

145. A six-digit repeating integer can take three different forms: a three-digit sequence occurring twice, a two-digit sequence occurring three times or one digit occurring six times. There are 9 × 10 × 10 = 900 ways to choose a three-digit repeating sequence, and there are 9 × 10 = 90 ways to choose a two-digit repeating sequence, for a total of 900 + 90 = 990 repeating integers. But the 9 sequences of one digit occurring six times (111111 to 999999) have been counted twice, which means that 990 − 9 = **981** numbers are six-digit repeating integers.



146. In the figure shown, each circle is divided into quarters. The arrows show how the pieces can be rearranged to form four congruent shaded rectangles. Each rectangle is 2 cm by 4 cm, so the total area of the shaded regions is 4 × (2 × 4) = **32** cm^2.

147. The two-digit representations of all the months are valid days. The two-digit representation of only 12 days represent valid months. Thus, there would be 12 × 12 = 144 dates for which both interpretations are valid. But we need to exclude the 12 dates for which the month and day are the same. That leaves 144 − 12 = **132** dates.

148. We can set the two expressions equal to one another and solve for x. We have $6x - 6 = 2x + 14 \rightarrow 4x = 20 \rightarrow x = 5$. The square base of the pyramid, then, has side length $2x + 14 = 2 \times 5 + 14 = 10 + 14 = 24$ units and area 576 $units^2$. The four triangular sides of the pyramid each have side length 24 units, height $12\sqrt{3}$ units and area $(1/2) \times 24 \times 12\sqrt{3} = 144\sqrt{3}$ $units^2$. The surface area of the pyramid is $4 \times 144\sqrt{3} + 576 = 576\sqrt{3} + 576$, which can be rewritten as $24^2(\sqrt{3} + 1)$. Therefore, $a = 24$, $b = 3$, $c = 1$ and $a + b + c = 24 + 3 + 1 =$ **28**.

149. It's probably safe to assume that D doesn't represent 0, 1, 2, 5, 7 or 8 since those digits are shown in the equation. That leaves 3, 4, 6 and 9 as the possible digit that D represents. Since there are only four possibilities, we could start trying them to see which one makes the equation true. If D represents 3, we get 23 × 351 = 8073. If D represents 4, 6 or 9, the equation is not true. Therefore, D must represent the digit **3**.

150. Using only the digits 2 through 9, the number of unique three-digit extensions that can be assigned is 8 × 8 × 8 = **512** extensions.

Warm-Up 10

151. The rectangular prism should be as close to a cube as possible, given the available factors. The prime factorization of 2016 is $2^5 \times 3^2 \times 7$. We want to assign these prime factors to three groups that have as close to the same product as possible. One triple with a product of 2016 is 8 × 14 × 18, which makes the sum of the dimensions 8 + 14 + 18 = 40. Another triple is 9 × 14 × 16, for which 9 + 14 + 16 = 39. The triple with the lowest sum, however, is 12 × 12 × 14, with a sum of 12 + 12 + 14 = **38**.

152. There are two pairs of digits that have to have a sum of 10, and the sum of all the digits has to be a multiple of 10. Since there are only five digits, this means that one of the digits has to be a zero. Also, since the leftmost digit of the number is 7 and we need two pairs of adjacent numbers that differ by 1, it makes sense for the zero to be the rightmost digit. For the first and second digits to differ by 1, the second digit needs to be 6 or 8. The 8 is more promising since it is also a multiple of 4 and allows us to put 2, which is 8 ÷ 4, in the third place. So far we have 782_0. The missing digit adjacent to the 2 needs to be 1 or 3. The sum of that digit and 7 has to be 10, so the missing digit is 3, and the ZIP code is **78230**.

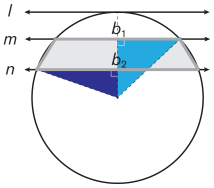
153. We might consider trying to find the prime factorization of 140,209, but it's not divisible by 2, 3, 5, 7 or 11, so this approach could take a while. (It turns out that ABC and CBA are the two prime factors of 140,209.) Another approach is to consider the units digit 9. Only 1 × 9, 3 × 3 and 7 × 7 yield a units digit of 9. In our problem, A, B and C must each represent a different digit, so this means A and C can only be 1 and 9. At this point, we can just try different values of B, and it turns out that 149 × 941 = 140,209. The value of A + B + C is 1 + 4 + 9 = **14**.

154. Since all the triangles are similar, corresponding angles are congruent. So we can let x represent the measure of each of the base angles of each triangle. Then the vertex angle has degree measure $(x + x) \times 2 = 4x$. The sum of the measures of the angles in a triangle is 180 degrees. We have $x + x + 4x = 180$, so $6x = 180$ and $x = 30$. We can see that 8 of these base angles come together at the center of the fan, leaving a total of 360 − 8 × 30 = 360 − 240 = 120 degrees for the congruent angles in the spaces between the blades. Angle α measures 120 ÷ 4 = **30** degrees.

155. The twelve couples amount to 24 participants. So when forming a team of three people, there are 24 choices for the first team member. Since the individual with whom the first team member is coupled is no longer an option for this team, there are 22 choices for the second team member. Again, the individuals with whom the first two team members are coupled cannot be part of this team. That means there are 20 choices for the third team member, which gives us a total of 24 × 22 × 20 = 10,560 ways to form a team of three. But this includes the 6 orders in which each team can be assembled. Therefore, the number of different teams of three that can be formed is 10,560 ÷ 6 = **1760** teams.

156. The algebra below shows one way to find the fraction that is equivalent to the repeating decimal. Let x represent the common fraction equivalent to $0.3\overline{27}$. Then $10x = 3.\overline{27}$ and $1000x = 327.\overline{27}$. Subtracting these two equations yields the following:

$$\begin{aligned} 1000x &= 327.\overline{27} \\ \underline{-10x} &\underline{= \quad 3.\overline{27}} \\ 990x &= 324 \\ x &= \frac{324}{990} = \mathbf{\frac{18}{55}}. \end{aligned}$$



157. The area of a trapezoid with height h and bases of length b_1 and b_2 is $1/2 \times h \times (b_1 + b_2)$. The figure shows the trapezoid in the circle. To find b_2, we can construct the right triangle shown on the left in the circle. This triangle has a hypotenuse of length 3 cm and a short leg of length 1 cm. The long leg, then, has length $\sqrt{(3^2 - 1^2)} = \sqrt{8}$ cm. So, $b_2 = 2\sqrt{8}$ cm. To find b_1, we can construct the right triangle shown on the right in the circle. This triangle has a hypotenuse of length 3 cm and a short leg of length 2 cm. The long leg, then, has length $\sqrt{(3^2 - 2^2)} = \sqrt{5}$ cm. So, $b_1 = 2\sqrt{5}$ cm. We know that the distance between the bases is 1 cm, so the area of the trapezoid is $1/2 \times h \times (b_1 + b_2) = 1/2 \times 1 \times (2\sqrt{8} + 2\sqrt{5}) = \sqrt{8} + \sqrt{5}$ cm^2. Therefore, $ab = 8 \times 5 =$ **40**.

158. Let $n = 2017$ and substitute to get the following: $(n^2 + 11n - 42)/(n - 3)$. Now we can simplify the numerator by factoring. For the fraction, we get $(n + 14)(n - 3)/(n - 3)$, which simplifies to $n + 14$. Substituting 2017 for n yields the answer: $n + 14 = 2017 + 14 =$ **2031**.

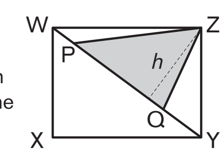
159. We need to know how many 2s we can factor out of $32! = 32 \times 31 \times 30 \times \ldots \times 3 \times 2 \times 1$. We can factor a 2 out of each of the 16 multiples of 2 from 1 to 32; we can factor another 2 out of each of the 8 multiples of 4; we can factor another 2 out of each multiple of 8, and so forth. There are $16 + 8 + 4 + 2 + 1 = 31$ factors of 2, so $n =$ **31**.

160. First, we should note that the total value of the coins will be less than \$1 if all the allowed 3 quarters are not used. We now have an easier problem to solve: how many combinations of dimes, nickels and pennies are there with a total value of at least 25¢, using no more than three of any single denomination? If all 3 dimes are used, then combining them with any combination of 0, 1, 2 or 3 nickels and 0, 1, 2 or 3 pennies gives us a total value greater than \$1, and that's $4 \times 4 = \underline{16}$ combinations. If 2 dimes are used, then there must be at least 1 nickel, and combined with any combination of 0, 1 or 2 of the remaining nickels and 0, 1, 2 or 3 pennies, the total value is greater than or equal to \$1, for another $3 \times 4 = \underline{12}$ combinations. If 1 dime is used, then all 3 nickels must be used. Adding to that, 0, 1, 2 or 3 pennies yields another $\underline{4}$ combinations. In all, there are $16 + 12 + 4 =$ **32** combinations.

Warm-Up 11

161. There are $2 \times 2 \times 2 = 8$ possible outcomes when three coins are flipped. The outcome of 0 tails can occur 1 way and has a probability of 1/8. The outcome of 1 tail can occur 3 ways, so the probability of this outcome is 3/8. The outcome of 2 tails can also occur 3 ways, so the probability of this outcome is also 3/8. The outcome of 3 tails can occur 1 way and has a probability of 1/8. So, the probability that all three students get 0 tails is $1/8 \times 1/8 \times 1/8 = 1/512$, as is the probability that all three get 3 tails. The probability that all three get 1 tail is $3/8 \times 3/8 \times 3/8 = 27/512$, as is the probability that all three get 2 tails. Therefore, the probability that all three get the same number of tails is $(1 + 1 + 27 + 27)/512 = 56/512 =$ **7/64**.

162. The earliest possible sum date is 01/01/02, but the year 2002 will have *only* this one sum date. When determining the earliest year with 12 sum dates, we have to make sure that December has a sum date as well. This doesn't happen until 12/01/13. The latest possible sum date is 12/31/43, but this is again the *only* sum date for the year. When finding the latest year with 12 sum dates, we have to make sure that January and February have sum dates. The year 2032 works for January (01/31/32), but February 29th for this leap year will only give us $2 + 29 = 31$. The year 2031 is not a leap year, so February will not have a sum date. We have to go back to 2030 for February to give us $2 + 28 = 30$. This means the years from 2013 to 2030 all have 12 sum dates, one for each month. That's a total of $2030 - 2012 =$ **18** years.

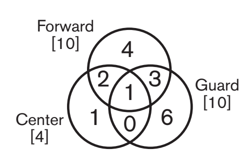


163. Since 3(WP + QY) = 2PQ, we know that (WP + QY)/PQ = 2/3 and that PQ = $3/5 \times$ WY. Diagonal WY divides rectangle WXYZ into two congruent right triangles, each with area equal to half that of WXYZ, or $90 \div 2 = 45$ cm^2. We can write the following equation for the area of $\triangle$WYZ: $1/2 \times \text{WY} \times h = 45$, where h is the altitude from Z to WY, as shown. The area of $\triangle$PQZ, then, is $1/2 \times \text{PQ} \times h = 1/2 \times 3/5 \times \text{WY} \times h = 3/5 \times 45 = 3 \times 9 =$ **27** cm^2.

164. Starting at the point number 1 and drawing segments between points that are 8 spaces apart, we see that the 23rd segment drawn will be to point number $(1 + 23 \times 8) \bmod 50 = 185 \bmod 50 = 35$. The other endpoint of the segment will be point number $35 - 8 = 27$. The sum of these numbers is $27 + 35 =$ **62**.

165. If the figure could not be rotated, there would be $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ ways to assign the colors. Each distinct coloring can be rotated 0, 90, 180 or 270 degrees, which makes the number of distinct colorings $120 \div 4 =$ **30** colorings.

166. Suppose the number is ABBA. Its value is 1001A + 110B. Because 1001 is divisible by 7 but 110 is not, B must be divisible by 7, which means that it must be 0 or 7. If B = 7, then the only way for A77A to be divisible by 8 is for A to be 6, and the four-digit number 6776 is a palindrome divisible by both 7 and 8. Otherwise B = 0, and the only way for A00A to be divisible by 8 is for A to be 8, and the four-digit number 8008 is a palindrome divisible by both 7 and 8. The larger of the two palindromes that meet the conditions is **8008**.



167. The Venn diagram shows how the 17 players tried out for the different positions. We started with the 1 player who tried out for all three positions and then filled in the numbers for the players who tried out for exactly two positions. There must have been **6** players who tried out only for guard.

168. We can ignore the area of the base of the cylinder, because the height of the water will drop in proportion with the drop in volume. The height will be $100\% - 8\% = 92\%$ of the original 10 cm, so it will be $0.92 \times 10 =$ **9.2** cm.

169. The Triangle Inequality tells us that in a triangle, the longest side length is always less than the sum of the other two side lengths. In other words, the following three inequalities must be true: $2x + 3x + 7 > 6x - 5$, $2x + 6x - 5 > 3x + 7$, $3x + 7 + 6x - 5 > 2x$. Simplifying each inequality yields the following: $x < 12$, $x > 12/5$, $x > -2/7$. So, $12/5 < x < 12$. Therefore, 3, 4, 5, 6, 7, 8, 9, 10 and 11 are the **9** integer values of x for which a triangle exists with the resulting side lengths.

x	y	$(x+y)^2$
−10	1	81
−5	2	9
−2	5	9
−1	10	81
1	−10	81
2	−5	9
5	−2	9
10	−1	81

170. If we expand $(x + y)^2 = 9$, we get $x^2 + 2xy + y^2 = 9$. Since we are told that $x^2 + y^2 = 29$, we can substitute to get $2xy + 29 = 9$. So $2xy = -20$ and $xy = -10$. The table shows the values of $(x + y)^2$ for ordered pairs of integers whose product is -10. Notice that though there are four ordered pairs for which $(x + y)^2 = 9$, only two have $x > y$, and the smallest value of x is **2**.

Workout 7

171. The curved part of the perimeter of each sector of pizza is $C/8$, and $C = 2 \times \pi \times r$. So, the curved part of the perimeter is $C/8 = (2 \times \pi \times r)/8 = (\pi \times r)/4$. The total perimeter of each sector is $r + r + (\pi \times r)/4 = (8r + \pi r)/4 = r(8 + \pi)/4$. Since this perimeter is 10 inches, the value of r must be $10 \div (8 + \pi)/4 = 10 \times 4/(8 + \pi) \approx 3.59$ inches. Now to find the area of the pizza, we square the radius and multiply by π, which is $3.59^2 \times \pi \approx$ **40.5** in^2.

172. Let's say that Alana usually travels at r mi/h and takes t hours to drive the 18 miles to work. Distance = rate × time, so we have $18 = rt$. With the traffic, she averaged $r - 9$ mi/h and took $t + 1/15$ hour. Since the same distance was covered, we can set two expressions for the distance equal to each other to get $rt = (r - 9)(t + 1/15) \rightarrow rt = rt - 9t + r/15 - 9/15 \rightarrow 9t + 9/15 = r/15 \rightarrow 135t + 9 = r$. Now we can substitute this for r in the equation $18 = rt$ to get $18 = (135t + 9)t \rightarrow 18 = 135t^2 + 9t \rightarrow 0 = 15t^2 + t - 2 \rightarrow 0 = (5t + 2)(3t - 1)$. So $5t + 2 = 0$ and $t = -2/5$, or $3t - 1 = 0$ and $t = 1/3$. The measure of time cannot be a negative number, which means it would normally take Alana 1/3 hour = 20 minutes to get to work. On this particular day with the traffic jam, it took her 20 + 4 = **24** minutes.

173. First we convert 128 miles per hour to feet per second as follows: $\dfrac{128 \text{ }\cancel{\text{miles}}}{1 \text{ }\cancel{\text{hour}}} \times \dfrac{5280 \text{ feet}}{1 \text{ }\cancel{\text{mile}}} \times \dfrac{1 \text{ }\cancel{\text{hour}}}{60 \text{ }\cancel{\text{minutes}}} \times \dfrac{1 \text{ }\cancel{\text{minute}}}{60 \text{ seconds}} =$

$\dfrac{128 \times 5280 \text{ feet}}{60 \times 60 \text{ seconds}} = \dfrac{675{,}840 \text{ feet}}{3600 \text{ seconds}}$. It takes 3.5 seconds to reach this speed, so the acceleration must be $675{,}840/3600 \times 1/3.5 =$

$675{,}840/12{,}600$ feet/second^2. This is faster than the acceleration due to gravity. In fact, it is about $675{,}840/12{,}600 \div 32 \approx$ **1.7** g_n.

174. Half of all positive integers are multiples of 2, but we need to focus on the other half, which are not multiples of 2. Of these numbers, one-third are multiples of 3 and the other two-thirds are not. Putting these two ideas together, we can state that $1/2 \times 2/3 = 1/3$ of positive integers are not divisible by 2 or 3. Similar reasoning suggests that we keep the 4/5 of these numbers that are not divisible by 5. We don't need to check for multiples of 4 since they are multiples of 2, and multiples of 6, since they are multiples of 2 and 3, which we have already considered. Therefore, $1/2 \times 2/3 \times 4/5 = 4/15 \approx$ **27**% of the positive integers are not multiples of 2, 3, 4, 5 or 6.

175. Let's use the method of subtracting the probability of the complement of the desired outcome from 1, since it's easier to calculate the probability that no ball is selected more than once. Thus, the probability that at least one ball is selected more than once is $1 - 75/75 \times 74/75 \times 73/75 \times \cdots \times 57/75 \times 56/75 \approx$ **0.94**.

176. The probability that they both get a hit is $8/20 \times 6/16 =$ **3/20**.

177. If we let the width of the flag be 2 units, then the semicircle has radius 1 unit, and based on properties of 30-60-90 right triangles, the triangle has height $\sqrt{3}$ units. Therefore, the flag has a length of $1 + \sqrt{3}$ units and a total area of $2 \times (1 + \sqrt{3}) = 2 + 2\sqrt{3}$ units^2. The area of the semicircle is $1/2 \times \pi \times r^2 = 1/2 \times \pi \times 1^2 = \pi/2$ units^2. The area of the triangle is $1/2 \times b \times h = 1/2 \times 2 \times \sqrt{3} = \sqrt{3}$ units^2. Therefore, the painted area covers $100\% \times (\pi/2 + \sqrt{3})/(2 + 2\sqrt{3}) \approx$ **60**%.

178. Using units cancellation, we get $\dfrac{100 \text{ }\cancel{\text{bars}}}{11.53 \text{ }\cancel{\text{ticks}}} \times \dfrac{4 \text{ meters}}{5 \text{ }\cancel{\text{bars}}} \times \dfrac{100 \text{ }\cancel{\text{ticks}}}{1 \text{ }\cancel{\text{minute}}} \times \dfrac{1 \text{ }\cancel{\text{minute}}}{60 \text{ seconds}} = \dfrac{100 \times 4 \times 100 \text{ meters}}{11.53 \times 5 \times 60 \text{ seconds}} = \dfrac{40{,}000 \text{ meters}}{3459 \text{ seconds}}$.

Usain Bolt ran 100 meters in 9.58 seconds. So, if $(100/9.58)k = 40{,}000/3459$, then $k = 40{,}000/3459 \times 9.58/100 \approx$ **1.11**.

179. To find the equation of Jason's line, we will use the slope $-1/3 = m$ and the point $(5, -2) = (x, y)$. Substituting these values into the point-slope form $y - y_1 = m(x - x_1)$, we see that the equation for Jason's line is $y - (-2) = (-1/3)(x - 5) \rightarrow y + 2 = (-1/3)x + 5/3 \rightarrow y = (-1/3)x - 1/3$. Amisha's line is perpendicular to Jason's, so its slope is the negative reciprocal of $-1/3$, which is 3. We can find the equation of Amisha's line by using the slope $3 = m$ and the point $(4, 1) = (x, y)$. Again, substituting these values into the point-slope form, we see that the equation of Amisha's line is $y - 1 = 3(x - 4) \rightarrow y - 1 = 3x - 12 \rightarrow y = 3x - 11$. Now we can set these two expressions for y equal to each other to get $(-1/3)x - 1/3 = 3x - 11 \rightarrow -1x - 1 = 9x - 33 \rightarrow 32 = 10x \rightarrow x = 32/10 = 16/5$. Now, let's substitute this value for x in Amisha's equation to get the value of the y-coordinate. We have $y = 3 \times 16/5 - 11 = 48/5 - 11 = (48 - 55)/5 = -7/5$. The sum $x + y$ is $16/5 + (-7/5) =$ **9/5**.

180. At each vertex of the triangle, the sector of the circle is 1/6 of a full circle. The three sectors have a combined area equivalent to $3/6 = 1/2$ of a circle with a radius of 4 units. That area is $1/2 \times \pi \times r^2 = 1/2 \times \pi \times 4^2 = 8\pi$ units^2. The triangle has side length equal to twice the radius of a circle, or 8 units, and its altitude is $4\sqrt{3}$ units. Thus, the triangle has area $1/2 \times b \times h = 1/2 \times 8 \times 4\sqrt{3} = 16\sqrt{3}$ units^2. The shaded region within the triangle but outside the circles has area $16\sqrt{3} - 8\pi \approx$ **2.6** units^2.

Workout 8

181. Triangle XYV is $1/2 \times 1/2 = 1/4$ of the area of square WXYZ. The area of the square must be $4 \times 4/5 =$ **16/5** units^2.

182. We know there is at least one of each denomination, which amounts to $5 + 11 + 19 = 35$ points. That leaves $56 - 35 = 21$ points to account for. There cannot be another 19-point token. The remaining 21 points can only include two 5-point tokens and one 11-point token. To verify, we have $3 \times 5 + 2 \times 11 + 1 \times 19 = 15 + 22 + 19 = 56$ points and a total of $3 + 2 + 1 =$ **6** tokens.

183. Each round of this "game" consists of Xera rolling a die followed by Yeta picking a card, with replacement. In any round, Yeta can win only after Xera first rolls a number other than four. The probability that Yeta wins in any round, then, is $(5/6) \times (1/13) = 5/78$. It is possible for there to be no winner in a round, but we are interested in the probability that when there is a winner, the winner is Yeta. Since the probability that there is no winner in a round is $(5/6) \times (12/13) = 60/78$, it follows that $1 - (60/78) = 18/78$ is the probability that there is a winner in a round. Therefore, the probability that Yeta will win the game is the $(5/78)/(18/78) =$ **5/18**.

184. The volume of a cone is $1/3 \times \pi \times r^2 \times h$. Notice that we multiply by the height h and the square of the radius r. This is why doubling the radius can compensate for reducing the height by one-fourth. We get the same volume because $2^2 \times 1/4 = 4/4 = 1$. If we decrease the height to 1/3 of what it was, the radius has to increase by a number that when squared is 3. That number is $\sqrt{3}$. Since $\sqrt{3} \approx 1.73$, the radius must increase by a factor of $1.73 - 1 = 0.73$, or **73**%.

185. The expression $(n^2 + 9)/(n^2 + 4)$ can be factored to get $((n - 3)(n + 3))/((n - 2)(n + 2))$. Since $n - 3$ and $n - 2$, as well as $n + 3$ and $n + 2$, each differ by 1, $(n - 3)/(n - 2)$ and $(n + 3)/(n + 2)$ are both common fractions. We need to look at $(n - 3)/(n + 2)$ and $(n + 3)/(n - 2)$ to see when either of these is not a common fraction, which will also make the expression $((n - 3)(n + 3))/((n - 2)(n + 2))$ not a common fraction. The pair $n - 3$ and $n + 2$, as well as $n + 3$ and $n - 2$, each differ by 5. When the value of $n - 3$ is a multiple of 5, the value of $n + 2$ will also be a multiple of 5, as is the case with $n + 3$ and $n - 2$. So when any of these factors is a multiple of 5, there will be a multiple of 5 in both the numerator and denominator, and the expression will not be a common fraction. Each multiple of 5 in our range can be assigned to the pair $n - 3$ and $n + 2$, as well as the pair $n + 3$ and $n - 2$. The smallest multiple of 5 we consider is $1 \times 5 = 5$ and the largest is $403 \times 5 = 2015$. So we have 403 integers that can be used twice, and the number of integers n from 1 to 2016 that yield a GCF greater than 1 for the numerator and denominator is $403 \times 2 =$ **806**.

186. In ascending order, the five known numbers are 4, 5, 5, 6 and 7. In an ordered list of six numbers, the median is the mean of the third and fourth values. If $x \le 5$, then the third and fourth values will have a mean of 5. If $x \ge 6$, then the mean of the third and fourth values is $(5 + 6)/2 = 5.5$. Since there are only two possible values for the median, let's see if there are two values of x that will give us the same overall mean and mean of modes as those two values. The sum of the five known numbers is $4 + 5 + 5 + 6 + 7 = 27$. In order for the mean of the six values to be 5, the sum of all six values would have to be $6 \times 5 = 30$, so $x = 3$. The six values 3, 4, 5, 5, 6 and 7 have a mode of 5, so the mean, median and mean of modes are all 5 when $x = 3$. Similarly, in order for the six numbers to have a mean of 5.5, their sum must be $5.5 \times 6 = 33$, so $x = 6$. The six values 4, 5, 5, 6, 6 and 7 have modes 5 and 6, making the mean of the modes $(5 + 6)/2 = 5.5$. So the mean, median and mean of the modes are all 5.5 when $x = 6$. Therefore, 3 and 6 are the **2** possible values of x.

187. There are 13 possible sets of three consecutive numbers in the range 1 to 15. The first starts with 1 and the last starts with 13. There are $15 \times 14 \times 13$ ways to randomly draw three of the slips. The probability is, thus, $13/(15 \times 14 \times 13) = 1/(15 \times 14) =$ **1/210**.

188. Construct a rectangle with length a inches, width b inches and diagonal 8 inches. It has area $a \times b = 26$ in^2 and perimeter $2a + 2b = 2(a + b)$. We have a right triangle with legs a and b inches and hypotenuse 8 inches. Using the Pythagorean Theorem, we can write $a^2 + b^2 = 8^2$. Recall that $(a + b)^2 = a^2 + 2ab + b^2$, which can be rewritten as $a^2 + b^2 + 2ab$. Substituting $8^2 = 64$ for $a^2 + b^2$ and 26 for ab, we get $a^2 + b^2 + 2ab = 64 + 2 \times 26 = 64 + 52 = 116 = (a + b)^2$ and $a + b = \sqrt{116} = 2\sqrt{29}$. The perimeter of the rectangle, then, is $2(a + b) = 2 \times 2\sqrt{29} =$ $\mathbf{4\sqrt{29}}$ inches.

189. There are four similar triangles in the figure, so we will be able to use proportional reasoning to find some of the unknown lengths. Triangles CDF and ECF are similar, so if we let CF = x, we can write and solve the following proportion: $x/8 = 26/x \rightarrow x^2 = 8 \times 26 \rightarrow x^2 = 208 \rightarrow x = \sqrt{208} = 4\sqrt{13}$ units. If we let CE = y, we can use the Pythagorean Theorem with triangle CEF as follows: $8^2 + y^2 = (\sqrt{208})^2 \rightarrow y^2 = (\sqrt{208})^2 - 8^2 \rightarrow y^2 = 144 \rightarrow y = 12$. Now, using similar triangles ECF and BCA, we can find the length of side BC. If we let BC = z, we can write and solve the following proportion: $4\sqrt{13}/12 = 26/z \rightarrow 4\sqrt{13} \times z = 12 \times 26 \rightarrow z = (12 \times 26)/(4\sqrt{13}) \times (4\sqrt{13})/(4\sqrt{13}) = (312 \times 4\sqrt{13})/(16 \times 13) =$ $\mathbf{6\sqrt{13}}$ units.

190. Since Pump P takes 12 hours to fill the tank, it must fill 1/12 of the tank each hour. Similarly, since Pump Q takes 15 hours, it must fill 1/15 of the tank each hour. Together the two pumps fill $1/12 + 1/15 = 5/60 + 4/60 = 9/60 = 3/20$ of the tank each hour. At this rate, it will take 20/3 hours to fill the tank. When Pump P was turned off, 60% of the total time, or $0.6 \times 20/3 = 4$ hours had elapsed. To fill the remaining 40% of the tank, Pump Q needed another $0.4 \times 15 = 6$ hours. It must have taken a total of $4 + 6 =$ **10** hours to completely fill the tank.

Warm-Up 12

191. From left to right, we'll label the four spaces on the clock (two for the hour and two for the minutes) 1, 2, 3 and 4 so they are easily referenced in this solution. For 5 minutes every hour, there is a 0 in the 4th space of the clock's display when the 3rd space does not show 0 (:10, :20, :30, :40 and :50). For the first 10 minutes of every hour, there is a 0 in the 3rd space of the clock's display (:00 to :09). Not including the hour from 10:00 to 10:59, which we will consider separately, that's $10 + 5 = 15$ minutes every hour for 11 hours a day, which is $15 \times 11 = 165$ minutes. From 10:00 to 10:59, there is a 0 in the 2nd space of the clock's display for all 60 minutes (10:). There is never a 0 in the 1st space of the clock's display. Our total is $165 + 60 =$ **225** minutes.

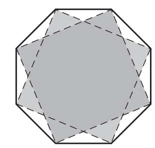
192. Rather than try to subtract with repeating decimals, we should convert each of these numbers to fractions. If we let $x = 1.\overline{18}$ and $y = 2.3\overline{6}$, we can convert these repeating decimals as follows:

$$\begin{array}{rl} 100x = & 118.\overline{18} \\ -\quad x = & 1.\overline{18} \\ \hline 99x = & 117 \\ x = & \frac{117}{99} = \frac{13}{11} \end{array} \qquad \begin{array}{rl} 100y = & 236.\overline{6} \\ -10y = & 23.\overline{6} \\ \hline 90y = & 213 \\ y = & \frac{213}{90} = \frac{71}{30} \end{array}$$

Therefore, $\left|1.\overline{18} - 2.3\overline{6}\right| = \left|\frac{13}{11} - \frac{71}{30}\right| = \left|\frac{13 \times 30 - 71 \times 11}{11 \times 30}\right| = \left|\frac{390 - 781}{330}\right| = \mathbf{\frac{391}{330}}$.

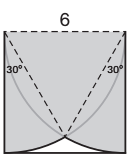
193. It should be noted that this dissection and rearrangement of one square into two other squares is essentially a proof of the Pythagorean Theorem. The area of square DEFP' is 1 × 1 = 1 in^2. Square FPBE' is said to have twice the area, so it has area 2 in^2 and side length $\sqrt{2}$ inches. It follows, then, that the original square has area 1 + 2 = 3 in^2 and side length $\mathbf{\sqrt{3}}$ inches.

194. There would be 8! arrangements of an eight-letter word if all the letters were different. Since the three identical Ss in TRESPASS can be rearranged in 3! = 3 × 2 × 1 ways, we have to divide 8!/3! = 8 × 7 × 6 × 5 × 4 = 6720. Thus, there are 6720 arrangements of the letters in TRESPASS. Since 5 of the 8 letters are not S, there must be 5/8 × 6720 = **4200** arrangements that do not have S as the final letter. (This is actually true for any position in the word.) Alternatively, there are $_7C_3$ = 7!/(4! × 3!) = (7 × 6 × 5)/(3 × 2 × 1) = 210/6 = 35 ways to place the Ss, and there are 5! = 120 ways to permute the remaining letters. Again, there are 35 × 120 = **4200** arrangements.



195. There are $_8C_4$ = (8 × 7 × 6 × 5) ÷ (4 × 3 × 2 × 1) = 70 different ways to choose 4 of the 8 vertices in a regular octagon. Only 2 of these ways will be the vertices of a square, as shown. So the probability is 2/70 = **1/35**.

196. If we multiply the plane's speed on the return trip by the time it took, we find that the distance traveled was 450 × 2.5 = 1125 miles. To travel this same distance took 3 hours on the first trip, so the average speed was 1125 ÷ 3 = 375 mi/h. If we let r be the rate of the plane and w be the rate of the wind, then for the first trip we have $r - w = 375$ and for the return trip we have $r + 2w = 450$. Solving this system of equations yields $(r + 2w) - (r - w) = 450 - 375 \rightarrow 3w = 75 \rightarrow w = 25$, so the speed of the original headwind was **25** mi/h.

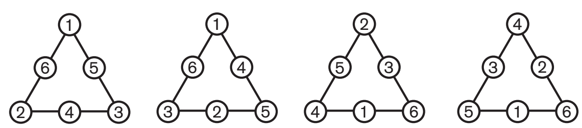


197. The figure shows how we can construct an equilateral triangle from three radii of the quarter-circles. Since the area of an equilateral triangle with side length s is $s^2\sqrt{3}/4$, this triangle of side length 6 cm has area $6^2\sqrt{3}/4 = 36\sqrt{3}/4 = 9\sqrt{3}$ cm^2. The remaining sectors each have a combined degree measure of 30 + 30 = 60, so their combined area is 1/6 the area of a circle of radius 6 cm, or 1/6 × π × r^2 = 1/6 × π × 6^2 = 6π cm^2. The total area of the shaded region, then, is $6\pi + 9\sqrt{3}$ cm^2, so the value of $a + b + c$ is 6 + 9 + 3 = **18**.

198. We need to write 60 as a product of three factors representing the prism's length l, width w and height h such that $l \geq w \geq h$. We have the following organized list of dimensions: 60 × 1 × 1, 30 × 2 × 1, 20 × 3 × 1, 15 × 4 × 1, 15 × 2 × 2, 12 × 5 × 1, 10 × 6 × 1, 10 × 3 × 2, 6 × 5 × 2 and 5 × 4 × 3. These are the dimensions for the **10** prisms that are possible.

199. There are $_6C_3$ = 20 ways to select the three numbers for the circles along a side of the triangle. For each of the six circles to be filled with a different number from 1 to 6, the largest sum that can include the number 1 is 1 + 5 + 6 = 12, and the smallest sum that can include the number 6 is 1 + 2 + 6 = 9. If the sums of the numbers along all three sides are to be equal, these sums must range from 9 to 12. Each of the sums 9, 10, 11 and 12 can be obtained in three ways as follows:

9 = 1 + 2 + 6 = 1 + 3 + 5 = 2 + 3 + 4
10 = 1 + 3 + 6 = 1 + 4 + 5 = 2 + 3 + 5
11 = 1 + 4 + 6 = 2 + 3 + 6 = 2 + 4 + 5
12 = 1 + 5 + 6 = 2 + 4 + 6 = 3 + 4 + 5



The figure shows filled-in figures for these **4** solutions.

200. Recall that the infinite series 1/2 + 1/4 + 1/8 + 1/16 + … converges to 1. Thus, the time it takes to run 1 length of the field is $t = 1 + 2/3 + (2/3)^2 + (2/3)^3 + \cdots$. If we multiply both sides of the equation by 3/2, we get $(3/2)t = 3/2 + 1 + 2/3 + (2/3)^2 + (2/3)^3 + \cdots$. Our infinite series appears again. Substituting, we get $(3/2)t = 3/2 + t \rightarrow (1/2)t = 3/2 \rightarrow t =$ **3** minutes.

Warm-Up 13

201. We can factor the difference of cubes to be $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$. We know the values of $a - b$ and $a^2 + b^2$. To find ab, let's square both sides of the first equation to get $a - b = 3 \rightarrow a^2 - 2ab + b^2 = 9$. Substituting 65 for $a^2 + b^2$, we get $65 - 2ab = 9 \rightarrow 2ab = 56 \rightarrow ab = 28$. We now have the values for $a^2 + b^2$ and ab. Substituting in the factorization of $a^3 - b^3$, we see that $a^3 - b^3$ is 3 × (65 + 28) = 3 × 93 = **279**.

111	222	333	444
555	666	777	888
999			
102	204	306	408
123	135	147	159
234	246	258	
345	357	369	
456	468		
567	579		
678			
789			

202. To solve this problem, we will consider three cases.

Case I: The first case includes numbers with all three digits the same, of which there are 9.

Case II: The next case includes numbers with 0 as one of the three digits. The digits in each of these 4 numbers can be arranged to form 4 different numbers that meet our criteria, for a total of 4 × 4 = 16.

Case III: The final case includes numbers with three distinct digits from 1 to 9. The digits in each of these 16 numbers can be arranged to form 6 different numbers that meet our criteria, for a total of 16 × 6 = 96.

Altogether, that's 9 + 16 + 96 = **121** integers.

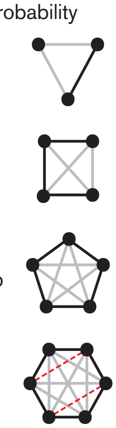
203. There must be more green marbles than red marbles since the probability is greater that the two marbles will be green. Since there are at least two marbles of each color, the smallest case we can try is 3 green marbles and 2 red marbles. The probability that the marbles are both green is 3/5 × 2/4 = 3/10 and the probability that they are both red is 2/5 × 1/4 = 1/10. This makes 2 reds one-third as likely as 2 greens. Increasing the number of green marbles with just 2 red marbles makes the probability of 2 reds even less likely than 2 greens. So, let's try 4 green and 3 red marbles. The probabilities are p(2 greens) = 4/7 × 3/6 = 2/7 and p(2 reds) = 3/7 × 2/6 = 1/7. This has 2 reds half as likely as 2 greens. The minimum number of marbles is **7**. Some students may work this out as a combinatorics question: If you have 4 greens and 3 reds, then you have "4 choose 2" = 4 × 3 ÷ 2 = 6 ways to choose 2 greens and "3 choose 2" = 3 × 2 ÷ 2 = 3 ways to choose 2 reds, which is half as many ways.

204. Let's find the three points of intersection. If we add our first equation to the third equation, we get $x + 2y + x - 2y = 8 + 0 \rightarrow 2x = 8 \rightarrow x = 4$. Substituting this for x in the first equation, we get $4 + 2y = 8 \rightarrow 2y = 4 \rightarrow y = 2$. So, one point of intersection is (4, 2). Subtracting the first equation from the second equation, we get $5x + 2y - (x + 2y) = 48 - 8 \rightarrow 4x = 40 \rightarrow x = 10$. Substituting this for x in the first equation gives us $10 + 2y = 8 \rightarrow 2y = -2 \rightarrow y = -1$ and a second intersection point of (10, −1). For the last point of intersection, adding the second equation and the third equation yields $5x + 2y + x - 2y = 48 + 0 \rightarrow 6x = 48 \rightarrow x = 8$. Substituting into the third equation, we see that $8 - 2y = 0 \rightarrow -2y = -8 \rightarrow y = 4$ and the third point of intersection is (8, 4). We need to determine which is the longest side. The distance between two points is determined by the formula $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$. But we can tell by looking at a plot of the points of intersection, without actually calculating the distances between them, that the greatest distance will be between the points (4, 2) and (10, − 1). The midpoint of the segment joining (4, 2) and (10, −1) is the point whose coordinates are the averages of the coordinates of the endpoints ((4 + 10)/2, (2 + (−1))/2) = (7, 1/2). Therefore, the sum of the coordinates of the midpoint of the longest side is 7 + 1/2 = **15/2.**

205. If there were eight girls and eight boys, there would be the same number of ways to choose three of each. If there are nine girls and seven boys there are $_9C_3$ = (9 × 8 × 7) ÷ (3 × 2 × 1) = 84 ways to choose three girls and $_7C_3$ = (7 × 6 × 5) ÷ (3 × 2 × 1) = 35 ways to choose three boys, but 84 ÷ 35 ≠ 6. With ten girls and six boys, then there are $_{10}C_3$ = (10 × 9 × 8) ÷ (3 × 2 × 1) = 120 ways to choose three girls and $_6C_3$ = (6 × 5 × 4) ÷ (3 × 2 × 1) = 20 ways to choose three boys. This works since 120 ÷ 20 = 6. The ratio of girls to boys, then, is 10/6 = **5/3**.

206. We will start with the prime factorization of 432, which is 3 × 3 × 3 × 2 × 2 × 2 × 2. We will use these factors to make four single-digit factors of 432, starting with the greatest possible four-digit number, which gives us 9 × 8 × 6 × 1 = 432. Unfortunately, 9 + 8 + 6 + 1 = 24, not 20. The next largest four-digit possibility gives us 9 × 8 × 3 × 2 = 432, but 9 + 8 + 3 + 2 = 22, not 20. Let's now try 9 × 6 × 4 × 2 = 432, which gives us 9 + 6 + 4 + 2 = 21, not 20. On the next try, we get 9 × 4 × 4 × 3 = 432 and 9 + 4 + 4 + 3 = 20. This means 9443 is the greatest number we can make. At the other extreme, we can make the number 2666, which meets the requirements that 2 × 6 × 6 × 6 = 432 and 2 + 6 + 6 + 6 = 20. The difference between these two numbers is 9443 − 2666 = **6777**.

207. The probability that Jebediah picks the fair coin is 1/2. The probability of flipping heads on a fair coin is 1/2. Therefore, the probability that he picks the fair coin and then flips three heads in a row is 1/2 × 1/2 × 1/2 × 1/2 = 1/16. The probability that Jebediah picks the two-headed coin is also 1/2. But the probability of flipping heads on the two-headed coin is 1. So the probability that he picks the two-headed coin and then flips three heads in a row is 1/2 × 1 × 1 × 1 = 1/2. The probability of flipping three heads in a row, then, is 1/16 + 1/2 = (1 + 8)/16 = 9/16. The probability this outcome occurs as a result of picking the fair coin is (1/16)/(9/16) = 1/16 × 16/9 = **1/9**.



208. One way to visualize this is using dots to represent people and colored lines between them to represent their relationship. A gray line is used to indicate that 2 people do not know each other, while a black line indicates that they do. We want to be able to color all of the lines connecting the dots without creating any monochromatic triangles (a connected group of three dots in which all three lines are the same color). Let's see if there is a scenario in which no 3 people all know each other and no 3 people are all strangers for 3, 4, 5, 6, … people in a room. The first three figures show an example of one such scenario involving 3, 4 and 5 people in a room. At 6 people it becomes difficult to find one such example. As the last figure shows, we can draw seven of the nine connections among 6 people without producing any monochromatic triangles. But either way we color the final two connections results in two monochromatic triangles. Let's verify that with 6 people there must be at least 3 people who all know each other or who all do not know each other. For a 6-person group every individual has a connection to 5 others. Of the 5 lines leaving each individual, at least 3 must be the same color. If we look at these 3 lines and 4 dots and draw in the other connections, we see that it is impossible to choose a color without creating a 3-person group that either are all connected by black lines or all connected by gray lines. Thus, the minimum number of people must be **6**. (Note: This is a special case of Ramsey's Theorem.)

209. We are given the formula $F = (9/5)C + 32$, but we need to convert Fahrenheit to Celsius, so $C = (5/9)(F - 32)$. Converting 80 °F to °C, we get (5/9)(80 − 32) = 240/9 °C. Converting 40 °F to °C, we get (5/9)(40 − 32) = 40/9 °C. To each of these numbers we need to add 273.15 to establish how far the temperature is from absolute zero. The percent decrease is (240/9 − 40/9)/(240/9 + 273.15) × 100% ≈ **7**%.

210. Three of the four faces of the tetrahedron are right triangles. Angles ADC, ADB and DBC are all 90-degree angles. The distance from B to the face ACD can be thought of as an altitude of triangle BDC drawn from B to side DC. The side lengths of triangle BDC are 12, 16, and 20 units, a multiple of the 3-4-5 Pythagorean Triple. The area of right-triangle BDC is 1/2 × 12 × 16 = 96 units^2. The length of side DC is 20 units, so for altitude h, we have $96 = 1/2 \times 20 \times h \rightarrow 96 = 10h \rightarrow h = 96/10 =$ **48/5** units.

Warm-Up 14

211. The number of chords that can be drawn in this circle with 8 points is $_8C_2$ = 8!/(6! × 2!) = (8 × 7)/(2 × 1) = **28** chords.

212. There are $_{10}C_3$ = 10!/(7! × 3!) = (10 × 9 × 8)/(3 × 2 × 1) = 120 ways to choose three cards from the deck of ten numbered cards. There are 8 sets of three consecutive numbers: 1-2-3, 2-3-4, 3-4-5, …, 7-8-9, 8-9-10. There are 9 sets of two consecutive numbers:1-2, 2-3, 3-4,…, 8-9, 9-10. Each of these pairs of consecutive numbers has 8 choices for the third number in the set, for a total of 9 × 8 = 72, but the sets of three consecutive numbers have been counted twice here. So, in all, there are 72 – 8 = 64 sets of three cards with two or more consecutive integers. Therefore, the probability of not drawing one of these sets is 1 – 64/120 = 56/120 = **7/15**.

213. It would take the original 12 people another 14 days to finish shearing the other 2/3 of the field of pine trees. The goal is to accomplish this in 6 days instead of 14 days, so 14/6 times as many people are needed. That would be 14/6 × 12 = 28 people, which means that 28 – 12 = **16** people must be added to the crew.

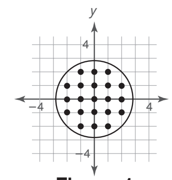


Figure 1

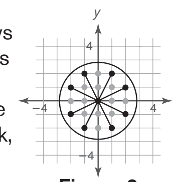
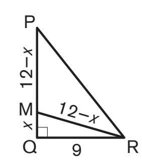


Figure 2

214. The general equation of a circle with center (0, 0) and radius r is $x^2 + y^2 = r^2$. So the circle's radius is $\sqrt{8} = 2\sqrt{2}$ units. This is the length of the diagonal of a square with side length 2 units, so our circle passes through the point (2, 2). Figure 1 shows the 21 lattice points that are in the interior of this circle. There are $_{21}C_3$ = 21!/(18! × 3!) = (21 × 20 × 19)/(3 × 2 × 1) = 7980/6 = 1330 ways to choose 3 points. That's the total number of outcomes possible. To determine the number of favorable outcomes, we will use complementary counting and find the sets of three points that *cannot* make a triangle. A triangle cannot be made if the three chosen points are collinear. Considering first the horizontal rows of points, there is $_3C_3$ = 1 way to choose all three points in both the top and bottom rows, and there are $_5C_3$ = 10 ways to choose three points from the three middle rows. That's 1 + 1 + 10 + 10 + 10 = 32 ways so far. By symmetry, there are also 32 ways to choose three collinear points from the vertical columns. Now let's consider the diagonals from the top left to the bottom right at a 45-degree angle. There are 1 + 4 + 1 + 4 + 1 = 11 ways to choose three collinear points on these diagonals. Using symmetry again, we can say there are another 11 ways on the other 45-degree diagonals. Besides these, there are another 4 sets of three collinear points that are shown in Figure 2. The total is 32 + 32 + 11 + 11 + 4 = 90 ways that do not work, so the other 1330 – 90 = 1240 sets of three points must make triangles. The probability, then, is 1240/1330 = **124/133**.

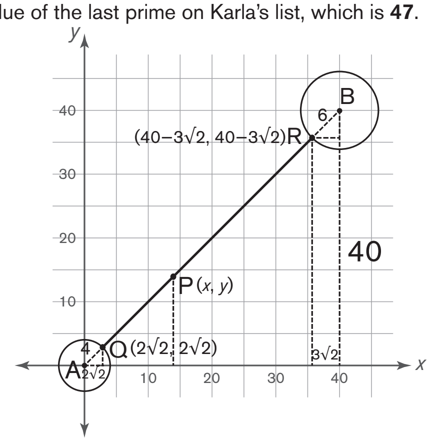
215. There is no four-digit number with all the digits the same for which the sum of the digits equals the product of the digits. So we are looking for a four-digit number with at least two distinct digits. Consider the four-digit number ABCD, where A > B ≥ C ≥ D and let's assume that A × B × C × D = A + B + C + D. Since B, C and D are all less than A, we can write the following inequality: A × B × C × D < A + A + A + A → A × B × C × D < 4 × A. Dividing both sides by A, we get B × C × D < 4. So, the product B × C × D can have a value of 1, 2 or 3. Given that B ≥ C ≥ D, the only possibilities for BCD, then, are 311, 211 and 111. Let's test these three options. If BCD = 311, we have A × 3 × 1 × 1 = A + 3 + 1 + 1 → 3 × A = A + 5 → 2 × A = 5 → A = 5/2, which cannot be true. If BCD = 211, we have A × 2 × 1 × 1 = A + 2 + 1 + 1 → 2 × A = A + 4 → A = 4, so the four-digit combination 4211 works. Finally, if BCD = 111, we have A × 1 × 1 × 1 = A + 1 + 1 + 1 → A = A + 3, which cannot be true. Therefore, the only combination of four digits that exists for which the sum of the digits equals the product of the digits is 4, 2, 1, 1. Since two of the four digits are the same, there are 4!/2! = (4 × 3 × 2 × 1)/ (2 × 1) = 24/2 = 12 possible arrangements of these four digits. Therefore, the number of four-digit numbers with this property is **12** numbers.

216. The three blue faces can intersect at one vertex, or they can be in a row of three adjacent faces of the cube. If the three blues intersect at a vertex, then the red, yellow and green sides can be placed in <u>2</u> different orientations that are mirror images of each other. If the three blues are in a row, then there are <u>3</u> ways the red, green and yellow faces can be arranged. That's a total of 2 + 3 = **5** cubes.



217. Let MQ = x . Since PQ = 12, it follows that MP = MR = 12 − x. Given that QR = 9, we can use the Pythagorean Theorem to find the side lengths of right triangle MQR. We have $x^2 + 9^2 = (12 - x)^2 \rightarrow x^2 + 81 = 144 - 24x + x^2 \rightarrow 81 = 144 - 24x \rightarrow 24x = 63 \rightarrow$ x = 63/24 = 21/8. Therefore, MQ = **21/8** units.

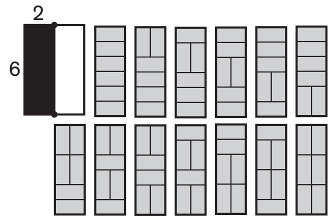
218. Karla could not have the number 2 on her list of six different primes since 2 would make the product of the first three primes even and the sum of the last three primes, which are all odd, must be odd. We are looking for a least possible value for the greatest number on Karla's list, so it makes sense to look at the least possible odd product of primes, which is 3 × 5 × 7 = 105. Now, let's look for three different primes with a sum of 105. Their average would be 105 ÷ 3 = 35. We should try to use two primes slightly above the average and one prime well below the average. After a bit of guessing and checking, we find that 17 + 41 + 47 = 105. When we consider the products 3 × 5 × 11 = 165 and 3 × 5 × 13 = 195, after more guessing and checking, we see that there aren't three primes between 11 and 47 whose sum is 165 or three primes between 13 and 47 whose sum is 195. So, we conclude that starting with the product 3 × 5 × 7 will yield the least possible value of the last prime on Karla's list, which is **47**.



219. The graph shows the initial positions of the two circles, with the segment from Q on circle A to R on circle B. Based on properties of 45-45-90 right triangles, Q has coordinates $(2\sqrt{2}, 2\sqrt{2})$ and R has coordinates $(40 - 3\sqrt{2}, 40 - 3\sqrt{2})$. Since the radius of circle B grows at twice the rate that the radius of circle A does, it follows that the distance along segment QR from Q to the point of tangency P is one-third the distance from Q to R. Similarly, for P(x, y), x must be

$$\frac{1}{3} \times \left((40 - 3\sqrt{2}) - 2\sqrt{2}\right) + 2\sqrt{2} = \frac{40 - 5\sqrt{2}}{3} + 2\sqrt{2} = \frac{40 - 5\sqrt{2} + 6\sqrt{2}}{3} = \mathbf{\frac{40 + \sqrt{2}}{3}}.$$

220. Since none of the 1 × 2 rectangles can cross the center line, let's consider the number of arrangements of tiles on each side of the line. As the figure shows, there are 13 different ways to tile one side of the 6 × 4 rectangle. So, with 13 ways to tile the left side and 13 ways to tile the right side, there are 13 × 13 = **169** ways to tile the whole 6 × 4 rectangle.



Fractions Stretch

221. Dividing, we see that 25/100 = **1/4**.

222. Dividing, we see that 9/16 ÷ 3/8 = 9/16 × 8/3 = **3/2**.

223. Start by simplifying the expression under the radical symbol. We have $\sqrt{(3/11 \div 11/12)} = \sqrt{(3/11 \times 12/11)} = \sqrt{(36/11^2)}$ = **6/11**.

224. Of the 20 unit squares, 6 are shaded. That represents 6/20 = **3/10** of the squares.

225. The area of the triangle is 1/2 × (3/2)w × w = (3/4)w^2. The area of the rectangle is 2w^2. Thus, the triangle's area is (3/4)/2 = 3/4 × 1/2 = **3/8** of the rectangle's area.

226. The difference between 3/4 and 1/2 is 3/4 − 2/4 = 1/4, and 3/4 of that is 3/4 × 1/4 = 3/16. The common fraction that is 3/16 more than 1/2 is 8/16 + 3/16 = **11/16**.

227. The reciprocal of 1/(2 + 1/3) = 2 + 1/3 = 6/3 + 1/3 = **7/3**.

228. Let $x = 0.7\overline{5}$. Then $10x = 7.\overline{5}$ and $100x = 75.\overline{5}$. Subtracting, we see that $100x - 10x = 75.\overline{5} - 7.\overline{5} \rightarrow 90x = 68 \rightarrow x = 68/90 =$ **34/45**.

229. Let's start by simplifying the denominator. We get $\dfrac{1}{\frac{1}{n}+\frac{1}{3}} + \dfrac{1}{\frac{1}{3}+\frac{1}{n}} = \dfrac{1}{\frac{n+3}{3n}} + \dfrac{1}{\frac{n+3}{3n}} = \dfrac{2}{\frac{n+3}{3n}} = 2 \times \dfrac{3n}{n+3} = \dfrac{6n}{n+3}$. Since 1 divided by this fraction is simply the reciprocal $(n + 3)/(6n)$, we now have $(n + 3)/(6n) = 5/12$. Cross-multiplying gives us $12n + 36 = 30n \rightarrow 18n = 36 \rightarrow n =$ **2**.

230. Simplifying the left-hand side of the equation, we get $(2x - 2(x - 3))/(x - 3) = 4/(x + 2) \rightarrow 6/(x - 3) = 4/(x + 2)$. Cross-multiplying, we see that $6(x + 2) = 4(x - 3) \rightarrow 6x + 12 = 4x - 12 \rightarrow 2x = -24 \rightarrow x =$ **−12**.

Angles and Arcs Stretch

231. Vertex H is on the circle, so the measure of inscribed angle AHC is half the measure of intercepted $\widehat{AC}$. Since the regular nonagon divides the 360° of the circle into nine 40° arcs, it follows that $m\widehat{AC}$ = 80°, making $m\angle$AHC = 1/2 × $m\widehat{AC}$ = 1/2 × 80 = **40°**.

232. Vertex B is on the circle, so the measure of inscribed ∠CBD is half the measure of intercepted minor $\widehat{BC}$. Since the measure of major $\widehat{BC}$ is 230°, it follows that minor $\widehat{BC}$ measures 360 − 230 = 130°, making $m\angle$CBD = 1/2 × $m\widehat{BC}$ = 1/2 × 130 = **65°**.

233. The measure of inscribed ∠ABE is 35°, so the measure of intercepted $\widehat{AE}$ is 2 × $m\angle$ABE = 2 × 35 = 70°. Angle AXE has measure 15° and intercepts arcs AE and CD. So, $m\angle$AXE = 1/2 × ($m\widehat{AE}$ − $m\widehat{CD}$) → 15 = 1/2 × (70 − $m\widehat{CD}$) → 30 = 70 − $m\widehat{CD}$ → $m\widehat{CD}$ = 70 − 30 = **40°**.

234. The measure of angle AXB, which intercepts major and minor $\widehat{AB}$, is 50°. If $m\widehat{AB}$ represents the measure of minor $\widehat{AB}$, then $m\angle$AXB = 1/2 × ($m\widehat{AB}$ − (360 − $m\widehat{AB}$)) → 50 = 1/2 × (2 × $m\widehat{AB}$ − 360) → 100 = 2 × $m\widehat{AB}$ − 360 → 2 × $m\widehat{AB}$ = 460 → $m\widehat{AB}$ = 460 ÷ 2 = 230. Therefore, major $\widehat{AB}$ has measure **230°**.

235. Inscribed ∠ABD intercepts $\widehat{AD}$, which has measure 125°. Therefore, $m\angle$ABD = 1/2 × $m\widehat{AD}$ = 1/2 × 125 = **62.5°**.

236. Since $\overline{AC}$ is a diameter of circle O, we know that $m\widehat{AB}$ + $m\widehat{BC}$ = 180°. We are told that inscribed ∠BDC, which intercepts $\widehat{BC}$, has measure 40°. It follows, then, that $m\widehat{BC}$ = 2 × 40 = 80°, and $m\widehat{AB}$ = 180 − 80 = **100°**.

237. Inscribed ∠BAE intercepts $\widehat{ADB}$, so $m\angle$BAE = 1/2 × $m\widehat{ADB}$. Arc ADB is composed of arcs ADC (a semicircle) and BC. From the previous problem, we know that $m\widehat{BC}$ = 80°, so $m\widehat{ADB}$ = 180 + 80 = 260°. It follows that $m\angle$BAE = 1/2 × 260 = **130°**.

238. Angles CFD and AFB intercept arcs CD and AB, respectively. These two angles are congruent and have measure equal to $1/2 \times (m\overset{\frown}{CD} + m\overset{\frown}{AB})$. Semicircle ADC is composed of arcs AD and CD of measure 125° and 180 − 125 = 55°, respectively. From Problem 236, we know that $m\overset{\frown}{AB} = 100°$. Therefore, $m\angle CFD = m\angle AFB = 1/2 \times (55 + 100) = 1/2 \times 155 =$ **77.5°**.

239. Angles CXD and AXB intercept arcs CD and AB, respectively. The measure of these two congruent angles is $1/2 \times (m\overset{\frown}{CD} + m\overset{\frown}{AB})$. We are told that $m\overset{\frown}{AB} = 110°$, and because AB = BD, it follows that $m\overset{\frown}{AB} = m\overset{\frown}{BCD} = 110°$. In addition, since $\overset{\frown}{BCD}$ is composed of arcs BC and CD, we have $m\overset{\frown}{BCD} = m\overset{\frown}{BC} + m\overset{\frown}{CD} \rightarrow 110 = 60 + m\overset{\frown}{CD} \rightarrow m\overset{\frown}{CD} = 110 - 60 = 50°$. Now that we have the measures of arcs AB and CD, we have $m\angle CXD = 1/2 \times (50 + 110) = 1/2 \times 160 =$ **80°**.

240. Angle DAC intercepts arcs CD and FG of circles O and P, respectively. So $m\overset{\frown}{FG} = m\overset{\frown}{CD} = 50°$. Semicircle AFG is composed of arcs AF and FG, so we have $m\overset{\frown}{AF} + 50 = 180 \rightarrow m\overset{\frown}{AF} = 180 - 50 = 130°$. Since ∠AOB, which intercepts arcs AF and FG, is in the exterior of circle P, it follows that $m\angle AOB = 1/2 \times (m\overset{\frown}{AF} - m\overset{\frown}{FG}) = 1/2 \times (130 - 50) = 1/2 \times 80 = 40°$. In addition, ∠AOB is a central angle of circle O that intercepts $\overset{\frown}{AB}$, so $m\overset{\frown}{AB} = 40°$ as well. Diameter AD of circle O creates a semicircle composed of arcs AB, BC and CD. Therefore, $m\overset{\frown}{AB} + m\overset{\frown}{BC} + m\overset{\frown}{CD} = 180 \rightarrow 40 + m\overset{\frown}{BC} + 50 = 180 \rightarrow 90 + m\overset{\frown}{BC} = 180 \rightarrow m\overset{\frown}{BC} = 180 - 90 =$ **90°**.

Bases Stretch

241. For base 9, the place values for a two-digit numeral are $9^1 = 9$ and $9^0 = 1$. Therefore, $24_9 = 2(9^1) + 4(9^0) = 2(9) + 4(1) = 18 + 4 =$ **22**.

242. For base 8, the place values for a two-digit numeral are $8^1 = 8$ and $8^0 = 1$. Therefore, $24_8 = 2(8^1) + 4(8^0) = 2(8) + 4(1) = 16 + 4 =$ **20**.

243. For base 7, the place values for a two-digit numeral are $7^1 = 7$ and $7^0 = 1$. Therefore, $24_7 = 2(7^1) + 4(7^0) = 2(7) + 4(1) = 14 + 4 =$ **18**.

244. The greatest power of nine that goes into 24 is 9, and $24 \div 9 = \underline{2}$ r $\underline{6}$. Therefore, $24 = 2(9^1) + 6(9^0) =$ **26** base 9.

245. The greatest power of eight that goes into 24 is 8, and $24 \div 8 = \underline{3}$ r $\underline{0}$. Therefore, $24 = 3(8^1) + 0(8^0) =$ **30** base 8.

246. The greatest power of seven that goes into 24 is 7, and $24 \div 7 = \underline{3}$ r $\underline{3}$. Therefore, $24 = 3(7^1) + 3(9^0) =$ **33** base 7.

247. For base 12, the place values for a four-digit numeral are $12^3 = 1728$, $12^2 = 144$, $12^1 = 12$ and $12^0 = 1$. The greatest power of 12 that goes into 4991 is 1728, and $4991 \div 1728 = \underline{2}$ r 1535. The greatest power of 12 that goes into 1535 is 144, and $1535 \div 144 = \underline{10}$ r 95. The greatest power of 12 that goes into 95 is 12, and $95 \div 12 = \underline{7}$ r $\underline{11}$. Recall, in base 12, that 10 = A and 11 = B. Therefore, 4991 = **2A7B** base 12.

248. In base 12, the first three place values are $12^2 = 144$, $12^1 = 12$ and $12^0 = 1$, and B = 11. So, $3BB_{12} = 3(144) + 11(12) + 11(1) = 432 + 132 + 11 = 575$ base 10. We now need to convert this to base 6. The greatest power of 6 that goes into 575 is $6^3 = 216$, and $575 \div 216 = \underline{2}$ r 143. The greatest power of 6 that goes into 143 is $6^2 = 36$, and $143 \div 36 = \underline{3}$ r 35. The greatest power of 6 that goes into 35 is 6, and $35 \div 6 = \underline{5}$ r $\underline{5}$. Therefore, $3BB_{12} =$ **2355** base 6.

249. If $523_b = 262$, then $5 < b < 10$ (since 262 is less than 523 and since 5 is a digit in base b). So the possible values for b are 6, 7, 8 and 9. If we try $b = 6$, we get $523_6 = 5(36) + 2(6) + 3(1) = 180 + 12 + 3 = 195$. If $b = 7$, we get $523_7 = 5(49) + 2(7) + 3(1) = 245 + 14 + 3 = 262$. Thus, $b =$ **7**. Alternatively, since $523_b = 5(b^2) + 2(b) + 3(1) = 5b^2 + 2b + 3$, we can solve the following quadratic equation: $5b^2 + 2b + 3 = 262 \rightarrow 5b^2 + 2b - 259 = 0 \rightarrow (5b + 37)(b - 7) = 0 \rightarrow 5b + 37 = 0$ or $b - 7 = 0 \rightarrow b = -37/5$ or $b = 7$. Since b must be positive and an integer, we conclude that the answer is $b =$ **7**.

250. Since 5 is a digit in base b, we know that $b > 5$, and we are told that $b < 10$. The possible values of b, then, are 6, 7, 8 and 9. We could try each of these to determine for which value of b both of the given equations hold true. Let's, instead, solve this problem algebraically. We know that $441_b = 4(b^2) + 4(b^1) + 1(b^0) = 4b^2 + 4b + 1$ and $351_b = 3(b^2) + 5(b^1) + 1(b^0) = 3b^2 + 5b + 1$, so we can write the following equations: $4b^2 + 4b + 1 = n^2$ and $3b^2 + 5b + 1 = (n - 2)^2$. Factoring the left-hand side of the first equation yields $(2b + 1)^2 = n^2$, so $n = 2b + 1$. Substituting this for n in the second equation, we get $3b^2 + 5b + 1 = (2b + 1 - 2)^2 \rightarrow 3b^2 + 5b + 1 = (2b - 1)^2 \rightarrow 3b^2 + 5b + 1 = 4b^2 - 4b + 1 \rightarrow b^2 - 9b = 0 \rightarrow b(b - 9) = 0 \rightarrow b = 0$ or $b - 9 = 0 \rightarrow b = 9$. Therefore, $b = 9$, and we have $441_9 = 4(81) + 4(9) + 1(1) = 324 + 36 + 1 = 361 = 19^2$, so $n = 19$. If we substitute this value for n in $(n - 2)^2$, we confirm that $(19 - 2)^2 = 17^2 = 289 = 3(81) + 5(9) + 1(1) = 351_9$. So $n =$ **19**.

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