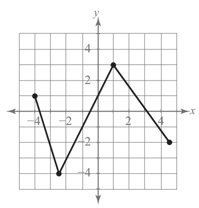
2016 Chapter Competition

<u>Sprint Round</u>

1. *Given:* Count backwards from 155 by 4.
   Find: The 9th number.
   The first number is 155.
   The second number is 151, or 4 less.
   The ninth number is $4 \times 8 = 32$ less.
   155 – 32 = 123 **<u>Ans.</u>**

2. *Given:* The graph, as displayed below.
   Find: The difference between the maximum and minimum values.



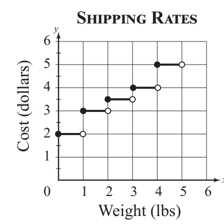
   The highest point in terms of y is the third point from the right with coordinates (1, 3). The lowest point in terms of y is the second point from the right with coordinates (−2.5, −4) – at least it looks like around −2.5. The difference in the y-coordinates is $3 - (-4) = 7$ **<u>Ans.</u>**

3. *Given:* $\frac{1}{b} = \frac{b}{a}, b = -1$
   Find: a
   Substituting −1 for b, we get 1/−1 = −1/a.
   Cross-multiplying, we find that
   a = (−1)(−1) = 1 **<u>Ans.</u>**

4. *Find:* The sum of all two-digit multiples of 3 that have a units digit of 1.
   21 is the first two-digit multiple of 3 which has a units digit of 1. Each successive two-digit multiple of 3 with a units digit of 1 is 30 larger. There are three in all: 21, 51 and 81. Their sum is
   21 + 51 + 81 = 153 **<u>Ans.</u>**

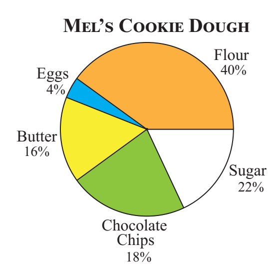
5. *Find:* the sum of the first 8 terms of the sequence beginning -4, 5 … where each term is the sum of the previous two terms.
   The first eight terms of the sequence are −4, 5, 1, 6, 7, 13, 20 and 33. Their sum is
   −4 + 5 + 1 + 6 + 7 + 13 + 20 + 33 = 81 **<u>Ans.</u>**

6. *Given:* The graph displayed below.
   Find: the total cost to ship 3 packages weighing 1.8 lbs., 2 lbs. and 4.4 lbs.



   It costs \$3 to ship the 1.8-lb. package, \$3.50 to ship the 2-lb. package and \$5 to ship the 4.4-lb. package. The total cost to mail all three packages is $3 + 3.50 + 5 =$ 11.50 **<u>Ans.</u>**

7. *Given:* the graph below.
   Find: the degree measure of the portion of the cookie that is flour.



   The cookie is made of 40% flour and.
   40% of 360° is $360 \times 0.4 = 144$ **<u>Ans.</u>**

8. *Given:* 3 zoguts and 4 gimuns costs \$18.
   2 zoguts and 3 gimuns costs \$13.
   Find: the cost of 1 zogut and 1 gimun
   Let z = the cost of a zogut and g = the cost of a gimun. We can write the equations

$3z + 4g = 18$ (Eq. 1)
$2z + 3g = 13$ (Eq. 2)
Subtracting Eq. 2 from Eq. 1, we get
$1z + 1g = 18 - 13 = 5$ **Ans.**

9. *Given:* A rectangular piece of paper with a 40-inch length. It is folded in half. The ratio of the long side of the original sheet to the short side of the original sheet is the same as the ratio of the long side of the folded sheet to the short side of the folded sheet.
Find: the length of the short side of the original sheet.
Let s = the length of the short side of the original sheet, which is also the long side of the folded sheet. The length of the short side of the folded sheet is 40/2 = 20.
We have the ratio $\frac{40}{s} = \frac{s}{20}$. Cross-multiplying yields $s^2 = 800$. So the length of the short side of the original sheet is $s = \sqrt{800} = 10\sqrt{8} = 20\sqrt{2}$ **Ans.**

10. *Find:* the greatest multiple of 3 that can be formed using one or more of the digits 2, 4, 5 and 8, using each digit only once.
A number is a multiple of 3 if the sum of its digits is divisible by 3. Since 8 + 5 + 4 + 2 = 19, which is not divisible 3, we know that no number formed using all four digits will be a multiple of 3. The four combinations of three digits that can be used to form 3-digit numbers are 8, 5, 4; 8, 5, 2; 8, 4, 2; and 5, 4, 2. Since 8 + 5 + 4 = 17, 8 + 4 + 2 = 14 and 5 + 4 + 2 = 11, none of which is divisible by 3, we know that no 3-digit number formed with these combinations of digits will be a multiple of 3. But 8 + 5 + 2 = 15, which is divisible by 3. The greatest multiple of 3 formed using the given numbers is 852 **Ans.**

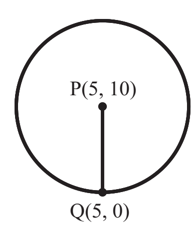
11. *Given:* $\frac{a^6 b}{a^3 b^5} = \frac{a^m}{b^n}$
Find: $m + n$
Simplifying the expression, we get
$\frac{a^6 b}{a^3 b^5} = \frac{a^3}{b^4}$
So $m = 3$, and $n = 4$. Therefore,
$m + n = 3 + 4 = 7$ **Ans.**

12. *Given:* 8% of x% of 200 = 4
Find: x
Rewriting the percents as fractions, the problem statement can be expressed algebraically as
$\frac{8}{100}\left(\frac{x}{100} \times 200\right) = 4$
Simplifying and solving for x, we get
$16x = 400$ and $x = 25$ **Ans.**

13. *Given:* There are 192 9th-graders and 136 10th-graders entered in a drawing.
Find: the probability a 10th-grader wins the drawing.
The probability is $\frac{136}{192+136} = \frac{136}{328} = \frac{17}{41}$ **Ans.**

14. *Given:* $2^2 \cdot 4^4 = 2^k$
Find: k
$2^k = 2^2 \cdot 4^4 = 2^2 \cdot (2^2)^4 = 2^2 \cdot 2^8 = 2^{10}$.
Therefore, $k = 10$ **Ans.**

15. *Given:* A circle with center P(5, 10) intersects the x-axis at Q(5, 0).
Find: the area of the circle.



The radius of the circle drawn from P to Q, as shown, has length. This is the radius 10 – 0 = 10. The area is $\pi r^2 = 100\pi$ **Ans.**

16. *Given:* Subtract 3 from $2x$ and divide the difference by 5. The result is 7.
Find: x
The problem statement can be expressed algebraically as $\frac{2x-3}{5} = 7$. Solving for x, we see that $2x - 3 = 35$ and $2x = 38$. So, $x = 19$ **Ans.**

17. *Given:* Melina's ratio of 2-point shot attempts to 3-point shot attempts is 4:1.
Find: the percent of Melina's attempted shots that are 3-point shots.
Of every 5 shot attempts, 1 is a 3-point shots and 4 are 2-point shots. So, the 3-point shots account for $\frac{1}{5} = 20\%$ **Ans.**

18. *Find:* the sum of the distinct prime factors of 2016.
The prime factorization of 2016 is $2^5 \times 7 \times 3^2$. The distinct prime factors 2, 3 and 7 have the sum $2 + 3 + 7 = 12$ **Ans.**

19. *Given:* The opposite faces of a six-sided die add up to 7. Two identical six-sided dice are placed as shown.
Find: the sum of the number of dots on the two faces that touch each other.

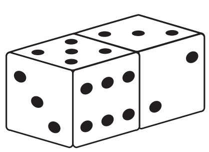


Figure 1

Since the dots on opposite faces of a die add to 7, it follows that the opposing sides must be 6 and 1, 5 and 2, 4 and 3. As Figure 1 shows, the first die is positioned so that the top face has 5 dots and the bottom face has 2 dots. The right face has 6 dots, and the left face has 1 dot. The front face has 3 dots, and the back face, which touches the front face of the second die, has 4 dots.

Now let's try to rotate the first die so it is positioned exactly like the second die so that corresponding sides on the dice have the same number of dots and are aligned the same way. If we rotate the first die so that the front face is now positioned on the top, we obtain the arrangement shown in Figure 2.

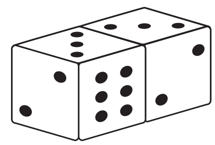


Figure 2

If we rotate the first die again so that the current front face is now on the right, we get the arrangement in Figure 3.

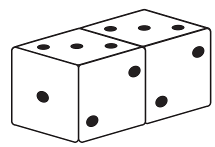


Figure 3

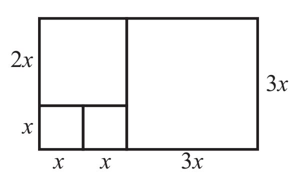
Now that all corresponding faces on the dice are the same, we can see that on the second die, the front face, which touches the back face of the first die, has 1 dot. The sum of these two values, then, is $4 + 1 = 5$ **Ans.**

20. *Find:* how many sets of two or three distinct positive integers have a sum of 8?
There are 3 sets of two integers that have a sum of 8: {1, 7}, {2, 6}, {3, 5}. The pair {4, 4} doesn't make it because the values aren't distinct. There are 2 sets of three integers that have a sum of 8: {1, 2, 5}, {1, 3, 4}. That makes the total number of sets $3 + 2 = 5$ **Ans.**

21. *Given:* $\frac{a+b}{2} = 3, \frac{b+c}{2} = 4, \frac{a+c}{2} = 5$
Find: $a + b + c$
The three given equations can be rewritten as
$a + b = 6$
$b + c = 8$
$a + c = 10.$
Adding these three equations yields $2a + 2b + 2c = 24$. Dividing both sides by 2, we get $a + b + c = 12$ **Ans.**

22. *Given:* The mean of a set of 5 numbers is $3k$. Add a sixth number to the set and the mean increases by k.
Find: the ratio of the sixth number to the sum of the first 5 numbers.
Let S be the sum of the first five numbers. Then we have $\frac{S}{5} = 3k$ and $S = 15k$. Let x be the sixth number. Then $\frac{S+x}{6} = 3k + k$ and $S + x = 24k$. Substituting $15k$ for S in $S + x = 24k$ yields $15k + x = 24k$. So, $x = 9k$. Therefore, the ratio $\frac{x}{S}$ is $\frac{9k}{15k} = \frac{3}{5}$ **Ans.**

23. *Given:* a rectangle composed of 4 squares. The area of the rectangle is 240.
Find: the perimeter.



Let x = the side length of the smallest square. The side length of the medium square is $2x$, and the side length of the largest square is $3x$. The rectangle has length $x + x + 3x = 5x$ and width $3x$. The area is $(5x)(3x) = 240$. Solving for x, we get $15x^2 = 240$, so $x^2 = 16$ and $x = 4$ (x can't be -4). Thus, the rectangle has length $5(4) = 20$, width $3(4) = 12$. The perimeter, then, is $2(20 + 12) = 64$ **Ans.**

24. *Find:* the least positive integer n such that $n!$ is divisible by 1000.
Since $1000 = 2^3 \times 5^3$, we are looking for n such that $n! = 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times k$. With factors of 2 and 4 we have three 2s. With the factors of 5, 10 and 15, we have three 5s. The least value is $n! = 1 \times 2 \times 3 \times \cdots \times 14 \times 15$. So, $n = 15$ **Ans.**

25. *Given:* $\triangle$ABC is an isosceles triangle. AB = AC, $m\angle A = 32°$. Triangles ABC and PQR are congruent, and $m\angle PXC = 114°$.
Find: the degree measure of $\angle PYC$.

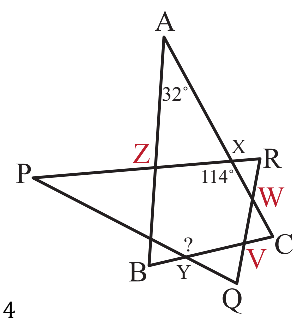


Figure 4

Since isosceles triangles ABC and PQR are congruent, AB = AC = PQ = PR, $m\angle P = m\angle A = 32°$ and $m\angle B = m\angle C = m\angle Q = m\angle R = \frac{180-32}{2} = \frac{148}{2} = 74°$. Since $m\angle PXC = 114°$, it follows that $m\angle AXZ = 66°$.

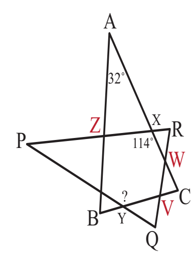


Figure 5

Now let's look at $\triangle$RXW and $\triangle$WCV, shown in Figure 5. By properties of vertical angles, $m\angle RXW = m\angle AXZ$. We also know that $m\angle RWX = 180 - (74 + 66) = 180 - 140 = 40°$. Again, by properties of vertical angles, $m\angle VWC = m\angle RWX = 40°$.

Since $m\angle RWX = 40°$ and $m\angle C = 74°$, it follows that $\triangle RXW \sim \triangle WCV$ (Angle-Angle) and $m\angle CVW = 66°$.

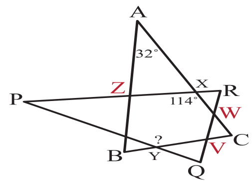


Figure 6

Now let's look at and $\triangle YVQ$, shown in Figure 6. Once again, by properties of vertical angles $m\angle YVQ = m\angle CVW = 66°$. Since $m\angle YVQ = 66°$ and $m\angle Q = 74°$, it follows that $\triangle WCV \sim \triangle YVQ$ and $m\angle VYQ = 40°$. Since angles PYC and VYQ are supplementary, we know that $m\angle PYC = 180 - 40 = 140°$ **<u>Ans.</u>**

26. *Given:* The integers 1-66 are arranged as shown.
Find: sum of the numbers in column D.

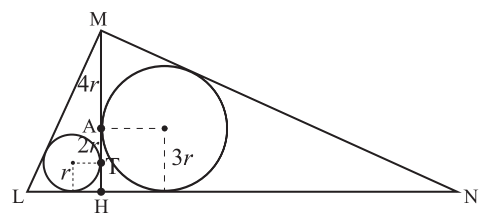
A	B	C	D	E
1	2	3		
		6	5	4
7	8	9		
		12	11	10
⋮	⋮	⋮	⋮	⋮

The numbers in column D all differ by 6 and range from 5 to 65. The sum of these numbers is $5 + 11 + \cdots + 59 + 65 =$
$70 \times \frac{11}{2} = 35 \times 11 = 385$ **<u>Ans.</u>**

27. *Given:* On Monday, a single worker painted a fence alone for two hours. Then two more painters came and, together, they finished the job 1.5 hours later. On Tuesday, a single worker began painting an identical fence at 8:00 a.m. Later 2 more workers showed up and, together, the 3 workers finished the fence at 10:54.
Find: the number of minutes the first worker painted alone on Tuesday.

All three workers paint at the same rate; we will call this R units of fences per hour. For Monday, when the entire job was completed in 3.5 hours, we have
$R \times 2 + 3R \times 1.5 = 6.5R$.
Now let T be the time worker 1 painted alone. For Tuesday, when the entire job was completed in 2.9 hours, we have
$R \times T + 3R \times (2.9 - T) = 8.7R - 2RT$.
Setting these two equations equal to each other and solving for T, we get
$6.5R = 8.74R - 2RT$ and $2.2R = 2RT$ so
$T = 1.1$ hours. Converting to minutes,
$1.1 \times 60 = 66$ **<u>Ans.</u>**

28. *Given:* $\triangle LMN$ has altitude MH. Circles are inscribed in triangles MNH and MLH, tangent to altitude MH.
MA:AT:TH = 4:2:1
Find: the ratio of the smaller circle's area to the larger circle's area.



Let r be the radius of the smaller circle. So TH = r, AT = $2r$ and MA = $4r$. It follows, then, that the radius of the larger circle has length AH = $2r + r = 3r$. The areas of the two circles are πx^2 and $\pi(3x)^2 = 9\pi x^2$, and the ratio of the areas is
$\frac{\pi x^2}{9\pi x^2} = \frac{1}{9}$ **<u>Ans.</u>**

29. *Given:* A bug crawls n miles at $n + 1$ mi/h on one day. The next day it crawls $2n + 1$ miles at $n^2 + n$ mi/h. The total time for the trip is 6 hours.
Find: the bug's average speed.
On the first day the bug spends $\frac{n}{n+1}$ hours crawling. On the second day, the bug spends $\frac{2n+1}{n^2+n}$ hours crawling. Since the entire trip took a total of 6 hours over the two days, we have $\frac{n}{n+1} + \frac{2n+1}{n^2+n} = 6$.
Solving for n, we get

$$\frac{n}{n+1} + \frac{2n+1}{n(n+1)} = 6$$
$$n + \frac{2n+1}{n} = 6(n+1)$$
$$n^2 + 2n + 1 = 6n^2 + 6n$$
$$5n^2 + 4n - 1 = 0$$
$$(5n-1)(n+1) = 0$$
$$n = \frac{1}{5} \; (n \text{ can't be } -1)$$

Substituting, we find that, on the first day, the bug crawled $\frac{1}{5}$ mile at $n + 1 = 1\frac{1}{5}$ mi/h. On the second day, it crawled $2n + 1 = 1\frac{2}{5}$ miles at $n^2 + n = \frac{1}{25} + \frac{1}{5} = \frac{6}{25}$ mi/h. This is a total of $\frac{1}{5} + 1\frac{2}{5} = 1\frac{3}{5}$ miles in 6 hours. So the average speed was $1\frac{3}{5} \div 6 = \frac{8}{5} \times \frac{1}{6} = \frac{8}{30} = \frac{4}{15}$ **Ans.**

30. *Find:* the greatest 5-digit palindrome n such that 7n is a 6-digit palindrome.
Start by looking at a 5-digit palindrome with leading digit 9 of the form 9__ __ __9.
Since 9 × 7 = 63, the last digit in the 6-digit product will be a 3, but there in no way to have the leading digit be a 3.
Next we check those with leading digit 8 of the form 8__ __ __8.
Since 8 × 7 = 56, the last digit in the 6-digit product will be a 6, and the leading digit could possibly be a 6 depending on what the other digits are.
The greatest palindrome we can form is 89998, but 7 × 89998 = 629986, which is not a palindrome. We continue looking at palindromes of the form 89__98 in descending order. Notice that with each successive palindrome of less value, the resulting 6-digit product of 7 and the palindrome decreases by 700. Also, the second digit in the product will always remain a 2 and the fifth digit will always be an 8. Next, looking at 88__88, the greatest palindrome we can form is 88988, but 7 × 88988 = 622916, which is not a palindrome. The fifth digit will always be a 1, so we are looking to bring the second digit down to a 1. We are subtracting 700 each time. We look at the second, third and fourth digit to figure out what multiple of 3 makes the second digit a 1 and the third and fourth digit equal. 229 − 63 = 166. We can get to the 6-digit palindrome 616616 by multiplying 7 and 88088 **Ans.**