6/26/2018

2017 Kansas Mathematics Standards

Flip Book

1st Grade

This project used work created by the Departments of Education in Ohio, North Carolina, Georgia and resources created by Achieve the Core, EngageNY, Illustrative Mathematics, and NCTM.

About the Flip Books

This project attempts to organize some of the most valuable resources that help develop the intent, understanding and implementation of the 2017 Kansas Mathematics Standards. These documents provide a starting point for teachers and administrators to begin discussion and exploration into the standards. It is not the only resource to support implementation of the 2017 Kansas Mathematics Standards.

This project is built on the previous work started in the summer of 2012 from Melisa Hancock (Manhattan, KS), Debbie Thompson (Wichita, KS) and Patricia Hart (Wichita, KS) who provided the initial development of the "flip books." The "flip books" are based on a model that Kansas had for earlier standards; however, this edition specifically targets the Kansas Mathematics Standards that were adopted in the summer of 2017. These flip books incorporate the resources from other state departments of education, the mathematics learning progressions, and other reliable sources including The National Council of Teachers of Mathematics and the National Supervisors of Mathematics. In addition, mathematics educators across the country have suggested changes/additions that could or should be made to further enhance its effectiveness. The document is posted on the KSDE Mathematics website at http://community.ksde.org/Default.aspx?tabid=5646 and will continue to undergo changes periodically. When significant changes/additions are implemented, the modifications will be posted and dated.

Planning Advice - Focus on the Clusters

The (mathematics standards) call for a greater focus. Rather than racing to cover topics in today's mile-wide, inch-deep curriculum, we need to use the power of the eraser and significantly narrow and deepen how time and energy is spent in the mathematics classroom. There is a necessity to focus deeply on the major work of each grade to enable students to gain strong foundations: solid conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the mathematics they know to solve problems both in and out of the mathematics classroom. (*www.achievethecore.org*)

Not all standards should have the same instructional emphasis. Some groups of standards require a greater emphasis than others. In order to be intentional and systematic, priorities need to be set for planning, instruction, and assessment. "Not everything in the Standards should have equal priority" (Zimba, 2011). Therefore, there is a need to elevate the content of some standards over that of others throughout the K-12 curriculum.

When the Standards were developed the following were considerations in the identification of priorities: 1) the need to be qualitative and well-articulated; 2) the understanding that some content will become more important than other; 3) the creation of a focus means that some essential content will get a greater share of the time and resources "while the remaining content is limited in scope." 4) a "lower" priority does not imply exclusion of content, but is usually intended to be taught in conjunction with or in support of one of the major clusters.

"The Standards are built on the progressions, so priorities have to be chosen with an eye to the arc of big ideas in the Standards. A prioritization scheme that respects progressions in the Standards will strike a balance between the journey and the endpoint. If the endpoint is everything, few will have enough wisdom to walk the path, if the endpoint is nothing, few will understand where the journey is headed. Beginnings and the endings both need particular care. … It would also be a mistake to identify such standard as a locus of emphasis. (Zimba, 2011)

The important question in planning instruction is: "What is the mathematics you want the student to walk away with?" In order to accomplish this, educators need to think about "grain size" when planning instruction. Grain size corresponds to the knowledge you want the student to know. Mathematics is simplest at the right grain size. According to Phil Daro (*Teaching Chapters, Not Lessons—Grain Size of Mathematics*), strands are too vague and too large a grain size, while lessons are too small a grain size. Units or chapters produce about the right "grain size". In the planning process educators should attend to the clusters, and think of the standards as the ingredients of a cluster. Coherence of mathematical ideas and concepts exists at the cluster level across grades.

A caution--Grain size is important but can result in conversations that do not advance the intent of this structure. Extended discussions among teachers where it is argued for "2 days" instead of "3 days" on a topic because it is a lower priority can detract from the overall intent of suggested priorities. The reverse is also true. As Daro indicates, focusing on lessons can provide too narrow a view which compromises the coherence value of closely related standards.

The video clip Teaching Chapters, Not Lessons—Grain Size of Mathematics presents Phil Daro further explaining grain size and the importance of it in the planning process. (Click on photo to view video.)

Along with "grain size", clusters have been given **priorities** which have important implications for instruction. These priorities should help guide the focus for teachers as they determine allocation of time for both planning and instruction. The priorities provided help guide the focus for teachers as they determine distribution of time for both planning and instruction, helping to assure that students really understand mathematics before moving on. Each cluster has been given a priority level. As professional educators begin planning, developing and writing units, these priorities provide guidance in assigning time for instruction and formative assessment within the classroom.

Each cluster within the standards has been given a priority level influenced by the work of Jason Zimba. The three levels are referred to as — **Major, Supporting** and **Additional**. Zimba suggests that about 70% of instruction should relate to the **Major** clusters. The lower two priorities (**Supporting** and **Additional**) can work together by supporting the **Major** priorities. You can find the grade Level Focus Documents for the 2017 Kansas Math Standards at: http://community.ksde.org/Default.aspx?tabid=6340.

Recommendations for Cluster Level Priorities

Appropriate Use:

- Use the priorities as guidance to inform instructional decisions regarding time and resources spent on clusters by varying the degrees of emphasis.
- Focus should be on the major work of the grade in order to open up the time and space to bring the Standards for Mathematical Practice to life in mathematics instruction through sense-making, reasoning, arguing and critiquing, modeling, etc.
- Evaluate instructional materials by taking the cluster level priorities into account. The major work of the grade must be presented with the highest possible quality; the additional work of the grade should support the major priorities and not detract from them.
- Set priorities for other implementation efforts such as staff development, new curriculum development, and revision of existing formative or summative testing at the state, district or school level.

Things to Avoid:

- Neglecting any of the material in the standards. Seeing Supporting and Additional clusters as optional.
- Sorting clusters (from Major to Supporting to Additional) and then teaching the clusters in order. This would remove the coherence of mathematical ideas and create missed opportunities to enhance the major work of the grade with the other clusters.
- Using the cluster headings as a replacement for the actual standards. All features of the standards matter—from the practices to surrounding text, including the particular wording of the individual content standards. Guidance for priorities is given at the cluster level as a way of thinking about the content with the necessary specificity yet without going so far into detail as to comprise the coherence of the standards (grain size)

Mathematics Teaching Practices

(High Leverage Teacher Actions)

[National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: National Council of Teachers of Mathematics.]

The eight Mathematics Teaching Practices should be the foundation for mathematics instruction and learning. This framework was informed by over twenty years of research and presented in *Principles to Actions* by the National Council of Teachers of Mathematics (NCTM). If teachers are guided by this framework, they can move “toward improved instructional practice” and support “one another in becoming skilled at teaching in ways that matter for ensuring successful mathematics learning for all students” (NCTM, 2014, p. 12).

1. **Establish mathematics goals to focus learning.**
   Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
2. **Implement tasks that promote reasoning and problem solving.**
   Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
3. **Use and connect mathematical representations.**
   Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
4. **Facilitate meaningful mathematical discourse.**
   Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
5. **Pose purposeful questions.**
   Effective teaching of mathematics uses purposeful questions to assess and advance students’ reasoning and sense making about important mathematical ideas and relationships.
6. **Build procedural fluency from conceptual understanding.**
   Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
7. **Support productive struggle in learning mathematics.**
   Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
8. **Elicit and use evidence of student thinking.**
   Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Standards for Mathematical Practice in First Grade

The Standards for Mathematical Practice are practices expected to be integrated into every mathematics lesson for all students grades K-12. Below are a few examples of how these Practices may be integrated into tasks that Grade 1 students complete.

Practices	Explanations and Examples
1) Make sense of problems and persevere in solving them.	Mathematically proficient students in First Grade continue to develop the ability to focus attention, test hypotheses, take reasonable risks, remain flexible, try alternatives, exhibit self-regulation, and persevere (Copley, 2010). As the teacher uses thoughtful questioning and provides opportunities for students to share thinking, First Grade students become conscious of what they know and how they solve problems. They make sense of task-type problems, find an entry point or a way to begin the task, and are willing to try other approaches when solving the task. For example, to solve a problem involving multi-digit numbers, they might first consider similar problems that involve multiples of ten. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. They often check their answers to problems using a different method or approach and lastly, mathematically proficient students complete a task by asking themselves the question, "Does my answer make sense?" First Grade students' conceptual understanding builds from their experiences in Kindergarten as they continue to rely on concrete manipulatives and pictorial representations to solve a problem, eventually becoming fluent and flexible with mental math as a result of these experiences.
2) Reason abstractly and quantitatively.	Mathematically proficient students in First Grade recognize that a number represents a specific quantity. They use numbers and symbols to represent a problem, explain thinking, and justify a response. This involves two processes - decontextualizing and contextualizing. In Grade 1, students represent situations by decontextualizing tasks into numbers and symbols and contextualizing numbers and symbols into situations. For a contextualizing example, when a student sees the expression 40-26, she might visualize this problem by thinking, "If I have 26 marbles and Melisa has 40, how many more do I need to have as many as Melisa?" Then, in that context, she thinks, "4 more will get me to a total of 30, and then 10 more will get me to 40, so the answer is 14." In this example, the student uses a context to think through a strategy for solving the problem, using the relationship between addition and subtraction and decomposing and recomposing the quantities. For a decontextualizing example there is this problem: Melisa has 15 pieces of candy and gives 7 away to her friend, Deb. Students may think about the number they have to add to 7 to get to 15 and write: 15 – 7 = 8.
3) Construct viable arguments and critique the reasoning of others.	Mathematically proficient students in First Grade continue to develop their ability to clearly express, explain, organize and consolidate their math thinking using both verbal and written representations. Their understanding of grade appropriate vocabulary helps them to construct arguments about mathematics that are accurate and make sense. For example, when justifying why a particular shape isn't a square, a first grade student may hold up a picture of a rectangle, point to the various parts, and reason, "It can't be a square because, even though it has 4 sides and 4 angles, the sides aren't all the same size." In a classroom where risk-taking and varying perspectives are encouraged, mathematically proficient students are willing and eager to share their ideas with others, consider other ideas proposed by classmates, and question ideas that don't seem to make sense.

4) Model with mathematics.	Mathematically proficient students in First Grade model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. They also use tools, such as tables, to help collect information, analyze results, make conclusions, and review their conclusions to see if the results make sense and revising as needed. Grade 1 students still rely on concrete manipulatives and pictorial representations while solving problems, but the expectation is that they will also write an equation to model problem situations. Also, Grade 1 students are expected to create an appropriate problem situation from an equation. For example, students are expected to create a story problem for the equation $24 + 17 - 13 = ?$ (*See Table 1 in Appendix for Addition/Subtraction "Situations".)*
5) Use appropriate tools strategically.	Mathematically proficient students in First Grade have access to a variety of concrete (e.g. 3-dimensional solids, ten frames, number balances, number lines) and technological tools (e.g., virtual manipulatives, calculators, interactive websites) and use them to investigate mathematical concepts. They select tools that help them solve and/or illustrate solutions to a problem. They recognize that multiple tools can be used for the same problem- depending on the strategy used. For example, a child who is in the counting stage may choose connecting cubes to solve a problem. While, a student who understands parts of number, may solve the same problem using ten-frames to decompose numbers rather than using individual connecting cubes. As the teacher provides numerous opportunities for students to use educational materials, first grade students' conceptual understanding and higher-order thinking skills are developed.
6) Attend to precision.	Mathematically proficient students in First Grade attend to precision in their communication, calculations, and measurements. They are able to describe their actions and strategies clearly, using grade-level appropriate vocabulary accurately. Their explanations and reasoning regarding their process of finding a solution becomes more precise. In varying types of mathematical tasks, first grade students pay attention to details as they work. For example, as students' ability to attend to position and direction develops, they begin to notice reversals of numerals and self-correct when appropriate. When measuring an object, students check to make sure that there are not any gaps or overlaps as they carefully place each unit end to end to measure the object (iterating length units). Mathematically proficient first grade students understand the symbols they use (=, ≠, >, <) and use clear explanations in discussions with others concerning the meaning of those symbols. For example, for the sentence 4 > 3, a proficient student who is able to attend to precision states, "Four is more than 3" or "Four is greater than 3", rather than "The alligator eats the four so, it's bigger." The second statement is not mathematically precise.
7) Look for and make use of structure.	Mathematically proficient students in First Grade carefully look for patterns and structures in the number system and other areas of mathematics. For example, while solving addition problems using a number balance, students recognize that regardless whether you put the 7 on a peg first and then the 4, or the 4 on first and then the 7, they both equal 11 (commutative property). When decomposing two-digit numbers, students realize that the number of tens they have constructed 'happens' to coincide with the digit in the tens place. When exploring geometric properties, first graders recognize that certain attributes are critical (number of sides, angles), while other properties are not (size, color, orientation).
8) Look for and express regularity in repeated reasoning.	Mathematically proficient students in First Grade begin to look for regularity in problem structures when solving mathematical tasks. For example, when adding three one-digit numbers and by making tens or using doubles, students engage in future tasks looking for opportunities to employ those same strategies. So when solving 8+7+2, a student may say, "I know that 8 and 2 equal 10 and then I add 7 more. That makes 17. It helps to see if I can make a 10 out of 2 numbers when I start." Also, students use repeated reasoning while solving a task with multiple correct answers. For example, in the task "There are 12 crayons in the box. Some are red and some are blue. How many of each could there be?" First Grade students realize that the 12 crayons could include 6 of each color (6+6 = 12), 7 of one color and 5 of another (7+5 = 12), etc. In essence, students repeatedly find numbers that add up to 12.

Adapted from the State Department of Education of North Carolina.

Implementing Standards for Mathematical Practice

This guide was created to help educators implement these standards into their classroom instruction. These are the practices for the **students,** and the teacher can assist students in using them efficiently and effectively.

#1 – Make sense of problems and persevere in solving them.

Summary of this Practice:

- Interpret and make meaning of the problem looking for starting points. Analyze what is given to explain to themselves the meaning of the problem.
- Plan a solution pathway instead of jumping to a solution.
- Monitor their progress and change the approach if necessary.
- See relationships between various representations.
- Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another.
- Continually ask themselves, "Does this make sense?"
- Understand various approaches to solutions.

Student Actions	Teacher Actions
• Actively engage in solving problems and thinking is visible (doing mathematics vs. following steps or procedures with no understanding). • Relate current "situation" to concepts or skills previously learned, and checking answers using different methods. • Monitor and evaluate their own progress and change course when necessary. • Always ask, "Does this make sense?" as they are solving problems.	• Allow students time to initiate a plan; using question prompts as needed to assist students in developing a pathway. • Constantly ask students if their plans and solutions make sense. • Question students to see connections to previous solution attempts and/or tasks to make sense of the current problem. • Consistently ask students to defend and justify their solution(s) by comparing solution paths.

What questions develop this Practice?

- How would you describe the problem in your own words? How would you describe what you are trying to find?
- What do you notice about...?
- What information is given in the problem? Describe the relationship between the quantities.
- Describe what you have already tried. What might you change? Talk me through the steps you've used to this point.
- What steps in the process are you most confident about? What are some other strategies you might try?
- What are some other problems that are similar to this one?
- How might you use one of your previous problems to help you begin? How else might you organize...represent...show...?

What are the characteristics of a good math task for this Practice?

- Requires students to engage with conceptual ideas that underlie the procedures to complete the task and develop understanding.
- Requires cognitive effort - while procedures may be followed, the approach or pathway is not explicitly suggested by the task, or task instructions and multiple entry points are available.
- Encourages multiple representations, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations to develop meaning.
- Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task.

#2 – Reason abstractly and quantitatively.

Summary of this Practice:

- Make sense of quantities and their relationships.
- Decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships.
- Understand the meaning of quantities and are flexible in the use of operations and their properties.
- Create a logical representation of the problem.
- Attend to the meaning of quantities, not just how to compute them.

Student Actions	Teacher Actions
• Use varied representations and approaches when solving problems. • Represent situations symbolically and manipulating those symbols easily. • Give meaning to quantities (not just computing them) and making sense of the relationships within problems.	• Ask students to explain the meaning of the symbols in the problem and in their solution. • Expect students to give meaning to all quantities in the task. • Question students so that understanding of the relationships between the quantities and/or the symbols in the problem and the solution are fully understood.

What questions develop this Practice?

- What do the numbers used in the problem represent? What is the relationship of the quantities?
- How is_____related to______?
- What is the relationship between _____ and _____?
- What does ______ mean to you? (e.g. symbol, quantity, diagram)
- What properties might you use to find a solution?
- How did you decide that you needed to use _____? Could we have used another operation or property to solve this task? Why or why not?

What are the characteristics of a good math task for this Practice?

- Includes questions that require students to attend to the meaning of quantities and their relationships, not just how to compute them.
- Consistently expects students to convert situations into symbols in order to solve the problem; and then requires students to explain the solution within a meaningful situation.
- Contains relevant, realistic content.

#3 – Construct viable arguments and critique the reasoning of others.

Summary of this Practice:

- Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments.
- Justify conclusions with mathematical ideas.
- Listen to the arguments of others and ask useful questions to determine if an argument makes sense.
- Ask clarifying questions or suggest ideas to improve/revise the argument.
- Compare two arguments and determine correct or flawed logic.

Student Actions	Teacher Actions
• Make conjectures and exploring the truth of those conjectures. • Recognize and use counter examples. • Justify and defend all conclusions and using data within those conclusions. • Recognize and explain flaws in arguments, which may need to be demonstrated using objects, pictures, diagrams, or actions.	• Encourage students to use proven mathematical understandings, (definitions, properties, conventions, theorems etc.), to support their reasoning. • Question students so they can tell the difference between assumptions and logical conjectures. • Ask questions that require students to justify their solution and their solution pathway. • Prompt students to respectfully evaluate peer arguments when solutions are shared. • Ask students to compare and contrast various solution methods • Create various instructional opportunities for students to engage in mathematical discussions (whole group, small group, partners, etc.)

What questions develop this Practice?

- What mathematical evidence would support your solution? How can we be sure that...? How could you prove that...?
- Will it still work if...?
- What were you considering when...? How did you decide to try that strategy?
- How did you test whether your approach worked?
- How did you decide what the problem was asking you to find? (What was unknown?)
- Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not?
- What is the same and what is different about...? How could you demonstrate a counter-example?

What are the characteristics of a good math task for this Practice?

- Structured to bring out multiple representations, approaches, or error analysis.
- Embeds discussion and communication of reasoning and justification with others.
- Requires students to provide evidence to explain their thinking beyond merely using computational skills to find a solution.
- Expects students to give feedback and ask questions of others' solutions.

#4 – Model with mathematics.

Summary of this Practice:

- Understand reasoning quantitatively and abstractly (able to decontextualize and contextualize).
- Apply the math they know to solve problems in everyday life.
- Simplify a complex problem and identify important quantities to look at relationships.
- Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation.
- Reflect on whether the results make sense, possibly improving/revising the model.
- Ask themselves, "How can I represent this mathematically?"

Student Actions	Teacher Actions
• Apply mathematics to everyday life. • Write equations to describe situations. • Illustrate mathematical relationships using diagrams, data displays, and/or formulas. • Identify important quantities and analyzing relationships to draw conclusions.	• Demonstrate and provide students experiences with the use of various mathematical models. • Question students to justify their choice of model and the thinking behind the model. • Ask students about the appropriateness of the model chosen. • Assist students in seeing and making connections among models.

What questions develop this Practice?

- What number model could you construct to represent the problem?
- How can you represent the quantities?
- What is an equation or expression that matches the diagram..., number line…, chart..., table…?
- Where did you see one of the quantities in the task in your equation or expression?
- What math do you know that you could use to represent this situation?
- What assumptions do you have to make to solve the problem?
- What formula might apply in this situation?

What are the characteristics of a good math task for this Practice?

- Structures represent the problem situation and their solution symbolically, graphically, and/or pictorially (may include technological tools) appropriate to the context of the problem.
- Invites students to create a context (real-world situation) that explains numerical/symbolic representations.
- Asks students to take complex mathematics and make it simpler by creating a model that will represent the relationship between the quantities.

#5 – Use appropriate tools strategically.

Summary of this Practice:

- Use available tools recognizing the strengths and limitations of each.
- Use estimation and other mathematical knowledge to detect possible errors.
- Identify relevant external mathematical resources to pose and solve problems.
- Use technological tools to deepen their understanding of mathematics.
- Use mathematical models for visualize and analyze information

Student Actions	Teacher Actions
• Choose tools that are appropriate for the task. • Know when to use estimates and exact answers. • Use tools to pose or solve problems to be most effective and efficient.	• Demonstrate and provide students experiences with the use of various math tools. A variety of tools are within the environment and readily available. • Question students as to why they chose the tools they used to solve the problem. • Consistently model how and when to estimate effectively, and requiring students to use estimation strategies in a variety of situations. • Ask student to explain their mathematical thinking with the chosen tool. • Ask students to explore other options when some tools are not available.

What questions develop this practice?

- What mathematical tools could we use to visualize and represent the situation?
- What information do you have?
- What do you know that is not stated in the problem? What approach are you considering trying first?
- What estimate did you make for the solution?
- In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative? Why was it helpful to use...?
- What can using a_____show us that _____may not?
- In what situations might it be more informative or helpful to use...?

What are the characteristics of a good math task for this Practice?

- Lends itself to multiple learning tools. (Tools may include; concrete models, measurement tools, graphs, diagrams, spreadsheets, statistical software, etc.)
- Requires students to determine and use appropriate tools to solve problems.
- Asks students to estimate in a variety of situations:
    - a task when there is no need to have an exact answer
    - a task when there is not enough information to get an exact answer
    - a task to check if the answer from a calculation is reasonable

#6 – Attend to precision.

Summary of this Practice:

- Communicate precisely with others and try to use clear mathematical language when discussing their reasoning.
- Understand meanings of symbols used in mathematics and can label quantities appropriately.
- Express numerical answers with a degree of precision appropriate for the problem context.
- Calculate efficiently and accurately.

Student Actions	Teacher Actions
• Use mathematical terms, both orally and in written form, appropriately. • Use and understanding the meanings of math symbols that are used in tasks. • Calculate accurately and efficiently. • Understand the importance of the unit in quantities.	• Consistently use and model correct content terminology. • Expect students to use precise mathematical vocabulary during mathematical conversations. • Question students to identify symbols, quantities and units in a clear manner.

What questions develop this Practice?

- What mathematical terms apply in this situation? How did you know your solution was reasonable?
- Explain how you might show that your solution answers the problem.
- Is there a more efficient strategy?
- How are you showing the meaning of the quantities?
- What symbols or mathematical notations are important in this problem?
- What mathematical language..., definitions..., properties can you use to explain...?
- How could you test your solution to see if it answers the problem?

What are the characteristics of a good math task for this Practice?

- Requires students to use precise vocabulary (in written and verbal responses) when communicating mathematical ideas.
- Expects students to use symbols appropriately.
- Embeds expectations of how precise the solution needs to be (some may more appropriately be estimates).

#7 – Look for and make use of structure.

Summary of this Practice:

- Apply general mathematical rules to specific situations.
- Look for the overall structure and patterns in mathematics.
- See complicated things as single objects or as being composed of several objects.

Student Actions	Teacher Actions
• Look closely at patterns in numbers and their relationships to solve problems. • Associate patterns with the properties of operations and their relationships. • Compose and decompose numbers and number sentences/expressions.	• Encourage students to look for something they recognize and having students apply the information in identifying solution paths (i.e. compose/decompose numbers and geometric figures, identify properties, operations, etc.) • Expect students to explain the overall structure of the problem and the big math idea used to solve the problem.

What questions develop this Practice?

- What observations do you make about...? What do you notice when...?
- What parts of the problem might you eliminate..., simplify...?
- What patterns do you find in...?
- How do you know if something is a pattern?
- What ideas that we have learned before were useful in solving this problem?
- What are some other problems that are similar to this one? How does this relate to...?
- In what ways does this problem connect to other mathematical concepts?

What are the characteristics of a good math task for this Practice?

- Requires students to look for the structure within mathematics in order to solve the problem. (i.e. – decomposing numbers by place value; working with properties; etc.)
- Asks students to take a complex idea and then identify and use the component parts to solve problems. i.e. Building on the structure of equal sharing, students connect the understanding to the traditional division algorithm. When "unit size" cannot be equally distributed, it is necessary to break down into a smaller "unit size". (example below)

```
4 )351
  -32
   31
  -28
    3
```

3 *hundreds* units cannot be distributed into 4 equal groups. Therefore, they must be broken down into *tens* units.

There are now 35 *tens* units to distribute into 4 groups. Each group gets 8 sets of tens, leaving 3 extra *tens* units that need to become *ones* units.

This leaves 31 *ones* units to distribute into 4 groups. Each group gets 7 *ones* units, with 3 *ones* units remaining. The quotient means that each group has 87 with 3 left.

- Expects students to recognize and identify structures from previous experience(s) and apply this understanding in a new situation. i.e. $7 \times 8 = (7 \times 5) + (7 \times 3)$ OR $7 \times 8 = (7 \times 4) + (7 \times 4)$ new situations could be, distributive property, area of composite figures, multiplication fact strategies.

#8 – Look for and express regularity in repeated reasoning.

Summary of this Practice:

- See repeated calculations and look for generalizations and shortcuts.
- See the overall process of the problem and still attend to the details.
- Understand the broader application of patterns and see the structure in similar situations.
- Continually evaluate the reasonableness of their intermediate results.

Student Actions	Teacher Actions
• Notice if processes are repeated and look for both general methods and shortcuts. • Evaluate the reasonableness of intermediate results while solving. • Make generalizations based on discoveries and constructing formulas when appropriate.	• Ask what math relationships or patterns can be used to assist in making sense of the problem. • Ask for predictions about solutions at midpoints throughout the solution process. • Question students to assist them in creating generalizations based on repetition in thinking and procedures.

What questions develop this Practice?

- Will the same strategy work in other situations?
- Is this always true, sometimes true or never true? How would we prove that...?
- What do you notice about...?
- What is happening in this situation? What would happen if...?
- Is there a mathematical rule for...?
- What predictions or generalizations can this pattern support? What mathematical consistencies do you notice?

What are the characteristics of a good math task for this Practice?

- Present several opportunities to reveal patterns or repetition in thinking, so students can make a generalization or rule.
- Requires students to see patterns or relationships in order to develop a mathematical rule.
- Expects students to discover the underlying structure of the problem and come to a generalization.
- Connects to a previous task to extend learning of a mathematical concept.

Critical Areas for Mathematics in First Grade

In Grade 1, instructional time should focus on **four** critical areas:

1. **Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20.**
   Students develop strategies for adding and subtracting whole numbers based on their prior work from Kindergarten with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition (e.g., Commutative Property and Associative Property) to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens" and "doubles +1") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

2. **Developing understanding of whole number relationships and place value, including grouping in tens and ones.**
   Students develop, discuss, and use *efficient, accurate,* and *generalizable* methods (students are expected to use more than just the traditional algorithms) to add within 100 and subtract multiples of 10. They compare whole numbers (at <u>least</u> to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

3. **Developing understanding of linear measurement and measuring lengths as iterating length units.**
   Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement. (Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need <u>not</u> use this technical term.)

4. **Reasoning about attributes of, and composing and decomposing geometric shapes.**
   Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

Dynamic Learning Maps (DLM) and Essential Elements

The Dynamic Learning Maps and Essential Elements are knowledge and skills linked to the grade-level expectations identified in the Common Core State Standards. The purpose of the Dynamic Learning Maps Essential Elements is to build a bridge from the content in the Common Core State Standards to academic expectations for students with the most significant cognitive disabilities.

For more information please visit the Dynamic Learning Maps and Essential Elements website.

Growth Mindset

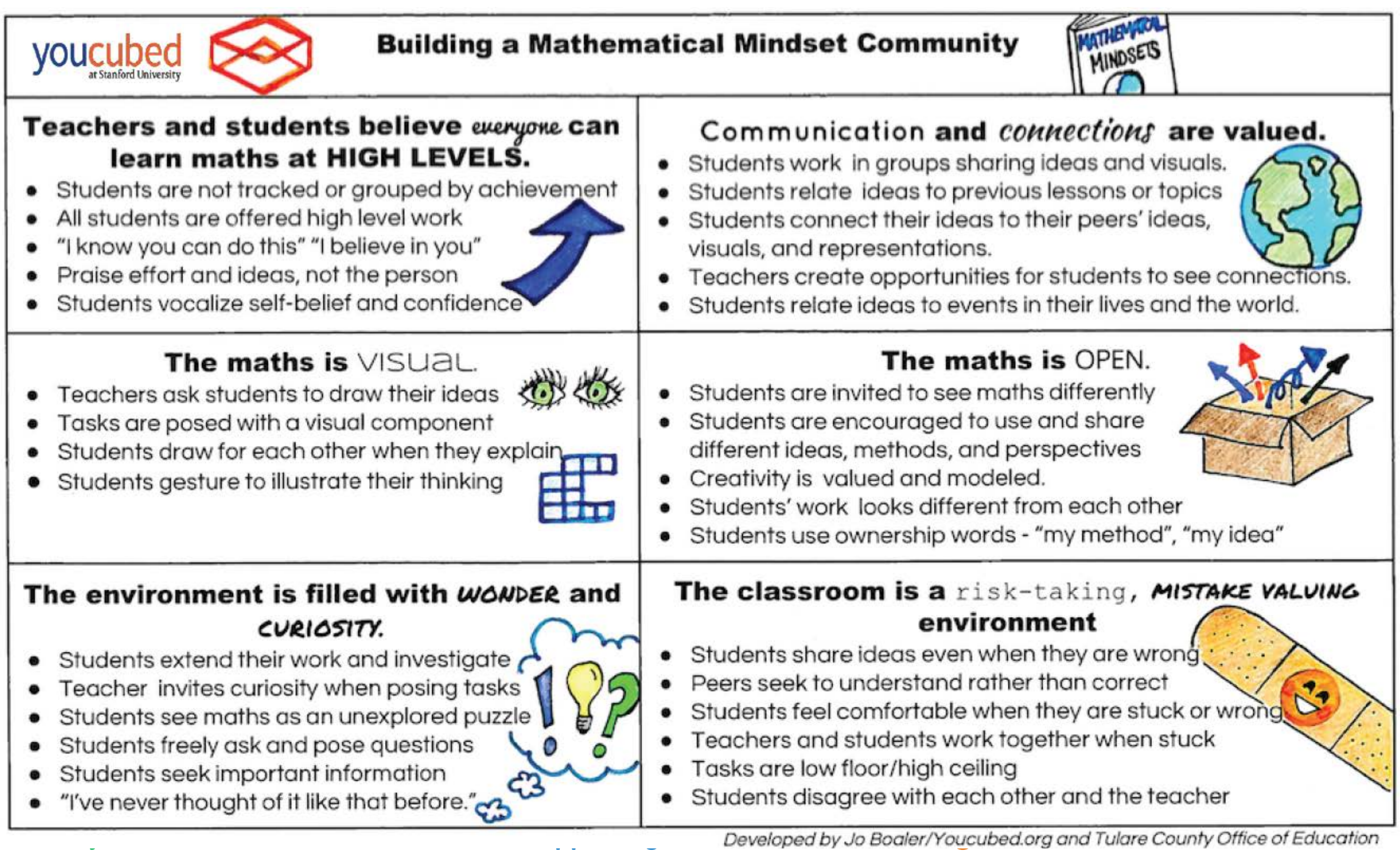
The term "growth mindset" comes from the groundbreaking work of Carol Dweck. She identified that everyone holds ideas about their own potential. Some people believe that their intelligence is more or less fixed in math – that you can do math or you can't, while others believe they can learn anything and that their intelligence can grow.

In a fixed mindset, people believe their basic qualities, like their intelligence or talent, are simply fixed traits. They spend their time documenting their intelligence or talent instead of developing it. They also believe that talent alone creates success—without effort. Students with a fixed mindset are those who are more likely to give up easily.

In a **growth mindset**, people believe that their most basic abilities can be developed through dedication and hard work—brains and talent are just the starting point. This view creates a love of learning and a resilience that is essential for great accomplishment. Students with a growth mindset are those who keep going even when work is hard, and who are persistent.

It is possible to change mindsets and to shift students' mindsets from fixed to growth and cause higher mathematics achievement and success in life. Watch this short video to get a better understanding of what Growth Mindset is and the benefits it can bring our students.

You can find a variety of resources related to **Growth Mindset** at: http://community.ksde.org/Default.aspx?tabid=6383.



Grade 1 Content Standards Overview

Operations and Algebraic Thinking (1.OA)

A. Represent and solve problems involving addition and subtraction.
   OA.1 **OA.2**
B. Understand & apply properties of operations and the relationship between addition & subtraction.
   OA.3 **OA.4**
C. Add and subtract within 20.
   OA.5 **OA.6**
D. Work with addition and subtraction equations.
   OA.7 **OA.8**

Number and Operations in Base Ten (1.NBT)

A. Extend the counting **sequence**.
   NBT.1
B. Understand place value.
   NBT.2 **NBT.3**
C. Use place value understanding and properties of operations to add and subtract.
   NBT.4 **NBT.5** **NBT.6**

Measurement and Data (1.MD)

A. Measure lengths indirectly and by iterating length units.
   MD.1 **MD.2**
B. Tell and write time.
   MD.3
C. Represent and interpret data.
   MD.4

Geometry (1.G)

A. Reason with shapes and their attributes.
   G.1 **G.2** **G.3**

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

▶ **Major Clusters** ◆ **Supporting Clusters** ● **Additional Clusters**

Domain: Operations and Algebraic Thinking (OA)

▶ **Cluster A:** Represent and solve problems involving addition and subtraction.

Standard: 1.OA.1

Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, *(e.g. by using objects, drawings, and situation equations and/or solution equations with a symbol for the unknown number to represent the problem.)* **(1.OA.1)**

For Example: *A clown had 20 balloons. He sold some and has 12 left. How many did he sell?* Situation Equation: 20 – ? = 12 Solution Equation: 20 -12 = ?

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

- This cluster is connected to developing understanding of addition, subtraction, and strategies for addition and subtraction within 20.
- This cluster is connected to "Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from" the Kindergarten standards, to "Work with addition and subtraction equations" in Grade 1, and to "Represent and solve problems involving addition and subtraction" and "Add and subtract within 20" in Grade 2.

Explanation and Examples:

This standard builds on the work from Kindergarten by having students use a variety of mathematical representations (e.g., objects, drawings, and equations) during their work. The unknown symbols should include boxes, pictures or ? - not letters for this grade level.

Teachers should be aware of all three situation types as shown in Table 1 (Appendix) and provide multiple experiences for students solving ALL of these types. These problem types are: **Result Unknown, Change Unknown, and Start Unknown**.

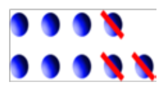
Use informal language first to describe joining situations (putting together) and separating situations (breaking apart).

Use the addition symbol (**+**) to represent joining situations, the subtraction symbol (**-**) to represent separating situations, and the equal sign (**=**) to represent an equal relationship regarding the quantities between one side of the equation and the other. The **+** and **–** symbols show action (operational symbols). The **=** symbol represents equality (relational symbol). Students don't need to know these terms, but you need to make sure you are clear in your use of them.

A helpful strategy for students in recognizing sets of objects with common patterned arrangements (0-10) without counting is the skill of subitizing. Refer to the Counting and Cardinality section of the OA progression for details.

Contextual problems that are closely connected to students' lives should be used to develop fluency with addition and subtraction. Table 1 in the Appendix, describes the four different addition and subtraction situations and their relationship to the position of the unknown. Students use objects or drawings to represent the different situations.

- **Take From example**: Avery has 9 balls. She gave 3 to Susan. How many balls does Avery have now? *A student can use 9 objects to represent Avery's ball and then remove 3 of them that were given to Susan.*



- **Compare example**: Avery has 9 balls. Susan has 3 balls. How many more balls does Avery have than Susan? *A student can use 9 objects to represent Avery's 9 balls and 3 objects to represent Susan's 3 balls. They can then compare the two sets of objects.*

The equation to represent each situation can be different – the Take From example (9 – 3 = ?) and the Compare example (9 – 3 = ? OR 3 + ? = 9). Even though the equations to *show* or *represent* each situation can be different, each problem would have the same solution equation (9 – 3 = ?). Students should be making sense of the situation first and then determining the best way to solve the problem after understanding the situation.

It is important to attend to the difficulty level of the problem situations in relation to the position of the unknown.

- Result Unknown, Total Unknown, and Both Addends Unknown problems are the least complex for students.
- The next level of difficulty includes Change Unknown, Addend Unknown, and Difference Unknown
- The most difficult are Start Unknown and versions of Bigger and Smaller Unknown (compare problems).

More Examples:

Result Unknown	**Change Unknown**	**Start Unknown**
There are 9 students on the playground. Then 8 more students showed up. How many students are there now? $9 + 8 = ?$	There are 9 students on the playground. Some more students showed up. There are now 17 students. How many students came? $9 + ? = 17$	Here are some students on the playground. Then 8 more students came. There are now 17 students. How many students were on the playground at the beginning? $? + 8 = 17$

Please see Table 1 in the Appendix for additional examples. See Table 6 for the level of difficulty for these problems can be differentiated by using smaller numbers (up to 10) or larger numbers (up to 20).

Instructional Strategies: (1.OA.1 and 1.OA.2)
Learning to ***mathematize*** (*the process of seeing and focusing on the mathematical aspects and ignoring the nonmathematical aspects)* is important in first grade. Mathematizing in first grade involves solving problems, reasoning, communicating, connecting, and representing Ideas.
Modeling addition and subtraction situations with objects, fingers, and drawings is the foundation for algebraic problem solving. More difficult types of problems situations (change and start unknown situations) should be given to students from grade 1 on.

Provide opportunities for students to participate in shared problem-solving activities to solve word problems. Allow students to collaborate in small groups to develop problem-solving strategies using a variety of models, such as drawings, words, and equations with symbols for the unknown numbers, to find the solutions. Additionally students need the opportunity to explain, write and reflect on their problem-solving strategies in order to make sense of them.

The situations for the addition and subtraction story problems should involve sums and differences less than or equal to 20. They need to align with the twelve situations found in Table 1 in the appendix of this document.

It is important to emphasize the most critical problem-solving strategy—**understand the situation** and represent the problem (e.g. use counters, cubes, or drawings). Key-word strategies in which children focus only on one or a few words will not work with algebraic problems. Key words are only minimally helpful for one-step word problems and since students are to begin solving two-step word problems in second grade, this is not helpful at all. Teachers will want to help all children move beyond the key word limiting strategy by emphasizing **understanding** the situation and then **representing** the situation.

Literature is an excellent way to incorporate problem-solving in a context that young students can understand. Many literature books that include mathematical ideas and concepts have been written in recent years. For Grade 1, the incorporation of books that contain a problem situation involving addition and subtraction with numbers 0 to 20 should be included in the curriculum. Use the situations found in Table 1 in the Appendix for guidance in selecting appropriate books. As the story is read aloud, the students can use a variety of manipulatives, drawings, or equations to model and find the solution to problems from the story. (Visit K-5 Math Teaching Resources site to get task cards for great books. Sample link to one math task card).

Tools/Resources

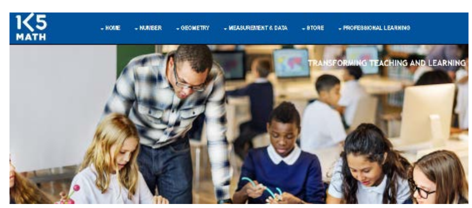
For detailed information see Operations & Algebraic Thinking Learning Progression.

Illustrative Mathematics tasks:

- 1.OA At the Park
- 1.OA Boys and Girls, Variation 1
- 1.OA Maria's Marbles
- 1.OA Sharing Markers
- 1.OA Finding a Chair
- 1.OA Boys and Girls, Variation 2
- 1.OA The Pet Snake
- 1.OA Measuring Blocks
- 1.MD Growing Bean Plants
- 1.OA 20 Tickets
- 1.OA Field Day Scarcity
- 1.OA School Supplies
- 1.OA Link-Cube Addition
- 1.OA Measuring Blocks
- 1.OA Peyton's Books

Dr. Douglas Clements and Dr. Julia Sarama are well respected early numeracy educators and researchers. Their book, Learning and Teaching Early Math: The Learning Trajectories Approach, provides a fabulous learning trajectory for *Addition and Subtraction* from the ages of 1 to 7. These researchers explain more about the different situations that students are expected to know and solve. In addition to this trajectory, they have others that are useful when diagnosing students' gaps in mathematical understanding. Clements and Sarama have provided educators access to their online trajectories. Make sure you register so you can use their developmental progressions and the activities and lessons to go with each stage of learning.

Visit **K-5 Math Teaching Resources** section showing the developmental stages of addition and subtraction with corresponding games and activities.



These links from the **K-5 Math Teaching Resources** site offer word problems with change unknown for both addition and subtraction problems.

Greg Tang's **Word Problem Generator** is a fabulous resource to make sure you are providing your students all situations subtypes for addition and subtraction.

Thinking Blocks on Math Playground allows students several ways to model problems.

Common Misconceptions:

Many children misunderstand the meaning of the equal sign. The equal sign means “has the same value as” or “balance”. However, most primary students believe the equal sign tells you that the “answer is coming up” to the right of the symbol. This misconception is over-generalized by only seeing examples of number sentences with an operation to the left of the equal sign and the total on the right. They also focus so much on the operational symbols (**+**, **-**) that they believe the **=** symbol also is an operational symbol instead of a relational symbol. First graders need to see equations written multiple ways, for example $5 + 7 = 12$ and $12 = 5 + 7$.

A second misconception that many students have is that it is valid to assume that a key word or phrase in a problem suggests the same operation will be used every time. For example, they might assume that the word *left* always means that subtraction must be used to find a solution. Providing problems in which key words like this are used to represent different operations is essential. For example, the use of the word *left* in this problem does not indicate subtraction as a solution method: *Jose took the 8 stickers he no longer wanted and gave them to Anna. Now Jose has 11 stickers left. How many stickers did Jose have to begin with?*

Domain: Operations and Algebraic Thinking (OA)

▶ **Cluster A:** Represent and solve problems involving addition and subtraction.

Standard: 1.OA.2

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, *(e.g. by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.)* **(1.OA.2)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 1. OA.1

Explanation and Examples:

This standard asks students to add (join) three numbers whose sum is less than or equal to 20, using a variety of mathematical representations. This objective does address the beginnings of multi-step word problems.

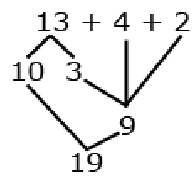
Example:

There are lots of cookies on my plate. I have 4 oatmeal raisin cookies, 5 chocolate chip cookies, and 6 gingerbread cookies. How many cookies do I have?

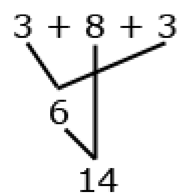
Student 1 **Adding with a Ten Frame and Counters**	**Student 2** **Look for ways to make 10**	**Student 3** **Number Path (See *Number Path* explanation in the following section)**
I put 4 counters on the 10 Frame for the oatmeal raisin cookies. Then, I put 5 different color counters on the 10-Frame for the chocolate chip cookies. Then, I put another 6 color counters out for the gingerbread cookies. Only one of the gingerbread cookies fit, so I had 5 left over. One 10-Frame and five leftover makes 15 cookies. (Students use concrete models).	I know that 4 and 6 equal 10, so the oatmeal raisin and gingerbread equals 10 cookies. Then, I add the 5 chocolate chip cookies and get 15 total cookies.	I counted on the number path. First, I counted 4, and then I counted 5 more and landed on 9. Then, I counted 6 more and landed on 15. So there were 15 total cookies.

To further students' understanding of the concept of addition allow them to create word problems with three addends. They use properties of operations and different strategies to find the sum of three whole numbers such as:

- *Counting on* and *counting on* again (e.g., to add $3 + 2 + 4$ a student writes $3 + 2 + 4 =?$ and thinks, "3, 4, 5, that's 2 more, 6, 7, 8, 9 that's 4 more so $3 + 2 + 4 = 9$."
- *Making tens* (e.g., $4 + 8 + 6 = 4 + 6 + 8 = 10 + 8 = 18$)
- Using "*plus 10, minus 1*" to add 9 (e.g., $3 + 9 + 6$ A student thinks, "9 is close to 10 so I am going to add 10 plus 3 plus 6 which gives me 19. Since I added 1 too many, I need to take 1 away so the answer is 18).
- **Decomposing numbers** between 10 and 20 into 1 ten plus some ones to facilitate adding the ones



- Using **doubles**



Students will use different strategies to add the 6 and 8.

- Using **near doubles** (e.g.,$5 + 6 + 3 = 5 + 5 + 1 + 3 = 10 + 4 = 14$)

You will want students to display their combining strategies (on the board or under a document camera). This gives them the opportunity to communicate and justify their thinking (math practice standards).

Instructional Strategies:

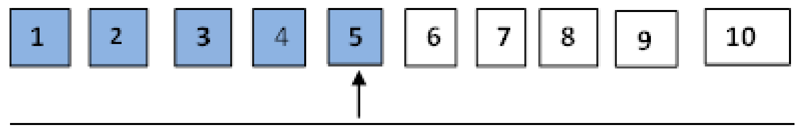
Children need many opportunities to use a variety of models, including discrete objects, length-based models (e.g., lengths of connecting cubes), and number paths, to model "part-whole", "adding to," "taking away from", and "comparing situations" to develop an understanding of the meanings of addition and subtraction and strategies to solve such arithmetic problems. Children need to understand the connections between counting and the operations of addition and subtraction (e.g., adding two is the same as "counting on" two).

"Number paths" were used in the examples rather than "number lines". A great deal of confusion can arise about what the term *number line* means. It is recommended that number lines not be used until grade 2 or the end of 1st grade because the *continuous unit* can be difficult for young learners. In early childhood materials, the term *number line or mental number line* often really means a **number path**, where numbers are put on squares and children move along the numbered path. Such number paths are counting models in which things are counted. Each square is a thing that can be counted (a *discrete unit*), so these are appropriate for children age two through grade 1.

A number path and a number line are shown below along with the meanings that children must understand and relate when using these models. A number line is a **length model** such as a ruler or a bar graph in which numbers are represented by the length from zero along a line segmented into equal lengths (a *continuous unit*). Children need to count the **length units** on a number line, not the numbers.

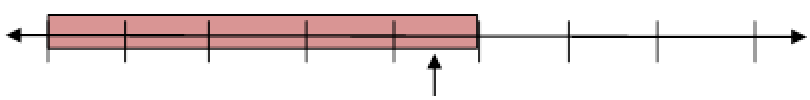
Young children can have difficulties with such a number line representation because they have difficulty seeing the units since it is continuous — they may need to see discrete things, so they focus on the numbers or the segmenting marks instead of on the lengths. Thus, they may count the starting point 0 and then be off by one, or they may focus on the spaces and be confused by the location of the numbers at the ends of the spaces.

A Number Path: Counting Things (discrete units)



Count and Cardinal Word Meanings When Counting Things in a Number Path
Count word reference: *"The arrow points to the square where I say five."*
Cardinal word reference: *"These are five squares."* (shaded squares)

A Number Line: Counting Length Units (continuous unit)



Count and Measure Word Meanings When Counting Unit Lengths on a Number Line
Count word reference: *"The arrow points to the **unit length** where I say five*".
Measure word reference: "These are five ***unit lengths***." (shaded lengths)

Consider also teaching *subtraction as an unknown addend*. Students can solve subtraction problems by a forward counting method that finds the unknown. The counting down or back is difficult and error-prone. Help children learn and use the more accurate forwards methods.

Students also need the opportunity of writing and solving story problems involving **three addends** with a sum that is less than or equal to 20. Students can write or draw a problem in which three whole things are being combined. The students exchange their problems with each other, solve them individually and then discuss their models and solution strategies. Finally have the pairs of students work together to solve each problem using a different strategy.

Resources/Tools

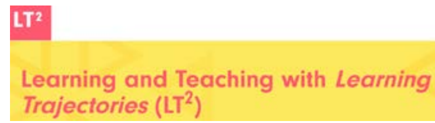
Illustrative Mathematics tasks:

- 1.OA Daisies in vases
- 1.OA, NBT The Very Hungry Caterpillar

Georgia Department of Education

- "Creating Story Problems" – Student will apply comprehensions skills to story problems and the understanding of addition and subtraction situations and operations to solve and to write problems that include: part-part-whole, comparing, grouping, doubling, counting on and counting back situations.

Focus in Grade 1, NCTM



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Visit **K-5 Math Teaching Resources** section showing the developmental stages of addition and subtraction with corresponding games and activities. Also click on the **Number** link at the top of the web page, click on **1st Grade** and access activities for this standard.

Thinking Blocks on Math Playground allows students several ways to model problems.

Common Misconceptions:

As mentioned earlier, many children misunderstand the meaning of the equal sign (a relational symbol – not an operational symbol). Emphasize the conceptual understanding of the equal sign, e.g., "has the same value as" or "balance". Most primary students believe the equal sign tells them that the "answer is coming up" on the right side of the equal sign. They also believe the equal sign can only be on the right side of an equation. This misconception is over-generalized by only seeing examples of number sentences with an operation to the left of the equal sign and the answer on the right. It is important that first graders see equations written multiple ways.

Example: $5+7=12$ and $12=5+7$ and $3+2+7=7+5$

See **Number Line and Number Path** explanations and examples above for misconceptions on use of the Number Line (linear/length model) for counting, adding, etc.

Domain: Operations and Algebraic Thinking (OA)

▶ **Cluster B**: Understand and apply properties of operations and the relationship between addition and subtraction.

Standard: 1.OA.3

Apply (not necessary to name) properties of operations as strategies to add and subtract. *Examples:* $8 + 3 = 11$ *is known, then* $3 + 8 = 11$ *is also known. (Commutative property of addition.) To add* $2 + 6 + 4$*, the second two numbers can be added to make a ten, so* $2 + 6 + 4 = 2 + 10 = 12$*. (Associative property of addition.) To add 0 to any number, the answer is that number* $7 + 0 = 7$ *(Additive identity property of 0).* Students need not use formal terms for these properties. **(1.OA.3)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

- This cluster is connected to *Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from* in Kindergarten, to *Add and subtract within 20* and *Use place value understanding and properties of operations to add and subtract* in Grade 1 and to *Use place value understanding and properties of operations to add and subtract* in Grade 2.

Explanation and Examples:

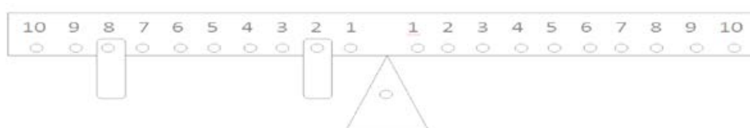
This standard asks students to apply properties of operations as strategies to **add** and **subtract**. Students do not need to use formal terms for these properties. Students should use mathematical tools, such as cubes and counters, and representations, such as the number path and a 100 chart, to model these ideas.

Students use properties of addition (the commutative and associative properties) to add whole numbers, and they create and use increasingly sophisticated strategies based on these properties (e.g., making tens, near doubles, etc.) to solve addition and subtraction problems involving basic facts. By comparing a variety of solution strategies, children relate addition and subtraction as inverse operations.

Example:
Student can build a tower of 8 green cubes and 3 yellow cubes and another tower of 3 yellow and 8 green cubes to show that order does not change the result in the operation of addition. Students can also use cubes of 3 different colors to prove that $(2 + 6) + 4$ is equivalent to $2 + (6 + 4)$ and then to prove $2 + 6 + 4 = 2 + 10$. Students should understand the **important ideas** of the following properties:

- Identity property of addition (e.g., $6 = 6 + 0$)
- Identity property of subtraction (e.g., $9 - 0 = 9$)
- Commutative property of addition - Order does not matter when you **add** numbers. (e.g. $4 + 5 = 5 + 4$)
- Associative property of addition - When adding a string of numbers you can add any two numbers first. (e.g., $3 + 9 + 1 = 3 + 10 = 13$)

Using a number balance to investigate the commutative property can provide a wonderful visual for students. If I put a weight on 8 *first* and *then* 2, will it balance if I put a weight on 2 *first* this and *then* on the 8?



Instructional Strategies: 1.OA.3-4
Instruction needs to focus on lessons that help students to discover and apply the commutative and associative properties as strategies for solving addition problems. It is not necessary for students to learn the names for these properties. It is important for students to represent, share, discuss, and compare their strategies as a class.

Another focus is the relationship between addition and subtraction as a strategy to solve unknown-addend problems. Many students naturally connect counting on to solving subtraction problems so this strategy should be explored. For the problem "$15 - 7 =?$" they think about the number they need to add to 7 to get to 15. This works well with a number line. The **difference** between the numbers on the number line is the solution to the subtraction problem. First graders should be working with sums and differences less than or equal to 20 using the numbers 0 to 20.

Provide investigations that require students to identify and then apply a pattern or structure of mathematics.

- For example, pose a string of addition and subtraction problems involving the same three numbers chosen from the numbers 0 to 20, like $4 + 13 = 17$ and $13 + 4 = 17$.
- Students analyze number patterns to see how the properties can assist in mentally solving problems. Have students choose combinations of three numbers and explore the associative property.
- Students should constantly share and discuss their reasoning to you and others. Be sure to highlight students' uses of the commutative and associative properties and the relationship between addition and subtraction.

Provide multiple opportunities for students to study the relationship between addition and subtraction in a variety of ways, including games, modeling and real-world situations. Students need to understand that addition and subtraction are related, and that subtraction can be used to solve problems where the addend is unknown.

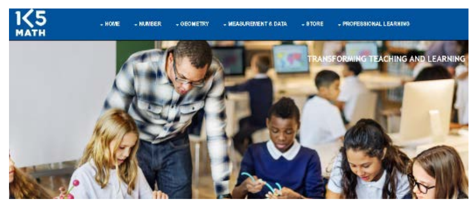
Decomposing numbers is the foundational skill for understanding and using the properties of operations. If your students are struggling with decomposing numbers then you may need to work on the skill of **subitizing**. See the Counting and Cardinality section of the OA progression for more details about this important concept.

Tools/Resources

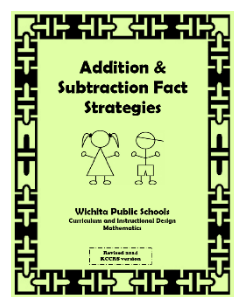
Illustrative Mathematics tasks:

- 1.OA Fact Families
- 1.OA Fact Families with Pictures
- 1.OA Domino Addition

Dr. Douglas Clements and Dr. Julia Sarama are well respected early numeracy educators and researchers. Their book, Learning and Teaching Early Math: The Learning Trajectories Approach, provides a fabulous learning trajectory for *Addition and Subtraction* from the ages of 1 to 7. These researchers explain more about the different situations that students are expected to know and solve. In addition to this trajectory, they have others that are useful when diagnosing students' gaps in mathematical understanding. Clements and Sarama have provided educators access to their online trajectories. Make sure you register so you can use their developmental progressions and the activities and lessons to go with each stage of learning.



Visit **K-5 Math Teaching Resources** section showing the developmental stages of addition and subtraction with corresponding games and activities. Also click on the **Number** link at the top of the web page, click on **1st Grade** and access activities for this standard.



Access the Addition and Subtraction Fact Strategy book from the KSDE Mathematics website for lessons, activities, and games that center on using the properties of operations to build fact fluency.

Common Misconceptions:

A common misconception is that the commutative property applies to subtraction. After students have discovered and applied the commutative property for addition, ask them to investigate whether this property works for subtraction. Have students share and discuss their reasoning and guide them to conclude that the commutative property does not apply to subtraction. Students usually need several experiences investigating this. The intent is not for students to experiment with negative numbers but only to recognize that taking 5 from 8 is not the same as taking 8 from 5. Students should recognize that they will be working with numbers later on that will allow them to subtract larger numbers from smaller numbers, but they will not be working on that in first grade.

Domain: Operations and Algebraic Thinking (OA)

▶ **Cluster B:** Understand and apply properties of operations and the relationship between addition and subtraction.

Standard: 1.OA.4

Understand subtraction as an unknown-addend problem. *For example, subtract* $10 - 8$ *by finding the number that makes 10 when added to 8.* **(1.OA.4)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 1.OA.3

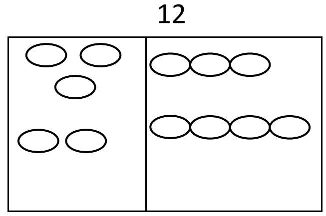
Explanation and Examples:

This standard asks for students to use subtraction in the context of unknown addend problems. When determining the answer to a subtraction problem, $12 - 5$, students think, "If I have 5, how many more do I need to make 12?" Encouraging students to record this symbolically, $5 + ? = 12$, will develop their understanding of the relationship between addition and subtraction.

Some strategies students may use are counting objects, creating drawings, counting up, using number path, number line or ten-frames to determine an answer. Refer to Table 1 type of strategies and Table 6 in the Appendix to consider the level of difficulty of this standard.

Example:

$12 - 5 = ?$ could be expressed as $5 + ? = 12$. Students should use cubes, counters and representations (such as the number path and the100 chart) to model and solve problems involving the inverse relationship between addition and subtraction.

Student 1	Student 2	Student 3
I used a ten frame. I started with 5 counters. I know that I had to have 12, which is one full ten frame and two left over. I needed 7 counters, so $12 - 5 = 7$	I used a part-part-whole diagram. I put 5 counters on one side, then I wrote 12 above the diagram. I put counters into the other side until there were 12 in all. I see that I put 7 counters on the other side, so $12 - 5 = 7$. 12  5 7	Draw a number path. I started at 5 and counted until I reached 12. I counted 7 numbers, so I knew that $12 - 5 = 7$.

Common Misconceptions:
Often students believe all of subtraction is take away and will not see the connection to addition. Showing the difference between the numbers on the number line and counting up to see the difference can be beneficial.

Domain: Operations and Algebraic Thinking (OA)

▶ **Cluster C**: Add and subtract within 20.

Standard: 1.OA.5

Relate counting to addition and subtraction (*e.g. by counting on 2 to add 2, counting back 1 to subtract 1*). **(1.OA.5)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

- This cluster is connected to all clusters in the Counting and Cardinality Domain, *Understand addition as putting together and adding to, and understanding subtraction as taking apart and taking from* and *work with numbers 11-19 to gain foundations for place value* in Kindergarten, to *Understand and apply properties of operations and the relationship between addition and subtraction in* Grade 1 and to *Add and subtract within 20* and *Use place value understanding and properties of operations to add and subtract* in Grade 2.

Explanation and Examples:

This standard asks for students to make a connection between counting and addition & subtraction. Students use various counting strategies, including **counting all and counting on**, with numbers up to 20. This standard calls for students to move beyond counting all (which they started in Kindergarten) and become comfortable at counting on (students will eventually move on from this to using fact strategies based in the properties of operations). The counting all strategy requires students to count an entire set. The **counting on** and **counting back strategies** occur when students are able to hold the "start number" in their head and count on or back from that number.

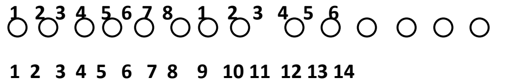
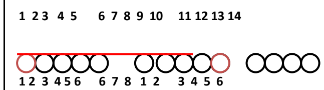
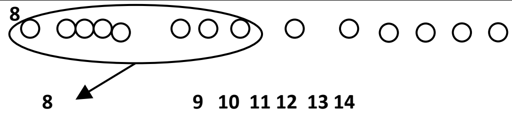
Students' multiple experiences with counting may hinder their understanding of **counting on** as connected to addition and subtraction. To help them make these connections when students count on 3 from 4, they should write this as $4 + 3 = 7$. When students count on for subtraction (3) from 7, they should connect this to $7 - 3 = 4$. Students write $7 - 3 =?$ and think I *count on* $3 + ? = 7$.

Additional Example: $\mathbf{5 + 3} =?$

Student 1	Student 2
Counts all $5 = 2+?$. The student counts five counters. The student adds two more. The student counts 1,2,3,4,5,6,7 to get the answer.	Counts On $5 + 2 =?$. Student count five counters. The student adds the first counters & says 6, then adds another counter & says 7. The student knows the answer is 7 since they counted on 2.

For more details see *Levels of Children's Addition and Subtraction Methods* (NCTM, *Focus in Grade 1*)

Levels of Children's' Addition and Subtraction Methods

	8 + 6 = 14	**14 – 8 = 6**
Level 1: **Count all**		
Level 2: **Count on**		To solve $14 - 8$: I count on $8 + ? = 14$ I took away 8. 8 to 14 is 6, so $14 - 8 + 6$.
Level 3: Recompose		$14 - 8$: I make a ten for $8 + ? = 14$  $8 + 2 + 4$ \ / 6 $8 + ? = 14$
Doubles ± *n*	$6 + 8 = ?$ $6 + 6 + 2 = ?$ $12 + 2 + 14$	

Note: Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful when counting; it makes subtraction as easy as addition. The use of "touch points or touch math" should be avoided since it encourages students to stay at Level 1 and continually use counting all. See Table 6 for level descriptions.

Instructional Strategies: (Grade 1.OA.5-6)

Provide many experiences for students to construct strategies to solve the different problem types illustrated in Table 1 in the Appendix. These experiences should help students combine their procedural and conceptual understandings.

Have students invent and refine their strategies for solving problems involving sums and differences less than or equal to 20. Ask them to explain and compare their strategies as a class and discuss which ones are more efficient.

Provide multiple and varied experiences that will help students develop a strong sense of numbers based on comprehension – **not rules and procedures**. Number sense is a blend of comprehension of numbers and operations and fluency with numbers and operations. Students gain computational fluency (using efficient and accurate methods for computing) as they come to understand the role and meaning of arithmetic operations in number systems.

Primary students should come to understand addition and subtraction as they connect counting and number sequence to these operations. Addition and subtraction also involve part to whole relationships. Students' understanding that the whole is made up of parts is connected to decomposing and composing numbers.

Provide numerous opportunities for students to use the *counting on* strategy for solving addition and subtraction problems. For example, provide a ten frame showing 5 colored dots in one row. Students add 3 dots of a different color to the next row and write $5 + 3$. Ask students to count on from 5 to find the total number of dots (or some students may be able to subitize this amount and not have to count on). Then have them add an equal sign and the number eight to $5 + 3$ to form the equation $5 + 3 = 8$.

Ask students to verbally explain how counting on helps to add one part to another part to find a sum. Discourage students from inventing a counting back strategy for subtraction because it is difficult and leads to errors.

Instructional Resources/Tools

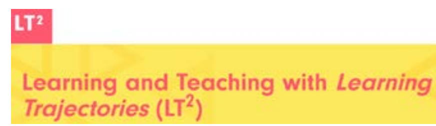
Illustrative Mathematics tasks:

1.OA, NBT The Very Hungry Caterpillar

Five-frame and Ten-frame activities

A variety of objects for modeling and solving addition and subtraction problems
See Table 1 in the Appendix

Dr. Douglas Clements and Dr. Julia Sarama are well respected early numeracy educators and researchers. Their book, Learning and Teaching Early Math: The Learning Trajectories Approach, provides a fabulous learning trajectory for *Addition and Subtraction* from the ages of 1 to 7. These researchers explain more about the different situations that students are expected to know and solve. In addition to this trajectory, they have others that are useful when diagnosing students' gaps in mathematical understanding. Clements and Sarama have provided educators access to their online trajectories. Make sure you register so you can use their developmental progressions and the activities and lessons to go with each stage of learning.



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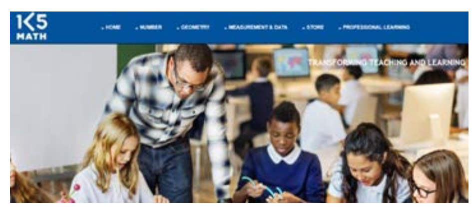


Common Misconceptions

Students ignore the need for regrouping when subtracting with numbers 0 to 20 and think that they should always subtract a smaller number from a larger number. For example, students solve 15 – 7 by subtracting 5 from 7 and 0 (0 tens) from 1 to get 12 as the incorrect answer. Students need to relate their understanding of place-value concepts and grouping in tens and ones to their steps for subtraction. They need to show these relationships for each step using mathematical drawings, ten-frames or base-ten blocks so they can understand an efficient strategy for multi-digit subtraction.

Domain: Operations and Algebraic Thinking (OA)

▶ **Cluster C**: Add and subtract within 20.

Standard: 1.OA.6

Add and subtract within 20, demonstrating fluency (efficiently, accurately, and flexibly) for addition and subtraction within 10. Use mental strategies such as **counting on**; **making *ten*** *(e.g.* $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$*)*; **decomposing a number leading to a ten** *(e.g.* $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$*)*; **using the relationship between addition and subtraction** *(e.g. knowing that* $8 + 4 = 12$*, one knows* $12 - 8 = 4$); and **creating equivalent but easier or known sums** *(e.g. adding* $6 + 7$ *by creating the known equivalent* $6 + 6 + 1 = 12 + 1 = 13$*)*. **(1.OA.6)**

This is a required fluency by the end of Grade 1.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 1.OA.6

This standard is strongly connected to all the standards in this domain. It focuses on students being able to fluently add and subtract numbers to 10 and having experiences adding and subtracting within 20.

Explanation and Examples:

By studying patterns and relationships in addition facts and relating addition and subtraction, students build a foundation for fluency with addition and subtraction facts. Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly (use of different strategies), accurately, and efficiently.

The use of objects, diagrams, number lines and fact strategies will help students develop fluency. It is important for students to be able to use a **variety** of strategies when adding and subtracting numbers within 20. Students should have ample experiences modeling these operations before working on automaticity. **Current research** (Clements & Sarama; Boaler) **has found that timing students before they know the answers to their facts is detrimental to understanding and long-term retention of those facts. Focusing on fact strategies to find solutions has shown to lead to fluency**.

Instructional Strategies:
The use of objects, diagrams, or interactive whiteboards and various strategies will help students develop fluency. It is important for students to be able to use a **variety** of strategies when adding and subtracting numbers within 20. Students should have ample experiences modeling these operations before working on fluency.

It is important to move *beyond the strategy of counting on*, which is a Level 2 strategy. You want to move on to Level 3 strategies to develop fluency of facts. Many teachers think that counting on is all a child needs in order to solve facts, when in reality, it is just a step above counting all and can becomes a hindrance when working with larger numbers. To develop real fluency with operations, students need to be **using Level 3 strategies** see Table 6.

Example: $8 + 7 = ?$ **AND 14 – 6 = ?**

Student 1	Student 2
Making 10 and Decomposing a Number I know that 8 plus 2 is 10 so I decomposed (broke) the 7 up into a 2 and a 5. First I added 8 and 2 to get 10 and then added the 5 to get 15, $8 + 7 = (8 + 2) + 5 = 10 + 5 = 15$	**Creating an Easier Problem with Known Sums** I know 8 + 7 is close to a double, 7 + 7. I that 7 and 7 equals 14, then I add 1 more (to make up the 8) to get 15. $8 + 7 = (7 + 7) + 1 = 15$
Student 1	**Student 2**
Decomposing the Numbers when You Subtract I know that 14 minus 4 is 10 so I broke the 6 up into 4 and 2. 14 minus 4 is 10, Then I take away 2 more to get 8. $14 - 6 = (14 - 4) - 2 + 10 - 2 + 8$	**Relationship Between Addition and Subtraction** 14 – 6 = ?. I know that 6 plus 8 is 14, so that means 14 minus 6 is 8. $6 + 8 = 14$ so $14 - 6 + 8$

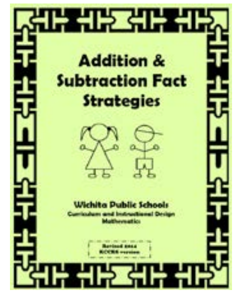
*Algebraic ideas underlie what students are doing when they create equivalent expressions in order to solve a problem or when they use addition combinations they know to solve more difficult problems. Students begin to consider the relationship between the parts. For example, students notice that the whole remains the same, as one part increases the other part decreases.

Tools/Resources:
For detailed information, see Learning Progressions Operations and Algebraic Thinking:

Illustrative Mathematics tasks:
- 1.OA $20 Dot Map
- 1.OA Making a ten

See also: "Ten is Our Friend" from NSCM's, Great Tasks for Mathematics K-5, (2013).

Access the Addition and Subtraction Fact Strategy book from the KSDE Mathematics website for lessons, activities, and games that center on using the properties of operations to build fact fluency:



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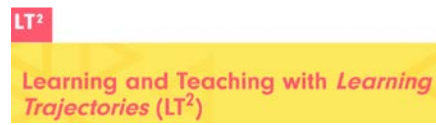

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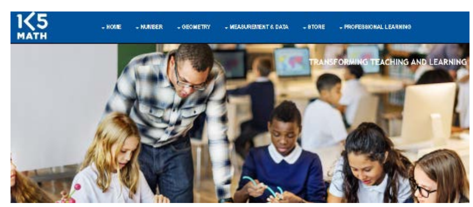


Common Misconceptions: See 1.OA.5
It is unfortunate that many educators and parents believe that giving students timed tests will improve fact fluency and retention of facts. This is actually the exact opposite. **Current research** (Clements & Sarama; Boaler) **has found that timing students before they know the answers to their facts is detrimental to understanding and long-term retention of those facts. Focusing on fact strategies to find solutions has shown to lead to fluency**.

For more information please visit Jo Boaler's website (YouCubed.org) for research and videos explaining fact fluency.

Also, access this white paper about fluency from KSDE for more information.

Domain: Operations and Algebraic Thinking (OA)

▶ **Cluster D:** Work with addition and subtraction equations.

Standard: 1.OA.7

Understand the meaning of the equal sign (the value is the same on both sides of the equal sign), and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false?*

$6 = 6;\ \ 7 = 8 - 1;\ \ 5 + 2 = 2 + 5;\ \ 4 + 1 = 3 + 2;\ \ 7 - 1 = 4;\ \ 5 + 4 = 7 - 2$ **(1.OA.7)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections:

- This cluster is connected to *Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from* in Kindergarten, to *Represent and solve problems involving addition and subtraction* in Grade 1, and to *Represent and solve problems involving addition and subtraction* and *Add and subtract within 20* in Grade 2.

Explanation and Examples:

This standard expects students to work with the *concept of equality* by identifying whether **equations** are **true** or **false.** Therefore, students need to understand that the equal sign does not mean "the answer comes next", but rather that the **equal sign signifies a relationship that is balanced between the left and right side of the equation**. Interchanging the language of "equal to" and "the same value as" as well as "not equal to" and "not the same value as" will help students grasp the meaning of the equal sign. Students should understand that *"*equality*"* means "the same value as".

In order for students to avoid the common pitfall that the equal sign means "to do something" or that the equal sign means "the answer is," they need to be able to:

- Express their understanding of the meaning of the equal sign.
- Accept sentences other than a + b = c as true ($a = a, c = a + b, a = a + 0, a + b = b + a$).
- Know that the equal sign represents a relationship between two equal quantities.
- Compare expressions without calculating.

The number sentence $4 + 5 = 9$ can be read as, "Four plus five has the same value as nine."

In addition, Students should be exposed to various representations of equations, such as:

- an operation on the left side of the equal sign and the solution on the right side ($5 + 8 = 13$) "Compose"
- an operation on the right side of the equal sign and the solution on the left side ($13 = 5 + 8$) "Decompose"
- numbers on both sides of the equal sign ($6 = 6$)
- number sentences on both sides of the equal sign ($5 + 2 = 4 + 3$ OR $5 + 2 = 9 - 2$).

Students need many opportunities to model equations using cubes, counters, drawings, etc.

Experiences determining if equations are true or false help student develop the understanding of the equal sign. Initially, students develop an understanding of the meaning of equality by using models. However, the goal is for students to reason at a more abstract level. At all times students should justify their answers, make conjectures (e.g., if you add a number and then subtract that same number, you always get zero), and make estimations.

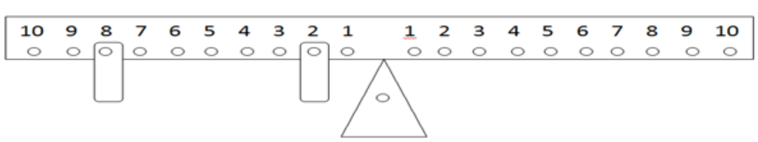
Once students have a solid foundation of the key skills listed above, they can begin to rewrite false statements using the symbols, **< and >** and explain how these symbols show inequality (students explain that the statements aren't equal so they must use greater than or less than symbols).

Examples of **true** and **false** statements:

- $7 = 8 - 1$
- $8 = 8$
- $1 + 1 + 3 = 7$
- $4 + 3 = 3 + 4$
- $6 - 1 = 1 - 6$
- $12 + 2 - 2 = 12$
- $9 + 3 = 10$
- $5 + 3 = 10 - 2$
- $3 + 4 + 5 = 3 + 5 + 4$
- $3 + 4 + 5 = 7 + 5$
- $13 = 10 + 4$
- $19 + 9 + 1 = 19$

Instructional Strategies:

Provide opportunities for students use objects of equal weight and a number balance to model equations for sums and differences less than or equal to 20 using the numbers 0 to 20. Give students equations in a variety of forms that are true and false. Include equations that use the identity property, commutative property of addition, and associative property of addition.



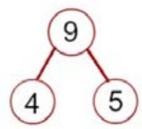
Students need not use formal terms for these properties.

- $13 + 0 = 13$ Identity Property
- $8 + 6 = 6 + 8$ Commutative Property for Addition
- $3 + (7 + 4) = (3 + 7) + 4$ Associative Property for Addition

When asking students to determine whether the equations are true or false have them record their work with drawings. Students then compare their answers as a class and discuss their reasoning. Present equations recorded in a nontraditional way, like $13 = 16 - 3$ and $9 + 4 = 18 - 5$, then ask, “Is this true?.” Have students decide if the equation is true or false. Then as a class, students discuss their thinking that supports their answers.

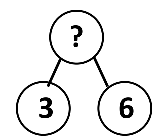
Provide situations relevant to first graders for the problem types illustrated in Table 1 in the Appendix: **Add to/Result Unknown**, **Take from/Start Unknown**, and **Add to/Result Unknown**.

Demonstrate how students can use graphic organizers such as the *number bond* (shown below) to help them think about problems. The *number bond* shows a sum with diagonal lines going down to connect with the two addends, forming a triangular shape.



It shows two known quantities and one unknown quantity. Use various symbols, such as a square, to represent an unknown sum or addend in an equation. For example, here is a **Take from/Start Unknown problem situation**: *Some markers were in a box. Matt took 3 markers to use. There are now 6 markers in the box. How many markers were in the box before?* The situation equation is **? – 3 = 6**.

A number bond can be used to assist in seeing the relationships between the numbers.



Have students practice using *number bonds* to organize their solutions to problems involving sums and differences less than or equal to 20 with the numbers 0 to 20. Then ask them to share their reactions to using a *number bond.*
Provide numerous experiences for students to compose and decompose numbers less than or equal to 20 using a variety of manipulatives. Have them represent their work with drawings, words, and numbers. Ask students to share their work and thinking with their classmates. Then ask the class to identify similarities and differences in the students’ representations.

Tools/Resources:

Illustrative Mathematics tasks:

- 1.OA Valid Equalities?
- 1.OA Using lengths to represent equality
- 1.OA Equality Number Sentences
- 1.OA, NBT The Very Hungry Caterpillar
- 1.OA 20 Tickets

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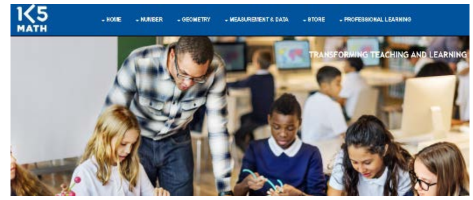


Visit **K-5 Math Teaching Resources** section showing the developmental stages of addition and subtraction with corresponding games and activitieshttps://www.k-5mathteachingresources.com/addition-and-subtraction-centers.html.
Also click on the **Number** link at the top of the web page, click on 1st Grade and access activities for this standard.

Common Misconceptions:

Many students think that the equals sign means that an operation must be performed on the numbers on the left and the result of this operation is written on the right. Students often ignore the equal sign in equations that are written in a nontraditional way. For instance, students find the incorrect value for the unknown in the equation $9 = \Delta - 5$ by thinking $9 - 4 = 5$. It is important to provide equations with a single number on the left as in $18 = 10 + 8$ in order to students to fix their misconceptions about the equal symbol. Showing pairs of equations such as $11 = 7 + 4$ and $7 + 4 = 11$ gives students experiences with understanding that the meaning of the equal sign as *has the same value as*.

Domain: Operations and Algebraic Thinking (OA)

▶ **Cluster D:** Work with addition and subtraction equations.

Standard: 1.OA.8

Using related equations, determine the unknown whole number in an addition or subtraction equation. *For example, determine the unknown number that makes the equation true in each of the equations* $\blacksquare - 3 = 7$; $7 + 3 = \blacksquare$. **(1.OA.8)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See **1.OA.7**

Explanation and Examples:

This standard extends the work that students do in 1.OA.4 by relating addition and subtraction as related operations for situations with an unknown. This standard builds upon the "think addition" for subtraction problems as explained by the example of **Student 2 in 1.OA.6.**

Student 2
14 – 6 = ?. I know that 6 plus 8 is 14, so that means 14 minus 6 is 8. $6 + 8 = 14$ so $14 - 6 + 8$

Students should work on related equations to fully understand the relationship between addition and subtraction. Related equations are similar to fact families but extend that understanding to include **composing equations** and **decomposing equations**. When thinking about solving an equation there are 7 other related equations for a total of **8 related equations**. For example; the equation **14 – 6 = 8** has seven other related equations:

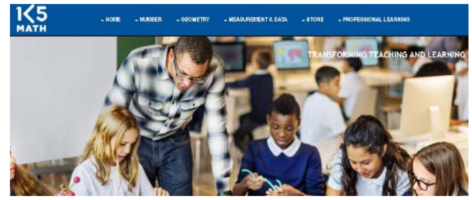
Composing equations	Decomposing equations
6 + 8 = 14	14 = 6 + 8
8 + 6 = 14	14 = 8 + 6
14 – 8 = 6	8 = 14 – 6
	6 = 14 - 8

Tools/Resources

Illustrative Mathematics tasks:

- 1.OA Find the Missing Number
- 1.OA Kiri's Mathematics Match Game

Dr. Douglas Clements and Dr. Julia Sarama are well respected early numeracy educators and researchers. Their book, Learning and Teaching Early Math: The Learning Trajectories Approach, provides a fabulous learning trajectory for *Place Value* from the age of 1 to 7. They also have provided educators access to their online trajectories. Make sure you register to gain access to their developmental progressions and the activities and lessons to go with each stage of learning.



Visit K-5 Math Teaching Resources click on **Number**, then on **1st Grade**, then scroll down to 1.OA.8 to access resources specifically for this standard.

Instructional Strategies: See **1.OA.7**

Common Misconceptions: See **1.OA.7**

Domain: Number and Operations in Base 10 (NBT)

▶ **Cluster A**: Extend the counting sequence.

Standard: 1.NBT.1

Count to 120 (recognizing growth and repeating patterns), starting at any number less than 120. In this range, read and write **numerals** and represent a number of objects with a written numeral. **(1.NBT.1)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

- This cluster is connected to *Know number names and the count sequence* and *Compare numbers* in Kindergarten, and to *Understand place value* in Grade 2.

Explanation and Examples:

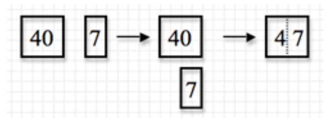
This standard expects students to rote count forward to 120 by *Counting On* from any number less than 120. To assist in developing understanding and fluency, students should have ample experiences with the hundred chart to see patterns between numbers (for example; all of the numbers in a column on the hundred chart have the same digit in the ones place and all of the numbers in a row have the same digit in the tens place). Digital hundred charts are an excellent tool to use when exploring these patterns. The National Library of Virtual Manipulatives has one that is easy to use:

This standard also calls for students to read, write and represent a number of objects with a written numeral (number form or standard form). These representations can include connecting cubes, place value blocks, pictorial representations or other concrete materials. Students should use objects, words, and/or symbols to express their understanding of numbers.

As students are developing accurate counting strategies they are also building an understanding of how the numbers in the counting sequence are related—each number is one more (or one less) than the number before (or after). They extend their counting beyond 100 to count up to 120 by counting by 1s.

Some students may begin to count in groups of 10 (while other students may use groups of 2s or 5s to count). Counting in groups of 10 as well as grouping objects into 10 groups of 10 will develop students understanding of place value concepts.

After counting objects, students write the numeral or use numeral cards to represent the number. Given a numeral, students read the numeral, identify the quantity that each digit represents using numeral cards, and count out the given number of objects.



Students should experience counting from different starting points (e.g., start at 83; count to 120). To extend students' understanding of counting, they should be given opportunities to count backwards by ones and tens. They should also investigate patterns in the base 10 system.

Instructional Strategies:
In first grade, students build on their counting to 100 by ones and tens beginning with numbers other than 1 as they learned in Kindergarten. Students can start counting at any number less than 120 and continue to 120. It is important for students to connect different representations for the same quantity or number.

Students use materials to count by ones and tens to a build models that represent a number, then they connect this model to the number word and its representation as a written numeral. Students learn to use numerals to represent numbers by relating their place-value notation to their models. **Caution**: If students have not had enough experience with groupable models (connecting cubes, ten frames, sticks in bundles, etc.) for place value then their understanding of place value will be limited. If tradeable models (place value blocks or base-ten blocks) are used too soon, students will not have built a solid understanding that ten ones create one ten and this can lead to serious misunderstandings. For example, students may not understanding that 31 can be shown by 3 tens and 1 one OR 2 tens and 11 ones OR 1 ten and 21 ones OR 31 ones.

Students represent the quantities shown in the models by placing numerals in labeled hundreds, tens and ones columns. They eventually move to representing the numbers in standard form, where the group of hundreds, tens, then singles shown in the model matches the left-to-right order of digits in numbers.

Listen as students orally count to 120 and focus on their transitions between decades and the century number. These transitions will be signaled by a 9 and require new rules to be used to generate the next set of numbers. Students need to listen to their rhythm and pattern as they orally count so they can develop a strong number word list.

Extend the hundred chart by attaching a blank hundred chart and writing the numbers 101 to 120 in the spaces following the same pattern as in the hundreds chart. Students can use these charts to connect the number symbols with their count words for numbers 1 to 120.

For some students a **vertical number scroll** makes the patterns of the ones and tens pop out for them. It would be worth the time to create a classroom number scroll and ask what the students notice as they numbers are written.

Do your students have the foundations of Counting and Cardinality? Try these examples to assess students:
Example 1: Place a handful of objects in front of your students and see how they count all. Do they have a system for keeping track of the items that have been counted? Are they demonstrating one-to-one correspondence?

Example 2: Tell the student that you have 18 objects under a cup. Hand them some more and have them count to see how many there are all together. Can the student start with 19 and count up? This is a stepping stone to adding, and understanding stories in context where one part is unknown. (Table 1 in the Appendix)

To develop concepts of number, have students estimate how many of something before counting. For example, before getting crayons have students estimate how many crayons are in the box. After estimating, partners should count to find a total. When students are first estimating, ask them if the amount is more than ten or less than ten, or more than 20 less than 20, etc. This helps them begin to get a sense of quantity.

Resources/Tools:

For detailed information, see Learning Progression Numbers and Operations in Base Ten:

Illustrative Mathematics tasks:

- 1.NBT Counting Circles II
- 1.NBT "Crossing the Decade" Concentration
- 1.NBT Start/Stop Counting II
- 1.NBT Choral Counting II
- 1.NBT Hundred Chart Digit Game
- 1.NBT Number of the Day
- 1.NBT Where Do I Go?

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Also see: ***Developing Counting***, Table 6 in the Appendix

Common Misconceptions:

Some students do not have a well-established concept of place value. For example, when shown the number 16 the student will be able to show the 6 with six objects but when asked to show the one ten, the student will lay out only one additional object as a representation for the ten rather than 10 objects.

The NBT progression explains how some students misunderstand "sixteen" and "sixty" and provides some resources to use with your students.

Domain: Number and Operations in Base Ten (NBT)

Cluster B: Understand place value.

Standard: 1.NBT.2a-c

Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

1.NBT.2a. 10 can be thought of as a grouping of ten ones—called a "ten." **(1.NBT.2a)**

1.NBT.2b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. **(1.NBT.2b)**

1.NBT.2c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). **(1.NBT.2c)**

1.NBT.2d. Show flexibility in composing and decomposing tens and ones *(e.g. 20 can be composed from 2 tens or 1 ten and 10 ones, or 20 ones.)* **(2017)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

- This cluster is connected to *Work with numbers 11-19 to gain foundations for place value* in Kindergarten, and to *Understand place value* in Grade 2.

Explanation and Examples:

Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones).

Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

1.NBT.2a asks students to **unitize a group of ten ones as a whole unit - a ten**. This is the foundation of the place value system. So, rather than seeing a group of ten cubes as ten individual cubes, the student is now asked to see those ten cubes as a bundle - one bundle of ten.

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Students need to be flexible in seeing that 10 ones is the same in value as 1 ten and that 1 ten is the same in value as 10 ones.

1.NBT.2b asks students to extend their work from Kindergarten when they composed and decomposed numbers from 11 to 19 into ten ones and some more ones. In Kindergarten, everything was thought of as individual units - ones.

In First Grade, students are asked to **unitize** those ten individual ones as a whole unit - ***one* ten**. Students in first grade explore the idea that the teen numbers (11 to 19) can be expressed as *one* ten and some leftover ones. Ample experiences with ten frames, connecting cubes and other groupable models will help develop this concept. (Note: tradeable models (place value blocks) can eventually be introduced once you know your students understand that it isn't "magic" that makes the ten stick. Tradeable models used too soon can lead to serious misconceptions that have implications way beyond elementary school.)

Example:
For the number 12, do you have enough to make a ten? Would you have any leftover? If so, how many leftovers would you have?

Student 1	Student 2
I filled a ten frame to make one ten and had two counters left over. I had enough to make a ten with some leftover. The number 12 has 1 ten and 2 ones.	I counted out 12 connecting cubes. I had enough to put 10 cubes into a ten-rod (stick). I now have 1 ten-rod and 2 cubes left over. So the number 12 has 1 ten and 2 ones.

1.NBT.2c builds on the work of **1.NBT.2b.** Students should explore the idea that decade numbers (e.g. 10, 20, 30, 40) are groups of tens with no left over ones. Students can represent this with cubes or ten frames (place value blocks can eventually be used).

Understanding the concept of 10 is fundamental to children's mathematical development. Students need multiple opportunities ofcounting 10 objects and "bundling" them into one group of ten. They count between 10 and 20 objects and make a bundle of 10 with or without some left over (this will help students who find it difficult to write teen numbers). Finally, students count any number of objects up to 99, making bundles of 10s with or without leftovers.

As students are representing the various amounts, it is important that an emphasis is placed on the language associated with the quantity. For example, 53 should be expressed in multiple ways such as 53 ones or 5 groups of ten with 3 ones leftover.

When students read numbers, they should read them in standard form as well as using place value concepts. For example, 53 should be read as "fifty-three" as well as five tens and three ones. Reading 10, 20, 30, 40, 50 as "1 ten, 2 tens, 3 tens, etc." helps students see the patterns in the number system.

Instructional Strategies:
Essential skills for students to develop include making tens (composing) and breaking a number into tens and ones (decomposing). Composing numbers by tens is foundational for representing numbers with numerals by writing the number of tens and the number of leftover ones.

Decomposing numbers by tens builds number sense and the awareness that the order of the digits is important. Composing and decomposing numbers involves number relationships and promotes flexibility with mental computation. These skills are also foundational for understanding the properties of operations that will be used frequently in Grade 1 and on.

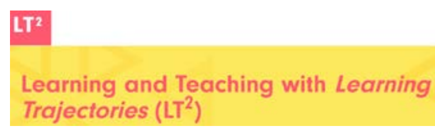
It is essential at this grade for students to see and use multiple representations of making tens using many different representations. Making the connections among the representations, the numerals, and the words are very important.

Tools /Resources
For detailed information, see Learning Progression Numbers and Operations in Base Ten:

Illustrative Mathematics tasks:
- Roll and Build
- The Very Hungry Caterpillar

See: "Ten is Our Friend", NSCM, Great Tasks for Mathematics K-5, (2013).

Also, see engageNY Modules



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Visit K-5 Math Teaching Resources click on **Number**, then on **1st Grade**, then scroll down to 1.NBT.2 to access resources specifically for this standard.

Common Misconceptions:
Students can get easily confused when thinking about 1 one and 1 ten. Each one is worth 1 of something. They need to build the understanding that the **unit** describing each 1 is different. The 1 ten is made up of 10 ones but their values are still the same. This is complex for young learners. Make sure you provide MANY opportunities for students to investigate this relationship. View this video from Christopher Danielson.

Domain: Number and Operations in Base Ten (NBT)

▶ **Cluster B:** Understand place value.

Standard: 1.NBT.3

Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the relational symbols >, <, =, and ≠. **(1.NBT.3)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 1.NBT.2a-c

Explanation and Examples:

This standard builds on the work of **1.NBT.1** and **1.NBT.2** by having students compare two numbers by examining the amount of tens and ones in each number. Students are introduced to the symbols greater than (>), less than (<), not equal to (≠), and equal to (=). From *Developing Essential Understanding of Number and Numeration* (NCTM) it is stated that students should really determine if quantities are equal or not equal before using the terms of less than or greater than. Once students are able to determine that quantities are not equal, then discussions can be had about "how" they are not equal – Which one is greater? Which one is less? If students are not able to determine equal or not equal then they will not be able to conceptualize greater than or less than.

Students should have ample experiences communicating their comparisons using words, models, and in context before using only symbols in this standard.

Example: 42__45

Student 1	Student 2
42 has 4 tens and 2 ones. 45 has 4 tens and 5 ones. They have the same number of tens, but 45 has more ones than 42. So 45 is greater than 42. So, $45 > 42$	42 is less than 45. I know this because when I count up I say 42 before I say 45. So, $42 < 45$

Students use concrete models that represent two sets of numbers. To compare, students first attend to the number of tens, then, if necessary, to the number of ones. Students may also use pictures, number paths, and spoken or written words to compare two numbers.

Comparative language includes, but is not limited to, more than, less than, greater than, most, greatest, least, same as, equal to and not equal to.

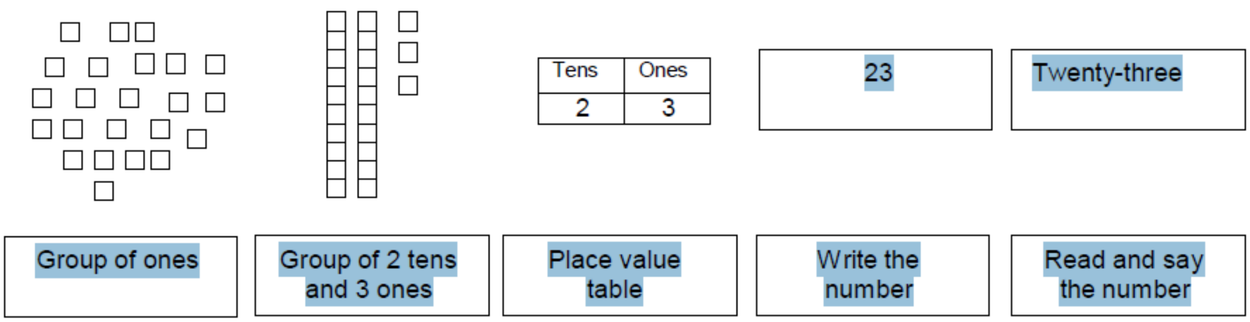
Instructional Strategies:

It is essential for students to develop the concepts of making tens (composing) and breaking a number into tens and ones (decomposing). Composing numbers by tens is foundational for representing numbers with numerals by writing the number of tens and the number of leftover ones. Decomposing numbers by tens builds number sense and the awareness that the order of the digits is important. Composing and decomposing numbers involves number relationships and promotes flexibility with mental computation.

Students continue their work in understanding that putting ten ones together makes a ten. They understand that there is a way to write that down so the same number is always understood. Instruction needs to help students move from counting by ones, to creating groups and ones, to tens and ones. They need to see and use multiple representations of making tens using bundles of tens and ones, ten frames and eventually base ten blocks.

Making the connections among the representations, the numerals, and the words are very important. Students need to connect these different representations for the numbers 0 to 99.

23 = Twenty-three



Students need to move through a progression of representations to learn a concept. They start with a concrete model, move to a pictorial or representational model, then an abstract model.

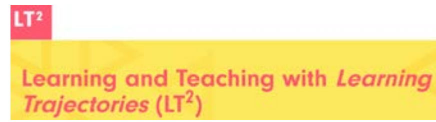
Example

1. Ask students to place a handful of small objects in one region and a handful in another region. Next have them draw a picture of the objects in each region. They can draw a likeness of the objects or use a symbol for the objects in their drawing.
2. They count the physical objects or the objects in their drawings in each region and use numerals to represent the two counts.
3. They also say and write the number word.
4. Now students can compare the two numbers using one of the relational symbols (=, ≠, <, >).

Tools/Resources:
For detailed information, see Learning Progression Numbers and Operations in Base Ten:

Illustrative Mathematics tasks:

- Ordering Numbers
- Where Do I Go?
- Comparing Numbers



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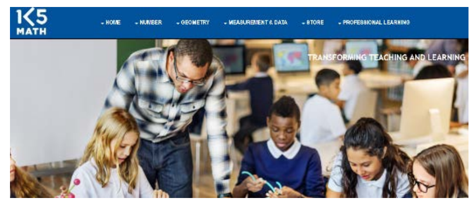


Visit K-5 Math Teaching Resources click on **Number**, then on **1st Grade**, then scroll down to 1.NBT.3 to access resources specifically for this standard.

See engageNY Modules

Common Misconceptions:

Often when students learn to use an aid (Pac Man, alligator, bird, etc.) for knowing which comparison sign (<, >) to use, the students don't associate the real meaning and name with the sign. (Often students get confused as to whether the bigger number is in the "stomach" of the aid or not, so they will use the wrong symbol.) The use of the learning aids must be accompanied by a connection to the names: < Less Than and > Greater Than. It is often helpful to place these symbols above your number line. The < points toward the lesser numbers on the number line and the > points toward the greater numbers. This is based in understanding the meaning in relation to other numbers.

Students need to understand that the symbols are shortcuts for writing down an unequal relationship. Finally, students need to begin to understand that both inequality symbols (<, >) can create true statements about any two numbers where one is greater or smaller than the other, ($15 < 28$ and $28 > 15$).

Domain: Number and Operations in Base Ten (NBT)

▶ **Cluster C:** Use place value understanding and properties of operations to add and subtract.

Standard: 1.NBT.4
Add within 100 using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used including: **(1.NBT.4)**

1.NBT.4a. Adding a two-digit number and a one-digit number **(1.NBT.4)**
1.NBT.4b. Adding a two-digit number and a multiple of 10 **(1.NBT.4)**
1.NBT.4c. Understanding that when adding two-digit numbers, combine like base-ten units such as tens and tens, ones and ones; and sometimes it is necessary to compose a ten. **(1.NBT.4)**

Suggested Standards for Mathematical Practice (MP):
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:
- This cluster connects to *Understand and apply properties of operations and the relationship between addition and subtraction* and *Understand place value* in Grade 1, and to *Add and subtract within 20*, *Use place value understanding and properties of operations to add and subtract* and *Relate addition and subtraction to length* in Grade 2.

Explanation and Examples:
This standard calls for students to use concrete models, drawings and place value strategies to add and subtract within 100. (Students not expected to learn or use the traditional standard algorithm in first grade).

Students extend their number fact and place value strategies to add within 100. They represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols. It is important for students to understand if they are adding a number that has 10s to a number with 10s, they will have more tens than they started with; the same applies to the ones.

Application of their place value skills is expected so students can decompose numbers in order to add or subtract. For example, $17 + 12$ can be thought of 1 ten and 7 ones plus 1 ten and 2 ones. Layered place value cards may help students decompose the numbers into 10s and 1s (these are described in the NBT progression on p. 10.

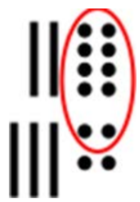
Students should be exposed to problems in context and out of context (word problems and equations) and presented in horizontal and vertical forms. As students are solving problems, it is important that they use language associated with proper place value (see example). They should always explain and justify their mathematical thinking both verbally and in a written format.

Estimating the solution prior to finding the answer focuses students on the meaning of the operation and helps students to attend to the actual quantities (this does not mean that students solve the problem and then round for the estimation – encourage them to mentally determine what will be close to the exact answer before they compute). This standard focuses on developing addition - the intent is not to introduce traditional algorithms or rules.

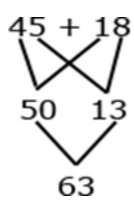
Examples:

- $43 + 36$
  Student counts the 10s (10, 20, 30…70 or 1, 2, 3…7 tens) and then the 1s.

- 28
  <u>+34</u>
  Student thinks: 2 tens plus 3 tens is 5 tens or 50. S/he counts the ones and notices there is another 10 plus 2 more. 50 and 10 is 60 plus 2 more or 62.



- $45 + 18$
  Student thinks: Four 10s and one 10 are 5 tens or 50. Then 5 and 8 is $5 + 5 + 3 (or\ 8 + 2 + 3)$ or 13. 50 and 13 is 6 tens plus 3 more or 63.



- 29
  <u>+14</u>
  Student thinks: "29 is almost 30. I added one to 29 to get to 30. 30 and 14 is 46. Since I added one to 29, I have to subtract one so the answer is 43."

- There are 37 children on the playground. 20more children show up. How many children are now on the playground? Student uses mental math. I started at 37 and counted on 3 to get to 40. Then, I added 20 which is 2 tens, to land on 60. So, there are 60 people on the playground.

- Same problem from above. I used a number path. I started on 37. Then I broke up 23 into 20 and 3 in my head. Next, I added 3 ones to get to 40. I then counted 10 to get to 50 and 10 more to get to 60. So, there are 60 children on the playground.

Instructional Strategies: **1.NBT.4-6**
It is important to provide multiple and varied experiences that will help students develop a strong sense of numbers based on **comprehension – not rules and procedures**.

Number sense is a blend of comprehension of numbers and operations along with fluency of numbers and operations. Students gain computational fluency (using efficient and accurate methods for computing) when they are flexible and have various strategies to choose from so they can select the most efficient one.

Students should solve problems using **concrete models** and **drawings** to support and record their solutions. It is important for them to share the reasoning that supports their solution strategies with their classmates. Students will usually move to using base-ten concepts, properties of operations, and the relationship between addition and subtraction to invent mental and written strategies for addition and subtraction. Help students share, explore, and record their invented strategies. Conduct discussions about how the various methods are similar and how they are using place value to solve the problems.

Recording the expressions and equations in the strategies horizontally encourages students to think about the numbers and the quantities they represent instead of instantly going to thinking about the traditional algorithm without meaning or understanding. Encourage students to try the mental and written strategies created by their classmates. Students eventually need to choose efficient strategies to use to find accurate solutions.

Students should use and connect different representations when they solve a problem. They should start by building a concrete model to represent a problem. This will help them form a mental picture of the model. Now students move to using pictures and drawings to represent and solve the problem. If students skip the first step, building the concrete model, they might use finger counting to solve the problem. Finger counting is an inefficient strategy for adding within 100 and subtracting within multiples of 10 between10 and 90.

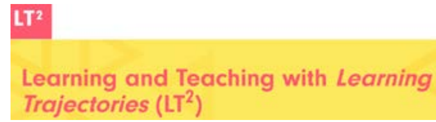
In a classroom environment where children know that they need to make sense of and explain their solution methods, children can invent methods for adding two-digit numbers with regrouping without having to have instruction about particular methods. Math drawings of tens and ones can serve as thinking tools in this process. It is vital that children do not use math drawings or manipulatives (such as base-ten blocks or cubes organized into tens and ones) in a rote manner just to get an answer. The explicit goal needs to be to develop a written method using numbers and to show the steps with numbers, as well as with the quantities, in the math drawings. The quantities of tens and ones need to be related to the written numerals in a solution method. Questions that can guide this process for children are these:

- *Will you get another ten or not?*
- *Where will you write this ten in your number problem?*

Tools/Resources :
For detailed information, see Learning Progression Numbers and Operations in Base Ten:

Visit Illustrative Mathematics to view great tasks for this standard.

- Ford and Logan Add 45 + 36



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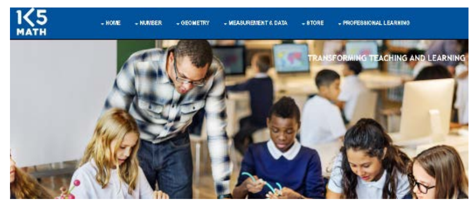


Visit K-5 Math Teaching Resources click on **Number**, then on **1st Grade**, then scroll down to 1.NBT.4 to access resources specifically for this standard.

Common Misconceptions:
Students have alternate concepts of multi-digit numbers and see them as numbers independent of place value. Example: When counting or adding numbers, student read the number 32 as “thirty-two” and count out 32 objects to demonstrate the value of the number, but when asked to write the number in expanded form, they write “3+2”. Or when asked the value of the digits in the number they respond that the values are “3” and “2”.

Domain: Number and Operations in Base Ten (NBT)

▶ **Cluster:** Use place value understanding and properties of operations to add and subtract.

Standard C: 1.NBT.5

Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. **(1.NBT.5)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 1.NBT.4

Explanation and Examples:

This standard builds on students' work with tens and ones and requires them to understand and apply the concept of 10 by mentally adding ten more or ten less than any number less than 100. This understanding leads to future place value concepts. It is critical for students to do this without counting or having to write down anything.

Prior use of models such as number paths, number lines, and 100 charts helps facilitate understanding. Ample experiences with ten frames will also help students see the pattern involved when adding or subtracting 10 and **use** these patterns to solve such problems.

Example:

There are 74 birds in the park. 10 birds fly away. How many are left?

Student 1: I used a 100s board. I started at 74. Then, because 10 birds flew away. I moved back one row. I landed on 64. So, there are 64 birds left in the park.

Student 2: I pictured 7 ten frames and 4 left over in my head. Since 10 birds flew away. I took one of the ten frames away. That left 6 ten frames and 4 left over. So, there are 64 birds left in the park.

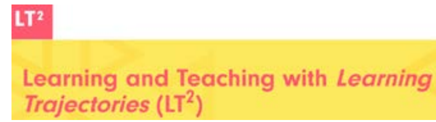
Instructional Strategies:

Counting Around the Circle is a great routine to help students practice adding or subtracting ten mentally. Everyone stands in a circle and the first person is given a number to start with (such as 43). Now each student in the circle will say 10 more than the previous person. Students should be allowed some think time without prompting. If this routine is too difficult at first for your students, you may decide to do Choral Counting by 10s starting at different numbers first and then move to Counting Around the Circle. Starting at a larger number and then subtracting 10 should also be introduced to your students. Variations are explain in Number Sense Routines by Jessica Shumway.

Tools/Resources:

Illustrative Mathematics tasks:

- Number Square



Dr. Douglas Clements and Dr. Julia Sarama are well respected early numeracy educators and researchers. Their book, Learning and Teaching Early Math: The Learning Trajectories Approach, provides a fabulous learning trajectory for *Place Value* from the age of 1 to 7. They also have provided educators access to their online trajectories. Make sure you register to gain access to their developmental progressions and the activities and lessons to go with each stage of learning.

Visit K-5 Math Teaching Resources click on Number, then on 1st Grade, then scroll down to 1.NBT.5 to access resources specifically for this standard.

Common Misconceptions:

Students lack the concept that 10 in any position (place) makes one (group) of the next place value.

Example:

If students are asked to add a collection of 12 hundreds , 2 tens and 13 ones, students write 12213, possibly squeezing the 2 and the 13 together or separating the three numbers with some space.

Domain: Number and Operations in Base Ten (NBT)

▶ **Cluster C:** Use place value understanding and properties of operations to add and subtract.

Standard: 1.NBT.6

Subtract multiples of 10 in the range 10 to 90 from multiples of 10 in the range 10 to 90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **(1.NBT.6)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 1.NBT.4

Explanation and Examples:

This standard calls for students to use concrete models, drawings and place value strategies to subtract multiples of 10 from decade numbers (e.g., 30, 40, 50).

This standard is foundation for future work in subtraction with greater numbers. Students should have multiple experiences representing numbers that are multiples of 10 (e.g. 90) with models or drawings. Then they subtract multiples of 10 (e.g. 20) using these representations or strategies based on place value. These opportunities develop fluency of addition and subtraction facts and reinforce counting up and back by 10s.

Examples:

- 70 - 30: Seven 10s take away three 10s is four 10s
- 80 - 50: 80, 70 (one 10), 60 (two 10s), 50 (three 10s), 40 (four 10s), 30 (five 10s)
- 60 - 40: I know that 4 + 2 is 6 so four 10s + two 10s is six 10s so 60 - 40 is 20

Student Thinking Examples:
There are 60 students in the gym. 30 students leave. How many students are still in the gym?

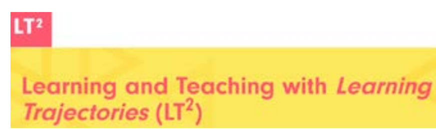
Student 1	Student 2	Student 3	Student 4
I used a hundreds chart and started at 60. I moved up 3 rows to land on 30. There are 30 students left.	I used connecting cubes (or place value blocks if understanding is solid) to build towers of 10. I started with 6 towers of 10 and removed 3. I had 3 towers left. 3 towers have a value of 30. There are 30 students left.	I know that 30 plus 30 is 60, so 60 minus 30 equals 30. There are 30 students left. (Student mentally applies knowledge of addition to solve this subtraction problem.)	I used a number path (or a number line). I started at 60 and moved back 3 tens and landed on 30. There are 30 students left.

Students may use interactive versions of models (hundred charts, number paths, number lines, base ten blocks, etc.) to demonstrate and justify their thinking. Some of these can be found on the National Library of Virtual Manipulatives website.

Instructional Strategies: See 1.NBT.4

Tools/Resources:
For detailed information, see Learning Progression Numbers and Operations in Base Ten:

Visit Illustrative Mathematics to view great tasks.



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Visit K-5 Math Teaching Resources click on Number, then on 1st Grade, then scroll down to 1.NBT.6 to access resources specifically for this standard.

Common Misconceptions: See 1.NBT.5

Domain: Measurement and Data (MD)

▶ **Cluster A:** Measure lengths indirectly and by iterating length units.

Standard: 1.MD.1

Order three objects by length; compare the lengths of two objects indirectly by using a third object. **(1.MD.1)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

- This cluster connects to *Describe and compare measurable attributes* in Kindergarten, and to *Measure and estimate lengths in standard unit*s and *Represent and interpret data* in Grade 2.

Explanation and Examples:

This standard calls for students to indirectly measure objects by comparing the length of two objects by using a third object (this third object is being used as a measuring tool). This concept is referred to as *transitivity*. This is connected to *conservation*. Students have to be able to conserve quantity and length in order to understand that the amount doesn't change just because I move the third object from one place to another in order to measure two objects.

In order for students to be able to compare objects, students need to understand that length is measured from one end point to another end point. They determine which of two objects is longer, by physically aligning the objects. Typical language of length includes taller, shorter, longer, and higher. When students use *bigger* or *smaller* as a comparison, they should explain what they mean by the word. Some objects may have more than one measurement that uses length, so students need to identify the length they are measuring. Both the height and the width of an object are measurements of length.

Examples for ordering:

- Order three students by their height
- Order pencils, crayons, and/or markers by length
- Build three towers (with cubes) and order them from shortest to tallest
- Three students each draw one line, then order the lines from longest to shortest

Example for comparing indirectly:

Two students each make their own dough "snake." Given one tower of cubes, each student compares his/her snake to the tower. Then students should then make comparison statements such as, "My snake is longer than the cube tower and your snake is shorter than the cube tower. So, my snake is longer than your snake."

Instructional Strategies:

The measure of an attribute is a count of how many units are needed to fill, cover or match the attribute of the object being measured. Students need to understand what a unit of measure is and how it is used to find a measurement. They need to predict the measurement, find the measurement and then discuss the estimates, errors and the measuring process.

It is important for students to measure the same attribute of an object with different sized units recognizing that different units will result in different measures.

It is beneficial to use informal units for beginning measurement activities at all grade levels because they allow students to focus on the attribute being measured. Numbers for the measurements can be kept manageable by simply adjusting the size of the units.

Experiences with informal, or nonstandard, units promote the need for measuring with standard units.

Measurement units should share the attribute being measured. Students need to use as many copies of the length unit as necessary to match the length being measured. For instance, use large footprints with the same size as length units. Place the footprints end to end, without gaps or overlaps, to measure the length of a room to the nearest whole footprint. Use language that reflects the approximate nature of measurement, such as the length of the room is about 19 footprints.

Students need experiences of measuring the lengths of curves and other distances that are not straight lines. Discussions concerning how they measured and what they discovered is essential in building understanding.

Students should to make their own measuring tools before using premade measurement tools. For instance, they can place paper clips end to end along a piece of cardboard, make marks at the endpoints of the clips and color in the spaces. Students can now see that the **spaces** represent the unit of measure, **not the marks** or numbers on a ruler.

Eventually they write numbers in the center of the spaces. Encourage students not to use the end of the ruler as a starting point. Compare and discuss two measurements of the same distance, one found by using a ruler and one found by aligning the actual units end to end, as in a chain of paper clips.

Students should measure lengths that are longer than their measuring tools. This will lead to some great problem solving opportunities about how they can measure something that is longer than their tool.

It is important for students to measure the same attribute of an object with different sized units recognizing that different units will result in different measures.

Use of the Transitive Property

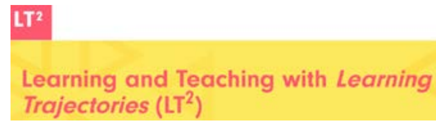
Have students use reasoning to compare measurements indirectly. For example; to order the lengths of Objects A, B and C, examine and then compare the lengths of Object A and Object B and the lengths of Object B and Object C. The results of these two comparisons allow students to use reasoning to determine how the length of Object A compares to the length of Object C.

For example, to order three objects by their lengths, reason that if Object A is shorter than Object B and Object B is shorter than Object C, then Object A has to be shorter than Object C. The order of objects by their length from smallest to largest would be Object A - Object B - Object C.

Resources/Tools

For detailed information, see Measurement Learning Progression (also called Geometric Measurement).

Visit Illustrative Mathematics to view great tasks.



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Visit K-5 Math Teaching Resources click on Measurement and Data, then on 1st Grade, then scroll down to 1.MD.1 to access resources specifically for this standard.

Georgia Department of Education

- "Measurement Matters," – Students make direct comparisons of two or more lengths, use non-standard units of length, and order lengths of objects in the classroom.

Common Misconceptions:

Some students may view the measurement process as a procedural counting task. They might count the markings on a ruler rather than the spaces between (the units of measure). Students need numerous experiences measuring lengths with student-made tapes or rulers with numbers in the center of the spaces before using premade measuring tools.

Domain: Measurement and Data (MD)

▶ **Cluster A:** Measure lengths indirectly and by iterating length units.

Standard: 1.MD.2

Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* **(1.MD.2)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 2.MD.1

Explanation and Examples:

Ask students to use multiple copies of one object to measure a larger object. This concept is referred to as *iteration*. Through numerous experiences and careful questioning by the teacher, students will recognize the importance of making sure that there are not any gaps or overlaps in order to get an accurate measurement. This concept is a foundational building block for the concept of area in 3rd Grade.

Example: How long is the paper in terms of paper clips?

Instructional Strategies:
Students use their counting skills while measuring with non-standard units. While this standard limits measurement to whole numbers of length, in a natural environment, not all objects will measure to an exact whole unit. When students determine that the length of a pencil is six to seven paperclips long, they can state that it is about six paperclips long.

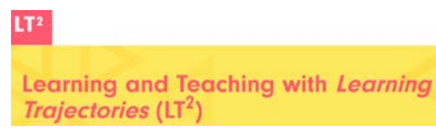
Example:

- Ask students to use multiple units of the same object to measure the length of a pencil. (How many paper clips will it take to measure how long the pencil is?)

Tools/Resources:
For detailed information, see Measurement Learning Progression (also called Geometric Measurement).

Visit Illustrative Mathematics to view great tasks.

- 1.MD How Long?
- 1.MD Measure Me!
- 1.MD Growing Bean Plants
- 1.OA Measuring Blocks



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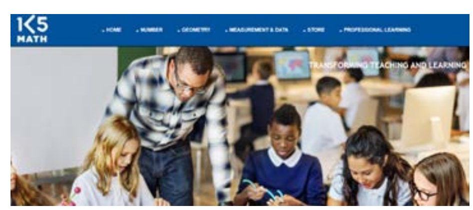


Visit K-5 Math Teaching Resources click on Measurement and Data, then on 1st Grade, then scroll down to 1.MD.2 to access resources specifically for this standard.

Common Misconceptions: See 1.MD.1

Domain: Measurement and Data (MD)

● **Cluster B:** Tell and write time.

Standard: 1.MD.3

Tell and write time in hours and half-hours using analog and digital clocks. **(1.MD.3)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

- This Cluster connects to *Work with time and money* in Grade 2.

Explanation and Examples:

This standard asks students to read both analog and digital clocks and then orally tell and write the time. Times should be limited to the hour and the half-hour.

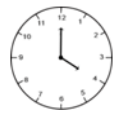
Students need experiences exploring the idea that when the time is at the half-hour the hour hand is between numbers and not on a number. Further, the hour is the number before where the hour hand is. For example, in the clock below, the time is 8:30. The hour hand is between the 8 and 9, but the hour is 8 since it is not yet on the 9.

See Instructional Strategies below for information about using a one-handed clock to differentiate the hour hand from the minute hand.

Ideas to support telling time:

- within a day, the hour hand goes around a clock twice (the hand moves only in one direction)
- when the hour hand points exactly to a number, the time is exactly on the hour
- time on the hour is written in the same manner as it appears on a digital clock
- the hour hand moves as time passes, so when it is half way between two numbers it is at the half hour
- there are 60 minutes in one hour; so halfway between an hour, 30 minutes have passed
- half hour is written with "30" after the colon

"It is 4 o'clock"



"It is halfway between 8 o'clock and 9 o'clock. It is 8:30."

The idea of 30 being "halfway" is difficult for students to grasp. Students can write the numbers from 0 - 60 counting by tens on a sentence strip. Fold the paper in half and determine that halfway between 0 and 60 is 30. A number path on an interactive whiteboard may also be used to demonstrate this.

Instructional Strategies:

Students are likely to experience some difficulties learning about time. On an analog clock, the short hand indicates approximate time to the nearest hour and the focus is on where it is pointing. The long (or big) hand shows minutes before and after an hour and the focus is on distance that it has gone around the clock or the distance yet to go for the hand to get back to the top.

One method of instruction comes from Dr. Van de Walle's book (Teaching Student-Centered Mathematics: PreK-2) and is called the **One-handed Clock**. The hour hand gives the most information about the time. To give students a better understanding of this you will need to buy two inexpensive clocks. Place both clocks in an area so all students can see them but are easy for you to access. Make sure both clocks are set to the same correct time and then remove the minute hand from one of the clocks. The clock with both hands should then be covered so that students will see just the one-handed clock. At various hours and half-hours, draw your students' attention to the one-handed clock and ask them to tell you the time. Then remove the cover from the two-handed clock to verify the time. Students will begin to see that the hour hand will be between two numbers when it is half-past the hour. Focusing on the hour hand first will provide them a firm foundation for more complex time reading in future grades.

Resources/Tools

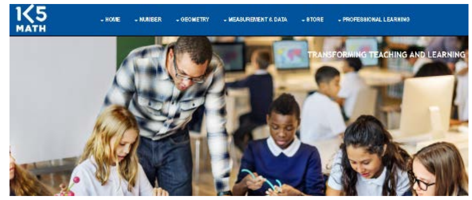
Visit Illustrative Mathematics to view great tasks.

- Making a clock

Georgia Department of Education

- "It's Time" – Students explore when it is important to know time and view a video. Using the interactive white board, students show times to the hour on a clock and model time on digital and analog clocks.

Visit K-5 Math Teaching Resources click on **Measurement and Data**, then on **1st Grade**, then scroll down to 1.MD.3 to access resources specifically for this standard.



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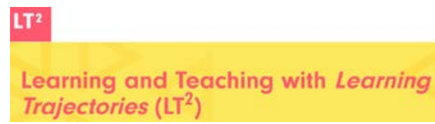


Common Misconceptions:

The use of "paper plate clocks" is discouraged for more than a "show me" tool. These do not keep the relationship between the hour hand and the minute hand. Essentially analog clocks operate on a ratio that students will learn about in later grades. Student clocks with gears work the best in keeping the hands moving correctly. Broken clocks that no longer run can provide models in the absence of manipulative clocks that are geared.

Domain: Measurement and Data (MD)

◆ **Cluster C:** Represent and interpret data.

Standard: 1.MD.4

Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. **(1.MD.4)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.

Connections:

- This cluster connects to *Classify objects and counts the number of objects in each category* in Kindergarten, and to *Represent and interpret data* in Grade 2.

Explanation and Examples:

This standard calls for students to create graphs and tally charts using data relevant to their lives (e.g. **categorical data**--favorite ice cream, eye color, pets, etc). Graphs may be constructed by groups of students as well as by individual students. Students work with the data by organizing, representing and interpreting data. They should have experiences posing a question with **three** possible responses (the three categories as indicated in the standard) and then work with the data that they collect.

Counting objects should be reinforced when collecting, representing, and interpreting data. Students describe the object graphs and tally charts they create. They should also ask and answer questions based on these charts or graphs that reinforce other mathematics concepts such as sorting and comparing. The data chosen or questions asked give students opportunities to reinforce their understanding of place value, identifying ten more and ten less, relating counting to addition and subtraction and using comparative language and symbols.

Example:
Students pose a question and the three possible responses.

- Which is your favorite flavor of ice cream? Chocolate, vanilla or strawberry?

Students collect their data by using tallies or another way of keeping track.
Students organize their data by totaling each category in a chart or table.

What is your favorite flavor of ice cream?	
Chocolate	12
Vanilla	5
Strawberry	6

Students interpret the data by comparing categories.

Examples of comparisons:
What does the data tell us? Does it answer our question?

- More people like chocolate than the other two flavors.
- Only 5 people liked vanilla.
- Six people liked Strawberry.
- 7 more people liked Chocolate than Vanilla.
- The number of people that liked Vanilla was 1 less than the number of people who liked Strawberry.
- The number of people who liked either Vanilla or Strawberry was 1 less than the number of people who liked chocolate.
- 23 people answered this question.

Instructional Strategies:

Ask students to sort a collection of items (up to three categories). Then ask questions about the number of items in each category and the total number of items. Students should also compare the number of items in each category, and if they have been introduced to the relational symbols, they should use those symbols when appropriate. The total number of items to be sorted should be less than or equal to 100 to allow for sums and differences less than or equal to 100.

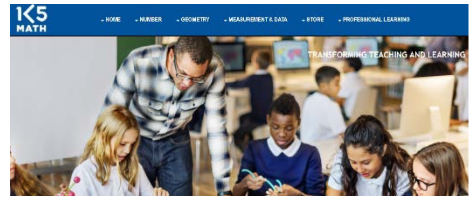
Connect to the geometry content being studied in Grade 1. Provide categories and have students sort identical collections of different geometric shapes. After the shapes have been sorted, ask these questions: How many triangles are in the collection? How many rectangles are there? How many triangles and rectangles are there? Which category has the most items? How many more? Which category has the least? How many less?

Students can create real or cluster graphs (Venn diagrams) after they have had multiple experiences with sorting objects according to given categories. The teacher should model a cluster graph several times before students make their own. A cluster graph in Grade 1 has two or three labeled loops or regions (categories). Students place items inside the regions that represent a category that they chose. Items that do not fit in a category are placed outside of the loops or regions. Students can place items in a region that overlaps the categories if they see a connection between categories. Ask questions that compare the number of items in each category and the total number of items inside and outside of the regions.

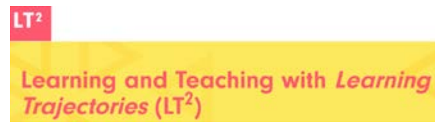
Resources/Tools

Visit Illustrative Mathematics to view great tasks.

- 1, 2.MD Favorite Ice Cream Flavor
- 1.MD Weather Graph Data



Visit K-5 Math Teaching Resources click on Measurement and Data, then on 1st Grade, then scroll down to 1.MD.3 to access resources specifically for this standard.



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- "Jelly Bean Graph" – Students make several grabs of jelly beans and record the information on a graphic organizer and decide if the number is more than, less than, or equal to ten and record it on their tally sheets. They discuss how to create a graph to show results and write questions to help others interpret their graphs.

Common Misconceptions:

Students may think that graphs are "pictures" of situations, rather than abstract representations.

Domain: Geometry (G)

● **Cluster A:** Reason with shapes and their attributes.

Standard: 1.G.1

Distinguish between defining attributes *(e.g. triangles are closed and three-sided)* versus non-defining attributes *(e.g. color, orientation, overall size);* build and draw shapes that possess defining attributes. **(1.G.1)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections:

- This cluster is connected to both clusters in the Geometry Domain in Kindergarten and *to Reason with shapes and their attributes* in Grade 2.

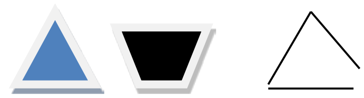
Explanation and Examples:

Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

This standard calls for students to determine which attributes of shapes (includes triangles, squares, rectangles, and trapezoids) are **defining** compared to those that are **non-defining**. Defining attributes are attributes that must always be present in order to create that shape. Non-defining attributes are attributes that do not always have to be present in order to create the shape. Defining attributes are attributes that help to define a particular shape (#angles, # sides, length of sides, etc.). Non-defining attributes are attributes that do not define a particular shape (color, position, location, etc.).

Example:
All triangles must be closed figures and have 3 sides. These are **defining** attributes. They create the shape of a triangle. Triangles can be different colors, sizes and be turned in different directions, but these are **non-defining**.

Student 1
Which figure is a triangle?
How do you know that it is a triangle?
"It has 3 sides. It's also closed."



Attributes refer to any characteristic of a shape. Students use attribute language to describe a given two-dimensional shape: number of sides, number of vertices/points, straight sides, closed. A child might describe a triangle as "right side up" or "red." These attributes are not defining because they are not relevant to whether a shape is a triangle or not. Students should articulate ideas such as, "A triangle is a triangle because it has three straight sides and is closed."

It is important that students are exposed to both **regular** and **irregular** shapes so that they can communicate defining attributes. Students should attend to precision and use attribute language to describe why these shapes are not triangles.

Students should also use appropriate language to describe a given three-dimensional shape: number of faces, number of vertices/points, and number of edges.

Example: A cylinder may be described as a solid that has two circular faces connected by a curved surface (which is not considered a face). Students may say, "It looks like a can."

Students should compare and contrast two-and three-dimensional figures using **defining** attributes.

Examples:
- List two things that are the same and two things that are different between a triangle and a cube.
- Given a circle and a sphere, students identify the sphere as being three-dimensional but both are round.
- Given a trapezoid, find another two-dimensional shape that has two things that are the same.

Instructional Strategies: (1.G.1-3)
Students can easily form 2-dimensional shapes on geoboards using colored rubber bands to represent the sides of a shape. Ask students to create a shape with four sides on their geoboard, and then copy the shape on dot paper (if you do not have dot paper, it can be created using *Dynamic Paper* on the Illuminations website). Students can share and describe their shapes as a class while the teacher records the different defining attributes discussed by the students.

Pattern block pieces can be used to model defining attributes for shapes. Ask students to create their own rule for sorting pattern blocks. Students take turns sharing their sorting rules with their classmates and showing examples that support their rule. The classmates then draw a new shape that fits this same rule after it is shared.

Students can use a variety of manipulatives and real-world objects to build larger shapes. The manipulatives can include paper shapes, pattern blocks, color tiles, triangles cut from squares (isosceles right triangles), tangrams, canned food (right circular cylinders) and gift boxes (cubes or right rectangular prisms).

Folding shapes made from paper enables students to physically feel the shape and form the equal shares. Ask students to fold circles and rectangles first into halves and then into fourths. They should observe and then discuss the change in the size of the parts.

Students may use interactive whiteboards or computer environments to move shapes into different orientations and to enlarge or decrease the size of a shape still keeping the same shape. They can also move a point/vertex of a triangle and identify that the new shape is still a triangle. When they move one point/vertex of a rectangle they should recognize that the resulting shape is no longer a rectangle. Access the Illuminations website to use interactives (listed below).

Resources/Tools

Visit Illustrative Mathematics to view great tasks.

- 1.G 3-D Shape Sort
- 1.G All vs. Only some

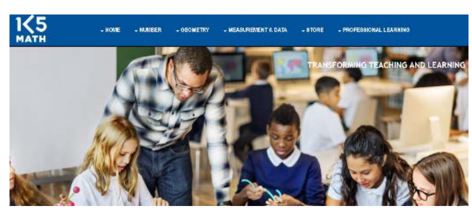
Georgia Department of Education:

- ”Which One Doesn’t Belong” – Students become familiar with the different ways to classify shapes by attributes (size, shape, color, number of sides)and self-select multiple shapes and classify them by a common attribute, and determine “which one doesn’t belong”.

NCTM Illuminations – NCTM has many great resources available to educators, some of these resources (i.e. interactives) are open to any educator while others (i.e. lessons) require an individual or institutional membership. If you find that a resource referenced in the flip books requires membership access, check with your school/district to see if they have an institutional membership which would grant you access all NCTM documents. If they do not have a membership, this would be a valuable resource to request.

- Geometric Solids
- Shape Tool
- Patch Tool
- Making Triangles
- Creating Polygons

Visit K-5 Math Teaching Resources click on Geometry, then on 1st Grade, then scroll down to 1.G.1 to access resources specifically for this standard.



Dr. Douglas Clements and Dr. Julia Sarama are well respected early numeracy educators and researchers. Their book, Learning and Teaching Early Math: The Learning Trajectories Approach, provides a fabulous learning trajectory for *Geometry* from the age of 1 to 7. They also have provided educators access to their online trajectories. Make sure you register to gain access to their developmental progressions and the activities and lessons to go with each stage of learning.

Common Misconceptions:

Students may think that a square that has been rotated 45-degrees is no longer a square but a rhombus or diamond. They need to have experiences with shapes in different orientations. To illustrate this, hold up a crayon or marker. Ask the students what it is. Then turn it and ask what it is. Then move over and turn it again and ask what it is. Tell them this is the same for shapes. Just because I have moved the shape doesn't mean the name has changed.

Domain: Geometry (G)

● **Cluster A:** Reason with shapes and their attributes.

Standard: 1.G.2

Compose two-dimensional shapes (rectangles, squares, **trapezoids**, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. Students do not need to learn formal names such as “right rectangular prism.” **(1.G.2)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See 1.G.1

Explanation and Examples:

In this standard students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build an understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different to develop the background for measurement and for initial understandings of properties such as congruence and symmetry. This standard includes shape puzzles in which students use objects (e.g., pattern blocks) to fill a larger region.

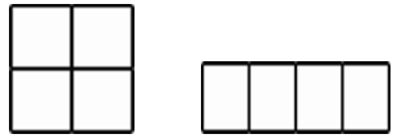
The ability to describe, use, and visualize the effect of composing and decomposing shapes is an important mathematical skill. It is relevant not only to geometry, but to children’s ability to compose and decompose numbers.

Examples:

- Show the different shapes that you can make by joining a triangle with a square.
- Show the different shapes you can make joining a trapezoid with a half-circle.
- Show the different shapes you can make with a cube and a rectangular prism.

Students may use pattern blocks, plastic shapes, tangrams, or computer environments to make new shapes. The teacher can provide students with cutouts of shapes and ask them to combine them to make a particular shape.

Example:



- What shapes can be made from four squares?

Students can make three-dimensional shapes with clay or dough, slice into two pieces (not necessarily congruent) and describe the two resulting shapes. For example, slicing a cylinder will result in two smaller cylinders.

Instructional Strategies: See 1.G.1

Resources/Tools

Visit Illustrative Mathematics to view great tasks.

- 1.G Overlapping Rectangles
- 1.G Counting Squares
- 1.G Make Your Own Puzzle
- 1.G Grandfather Tang's Story

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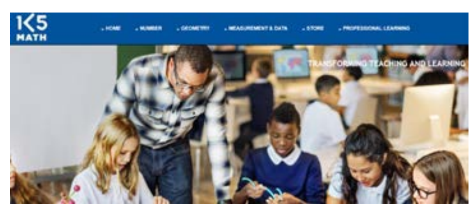
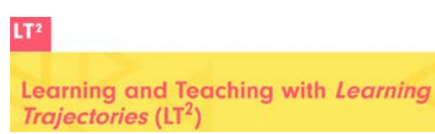


Visit K-5 Math Teaching Resources click on Geometry, then on 1st Grade, then scroll down to 1.G.2 to access resources specifically for this standard.



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Common Misconceptions: See 1.G.1

Domain: Geometry (G)

● **Cluster A:** Reason with shapes and their attributes.

Standard: 1.G.3

Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Note: fraction notation ($\frac{1}{2}, \frac{1}{4}$) is not expected at this grade level. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. **(1.G.3)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See 1.G.1

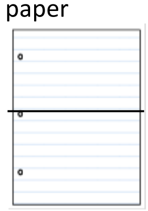
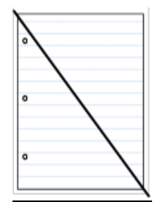
Explanation and Examples:

Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

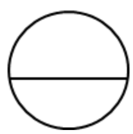
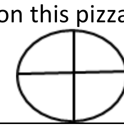
This is the first time students begin partitioning regions into equal shares using a context such as cookies, pies, pizza, etc. This is a foundational building block of fractions, which will be extended in future grades. Students should have ample experiences using the words, *halves, fourths,* and *quarters*, and the phrases *half of, fourth of,* and *quarter of*. Students should also work with the idea of the whole, which is composed of two halves, or four fourths or four quarters.

Example:

How can you and a friend share equally (partition) this piece of paper so that you both have the same amount of paper to paint a picture?

Student 1:	**Student 2**:
I would split the paper right down middle. That gives me 2 halves. I have half of the paper and my friend has the other half of the paper 	I would split it from corner to corner (diagonally). She gets half of the paper and I get half of the paper. See, if we cut here (along the line) the parts are the same size. 

Example:

Teacher:	**Teacher**:
There is pizza for dinner. What do you notice about the slice of the pizza? 	If we cut the same pizza into four slices (fourths) do you think the slices would be the same size, larger or smaller as the slices on this pizza? 

Student:	**Student**:
There are two slices on the pizza. Each slice is the same size. Those are big slices.	*When you cut the pizza into fourths. The slices are smaller than the other pizza. More slices mean that the slices get smaller. I want a slice from the first pizza.*

Students need many experiences with different sized circles and rectangles to recognize that when they cut something into two equal pieces, each piece will equal one half of its original whole. Children should recognize that halves of two different wholes are not necessarily the same size. Also they should reason that decomposing equal shares into more equal shares results in smaller equal shares.

More Examples:

- Student partitions a rectangular candy bar to share equally with one friend and thinks "I cut the rectangle into two equal parts. When I put the two parts back together, they equal the whole candy bar. One half of the candy bar is smaller than the whole candy bar."

- Student partitions an identical rectangular candy bar to share equally with 3 friends and thinks "I cut the rectangle into four equal parts. Each piece is one fourth of or one quarter of the whole candy bar. When I put the four parts back together, they equal the whole candy bar. I can compare the pieces (one half and one fourth) by placing them side-by-side. One fourth of the candy bar is smaller than one half of the candy bar.

- Students partition a pizza to share equally with three friends. They recognize that they now have four equal pieces and each will receive a fourth or quarter of the whole pizza.

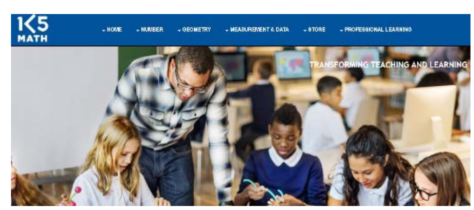
Instructional Strategies: See 1.G.1

Resources/Tools

Visit Illustrative Mathematics to view great tasks.

- Equal Shares

Visit K-5 Math Teaching Resources click on Geometry, then on 1st Grade, then scroll down to 1.G.3 to access resources specifically for this standard.



Common Misconceptions:

Some students may think that the size of the equal shares is directly related to the number of equal shares. For example, they think that fourths are larger than halves because there are four fourths in one whole and only two halves in one whole. Students need to focus on the change in the size of the fractional parts as recommended in the folding shapes strategy. (Focus on Concrete and Representational activities).

APPENDIX: TABLE 1. Common Addition and Subtraction Situations

Shading taken from OA progression

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Taken from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

Blue shading indicates the four Kindergarten problem subtypes. Students in grades 1 and 2 work with all subtypes and variants (blue and green). Yellow indicates problems that are the difficult four problem subtypes or variants that students in Grade 1 work with but do not need to master until Grade 2.

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

TABLE 2. Common Multiplication and Division Situations

Grade level identification of introduction of problem situations taken from OA progression

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	3 x 6 = ?	3 x ? = 18; 18 ÷ 3 = ?	? x 6 = 18; 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[4], Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	a × b = ?	a × ? = p, and p ÷ a = ?	? × b = p, and p ÷ b = ?

Multiplicative compare problems appear first in Grade 4 (green), with whole number values and with the "times as much" language from the table. In Grade 5, unit fractions language such as "one third as much" may be used. Multiplying and unit language change

TABLE 3. The Properties of Operations

the subject of the comparing sentence ("A red hat costs n times as much as the blue hat" results in the same comparison as "A blue hat is 1/n times as much as the red hat" but has a different subject.)

Name of Property	Representation of Property	Example of Property, Using Real Numbers
Properties of Addition		
Associative	$(a+b)+c=a+(b+c)$	$(78+25)+75=78+(25+75)$
Commutative	$a+b=b+a$	$2+98=98+2$
Additive Identity	$a+0=a$ *and* $0+a=a$	$9875+0=9875$
Additive Inverse	For every real number a, there is a real number $-a$ such that $a+-a=-a+a=0$	$-47+47=0$
Properties of Multiplication		
Associative	$(a\times b)\times c=a\times(b\times c)$	$(32\times 5)\times 2=32\times(5\times 2)$
Commutative	$a\times b=b\times a$	$10\times 38=38\times 10$
Multiplicative Identity	$a\times 1=a$ *and* $1\times a=a$	$387\times 1=387$
Multiplicative Inverse	*For every real number a,* $a\neq 0$, *there is a real number* $\frac{1}{a}$ *such that* $a\times\frac{1}{a}=\frac{1}{a}\times a=1$	$\frac{8}{3}\times\frac{3}{8}=1$
Distributive Property of Multiplication over Addition		
Distributive	$a\times(b+c)=a\times b+a\times c$	$7\times(50+2)=7\times 50+7\times 2$

(Variables a, b, and c represent real numbers.)
*Excerpt from NCTM's **Developing Essential Understanding of Algebraic Thinking**, grades 3-5 p. 16-17*

TABLE 4. The Properties of Equality

Name of Property	Representation of Property	Example of property
Reflexive Property of Equality	$a = a$	$3{,}245 = 3{,}245$
Symmetric Property of Equality	$If\ a = b, then\ b = a$	$2 + 98 = 90 + 10, then\ 90 + 10 = 2 + 98$
Transitive Property of Equality	$If\ a = b\ and\ b = c, then\ a = c$	$If\ 2 + 98 = 90 + 10\ and\ 90 + 10 = 52 + 48$ *then* $2 + 98 = 52 + 48$
Addition Property of Equality	$If\ a = b, then\ a + c = b + c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} + \frac{3}{5} = \frac{2}{4} + \frac{3}{5}$
Subtraction Property of Equality	$If\ a = b, then\ a - c = b - c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} - \frac{1}{5} = \frac{2}{4} - \frac{1}{5}$
Multiplication Property of Equality	$If\ a = b, then\ a \times c = b \times c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} \times \frac{1}{5} = \frac{2}{4} \times \frac{1}{5}$
Division Property of Equality	$If\ a = b\ and\ c \neq 0, then\ a \div c = b \div c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} \div \frac{1}{5} = \frac{2}{4} \div \frac{1}{5}$
Substitution Property of Equality	If $a = b$, then *b* may be substituted for *a* in any expression containing *a*.	$If\ 20 = 10 + 10$ *then* $90 + 20 = 90 + (10 + 10)$

(Variables a, b, and c can represent any number in the rational, real, or complex number systems.)

TABLE 5. The Properties of Inequality

Exactly one of the following is true: $a < b, a = b, a > b$.

$If\ a > b\ and\ b > c\ then\ a > c.$

$If\ a > b, then\ b < a.$

$If\ a > b, then - a < -b.$

$If\ a > b, then\ a \pm c > b \pm c.$

$If\ a > b\ and\ c > 0, then\ a \times c > b \times c.$

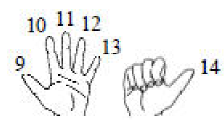
$If\ a > b\ and\ c < 0, then\ a \times c < b \times c.$

$If\ a > b\ and\ c > 0, then\ a \div c > b \div c.$

$If\ a > b\ and\ c < 0, then\ a \div c < b \div c.$

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

TABLE 6. Development of Counting in K-2 Children

Levels	8 + 6 = 14	14 – 8 = 6
Level 1: Count all	Count All a 1 2 3 4 5 6 7 8 b 1 2 3 4 5 6 ○○○○○ ○○○ ○○○○○ ○ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 c	Take Away a 1 2 3 4 5 6 7 8 9 10 11 12 13 14 ~~○○○○○ ○○○~~○○ ○○○○ 1 2 3 4 5 6 7 8 1 2 3 4 5 6 b c
Level 2: Count on	Count On 8 (○○○○○ ○○○) ○○○○○ ○ →8 9 10 11 12 13 14	To solve 14 – 8 I count on 8 + ? = 14 I took away 8  8 to 14 is 6 so 14 – 8 = 6
Level 3: Recompose Make a ten (general): one addend breaks apart to make 10 with the other addend Make a ten (from 5's within each addend)	Recompose: Make a Ten (○○○○○ ○○○ ○○)○○○ ○ 10 + 4 (○○○○○)(○○○) (○○○○○)(○) 10 + 4	14 – 8: I make a ten for 8 + ? = 14 ~~○○○○○ ○○○~~○○ ○○○○ 8 + 2 + 4 6 8 + 6 = 14
Doubles ± n	6 + 8 = 6 + 6 + 2 = 12 + 2 = 14	

Note: Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful with counting on; it makes subtraction as easy as addition.

Beginning--A child can count very small collections (1-4) collection of items and understands that the last word tells "how many" even. Beyond on small numbers the number words may be said without the lack of numerical understanding. This is often referred to as rote counting.

Level 1—The child uses one to one correspondence to find the number of objects in two sets. Even if the quantity is known for the first set, the child will start with the first set and continue counting on into the second set. The child begins the count with one. This also connects to Piaget's Hierarchical Inclusion – that in order to have 7 – you have to have 6, 5, 4, etc.

Level 2 – At this level the student begins the counting, starting with the known quantity of the first set and "counts on" from that point in the sequence to establish how many. This method is used to find the total in two sets. This counting is not rote. This level of counting requires several connections between cardinality and counting meanings of the number words.

Level 3 - At this level the student begins using known facts to solve for unknown facts. For example, the student uses "make a ten" where one addend breaks apart to make 10 with another addend OR a doubles plus/minus one strategy. Students begin to implicitly use the properties of operations.

Table 7: Cognitive Rigor Matrix/Depth of Knowledge (DOK)

The Kansas Math Standards require high-level cognitive demand asking students to demonstrate deeper conceptual understanding through the application of content knowledge and skills to new situations and sustained tasks. For each Assessment Target the depth(s) of knowledge (DOK) that the student needs to bring to the item/task will be identified, using the Cognitive Rigor Matrix shown below.

Depth of Thinking (Webb)+ Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	• Recall conversions, terms, facts			
Understand	• Evaluate an expression • Locate points on a grid or number on number line • Solve a one-step problem • Represent math relationships in words, pictures, or symbols	• Specify, explain relationships • Make basic inferences or logical predictions from data/observations • Use models/diagrams to explain concepts • Make and explain estimates	• Use concepts to solve non-routine problems • Use supporting evidence to justify conjectures, generalize, or connect ideas • Explain reasoning when more than one response is possible • Explain phenomena in terms of concepts	• Relate mathematical concepts to other content areas, other domains • Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	• Follow simple procedures • Calculate, measure, apply a rule (e.g., rounding) • Apply algorithm or formula • Solve linear equations • Make conversions	• Select a procedure and perform it • Solve routine problem applying multiple concepts or decision points • Retrieve information to solve a problem • Translate between representations	• Design investigation for a specific purpose or research question • Use reasoning, planning, and supporting evidence • Translate between problem & symbolic notation when not a direct translation	• Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	• Retrieve information from a table or graph to answer a question • Identify a pattern/trend	• Categorize data, figures • Organize, order data • Select appropriate graph and organize & display data • Interpret data from a simple graph • Extend a pattern	• Compare information within or across data sets or texts • Analyze and draw conclusions from data, citing evidence • Generalize a pattern • Interpret data from complex graph	• Analyze multiple sources of evidence or data sets
Evaluate			• Cite evidence and develop a logical argument • Compare/contrast solution methods • Verify reasonableness	• Apply understanding in a novel way, provide argument or justification for the new application
Create	• Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	• Generate conjectures or hypotheses based on observations or prior knowledge and experience	• Develop an alternative solution • Synthesize information within one data set	• Synthesize information across multiple sources or data sets • Design a model to inform and solve a practical or abstract situation

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