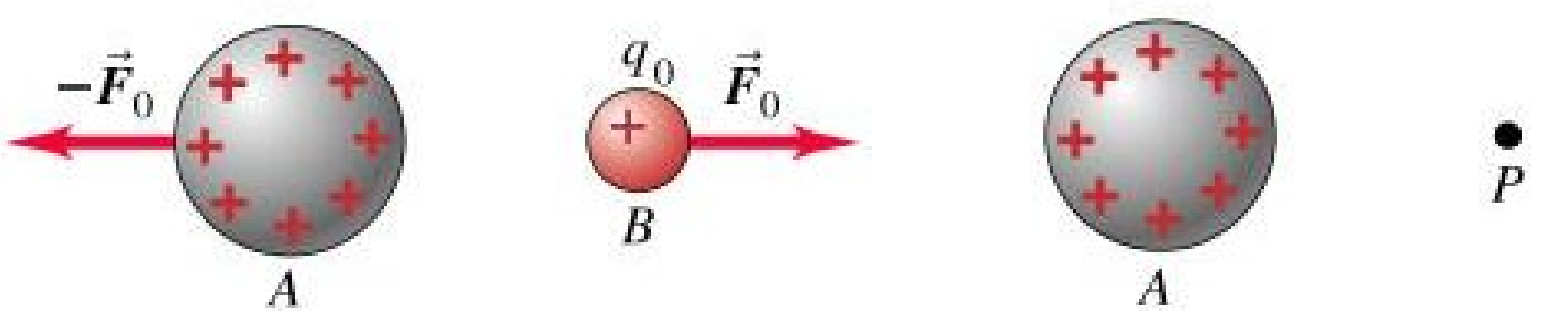
21.4 Electric Field and Electric Forces

How do charged particles interact in empty space?
How do they know the presence of each other?
What goes on in the space between them?



(a) How does charged body A exert a force on charged body B?

(b) Remove body B and label its former position as P

- **Body *A* produces an *electric field* at *P* as a consequence of the charge on body *A* only**

(c) Body A sets up an electric field $\vec{E}$ at point P:

- Place ***test charge*** ***q_0*** **at *P* – if *q_0* feels an electric force, then there is an electric field at that point**
- **The electric field is the intermediary through which *A* communicates its presence to *q_0***
- **The electric field that *A* produces exists at all points in the region around *A***

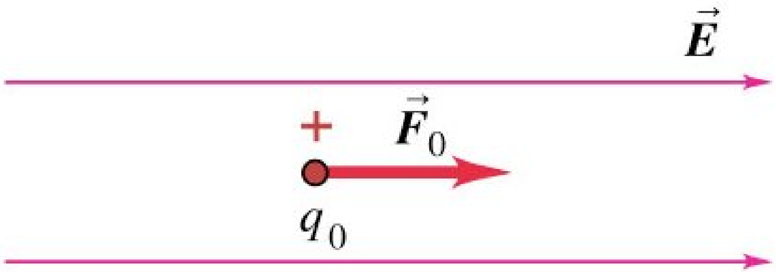
Definition: The Electric Field

The electric field at a point is the electric force experienced by a test charge q_0 at that point divided by the charge q_0.

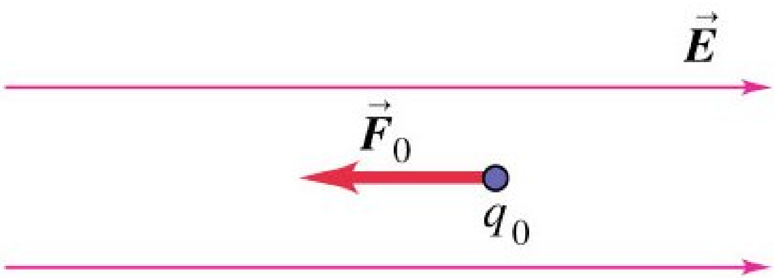
$$\vec{E} = \frac{\vec{F}_0}{q_0}$$

- **electric force per unit charge**
- **SI unit is N/C**
- **force exerted on q_0 by an electric field:**

$$\vec{F}_0 = q_0\vec{E}$$

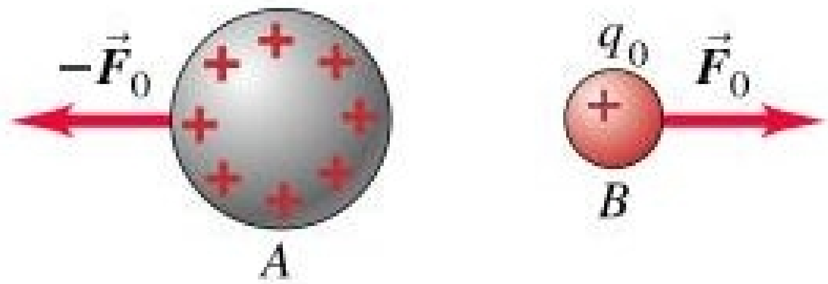


(a) Positive charge q_0 placed in an electric field: force on q_0 is in same direction as $\vec{E}$



(b) Negative charge q_0 placed in an electric field: force on q_0 is in opposite direction from $\vec{E}$

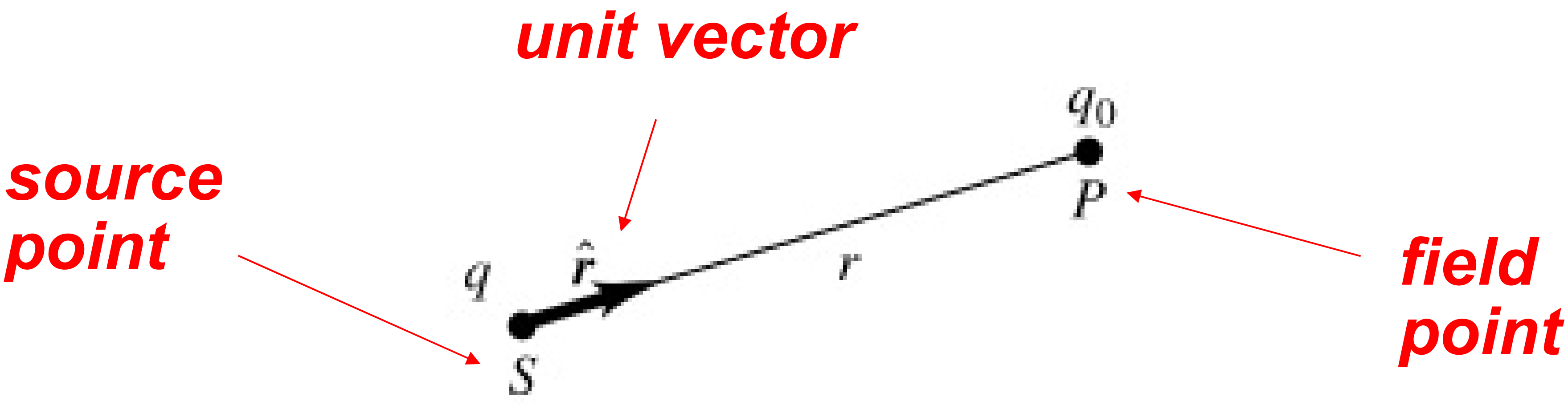
Note:



- force exerted by q_0 on *A* may cause the charge distribution on *A* to be shifted
- electric field around *A* may be different if q_0 is present
- if q_0 is very small, redistribution of charge on *A* is also very small
- therefore, correct definition of electric field is:

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}_0}{q_0}$$

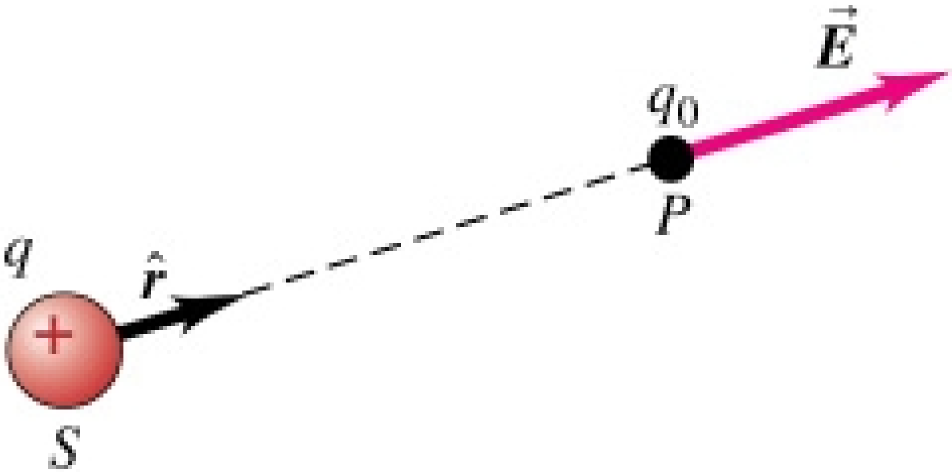
Some terminology:



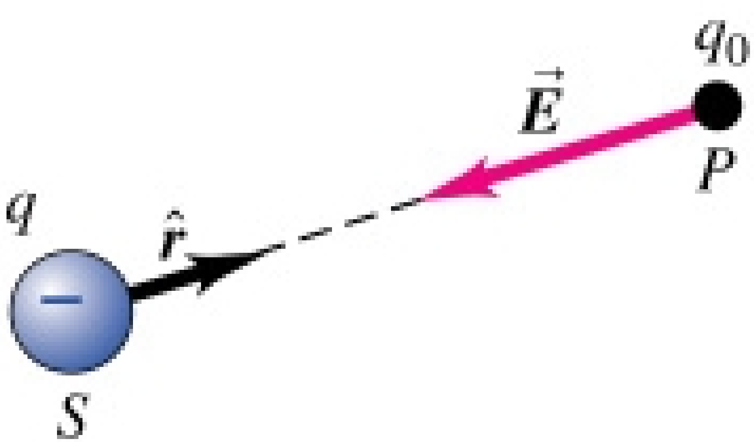
Reminder (from mechanics):

unit vector

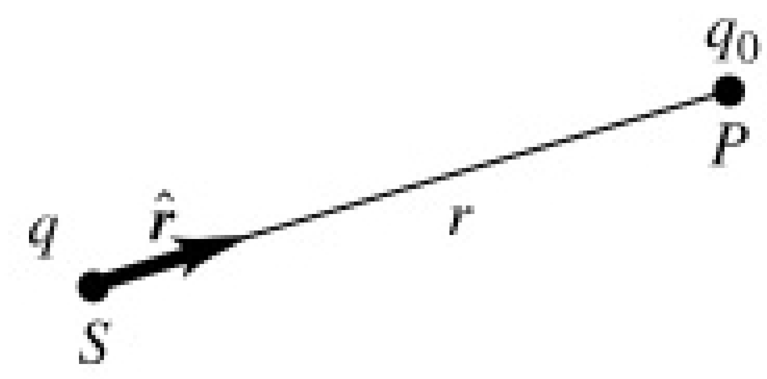
$$\hat{r} = \frac{\vec{r}}{r} \quad \textbf{where} \quad r = |\vec{r}|$$



field created by +ve charge ***q*** at ***P*** points away from ***q*** in the same direction as $\hat{r}$



field created by -ve charge ***q*** at ***P*** points toward ***q*** in the opposite direction from $\hat{r}$



magnitudes:

$$F_0 = \frac{1}{4\pi\epsilon_0}\frac{|qq_0|}{r^2}$$

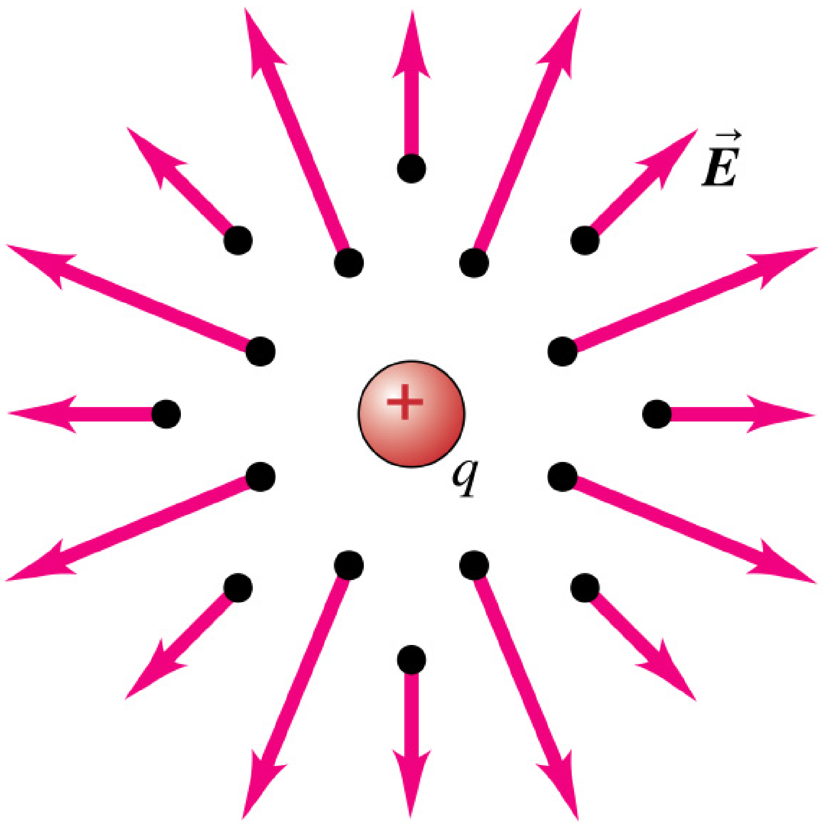
$$E = \frac{F_0}{q_0}$$

$$\Rightarrow \quad E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$$

The vector equation for the electric field of a point charge :

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}\ \hat{r}$$

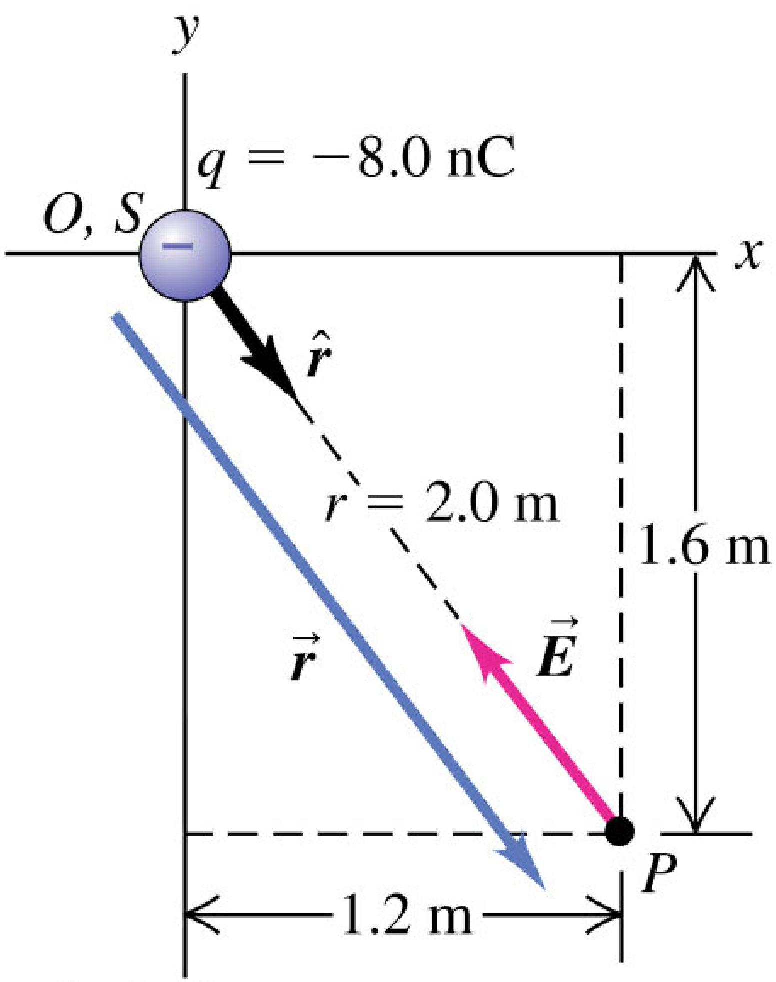
- E-field varies from point to point – is an infinite set of vector quantities →**vector field**
- field strength decreases with increasing distance



Field pattern for a positive charge *q*

Example: Electric-field vector for a point charge

A point charge q = -8.0 nC is located at the origin. Find the electric-field vector at the field point x = 1.2 m, y = -1.6 m.

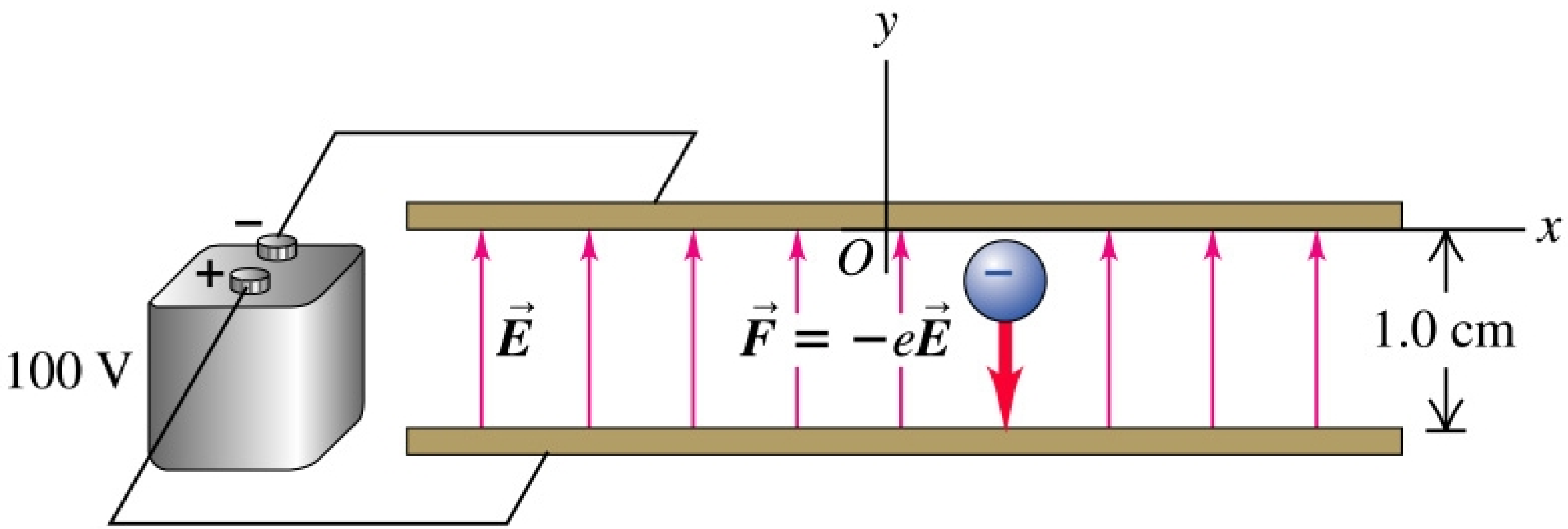


Example: Electron in a uniform *E*-field

Consider a uniform ***E***-field set up by the configuration below. The two horizontal parallel conducting plates are a distance 1.0 cm apart and are connected to a 100 V battery. The magnitude of the ***E***-field created is $E = 1.00 \times 10^4$ N/C and points vertically upwards. (neglect gravitational forces)

a) If an electron is released from rest at the upper plate, what is its acceleration?

(electron: charge $-e = -1.60 \times 10^{-19}$ C and mass $m = 9.11 \times 10^{-31}$ kg)

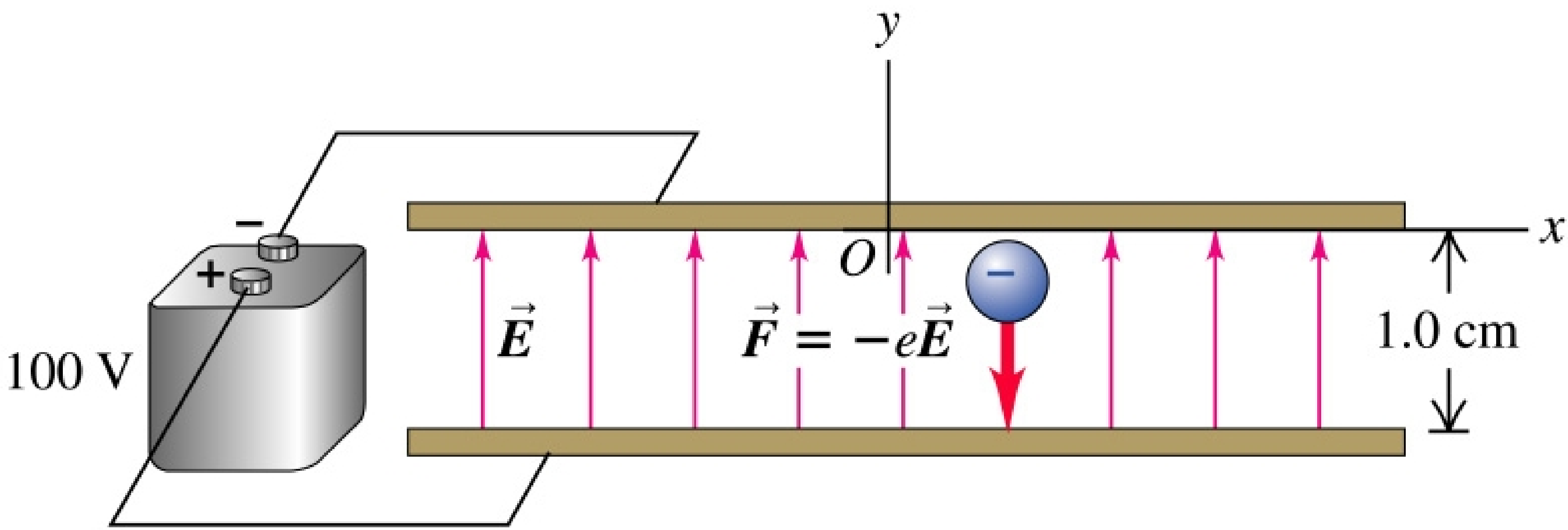


Example: Electron in a uniform *E*-field .... continued

b) What speed and kinetic energy does it acquire while travelling 1.0 cm to the lower plate?

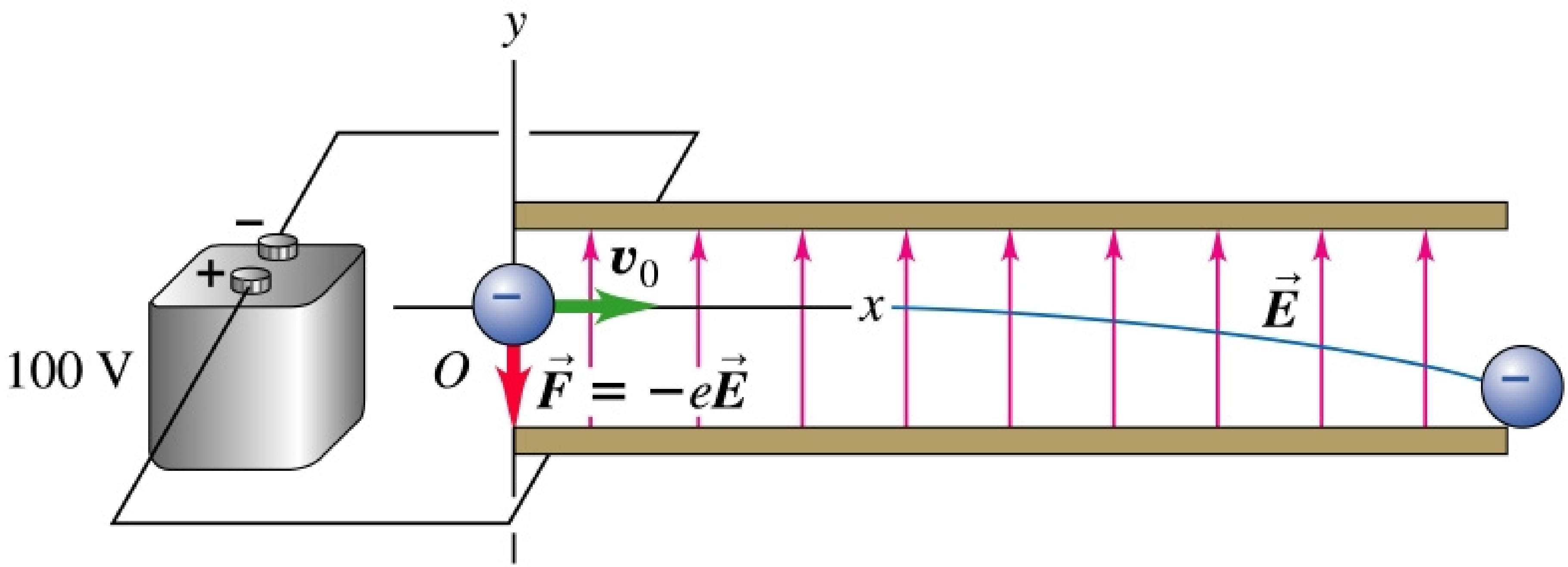
c) How much time is required for it to travel this distance?

(electron: charge $-e = -1.60 \times 10^{-19}$ C and mass $m = 9.11 \times 10^{-31}$ kg)



Example: Electron in a uniform *E*-field .... continued

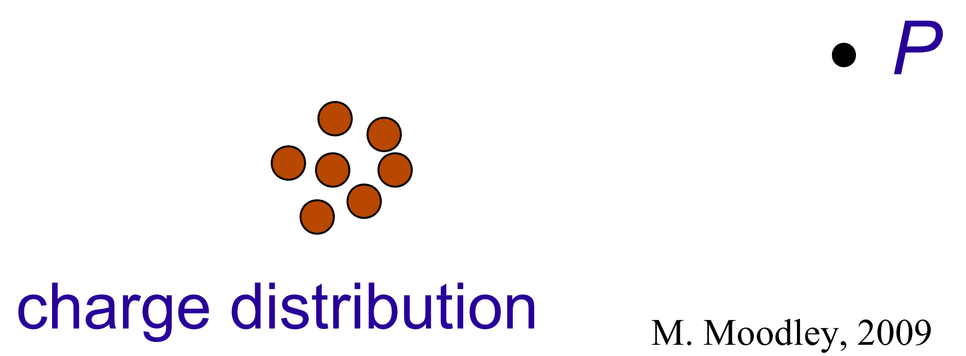
d) If an electron is launched into the *E*-filed with an initial horizontal velocity v_0, what is the equation of its trajectory?

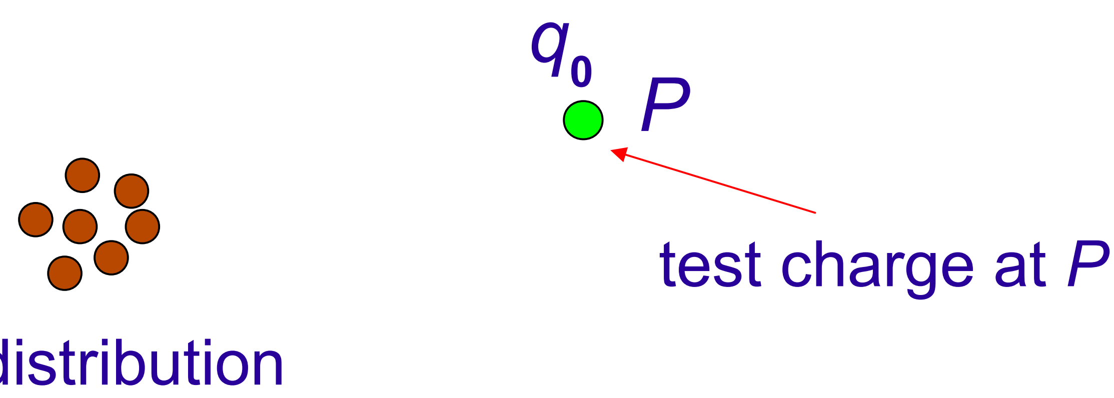


Parabolic trajectory of an electron in a uniform *E*-field

21.5 Electric-Field Calculations

- in realistic situations charge is distributed over space
- distribution made up of many point charges q_1, q_2, q_3 . . . .
- at each point P, each charge produces its own electric field $\boldsymbol{E}_1$, $\boldsymbol{E}_2$, $\boldsymbol{E}_3$ . . .
- so test charge q_0 placed at P experience forces $\boldsymbol{F}_1$, $\boldsymbol{F}_2$, $\boldsymbol{F}_3$ . . . due to q_1, q_2, q_3 . . . .





From superposition principle, total force $\boldsymbol{F}_0$ that charge distribution exerts on q_0 :

$$\vec{F}_0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = q_0\vec{E}_1 + q_0\vec{E}_2 + q_0\vec{E}_3 + \cdots$$

Total electric field $\boldsymbol{E}$ at point P :

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots$$

(principle of superposition of electric fields)

Different charge distributions:

line charge distribution

λ = linear charge density (charge per unit length, C/m)

surface charge distribution

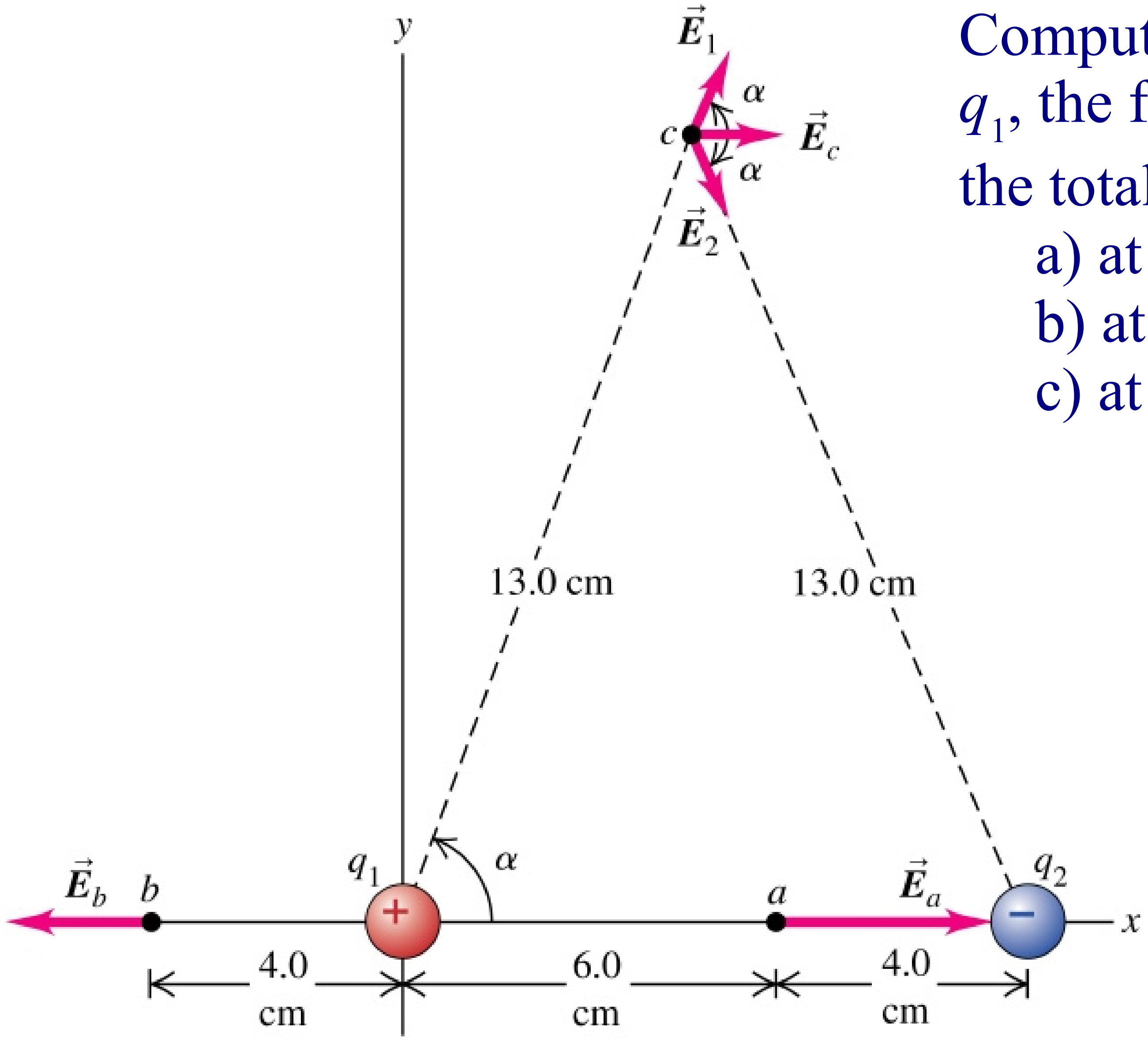
σ = surface charge distribution (charge per unit area, C/m^2)

volume charge distribution

ρ = volume charge density (charge per unit volume, C/m^3)

Example: Field of an electric dipole

Point charges q_1 = 12 nC and q_2 = -12 nC are placed 10 cm apart.



Compute the ***E***-field caused by q_1, the field caused by q_2, and the total field

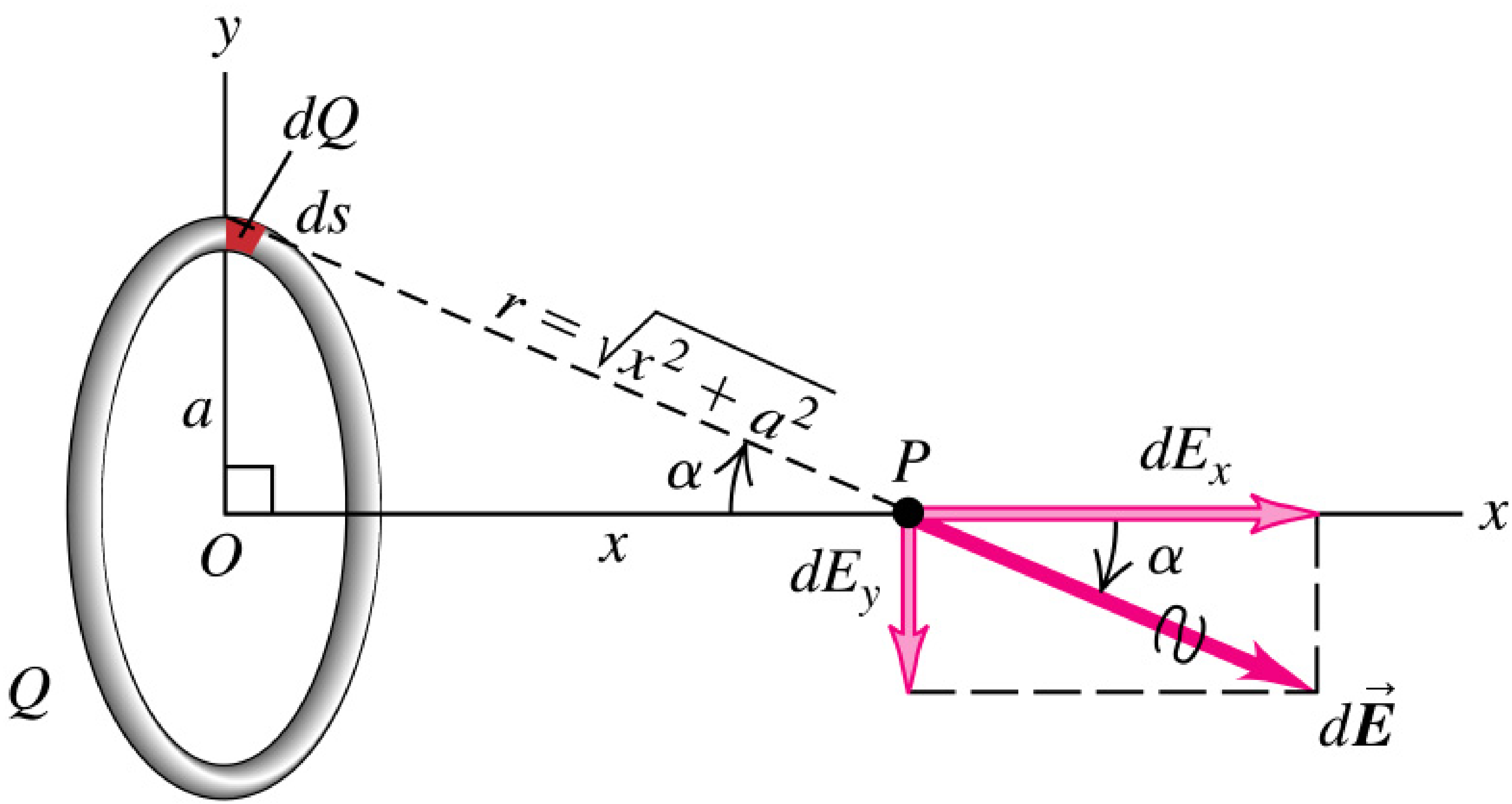
a) at point a;

b) at point b; and

c) at point c

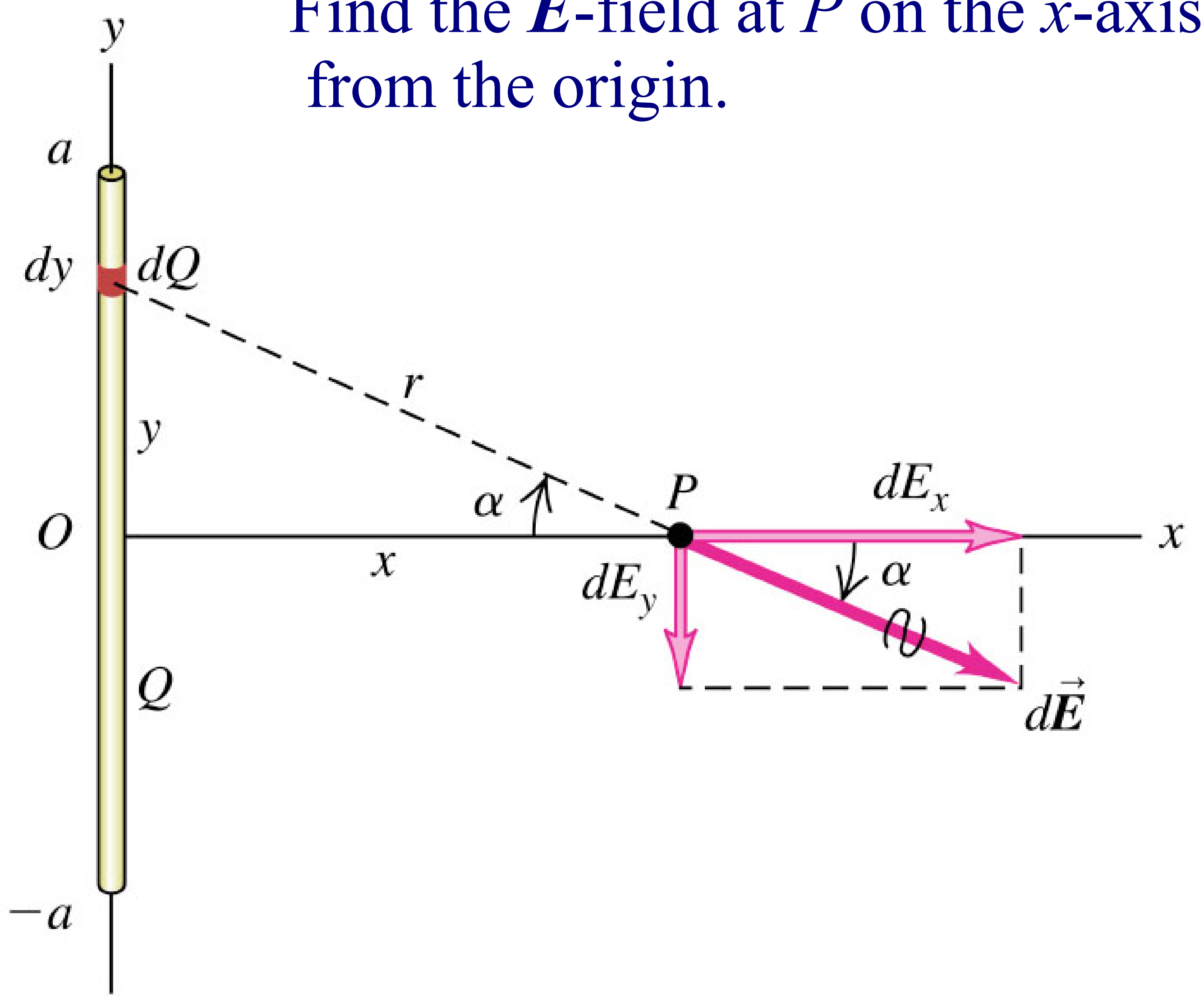
Example: Field of a ring of charge

A ring-shaped conductor with radius a carries a total charge Q uniformly distributed around it. Find the ***E***-field at a point P that lies on the axis of the ring at a distance x from its centre.



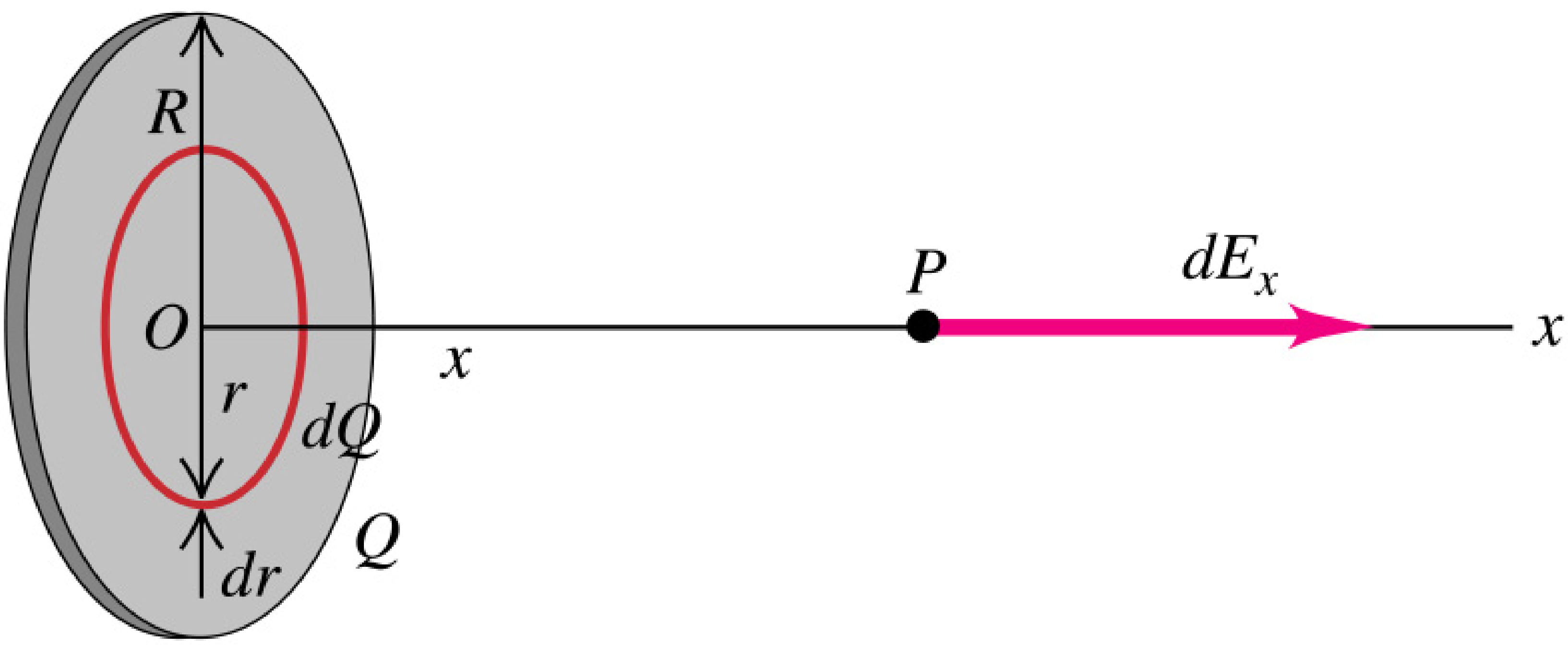
Example: Field of a line of charge

Positive electric charge Q is distributed uniformly along a line with length $2a$, lying along the y-axis between $y = -a$ and $y = +a$. Find the ***E***-field at P on the x-axis at a distance x from the origin.



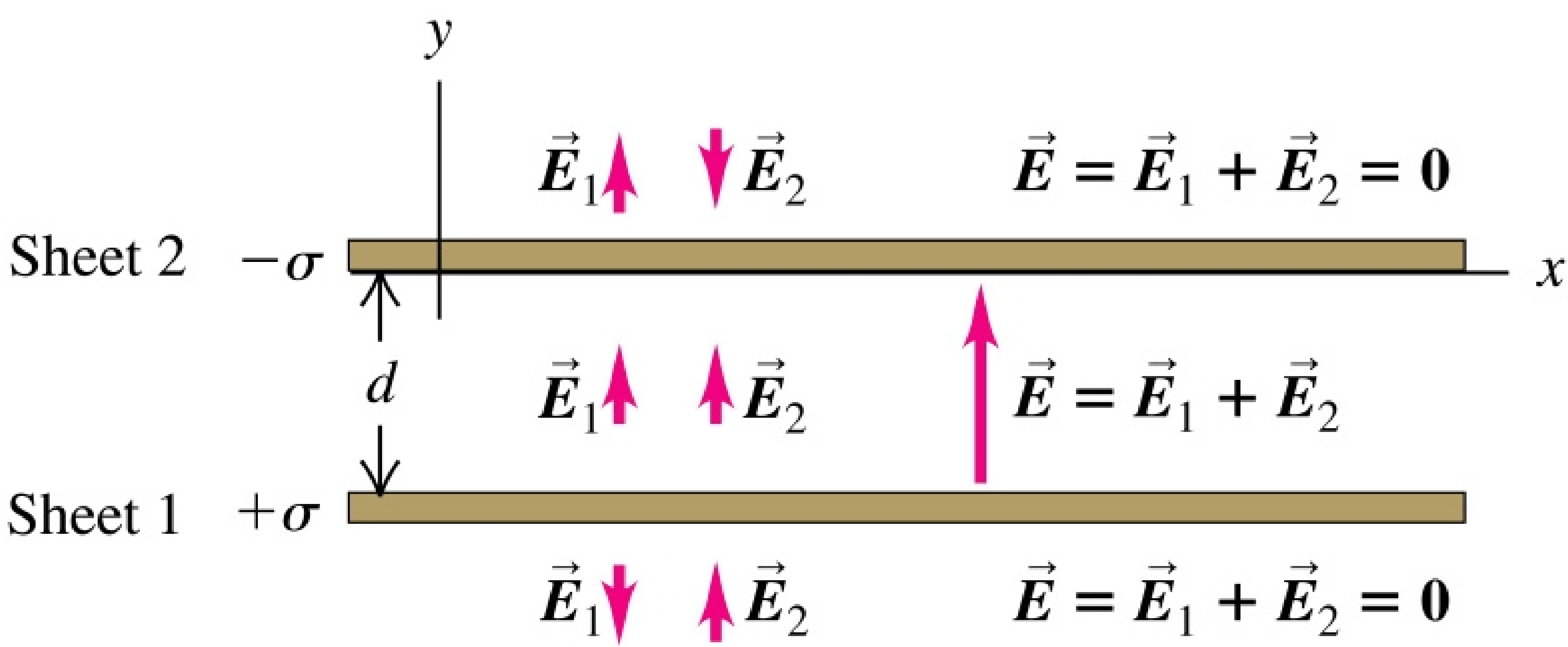
Example: Field of a uniformly charged disk

Find the ***E***-field caused by a disk of radius R with a uniform positive surface charge density σ, at a point along the axis of the disk a distance x from its centre.



Example: Field of two oppositely charged sheets

Two infinite plane sheets are placed parallel to each other, separated by a distance *d*. The lower sheet has a uniform positive surface charge density σ, and the upper sheet has a uniform negative surface charge density -σ with the same magnitude. Find the ***E***-field between the sheets, above the upper sheet and below the lower sheet.

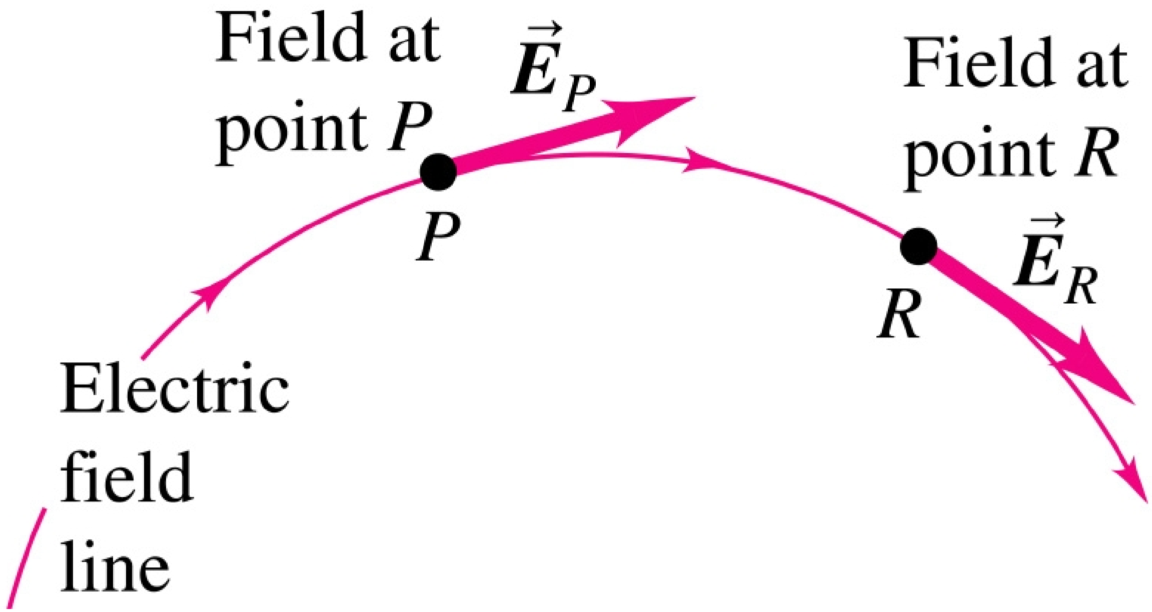


21.6 Electric Field Lines

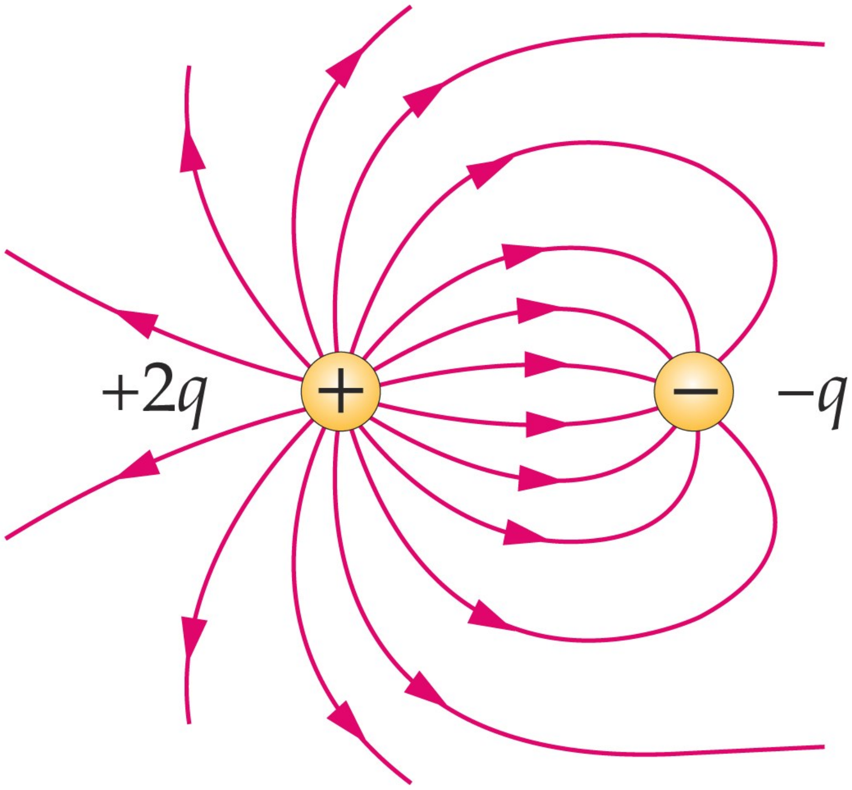
Help in the *visualization* of electric fields:

- electric fields can be represented by electric field lines at various points in space

- it is an *imaginary* line or curve drawn through a region of space so that its tangent at any point is in the direction of the electric field vector at that point

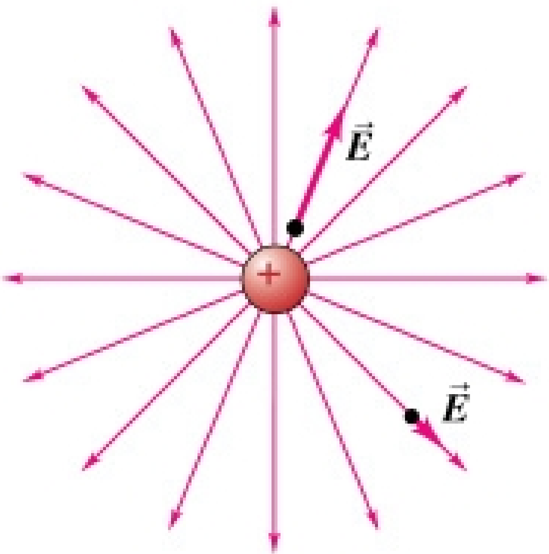


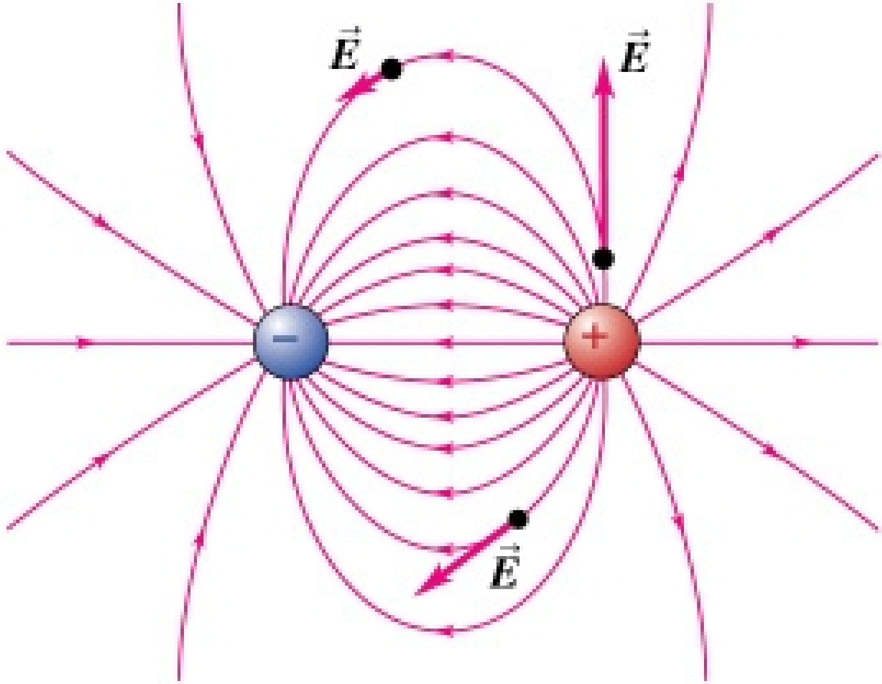
- These lines start on a positive charge and end on a negative charge
- The number of field lines starting (ending) on a positive (negative) charge is proportional to the magnitude of the charge
- The electric field is stronger where the field lines are closer together

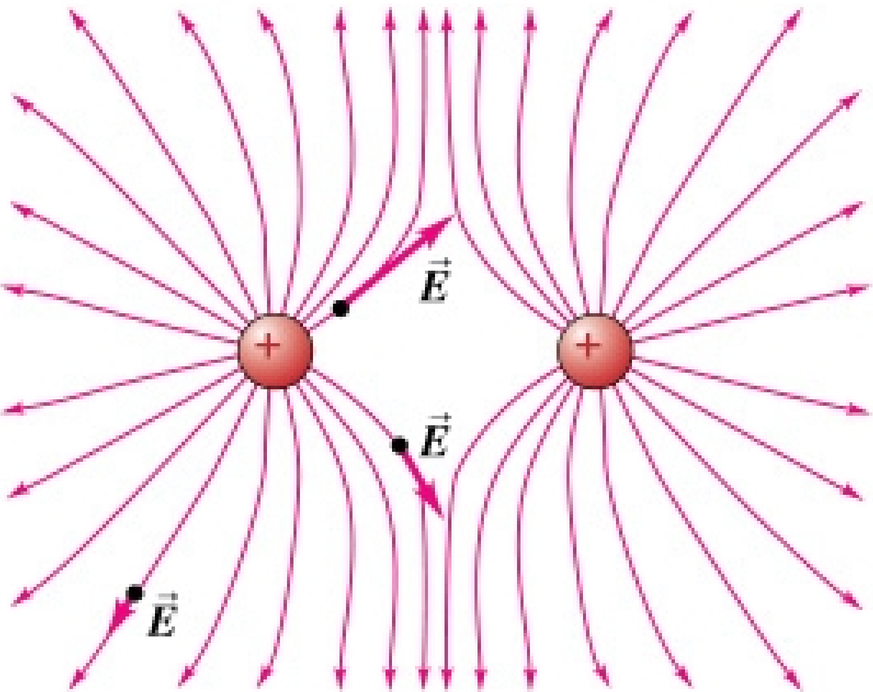


- only one field line can pass through each point of the field - field lines never intersect

Different electric field line configurations







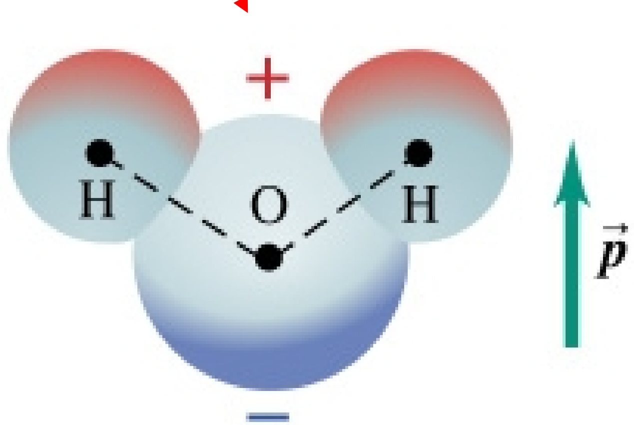
an electric dipole

21.7 Electric Dipoles

An electric dipole is a pair of point charges with equal magnitude and opposite charge separated by a distance *d*.

Water molecules (H_2O) behave like electric dipoles:

net positive charge

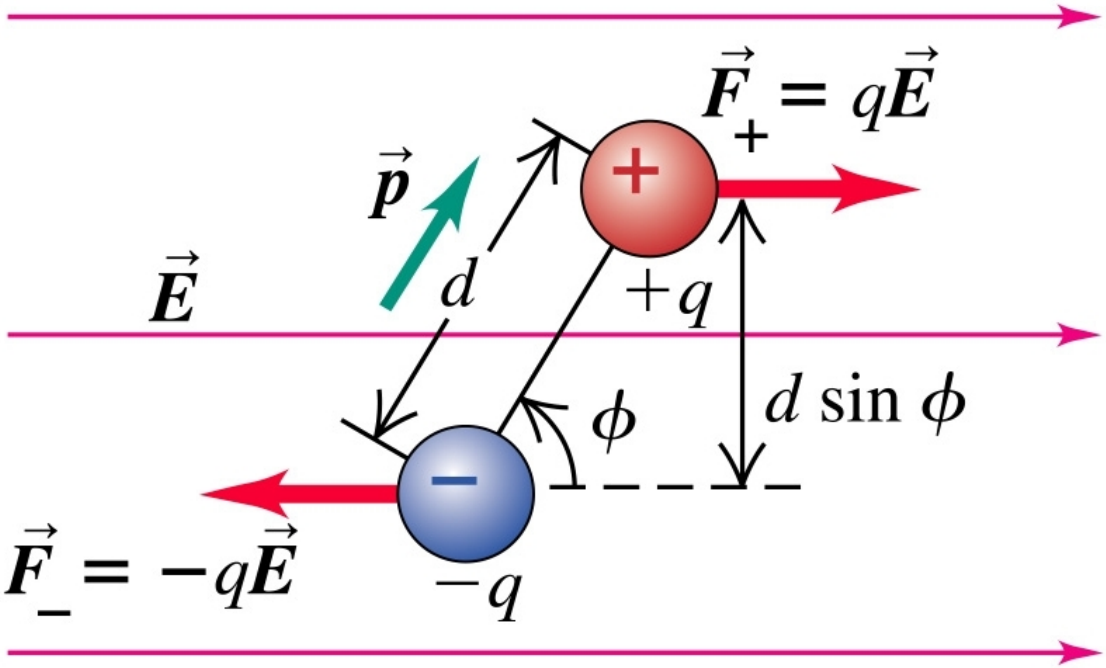


net negative charge

- it is electrically neutral
- chemical bonds cause a displacement of charge
- is an excellent solvent

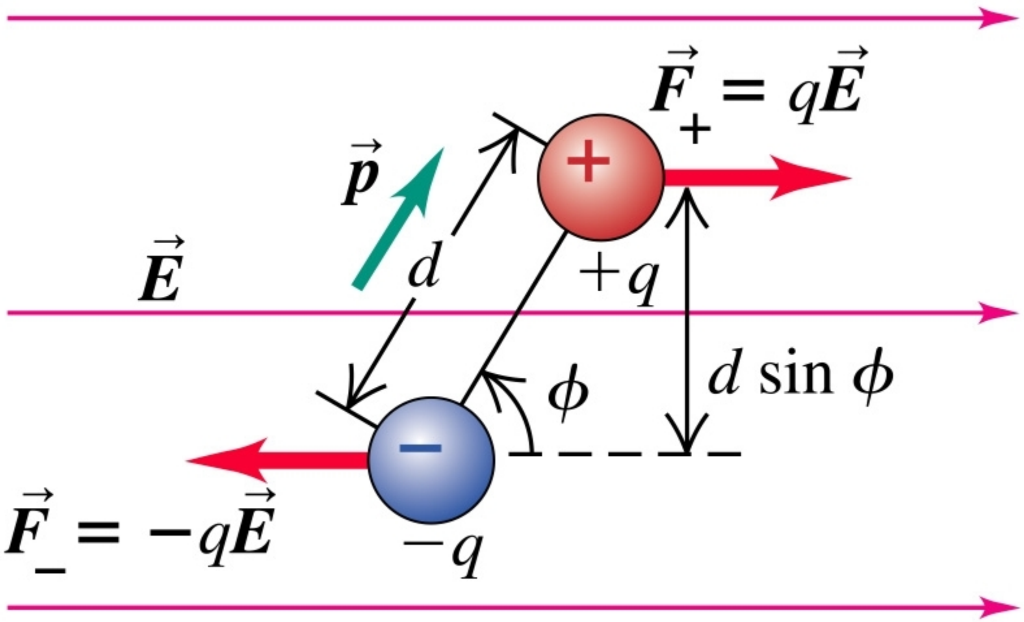
Force and Torque on an Electric Dipole

Consider an electric dipole in a uniform electric field ***E***



- the net force on it is zero
- different line of action implies torques don't add to zero

Let's calculate the torque:



- w.r.t. centre of dipole
- lever arm for both forces is $(d/2)\sin\phi$
- torque for both forces is $(qE)(d/2)\sin\phi$
- magnitude of the net torque: $\tau = (qE)(d\sin\phi)$
- is directed into the page since both torques rotate the dipole clockwise
- product of *q* and *d* is the magnitude of the electric dipole moment, denoted by *p*:

$$p = qd \quad \text{(unit of C.m)}$$

- is a vector, direction along axis of dipole (-ve to +ve)

In terms of *p*, the magnitude of the net torque is:

$$\tau = pE\sin\phi$$

- this is a vector product between vectors ***p*** and ***E*** :

$$\vec{\tau} = \vec{p} \times \vec{E}$$

- use right-hand rule to get direction of the torque
- torque is greatest when ***p*** is perpendicular to ***E***
- is zero when ***p*** and ***E*** are parallel or anti-parallel
- torque always tends to turn ***p*** to line up with ***E***
- $\phi = 0$ (***p*** $\parallel$ ***E***) $\Rightarrow$ position of stable equilibrium
- $\phi = \pi$ (***p*** anti-$\parallel$ ***E***) $\Rightarrow$ position of unstable equilibrium

When the dipole changes direction in an *E*-field, the torque does work on it.

Work: $dW = \tau d\phi$ for an infinitesimal displacement $d\phi$

- torque is in direction of decreasing ϕ implies

$$\tau = -pE\sin\phi$$

$$\Longrightarrow \quad dW = \tau d\phi = -pE\sin\phi \; d\phi$$

- In a finite displacement from ϕ_1 to ϕ_2 the total work done on the dipole is

$$\begin{aligned} W &= \int_{\phi_1}^{\phi_2} (-pE\sin\phi)\; d\phi \\ &= pE\cos\phi_2 - pE\cos\phi_1 \end{aligned}$$

Potential energy for a dipole in an *E*-field:

- work is the negative of the change in potential energy

$$W = U_1 - U_2$$

- potential energy can therefore be defined as

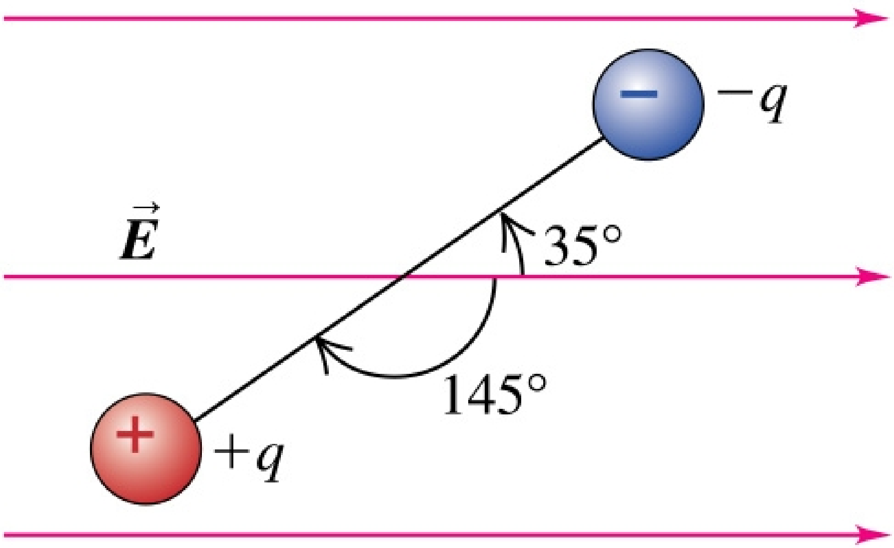
$$\begin{aligned} U(\phi) &= -pE\cos\phi \\ &= -\vec{p}\cdot\vec{E} \end{aligned}$$

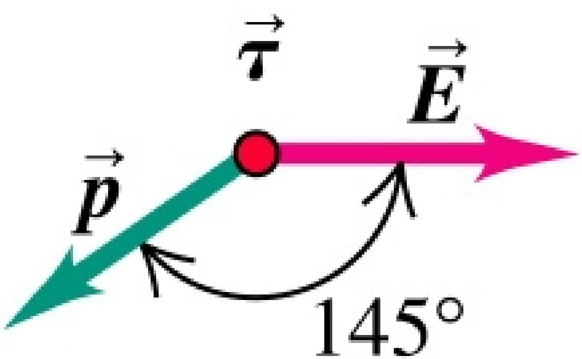
scalar product

- *U* is minimum at $\phi = 0$ (***p*** || ***E***) → *U = -pE*
- *U* is maximum at $\phi = \pi$ (***p*** anti-|| ***E***) → *U = +pE*
- *U* is zero at $\phi = \pi/2$ (***p*** ⊥ ***E***) → *U = 0*

Example: Force and torque on an electric dipole

Consider an electric dipole in a uniform *E*-field with magnitude 5.0×10^5 N/C. The two charges are of magnitude 1.6×10^{-19} C and are separated by a distance of 0.125 nm. Find





a) net force exerted by the field on the dipole

b) the electric dipole moment

c) the torque

d) potential energy of the system