26. A $50-\mathrm{kg}$ pole-vaulter running at $10 \mathrm{~m} / \mathrm{s}$ vaults over the bar. Her speed when she is above the bar is $1.0 \mathrm{~m} / \mathrm{s}$. Neglect air resistance, as well as any energy absorbed by the pole, and determine her altitude as she crosses the bar.

Initially, while running, the pole-vaulter has only kinetic energy (we choose to set $U_{g}=0$ at the ground level). As she goes over the bar, she has gravitational potential energy as well as some kinetic energy. There are no non-conservative forces to worry about in this problem.

$$
\frac{1}{2} m v_{i}^{2}=\frac{1}{2} m v_{f}^{2}+m g y_{f}
$$

Solve for $y_{f} \ldots$

$$
y_{f}=\frac{v_{i}^{2}-v_{f}^{2}}{2 g}=\frac{\left(10 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}-\left(1 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}{2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)}=5.05 \mathrm{~m}
$$

27. A child and sled with a combined mass of 50.0 kg slide down a frictionless slope. If the sled starts from rest and has a speed of 3.00 $\mathrm{m} / \mathrm{s}$ at the bottom, what is the height of the hill?

In this problem gravitational potential energy is converted into kinetic energy. If we set $\mathrm{Ug}=0$ at the bottom of the hill our energy equation simplifies to...

$$
m g y_{i}=\frac{1}{2} m v_{f}^{2}
$$

Solve for $y_{i} \ldots$

$$
y_{i}=\frac{v_{f}^{2}}{2 g}=\frac{\left(3 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}{2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)}=0.459 \mathrm{~m}
$$


28. A $0.400-\mathrm{kg}$ bead slides on a curved wire, starting from rest as shown in Figure P5.28. If the wire is frictionless, find the speed of the bead (a) at $\mathbf{A}$ and (b) at $\mathbf{B}$.

In this problem gravitational potential energy is converted into kinetic energy. If we set $U_{g}=0$ at point $A$ we can use a simple equation to find the speed at $A$. Set $U_{g}=0$ at point $B$ to find the speed at $B$.

$$
m g y_{i}=\frac{1}{2} m v_{f}^{2}
$$

Solving for $v_{f} \ldots$

$$
\begin{aligned}
& v_{f A}=\sqrt{2 g y_{i}}=\sqrt{2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(5 \mathrm{~m})}=9.90 \mathrm{~m} / \mathrm{s} \\
& v_{f B}=\sqrt{2 g y_{i}}=\sqrt{2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(3 \mathrm{~m})}=7.67 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

29. A gymnast swings on the high bar as shown in Figure P5.29. Starting from rest directly over the bar, he swings around the bar while keeping his arms and legs outstretched. Treating the gymnast as though his entire mass were concentrated at a point 1.20 m from the bar, determine his speed as he passes under the bar at position $\mathbf{A}$.

This is another problem where gravitational potential energy (at the top) gets converted into kinetic (at the bottom). Setting $\mathrm{U}_{\mathrm{g}}=0$ at the bottom...

$$
m g y_{i}=\frac{1}{2} m v_{f}^{2}
$$



Solving for $\mathrm{v}_{\mathrm{f}} \ldots$

$$
v_{f}=\sqrt{2 g y_{i}}=\sqrt{2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(2 \times 1.2 \mathrm{~m})}=6.86 \mathrm{~m} / \mathrm{s}
$$


30. A bead of mass $m=5.00 \mathrm{~kg}$ is released from rest and slides on the frictionless track shown. Determine (a) the bead's speed at points A and B and

In this problem (just like in \#28) gravitational potential energy is converted into kinetic energy. If we set $U_{g}=0$ at point $A$ we can use a simple equation to find the speed at $A$. Set $U_{g}=0$ at point $B$ to find the speed at $B$.

$$
m g y_{i}=\frac{1}{2} m v_{f}^{2}
$$

Solving for $v_{f} \ldots$

$$
\begin{aligned}
& v_{f A}=\sqrt{2 g y_{i}}=\sqrt{2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(1.8 \mathrm{~m})}=5.94 \mathrm{~m} / \mathrm{s} \\
& v_{f B}=\sqrt{2 g y_{i}}=\sqrt{2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(3 \mathrm{~m})}=7.67 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) the net work done by the force of gravity in moving the bead from the starting point to point $\mathbf{B}$.

The net work done by gravity is $W_{g}=-m g \Delta y$ (the negative is because if the object is going up [ $\Delta y$ is positive] work by gravity is negative. You can also calculate net work using $W_{\text {net }}=\Delta K$. Either way, you should get net work done by gravity to be...

$$
\begin{aligned}
& W_{\text {net }(A)}=\Delta K=K_{f}-K_{i}=K_{f}=\frac{1}{2} m v_{f}^{2}=(0.5)(5 \mathrm{~kg})\left(5.94 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}=88.2 \mathrm{~J} \\
& W_{\text {net }(A)}=\Delta K=K_{f}-K_{i}=K_{f}=\frac{1}{2} m v_{f}^{2}=(0.5)(5 \mathrm{~kg})\left(7.67 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}=147 \mathrm{~J}
\end{aligned}
$$

31. Tarzan swings on a $30.0-\mathrm{m}$-long vine initially inclined at an angle of $37.0^{\circ}$ with the vertical. What is his speed at the bottom of the swing (a) if he starts from rest?

In this problem, gravitational potential energy is being changed over to kinetic energy. You must determine the change in (vertical) height of Tarzan to determine his change in potential energy. The diagram below illustrates how you can determine the change in height. Define $U_{g}=0$ at the bottom point of the swing.

$\Delta y=L-L \cos \theta=30 m-30 m\left(\cos 37^{\circ}\right)=6.04 m$
Because Tarzan starts from rest, initially he has only gravitational potential energy. At the bottom of the swing, he has only kinetic energy.

$$
m g y=\frac{1}{2} m v_{f}^{2}
$$

Solve for $v_{f}$.

$$
v_{f}=\sqrt{2 g y}=\sqrt{2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(6.04 \mathrm{~m})}=10.9 \mathrm{~m} / \mathrm{s}
$$

b) If he started while moving at $4.00 \mathrm{~m} / \mathrm{s}$, what is his speed at the bottom of the swing?
$\frac{1}{2} m v_{i}^{2}+m g y=\frac{1}{2} m v_{f}{ }^{2}$
$v_{f}=\sqrt{v_{i}^{2}+2 g y}=\sqrt{\left(4 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(6.04 \mathrm{~m})}=11.6 \frac{\mathrm{~m}}{\mathrm{~s}}$
33. The launching mechanism of a toy gun consists of a spring of unknown spring constant, as shown in Figure P5.33a. If the spring is compressed a distance of 0.120 m and the gun fired vertically as shown, the gun can launch a $20.0-\mathrm{g}$ projectile from rest to a maximum height of 20.0 m above the starting point of the projectile. Neglecting all resistive forces, determine (a) the spring constant.

We are going to compare two points: the low point where the ball is against the compressed spring (all energy is elastic potential in the spring) and the highest point in the balls path in the air (where all energy is stored as gravitational potential).

$$
\begin{gathered}
\frac{1}{2} k x_{i}^{2}=m g y_{f} \\
k=\frac{2 m g y_{f}}{x_{i}^{2}}=\frac{2(0.020 \mathrm{~kg})\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(20.12 \mathrm{~m})}{(0.120)^{2}}=548 \frac{\mathrm{~N}}{\mathrm{~m}}
\end{gathered}
$$

(b) the speed of the projectile as it moves through the equilibrium position of the spring (where $x=0$ ), as shown in Figure P5.33b.

It is probably easiest to compare the max height reached (all energy is gravitational potential) and ball just as spring reaches equilibrium (all energy is kinetic).

$$
\begin{gathered}
\frac{1}{2} m v^{2}=m g y_{f} \\
v=\sqrt{2 g y_{f}}=\sqrt{2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(20 \mathrm{~m})}=19.8 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{gathered}
$$

39. A $70-\mathrm{kg}$ diver steps off a $10-\mathrm{m}$ tower and drops, from rest, straight down into the water. If he comes to rest 5.0 m beneath the surface, determine the average resistive force exerted on him by the water.

Compare the point just as the diver jumps (all gravitational potential) to the stopping point below the water (no mechanical energy left).

$$
\begin{gathered}
m g y_{i}+W_{n c}=0 \\
m g y_{i}+\left(F_{\text {water }}\right)(d)(\cos \theta)=0
\end{gathered}
$$

Theta $=180^{\circ}$, so after re-arranging $\ldots$

$$
\begin{gathered}
m g y_{i}=\left(F_{\text {water }}\right)(d) \\
F_{\text {water }}=\frac{m g y_{i}}{d}=\frac{(70 \mathrm{~kg})\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(15 \mathrm{~m})}{5 \mathrm{~m}}=2060 \mathrm{~N}
\end{gathered}
$$

41. A $2.1 \times 10^{3}-\mathrm{kg}$ car starts from rest at the top of a $5.0-\mathrm{m}$-long driveway that is sloped at $20^{\circ}$ with the horizontal. If an average friction force of $4.0 \times 10^{3} \mathrm{~N}$ impedes the motion, find the speed of the car at the bottom of the driveway.

At the top of the ramp all the energy is gravitational potential. Some of this energy goes to kinetic and some becomes thermal due to the negative work done by friction...

$$
\begin{gathered}
m g y_{i}+(f)(d)(\cos \theta)=\frac{1}{2} m v^{2} \\
v=\sqrt{2 g y_{i}-\frac{2(f)(d)}{m}}=\sqrt{2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)\left(5 \sin 20^{\circ}\right)-\frac{2(4000 \mathrm{~N})(5 \mathrm{~m})}{2100 \mathrm{~kg}}}=3.80 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{gathered}
$$

42. A $25.0-\mathrm{kg}$ child on a $2.00-\mathrm{m}$-long swing is released from rest when the ropes of the swing make an angle of $30.0^{\circ}$ with the vertical. (a) Neglecting friction, find the child's speed at the lowest position.

As in problem 31 above, the gravitational potential energy is being changed over to kinetic energy. You must determine the change in (vertical) height of the child to determine the child's change in potential energy. The diagram below illustrates how you can determine the change in height. Define $U_{g}=0$ at the bottom point of the swing.


$$
\Delta y=L-L \cos \theta=2 m-2 m\left(\cos 30^{\circ}\right)=0.268 m
$$

Because the child starts from rest, initially s/he has only gravitational potential energy. At the bottom of the swing, s/he has only kinetic energy.

$$
m g y=\frac{1}{2} m v_{f}^{2}
$$

Solve for $v_{f}$.

$$
v_{f}=\sqrt{2 g y}=\sqrt{2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(0.268 \mathrm{~m})}=2.29 \mathrm{~m} / \mathrm{s}
$$

(b) If the actual speed of the child at the lowest position is $2.00 \mathrm{~m} / \mathrm{s}$, what is the mechanical energy lost due to friction?

In this case there is work done by friction.

$$
m g y+W_{n c}=\frac{1}{2} m v_{f}^{2}
$$

Solving for $W_{n c} \cdots$

$$
W_{n c}=\frac{1}{2} m v_{f}^{2}-m g y=\frac{1}{2}(25 \mathrm{~kg})\left(2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}-(25 \mathrm{~kg})\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(0.268 \mathrm{~m})=15.6 \mathrm{~J}
$$

43. Starting from rest, a $10.0-\mathrm{kg}$ block slides 3.00 m down a frictionless ramp (inclined at $30.0^{\circ}$ from the floor) to the bottom. The block then slides an additional 5.00 m along the floor before coming to a stop. Determine (a) the speed of the block at the bottom of the ramp, (

In the first part of this problem gravitational potential energy is converted into kinetic energy. If we set $U_{g}=0$ at the bottom of the ramp we can use a simple equation to find the speed at the bottom of the ramp. Knowing the d along the ramp is 3.0 m and at a $30^{\circ}$ angle, you can find the height using $y=d \sin 30^{\circ}=1.5 \mathrm{~m}$.

$$
m g y_{i}=\frac{1}{2} m v_{f}^{2}
$$

Solving for $v_{f} \ldots$

$$
v_{f}=\sqrt{2 g y_{i}}=\sqrt{2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(1.5 \mathrm{~m})}=5.42 \frac{\mathrm{~m}}{\mathrm{~s}}
$$

b) the coefficient of kinetic friction between block and floor,

Now we are going to compare the point where the block was released from rest to the point where it comes to a stop. At the top of the ramp all the energy is gravitational potential (set $U_{g}=0$ at the bottom of the ramp). At the stopping point there is no mechanical energy left. So, our conservation of energy equation simplifies to ...

$$
\begin{gathered}
m g y_{i}+W_{n c}=0 \\
m g y_{i}+(f)(d)(\cos \theta)=0 \\
m g y_{i}=(f)(d)=\mu m g d \\
\mu=\frac{y}{d}=\frac{1.5 m}{5 m}=0.3
\end{gathered}
$$

Theta $=180^{\circ}$, so $\ldots$
(c) the mechanical energy lost due to friction.

All of the mechanical energy was lost due to friction. Wnc $=m g y_{i}=(10 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(1.5 \mathrm{~m})=147 \mathrm{~J}$
46. In a circus performance, a monkey is strapped to a sled and both are given an initial speed of $4.0 \mathrm{~m} / \mathrm{s}$ up a $20^{\circ}$ inclined track. The combined mass of monkey and sled is 20 kg , and the coefficient of kinetic friction between sled and incline is 0.20 . How far up the incline do the monkey and sled move?


We will compare the starting point at the bottom of the ramp (where we will define $U_{g}=0$ ) and the ending point at the highest point reached by the sled. So, kinetic energy at the bottom gets converted into gravitational potential energy at the top, as well as thermal energy produced by the non-conservative friction force.

$$
\frac{1}{2} m v_{i}^{2}+(f)(d)(\cos \theta)=m g y_{f}
$$

Note that the theta in the work equation is $180^{\circ}$ (not $20^{\circ}$ !).

$$
\frac{1}{2} m v_{i}^{2}=m g y_{f}+(f)(d)
$$

Friction (f) is kinetic friction, so can be replaced with $\mu n$. On an incline $n=m g \cos \Theta$. So, the equation above becomes...

$$
\frac{1}{2} m v_{i}^{2}=m g y_{f}+(\mu m g \cos (\theta))(d)
$$

Replacing yf with dsinQ (see diagram above) gives...

$$
\frac{1}{2} m v_{i}^{2}=m g d \sin \theta+(\mu m g \cos (\theta))(d)
$$

Canceling out the mass, factoring our $d$ and solving for it gives...

$$
d=\frac{v_{i}^{2}}{2 g(\sin \theta+\mu \cos \theta)}=1.54 \mathrm{~m}
$$

