CHAPTER 2

INDUCTORS, CAPASITORS AND ALTERNATING CURRENT CIRCUITS

2.0 INTRODUCTION

This chapter is explaining about the indictors, capasitors and AC circuits. The learning outcome for this chapter are the students should be able to apply correctly the basic principles of inductors, capasitors and AC circuits that contains R, L and C to slove problems.

2.1 INDUCTOR

Inductor, choke or coil is the electric component that has the characteristics against the change of the current. Inductor made by winding the wire/conductor around the ferromagnetic material. There are two types of inductor which is often used in electronic circuits: fixed type and variable type. The symbol for inductor is as show in Figure 2.1.

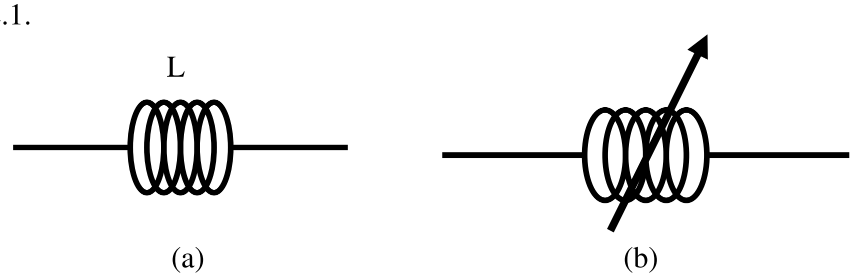


Figure 2.1: (a) Fixed type inductor (b) variable type inductor

Unit for inductor is Henry and the symbol is L. 1 Henry is equal to the total inductance of winding when the current is in the rate of 1 ampere per second and producing the induced voltage for 1 volt. Table 2.1 show the equivalent value and unit for inductor.

Table 2.1: Equivalent value and unit for inductor

Value		Units	
1	1	1 Henry	1 H
1000	1 x 10^3	1 kiloHenry	1 kH
0.001	1 x 10^{-3}	1 miliHenry	1 mH
0.000001	1 x 10^{-6}	1 mikroHenry	1 μH

Inductor is a spiral structure coil of wire which creates a magnetic field when current passes through it. The magnetic field through the middle of the coil is directed from left to right, and is highly intensified.

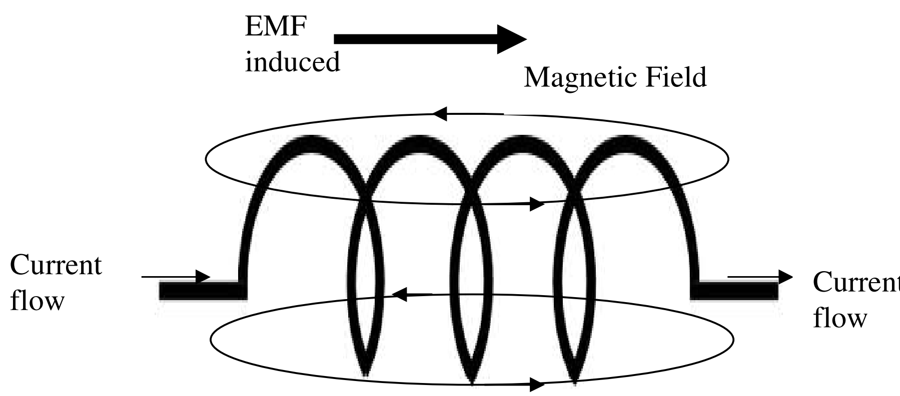


Figure 2.2: EMF Induced

This magnetic field gives the coil some interesting and useful properties known as inductance. Increase the current in a coil will create a changing in magnetic field that will generate an electromotive force (emf) in the coil. Generated emf is opposes the applied voltage. The current through an inductance can only change gradually, it cannot change instantaneously as it could with only resistors in the circuit. The coil will store or release energy in its magnetic field as rapidly as necessary to oppose any such change.

The effects of inductor as the electrical device in the circuit are to:

a) Smooth wave ripples in the DC circuit.
b) Improve the transmission characteristics of waves in the telephone line

2.2 INDUCTANCE

Inductance is a characterstic of the inductor that have oppose any change in current through itself. There are two types of inductance:

a) Self Inductance (L)
b) Mutual Inductance (M)

2.2.1 Self Inductance (L)

Self inductance occur when a current flow in the coil causing the changing of flux in the winding. The electromotive force (emf) produced is opposite direction with the direction of the applied voltage.

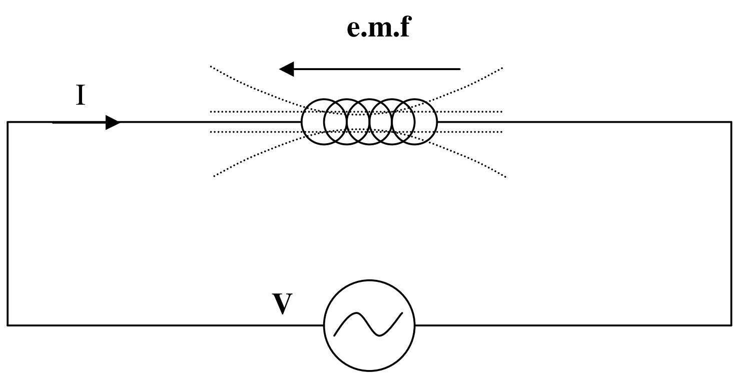


Figure 2.3: Self Inductance

The emf produce is opposite direction with the current flow.

EMF generates due to changes of magnetic flux,

$$e_1 = -N\frac{d\phi}{dt} \qquad (2.1)$$

EMF generates due to changes of current,

$$e_2 = -L\frac{di}{dt} \qquad (2.2)$$

Faraday's Law;

$$e_1 = e_2 \qquad (2.3)$$

$$-N\frac{d\phi}{dt} = -L\frac{di}{dt}$$

$$L = N\frac{d\phi}{dt} \bullet \frac{dt}{di}$$

Self Inductance, $L = N\dfrac{d\phi}{di}$

Where :

- L = Self Inductance
- N = Number of turns
- $\frac{d\phi}{dt}$ = flux change against time
- $\frac{di}{dt}$ = current change against time

2.2.2 Mutual Inductance (M)

Mutual inductance is the ability of a first coil to produce 1 emf in the nearest coil through induction or when the current in the first coil is changing.

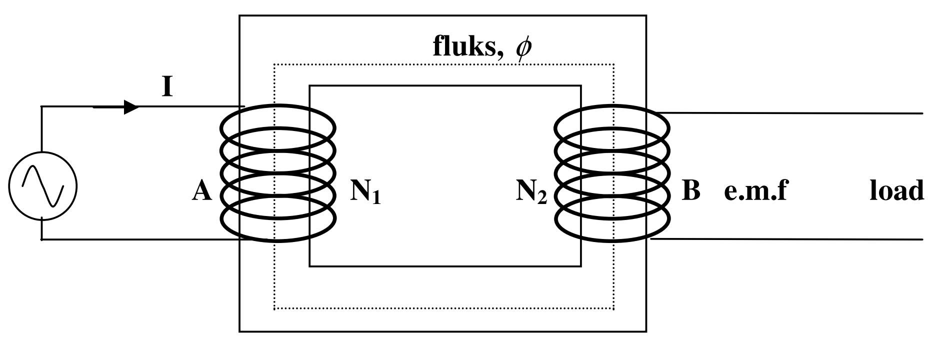


Figure 2.4: Mutual Inductance

When current flow in the first loop, flux will be produce in the first coil. The continuos current causes flux flow to the next coil and then generate emf in the second coil. Emf produced in second coil will cut the conductor and produce the voltage in second loop.

2.3 INDUCTOR CIRCUIT ANALYSIS

Inductors can be connected in two different ways. The two simplest of these are called series and parallel and occur very frequently.

2.3.1 Series Inductors

Inductors connected in series are connected along a single path, so the same current flows through all of the components. Figure 2.5 is the connection for series inductors. Total inductance (L_T) for a series circuit is the sum of all values of inductance in the circuit.

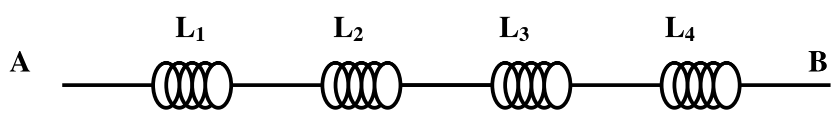


Figure 2.5: Series Inductors

$$L_T = L_1 + L_2 + L_3 + L_4 \qquad (2.4)$$

2.2.2 Parallel Inductors

Inductors connected in parallel are opposite to each other as in Figure 2.6. The same voltage is applied to each component but the total current will split into each branches. The total inductance of inductors in parallel is equal to the reciprocal of the sum of the reciprocals of their individual inductances. Total inductance for parallel circuit can calculate using equation (2.5).

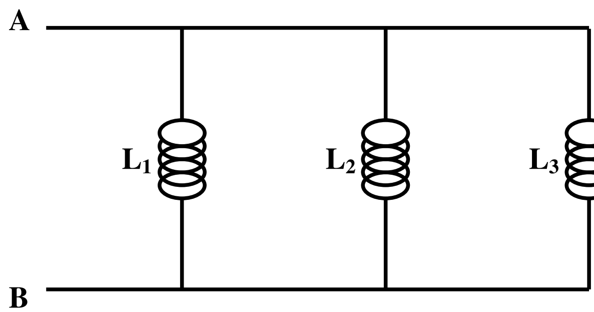


Figure 2.6: Parallel Inductors

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \qquad (2.5)$$

Example 2.1

Calculate the total inductance (L_T) for the three coil when the value of each inductor is 0.02H, 44mH, 400 µ H if the connection is in:

a) Series
b) Parallel

Solution 2.1

L_1 = 0.02H

L_2 = 44mH = 44 x 10^{-3} = 0.044H

L_3 = 400µH = 400 x 10^{-6} = 0.0004H

a) series

$$L_T = L_1 + L_2 + L_3$$

= 0.02 + 0.044 + 0.0004 = 0.0644H

b) parallel

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

$$= \frac{1}{0.02} + \frac{1}{0.044} + \frac{1}{0.0004}$$

$$= 2572.73$$

$$\therefore L_T = \frac{1}{2572.73} = \mathbf{389 \times 10^{-6} = 389\mu H}$$

2.4 INDUCTIANCE REACTANCE, X_L

The alternating current (AC) is changing continuously which in turn produced continuous opposed induces emf as well. The opposition to the current flow is called inductance reactance. The symbol for inductance reactance is X_L and the unit is Ohm (Ω). The value of Inductance reactance in a circuit depends on the inductance of the circuit due to the current change in the circuit. The rate of current change depends on the frequency of the supply voltage. Mathematically, equation (2.7) is use to calculate inductance reactance.

$$X_L = 2\pi f L \qquad (2.6)$$

where :
X_L = Inductance Reactance (Ω)
f = Frequency (Hz)
L = Inductor (Henry)

Example 2.2

A coil with 0.2H connected with AC 200V, 50Hz. Calculate the inductance reactance in the circuit.

Solution 2.2

$$X_L = 2\pi f L$$
$$= 2\pi(50)(0.2)$$
$$\mathbf{= 62.8\ \Omega.}$$

2.5 ENERGY IN INDUCTOR

Energy in the inductor can be calculate using the equation 2.4 below. The unit for energy is Joule (J)

$$E = \frac{1}{2} L I^2 \qquad (2.7)$$

2.6 CAPASITOR

Capacitor is an electrical device which is capable of storing electrical energy. Unit is Farad (F) and symbol is C. The quantity and duration of energy can be saved depends on the capacitance of the capacitor. Electrical energy stored in the capacitor is in a form of charge. A plate will has a negative charge (-ve) and the other plate is positive charge (+ve).

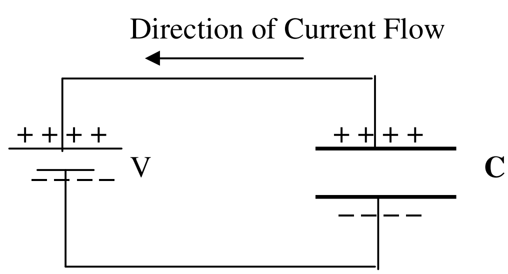
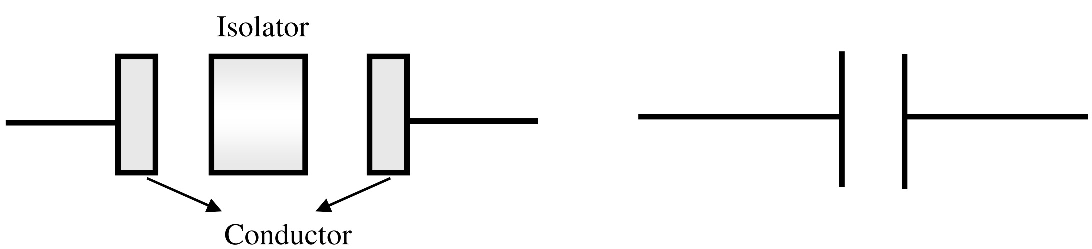


Figure 2.7 : Charges on the plate

Capacitor or condenser built with two-conductor or plate arranged opposite each other. It separated by insulating material called the dielectric as shown in Figure 6.2(a), Figure 6.2(b) show the symbol and unit for the capasitor.



(a) **(b)**

Figure 2.8 :Capasitor (a) Design Structure (b) Symbol Schematic

There are many types of capacitor which is Dielectric Air Convertible Capacitor, Paper Capacitor, Polyester Capacitor, Mica Capacitor, Ceramic Capacitor, Electrolytic Capacitor and Tantalum Capacitor

The effects of capacitor as the electrical device in the circuit are to:

1. Increasing the circuit power factor.
2. Reducing the fireworks during the switch is on inside the circuit.
3. Reduce radio interference test in the starter circuit pendaflour light.
4. Strengthen the electric current.
5. Store electrical charges.

2.7 CAPACITANCE

Capacitance is a characteristic of a capacitor to store electrical energy. It is define as the quantity or amount of electric charge needed to make a difference between the two plates. Capacitance of 1 Farad means a capacitor can store 1 coulomb of electrical charge when voltage is applied to the capacitor is 1 volt.

Capacitance, (Farad) $= \dfrac{Cas(Coulomb)}{Voltage(Volt)}$

$$C = \frac{Q}{V} \qquad (2.8)$$

Typically the unit use for capacitor are microFarad (μF) or pikoFarad (pF). Table 2.2 show the equivalent value and unit for capacitor.

Table 2.2: Equivalent value and unit for capacitor

Value		Units	
1	1	1 Farad	1 F
0.000001	1×10^{-6}	1 mikroFarad	1 μH
0.000000000001	1×10^{-12}	1 pikoFarad	1 pH

Three (3) factors affecting the value of the capacitance of a capacitor:

1. Area of the Plate, A

   Capacitance is directly proportional to the cross sectional area of the plates. Capacitance of a capacitor varies with the capacitor plate area. Area of large plates to accommodate many electrons, and can save a lot of charge.

$$C \propto A\,.$$

2. The Distance Between Two Plates, d

   Capacitance is inversely proportional to the distance between the plates. Capacitance of a capacitor change when the distance between the plates changes. Capacitance will increase when the plates when the plate is brought closer and less-plates removed.

$$C \propto \frac{1}{d}$$

3. Permeability, ε

   Capacitance is proportional to the permeability of the conductor.

$$C \propto \varepsilon$$

2.8 CAPACITOR CIRCUIT ANALYSIS

The method of the capacitor circuit analysis is different with the method of circuit analysis for inductance. There are 3 types of circuit analysis in capacitor:

1. Series
2. Parallel
3. Combination of series and parallel

2.8.1 Series Capacitors

When capacitors are connected in series, the total capacitance (C_T)is less than any one of the series capacitors' individual capacitances. If two or more capacitors are connected in series, the overall effect is that of a single (equivalent) capacitor having the sum total of the plate spacings of the individual capacitors.

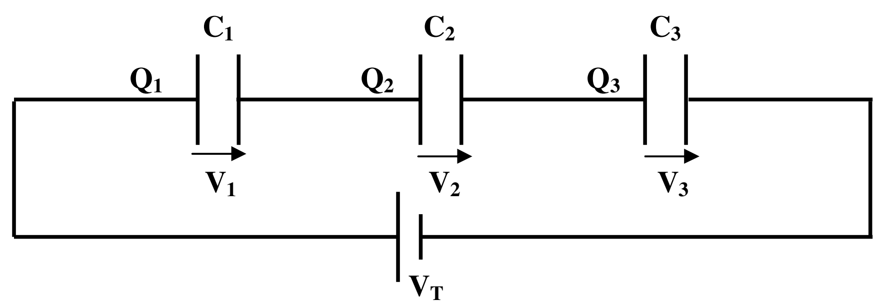


Figure 2.9 : Series Capacitors

The total capacitance is less than any one of the individual capacitors' capacitances. The formula for calculating the series total capacitance is as Equation 2.9. It is the same form as for calculating parallel resistances.

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad \text{(2.9)}$$

Equation 2.9 can be written as Equation 2.10 below.

$$C_T = \frac{C_1 C_2 C_3}{C_1 C_2 + C_1 C_3 + C_2 C_3} \quad \text{(2.10)}$$

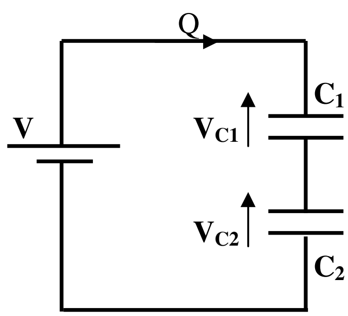
Charges for each capacitor connected in series are the same.

$$Q_1 = Q_2 = Q_3 = Q_T\,, \ \text{di mana} \ Q_T = C_T V_T \quad \text{(2.11)}$$

Voltage drop for each capacitors can be calculate using Equation 2.12).

$$V_{C1} = \frac{Q_T}{C_1}\ ,\quad V_{C2} = \frac{Q_T}{C_2}\quad \text{dan}\quad V_{C3} = \frac{Q_T}{C_3} \qquad \text{.............} \qquad (2.12)$$

Untuk dua(2) buah pemuat seperti litar Rajah 6.4 di bawah, kemuatan jumlah juga boleh dikira dengan menggunakan persamaan (6.9).



$$C_T = \frac{C_1 C_2}{C_1 + C_2} \qquad \text{.........} \qquad (2.13)$$

Figure 2.10: Series with Two Capacitors

Voltage drop for each capacitors can be calculate using Equation 2.14.

$$\left.\begin{aligned} V_{C1} &= \left(\frac{C_2}{C_1 + C_2}\right)V_T \\ V_{C2} &= \left(\frac{C_1}{C_1 + C_2}\right)V_T \end{aligned}\right\} \qquad \text{.............................} \qquad (2.14)$$

Charges are equal for each capacitors which connected in series. $Q_1 = Q_2 = Q_T$

2.8.2 Parallel Capacitors

The total capacitance of capacitors in parallel is equal to the sum of their individual capacitances.

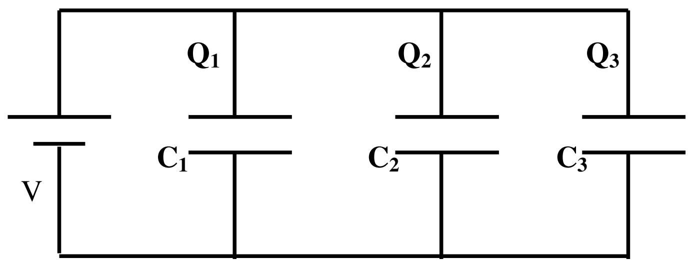


Figure 2.11: Parallel Capacitors

$$C_T = C_1 + C_2 + C_3 \quad \text{.......... (2.15)}$$

Voltage drop at each capacitors are equal.

$$V_{C1} = V_{C2} = V_{C3} = V_T \quad \text{.......... (2.16)}$$

Value of charges through each parallel capacitor are different and can be calculated using Equation 2.17.

$$Q_{C1} = C_1 V_T \,,\quad Q_{C2} = C_2 V_T \quad \text{and} \quad Q_{C3} = C_3 V_T \quad \text{.......... (2.17)}$$

Example 2.3

Calculate the total capacitance of the three (3) capacitor where the value of each capacitance is 120µF when it is connected in:

a. Parallel

b. Series

Solution 2.3

a. $C_T = C_1 + C_2 + C_3$ = (120+120+120) x 10^{-6} = 360 x 10^{-6} = 360 µF

b. $$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{120} + \frac{1}{120} + \frac{1}{120} = \frac{3}{120} = \frac{1}{40}$$

$$\therefore C_T = 40\mu F$$.

Example 2.4

Two capacitors each value is 6µF and 10µF is connected in series with a 200V power supply. Calculate;

a. Total capacitance

b. Charge in each capacitor

c. Voltage across each capacitor.

Solution 2.4

a. Total capacitance, C_T

$$C_T = \frac{C_1 C_2}{C_1 + C_2} = \frac{(6)(10)}{6+10} = 3.75\mu F$$

b. The value of charges for each capacitors in series are the same,

$$Q_T = C_T V_T = (3.75x10^{-6})(200) = 750x10^{-6} = 750\mu C$$

Therefore,

$$Q_1 = Q_2 = Q_T = 750\mu C$$

c. Voltage drop for each capacitors are different

$$V_1 = \frac{Q_T}{C_1} = \frac{750x10^{-6}}{6x10^{-6}} = 125V$$

$$V_2 = \frac{Q_T}{C_2} = \frac{750x10^{-6}}{10x10^{-6}} = 75V$$

2.9 CAPACITANCE REACTANCE, X_C

Capacitance reactance is the opposition to the flow of the current by the capacitor. Capacitance reactance value is inversely proportional to the frequency of the alternating current voltage. Symbol for capacitance reactance is X_C and the unit is ohm (Ω). Equation 2.18 is formula to calculate capacitance reactance.

$$X_C = \frac{1}{\omega C}, \text{ where } \omega = 2\pi f.$$

$$X_C = \frac{1}{2\pi f C} \qquad (2.18)$$

Where , C = *Capacitance (F)*
f = *Frequency (Hz)*
ω = *angular velocity (rads^{-1})*
2π = *constant*

Example 2.5

8μF capacitor connected to the supply of 240V, 50Hz. Calculate the value of capacitance reactance.

Solution 2.5

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(8x10^{-6})} = 397.9\Omega$$

2.10 ENERGY IN CAPASITOR

Energy can be calculate using Equation 2.19 below. The unit for energy is Joule.

$$E = \frac{1}{2}QV \qquad (2.19)$$

Equation 2.19 can be transform to another form in calculating energy by inserting Equation 2.8 in 2.19.

$$E = \frac{1}{2}CV^2 \text{ and} \qquad (2.20)$$

$$E = \frac{1}{2}(\frac{Q^2}{C}) \qquad (2.21)$$

Example 2.6

Capacitor with 8pF connected to the 600V power supply. Calculate the charge and energy that can be stored by the capacitor.

Solution 2.6

Charge, $Q = CV = (8x10^{-12})(600) = 4.8 \text{ x } 10^{-7}$ C

Energy, $E = \frac{1}{2}QV = (4.8x10^{-12})(600) = 2.88 \text{ x } 10^{-9}$ Joule

2.11 ALTERNATING CURRENT (AC)

Alternating Current is a current flowing in two conditions whether at negative or positive values. The current flows from zero to positive maximum, to zero again and further to negative maximum and back to zero.

Alternating voltage can be generated in 2 ways:

1. Conductors cut the magnetic flux which is the conductor is moving and the magnetic flux is stationary.
2. Magnetic flux cut the conductor where the flux is moving and conductor is stationary.

AC waveform is same as the form of sinus wave as shown in Figure 2.12

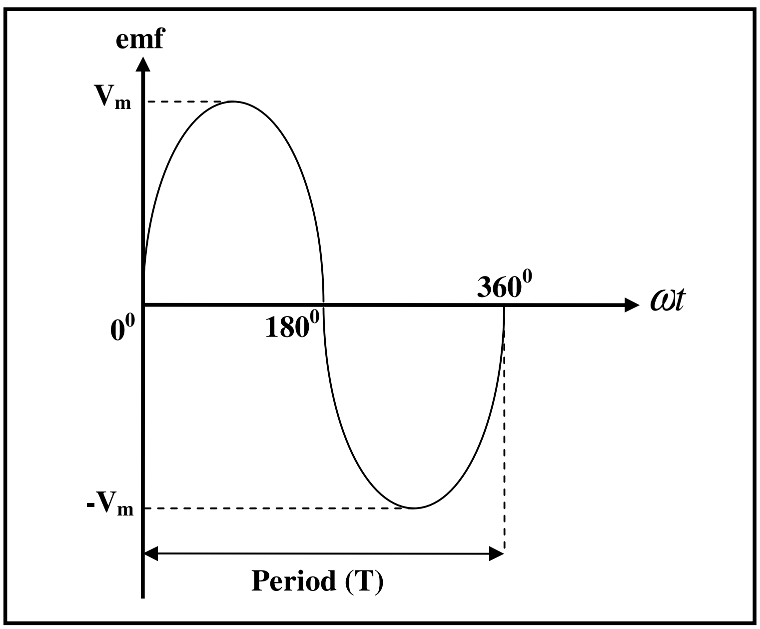


Figure 2.12: AC Waveform

$$v(t) = v_m \sin \omega t \qquad (2.22)$$

where $v(t) =$ Instantaneous voltage (volt)

$v_m =$ Maximum/peak voltage (volt)

$\omega t =$ Phase angle against time (rad/degree)

$T = \frac{2\pi}{\omega}$ (second)

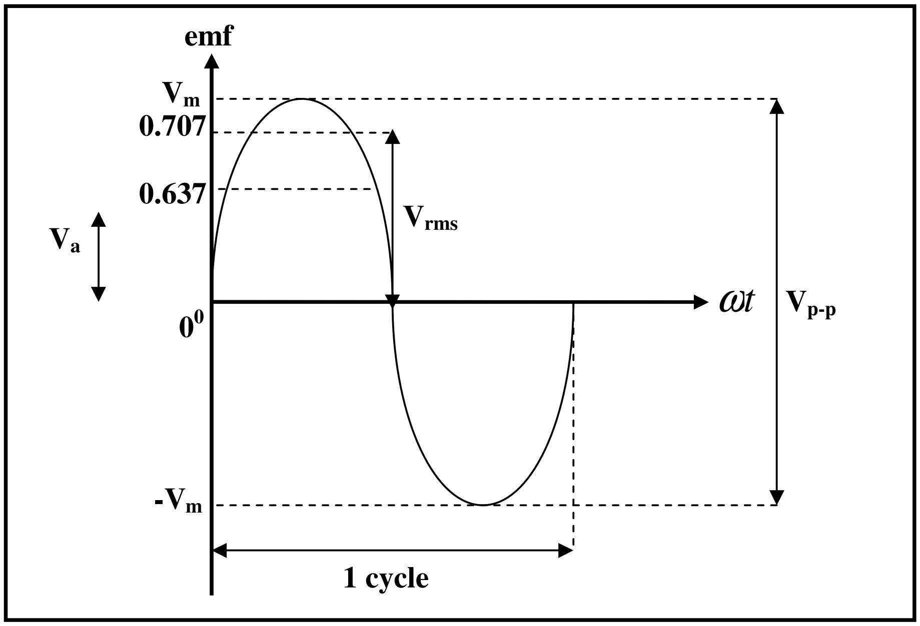


Figure 2.13: Terms in AC Waveform

A complete cycle/period of sine wave is 360^0 degree where 360º = 2Π radian. The terms related to the AC waveform:

a) V_P (peak voltage) is the maximum voltage (V_m) from the waveform.

$$V_P = V_m \qquad (2.23)$$

b) V_{PP} (peak to peak voltage) is the value that start from +ve maximum to –ve maximum.

$$V_{PP} = 2V_m \qquad (2.24)$$

c) V_a (average voltage) is the average value for sinus wave where the value is calculated for the area under ac wave line. The value is 63.7% of maximum voltage value.

$$V_a = 0.637\ V_m \qquad (2.25)$$

d) V_{rms} (root mean square voltage) is the important value in electric circuits. The most of meter indicate the reading of value in rms that equal to 70.7% of the ac peak voltage value.

$$V_{rms} = 0.707\ V_m \qquad (2.26)$$

2.12 TYPES OF AC WAVEFORM

There are 2 types of waveforms in AC:

1. In Phase Waveform
2. Different Phase Waveform

2.12.1 In Phase Waveform

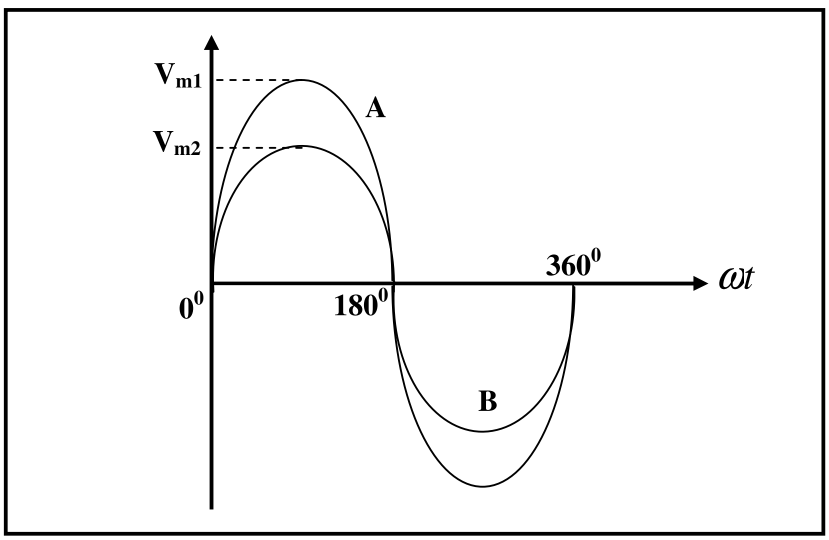


Figure 2.14: In Phase Waveform with Different Value of Vm

Sine wave for A and sine wave for B are in phase because there is no difference in phase angle between them. But, both waves have different value of maximum voltage. The maximum voltage for A is V_{m1} while the maximum voltage for B is V_{m2}. Therefore, both waves may described by use trigonometry Equation 2.27

$$\left.\begin{aligned} A: v(t) &= V_{m1}\sin\omega t \\ B: v(t) &= V_{m2}\sin\omega t \end{aligned}\right\} \qquad (2.27)$$

2.12.2 Different Phase Waveform

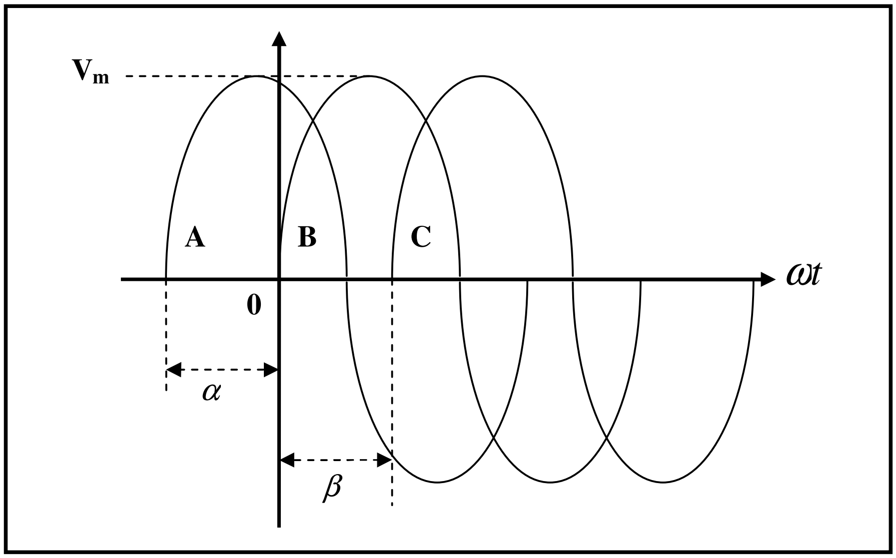


Figure 2.15: Different Phase with Same Value of Vm

In this case, all waves have the same maximum voltage (V_m), but reach at different period/time. Thus, there are phase differences between all waves. The phase difference is depend on the phase angle value (α and β). The wave through the 0^0 will be the reference point.

Therefore the trigonometry equations for three waves above are as below:

a) Wave B is the reference point for the three waves.

$$v(t) = V_m \sin \omega t$$

b) Wave A leads the wave B by a phase angle α

$$v(t) = V_m \sin(\omega t + \alpha) \qquad (2.28)$$

c) Wave C lags behind the wave B by a phase angle β

$$v(t) = V_m \sin(\omega t - \beta) \qquad (2.29)$$

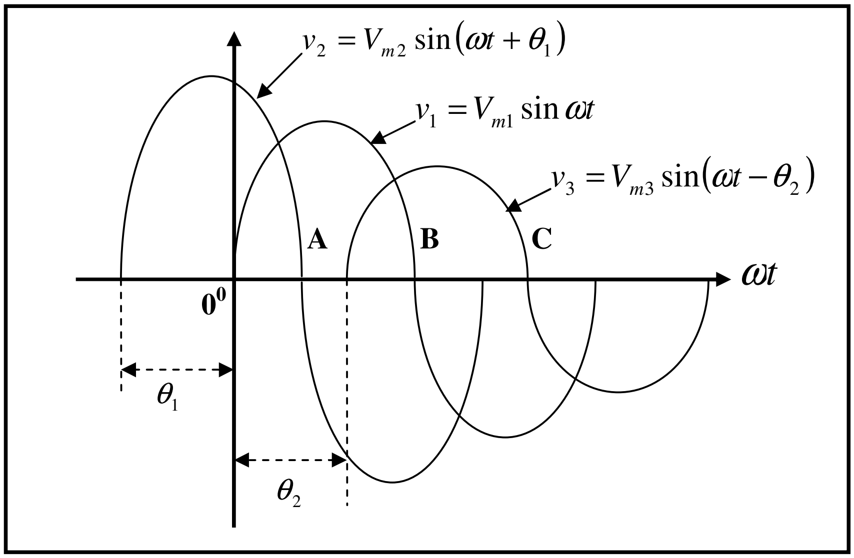


Figure 2.16: Different Phase with Different Value of Vm

2.13 VECTOR/PHASOR DIAGRAM

Vector diagram is a graft provides the information of the magnitude (amplitude) and direction (phase) of a sinusoidal wave. The vector diagram is drawn corresponding to a fix zero point or known as point of origin. A vectors magnitude is the peak value of the sinusoid while a phase magnitude is the rms value of the sinusoid.

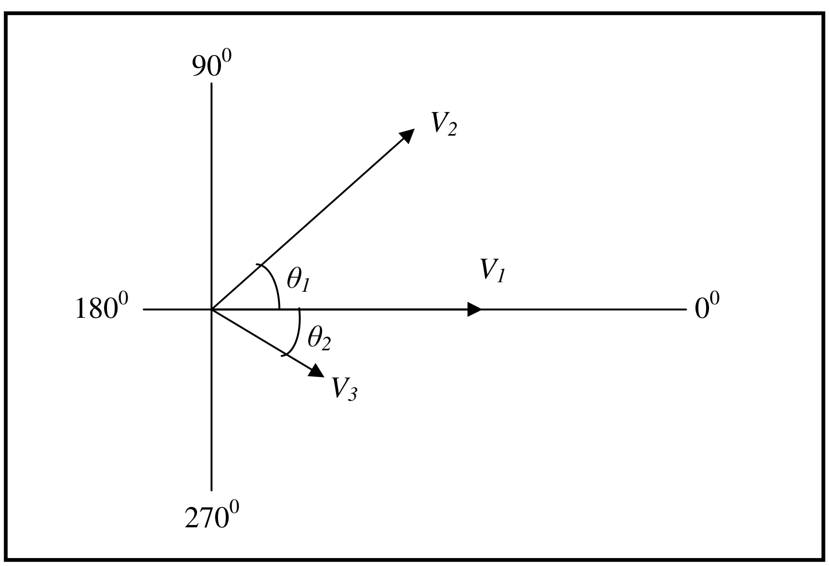


Figure 2.17: Vector Diagram

Figure 2.17 is a vector diagram for the AC wave in Figure 2.16. The length of the arrow is refers to the magnitude which depend on the peak value of each wave. Meanwhile the direction the vectors are located based on the phase different of each waves started at zero (0^0) or origin point.

2.14 BASIC TYPES OF AC CIRCUIT

There are 3 basic types of AC circuit:

1. Purely Resistance
2. Purely Inductance
3. Purely Capacitance

2.14.1 Purely Resistance

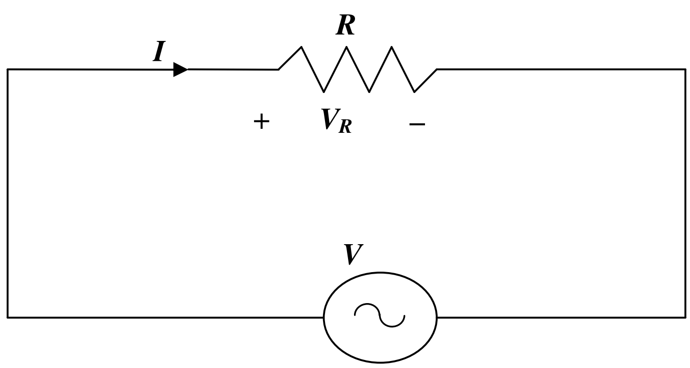


Figure 2.18: Purely Resistance Circuit

By applying an alternating voltage to a circuit that contain the resistor, the alternating current value in the circuit can be determined by Ohm's Law as equation 2.30

$$I = \frac{V}{R} \qquad (2.30)$$

In a purely resistance circuit, the current and the voltage are in phase because there is no difference angle. Hence, the wave diagram and the vector diagram are shown in Figure 2.19

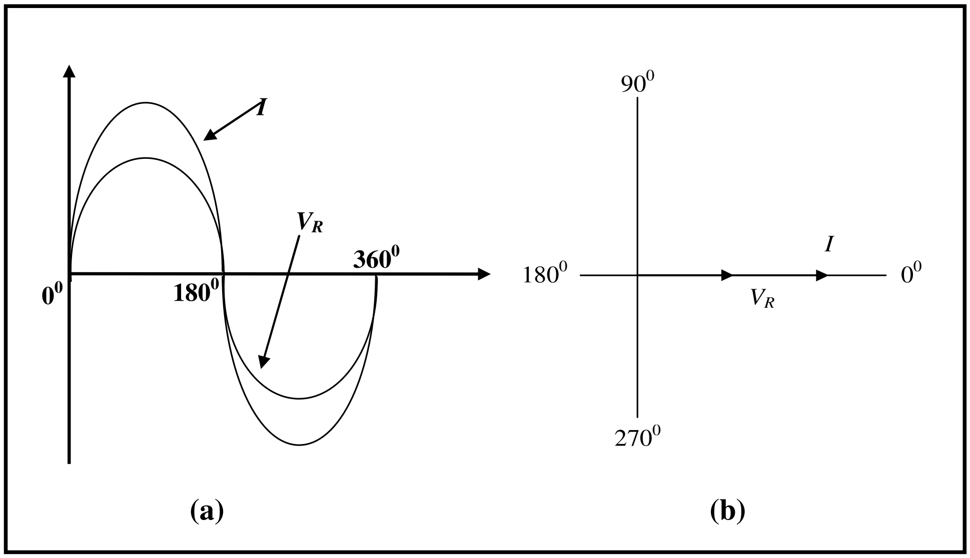


Figure 2.19: Purely Resistance (a) Waveform (b) Vector Diagram

The effects of resistance in AC circuit are:

a) If the resistance increases then the current decreases.
b) If the resistance decreases then the current increases.

AC current flow which flow in ac circuit with purely resistance is not influence by the value of the frequency.

2.14.2 Purely Inductance

Purely inductance ac circuit is a circuit containing only an inductor. When the currents flow in the inductance coil, the coil becomes an electromagnet. The electromagnet will generate the induced voltage that opposes the flowing of current in the coil circuit.

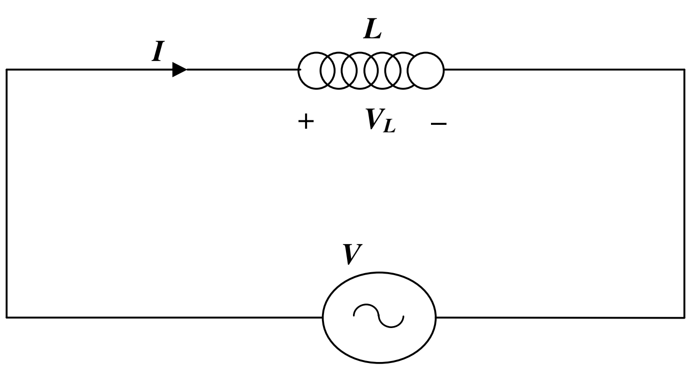


Figure 2.30: Purely Inductance Circuit

The current in purely inductance circuit lags behind the voltage by a phase angle of 90^0. Therefore, Figure 2.31 shows the waveform and vector diagram for purely inductance circuit.

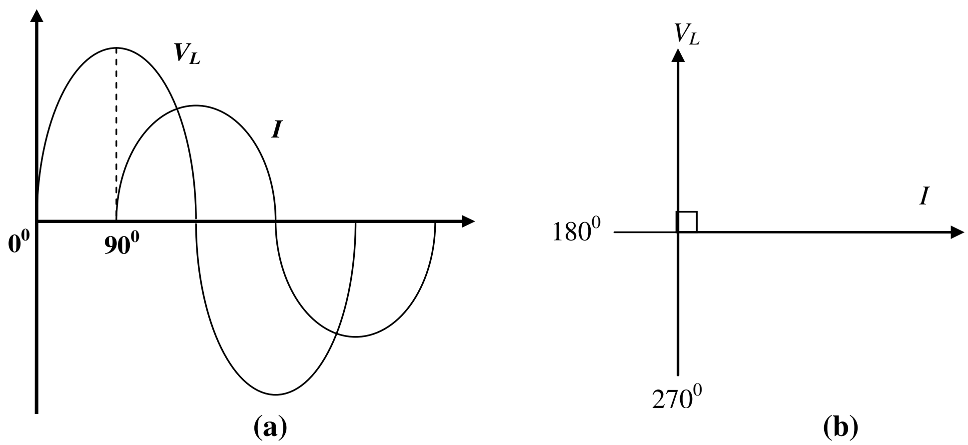


Figure 2.31: Purely inductance (a) Waveform (b) Vector Diagram

The effects of inductance in AC circuit are:

a) The value of inductance reactance is equal to resistance of resistor.
b) The inductance reactance is directly proportional to the frequency. When the frequency is increases, the voltage also increases and the reactance is increases too.

2.14.3 Purely Capacitance

Purely capacitance is an ac circuit containing only a capacitor.

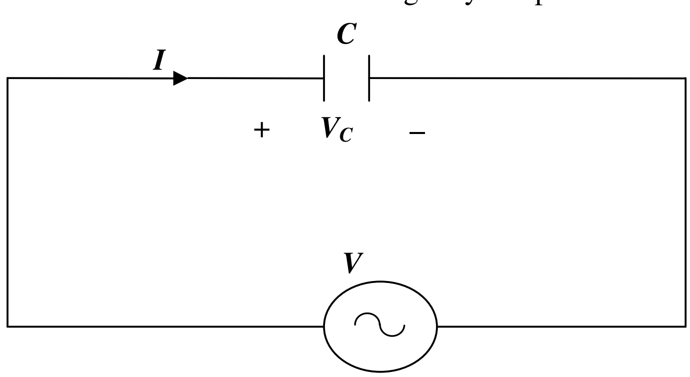
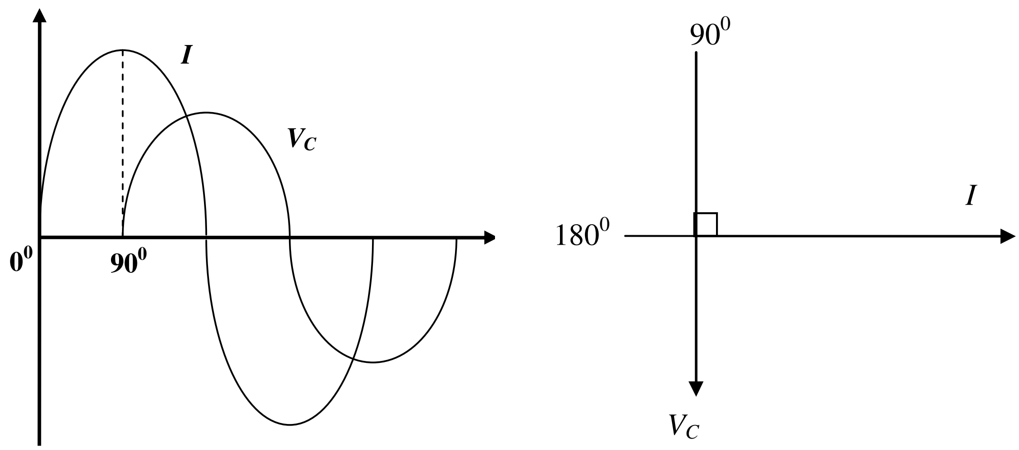


Figure 2.32: Purely Capacitance Circuit

In purely capacitance circuit, the current leads the voltage by a phase angle of 90^0.



(a) (b)

Figure 2.33: Purely capacitance (a) Waveform (b) Vector Diagram

The effects of capacitance in AC circuit are:

a) Capacitance reactance value is equal to the resistance value of resistor.
b) The capacitance reactance is directly proportional to the frequency. When frequency is increases, hence capacitance reactance is also increases.

NOTE:

C I V I L

C (CAPACITANCE) **– I V** (The current leads the voltage by a phase angle of 90^0)

L (INDUCTANCE) **– V I** (The voltage leads the current by a phase angle of 90^0)

Resistance (R) and reactance (X_L or X_L) are differentalthougt same in unit (Ω).

- ✓ Resistance is oppose to the current flow in both DC and AC circuits.
- ✓ Reactance is oppose to the current flow only in AC circuit
- ✓ Impedance (Z) also oppose to the current flow in only AC circuit.

2.15 AC CIRCUIT ANALYSIS

2.15.1 Series Resistance and Inductance (RL)

The RL circuit is the combination between resistor and inductor in series. In a series circuit, the current value is the same for each load. Thus, the current (I) become the reference factor in the vector diagram.

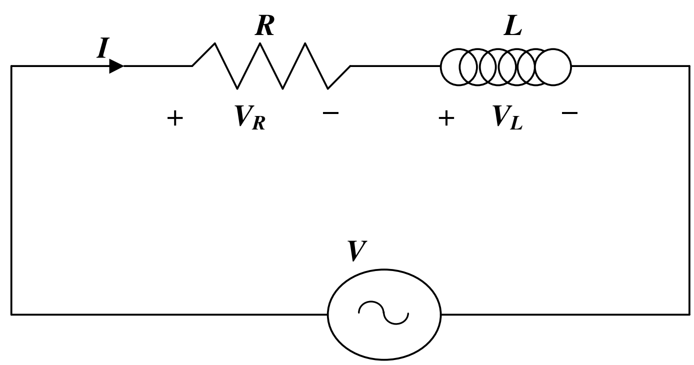


Figure 2.34: Series RL Circuit

The value of the current is limited by resistance and inductance reactance. The current flows through the resistance, R is in phase with the voltage but lags behind the voltage by a phase angle of 90^0 when flows through the inductance reactance, X_L.

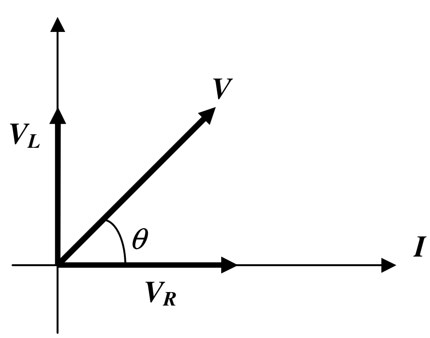


Figure 2.35: Vector Diagram for Series RL Circuit

Based on Figure 2.34, the magnitude of the supply voltage (V) can determine using Pythagoras theorem.

$$V = \sqrt{V_R^{\ 2} + V_L^{\ 2}} \quad \text{.................................} \quad (2.31)$$

The voltage drop at each components can be calculate using equations below;

$$V_R = IR_L \text{ and } V_L = IX_L \quad \text{.................................} \quad (2.32)$$

The impedance is the amount of impediment/resistance that exist in the ac circuit. The symbol for impedance is Z and unit is Ohm (Ω). An impedance triangle in Figure 2.36 show the relationship between the resistance (R), inductance reactance (X_L) and impedance (Z) can be generated based on the Figure 2.35.

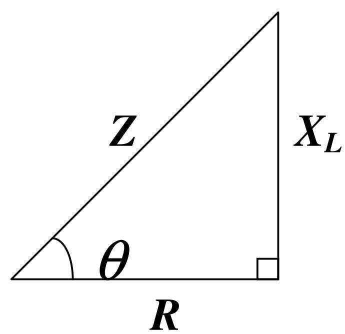


Figure 2.36: Impedance Triangle for RL

Impedance can be calculates using Equation 2.33.

$$Z = \sqrt{R^2 + X_L^{\ 2}} \quad \text{.........................} \quad (2.33)$$

Or

$$Z = R + jX_L$$

$$Z = r < \theta$$

Where $X_L = 2\Pi fL$

Phase angle and power factor for RL circuit can be calculates using equations below:

Phase angle, $\theta = \tan^{-1}\left(\dfrac{X_L}{R}\right)$ .... (2.34)

Power factor, pf = $\cos\theta = \dfrac{R}{Z}$ .... (2.35)

Example 2.7

The RL series circuit have 10Ω resistor, 0.2H inductor and supplied with 250v 50Hz AC supply. Calculates;

i. lmpedance, Z
ii. Current, I
iii. Phase angle, θ

Solution 2.7

i. lmpedance , Z

X_L = 2ΠfL = 2Π(50)(0.2) = 62.83 Ω

Z = R + jX_L
= 10 + j62.83
= 63.62 < 80.95º Ω

ii. Current, I

I = V/Z

$= \dfrac{250 < 0^\circ}{63.62 < 80.95^\circ}$

= 3.929 < -80.95º A

iii. Phase angle, θ

θ = tan $^{-1}$ (X_L/R)
= tan $^{-1}$ (62.83/10)
= 80.95º

2.15.2 Series Resistance and Capacitance (RC)

The RC circuit is the combination between resistor and capacitors in series. The current value is the same for each load. Thus, the current (I) become the reference factor in the vector diagram.

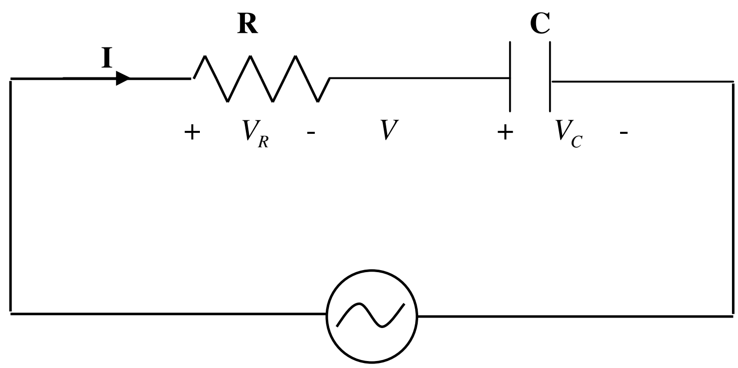


Figure 2.37: Series RC Circuit

The value of the current is limited by resistance, R and capacitance reactance, X_C. The current flows through R is in phase with the voltage but leading by a phase angle of 90^0 when flows through inductance reactance. Figure 2.38 is a vector diagram for series RC circuit.

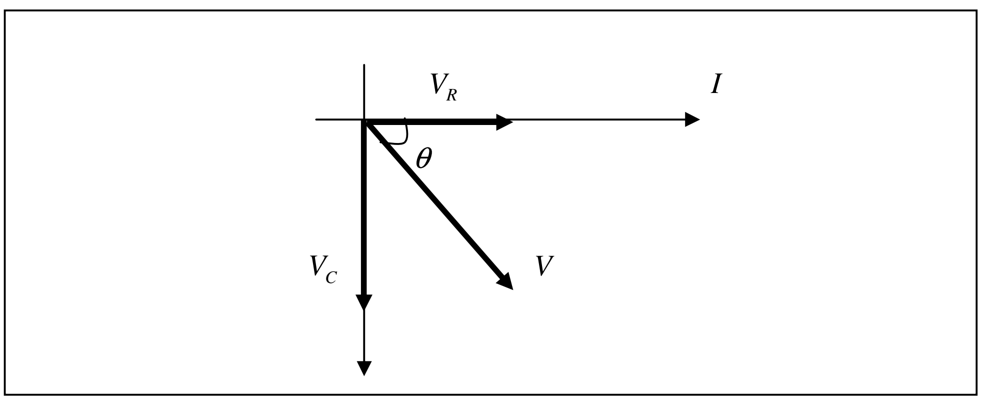


Figure 2.38: Vector Diagram for Series RC Circuit

Based on Figure 2.38, the magnitude of the supply voltage (V) can determine using Pythagoras theorem.

$$V = \sqrt{V_R^{\ 2} + V_L^{\ 2}} \quad \text{.........} \quad (2.36)$$

The voltage drop at each components can be calculate using equations below;

$$V_R = IR_L \text{ and } V_C = IX_C \qquad (2.37)$$

An impedance triangle in Figure 2.39 show the relationship between the resistance (R), capacitance reactance (X_C) and impedance (Z) which generated based on the Figure 2.38.

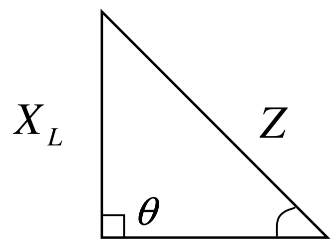


Figure 2.39: Impedance Triangle for RC

Impedance can be calculates using Equation 2.38.

$$Z = \sqrt{R^2 + X_C^2} \qquad (2.38)$$

Or

$$Z = R - jX_C$$

$$Z = r < -\theta$$

Where $X_C = 1 / (2\Pi fC)$

Phase angle and power factor for RC circuit can be calculates using equations below:

Phase angle, $\theta = \tan^{-1}\left(\frac{X_C}{R}\right)$ (2.39)

Power factor, pf = $\cos\theta = \frac{R}{Z}$ (2.40)

Example 2.8:

A 10Ω resistor and 200μF capasitor are connected in series across a 120V, 50V AC supply. Calculate:
i. Impedance, Z
ii. Current, I
iii. Power factor, pf

Solution 2.8:

Given: $R = 10\Omega$, $C = 200\mu F$, $V = 75V$ dan $f = 50Hz$.

$$\text{where, } X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(50)(200x10^{-6})} = 15.92\Omega$$

i. Impedance, $Z = R - jX_C$
$= 10 - j15.92$
$= 18.8 < -57.86° \Omega$

ii. Current, I $= V/Z$
$= \frac{75 < 0°}{18.8 < -57.86°}$
$= 3.989 < 57.86°$

iii. Power Factor,pf = $\cos\theta = \frac{R}{Z} = \frac{10}{15.92} = 0.628$

2.15.3 Series Resistance, Inductance and Capacitance (RLC)

The RLC circuit is the combination of resistor, indictor and capacitor in series with AC supply. The current value is the same for each load. Thus, the current (I) become the reference factor in the vector diagram.

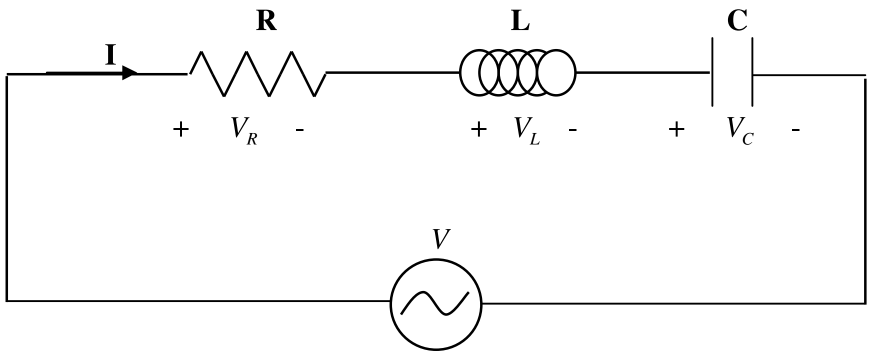


Figure 2.40: Series RLC Circuit

In RLC there are (2) conditions should to be consider;

a) inductance reactance is greather than capacitance reactance, $X_L > X_C$

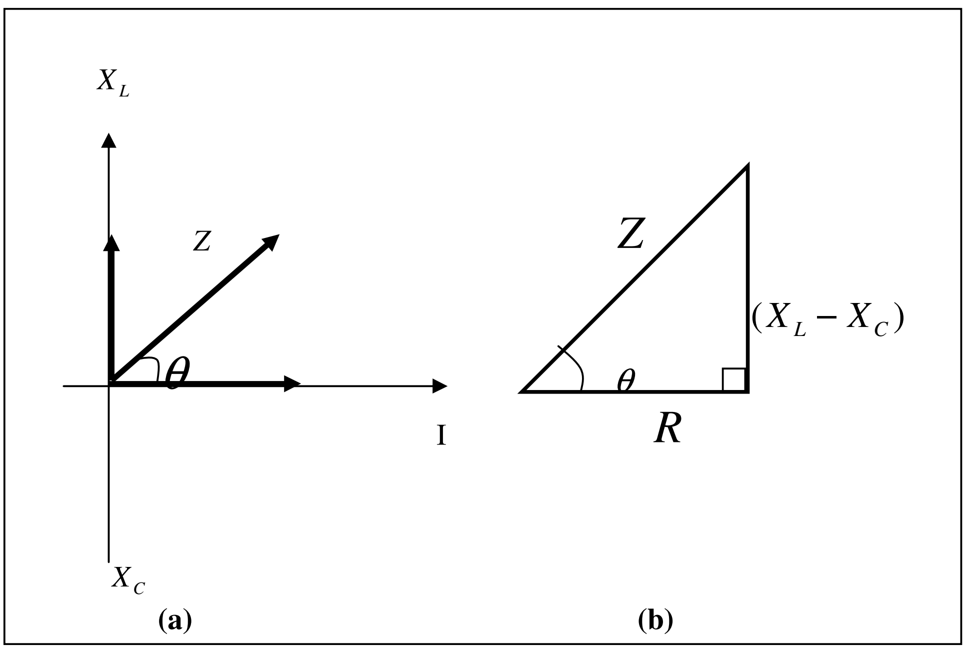


Figure 2.41: (a) Vector Diagram (b) Impedance Triangle for $X_L > X_C$

The impedance for RLC can be calculate using Equation 2.41 below:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad (2.41)$$

Or

$$Z = R - j\,(X_L - X_C)$$

$$Z = r < \theta$$

Voltages drop at each components can be calculate using equations below;

$V_R = IR$, $V_C = IX_C$ and $V_L = IX_L$ .... (2.42)

Phase angle and power factor for RC circuit can be calculates using equations below:

Phase angle, $\theta = \tan^{-1}(\frac{X_L - X_C}{R})$ .... (2.43)

Power factor, pf = $\cos\theta = \frac{R}{Z}$ .... (2.44)

b) Capacitance reactance is greather than inductance reactance, X_C **>** X_L

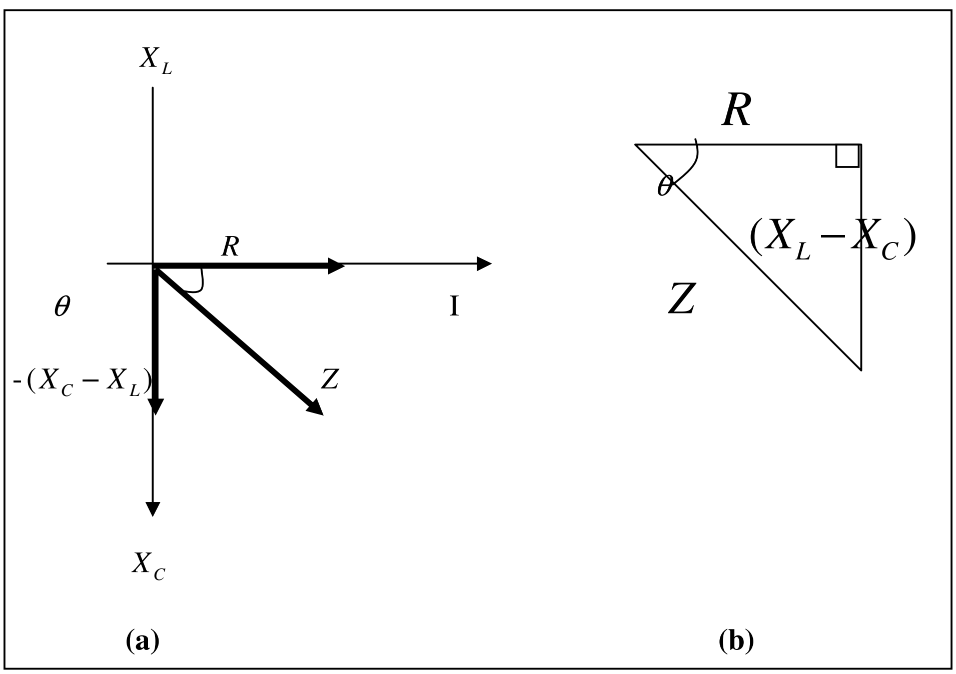


Figure 2.42: (a) Vector Diagram (b) Impedance Triangle for X_C **>** X_L

The analysis for calculating impedance, current, voltage drop at each components, phase angle and power factor are same as $X_L > X_C$. The different only at the value of phase which is –ve that show the direction of the angle.

$$Z = r < -\theta$$

2.16 POWER FACTOR

Power factor can be express in the form of percentage (%) or fractional numbers. It is known as cos θ and referred to as leading (lead) or lagging (lag) in which the phase angle between voltage and current.

a. Power factor is a ratio between real power and apparent power.

$$Cos\theta = \frac{P}{S} \quad \text{.......} \quad (2.45)$$

b. Power factor is a ratio between resistance and impedance.

$$Cos\theta = \frac{R}{Z} \quad \text{.......} \quad (2.46)$$

c. Leading power factor is voltage leading the current when voltage as refference factor and value of the voltage is positive.

d. Lagging power factor is voltage lagging the current the when voltage as refference factor and value of the voltage is negative.

e. Best value of power factor is where $Cos\theta = 1$ or nearly 1.

2.17 POWER IN AC CIRCUIT

There are 3 types of power in the ac circuit;

a) Apparent power, S
b) Real Power, P
c) Reactive Power, Q

2.17.1 Apparent power , S

Power is reduced due to the existence of the reactance that cause current and voltage is not in phase. The separation of current and voltage caused the power in the circuit will be reduced. The simbol is S and unit is Volt –Ampere (VA)

Apparent Power = Voltage x Current

$$S = VI \qquad (2.47)$$

2.17.2 Real Power, P

Real power or active power is the power consumed or absorbed by the resistor components in ac circuits. The symbol is P and the unit is watt (w).

Real Power = Voltage x Current x Power Factor

$$P = VI\cos\theta \qquad (2.48)$$

2.17.3 Reactive Power, Q

Reactive Power or Reactance Power is the power consumed or absorbed by the capacitor or inductor components in ac circuits . The simbol is Q and unit is Volt Ampere Reactive (VAR)

Reactive Power = Voltage x Current x Sinθ

$$Q = VI\sin\theta \qquad (2.49)$$

2.18 POWER TRIANGLE

Power triangle shows the relationship between the apparent power, real power and the reactive power.

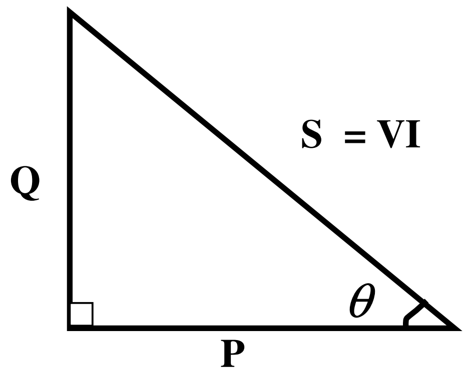


Figure 2.43: Power Triangle

Example 2.9:

A RLC circuit was connected in series with 100 ohms resistor, 200mikroF capacitor and 100mH inductor then supplied with AC power supply 240V, 50Hz. Calculate;

i. Impedance, Z
ii. Current, I
iii. Power factor and phase angle
iv. Power in kVA, kW and kVAR .

Solution 2.9:

$$X_L = 2\pi fL = 2\pi(50)(100x10^{-3}) = 31.42\Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(50)(200x10^{-6})} = 15.91\Omega$$

i. Impedance, $Z = R + j\ (X_L - X_C)$

$$Z = 100 + j\ (31.42 - 15.91)$$

Z = 100 + j 15.51

Z = 101.19 < 8.9º Ω

ii. $I = \frac{V}{Z}$

= $\frac{240 < 0^{o}}{101.19 < 8.9^{o}}$

= 2.37 < -8.9º

iii. Power factor,pf = $Cos\theta = \frac{R}{Z} = \frac{100}{101.2} = 0.988$

Phase Angle, $\theta = \cos^{-1}(\frac{R}{Z}) = \cos^{-1}(0.988) = 8.9^{o}$

iv. Apparent Power, S in kVA,

$$S = VI = (240)(2.37) = 568.8 = 0.57kVA$$

Real Power in kW,

$$P = VI\cos\theta = (568.8)(0.988) = 562 = 0.562kW$$

Reactive Power in kVAR,

$$Q = VI\sin\theta = (568.8)(\sin 8.9^{o}) = 88 = 0.09kVAR$$

REFERENCES

Hughes, E., 1960, "Electrical Technology 3rd Edition", University of Michigan, Longmans

Erickson, W.H., 1952, "Electrical engineering, theory and practice", University of Wisconsin – Madison, Wiley

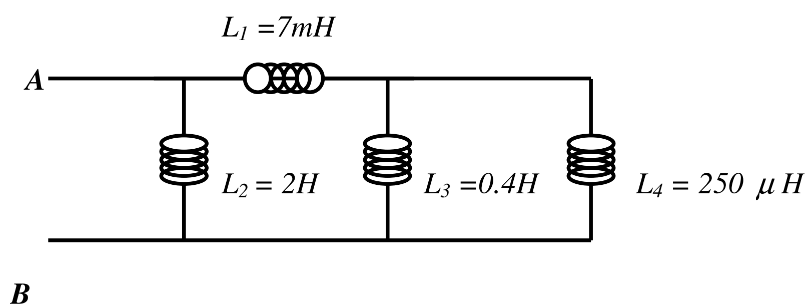
PROBLEMS

1. Calculate the total inductance (L_T) which connected in series and parallel for combination of three coils below:
   - i. 0.02H, 44mH and 400µH,
   - ii. 0.05H, 30mH and 755µH
   - iii. 0.08 H, 400mH and 400µH
   - iv. 15 mH, 50 mH and 75 mH.

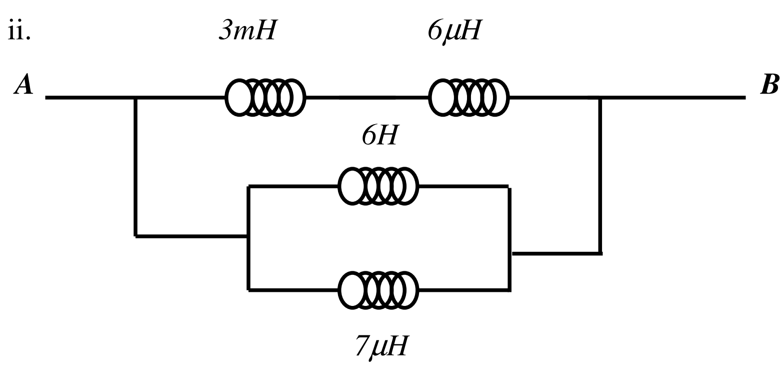
   (64.4mH, 388µH, 80.7mH, 725.7µH, 480mH, 397.6 µH, 140mH, 10mH)

2. Based on figure below, calculate the total inductance (L_T) if measured at point A and B.

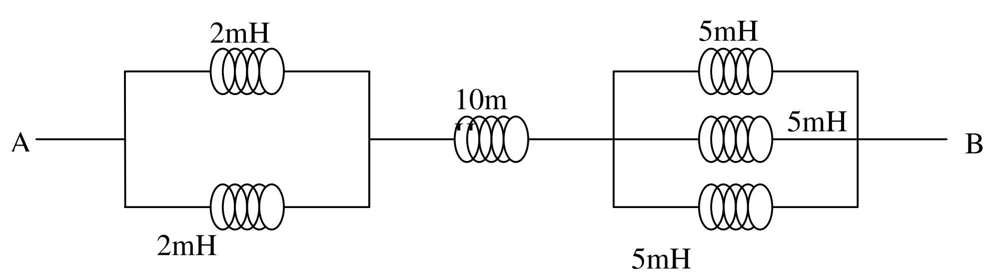
   i.



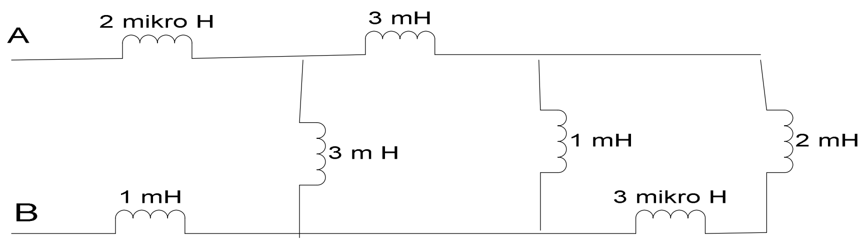
   ii.



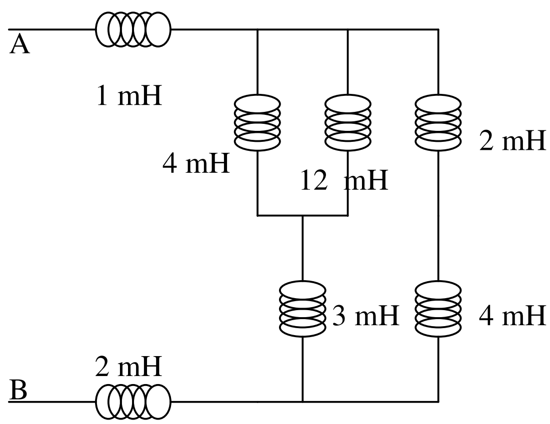
   iii.



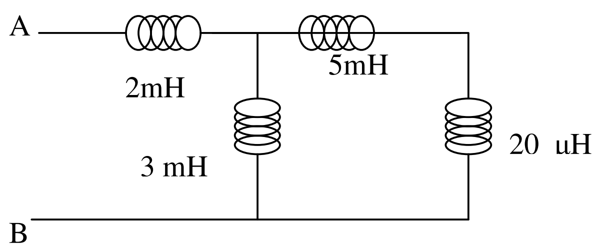
iv.



v.



vi.



(10mH, 6.98µH, 12.6mH, 4.65mH, 6mH, 3.88mH)

3. Coil with 0.2H connected with AC circuit 200V, 50Hz. Calculate the Inductave reactance.

(62.8Ω)

4. A coil with 6H connected to AC 12 V 50 Hz. Calculate the current flow.

(0.19A)

5. A capacitor with 50µF connected to AC 115 V 60 Hz. Calculate capacitance reactance and current flow.

(53.1Ω, 2.167A)

6. A capacitor with 120µF connected with 500 V 50 Hz. Calculate capacitance reactance and current flow.

(26.52Ω, 18.8A)

7. A capacitor 1000µF connected to AC 20 V 50 Hz.
   i. Calculate current flow
   ii. What is the effect of the current if the frequency change to 1000Hz.

(6.28A, increasing)

8. Capacitor with 50µF connected with 240V power supply. Calculate the charge and energy stored in the capacitor.

(0.012 C, 1.44 J)

9. Capacitor with 8µF connected with 240V, 50Hz power supply. Calculate the Capacitance Reactance.

(397.88Ω)

10. Calculate the total capacitance (C_T) which connected in series and parallel for combination of three capacitors below:
    i. $120\mu F$, $240\ \mu F$ dan $360\ \mu F$

(65.45µF, 720µF)

11. Calculate the value of capacitor which is connected in series with other 60µF capasitor, where the total capacitance is 15µF.

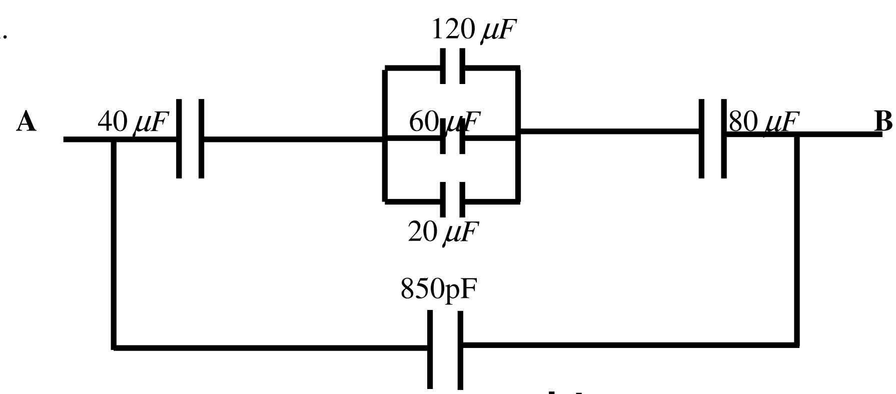
(20µF)

12. 2 capacitors with values 6µF and 10µF respectively, connected in series with 200V power supply. Calculate ;
    i. Total Capacitance
    ii. Charge at each capacitors
    iii. Voltage drop at each capacitors

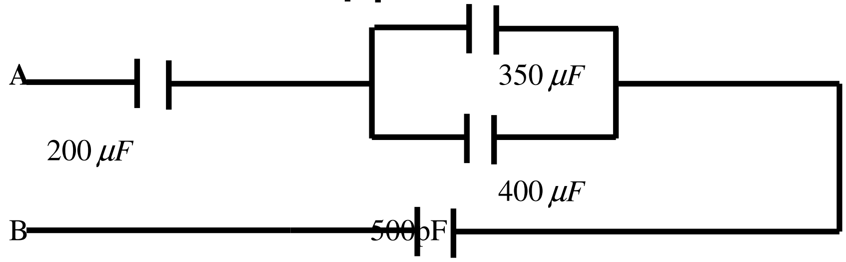
(3.75µF, 750x10^{-6}C, 125V and 75V)

13. Based on figure below, calculate the total capacitance (C_T)

i.



ii.



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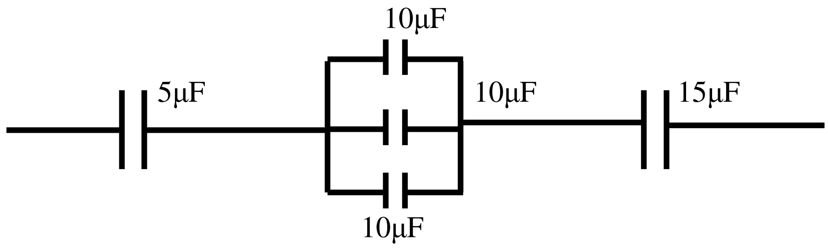

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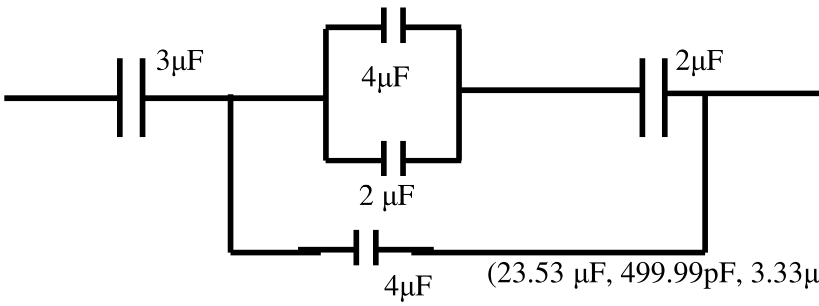
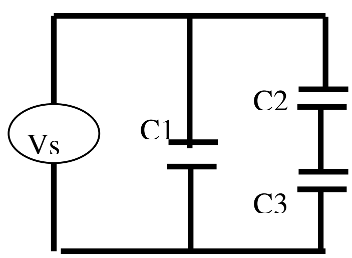


(23.53 µF, 499.99pF, 3.33µF, 1.94µF)

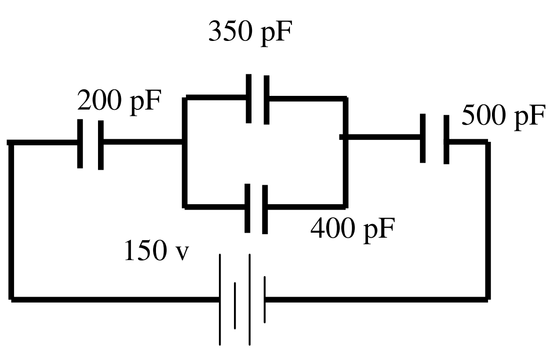
14. 3 capacitors C1 = 6 µF, C2 = 12 µF dan C3 = 16 µF connected with 60 v power supply as shown in figure below. Calculate:



i. Total Capacitance
ii. Charge at each capacitors
iii. Voltage drops at each components
iv. Energy used.

(12.85 µF, 3.6×10^{-4} C, 4.12×10^{-4} C, 60v, 34.28v, 25.72v, 0.023J)

15. Based on figure below, calculate total capacitance, C_T and total charge, Q_T.



(120pF, 18×10^{-9} C)

16. A circuit wit 3 capacitors C_1, C_2 and C_3 connected in series wit values 3µF, 4pF and 1µF respectively. If the voltage supply is 100v, calculate:
    i. Total capacitance, C_T
    ii. Total Charge, Q_T
    iii. Voltage drop at C_1

(3.99pF, 3.99x10^{-10}C, 0.133mV)

17. Series RL with 25Ω and 25mH connected to AC 60 V, 100Hz. Calculate the current and phase angle reffer to supplied voltege.

(2.03<0°v, 32.12°)

18. Series RLC with R=33Ω, L=50mH and C=10µF. Voltage supply 75 V, 200Hz. Calculate I, V_R , V_C ,V_L and phase angle reffer to supplied voltege.

(2.02<26.89°A, 66.7<26.89°V, 126.9<116.89°V, 150.7 <60.11°V)

19. A circuit with 25Ω resistance, 0.2 H inductance and 1µF capacitance connected in series to AC 100 V, 50 Hz. Calculate :
    i. Impedance, Z.
    ii. Current,I

(67.62<68.3°Ω, 1.47<-68.3°A)

20. Series RLC circuit with 20Ω, 0.1 H and 40µF respectively was connected to AC 230V, 50 Hz. Calculate:
    i. Impedance, Z.
    ii. Current, I
    iii. Voltage drop at each components V_R , V_C andV_L
    iv. Power factor
    v. Draw the vector diagram

(52.14<-67.44°Ω, 4.41<67.44°A, 88.2<67.44°V, 138.5<157.44°V, 350.9< -22.56°V, 0.384)

21. A circuit with resistance 50Ω, inductance 0.15 H and capacitance 100µF connected in series with AC 100 V, 50 Hz. Calculate:
    i. Inductice reactance, capacitice reactance and impedance
    ii. Current
    iii. Voltage drop at each components V_R , V_C andV_L

(94.24Ω, 15.9Ω, 92.9<57.43°Ω, 1.1<-57.43°A, 55<-57.43°V, 103.6<32.57°V, 17.49<-147.43°V)

22. AC circuit 200V, 50 Hz connected in series with resistance 40Ω, inductance reactance 20Ω and capacitance reactance 12Ω. Calculate:
    i. Impedance, Z.
    ii. Current, I

iii. Voltage drop at each components V_R , V_C andV_L

iv. Phase angle

v. Power factor

vi. Faktor kuasa

vii. Draw the Vector diagram

(40.79<11.3°Ω, 4.9<-11.3°A, 196<11.3°V, 98<101.3°V, 58.8<-78.7°V)

23. The RLC series circuit connected to AC 250V, 50 Hz with reactance 40Ω, inductance 0.4H and capacitance 150μF. Calculate:

i. Impedance, Z.

ii. Current, I

iii. Voltage drop at each components V_R , V_L andV_C

iv. Phase angle

v. Power factor

vi. Apparent power

(111.83<69°Ω, 2.23<-69°A, 89.2<69V, 280.22<159°V, 47.32<-21°V, 69°, 0.358, 557.5VA)

24. Series RLC circuit with 50Ω, 10mH and 100μF. Supplied with AC 240 <30˚ V 50 Hz. Calculate:

i. Impedance, Z.

ii. Current, I

iii. Voltage drop at each components V_R , V_L and V_C

iv. Power factor

(57.64<-29.84°Ω, 4.16<59.84°A, 208.18<59.84°V, 13.06<149.84°V, 132.41<
-30.16°V, 0.867)