2$^{\text{nd}}$ Meeting IST - IME

STABILITY THEORY AND NONUNIFORM HYPERBOLICITY
Luis Barreira

IST

I will survey recent results obtained jointly with Claudia Valls on the stability theory of nonautonomous equations, in the general context of nonuniform hyperbolicity. In particular, after explaining why this context is so general, I will discuss the Lyapunov stability of solutions, the existence and smoothness of invariant manifolds, and the construction and regularity of conjugacies, always in the infinite-dimensional setting. The necessary material from smooth ergodic theory will be recalled along the way.

ON THE ESSENTIAL SPECTRUM OF MARTIN-MORALES-NADIRASHVILI'S MINIMAL SURFACES
Gregorio Pacelli Bessa

Universidade Federal do Ceará

We prove that the essential spectrum of complete minimal surfaces properly immersed in an open ball is empty. This is a joint work with J. Fabio Montenegro and L. Jorge.

CHARACTERIZATION OF SOME NON-AUTONOMOUS ATTRACTORS
Alexandre Carvalho

Universidade de São Paulo

We introduce the concept of a gradient-like nonlinear semigroup as an intermediate concept between a gradient nonlinear semigroup (those possessing a Lyapunov function), and a nonlinear semigroup possessing a gradient-like attractor. We prove that a perturbation of a gradient-like nonlinear semigroup remains a gradient-like nonlinear semigroup. Moreover, for non-autonomous dynamical systems we introduce the concept of a gradient-like evolution process and prove that a non-autonomous perturbation of a gradient-like nonlinear semigroup is a gradient-like evolution process. For gradient-like nonlinear semigroups and evolution processes, we prove continuity, characterization and (pullback and forwards) exponential attraction of their attractors under perturbation extending the results of on characterization and of on exponential attraction.

FRACTAL DIMENSION OF BI-SPACE ATTRACTORS
Radoslaw Czaja
Uniwersytet Slaski

We consider bi-space global and exponential attractors for the time continuous dynamical systems and discuss the bounds on their fractal dimension in the context of the smoothing properties of the system between appropriately chosen function spaces.

In particular we consider global attractors with bounded fractal dimension for a semigroup governed by an abstract semilinear parabolic equation in a Banach space. As specific applications we present a strongly damped wave equation, including the case when the resolvent operators corresponding to a linear operator are non-compact and nonlinear term satisfies a critical growth condition, and the higher order parabolic problems involving $2m-$th order elliptic operators in the main part and fast growing nonlinearities.

More examples can be found in

[1] J. Cholewa, R. Czaja, G. Mola, Remarks on the fractal dimension of bi-space global and exponential attractors, Bollettino U.M.I. (9) I (2008), 121–145.

LOCAL "SUPERLINEARITY" AND "SUBLINEARITY" FOR THE p-LAPLACIAN
Djairo G. de Figueiredo
Universidade Estadual de Campinas

Joint work with Jean-Pierre Gossez and Pedro Ubilla. We present results on existence, nonexistence and multiplicity of positive solutions for a family of problems $-\Delta_p u = f_\lambda(x, u)$, $u \in W^{1,p}(\Omega)$, where Ω is a bounded domain in R^N, $N > p$, and $\lambda > 0$ is a parameter. This family includes the nonlinearities of Ambrosetti-Brezis-Cerami type in a more general form. We discuss some auxiliary questions: extension to the p-Laplacian context of the Brezis-Nirenberg result on local minimization in $W_0^{1,p}$ and C_0^1; a $C^{1,\alpha}$ estimate for equations of the form $-\Delta_p u = h(x, u)$ with h of critical growth; a strong comparison result for the p-Laplacian, and a variational approach to the method of upper-lower solutions for the p-Laplacian.

Elliptic Systems in Dimension Two
Marcos do Ó
Universidade Federal da Paraíba

We investigated the existence of positive solutions for two classes of elliptic systems defined in smooth domains of Euclidean plane with nonlinearities involving exponential growth.

Mixmaster cosmology - a self-defeating hypothesis?
Bernold Fiedler
Institut für Mathematik I, Freie Universität Berlin

Bianchi cosmology according to Gödel and Taub is a reduction of the Einstein equations in the big bang limit to few ordinary differential equations. Asymptotic dynamics in the backwards time limit is described by a quite elementary ergodic map induced by heteroclinic orbits on the Kasner circle of equilibria. That description, however, is purely formal.

The Belinskii-Khalatnikov-Lifshitz conjecture addresses the mixmaster question: can we "see" all the universe, in the distant backwards big bang limit. While the Kasner circle map suggests that "yes, we can" the actual answer may mostly be "no".

Equivariant entire solutions to elliptic systems with variational structure
Giorgio Fusco
Universitá di L'Áquila

We consider the elliptic system

$$\Delta u = W_u(u), \quad x \in \mathbb{R}^n \ ,$$

for a class of potentials $W : \mathbb{R}^n \to \mathbb{R}$ that possess several global minima and are invariant under a finite or discrete reflection group G acting on $\mathbb{R}^n$. We establish existence of nontrivial G-equivariant entire solutions $u : \mathbb{R}^n \to \mathbb{R}^n$ connecting the global minima of W. If G is a discrete (infinite) group the solution u has a kind of crystalline periodic structure and existence is ensured provided the elementary cell contains a ball of radius R^* with R^* a constant that depends only on W.

Dynamic and Geometry of Principal Curvature Lines
Ronaldo Garcia
Universidade Federal de Goiás

In this talk we will present results about stability and bifurcations (local and global) of principal curvature lines on immersed surfaces of R^3. In particular, it will be shown that there are analytic deformations of the torus of revolution such that both principal foliations have dense leaves. Also, we will comment on some open problems.

An inverse problem for the Schrödinger operator
Augusto Gerolin
Ecole Normale Supérieur de Lyon

The goal of this talk is to well pose and solve the following semi-classical inverse problem: The potential $V(x)$ in the Schrödinger operator $\hat{H} = -h\dfrac{d^2}{dx^2} + V(x)$ can be recovered, under some weak genericity assumptions, from the spectrum of $\hat{H}$ modulo $o(h^3)$. This work was supervised by Yves Colin de Verdière.

On a diffusive logistic equation with harvesting
Pedro Girão
IST

We construct some global solution curves for a diffusive logistic equation with harvesting, under Dirichlet boundary conditions and other appropriate hypotheses. Our analysis provides new information on the number of solutions of the equation.

Bernstein estimates for weakly coupled fully non-linear elliptic systems
Diogo Gomes
IST

In this talk we discuss Bernstein estimates for weakly fully non-linear elliptic systems. We are particularly interested in systems that arise in the stochastic optimal control problems of hybrid systems. For these a generalization of Bernstein estimates for first and second derivatives of classical solutions will be presented.

Jair Koiller
Fundação Getúlio Vargas, Rio de Janeiro

One hundred and fifty years after Helmholtz' "Wirbel" paper the study of vortices on surfaces has been restricted basically to the sphere or surfaces of revolution. An intrinsic Hamiltonian formulation for the motion of point vortices on a closed (compact, boundaryless, orientable) surface with an arbitrary riemannian metric is in order. We hope to fill this gap. For the full version: arXiv:0802.4313v1 [math.SG].

THE PRESENT OF THE DARBOUX THEORY OF INTEGRABILITY
Jaume Llibre
Universitat Autònoma de Barcelona

For a differential system or a vector field defined on the real or complex plane the existence of a first integral determines completely its phase portrait, of course working with real or complex time respectively. Since for such vector fields the notion of integrability is based on the existence of a first integral the following natural question arises: Given a vector field on the real or complex plane, how to recognize if this vector field has a first integral? The more easiest planar vector fields having a first integral are the Hamiltonian ones. The integrable planar vector fields which are not Hamiltonian are, in general, very difficult to detect. Many different methods have been used for studying the existence of first integrals for non–Hamiltonian vector fields. In this talk we will do a survey (including new results) of the Darboux theory of integrability, probably the best theory of integrability for studying the integrability of planar polynomial vector fields. We also present the results of this theory in arbitrary dimension.

PIECEWISE ANALYTIC SUBACTIONS FOR ANALYTIC DYNAMICS
Artur Lopes
Universidade Federal do Rio Grande do Sul

Joint work with E.R. Oliveira and D. Smania.
We consider a piecewise analytic expanding map $f : [0; 1] \to [0; 1]$ of degree d, and an analytic positive potential $g : [0; 1] \to R$. We denote

$$m(\log g) = \max_{\nu \text{ an invariant probability for } f} \int \log g(x) d\nu(x);$$

and μ_∞ any probability which attains the maximum value. Any one of these probabilities μ_∞ is called a maximizing probability for $\log g$. In order to analyze ergodic properties of such probability μ_∞, it is natural to associate to such f a bijective transformation $\hat{\sigma}$, which acts on $\hat{\Sigma} = \Sigma \times [0;1]$, where $\Sigma = \{1, 2, \ldots; d\}^N$. One can consider W the involution kernel associated to $\log g$, where $W : \hat{\Sigma} \to \mathbb{R}$, and $W(w, x)$ is defined for all $w \in \Sigma$ and $x \in [0, 1]$. We show the existence of an analytic involution kernel for $\log g$ (in the sense that it is analytic in the second variable, for w fixed) and a interesting relation with the dual potential $(\log g)^*$ defined in the Bernoulli space Σ. Using the above results we show that when μ_∞ is unique and has support in a periodic orbit, for an open and dense set of analytic g, there exists piecewise analytic calibrated sub-action $V : [0;1] \to \mathbb{R}$ for the potential $\log g$, that is a function V such that

$$\sup_{y \text{ such that } f(y)=x} \{V(y) + \log g(y) - m(\log g)\} = V(x).$$

We assume a twist condition in some of the proofs we present here. An interesting case where the theory can be applied is when $\log g(x) = -\log f'(x)$. In this case we relate the involution kernel to the so called scaling function. Another way to see the problem we consider here is the following: for a given analytic potential $\beta \log g$, where β is a real constant, it is known that there exists an analytic eigenfunction ϕ_β for the Ruelle operator. One can ask: what happen with ϕ_β when β goes to infinity. The correct question is to ask if $\frac{1}{\beta} \log \phi_\beta$ is analytic in the limit. Under a uniqueness assumption, this limit, when $\beta \to \infty$, is in fact a calibrated subaction V (as above). We show here that under certain conditions and for a certain class of potentials this continuous function is piecewise analytic (but not analytic). In a few examples one can get that the subaction is analytic (we need at least to assume that the maximizing probability has support in a unique fixed point). Finally we present a result that shows that our hypothesis are true for generic analytic potentials.

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SUFFICIENT CONDITIONS FOR EXISTENCE OF SOLUTIONS
OF A LOWER ORDER VARIATIONAL PROBLEM

José Matias

IST

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We derive conditions on a function $f : \Omega \to \mathbb{R}^d$ and on its quasiconvex envelope that are sufficient to ensure existence of solutions of a lower order

<div align="center">6</div>

variational problem with corresponding energy density $\bar{f}$ determined through a dimension reduction process.

On the Existence and Concentration of Positive Solutions to a Class of Quasilinear Elliptic Problems on $\mathbb{R}$

Olimpio Miyagaki
Universidade Federal de Viçosa

This paper is concerned with the existence and concentration of positive solutions for the following quasilinear equation

$$\epsilon^2 v'' - V(x)v + |v|^{q-1}v + \epsilon^2 k(|v|^2)''v = 0, \quad x \in \mathbb{R}.$$

The proof relies on variational methods by using directly the functional associated with the problem in an appropriate Sobolev space. It was found a family of solutions $\{u_\epsilon\}$ which concentrate around a local minimum of V as ϵ tends to zero.

Existence of solutions for singular fully nonlinear equations

Marcelo Montenegro
Universidade Estadual de Campinas

We are going to show how to solve equations of form $F(D^2 u) = |u|^p g(\text{grad } u)$ with $p < 0$ and $u = f$ on the boundary of the domain.

On the Stability Problem of Stationary Solutions for the Euler Equation on a 2-dimensional Torus

Piero Negrini
Sapienza Università di Roma

Let $L \in (0, \infty), \tau^2(L) := \{(x, y) \in \mathbb{R}^2, \mod (2\pi L, 2\pi)\}$. The Euler Equation written in terms of the stream function $\phi \in H^2(\tau^2(L), R)$ is:

$$\frac{\partial \Delta \phi}{\partial t} + \partial_x \phi \partial_y \nabla \Delta \phi - \partial_y \phi \partial_x \nabla \Delta \phi = 0.$$

Any regular function defined on $\tau^2(L)$, depending either on the y variable or on the x variable is a stationary solution. In particular we analyze the dependence on L of the stability properties of the stationary solution $\phi^* = \cos y$.

DYNAMICS AND NUMERICS OF A REACTION DIFUSION EQUATION WITH DELAYS
Sérgio Oliva
Universidade de São Paulo

The purpose of this talk is to present a simple model of a reaction diffusion equation which include delay effects both in the interior and the boundary domain. These kinds of models are used as prototype equations to study some population models.

We present two results, in the first one, some analytical tools to study a discretization scheme for this prototype and, in the second result, we prove a bifurcation result when we vary the delay. We point out that this bifurcation result shows a different kind of behavior when the delay is in the interior from the behavior when it is in the boundary.

PHASE PORTRAIT OF POLYNOMIAL QUADRATIC VECTOR FIELDS
HAVING A FIRST INTEGRAL OF DEGREE 3
Regilene Oliveira
Universidade de São Paulo

Joint work with J. Llibre. In this talk we discuss the classification of all the global phase portraits, in the Poincaré disc, of the quadratic vector fields having a rational first integral of degree 3.

SELFADJOINTNESS AND OPTIMAL DECAY RATES IN OVERDAMPED EQUATIONS
Marta Pellicer
Universitat de Girona, Catalunya

This is joint work with J. Solà-Morales. We study an abstract form for a linear strongly damped wave equation, namely

$$u_{tt} + Bu_t + Au = 0$$

where A, B represent operators in a Hilbert space. This type of equations can be used to model the behavior of some mechanical systems (take, for instance, A and B to be $-\Delta$). In studying the large time behavior of these equations, one can see the interest of the so-called overdamped regime, which essentially happens when α (the strong internal damping) is taken sufficiently large. In this work we will present two interesting phenomena that happen under this overdamped regime. First, and under the appropriate conditions, we will show the existence of an explicit inner product for this problem that

makes it selfadjoint. Secondly, and for the case $A = B$, we will be able to improve the decay rate for more regular solutions. Actually, we will show that it is of an exponential-polynomial type and that it is optimal for this class of regular solutions.

The results we will present have been recently published in M. Pellicer, J. Solà-Morales. Optimal decay rates and the selfadjoint property in overdamped systems. J. Differential Equations 246 (2009), n. 7, 2813-2828.

SOME RESULTS ON THE DYNAMICS OF A NON LOCAL EVOLUTION EQUATION
António Luiz Pereira
Universidade de São Paulo

We discuss some questions related to the asymptotic behavior of the flow generated by the nonlocal evolution equation

$$\frac{\partial m(r;t)}{\partial t} = -m(r;t) + g(\beta J * m(r;t) + \beta h), \quad h, \beta \geq 0,$$

in both bounded and unbounded domains. We show that the flow is gradient and prove the existence of global compact attractors in appropriate phase spaces, which depend continuously on the parameters present in the equation. We also discuss the (equivariant) bifurcation of equilibria from the trivial equilibrium and show that, under certain conditions, those equilibria persist for all values of the relevant parameter. (Part of this work has been done in collaboration with Severino Horácio da Silva and Andréia da Silva Coutinho).

BIFURCATION OF CONSTANT MEAN CURVATURE TORI IN EUCLIDEAN SPHERES
Paolo Piccione
Universidade de São Paulo

We use equivariant bifurcation theory to show the existence of infinite sequences isometric embeddings of tori with constant mean curvature in Euclidean spheres that are not isometrically congruent to the CMC Clifford tori, and accumulating at some CMC Clifford torus. Joint work with L. Alias

Golden Tilings
Alberto Pinto
Universidade do Minho

In this talk we present the definition of a golden sequence frigi2N. These golden sequences have the property of being Fibonacci quasi-periodic and determine a tiling in the real line. We prove a one-to-one correspondence between: (i) affine classes of golden tilings; (ii) smooth conjugacy classes of Anosov diffeomorphisms, with an invariant measure absolutely continuous with respect to the Lebesgue measure, that are topologically conjugate to the Anosov automorphism $GA(x; y) = (x + y; x)$ (iii) solenoid functions. A. Pinto and D. Sullivan developed a theory relating 2-adic sequences (Pinto-Sullivan tilings in the real line) with smooth conjugacy classes of doubling expanding circle maps. The solenoid functions give a parametrization of the infinite dimensional space consisting of the mathematical objects described in the above equivalences.

References and Literature for further reading:

[1] A.A. Pinto, J.P. Almeida and A. Portela, Golden tilings, submitted.

[2] A.A. Pinto, D. Rand and F. Ferreira, Fine Structures of Hyperbolic Diffeomorphisms, Springer Monograph in Mathematics (2009).

[3] A.A. Pinto, D. Rand, Solenoid functions for hyperbolic sets on surfaces, Recent Progress in Dynamics. MSRI Publications, 54, 145-178, (2007).

[4] A.A. Pinto, D. Rand, Smoothness of holonomies for codimension 1 hyperbolic dynamics, Bull. London Math. Soc. 34, 341-352, (2002).

[5] A.A. Pinto, D. Rand, Rigidity of hyperbolic sets on surfaces, J. London Math. Soc. 2, 1-22, (2004).

[6] A.A. Pinto, D. Sullivan, The circle and the solenoid, Dedicated to A. Katok on the occasion of his 60th birthday, DCDS-A, 16 (2), 463-504, (2006).

Global normal forms for scalar second order ordinary differential equations
Clodoaldo Ragazzo
Universidade de São Paulo

In the present talk we give an answer to the following question posed (in a slightly different form) by Carlos Rocha: When is the equation $u'' = f(u, u')$, with $f(u, -v) = f(u, v)$, "conjugated" to $y'' = g(y)$?

Boundedness and segregation of solutions for a Bose-Einstein type system

Miguel Ramos

CMAF/University of Lisbon

We prove the existence of positive solutions for a system of the form $-\Delta u = u + u^3 - \beta u v^2$, $-\Delta v = v + v^3 - \beta u^2 v$, in a smooth bounded domain of R^3, with Dirichlet boundary conditions, as well as their segregation as the parameter β tends to infinity.

Generic dynamical properties of the scalar reaction-diffusion equations

Genevieve Raugel

Faculté des Sciences d'Orsay, Université Paris-Sud 11 and CNRS

In this talk, we first consider the scalar reaction-diffusion equation on the circle $S1$. In a recent work, Czaja and Rocha have proved that any connecting orbit, which connects two hyperbolic periodic orbits, is transverse and that there does not exists any homoclinic orbit, connecting a hyperbolic periodic orbit to itself. Here we complete these results by showing that any connecting orbit between two hyperbolic equilibria with distinct Morse indices or between a hyperbolic equilibrium and a hyperbolic periodic orbit is automatically transverse. We also show that, generically in the non-linearity, there does not exist connections between hyperbolic equilibria with the same Morse index. Finally, we prove that, generically in the non-linearity, the scalar reaction-diffusion equations on the circle, have the Morse-Smale property (joint work with Romain Joly).

In higher dimensions, we show that, generically in the non-linearity, the scalar reaction-diffusion equations have the Kupka-Smale property (joint work with P. Brunovsky and R. Joly).

On some variational problems for second and fourth order non autonomous equations in unbounded intervals

Luís Sanchez

CMAF/University of Lisbon

Our motivation is recent research on elliptic problems yielding stationary states of Klein-Gordon or Schrödinger equations. We concentrate on two simple one-dimensional problems (not radial problems originating in higher

dimensions) involving non-autonomous equations in the half-line: a second order problem with bounded nonlinearity and a fourth order problem with a power as nonlinearity.

The first problem is the following:

$$u'' = a(x)u - g(u), \quad u'(0) = u(+\infty) = 0 \tag{1}$$

where $g(u) = \mathrm{o}(u)$ at $u = 0$ and g is bounded and $a(x) \to a$ as $x \to \infty$, $0 < a < \infty$. We give a condition for the existence of a positive solution in terms of quantities related to the corresponding autonomous problem (with the constant a in place of $a(x)$).

The second problem is a fourth order analogue of the first but with a nonlinearity of power type. We consider

$$u^{(4)} - c\,u'' + a(x)\,u = |u|^{p-1}\,u, \quad u'(0) = u'''(0) = 0, \quad u(+\infty) = u'(+\infty) = 0 \tag{2}$$

where $a(x)$ is a nondecreasing function with $\lim_{x \to +\infty} a(x) = a > 0$, c is a positive constant and $p > 1$.

We prove the existence of a nontrivial solution by constrained minimization. In the autonomous case $(a(x) \equiv a)$ we show that indeed there exists a positive solution provided that $c^2 > 4a$.

This is joint work with R. Enguiça and A. Gavioli.

KADOMTSEV-PETVIASHVILI II EQUATIONS, WITH GENERALIZED DISPERSION, ON TWO AND THREE DIMENSIONAL CYLINDERS

Jorge Drumond Silva

IST

We will present recent results for the KP-II equations with generalized dispersion terms, in two and three spacial dimensions, periodic only in the x variable. We will start by showing how the solutions to the linearized equations satisfy bilinear Strichartz-type estimates, which are independent of the dispersion. We then use these estimates to establish local well-posedness for the Cauchy problem associated to the equations for low regularity data, in the framework of Bourgain spaces. For certain ranges of dispersion, these local results are optimal.

This is joint work with A. Grünrock and M. Panthee.

On an Initial Value Problem to Construct Area-Cartograms

Joan Solà-Morales

Universitat Politècnica de Catalunya

From the mathematical viewpoint, area-cartograms are defined by maps $\Phi : \Omega \to \Omega$, where Ω is usually a rectangle in the plane, such that

$$\det D\Phi = f(x) , \qquad (1)$$

where $f(x)$ is a given function. In our work [1] we proposed the following algorithm

$$u_t = \Delta u \text{ in } \Omega \text{ and for } t > 0,$$

$$u = f \text{ for } t = 0,$$

$$u_\nu = 0 \text{ on } \partial\Omega,$$

$$y'(t) = -\frac{\nabla u(y(t), t)}{u(y(t), t)} \text{ with } y(0) = y_0,$$

$$\Phi(y_0) := \lim_{t \to \infty} y(t)$$

that includes the solution of an initial value problem for a non-autonomous system of ODE's that is very singular at $t = 0$. In [1] we proved that this algorithm defines a solution Φ of (1) of class $C^{1+\alpha}$ when f of class C^α, if $\alpha > 0$. The publication [2] contributed to increase the interest on the case where $f(x)$ is piecewise constant, a case that was not covered by the results in [1]. By analyzing with detail some significative cases, we will present reasons to believe that the result will remain true even if $f(x)$ belongs to this new class.

[1] A. Avinyó, J. Solà-Morales and M. València: On maps with given jacobians involving the heat equation. Z. Angew. Math. Phys. 54 (2003), n. 6, 919-936.
[2] M.T. Gastner and M.E.J. Newman: Diffusion-based method for producing density-equalizing maps. Proc. National Academy of Sciences USA 101 (2004), n. 20, 7499-7504.

ON THE KALDOR-KALECKI TRADE CYCLE MODEL WITH FIXED DELAY
Plácido Z. Táboas
Universidade de São Paulo

N. Kaldor's 1940 trade model combined with the M. Kalecki's remark (1935) pointing out the existence of a time lag between a decision of investment and its effect on the capital stock leads to the system

$$
\begin{aligned}
\dot{Y}(t) &= \alpha[I(Y(t), K(t)) - S(Y(t), K(t))], \\
\dot{K}(t) &= I(Y(t-1), K(t)) - \delta K(t),
\end{aligned}
$$

proposed by Krawiec and Szydlowski in 1999, where I and S are the investment and the savings functions, respectively, Y is the national gross product, K is the capital stock, α is an adjustment coefficient in the goods market, and δ is the depreciation rate of the capital stock.

Under some conditions assumed by Krawiec and Szydlowski we investigate the occurrence of periodic orbits in the case where the savings and the investment have the same rate with respect to Y at $Y = 0$. We take the delay as a constant inherent to the focused economy rather than what is current in the related literature, where the delay varies as a bifurcation parameter. This is a joint work with Miguel V.S. Frasson, Marta C. Gadotti, and Selma H.J. Nicola.

AN INVERSE PROBLEM FOR VAKONOMIC MECHANICS
Gláucio Terra
Universidade de São Paulo

We study, in the context of Vakonomic Mechanics, an inverse problem for a system of mixed first and second order ordinary differential equations. Joint work with W. Oliva.