Unit 2

3.1 & 3.2 -Triangle Sum Theorem & Isosceles Triangles

Background for Standard G.CO.10: Prove theorems about triangles.

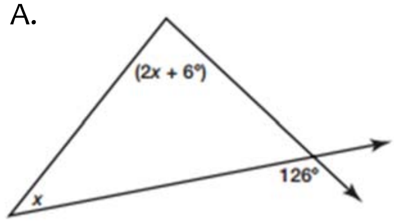
Objective: By the end of class, I should…

TRIANGLE SUM THEOREM: Draw any triangle on a piece of paper. Tear of the triangle's three angles. Arrange the angles so that they are adjacent angles. What do you notice about the sum of these three angles?

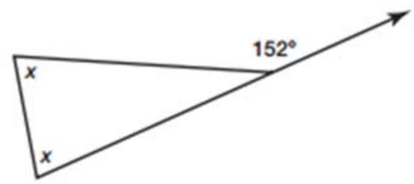
The sum of the measures of the interior angles of any triangle is __________.

Example 1: Use the triangle sum theorem to solve for x in each diagram.

A.



B.



Example 2: Describe the following classifications of triangles:

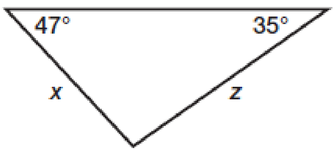
By Their Sides	By Their Angles
• Scalene • Isosceles • Equilateral	• Acute • Right • Obtuse

Example 3: Use a straight edge to draw a **LARGE** scalene triangle in the space below. Label the sides of the triangle S, M and L for small, medium and large. Use a protractor to measure and record the size of each interior angle of the triangle and label the angles S, M and L. Compare your results with your partner and the class.

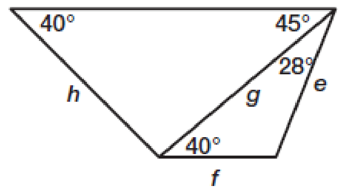
What conclusion can we draw about the **relationship between the lengths of the sides of a triangle and the measure of the interior angles**?

Example 4: List the sides from shortest to longest. Complete the problems below, then compare with your partner.

a.

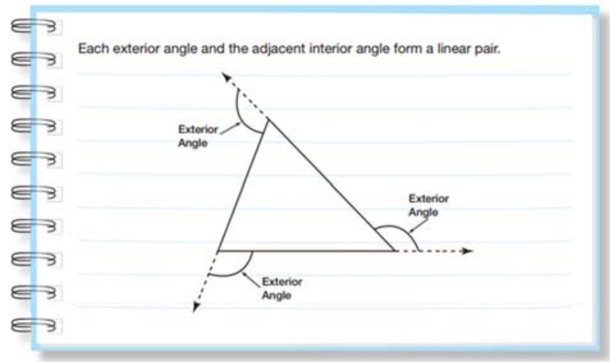


c.

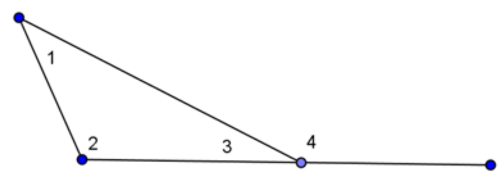


The **REMOTE INTERIOR ANGLES** of a triangle are the two angles that are non-adjacent to the specified angle.

The **EXTERIOR ANGLE THEOREM** says: The measure of the exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle.



Example 5: Prove the Exterior Angle Theorem.

Given: Triangle ABC with exterior angle $\angle 4$ Prove: $m\angle 1 + m\angle 2 = m\angle 4$	
STATEMENTS	REASONS
	2. Triangle sum theorem
	3. Linear pairs are supplementary
	5. Subtraction property

The **EXTERIOR ANGLE INEQUALITY THEOREM** says: an exterior angle must be larger than either remote interior angles. Use the diagram below to discuss this theorem as a class:

Pasta Activity: Sarah thinks any three lengths can represent the lengths of the sides of a triangle. Sam does not agree. Let's explore. Take your piece of pasta and break it a two random points so the strand is divided into three pieces. Measure each of your three pieces in centimeters to the tenths place. Try to form a triangle from your three pieces of pasta. List your three lengths below and state whether or not the lengths could form a triangle.

Random sample of class measurements:

Piece 1 (cm)	Piece 2 (cm)	Piece 3 (cm)	Forms a triangle? (yes/no)

With your partner write a hypothesis for what must be true for the 3 lengths to be able to form a triangle.

Example 7: Is it possible to form a triangle using segments with the following measurements? Sketch a diagram and explain your answers.

a. 1.9 cm, 5.2 cm, 2.9 cm

b. 152 cm, 73 cm, 79 cm

The **TRIANGLE INEQUALITY THEOREM** states: The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Compare this statement with the hypothesis your and your partner made.

Unit 2

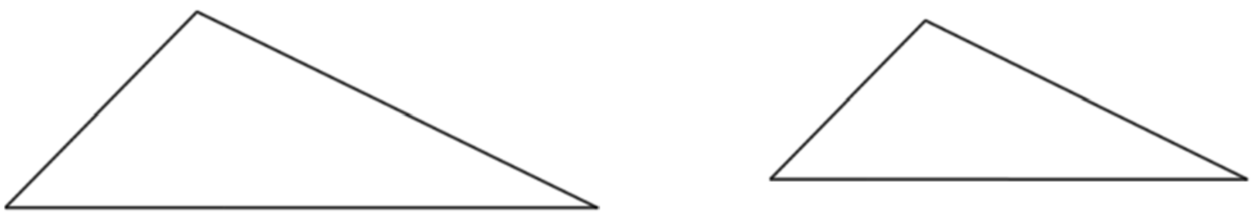
4.1 & 4.2 -Similar Triangle Theorems

Standard G.SRT.2: Given two figures, use the definition of similarity in terms of similarity transformations to decide whether they are similar.

Standard G.SRT.3: Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Objective: By the end of class, I should…

Example 1: Drawn below are a pair of triangles that have the same shape (corresponding angles are congruent) but that are not the same size.



Investigating Similar Triangles and Understanding Proportionality

Identify the two triangles in your picture, ΔABC (the larger triangle) and ΔADE (the smaller triangle). You will be asked to identify and record certain measurements from each triangle in the chart below.

1. Using your ruler, measure the lengths of the sides of your larger triangle, ΔABC, in centimeters. You will also be measuring sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$. Round to the nearest tenth of a centimeter. Record the measurements below.

2. Using your ruler, measure the lengths of the sides of the smaller triangle $\overline{AD}$, $\overline{DE}$, and $\overline{AE}$ in centimeters. Round to the nearest tenth of a centimeter. Record the measurements below.

3. Record the angle measures of your larger triangle, ΔABC. You will be recording $m\angle A$, $m\angle B$, and $m\angle C$. Verify that the sum of the angles is 180°.

Steps 1-6: Record Measurements Here!

Measurements for ΔABC		Measurements for ΔADE	
AB	$m\angle A$	AD	$m\angle DAE$
BC	$m\angle B$	DE	$m\angle ADE$
AC	$m\angle C$	AE	$m\angle DEA$

4. In the table below, identify and list the corresponding sides and the corresponding angles of your two triangles. Also, list each of the side lengths and angle measures on the two pictures.

Corresponding Sides	Corresponding Angles

5. Create ratios (fractions) using the corresponding sides of two triangles. Refer to your chart on the previous page for the lengths of the requested sides. Write the fractions as shown in the table below. Once you have set up the ratios, find the quotient (use your calculator to divide). Round your answer to three decimal places.

Ratio #1: $\frac{AB}{AD}$	**Ratio #2:** $\frac{BC}{DE}$	**Ratio #3:** $\frac{AC}{AE}$

6. What do you notice about the ratios of the corresponding sides?

The sides are **PROPORTIONAL** because the ratios of the corresponding sides are ______________________.

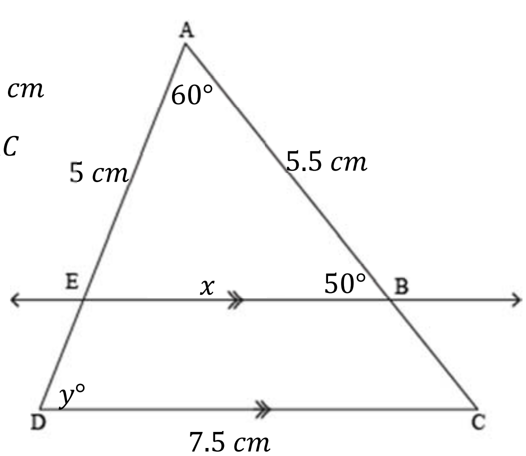
7. What did you notice about the measures of corresponding angles?

8. What do you now know about similar triangles?

Example 2: For what values of x, y, and z are the two triangles similar? [Hint: The sides must be proportional; you will have to write and solve two different proportions.]

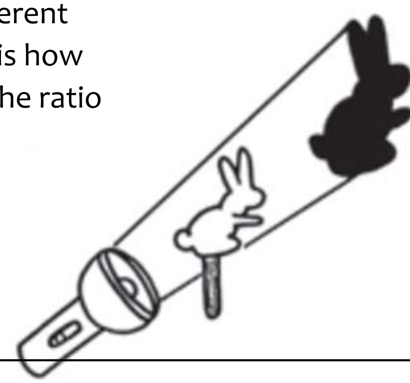
$AD = 7\ cm$

$z = AC$



Similar triangles are created through a dilation. A **DILATION** is a transformation that produces an image that is the same shape as the original, but is a different size. A dilation stretches or shrinks the original figure. The **SCALE FACTOR** is how much bigger/smaller the dilated figured is compared to the original—it's the ratio of the sides.

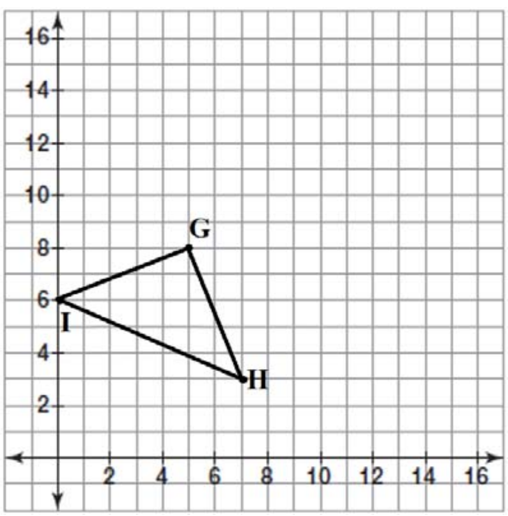
Think of a shadow puppet. In the picture to the right, the shape of the shadow bunny and the original bunny is the same, but the shadow bunny is **larger** than the original bunny.



DILATION vocabulary to know:

Scale Factor **Point of dilation** **Corresponding parts** **Proportional**

Example 3: *You can use your compass and a straightedge to perform a dilation.*
Consider ΔGHJ shown on the coordinate plane. You will dilate the triangle by using the origin as the center and by using a scale factor of 2



1. How will the distance from the center of dilation to a point on the image of $\Delta G'H'J'$ compare to the distance from the center of dilation to a corresponding point on ΔGHJ? Explain your reasoning.

2. For each vertex of ΔGHJ, draw a ray that starts and the origin and passed through the vertex.

3. List the coordinates of the vertices of ΔGHJ and $\Delta G'H'J'$. How do the coordinates of the image compare to the coordinates of the pre-image?

Example 4: Mappings and scale factors as used to transform geometric figures.

A) What mapping was used to dilate $A(2,-10)$ to $A'(5,-25)$?

B) Dilate $G(-1,5)$ using the maping $(x,y) \rightarrow (2x,2y)$

C) ΔMOB, with vertices $M(0,-3)$, $O(-12,6)$ and $B(5,4)$, has been dilated by a factor of $\frac{1}{3}$. What are the new vertices?

Proving Triangles are Similar

How could we describe similar triangles?

The **corresponding angles** are ________________________.

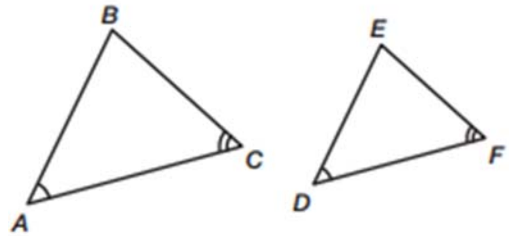
The **corresponding sides** are _________________.

So, if $\Delta COW \sim \Delta PIG$, what do we know?

Angle-Angle Similarity Theorem: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

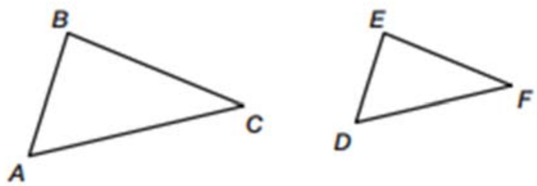
If $\angle A \cong \angle D$ and $\angle C \cong \angle F$, then $\Delta ABC \sim \Delta DEF$.

If $m\angle A = m\angle D$ and $m\angle C = m\angle F$, then $\Delta ABC \sim \Delta DEF$.



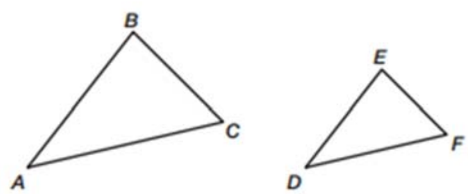
Side-Side-Side (SSS) Similarity Theorem: If all three corresponding sides of two triangles are proportional, then the triangles are similar.

If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\Delta ABC \sim \Delta DEF$.

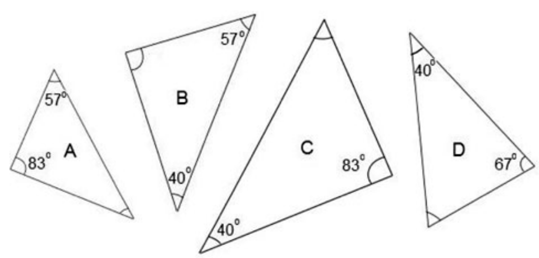


Side-Angle-Side (SAS) Similarity Theorem: If two of the corresponding sides of two triangles are proportional and the included angles are congruent, then the triangles are similar.

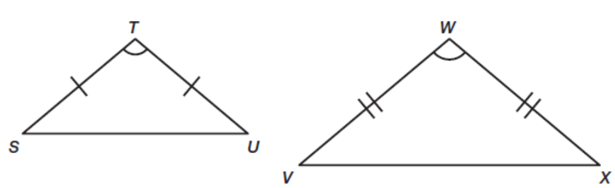
If $\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle A \cong \angle D$, then $\Delta ABC \sim \Delta DEF$.



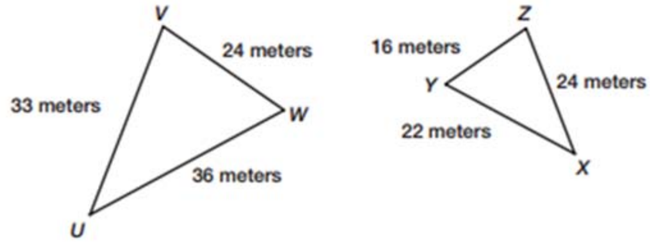
Example 5: In the following set of triangles, which one is not similar to the other three? Write an explanation with your answer.



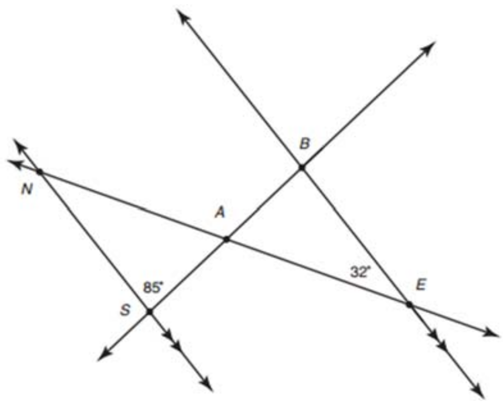
Example 6: Determine whether the given triangles are similar. If so, use symbols to write a similarity statement AND state which theorem you used.



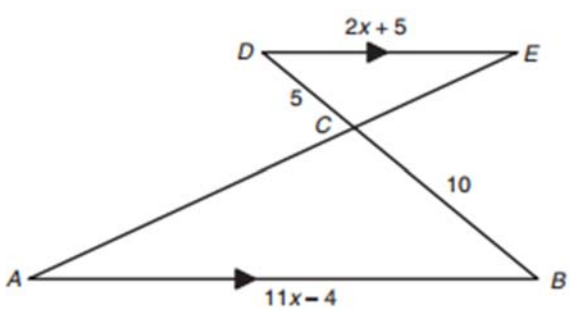
Example 7: Determine whether ΔUVW is similar to ΔXYZ. If so, use symbols to write a similarity statement.



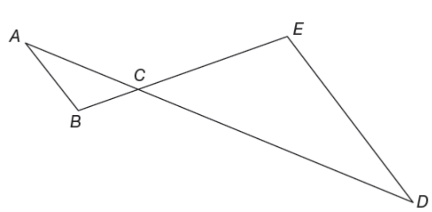
Example 8: In the figure, $\overleftrightarrow{NS} \parallel \overleftrightarrow{BE}$. Use the information given in the figure to determine the $m\angle SNA$, $m\angle NAS$, $m\angle ABE$, and $m\angle BAE$. Is ΔNSA similar to ΔEBA? If the triangles are similar, write a similarity statement.



Example 9: In the figure shown, segments AB and DE are parallel. The length of segment BC is 10 units and the length of segment CD is 5 units. List the corresponding sides as a proportion. Then use this information to calculate the value of x. Determine whether the given triangles are similar. If so, use symbols to write a similarity statement AND state which theorem you used.



Example 10: Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{ED}$, Prove: $\Delta ABC \sim \Delta DEC$



UNIT 2

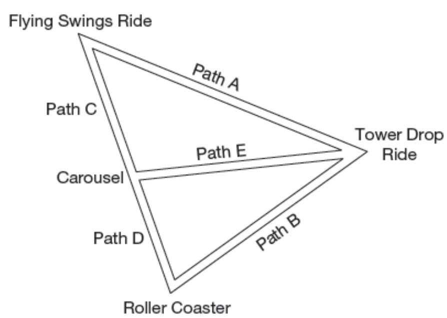
4.3 & 4.4 -MORE SIMILAR TRIANGLES

Standard G.SRT.3: Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Objective: By the end of class, I should…

The **ANGLE BISECTOR/PROPORTIONAL SIDE THEOREM** says: A bisector of an angle in a triangle divides the opposite side into two segments whose lengths are in the same ratio as the lengths of the sides adjacent to the angle.

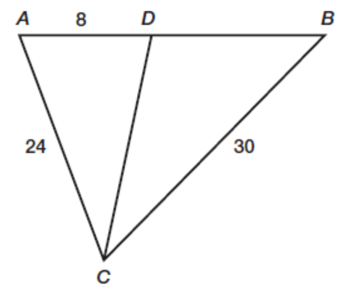
Example 1:



Path E bisects the angle formed by path A and path B.
Path A is 143 feet long.
Path C is 65 feet long.
Path D is 55 feet long.

Find the length of path B. Hint: $\frac{big}{small} = \frac{big}{small}$

Example 2:



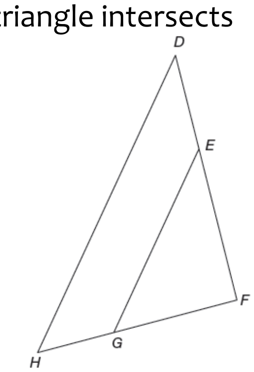
$\overline{CD}$ bisects $\angle C$. Solve for DB.

The **TRIANGLE PROPORTIONALITY THEOREM** says: If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

Example 3: GE ∥ HD, DE = 30, EF = 45, GH = 25, FG = ?. Solve using the theorem above.

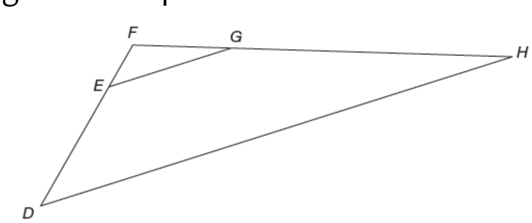
Hint: $\frac{big}{small} = \frac{big}{small}$

Are the triangles **similar**? Justify



Example 4: Write the **CONVERSE** of the **Triangle Proportionality Theorem**.

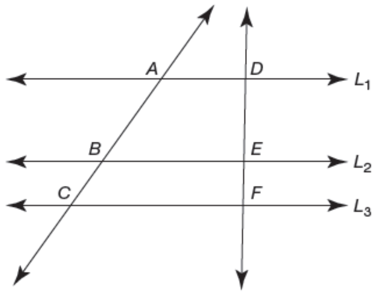
The **CONVERSE OF THE TRIANGLE PROPORTIONALITY** theorem can be used to test whether two lines segments are parallel.



Given: DE = 33, EF = 11, GH = 66, FG = 22.

Is $\overline{DH} \parallel \overline{EG}$? Justify using proportions.

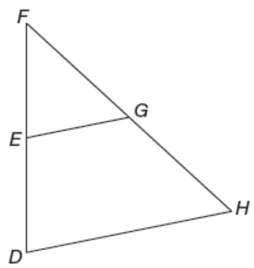
The **PROPORTIONAL SEGMENTS THEOREM** says: If three parallel lines intersect two transversals, then they divide the transversals proportionally.



a) Given: $L_1 \parallel L_2 \parallel L_3$, AB = 52, BC = 26, DE = 40, find EF.

b) Given: $L_1 \parallel L_2 \parallel L_3$, AB = 90, EF=15, DE = 75, find BC.

The **TRIANGLE MIDSEGMENT THEOREM** says: The midsegment of a triangle is parallel to the third side of the triangle and half the measure of the third side of the triangle.



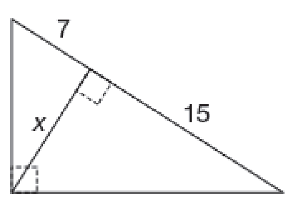
Use a ruler to measure the following segments in millimeters.

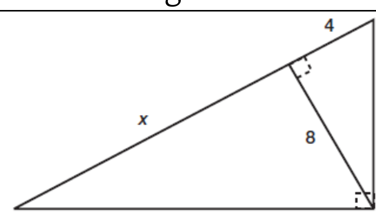
FE = ED = FG = GH = DH = EG =

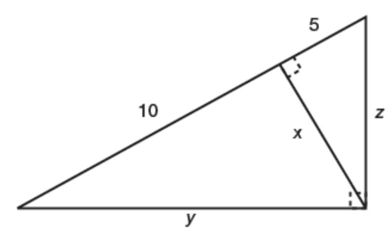
Is the midsegment length half the measure of the third side?

Are these triangles similar? Justify

Using the **RIGHT TRIANGLE/ALTITUDE THEOREM**. If an altitude is drawn from the vertex of a right angle to the hypotenuse, then three **similar** right triangles are formed! Use proportions to solve for the missing lengths. Can you see the three similar triangles? Solve for all variables.







Unit 2

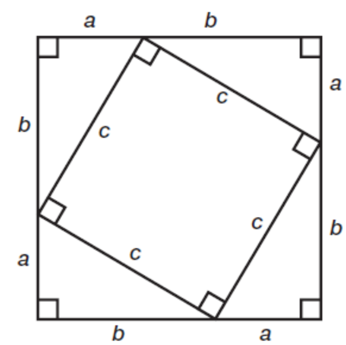
4.5 & 4.6 -Proving the Pythagorean Theorem
-Application of Similar Triangles

Standard G.SRT.4: Prove theorems about triangles, including the Pythagorean Theorem.

Standard G.SRT.5: Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Objective: By the end of class, I should…

Example 1: Proving the Pythagorean Theorem $a^2 + b^2 = c^2$



1) What is the area of the larger square?

2) What is the total area of the four right triangles?

3) What is the area of the smaller square?

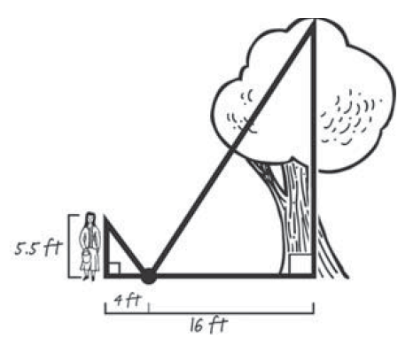
4) What is the relationship between the area of the four right triangles, the area of the smaller square, and the area of the larger square?

What is the **converse** of the Pythagorean Theorem?

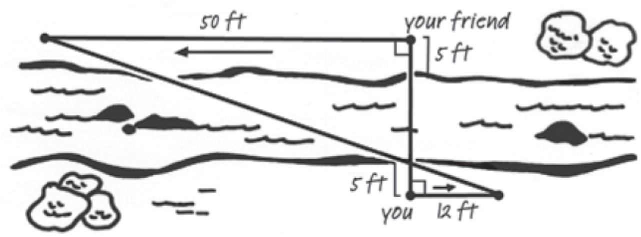
Example 2: You go to the park and place a mirror on the ground so you can see the top of a tree. You then gather enough information to calculate the height of one of the oak trees. The figure shows your measurements.

a. Are the triangles similar? How do you know?

b. Calculate the height of the tree.



Example 3: You stand on one side of the creek and your friend stands directly across the creek from you on the other side as shown in the figure. Your friend is standing 5 feet from the creek and you are standing 5 feet from the creek. You and your friend walk away from each other in opposite parallel directions. Your friend walks 50 feet and you walk 12 feet.



a. Label any angle measure and any angle relationships that you know on the diagram. Explain how you know.

b. How do you know the triangles formed are similar?

c. Calculte the distance from your friends starting point to your side of the creek.

d. What is the width of the creek? Explain your reasoning.

Example 4: Natasha stood 420 meters away from the tallest building in Singapore. She held a piece of wood 40 centimeters long at arm's length, 60 centimeters away from her eye. The piece of wood, held vertically, just blocked the building from my view. Use a proportion to calculate the height, h, in meters, of the building.