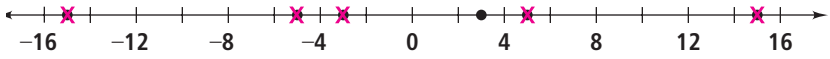
3-1 Inequalities and Their Graphs

Vocabulary

Review

1. Cross out any points that are NOT solutions of the *equation* $-5g = -15$.



2. Write two *equations* using 3, 13, and k. **Answers may vary. Samples are given.**

$3 + 13 = k$ $\qquad$ $13 - k = 3$

Vocabulary Builder

inequality (noun) **in ee KWAL uh tee**

inequality symbols
$<, >, \geq, \leq, \neq$

Definition: An **inequality** is a mathematical sentence that uses an **inequality** symbol to compare the values of two expressions.

Examples: The mathematical sentences $3 > 2$, $5 + 7 < 21$, and $x - 2 \leq 4$ are **inequalities.**

Nonexamples: The mathematical sentences $4 + 4 = 8$ and $x - 5 = 1$ are *not* **inequalities**. They are equations.

Use Your Vocabulary

3. Where does each mathematical sentence belong? Write each *inequality* or equation in the correct box.

$17 \geq 5$ $\qquad$ $3x = 9$ $\qquad$ $2x > 5$ $\qquad$ $2(4) = 8$ $\qquad$ $8 < 3x$

Inequality	Equation
$17 \geq 5$	$3x = 9$
$2x > 5$	$2(4) = 8$
$8 < 3x$	

4. Complete each *inequality* with $<$, $>$, $\leq$, or $\geq$.

$z < 4$, so 4 **>** z $\qquad$ $g \geq -2$ so -2 **≤** g $\qquad$ $m < 7$, so 7 **>** m

Problem 1 Writing Inequalities

Got It? What is an inequality that represents the verbal expression?

all real numbers p greater than or equal to 1.5

Use the verbal expression. Write T for *true* or F for *false*.

F **5.** A real number p could be less than 1.5.

T **6.** A real number p could be equal to 1.5.

T **7.** A real number p could be greater than 1.5.

8. Circle the symbol that represents "greater than or equal to."

$<$ $>$ $\leq$ (≥)

9. Complete the inequality that represents the verbal phrase.

p ≥ 1.5

Problem 2 Identifying Solutions by Evaluating

Got It? **Consider the numbers −1, 0, 1, and 3. Which are solutions of $13 - 7y \leq 6$?**

10. Check whether -1 is a solution of the inequality. Complete the steps below.

$13 - 7y \leq 6$	Write the original inequality.
$13 - 7 \cdot$ **−1** $\stackrel{?}{\leq} 6$	Substitute the value for y.
$13 -$ **−7** $\stackrel{?}{\leq} 6$	Multiply.
20 $\stackrel{?}{\leq} 6$	Simplify.

11. Underline the correct word(s) to complete each sentence.

When y is replaced with -1, the inequality is true / <u>false</u>.

So, -1 is / <u>is not</u> a solution.

12. Repeat with the other possible solutions: 0, 1, and 3.

$13 - 7(0) \stackrel{?}{\leq} 6$
$13 - 0 \stackrel{?}{\leq} 6$
$13 \not\leq 6$

$13 - 7(1) \stackrel{?}{\leq} 6$
$13 - 7 \stackrel{?}{\leq} 6$
$6 \leq 6$

$13 - 7(3) \stackrel{?}{\leq} 6$
$13 - 21 \stackrel{?}{\leq} 6$
$-8 \leq 6$

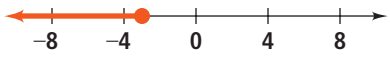
13. Circle the numbers that are solutions of $13 - 7y \leq 6$.

-1 0 (1) (3)

A closed dot on a graph means the number is part of the solution. An open dot on a graph means the number is *not* part of the solution.

14. The endpoint of each graph is -3. Is -3 a solution of the inequality represented by the graph? Explain why or why not.

No. The dot is open.



Yes. The dot is closed.

15. Write the endpoint of each graph of an inequality. Then explain why the endpoint *is* or *is not* a solution of the inequality.

Sample: -5; -5 is a solution because the dot is closed.

Sample: 7; 7 is not a solution because the dot is open.

Problem 3 Graphing an Inequality

Got It? **What is the graph of the inequality $x > -4$?**

16. Circle the words that complete the sentence.

The solutions of the inequality $x > -4$ are all numbers ? -4.

(greater than) | less than | greater than or equal to | less than or equal to

17. Underline the correct word or words to complete each sentence.

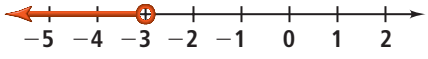
The graph of $x > -4$ includes / does not include -4.

The graph of $x > -4$ will have an open / a closed dot at -4.

18. Graph the solutions of the inequality $x > -4$ on the number line.

Problem 4 Writing an Inequality From a Graph

Got It? **What inequality represents the graph?**



19. Circle all statements that describe the graph.

(open dot) | shaded to the right of -3 | numbers greater than -3 are included

closed dot | (shaded to the left of -3) | (numbers less than -3 are included)

20. **Multiple Choice** Which inequality represents the graph?

(A) $x > -3$ (B) $x < -3$ (C) $x \geq -3$ (D) $x \leq -3$

Problem 5 Writing Real-World Inequalities

Got It? **Reasoning** **The inequality $s \leq 8$ describes a situation where s is a legal speed. Can the speed be *all* real numbers less than or equal to 8? Explain.**

21. Write *stopped, moving,* or *doesn't make sense* to describe each speed. Then circle a word to answer the question.

Speed	Description	Is it possible?
5	moving	(Yes) / No
0	stopped	(Yes) / No
−3	doesn't make sense	Yes / (No)

22. Can the speed be *all* real numbers less than or equal to 8? Explain. **Sample explanation:**

No. You cannot have a negative speed, so negative numbers are excluded.

Lesson Check • Do you UNDERSTAND?

Compare and Contrast **What are some situations you could model with $x \geq 0$? How do they differ from situations you could model with $x > 0$?**

23. Use the situations at the right. Write each one on the correct line.

$x \geq 0$: baseball team's score, whole numbers, inches of rain

$x > 0$: counting numbers, length of a poster, distance to a park

- Counting numbers
- Length of a poster
- One baseball team's score
- Whole numbers
- Distance from your home to a park
- Inches of rain

24. Describe how the situations for $x \geq 0$ differ from the situations for $x > 0$.

Sample: The situations for $x \geq 0$ are sometimes zero. The situations for $x > 0$ are never zero.

Math Success

Check off the vocabulary words that you understand.

☐ inequality ☐ solution of an inequality ☐ graph of an inequality

Rate how well you can *write and graph inequalities*.

Need to review — 0 2 4 6 8 10 — Now I get it!

3-2 Solving Inequalities Using Addition or Subtraction

Vocabulary

Review

1. Write an *inequality symbol* to represent each verbal description.

Symbol	Verbal Description	Symbol	Verbal Description
$<$	• less than, fewer than	$\leq$	• less than or equal to • at most, no greater than • as much as, no more than
$>$	• greater than, more than	$\geq$	• greater than or equal to • at least, no less than • as little as, no fewer than

Vocabulary Builder

equivalent (adjective) **ee KWIV uh lunt**

Related Word: equal

Main Idea: Numbers or expressions are **equivalent** when they have equal values.

Examples: $\frac{12}{4}$ is **equivalent** to 3.

The expression $1 + 6$ is **equivalent** to $9 - 2$.

Use Your Vocabulary

Equivalent inequalities **are inequalities that have the same solutions. Write an inequality that is *equivalent* to the inequality that is given.**

2. Since $10 \geq -3, -3$ $\leq$ 10.

3. Since $-7 < -1, -1$ $>$ -7.

4. If $b > -10$, then -10 $<$ b.

5. If $h \leq 0$, then 0 $\geq$ h.

6. Cross out the equations that are NOT *equivalent* to $x = 3$.

$3 = x$ ~~$x = \frac{1}{3}$~~ $x + 2 = 5$ ~~$x + 2 = 5 - 2$~~

7. Cross out the inequalities that are NOT *equivalent* to $x \leq 3$.

$3 \geq x$ ~~$x \leq \frac{1}{3}$~~ ~~$x + 2 \geq 5$~~ ~~$x + 2 \leq 5 - 2$~~

Take note

Key Concept Addition and Subtraction Properties of Inequality

When you add or subtract the same number on each side of an inequality, the relationship between the two sides does not change.

Complete each inequality using either the *Addition Property of Inequality* or the *Subtraction Property of Inequality*.

8. Since $3 > -1$, $3 + 5 > -1 +$ **5**.

9. Since $4 \leq 9$, $4 + n \leq 9 +$ **n**.

10. If $z < 8$, then $z - (-4) < 8 -$ **(−4)**.

11. If $w \geq k$, then $w - t \geq k -$ **t**.

Problem 1 Using the Addition Property of Inequality

Got It? **What are the solutions of $n - 5 < -3$? Graph the solutions.**

12. First add 5 to both sides of $n - 5 < -3$. Then simplify.

$$n - 5 + \mathbf{5} < -3 + \mathbf{5}$$

$$n < \mathbf{2}$$

13. Circle the graph that shows the solutions of $n - 5 < -3$.

Problem 2 Solving an Inequality and Checking Solutions

Got It? **What are the solutions of $m - 11 \geq -2$? Graph and check the solutions.**

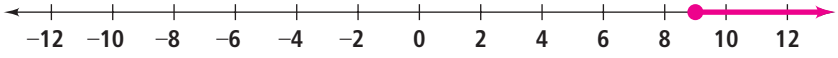
14. Underline the correct words to complete the sentence.

To isolate the variable, add 11 to / subtract −2 from each side of the equation.

15. Solve the inequality.

$$m - 11 \geq -2$$
$$m - 11 + 11 \geq -2 + 11$$
$$m \geq 9$$

16. Graph the inequality on the number line below.



17. Check the related equation. Substitute for m in $m - 11 = -2$.

$9 - 11 \stackrel{?}{=} -2$

$-2 = -2$

Does it check? (Yes) / No

18. Check the inequality symbol by replacing m with one of your solutions to Exercise 16. **Sample: Test 10.**

$m - 11 \geq -2$

$10 - 11 \stackrel{?}{\geq} -2$

$-1 \geq -2$

Does it check? (Yes) / No

Problem 3 Using the Subtraction Property of Inequality

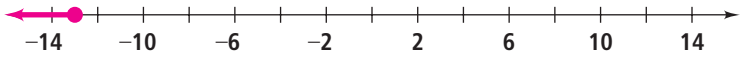
Got It? **What are the solutions of $-1 \geq y + 12$? Graph the solutions.**

19. Subtract 12 from both sides of the inequality. Then simplify.

$-1 - 12 \geq y + 12 - 12$

$-13 \geq y$

20. Graph the inequality on the number line.



21. Check your solution in the related equation and inequality to make sure it is correct.

Substitute for y in $-1 = y + 12$.

$-1 \stackrel{?}{=} -13 + 12$

$-1 = -1$ ✓

Substitute for y in $-1 \geq y + 12$.

$-1 \stackrel{?}{\geq} -15 + 12$

$-1 \geq -3$ ✓

Is your solution correct? (Yes) / No

Problem 4 Writing and Solving an Inequality

Got It? **A club has a goal to sell at least 25 plants for a fundraiser. Club members sell 8 plants on Wednesday and 9 plants on Thursday. What are the possible numbers of plants the club can sell on Friday to meet their goal?**

22. Circle the inequality that represents *at least*.

$<$ $>$ $\leq$ (≥)

23. Complete the model below.

Relate plants sold Wednesday plus plants sold Thursday plus plants sold Friday is at least 25

Define Let $p =$ plants sold Friday.

Write $8 + 9 + p \geq 25$

24. Simplify and solve the inequality.

$$\begin{aligned} 8 + 9 + p &\geq 25 \\ 17 + p &\geq 25 \\ 17 - 17 + p &\geq 25 - 17 \\ p &\geq 8 \end{aligned}$$

25. Club members must sell at least **8** plants on Friday to meet their goal.

Lesson Check • Do you UNDERSTAND?

Reasoning **What can you do to $x + 4 \leq 10$ to get $x \leq 6$?**

26. Circle the operation in the first inequality.

addition division multiplication subtraction

27. Circle the operation you can use to undo the operation you circled in Exercise 26.

addition division multiplication subtraction

28. Explain what you can do to $x + 4 \leq 10$ to get $x \leq 6$.

Explanations may vary. Sample: Subtraction undoes addition, so subtract 4 from each side of the inequality.

Math Success

Check off the vocabulary words that you understand.

☐ equivalent inequalities ☐ Addition and Subtraction Properties of Inequality

Rate how well you can *solve inequalities by adding or subtracting.*

3-3 Solving Inequalities Using Multiplication or Division

Vocabulary

Review

1. Circle each *inequality*.

$5 + x < 6$ $\quad 7 - y \geq 2$ $\quad a + b = 12$

$m - 3 > 5$ $\quad 2 + 3 = 3 + 2$ $\quad n + 3 \neq 8$

2. Draw a line from each *inequality* in Column A to an equivalent *inequality* in Column B.

Column A	Column B
$x + 7 < 11$	$11 \geq x + 7$
$x + 7 \geq 11$	$11 > x + 7$
$7 + x \leq 11$	$11 \leq x + 7$
$7 + x > 11$	$11 < x + 7$

Vocabulary Builder

reverse (verb) **rih VURS**

Related Words: flip (verb), reversible (adjective), reverse (noun)

Definition: To **reverse** something means to turn it in an opposite direction.

Example: The Supreme Court has the power to **reverse** or uphold decisions made by the lower courts.

Use Your Vocabulary

3. *Reverse* each inequality symbol below. Write the new symbol in the box.

$\leq$ **≥** $\quad > $ **<** $\quad <$ **>** $\quad \geq$ **≤**

4. If the inequality symbol in $x < 2$ is *reversed*, tell how the solutions change.

Answers may vary. Sample: The solutions change from all numbers less than 2 to all numbers greater than 2.

take note

Key Concept Multiplication Property of Inequality

When you multiply each side of an inequality by a positive number, *do not reverse* the direction of the inequality symbol.

Let a, b, and c be real numbers with $c > 0$.

If $a > b$, then $ac > bc$. If $a < b$, then $ac < bc$.

Write $<$, $>$, $\leq$, or $\geq$ to complete each inequality.

5. $6 > 4$

$\frac{6}{2}$ [>] $\frac{4}{2}$

3 [>] 2

6. $2 \leq 3$

$2(5)$ [≤] $3(5)$

10 [≤] [15]

7. $-12 < -9$

$\frac{-12}{3}$ [<] $\frac{-9}{3}$

[−4] [<] -3

8. If $p < q$ and $r > 0$, then pr [<] qr.

When you multiply each side of an inequality by a negative number, *reverse* the direction of the inequality symbol.

Let a, b, and c be real numbers with $c < 0$.

If $a > b$, then $ac < bc$. If $a < b$, then $ac > bc$.

Write $<$, $>$, $\leq$, or $\geq$ to complete each inequality.

9. $6 > 4$

$\frac{6}{-2}$ [<] $\frac{4}{-2}$

-3 [<] -2

10. $2 \leq 3$

$2(-5)$ [≥] $3(-5)$

-10 [≥] [−15]

11. $-12 < -9$

$\frac{-12}{-3}$ [>] $\frac{-9}{-3}$

[4] [>] 3

12. If $g < h$ and $k < 0$, then $\frac{g}{k}$ [>] $\frac{h}{k}$.

Problem 1 Multiplying by a Positive Number

Got It? **What are the solutions of $\frac{c}{8} > \frac{1}{4}$? Graph the solutions.**

13. Circle the first step in solving $\frac{c}{8} > \frac{1}{4}$.

Add 8 to each side.	Subtract 8 from each side.
Multiply each side by 8.	Divide each side by 8.

14. What must you do to the inequality symbol when solving $\frac{c}{8} > \frac{1}{4}$? Explain.

Nothing. Explanations may vary. Sample: 8 is positive.

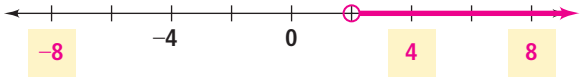
15. Solve the inequality.

$\frac{c}{8} > \frac{1}{4}$

$\frac{c}{8}(8) > \frac{1}{4} \cdot$ [8]

c [>] [2]

16. Graph the inequality on the number line.



Problem 2 Multiplying by a Negative Number

Got It? **What are the solutions of $-\frac{n}{3} < -1$? Graph and check.**

17. Circle the first step in solving $-\frac{n}{3} < -1$.

Add -3 to each side.	Subtract -3 from each side.
Multiply each side by -3.	Divide each side by -3.

18. What must you do to the inequality symbol when solving $-\frac{n}{3} < -1$? Explain.

Explanations may vary for # 18.

Sample: You reverse the symbol because you are multiplying by a negative number.

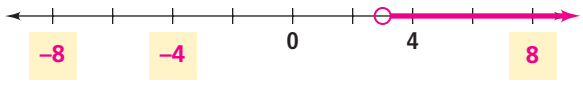
19. Solve the inequality.

$$-\frac{n}{3} < -1$$

$$-\frac{n}{3}(-3) > -1 \cdot \boxed{-3}$$

$$n \boxed{>} \boxed{3}$$

20. Graph the inequality.



21. Check the solution by following the steps below. Answers may vary. Sample is given.

$-\frac{n}{3} < -1$ Write the original inequality.

$-\frac{\boxed{6}}{3} \overset{?}{<} -1$ Substitute a value that makes the simplified inequality true.

$\boxed{-2} \overset{?}{<} -1$ Simplify and check.

22. Is your solution correct? (Yes) / No

Problem 3 Dividing by a Positive Number

Got It? **A student club plans to buy food for a soup kitchen. A case of vegetables costs $10.68. The club can spend at most $50 for this project. What are the possible numbers of cases the club can buy?**

23. Complete the model below.

Relate	the cost of a case of vegetables	times	the number of cases	is at most	$50
Define	Let c = the number of cases the club can buy.				
Write	10.68	·	c	≤	$50

24. Now solve the inequality.

$10.68c \leq 50$

$\frac{10.68c}{10.68} \leq \frac{50}{10.68}$

$c \leq 4.68$

25. Circle the possible numbers of cases of vegetables the club can buy.

(1) (2) (3) (4) 5

Problem 4 Dividing by a Negative Number

Got It? What are the solutions of $-5x > -10$? Graph the solutions.

26. To solve $-5x > -10$, will you reverse the inequality symbol? Why or why not?

Yes. Explanations may vary. To solve the inequality, you must divide by a negative number, so you must reverse the inequality symbol.

27. Solve the inequality.

$-5x > -10$

$\frac{-5x}{-5} < \frac{-10}{-5}$

$x < 2$

28. Graph the inequality on the number line.

−8 −4 8

Lesson Check • Do you UNDERSTAND?

Error Analysis Describe and correct the error in the solution.

$-\frac{n}{5} > 2$

$-5\left(-\frac{n}{5}\right) > -5(2)$

$n > -10$

29. Describe the student's first step.

Answers may vary. Sample: The student multiplied each side by −5.

30. What is the error in the student's solution?

Sample: The student did not reverse the inequality symbol.

31. What is the correct solution?

$n < -10$

Math Success

Check off the vocabulary words that you understand.

☐ inequality ☐ Multiplication Property of Inequality ☐ Division Property of Inequality

Rate how well you can *solve inequalities by multiplying or dividing.*

Need to review 0 2 4 6 8 10 Now I get it!

3-4 Solving Multi-Step Inequalities

Vocabulary

Review

1. **Multiple Choice** You must *reverse* the direction of an inequality symbol when you multiply both sides of an inequality by a number that is __?__.

(A) less than 0 (B) greater than 0 (C) less than or equal to 0 (D) greater than or equal to 0

2. Write *reverse* if the inequality symbol will change when you solve the inequality. Write *same* if the symbol will remain the same.

$3t < 6$	$-8s < 4$	$5x \leq -10$	$-4 \leq -2a$
same	reverse	same	reverse

Vocabulary Builder

at least (adverbial phrase) **at leest**; **at most** (adverbial phrase) **at mohst**

Main Idea: The phrase **at least** describes the least possible number that can be used. The phrase **at most** describes the greatest possible number that can be used.

Using Symbols: The inequality $x \geq 5$ means "x is **at least** 5." The inequality $x \leq 5$ means "x is **at most** 5."

Use Your Vocabulary

3. Complete each sentence with the words *at least* or *at most*.

You must be __?__ 18 years of age to vote in a national election. — at least

An elevator can safely carry __?__ 15 people. — at most

When water boils, you know the temperature is __?__ 212°F. — at least

If all books cost \$3 and Jane has \$20, she can buy __?__ 6 books. — at most

4. Use your answers to Exercise 3. Complete each inequality with $\leq$ or $\geq$.

$y \geq 18$	$e \leq 15$	$w \geq 212$	$3b \leq 20$

Problem 1 Using More Than One Step

Got It? **What are the solutions of the inequality $-6a - 7 \leq 17$? Check your solutions.**

5. Circle the first step in solving the inequality. Then underline the second step.

(Add 7 to each side.)	Divide each side by 6.	Divide each side by -6 and reverse the inequality.
Subtract 7 from each side.	Multiply each side by 6.	Multiply each side by -6 and reverse the inequality.

6. Use your answers to Exercise 5 to solve the inequality.

$$\begin{aligned} -6a - 7 &\leq 17 \\ -6a - 7 + 7 &\leq 17 + 7 \\ -6a &\leq 24 \\ \frac{-6a}{-6} &\geq \frac{24}{-6} \\ a &\geq -4 \end{aligned}$$

7. Check the endpoint by substituting into the related equation, $-6a - 7 = 17$.

$$\begin{aligned} -6a - 7 &= 17 \\ -6(-4) - 7 &\stackrel{?}{=} 17 \\ 24 - 7 &\stackrel{?}{=} 17 \\ 17 &= 17 \checkmark \end{aligned}$$

8. Check the inequality symbol by substituting into the original inequality, $-6a - 7 \leq 17$.

Sample: Test 3.

$$\begin{aligned} -6a - 7 &\leq 17 \\ -6(3) - 7 &\stackrel{?}{\leq} 17 \\ -18 - 7 &\stackrel{?}{\leq} 17 \\ -25 &\leq 17 \checkmark \end{aligned}$$

Problem 2 Writing and Solving a Multi-Step Inequality

Got It? **You want to make a rectangular banner that is 18 ft long. You have no more than 48 ft of trim for the banner. What are the possible widths of the banner?**

9. Circle the formula for the perimeter of a rectangle.

$C = 2\pi r$	$A = \ell w$	$d = rt$	$P = 2\ell + 2w$

10. Write an algebraic expression to describe the distance around a rectangular banner with a length of 18 ft and a width of w ft.

$2(18) + 2w$

11. The distance around the banner should be at least / at most 48 feet.

12. Use the expression you wrote in Exercise 10 and the information from Exercise 11. Write an inequality to represent the situation described in the problem. Then solve your inequality.

$$\begin{aligned} 2(18) + 2w &\leq 48 \\ 36 + 2w &\leq 48 \\ 2w &\leq 12 \\ w &\leq 6 \end{aligned}$$

13. The width of the banner should be at most 6 feet.

Problem 3 Using the Distributive Property

Got It? What are the solutions of $15 \leq 5 - 2(4m + 7)$? Check your solutions.

14. Use the justifications at the right to solve the inequality.

$15 \leq 5 - 2(4m + 7)$	Write the original inequality.
$15 \leq 5 - $ **8m** $ - $ **14**	Distributive Property
$15 \leq -8m - $ **9**	Subtract.
$15 + $ **9** $ \leq -8m - 9 + $ **9**	Addition Property of Inequality
$24 \leq $ **−8m**	Add.
$\frac{24}{\textbf{−8}} $ **≥** $ \frac{-8m}{\textbf{−8}}$	Division Property of Inequality
$-3 $ **≥** $ m$	Simplify.

15. Check your solutions by following the steps below. **Values for *m* may vary. Sample is shown.**

$15 \stackrel{?}{\leq} 5 - 2(4 \cdot $ **−4** $ + 7)$	Substitute one of your solutions to Exercise 14.
$15 \stackrel{?}{\leq} 5 - 2($ **−16** $ + 7)$	Multiply within parentheses.
$15 \stackrel{?}{\leq} 5 - 2 \cdot $ **−9**	Add within parentheses.
$15 \stackrel{?}{\leq} 5 - $ **−18**	Multiply.
$15 \leq $ **23**	Simplify.

Problem 4 Solving an Inequality With Variables on Both Sides

Got It? What are the solutions of $3b + 12 > 27 - 2b$? Check your solutions.

16. The inequality is solved below. Write a justification for each step.

$3b + 12 > 27 - 2b$	**Write the original inequality.**
$2b + 3b + 12 > 27 - 2b + 2b$	**Addition** Property of Inequality
$5b + 12 - 12 > 27 - 12$	**Subtraction** Property of Inequality
$\frac{5b}{5} > \frac{15}{5}$	**Division** Property of Inequality
$b > 3$	**Simplify.**

17. Check your solutions in the original inequality.

Values for *b* may vary. Sample:
$3(4) + 12 \stackrel{?}{\geq} 27 - 2(4)$
$12 + 12 \stackrel{?}{\geq} 27 - 8$
$24 > 19$ ✓

18. Are your solutions correct? (Yes) / No

Problem 5 Inequalities With Special Solutions

Got It? What are the solutions of the inequality $9 + 5n \leq 5n - 1$?

19. Solve the inequality $9 + 5n \leq 5n - 1$.

$$9 + 5n \leq 5n - 1$$
$$9 + 5n - 5n \leq 5n - 5n - 1$$
$$9 \leq -1$$

20. The inequality $9 + 5n \leq 5n - 1$ is always / never true.

So, the solution is all real numbers / there is no solution.

Lesson Check • Do you UNDERSTAND?

Error Analysis Your friend says that the solutions of the inequality $-2(3 - x) > 2x - 6$ are all real numbers. Do you agree with your friend? Explain. What if the inequality symbol were $\geq$?

21. The inequality $-2(3 - x) > 2x - 6$ is solved below. Write a justification from the box for each step.

Distributive Property
Simplify.
Subtraction Property of Inequality
Write the original inequality.

$-2(3 - x) > 2x - 6$	Write the original inequality.
$-6 + 2x > 2x - 6$	Distributive Property
$-6 + 2x - 2x > 2x - 6 - 2x$	Subtraction Property of Inequality
$-6 > -6$	Simplify.

22. Look at the final inequality in Exercise 21. Is the inequality ever true? Yes / No

23. Do you agree with your friend? Explain. What if the inequality symbol were $\geq$?

No. Explanations may vary. Sample: The original inequality has no solutions (a number is never greater than itself). If the inequality symbol were $\geq$, I would agree with my friend.

Math Success

Check off the vocabulary words that you understand.

☐ multi-step inequalities ☐ properties of inequality ☐ solutions

Rate how well you can *solve multi-step inequalities.*

Need to review 0 2 4 6 8 10 Now I get it!

3-5 Working With Sets

Vocabulary

Review

1. Cross out the numbers below that are NOT *whole numbers*.

12 ~~$\frac{2}{3}$~~ 108 ~~6.3~~ 2

2. Describe the relationship between the set of *whole numbers* and the set of *real numbers*.

Answers may vary. Sample: All whole numbers are real numbers, but not all real numbers are whole numbers.

Vocabulary Builder

set (noun) **set**

The **set** {0, 1, 2} contains the elements 0, 1, and 2.

Definition: A **set** is a collection of distinct objects or elements. The *complement* of a **set** is the set of all objects or elements *not* in the set.

Using Symbols: The **set** of the first three whole numbers can be written in roster form as $\{0, 1, 2\}$. It can be written in set-builder form as $H = \{x \mid \text{whole numbers}, x < 3\}$.

Examples: The universal **set** of all meals in a day is {breakfast, lunch, snack, dinner}. Let A be the **set** {breakfast, lunch}. The *complement* of set A, written A', is the **set** of all meals *not* in A. So, $A' = \{\text{snack, dinner}\}$.

Use Your Vocabulary

Complete each *set* with another element. Elements for Exercises 3–5 may vary. Samples are given.

3. $A = \{\text{eyes, ears, nose, } \underline{?}\ \}$ mouth

4. $B = \{\text{mother, father, brother, } \underline{?}\ \}$ sister

5. $C = \{\text{A, E, } \underline{?}\text{, O, U}\}$ I

6. Suppose that the universal set of coins is {penny, nickel, dime, quarter}. Let D be the set {penny, quarter}. What is the *complement* of set D?

$D' = \{$ nickel, dime $\}$

Problem 1 Using Roster Form and Set-Builder Notation

Got It? *N* is the set of even natural numbers that are less than or equal to 12. How do you write *N* in roster form? In set-builder notation?

7. Circle the even natural numbers that are less than or equal to 12.

1 (2) 3 (4) 5 (6) 7 (8) 9 (10) 11 (12)

8. Write the numbers you circled in roster form below.

$N = \{$ 2 , 4 , 6 , 8 , 10 , 12 $\}$

9. Complete the set-builder notation below. Use the description of the circled numbers from Exercise 7 to help you.

$N = \{x|$ even natural numbers $, x \leq 12\}$

Problem 2 Inequalities and Set-Builder Notation

Got It? In set-builder notation, how do you write the solutions of $9 - 4n > 21$?

10. Solve the inequality.

$9 - 4n > 21$
$-4n > 12$
$n < -3$

11. In set-builder notation, the solutions are

$\{n|$ $n < -3$ $\}$.

The *empty set,* written $\{\ \}$, is the set that contains no elements. It is a *subset* of every set.

Problem 3 Finding Subsets

Got It? What are the subsets of the set $P = \{a, b\}$? Of the set $S = \{a, b, c\}$?

12. List all of the subsets of set *P*. The first one is done for you.

The empty set: $\{\}$

Two sets with one letter each: $\{$ a $\}, \{$ b $\}$

The original set: $\{$ a , b $\}$

13. List all of the subsets of set *S*.

The empty set: { }

Three sets with one letter each: {a}, {b}, {c}

Three sets with two letters each: {a, b}, {a, c}, {b, c}

The original set: {a, b, c}

14. How many subsets does *P* have?

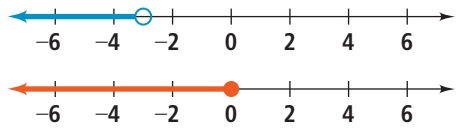
4

15. How many subsets does *S* have?

8

Got It? **Reasoning** Let $A = \{x | x < -3\}$ and $B = \{x | x \leq 0\}$. Is A a subset of B? Explain your reasoning.

Use the graphs of $x < -3$ and $x \leq 0$ for Exercises 16–20.



16. The top / bottom arrow represents the graph $x < -3$, or set A.

17. The top / bottom arrow represents the graph of $x \leq 0$, or set B.

18. Is the graph of $x < -3$ part of the graph of $x \leq 0$? Yes / No

19. Are the numbers from set A contained in set B? Yes / No

20. Is set A a subset of set B? Yes / No

Problem 4 Finding the Complement of a Set

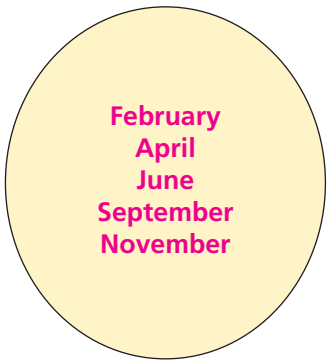
Got It? **Universal set $U = \{$months of the year$\}$ and set $A = \{$months with exactly 31 days$\}$. What is the complement of set A? Write your answer in roster form.**

21. Use the months from the universal set at the right. Write each month in the correct oval. Then label each set A or A'.

January
February
March
April
May
June
July
August
September
October
November
December

A = {exactly 31 days}

A′ = {not exactly 31 days}



22. Write the complement of set A in roster form.

$A' = \{$ February, April, June, September, November $\}$

Lesson Check • Do you know HOW?

Given the universal set $U = \{$seasons of the year$\}$ and $W = \{$winter$\}$, what is W'?

23. W' is ?. Circle your answer.

the universal set | the complement of W | a subset of W

24. Write the elements of U.

spring | summer | fall | winter

25. Use your answers to Exercise 24 and $W = \{\text{winter}\}$, to write W'.

$W' = \{$ spring, summer, fall $\}$

Lesson Check • Do you UNDERSTAND?

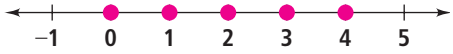
Error Analysis **A student says sets A and B below are the same. What error did the student make?**

$A = \{x \mid x \text{ is a whole number less than } 5\}$ $B = \{1, 2, 3, 4\}$

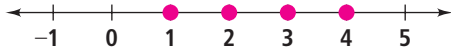
26. Describe the set of whole numbers.

Answers may vary. Sample: The set of whole numbers is the set of positive integers and zero.

27. Graph the numbers that are in set A on the number line below.



28. Graph the numbers that are in set B on the number line below.



29. What error did the student make?

Answers may vary. Sample: The student did not realize that the set of whole numbers begins with 0, not 1.

Math Success

Check off the vocabulary words that you understand.

☐ complement ☐ empty set ☐ universal set

☐ roster form ☐ set-builder notation

Rate how well you can *work with sets.*

Need to review 0 2 4 6 8 10 Now I get it!

3-6 Compound Inequalities

Vocabulary

Review

1. Write **I** if the math sentence is an *inequality*. Write **E** if it is an *equation*.

I $15 > -12$ I $18 \leq 35$ E $5x = 15$ I $9 > 3x$

2. Write the *inequality* symbol that matches each description.

greater than $>$ less than or equal to $\leq$ greater than or equal to $\geq$

Vocabulary Builder

compound (noun or adjective) KAHM pownd

Main Idea: A **compound** (noun) is a whole formed by a union of two or more parts. A **compound** inequality (adjective) consists of two distinct inequalities joined by the word *and* or the word *or*.

Example: You read $3 < x < 6$ as "x is less than 6 *and* greater than 3."

Use Your Vocabulary

3. Cross out each inequality that is NOT a *compound inequality*.

$-2 > x$ or $x > 3$ ~~$x \geq 0$~~ $5 < x \leq 10$ ~~$9 < x$~~

4. Draw a line from each description in Column A to the inequality it describes in Column B.

Column A	Column B
all numbers less than 9 or greater than 5	$x < 5$ or $x > 9$
all numbers less than 9 and greater than 5	$x < 9$ or $x > 5$
all numbers less than 5 and greater than 9	$x < 5$ and $x > 9$
all number less than 5 or greater than 9	$5 < x < 9$

5. **Reasoning** Which inequality in Exercise 4 describes an empty set? Explain.

$x < 5$ and $x > 9$. Explanations may vary. Sample: No number exists that is both less than 5 and greater than 9.

Problem 1 Writing a Compound Inequality

Got It? **Write a compound inequality to represent the phrase below. Graph the solutions.**

all real numbers that are greater than or equal to −4 and less than 6

6. Write the phrase as two phrases.

all real numbers that are **greater than or equal to −4** and all real numbers that are **less than 6**

7. Write an inequality to represent each statement from Exercise 6. Then write a compound inequality.

x **≥** **−4** x **<** **6**

x **≥** **−4** and x **<** **6**

8. Circle the graph of the compound inequality.

A solution of a compound inequality involving *and* is any number that makes *both* inequalities true.

Problem 2 Solving a Compound Inequality Involving *And*

Got It? **What are the solutions of $-2 < 3y - 4 < 14$? Graph the solutions.**

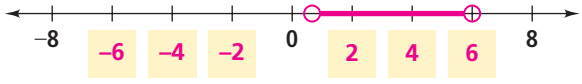
9. Use the justifications at the right to solve the compound inequality.

−2 $< 3y - 4$	and	**3y − 4** < 14	Write two inequalities joined by *and*.
−2 $+$ **4** $< 3y - 4 +$ **4**	and	**3y − 4** $+$ **4** $< 14 +$ **4**	Addition Property of Inequality
2 $< 3y$	and	**3y** < 18	Add.
$\frac{\mathbf{2}}{\mathbf{3}} < \frac{\mathbf{3y}}{\mathbf{3}}$	and	$\frac{\mathbf{3y}}{\mathbf{3}} < \frac{18}{\mathbf{3}}$	Division Property of Inequality
$\frac{\mathbf{2}}{\mathbf{3}} < y$	and	**y** $<$ **6**	Simplify.
$\frac{\mathbf{2}}{\mathbf{3}} < y <$ **6**			Write the solutions as a single inequality.

10. Underline the correct symbol(s) and words to complete the sentence.

Because the compound inequality includes $\underline{\leq}$ / $>$ / $\geq$ / $\leq$, the graph of the compound inequality will include closed dots / $\underline{\text{open dots}}$ / one closed and one open dot.

11. Graph the compound inequality on the number line at the right.



Problem 3 Writing and Solving a Compound Inequality

Got It? Reasoning **To earn a B in your algebra class, you must achieve an unrounded test average between 84 and 86, inclusive. You scored 78, 78, and 79 on the first three (out of four) tests. Is it possible for you to earn a B in the course? Assume that 100 is the maximum grade you can earn on the test. Explain.**

12. Let $x =$ the score of the fourth test. Write a compound inequality.

$84 \leq \frac{78 + 78 + 79 + x}{4} \leq 86$

13. Now solve the compound inequality.

$$84 \leq \frac{78 + 78 + 79 + x}{4} \leq 86$$

$$4(84) \leq 4\left(\frac{235 + x}{4}\right) \leq 4(86)$$

$$336 \leq 235 + x \leq 344$$

$$336 - 235 \leq 235 + x - 235 \leq 344 - 235$$

$$101 \leq x \leq 109$$

14. Is it possible for you to earn a B in the course? Explain.

No. Explanations may vary. Sample: To earn a B, the fourth test score must be between 101 and 109, inclusive. The highest test score possible is only 100.

A solution of a compound inequality involving *or* is any number that makes *either* inequality true.

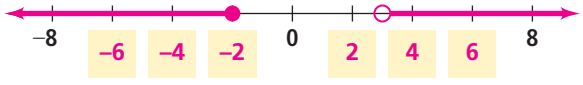
Problem 4 Solving a Compound Inequality Involving *Or*

Got It? What are the solutions of $-2y + 7 < 1$ or $4y + 3 \leq -5$? Graph the solutions.

15. Complete the steps to solve the inequalities.

$-2y + 7 < 1$	or	$4y + 3 \leq -5$
$-2y + 7 - \boxed{7} < 1 - \boxed{7}$	or	$4y + 3 - \boxed{3} \leq -5 - \boxed{3}$
$-2y < \boxed{-6}$	or	$4y \leq \boxed{-8}$
$\frac{-2y}{\boxed{-2}} > \frac{\boxed{-6}}{\boxed{-2}}$	or	$\frac{4y}{\boxed{4}} \leq \frac{\boxed{-8}}{\boxed{4}}$
$y \ \boxed{>} \ \boxed{3}$	or	$y \ \boxed{\leq} \ \boxed{-2}$

16. Graph the compound inequality on the number line below. (*Hint:* Will you use open dots, closed dots, or one of each?)



You can use an inequality to describe an *interval* along the number line. In *interval notation*, you use three special symbols.

brackets

Use [or] with $\leq$ or $\geq$ to indicate that the interval's endpoints are included.

parentheses

Use (or) with $<$ or $>$ to indicate that the interval's endpoints are *not* included.

infinity

Use ∞ when the interval continues forever in a *positive* direction.

Use $-\infty$ when the interval continues forever in a *negative* direction.

Problem 5 Using Interval Notation

Got It? **What is the graph of $(-2, 7]$? How do you write $(-2, 7]$ as an inequality?**

Underline the correct word or words to complete each sentence.

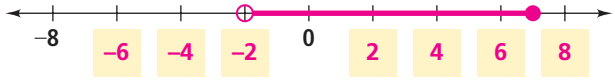
17. In $(-2, 7]$, the parenthesis to the left of -2 means -2 is / <u>is not</u> included in the interval.

18. In $(-2, 7]$, the bracket to the right of 7 means 7 <u>is</u> / is not included in the interval.

19. Use your answers to Exercises 17 and 18 to write a compound inequality.

$-2 < x \leq 7$

20. Graph the inequality.



Lesson Check • Do you UNDERSTAND?

Error Analysis **A student writes the inequality $x \geq 17$ in interval notation as $[17, \infty]$. Explain why this is incorrect.**

21. Circle the correct interval notation for the inequality $x \geq 17$.

$(17, \infty)$ $(17, \infty]$ $[17, \infty)$ $(-\infty, 17]$ or $[17, \infty)$

22. Explain the student's error. **Explanations may vary. Sample:**

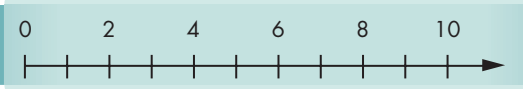
The student should have written a parenthesis after the ∞ symbol, not a bracket.

Math Success

Check off the vocabulary words that you understand.

☐ compound inequality ☐ inclusive ☐ interval ☐ interval notation

Rate how well you can *write and graph compound inequalities.*

Need to review  **Now I get it!**

3-7 Absolute Value Equations and Inequalities

Vocabulary

Review

Write T for *true* or F for *false*.

T **1.** To indicate the *absolute value* of -8, you write $|-8|$.

F **2.** The *absolute value* of -8 is -8, since -8 is 8 units to the left of 0 on the number line.

T **3.** The *absolute value* of -8 is 8, since -8 is 8 units away from 0 on the number line.

T **4.** According to the definition of absolute value, if $|r| = 3$, then $r = 3$ or $r = -3$.

Vocabulary Builder

expression (noun) **ek SPRESH un**

Related Words: express (verb), phrase (noun)

Main Idea: An **expression** is a word or phrase that communicates an idea. A mathematical **expression** is a mathematical phrase. A mathematical **expression** may be *numerical* or *algebraic*.

numerical expression
$18 \div (6 + 3)$

algebraic expression
$4k - 7$

Use Your Vocabulary

Write an *expression* for each word phrase.

5. m increased by 8

m + 8

6. y divided by 9

y ÷ 9

7. u more than 7

7 + u

8. Cross out the *expression* that is NOT algebraic.

$3y - 12$ ~~$4 \div 18 = 3$~~ $12 + x$

9. Cross out the *expression* that is NOT numeric.

$3 - 12$ ~~$4 \div 18q = 3$~~ $12 + 5$

Problem 1 Solving an Absolute Value Equation

Got It? What are the solutions of $|n| - 5 = -2$? Graph and check the solutions.

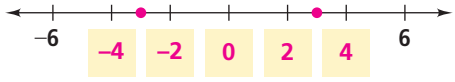
10. Complete the equation to solve for n.

$|n| - 5 + \boxed{5} = -2 + \boxed{5}$

$|n| = \boxed{3}$

$n = \boxed{3}$ or $n = \boxed{-3}$

11. Graph the solutions.



12. Check the solutions of the equation.

$|3| - 5 \stackrel{?}{=} -2$
$3 - 5 \stackrel{?}{=} -2$
$-2 = -2$

$|-3| - 5 \stackrel{?}{=} -2$
$3 - 5 \stackrel{?}{=} -2$
$-2 = -2$

take note

Key Concept Solving Absolute Value Equations

To solve an equation in the form $|A| = b$, where A represents a variable expression and $b > 0$, solve $A = b$ and $A = -b$.

Complete.

13. To solve $|b| = 3$, solve $b = \boxed{3}$ and $b = \boxed{-3}$.

14. To solve $|x - 5| = 6$, solve $x - 5 = \boxed{6}$ and $x - 5 = \boxed{-6}$.

15. To solve $|h + 7| = 2h$, solve $h + 7 = \boxed{2h}$ and $h + 7 = \boxed{-2h}$.

Problem 2 Solving an Absolute Value Equation

Got It? You are skating with a friend, The friend's distance d from you (in feet) after t seconds is given by $d = |80 - 5t|$.

16. a. Circle what the 80 in the equation represents.

The distance she travels.
How fast she travels.

How long she travels.
How far away she starts from you.

b. Circle what the 5 in the equation represents.

The distance she travels.
How fast she travels.

How long she travels.
How far away she is from you.

c. Why is the $5t$ subtracted from the 80?

Because her distance is getting smaller as she gets closer.

17. At what times is she 60 ft from you?

a. The 60 is a measure of distance / time and replaces the variable d / t.

b. To find the times she is 60 ft from you, solve the equation $\boxed{60} = |80 - 5\boxed{t}|$

Problem 3 Solving an Absolute Value Equation With No Solution

Got It? **What are the solutions of $|3x - 6| - 5 = -7$?**

19. To isolate the absolute value expression, you add 5 to each side of the equation. Circle the simplified value of the right side.

-12 -7 -6 -5 -3 (−2)

20. Underline the correct word to complete the sentence.

The absolute value of an expression cannot be negative / positive, so the inequality has no solution.

take note

Key Concept Solving Absolute Value Inequalities

Let A represent a variable expression and let $b > 0$.

To Solve an Inequality in the Form	Solve	
$\|A\| < b$	$-b < A < b$	(For $\|A\| \le b$, solve $-b \le A \le b$.)
$\|A\| > b$	$A < -b$ or $A > b$	(For $\|A\| \ge b$, solve $A \le -b$ or $A \ge b$.)

21. Circle the compound inequality you would use to solve $|5x| > 3$.

$-3 < 5x < 3$ $-3 \le 5x \le 3$ $5x < -3$ or $5x > 3$ $5x \le -3$ or $5x \ge 3$

22. Circle the compound inequality you would use to solve $|3x| < 5$.

$-5 < 3x < 5$ $-5 \le 3x \le 5$ $3x < -5$ or $3x > 5$ $3x \le -5$ or $3x \ge 5$

Problem 4 Solving an Absolute Value Inequality Involving ≥

Got It? **What are the solutions of $|2x + 4| \ge 5$? Graph the solutions.**

23. Write a compound inequality to solve the absolute value inequality.

$2x + 4$ ≤ -5 or $2x + 4$ ≥ 5

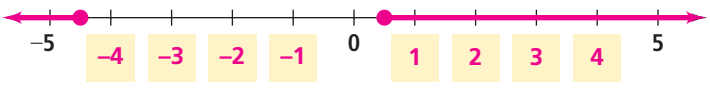
24. Solve the inequalities.

$2x + 4 \le -5$
$2x \le -9$
$x \le -4.5$

or

$2x + 4 \ge 5$
$2x \ge 1$
$x \ge 0.5$

25. Graph your solutions on the number line below.



Problem 5 Solving an Absolute Value Inequality Involving ≤

Got It? **A food manufacturer makes 32-oz boxes of pasta. Not every box weighs exactly 32 oz. The allowable difference from the ideal weight is at most 0.05 oz. Write and solve an absolute value inequality to find the range of allowable weights.**

24. Complete the model.

Relate	difference between ideal and actual weights	is at most	0.05 oz
Define	Let w = the actual weight.		
Write	$\lvert w - 32 \rvert$	$\leq$	0.05

25. Write the absolute value inequality as a compound inequality.

$-0.05 \leq w - 32 \leq 0.05$

26. Solve the compound inequality.

$-0.05 \leq w - 32 \leq 0.05$
$31.95 \leq w \leq 32.05$

27. A box of pasta must weigh between 31.95 oz and 32.05 oz, inclusive.

Lesson Check • Do you UNDERSTAND?

Reasoning **How many solutions do you expect to get when you solve an absolute value equation? Explain.**

28. Write how many solutions each absolute value equation has.

$\lvert x \rvert = 9$	$\lvert x \rvert = 0$	$\lvert x \rvert = -9$
2 solution(s)	1 solution(s)	0 solution(s)

29. Explain how many solutions are possible for any absolute value equation. Answers may vary.

Sample: An absolute value equation can have 0, 1, or 2 solutions.

Math Success

Check off the vocabulary words that you understand.

☐ absolute value ☐ equation ☐ inequality

Rate how well you can *solve absolute value equations and inequalities.*

Need to review 0 2 4 6 8 10 Now I get it!

3-8 Unions and Intersections of Sets

Vocabulary

Review

Write the elements of each *set*.

1. whole numbers less than 4

{0, 1, 2, 3}

2. even numbers between 1 and 9

{2, 4, 6, 8}

3. Two sets are *disjoint* when they have no elements in common.

Are the sets in Exercises 1 and 2 *disjoint*?

Yes / (No)

Vocabulary Builder

union (noun) YOON yuhn

Related Word: intersection (noun), Venn diagram (noun)

Definition: The **union** of two or more sets is the set that contains all the elements of the sets. The *intersection* of two or more sets is the set of elements that are in all of the sets.

$G \cup H$ means "the **union** of sets G and H."

$G \cap H$ means "the **intersection** of sets G and H."

Example: Set $A = \{\text{penny, nickel, dime}\}$ and set $B = \{\text{nickel, dime, quarter}\}$.

$A \cup B = \{\text{penny, nickel, dime, quarter}\}$ and $A \cap B = \{\text{nickel, dime}\}$

Use Your Vocabulary

Write *union* or *intersection* to describe each set.

4. $D = \{\text{cheese, milk, yogurt}\}$ and $F = \{\text{apple, banana, pear}\}$
the set {apple, banana, cheese, milk, pear, yogurt}

union

5. $D = \{\text{cheese, milk, yogurt}\}$ and $M = \{\text{bread, cheese, egg}\}$
the set {cheese}

intersection

6. $F = \{\text{apple, banana, pear}\}$ and $M = \{\text{bread, cheese, egg}\}$
the empty set

intersection

Problem 1 Union of Sets

Got It? Write sets P and Q below in roster form. What is $P \cup Q$?

$P = \{x \mid x \text{ is a whole number less than } 5\}$
$Q = \{y \mid y \text{ is an even natural number less than } 5\}$

7. The symbol $\cup$ means the union / intersection of the sets.

8. Circle the numbers in set P.

(0) (1) (2) (3) (4) 5

9. Write set P in roster form.

$P = \{$ 0 , 1 , 2 , 3 , 4 $\}$

10. Circle the numbers in set Q.

0 1 (2) 3 (4) 5

11. Write set Q in roster form.

$Q = \{$ 2 , 4 $\}$

12. Write $P \cup Q$.

$P \cup Q = \{$ 0 , 1 , 2 , 3 , 4 $\}$

13. Reasoning What is true about the union of two distinct sets if one set is a subset of the other? (Assume that the subset is not the original set.)

Answers may vary. Sample: Their union is the larger set.

Problem 2 Intersection of Sets

Got It? Let $A = \{2, 4, 6, 8\}$, $B = \{0, 2, 5, 7, 8\}$, and $C = \{n \mid n \text{ is an odd whole number}\}$. What is $A \cap B$?

14. The symbol $\cap$ means the union / intersection of the sets.

15. The numbers 2 and 8 are in both set A and set B.

16. Write $A \cap B$.

$A \cap B = \{$ 2 , 8 $\}$

17. Write $A \cap C$, and write $C \cap B$.

$A \cap C =$ { } $C \cap B =$ {5, 7}

18. Reasoning What is true about the intersection of two distinct sets if one set is a subset of the other? (Assume that the subset is not the original set.)

Answers may vary. Sample: Their intersection is the smaller set.

Problem 3 Making a Venn Diagram

Got It? **Let $A = \{x \mid x \text{ is one of the first five letters in the English alphabet}\}$, $B = \{x \mid x \text{ is a vowel}\}$, and $C = \{x \mid x \text{ is a letter in the word VEGETABLE}\}$. Which letters are in all three sets?**

19. List the elements of each set.

$A = \{\text{A, } \boxed{\text{B}}, \boxed{\text{C}}, \boxed{\text{D}}, \boxed{\text{E}}\}$ $B = \{\text{A, } \boxed{\text{E}}, \boxed{\text{I}}, \boxed{\text{O}}, \boxed{\text{U}}\}$

$C = \{\text{V, } \boxed{\text{E}}, \boxed{\text{G}}, \boxed{\text{T}}, \text{A, } \boxed{\text{B}}, \boxed{\text{L}}\}$

20. Write the correct letters to complete each statement.

The letters that are in set A but are *not* in any other set are ? . **C, D**

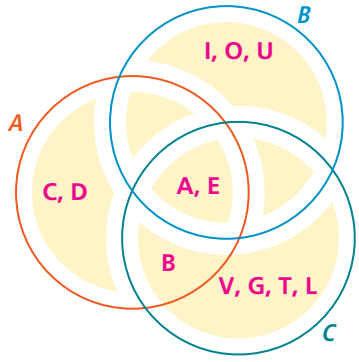
The letters that are in set B that are *not* in any other set are ? . **I, O, U**

The letters that are in set C that are *not* in any other set are ? . **V, G, T, L**

The letter that is in both sets A and C, but *not* in B is ? . **B**

The letters that are in all three sets are ? . **A, E**

21. Use your answers to Exercise 20 to complete the Venn diagram below.



Problem 4 Using a Venn Diagram to Show Numbers of Elements

Got It? **Of 30 students in student government, 20 are honor students and 9 are officers and honor students. All of the students are officers, honor students, or both. How many are officers but not honor students?**

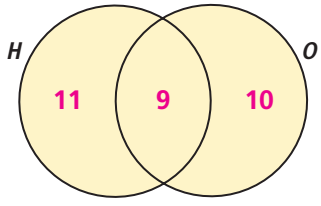
22. Use the information in the problem to complete each statement. Then complete the Venn diagram.

Let $H =$ **honor students** and let $O =$ **officers** .

$H \cap O$ has **9** students.

Only honor students: $20 - \boxed{9} = \boxed{11}$

Only officers: $30 - \boxed{20} = \boxed{10}$



23. The number of students who are officers but not honor students is **10** .

Problem 5 Writing Solutions of an Inequality

Got It? **Solve the inequality $8 \leq x + 5 < 11$. Write the solutions as either the union or the intersection of two sets.**

24. **Multiple Choice** What is the first step in solving the inequality?

(A) Add 5 to each expression.

(B) Subtract 5 from each expression.

(C) Add 8 to each expression.

(D) Subtract 8 from each expression.

25. When you isolate the variable, the inequality becomes $3 \leq x < 6$.

26. Write two inequalities.

$3 \leq x$ and $x < 6$

27. Now write the solutions of the inequality as the union or the intersection of two sets.

$\{x \mid 3 \leq x\} \cap \{x \mid x < 6\}$

Lesson Check • Do you UNDERSTAND?

Compare and Contrast **How are unions and intersections of sets different?**

28. Write **U** if the statement describes a *union*. Write **I** if the statement describes an *intersection*.

U — It contains the elements that belong to either set or both sets.

I — In a Venn diagram, it is the part of the circles that overlap.

29. Use your answers to Exercise 28 to explain how unions and intersections are similar and how they are different.

Answers may vary. Sample: Unions of sets include all elements in both sets. Intersections of sets include only the elements that the sets share.

Math Success

Check off the vocabulary words that you understand.

☐ union ☐ intersection ☐ disjoint sets ☐ subsets

Rate how well you can *find unions and intersections of sets.*

Need to review 0 2 4 6 8 10 Now I get it!