

Properties of Parallel Lines

1. Plan

What You'll Learn

- To identify angles formed by two lines and a transversal
- To prove and use properties of parallel lines

... And Why

To describe angles formed by an airport runway that crosses two parallel runways, as in Example 2



Check Skills You'll Need



page 30 or Skills Handbook page 758

x² Algebra Solve each equation.

1. $x + 2x + 3x = 180$ **30**

2. $(w + 23) + (4w + 7) = 180$ **30**

3. $90 = 2y - 30$ **60**

4. $180 - 5y = 135$ **9**

Write an equation and solve the problem.

5. The sum of $m\angle 1$ and twice its complement is 146. Find $m\angle 1$.

$m\angle 1 + 2(90 - m\angle 1) = 146; m\angle 1 = 34$

6. The measures of two supplementary angles are in the ratio 2 : 3. Find their measures. **72 and 108**



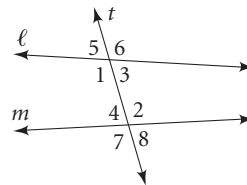
New Vocabulary

- transversal
- alternate interior angles
- same-side interior angles
- corresponding angles
- two-column proof
- alternate exterior angles
- same-side exterior angles

1

Identifying Angles

A **transversal** is a line that intersects two coplanar lines at two distinct points. The diagram shows the eight angles formed by a transversal t and two lines ℓ and m .

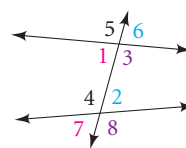
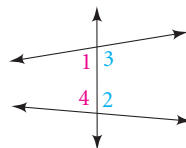
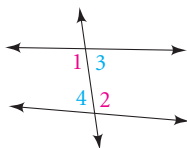


Pairs of the eight angles have special names as suggested by their positions.

$\angle 1$ and $\angle 2$ are **alternate interior angles.**

$\angle 1$ and $\angle 4$ are **same-side interior angles.**

$\angle 1$ and $\angle 7$ are **corresponding angles.**



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1 EXAMPLE Identifying Angles

Use the diagrams above. Name another pair of alternate interior angles and another pair of same-side interior angles.

- $\angle 3$ and $\angle 4$ are alternate interior angles. $\angle 2$ and $\angle 3$ are same-side interior angles.

Quick Check

- 1 Name three other pairs of corresponding angles in the diagrams above.
 $\angle 5$ and $\angle 4$, $\angle 6$ and $\angle 2$, $\angle 3$ and $\angle 8$

Objectives

- To identify angles formed by two lines and a transversal
- To prove and use properties of parallel lines

Examples

- Identifying Angles
- Real-World Connection
- Writing a Two-Column Proof
- Finding Measures of Angles
- Using Algebra to Find Angle Measures



Math Background

The Corresponding Angles Postulate is a variation of Euclid's famous Parallel Postulate, which subsequent mathematicians vainly hoped could be proved as a theorem. In the nineteenth century, altering the postulate enabled the invention of hyperbolic and elliptic geometries. Any geometry that obeys the Parallel Postulate is now known as a Euclidean geometry.

More Math Background: p. 124C

Lesson Planning and Resources

See p. 124E for a list of the resources that support this lesson.



Bell Ringer Practice

Check Skills You'll Need

For intervention, direct students to:

Solving Linear Equations
Algebra 1 Review, page 30
Skills Handbook, p. 758

Differentiated Instruction Solutions for All Learners

Special Needs L1

Students may not understand what is meant by the *interior* and *exterior* of two lines. Draw a diagram with two lines and a transversal. Shade and label the interior with one color and the exterior with another color.

learning style: visual

Below Level L2

Students can fold and cut a sheet of paper along a line not parallel to an edge and then match angles to confirm Theorem 3-1.

learning style: tactile

2. Teach

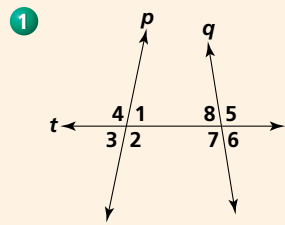
Guided Instruction

Tactile Learners

Provide straws for students to use to model parallel lines and transversals, and provide protractors to test the postulate and theorems in this lesson.

PowerPoint

Additional Examples



Use the diagram above. Identify which angle forms a pair of same-side interior angles with $\angle 1$. Identify which angle forms a pair of corresponding angles with $\angle 1$. **$\angle 8$; $\angle 5$**

2 Use the diagram from Example 2. Compare $\angle 2$ and the vertical angle of $\angle 1$. Classify them as alternate interior angles, same-side interior angles, or corresponding angles. **alternate interior angles**

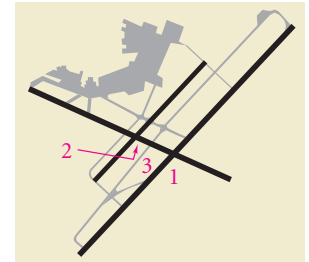
2 EXAMPLE Real-World Connection

Aviation In the diagram of Lafayette Regional Airport, the black segments are runways and the gray areas are taxiways and terminal buildings. Classify $\angle 1$ and $\angle 2$ as alternate interior angles, same-side interior angles, or corresponding angles.

- $\angle 1$ and $\angle 2$ are corresponding angles.



- Quick Check** 2 Classify $\angle 2$ and $\angle 3$ as alternate interior angles, same-side interior angles, or corresponding angles. **same-side int.** \triangle



Lafayette Regional Airport
Lafayette, Louisiana

2 Properties of Parallel Lines

In the photograph, the vapor trail of the high-flying aircraft suggests a transversal of the parallel trails of the low-flying aircraft.



Vocabulary Tip

Corresponding objects are related in a special way. Here, **corresponding angles** are angles that are in similar positions on the same side of a transversal.

The same-size angles that appear to be formed by the vapor trails suggest the postulate and theorems below.

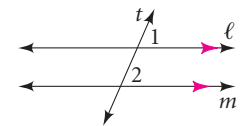


Key Concepts

Postulate 3-1 Corresponding Angles Postulate

If a transversal intersects two parallel lines, then corresponding angles are congruent.

$$\angle 1 \cong \angle 2$$

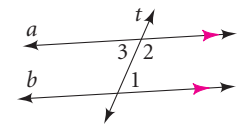


Key Concepts

Theorem 3-1 Alternate Interior Angles Theorem

If a transversal intersects two parallel lines, then alternate interior angles are congruent.

$$\angle 1 \cong \angle 3$$



Theorem 3-2 Same-Side Interior Angles Theorem

If a transversal intersects two parallel lines, then same-side interior angles are supplementary.

$$m\angle 1 + m\angle 2 = 180$$

Differentiated Instruction Solutions for All Learners

Advanced Learners L4

Ask students to explain how to find the other 15 angle measures in Example 4 when a different angle measure is given.

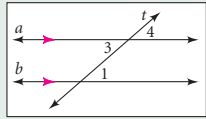
English Language Learners ELL

In Example 1, have partners discuss the vocabulary. Encourage them to use the words *alternate*, *corresponding*, *interior*, and *exterior* in nonmathematical contexts.

You can display the steps that prove a theorem in a **two-column proof**.

Given: $a \parallel b$ ← what you know

Prove: $\angle 1 \cong \angle 3$ ← what you must show



deductive steps that connect what you know to what you must show →

Statements	Reasons
1. ~~~~~	1. ~~~~~
2. ~~~~~	2. ~~~~~
3. ~~~~~	3. ~~~~~
4. ~~~~~	4. ~~~~~

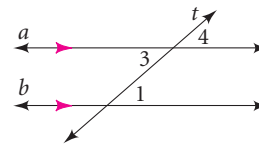
Proof

Proof of Theorem 3-1

If a transversal intersects two parallel lines, then alternate interior angles are congruent.

Given: $a \parallel b$

Prove: $\angle 1 \cong \angle 3$



Statements	Reasons
1. $a \parallel b$	1. Given
2. $\angle 1 \cong \angle 4$	2. If lines are \parallel , then corresponding angles are congruent.
3. $\angle 4 \cong \angle 3$	3. Vertical angles are congruent.
4. $\angle 1 \cong \angle 3$	4. Transitive Property of Congruence

You will prove Theorem 3-2 in Exercise 28.

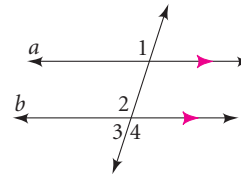
Proof

3 EXAMPLE Writing a Two-Column Proof

Study what is given, what you are to prove, and the diagram. Then write a two-column proof.

Given: $a \parallel b$

Prove: $\angle 1 \cong \angle 4$



Statements	Reasons
1. $a \parallel b$	1. Given
2. $\angle 1 \cong \angle 2$	2. If lines are \parallel , then corresponding angles are congruent.
3. $\angle 2 \cong \angle 4$	3. Vertical angles are congruent.
4. $\angle 1 \cong \angle 4$	4. Transitive Property of Congruence

1. $a \parallel b$ (Given)
2. $m\angle 2 + m\angle 3 = 180$
(Angle Add. Post.)
3. $m\angle 1 = m\angle 2$
(Corr. Δ Post.)
4. $m\angle 1 + m\angle 3 = 180$
(Substitute.)
5. $\angle 1$ and $\angle 3$ are supp.
(Def. of supp. Δ)



- 3 Using the same given information and diagram from Example 3, prove that $\angle 1$ and $\angle 3$ are supplementary. **See left.**

In Example 3, $\angle 1$ and $\angle 4$ are **alternate exterior angles**. $\angle 1$ and $\angle 3$ are **same-side exterior angles**. Example 3 and its Quick Check prove Theorems 3-3 and 3-4 as stated on the next page.

Guided Instruction

Error Prevention!

Students may try to apply the Corresponding Angles Postulate, Alternate Interior Angles Theorem, and Same-Side Interior Angles Theorem when lines are not parallel. Emphasize that the postulate and theorems apply only when a transversal intersects *parallel* lines.

Teaching Tip

Provide this summary of the steps to follow in a two-column proof.

- Draw and label a diagram.
- State the Given and the Prove in terms of the diagram.
- Develop a Plan for Proof.
- Write each step in the left column and the reason for each step in the right column.

3 EXAMPLE Alternative Method

After students study the Proof, point out that working backward can help them plan a proof.

4 EXAMPLE Technology Tip

You might want to use geometry software and the postulate and theorems in this lesson to find the measures of the angles.

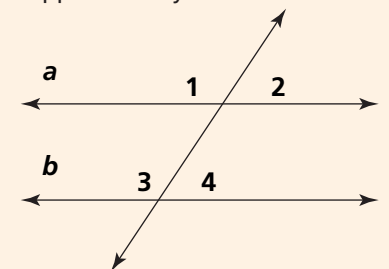
Visual Learners

Suggest that students draw a diagram of parallel lines and a transversal, use numbers to label the eight angles formed, and color-code the angles to indicate which are congruent.

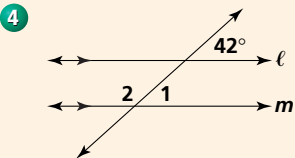


Additional Examples

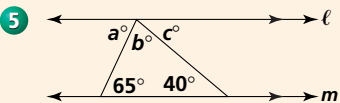
- 3 Use the given that $a \parallel b$ and the diagram to write a two-column proof that $\angle 1$ and $\angle 4$ are supplementary.



Additional Examples



In the diagram above, $l \parallel m$. Find $m\angle 1$ and then $m\angle 2$. $m\angle 1 = 42$; $m\angle 2 = 138$



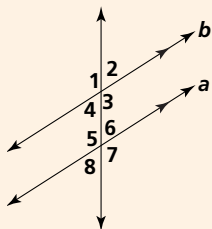
In the diagram above, $l \parallel m$. Find the values of a , b , and c . $a = 65$, $b = 75$, $c = 40$

Resources

- Daily Notetaking Guide 3-1 **L3**
- Daily Notetaking Guide 3-1—Adapted Instruction **L1**

Closure

In the diagram below, $a \parallel b$. Find all the angles that have equal measures.



$m\angle 1 = m\angle 3 = m\angle 5 = m\angle 7$;
 $m\angle 2 = m\angle 4 = m\angle 6 = m\angle 8$



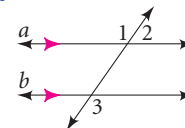
Key Concepts

Theorem 3-3

Alternate Exterior Angles Theorem

If a transversal intersects two parallel lines, then alternate exterior angles are congruent.

$$\angle 1 \cong \angle 3$$



Theorem 3-4

Same-Side Exterior Angles Theorem

If a transversal intersects two parallel lines, then same-side exterior angles are supplementary.

$$m\angle 2 + m\angle 3 = 180$$

When you see two parallel lines and a transversal, and you know the measure of one angle, you can find the measures of all the angles. This is illustrated in Example 4.



Test-Taking Tip

You can make marks on any diagrams shown on tests. This can help you keep track of known information.

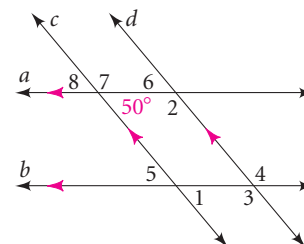
4

EXAMPLE Finding Measures of Angles

Find $m\angle 1$, and then $m\angle 2$. Which theorem or postulate justifies each answer?

Since $a \parallel b$, $m\angle 1 = 50$ because corresponding angles are congruent (Corresponding Angles Postulate).

Since $c \parallel d$, $m\angle 2 = 130$ because same-side interior angles are supplementary (Same-Side Interior Angles Theorem).



4

Find the measure of each angle. Justify each answer.

- a. $\angle 3$ b. $\angle 4$ c. $\angle 5$ **50; alt. int. \triangle are \cong .**
 d. $\angle 6$ e. $\angle 7$ f. $\angle 8$ **50; corr. \triangle are \cong or vert. \triangle are \cong .**
4a. 130; corr. \triangle are \cong . b. 130; vert. \triangle are \cong .
d. 50; alt. int. \triangle are \cong . e. 130; same-side int. \triangle are supp.

Sometimes you can use algebra to find angle measures.

5

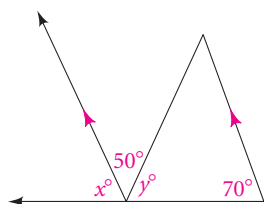
EXAMPLE Using Algebra to Find Angle Measures

Gridded Response Find the value of y in the diagram at the left.

$$x = 70 \quad \text{Corresponding angles of parallel lines are } \cong.$$

$$70 + 50 + y = 180 \quad \text{Angle Addition Postulate}$$

$$y = 60 \quad \text{Subtraction Property of Equality}$$

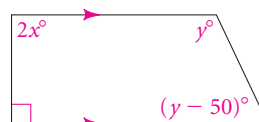


		6	0
.	/	/	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9



5

Find the values of x and y . Then find the measures of the angles.



$$x = 45, y = 115;$$

$$90, 90, 115, 65$$

5. $\angle 1$ and $\angle 2$: corr. \triangle
 $\angle 3$ and $\angle 4$: alt. int. \triangle
 $\angle 5$ and $\angle 6$: corr. \triangle

6. $\angle 1$ and $\angle 2$: same-side int. \triangle
 $\angle 3$ and $\angle 4$: corr. \triangle
 $\angle 5$ and $\angle 6$: corr. \triangle

7. $\angle 1$ and $\angle 2$: corr. \triangle
 $\angle 3$ and $\angle 4$: same-side int. \triangle
 $\angle 5$ and $\angle 6$: alt. int. \triangle

EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice*.

Practice and Problem Solving

A Practice by Example

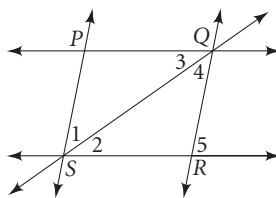
Examples 1, 2
(pages 127, 128)



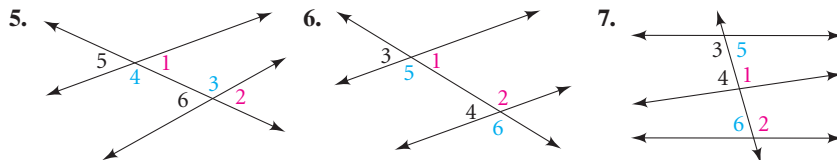
- \overleftrightarrow{PQ} and \overleftrightarrow{SR} with transversal \overleftrightarrow{SQ} ; alt. int. \triangle
- \overleftrightarrow{PS} and \overleftrightarrow{QR} with transversal \overleftrightarrow{SQ} ; alt. int. \triangle
- \overleftrightarrow{PS} and \overleftrightarrow{QR} with transversal \overleftrightarrow{PQ} ; same-side int. \triangle
- \overleftrightarrow{PS} and \overleftrightarrow{QR} with transversal \overleftrightarrow{SR} ; corr. \triangle

Name the two lines and the transversal that form each pair of angles. Then classify the pair of angles.

- $\angle 2$ and $\angle 3$
- $\angle 1$ and $\angle 4$
- $\angle SPQ$ and $\angle PQR$
- $\angle 5$ and $\angle PSR$



Classify each pair of angles labeled in the same color as *alternate interior angles*, *same-side interior angles*, or *corresponding angles*. 5–7. See margin.



- The boards securing this barn door suggest two parallel lines and a transversal. Classify $\angle 1$ and $\angle 2$ as alternate interior angles, same-side interior angles, or corresponding angles. alt. int. \triangle



Example 3
(page 129)

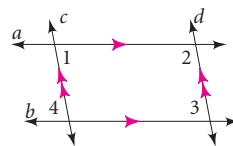
- $a \parallel b$ (Given)
- $\angle 1 \cong \angle 4$ (Alt. Int. \triangle Thm.)
- $c \parallel d$ (Given)
- $\angle 4 \cong \angle 3$ (Corr. \triangle Post.)
- $\angle 1 \cong \angle 3$ (Trans. Prop.)

- Developing Proof** Supply the missing reasons in this two-column proof.

Given: $a \parallel b, c \parallel d$

Prove: $\angle 1 \cong \angle 3$

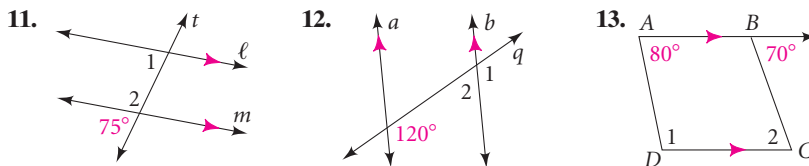
Statements	Reasons
1. $a \parallel b$	1. Given
2. $\angle 3$ and $\angle 2$ are supplementary.	2. Same-Side Int. Angles Thm.
3. $c \parallel d$	3. Given
4. $\angle 1$ and $\angle 2$ are supplementary.	4. Same-Side Int. Angles Thm.
5. $\angle 1 \cong \angle 3$	5. Congruent Supplements Thm.



- Proof** 10. Write a two-column proof for Exercise 9 that does not use $\angle 2$.

Example 4
(page 130)

Find $m\angle 1$, and then $m\angle 2$. Justify each answer. 11–13. See margin.



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- $m\angle 1 = 75$ because corr. \triangle of \parallel lines are \cong ; $m\angle 2 = 105$ because same-side int. \triangle of \parallel lines are suppl.
- $m\angle 1 = 120$ because corr. \triangle of \parallel lines are \cong ; $m\angle 2 = 60$ because same-side int. \triangle of \parallel lines are suppl.
- $m\angle 1 = 100$ because same-side int. \triangle of \parallel lines are suppl.; $m\angle 2 = 70$ because alt. int. \triangle of \parallel lines have = measure.

3. Practice

Assignment Guide

- 1 A B 1-8, 19-22, 26, 27
2 A B 9-18, 23-25, 28-30
C Challenge 31-36
- Test Prep 37-41
Mixed Review 42-50

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 6, 12, 24, 26, 28.

Exercises 1, 2 Go over these exercises as a large group to make sure that students understand which lines form the angles.

Connection to Discrete Math

Exercises 19–22 These exercises use the concept of combinations.

Differentiated Instruction Resources

GPS Guided Problem Solving	L3
Enrichment	L4
Reteaching	L2
Adapted Practice	L1
Practice	L3

Practice 3-1 Properties of Parallel Lines

Classify each pair of angles as alternate interior angles, same-side interior angles, or corresponding angles.

-
-
-
-
-
-

Use the figure on the right to answer Exercises 7–9.

- Name all pairs of corresponding angles formed by the transversal t and lines a and c .
- Name all pairs of alternate interior angles formed by the transversal t and lines a and c .
- Name all pairs of same-side interior angles formed by the transversal t and lines a and c .

Find $m\angle 1$ and then $m\angle 2$. Justify each answer.

-
-
-

Algebra Find the value of x . Then find the measure of each angle.

-
-
-

16. **Developing Proof** Supply the missing reasons in this two-column proof.

Given: $a \parallel b$
Prove: $\angle 1 \cong \angle 3$

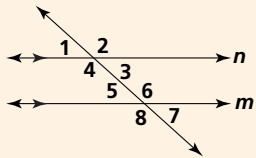
Statements	Reasons
1. $a \parallel b$	1. Given
2. $\angle 1 \cong \angle 2$	a. \triangle
3. $\angle 2 \cong \angle 3$	b. \triangle
4. $\angle 1 \cong \angle 3$	c. \triangle

4. Assess & Reteach

PowerPoint

Lesson Quiz

In the diagram below, $m \parallel n$. Use the diagram for Exercises 1–5.



- Complete: _____ and $\angle 4$ are alternate interior angles. $\angle 6$
- Complete: _____ and $\angle 8$ are corresponding angles. $\angle 4$
- Suppose that $m\angle 3 = 37$. Find $m\angle 6$. **143**
- Suppose that $m\angle 1 = x + 12$ and $m\angle 5 = 3x - 36$. Find x . **24**
- If a transversal intersects two parallel lines, then same-side exterior angles are supplementary. Write a Proof. **Given:** $m \parallel n$
Prove: $\angle 2$ and $\angle 7$ are supplementary. **Show that $m\angle 2 = m\angle 6$. Then show that $m\angle 6 + m\angle 7 = 180$, and substitute $m\angle 2$ for $m\angle 6$.**

Alternative Assessment

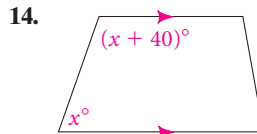
Have each student draw a diagram of two lines cut by a transversal. Then have students use their diagrams to write answers to these exercises.

- Define *alternate interior angles*, *corresponding angles*, and *same-side interior angles* in your own words.
- Summarize what you know about these angles when the transversal cuts parallel lines.

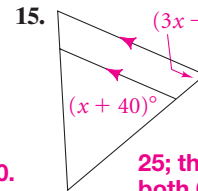
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Example 5 x^2 **Algebra** Find the value of x . Then find the measure of each labeled angle. (page 130)

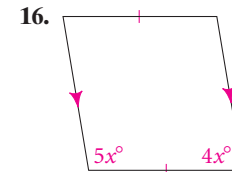
16. 20; the \triangle s are 100 and 80.



70; the \triangle s are 70 and 110.

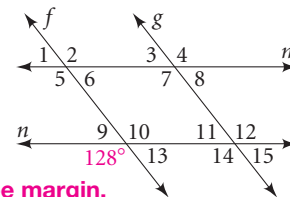


25; the \triangle s are both 65.



B Apply Your Skills

17. In the figure at the right, $f \parallel g$ and $m \parallel n$. Find the measure of each numbered angle. **See margin.**

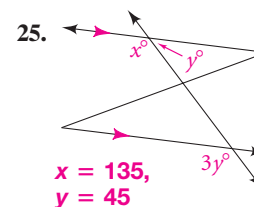
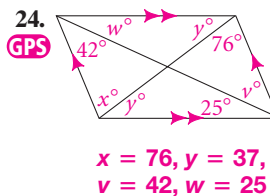
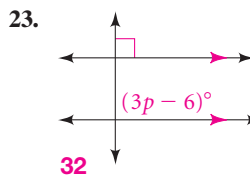


18. Two pairs of parallel segments form the “pound sign” on your telephone keypad. To find the measures of all the angles in the pound sign, how many angles must you measure? Explain. **See margin.**

Two lines and a transversal form how many pairs of the following?

- alternate interior angles **two**
- corresponding angles **four**
- same-side exterior angles **two**
- vertical angles **four**

x^2 **Algebra** Find the values of the variables.



Problem Solving Hint

In Exercise 24, turn your book so the other two parallel lines appear horizontal.



26. **Writing** Look up the meaning of the prefix *trans*. Explain how the meaning of the prefix relates to the word *transversal*. **Trans means across or over. A transversal cuts across other lines.**

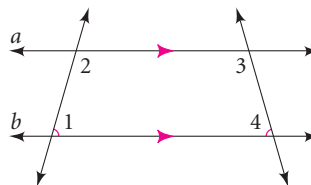
27. **Open-Ended** The letter Z illustrates alternate interior angles. Find at least two other letters that illustrate the pairs of angles presented in this lesson. Draw the letters, mark the angles, and describe them. **See margin p. 133.**

1. $a \parallel b$ (Given)
- $\angle 1$ and $\angle 2$ are suppl. (Same Side. Int. \triangle Thm.)
- $\angle 3$ and $\angle 4$ are suppl. (Same Side. Int. \triangle Thm.)
- $\angle 1 \cong \angle 4$ (Given)
- $\angle 2 \cong \angle 3$ (\cong Suppl. Thm.)

Proof 28. Write a two-column proof.

Given: $a \parallel b, \angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 3$

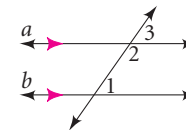


29. Prove Theorem 3-2. **See back of book.**

If a transversal intersects two parallel lines, then same-side interior angles are supplementary.

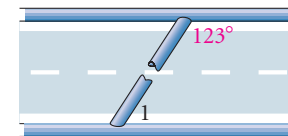
Given: $a \parallel b$

Prove: $\angle 1$ and $\angle 2$ are supplementary.



30. **Engineering** Engineers are laying pipe below ground on opposite sides of the street as shown here. To join the pipe, workers on each side of the street work towards the middle.

- If one team lays pipe at the angle shown, what should the other team use for $m\angle 1$? **57**
- Are these two angles alternate interior, same-side interior, or corresponding angles? **same-side int. \triangle**

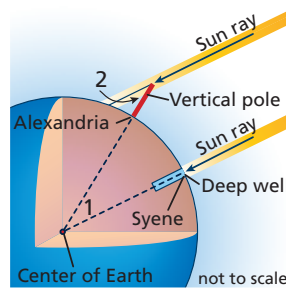


17. $m\angle 1 = m\angle 3 = m\angle 6 = m\angle 8 = m\angle 9 = m\angle 11 = m\angle 13 = m\angle 15 = 52$;
 $m\angle 2 = m\angle 4 = m\angle 5 = m\angle 7 = m\angle 10 = m\angle 12 = m\angle 14 = 128$

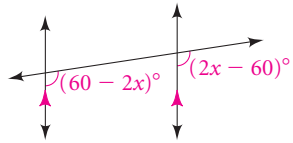
18. You must find the measure of one \angle . All \triangle s that are vert., corr., or alt. int. to that \angle will have that measure. All other \triangle s will be the suppl. of that measure.

C Challenge

31. History About 220 B.C., Eratosthenes estimated the circumference of Earth. He achieved this remarkable feat by using two locations in Egypt. He assumed that Earth is a sphere and that the sun's rays are parallel. He used the measures of $\angle 1$ and $\angle 2$ in his estimation.



- a. Classify $\angle 1$ and $\angle 2$ as alternate interior, same-side interior, or corresponding angles.
 b. How did Eratosthenes know that $\angle 1 \cong \angle 2$?
He knew that alt. int. \angle s of parallel lines are \cong .



- 33. Never; the two planes do not intersect.**
34. Sometimes; if they are \parallel .

32. Error Analysis The diagram at the left contains contradictory information. What is it? Why is it contradictory? **See margin.**

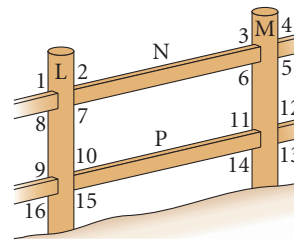
Line m is in plane A and line n is in plane B . Planes A and B are parallel. Complete each statement with *sometimes*, *always*, or *never*. Justify each answer.

- 33.** Lines m and n ? intersect. **34.** Lines m and n are ? coplanar.
35. Lines m and n are ? parallel. **36.** Lines m and n are ? skew.
Sometimes; they may be skew. **Sometimes; they may be \parallel .**



Test Prep

A fence on a hill uses vertical posts L and M to hold parallel rails N and P. Use the diagram for Exercises 37–41.



Multiple Choice

- 37.** $\angle 10$ and $\angle 14$ are alternate interior angles. Which is the transversal? **D**
 A. L B. M
 C. N D. P
- 38.** If $m\angle 1 = 115$, what is $m\angle 16$? **G**
 F. 35 G. 65 H. 85 J. 115
- 39.** If $m\angle 10 = x - 24$, what is $m\angle 7$? **D**
 A. $156 + x$ B. $204 + x$ C. $156 - x$ D. $204 - x$
- 40.** If $m\angle 1 = 6x$ and $m\angle 12 = 4x$, what is $m\angle 5$? **J**
 F. 54 G. 60 H. 72 J. 108

Short Response

- 41. a.** Describe a plan for showing that $\angle 1 \cong \angle 5$. **a-b. See margin.**
b. Explain why $\angle 1 \cong \angle 5$. Justify each step.

41. [2] a. First show that $\angle 1 \cong \angle 7$. Then show that $\angle 7 \cong \angle 5$. Finally, show that $\angle 1 \cong \angle 5$ (OR other valid solution plan).

b. $\angle 1 \cong \angle 7$ because vert. \angle s are \cong . $\angle 7 \cong \angle 5$ because corr. \angle s of \parallel lines are \cong . Finally, by the Transitive Prop. of \cong , $\angle 1 \cong \angle 5$.

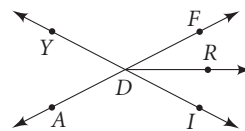
[1] incorrect sequence of steps OR incorrect logical argument

Mixed Review



Lesson 2-5

Find the measure of each angle if $m\angle YDF = 121$ and \overline{DR} bisects $\angle FDI$.



- 42.** $\angle IDA$ **43.** $\angle YDA$ **44.** $\angle RDI$
121 **59** **29.5**

Lesson 1-8

Coordinate Geometry Find the coordinates of the midpoint of \overline{AB} .

- 45.** $A(0, 9), B(1, 5)$ **46.** $A(-3, 8), B(2, -1)$ **47.** $A(10, -1), B(-4, 7)$
(0.5, 7) **(-0.5, 3.5)** **(3, 3)**

Lesson 1-1

Find a pattern for each sequence. Use the pattern to show the next two terms.

- 48.** 4, 8, 12, 16, ... **49.** 1, -2, 4, -8, ... **50.** 23, 16, 9, 2, ...
Add 4; 20, 24. **Multiply by -2; 16, -32.** **Subtract 7; -5, -12.**

27. Answers may vary.
1 Sample: E
2 illustrates corr. \angle s
3 ($\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$) and same-side
4 int. \angle s ($\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$);

1 2 / illustrates alt. int. \angle s ($\angle 1$ and $\angle 4$, $\angle 2$ and $\angle 3$) and same-side int. \angle s ($\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$).

32. The \angle s labeled are corr. \angle s and should be \cong . If you solve $2x - 60 =$

$60 - 2x$, you get $x = 30$. This would be impossible since the diagram shows they are positive \angle s and $2x - 60$ and $60 - 2x$ would equal 0.