3.1 Solving Quadratic Equations

COMMON CORE

Learning Standards
HSA-SSE.A.2
HSA-REI.B.4b
HSF-IF.C.8a

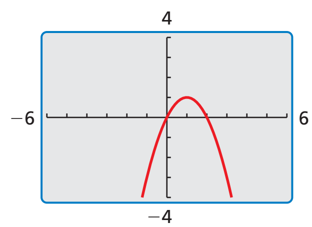
Essential Question How can you use the graph of a quadratic equation to determine the number of real solutions of the equation?

EXPLORATION 1 Matching a Quadratic Function with Its Graph

Work with a partner. Match each quadratic function with its graph. Explain your reasoning. Determine the number of *x*-intercepts of the graph.

a. $f(x) = x^2 - 2x$ **b.** $f(x) = x^2 - 2x + 1$ **c.** $f(x) = x^2 - 2x + 2$

d. $f(x) = -x^2 + 2x$ **e.** $f(x) = -x^2 + 2x - 1$ **f.** $f(x) = -x^2 + 2x - 2$

A.


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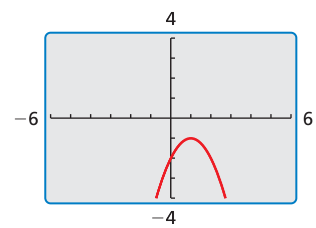

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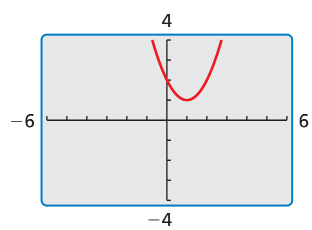

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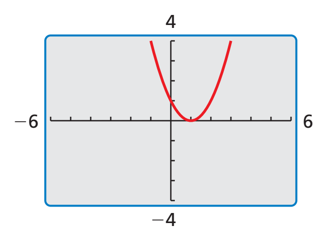

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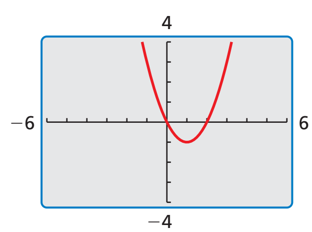

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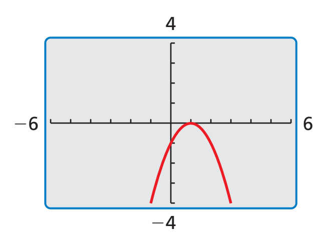


EXPLORATION 2 Solving Quadratic Equations

Work with a partner. Use the results of Exploration 1 to find the real solutions (if any) of each quadratic equation.

a. $x^2 - 2x = 0$ **b.** $x^2 - 2x + 1 = 0$ **c.** $x^2 - 2x + 2 = 0$

d. $-x^2 + 2x = 0$ **e.** $-x^2 + 2x - 1 = 0$ **f.** $-x^2 + 2x - 2 = 0$

MAKING SENSE OF PROBLEMS

To be proficient in math, you need to make conjectures about the form and meaning of solutions.

Communicate Your Answer

3. How can you use the graph of a quadratic equation to determine the number of real solutions of the equation?

4. How many real solutions does the quadratic equation $x^2 + 3x + 2 = 0$ have? How do you know? What are the solutions?

3.1 Lesson

Core Vocabulary

quadratic equation in one variable, *p. 94*
root of an equation, *p. 94*
zero of a function, *p. 96*

Previous
properties of square roots
factoring
rationalizing the denominator

What You Will Learn

- Solve quadratic equations by graphing.
- Solve quadratic equations algebraically.
- Solve real-life problems.

Solving Quadratic Equations by Graphing

A **quadratic equation in one variable** is an equation that can be written in the standard form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$. A **root of an equation** is a solution of the equation. You can use various methods to solve quadratic equations.

STUDY TIP
Quadratic equations can have zero, one, or two real solutions.

Core Concept

Solving Quadratic Equations

By graphing Find the x-intercepts of the related function $y = ax^2 + bx + c$.

Using square roots Write the equation in the form $u^2 = d$, where u is an algebraic expression, and solve by taking the square root of each side.

By factoring Write the polynomial equation $ax^2 + bx + c = 0$ in factored form and solve using the Zero-Product Property.

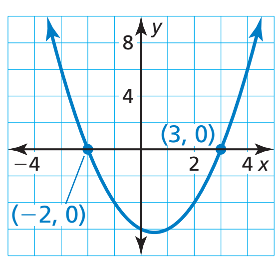
EXAMPLE 1 Solving Quadratic Equations by Graphing

Solve each equation by graphing.

a. $x^2 - x - 6 = 0$ **b.** $-2x^2 - 2 = 4x$

SOLUTION

a. The equation is in standard form. Graph the related function $y = x^2 - x - 6$.



The x-intercepts are -2 and 3.

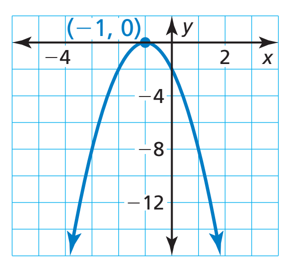
▶ The solutions, or roots, are $x = -2$ and $x = 3$.

Check

$x^2 - x - 6 = 0$
$(-2)^2 - (-2) - 6 \stackrel{?}{=} 0$
$4 + 2 - 6 \stackrel{?}{=} 0$
$0 = 0$ ✓

$x^2 - x - 6 = 0$
$3^2 - 3 - 6 \stackrel{?}{=} 0$
$9 - 3 - 6 \stackrel{?}{=} 0$
$0 = 0$ ✓

b. Add $-4x$ to each side to obtain $-2x^2 - 4x - 2 = 0$. Graph the related function $y = -2x^2 - 4x - 2$.



The x-intercept is -1.

▶ The solution, or root, is $x = -1$.

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Solve the equation by graphing.

1. $x^2 - 8x + 12 = 0$ **2.** $4x^2 - 12x + 9 = 0$ **3.** $\frac{1}{2}x^2 = 6x - 20$

Solving Quadratic Equations Algebraically

When solving quadratic equations using square roots, you can use properties of square roots to write your solutions in different forms.

When a radicand in the denominator of a fraction is not a perfect square, you can multiply the fraction by an appropriate form of 1 to eliminate the radical from the denominator. This process is called *rationalizing the denominator*.

EXAMPLE 2 Solving Quadratic Equations Using Square Roots

Solve each equation using square roots.

a. $4x^2 - 31 = 49$ **b.** $3x^2 + 9 = 0$ **c.** $\frac{2}{5}(x+3)^2 = 5$

SOLUTION

a. $4x^2 - 31 = 49$ Write the equation.

$4x^2 = 80$ Add 31 to each side.

$x^2 = 20$ Divide each side by 4.

$x = \pm\sqrt{20}$ Take square root of each side.

$x = \pm\sqrt{4} \cdot \sqrt{5}$ Product Property of Square Roots

$x = \pm 2\sqrt{5}$ Simplify.

▶ The solutions are $x = 2\sqrt{5}$ and $x = -2\sqrt{5}$.

b. $3x^2 + 9 = 0$ Write the equation.

$3x^2 = -9$ Subtract 9 from each side.

$x^2 = -3$ Divide each side by 3.

▶ The square of a real number cannot be negative. So, the equation has no real solution.

LOOKING FOR STRUCTURE

Notice that $(x+3)^2 = \frac{25}{2}$ is of the form $u^2 = d$, where $u = x + 3$.

STUDY TIP

Because $\frac{\sqrt{2}}{\sqrt{2}} = 1$, the value of $\frac{\sqrt{25}}{\sqrt{2}}$ does not change when you multiply by $\frac{\sqrt{2}}{\sqrt{2}}$.

c. $\frac{2}{5}(x+3)^2 = 5$ Write the equation.

$(x+3)^2 = \frac{25}{2}$ Multiply each side by $\frac{5}{2}$.

$x + 3 = \pm\sqrt{\frac{25}{2}}$ Take square root of each side.

$x = -3 \pm \sqrt{\frac{25}{2}}$ Subtract 3 from each side.

$x = -3 \pm \frac{\sqrt{25}}{\sqrt{2}}$ Quotient Property of Square Roots

$x = -3 \pm \frac{\sqrt{25}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ Multiply by $\frac{\sqrt{2}}{\sqrt{2}}$.

$x = -3 \pm \frac{5\sqrt{2}}{2}$ Simplify.

▶ The solutions are $x = -3 + \frac{5\sqrt{2}}{2}$ and $x = -3 - \frac{5\sqrt{2}}{2}$.

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Solve the equation using square roots.

4. $\frac{2}{3}x^2 + 14 = 20$ **5.** $-2x^2 + 1 = -6$ **6.** $2(x-4)^2 = -5$

When the left side of $ax^2 + bx + c = 0$ is factorable, you can solve the equation using the *Zero-Product Property*.

Core Concept

Zero-Product Property

Words If the product of two expressions is zero, then one or both of the expressions equal zero.

Algebra If A and B are expressions and $AB = 0$, then $A = 0$ or $B = 0$.

EXAMPLE 3 Solving a Quadratic Equation by Factoring

Solve $x^2 - 4x = 45$ by factoring.

SOLUTION

$x^2 - 4x = 45$		Write the equation.
$x^2 - 4x - 45 = 0$		Write in standard form.
$(x - 9)(x + 5) = 0$		Factor the polynomial.
$x - 9 = 0$ or	$x + 5 = 0$	Zero-Product Property
$x = 9$ or	$x = -5$	Solve for x.

▶ The solutions are $x = -5$ and $x = 9$.

UNDERSTANDING MATHEMATICAL TERMS

If a real number k is a zero of the function $f(x) = ax^2 + bx + c$, then k is an x-intercept of the graph of the function, and k is also a root of the equation $ax^2 + bx + c = 0$.

You know the x-intercepts of the graph of $f(x) = a(x - p)(x - q)$ are p and q. Because the value of the function is zero when $x = p$ and when $x = q$, the numbers p and q are also called *zeros* of the function. A **zero of a function** f is an x-value for which $f(x) = 0$.

EXAMPLE 4 Finding the Zeros of a Quadratic Function

Find the zeros of $f(x) = 2x^2 - 11x + 12$.

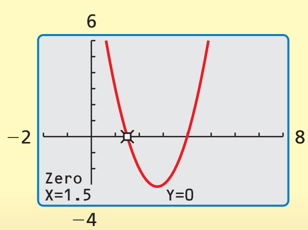
SOLUTION

To find the zeros of the function, find the x-values for which $f(x) = 0$.

$2x^2 - 11x + 12 = 0$		Set $f(x)$ equal to 0.
$(2x - 3)(x - 4) = 0$		Factor the polynomial.
$2x - 3 = 0$ or	$x - 4 = 0$	Zero-Product Property
$x = 1.5$ or	$x = 4$	Solve for x.

▶ The zeros of the function are $x = 1.5$ and $x = 4$. You can check this by graphing the function. The x-intercepts are 1.5 and 4.

Check



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Solve the equation by factoring.

7. $x^2 + 12x + 35 = 0$

8. $3x^2 - 5x = 2$

Find the zero(s) of the function.

9. $f(x) = x^2 - 8x$

10. $f(x) = 4x^2 + 28x + 49$

Solving Real-Life Problems

To find the maximum value or minimum value of a quadratic function, you can first use factoring to write the function in intercept form $f(x) = a(x - p)(x - q)$. Because the vertex of the function lies on the axis of symmetry, $x = \frac{p + q}{2}$, the maximum value or minimum value occurs at the average of the zeros p and q.

EXAMPLE 5 Solving a Multi-Step Problem

A monthly teen magazine has 48,000 subscribers when it charges \$20 per annual subscription. For each \$1 increase in price, the magazine loses about 2000 subscribers. How much should the magazine charge to maximize annual revenue? What is the maximum annual revenue?

SOLUTION

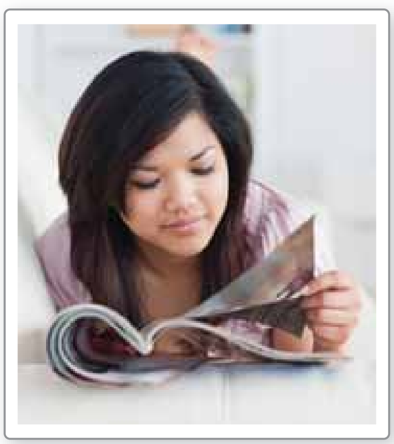
Step 1 Define the variables. Let x represent the price increase and $R(x)$ represent the annual revenue.

Step 2 Write a verbal model. Then write and simplify a quadratic function.

Annual revenue (dollars)	=	Number of subscribers (people)	•	Subscription price (dollars/person)
↓		↓		↓
$R(x)$	=	$(48{,}000 - 2000x)$	•	$(20 + x)$

$$R(x) = (-2000x + 48{,}000)(x + 20)$$

$$R(x) = -2000(x - 24)(x + 20)$$



Step 3 Identify the zeros and find their average. Then find how much each subscription should cost to maximize annual revenue.

The zeros of the revenue function are 24 and -20. The average of the zeros is $\frac{24 + (-20)}{2} = 2$.

To maximize revenue, each subscription should cost \$20 + \$2 = \$22.

Step 4 Find the maximum annual revenue.

$$R(2) = -2000(2 - 24)(2 + 20) = \$968{,}000$$

▶ So, the magazine should charge \$22 per subscription to maximize annual revenue. The maximum annual revenue is \$968,000.

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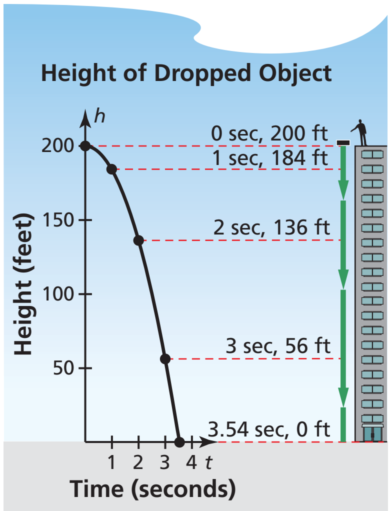
11. WHAT IF? The magazine initially charges \$21 per annual subscription. How much should the magazine charge to maximize annual revenue? What is the maximum annual revenue?



When an object is dropped, its height h (in feet) above the ground after t seconds can be modeled by the function $h = -16t^2 + h_0$, where h_0 is the initial height (in feet) of the object. The graph of $h = -16t^2 + 200$, representing the height of an object dropped from an initial height of 200 feet, is shown at the left.

The model $h = -16t^2 + h_0$ assumes that the force of air resistance on the object is negligible. Also, this model applies only to objects dropped on Earth. For planets with stronger or weaker gravitational forces, different models are used.

EXAMPLE 6 Modeling a Dropped Object

For a science competition, students must design a container that prevents an egg from breaking when dropped from a height of 50 feet.

a. Write a function that gives the height h (in feet) of the container after t seconds. How long does the container take to hit the ground?

b. Find and interpret $h(1) - h(1.5)$.

SOLUTION

a. The initial height is 50, so the model is $h = -16t^2 + 50$. Find the zeros of the function.

$h = -16t^2 + 50$	Write the function.
$0 = -16t^2 + 50$	Substitute 0 for h.
$-50 = -16t^2$	Subtract 50 from each side.
$\frac{-50}{-16} = t^2$	Divide each side by -16.
$\pm\sqrt{\frac{50}{16}} = t$	Take square root of each side.
$\pm 1.8 \approx t$	Use a calculator.

▶ Reject the negative solution, -1.8, because time must be positive. The container will fall for about 1.8 seconds before it hits the ground.

b. Find $h(1)$ and $h(1.5)$. These represent the heights after 1 and 1.5 seconds.

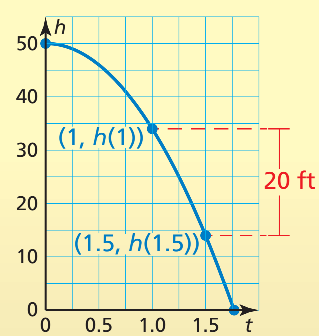
$$h(1) = -16(1)^2 + 50 = -16 + 50 = 34$$

$$h(1.5) = -16(1.5)^2 + 50 = -16(2.25) + 50 = -36 + 50 = 14$$

$$h(1) - h(1.5) = 34 - 14 = 20$$

▶ So, the container fell 20 feet between 1 and 1.5 seconds. You can check this by graphing the function. The points appear to be about 20 feet apart. So, the answer is reasonable.

Check



INTERPRETING EXPRESSIONS

In the model for the height of a dropped object, the term $-16t^2$ indicates that an object has fallen $16t^2$ feet after t seconds.

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12. WHAT IF? The egg container is dropped from a height of 80 feet. How does this change your answers in parts (a) and (b)?

3.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Explain how to use graphing to find the roots of the equation $ax^2 + bx + c = 0$.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

What are the zeros of $f(x) = x^2 + 3x - 10$?	What are the solutions of $x^2 + 3x - 10 = 0$?
What are the roots of $10 - x^2 = 3x$?	What is the y-intercept of the graph of $y = (x + 5)(x - 2)$?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–12, solve the equation by graphing. *(See Example 1.)*

3. $x^2 + 3x + 2 = 0$
4. $-x^2 + 2x + 3 = 0$
5. $0 = x^2 - 9$
6. $-8 = -x^2 - 4$
7. $8x = -4 - 4x^2$
8. $3x^2 = 6x - 3$
9. $7 = -x^2 - 4x$
10. $2x = x^2 + 2$
11. $\frac{1}{5}x^2 + 6 = 2x$
12. $3x = \frac{1}{4}x^2 + 5$

In Exercises 13–20, solve the equation using square roots. *(See Example 2.)*

13. $s^2 = 144$
14. $a^2 = 81$
15. $(z - 6)^2 = 25$
16. $(p - 4)^2 = 49$
17. $4(x - 1)^2 + 2 = 10$
18. $2(x + 2)^2 - 5 = 8$
19. $\frac{1}{2}r^2 - 10 = \frac{3}{2}r^2$
20. $\frac{1}{5}x^2 + 2 = \frac{3}{5}x^2$

21. **ANALYZING RELATIONSHIPS** Which equations have roots that are equivalent to the x-intercepts of the graph shown?

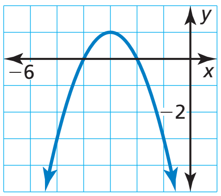
Ⓐ $-x^2 - 6x - 8 = 0$

Ⓑ $0 = (x + 2)(x + 4)$

Ⓒ $0 = -(x + 2)^2 + 4$

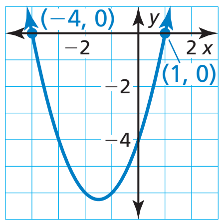
Ⓓ $2x^2 - 4x - 6 = 0$

Ⓔ $4(x + 3)^2 - 4 = 0$



22. **ANALYZING RELATIONSHIPS** Which graph has x-intercepts that are equivalent to the roots of the equation $\left(x - \frac{3}{2}\right)^2 = \frac{25}{4}$? Explain your reasoning.

Ⓐ



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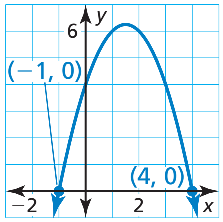
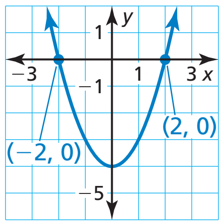


Ⓒ



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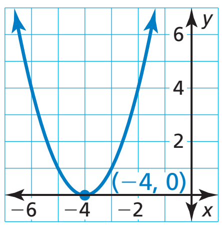


ERROR ANALYSIS In Exercises 23 and 24, describe and correct the error in solving the equation.

23. ✗

$$2(x + 1)^2 + 3 = 21$$
$$2(x + 1)^2 = 18$$
$$(x + 1)^2 = 9$$
$$x + 1 = 3$$
$$x = 2$$

24. ✗

$$-2x^2 - 8 = 0$$
$$-2x^2 = 8$$
$$x^2 = -4$$
$$x = \pm 2$$

25. **OPEN-ENDED** Write an equation of the form $x^2 = d$ that has (a) two real solutions, (b) one real solution, and (c) no real solution.

26. **ANALYZING EQUATIONS** Which equation has one real solution? Explain.

Ⓐ $3x^2 + 4 = -2(x^2 + 8)$

Ⓑ $5x^2 - 4 = x^2 - 4$

Ⓒ $2(x + 3)^2 = 18$

Ⓓ $\frac{3}{2}x^2 - 5 = 19$

In Exercises 27–34, solve the equation by factoring. *(See Example 3.)*

27. $0 = x^2 + 6x + 9$

28. $0 = z^2 - 10z + 25$

29. $x^2 - 8x = -12$

30. $x^2 - 11x = -30$

31. $n^2 - 6n = 0$

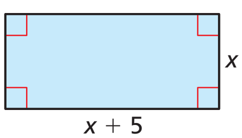
32. $a^2 - 49 = 0$

33. $2w^2 - 16w = 12w - 48$

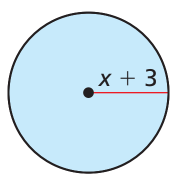
34. $-y + 28 + y^2 = 2y + 2y^2$

MATHEMATICAL CONNECTIONS **In Exercises 35–38, find the value of x.**

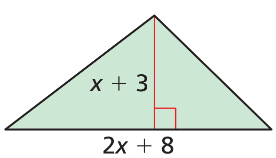
35. Area of rectangle = 36



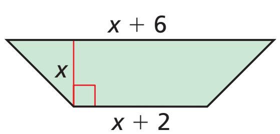
36. Area of circle = 25π



37. Area of triangle = 42



38. Area of trapezoid = 32



In Exercises 39–46, solve the equation using any method. Explain your reasoning.

39. $u^2 = -9u$

40. $\frac{t^2}{20} + 8 = 15$

41. $-(x + 9)^2 = 64$

42. $-2(x + 2)^2 = 5$

43. $7(x - 4)^2 - 18 = 10$

44. $t^2 + 8t + 16 = 0$

45. $x^2 + 3x + \frac{5}{4} = 0$

46. $x^2 - 1.75 = 0.5$

In Exercises 47–54, find the zero(s) of the function. *(See Example 4.)*

47. $g(x) = x^2 + 6x + 8$

48. $f(x) = x^2 - 8x + 16$

49. $h(x) = x^2 + 7x - 30$

50. $g(x) = x^2 + 11x$

51. $f(x) = 2x^2 - 2x - 12$

52. $f(x) = 4x^2 - 12x + 9$

53. $g(x) = x^2 + 22x + 121$

54. $h(x) = x^2 + 19x + 84$

55. **REASONING** Write a quadratic function in the form $f(x) = x^2 + bx + c$ that has zeros 8 and 11.

56. **NUMBER SENSE** Write a quadratic equation in standard form that has roots equidistant from 10 on the number line.

57. **PROBLEM SOLVING** A restaurant sells 330 sandwiches each day. For each \$0.25 decrease in price, the restaurant sells about 15 more sandwiches. How much should the restaurant charge to maximize daily revenue? What is the maximum daily revenue? *(See Example 5.)*

58. **PROBLEM SOLVING** An athletic store sells about 200 pairs of basketball shoes per month when it charges \$120 per pair. For each \$2 increase in price, the store sells two fewer pairs of shoes. How much should the store charge to maximize monthly revenue? What is the maximum monthly revenue?

59. **MODELING WITH MATHEMATICS** Niagara Falls is made up of three waterfalls. The height of the Canadian Horseshoe Falls is about 188 feet above the lower Niagara River. A log falls from the top of Horseshoe Falls. *(See Example 6.)*

a. Write a function that gives the height h (in feet) of the log after t seconds. How long does the log take to reach the river?

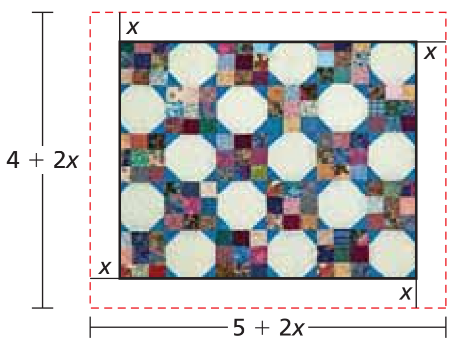
b. Find and interpret $h(2) - h(3)$.

60. **MODELING WITH MATHEMATICS** According to legend, in 1589, the Italian scientist Galileo Galilei dropped rocks of different weights from the top of the Leaning Tower of Pisa to prove his conjecture that the rocks would hit the ground at the same time. The height h (in feet) of a rock after t seconds can be modeled by $h(t) = 196 - 16t^2$.

a. Find and interpret the zeros of the function. Then use the zeros to sketch the graph.

b. What do the domain and range of the function represent in this situation?

61. **PROBLEM SOLVING** You make a rectangular quilt that is 5 feet by 4 feet. You use the remaining 10 square feet of fabric to add a border of uniform width to the quilt. What is the width of the border?

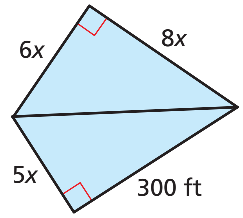


62. **MODELING WITH MATHEMATICS** You drop a seashell into the ocean from a height of 40 feet. Write an equation that models the height h (in feet) of the seashell above the water after t seconds. How long is the seashell in the air?

63. **WRITING** The equation $h = 0.019s^2$ models the height h (in feet) of the largest ocean waves when the wind speed is s knots. Compare the wind speeds required to generate 5-foot waves and 20-foot waves.

64. **CRITICAL THINKING** Write and solve an equation to find two consecutive odd integers whose product is 143.

65. **MATHEMATICAL CONNECTIONS** A quadrilateral is divided into two right triangles as shown in the figure. What is the length of each side of the quadrilateral?

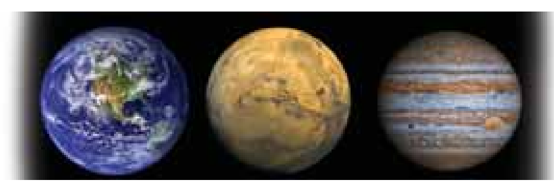


66. **ABSTRACT REASONING** Suppose the equation $ax^2 + bx + c = 0$ has no real solution and a graph of the related function has a vertex that lies in the second quadrant.

a. Is the value of a positive or negative? Explain your reasoning.

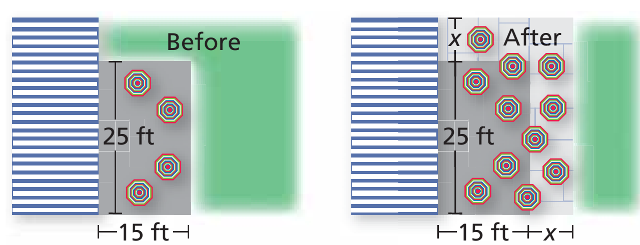
b. Suppose the graph is translated so the vertex is in the fourth quadrant. Does the graph have any x-intercepts? Explain.

67. **REASONING** When an object is dropped on *any* planet, its height h (in feet) after t seconds can be modeled by the function $h = -\frac{g}{2}t^2 + h_0$, where h_0 is the object's initial height and g is the planet's acceleration due to gravity. Suppose a rock is dropped from the same initial height on the three planets shown. Make a conjecture about which rock will hit the ground first. Justify your answer.



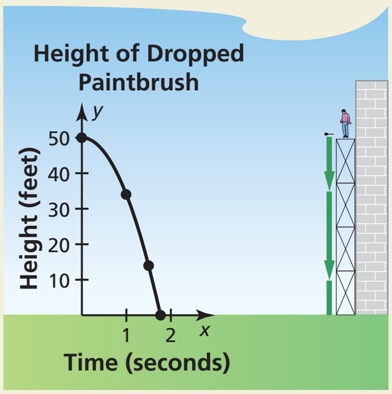
Earth: $g = 32$ ft/sec^2 | Mars: $g = 12$ ft/sec^2 | Jupiter: $g = 76$ ft/sec^2

68. **PROBLEM SOLVING** A café has an outdoor, rectangular patio. The owner wants to add 329 square feet to the area of the patio by expanding the existing patio as shown. Write and solve an equation to find the value of x. By what distance should the patio be extended?



69. **PROBLEM SOLVING** A flea can jump very long distances. The path of the jump of a flea can be modeled by the graph of the function $y = -0.189x^2 + 2.462x$, where x is the horizontal distance (in inches) and y is the vertical distance (in inches). Graph the function. Identify the vertex and zeros and interpret their meanings in this situation.

70. **HOW DO YOU SEE IT?** An artist is painting a mural and drops a paintbrush. The graph represents the height h (in feet) of the paintbrush after t seconds.



a. What is the initial height of the paintbrush?

b. How long does it take the paintbrush to reach the ground? Explain.

71. **MAKING AN ARGUMENT** Your friend claims the equation $x^2 + 7x = -49$ can be solved by factoring and has a solution of $x = 7$. You solve the equation by graphing the related function and claim there is no solution. Who is correct? Explain.

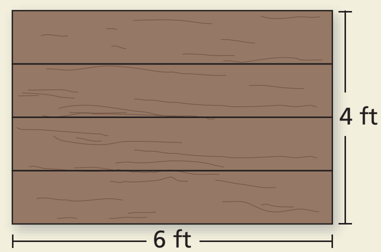
72. **ABSTRACT REASONING** Factor the expressions $x^2 - 4$ and $x^2 - 9$. Recall that an expression in this form is called a difference of two squares. Use your answers to factor the expression $x^2 - a^2$. Graph the related function $y = x^2 - a^2$. Label the vertex, x-intercepts, and axis of symmetry.

73. **DRAWING CONCLUSIONS** Is there a formula for factoring the *sum* of two squares? You will investigate this question in parts (a) and (b).

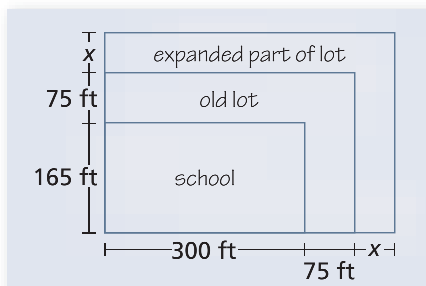
a. Consider the sum of squares $x^2 + 9$. If this sum can be factored, then there are integers m and n such that $x^2 + 9 = (x + m)(x + n)$. Write two equations that m and n must satisfy.

b. Show that there are no integers m and n that satisfy both equations you wrote in part (a). What can you conclude?

74. **THOUGHT PROVOKING** You are redesigning a rectangular raft. The raft is 6 feet long and 4 feet wide. You want to double the area of the raft by adding to the existing design. Draw a diagram of the new raft. Write and solve an equation you can use to find the dimensions of the new raft.



75. **MODELING WITH MATHEMATICS** A high school wants to double the size of its parking lot by expanding the existing lot as shown. By what distance x should the lot be expanded?



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the sum or difference. *(Skills Review Handbook)*

76. $(x^2 + 2) + (2x^2 - x)$

77. $(x^3 + x^2 - 4) + (3x^2 + 10)$

78. $(-2x + 1) - (-3x^2 + x)$

79. $(-3x^3 + x^2 - 12x) - (-6x^2 + 3x - 9)$

Find the product. *(Skills Review Handbook)*

80. $(x + 2)(x - 2)$

81. $2x(3 - x + 5x^2)$

82. $(7 - x)(x - 1)$

83. $11x(-4x^2 + 3x + 8)$