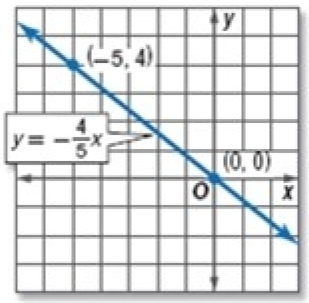
3-4 Direct Variation

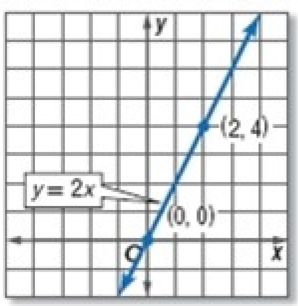
Name the constant of variation for each equation. Then find the slope of the line that passes through each pair of points.

1. 

SOLUTION:

The constant of variation is $-\frac{4}{5}$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4-0}{-5-0} \\ &= -\frac{4}{5} \end{aligned}$$

2. 

SOLUTION:

The constant of variation is 2.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4-0}{2-0} \\ &= \frac{4}{2} \\ &= 2 \end{aligned}$$

Graph each equation.

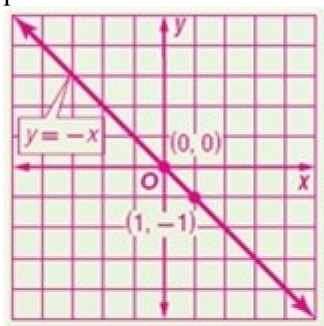
3. $y = -x$

SOLUTION:

The slope of $y = -x$ is -1. Write the slope as $\frac{\text{rise}}{\text{run}}$.

$-1 = \frac{-1}{1}$

Graph (0, 0). From there, move down 1 unit and right 1 unit to find another point. Then draw a line containing the points.



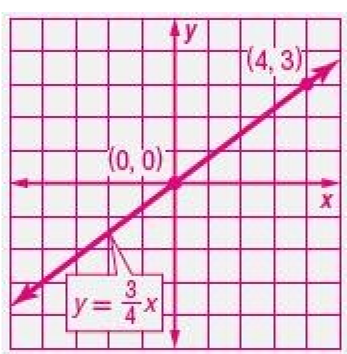
4. $y = \frac{3}{4}x$

SOLUTION:

The slope of $y = \frac{3}{4}x$ is $\frac{3}{4}$. Write the slope as $\frac{\text{rise}}{\text{run}}$.

$\frac{\text{rise}}{\text{run}} = \frac{3}{4}$

Graph (0, 0). From there, move up 3 units and right 4 units to find another point. Then draw a line containing the points.



3-4 Direct Variation

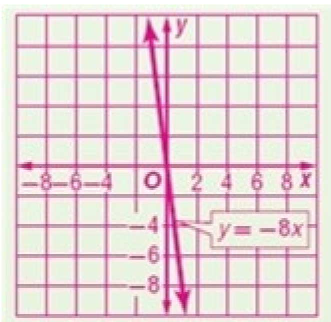
5. $y = -8x$

SOLUTION:

The slope of $y = -8x$ is -8. Write the slope as $\frac{\text{rise}}{\text{run}}$.

$$-8 = \frac{-8}{1}$$

Graph (0, 0). From there, move down 8 units and right 1 unit to find another point. Then draw a line containing the points.



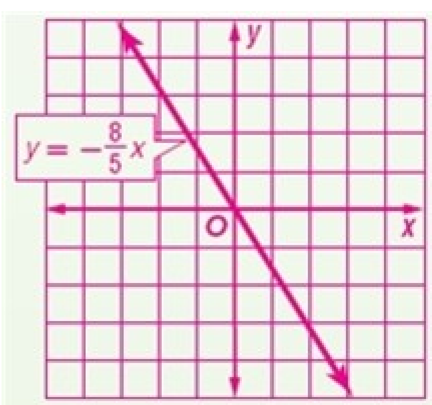
6. $y = -\frac{8}{5}x$

SOLUTION:

The slope of $y = -\frac{8}{5}x$ is $-\frac{8}{5}$. Write the slope as $\frac{\text{rise}}{\text{run}}$

$$\frac{\text{rise}}{\text{run}} = \frac{-8}{5}$$

Graph (0, 0). From there, move down 8 units and right 5 units to find another point. Then draw a line containing the points.



Suppose y varies directly as x. Write a direct variation equation that relates x and y. Then solve.

7. If $y = 15$ when $x = 12$, find y when $x = 32$.

SOLUTION:

$$y = kx$$
$$15 = k(12)$$
$$\frac{15}{12} = \frac{k(12)}{12}$$
$$\frac{5}{4} = k$$

So, the direct variation equation is $y = \frac{5}{4}x$. Substitute 32 for x and find y.

$$y = \frac{5}{4}x$$
$$y = \frac{5}{4}(32)$$
$$y = 40$$

So, $y = 40$ when $x = 32$.

8. If $y = -11$ when $x = 6$, find x when $y = 44$.

SOLUTION:

$y = kx$	Direct variation equation
$-11 = k(6)$	Replace y with -11 and x with 6.
$\frac{-11}{6} = \frac{k(6)}{6}$	Divide each side by 6.
$-\frac{11}{6} = k$	Simplify.

So, the direct variation equation is $y = -\frac{11}{6}x$. Substitute 44 for y and find x.

$y = -\frac{11}{6}x$	Direct variation formula
$44 = -\frac{11}{6}x$	Replace y with 44.
$-\frac{6}{11}(44) = -\frac{6}{11}\left(-\frac{11}{6}x\right)$	Multiply each side by $-\frac{6}{11}$.
$-24 = x$	Simplify.

So, $x = -24$ when $y = 44$.

9. **CCSS REASONING** You find that the number of messages you receive on your message board varies directly as the number of messages you post. When you post 5 messages, you receive 12 messages in return.
a. Write a direct variation equation relating your posts to the messages received. Then graph the equation.
b. Find the number of messages you need to post to receive 96 messages.

SOLUTION:

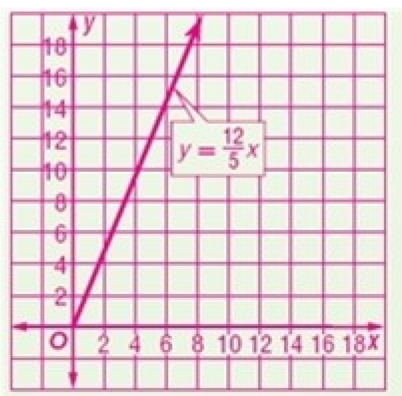
a.

$y = kx$	Direct variation formula
$12 = k(5)$	Replace y with 12 and x with 5.
$\frac{12}{5} = \frac{k(5)}{5}$	Divide each side by 5.
$\frac{12}{5} = k$	Simplify.

So, the direct variation equation is $y = \frac{12}{5}x$.
The slope of $y = \frac{12}{5}x$ is $\frac{12}{5}$. Write the slope as $\frac{\text{rise}}{\text{run}}$.

$$\frac{\text{rise}}{\text{run}} = \frac{12}{5}$$

Graph (0, 0). From there, move up 12 units and right 5 units to find another point. Then draw a line containing the points.

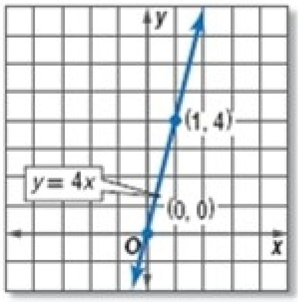


b. Substitute 96 for x and find y.

$y = \frac{12}{5}x$	Direct variation formula
$96 = \frac{12}{5}x$	Replace y with 96.
$\frac{5}{12}(96) = \frac{5}{12}\left(\frac{12}{5}x\right)$	Multiply each side by $\frac{5}{12}$.
$40 = x$	Simplify.

So, you need to post 40 messages to receive 96 messages.

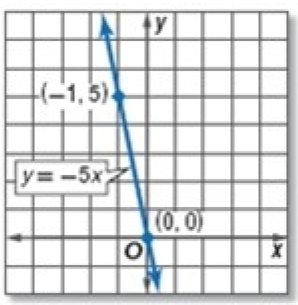
Name the constant of variation for each equation. Then find the slope of the line that passes through each pair of points.

10. 

SOLUTION:

The constant of variation is 4.

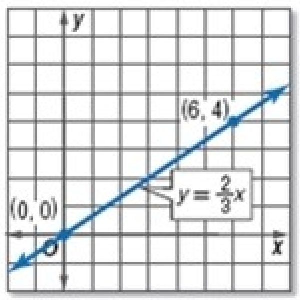
$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4-0}{1-0} \\ &= \frac{4}{1} \\ &= 4 \end{aligned}$$

11. 

SOLUTION:

The constant of variation is –5.

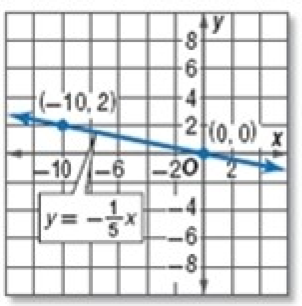
$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5-0}{-1-0} \\ &= \frac{5}{-1} \\ &= -5 \end{aligned}$$

12. 

SOLUTION:

The constant of variation is $\frac{2}{3}$.

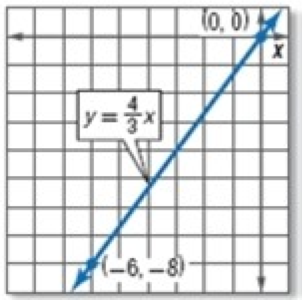
$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4-0}{6-0} \\ &= \frac{4}{6} \\ &= \frac{2}{3} \end{aligned}$$

13. 

SOLUTION:

The constant of variation is $-\frac{1}{5}$.

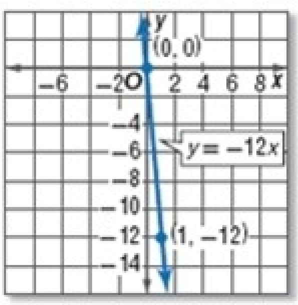
$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2-0}{-10-0} \\ &= \frac{2}{-10} \\ &= -\frac{1}{5} \end{aligned}$$

14. 

SOLUTION:

The constant of variation is $\frac{4}{3}$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-8 - 0}{-6 - 0} \\ &= \frac{-8}{-6} \\ &= \frac{4}{3} \end{aligned}$$

15. 

SOLUTION:

The constant of variation is −12.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-12 - 0}{1 - 0} \\ &= \frac{-12}{1} \\ &= -12 \end{aligned}$$

Graph each equation.

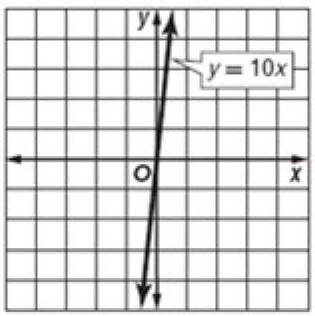
16. $y = 10x$

SOLUTION:

The slope of $y = 10x$ is 10. Write the slope as $\frac{\text{rise}}{\text{run}}$.

$10 = \frac{10}{1}$

Graph (0, 0). From there, move up 10 units and right 1 unit to find another point. Then draw a line containing the points.



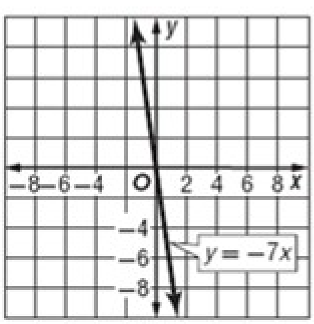
17. $y = -7x$

SOLUTION:

The slope of $y = -7x$ is –7. Write the slope as $\frac{\text{rise}}{\text{run}}$.

$-7 = \frac{-7}{1}$

Graph (0, 0). From there, move down 7 units and right 1 unit to find another point. Then draw a line containing the points.



3-4 Direct Variation

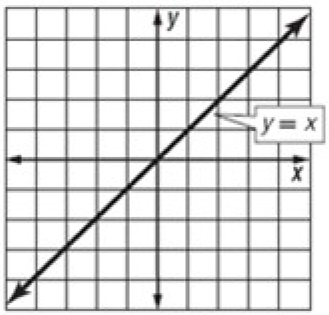
18. $y = x$

SOLUTION:

The slope of $y = x$ is 1.Write the slope as $\frac{\text{rise}}{\text{run}}$.

$1 = \frac{1}{1}$

Graph (0, 0). From there, move up 1 unit and right 1 unit to find another point. Then draw a line containing the points.



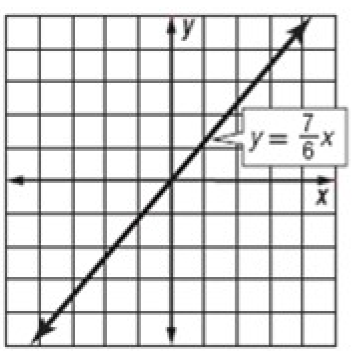
19. $y = \frac{7}{6}x$

SOLUTION:

The slope of $y = \frac{7}{6}x$ is $\frac{7}{6}$. Write the slope as $\frac{\text{rise}}{\text{run}}$.

$\frac{\text{rise}}{\text{run}} = \frac{7}{6}$

Graph (0, 0). From there, move down 7 units and left 6 units to find another point. Then draw a line containing the points.



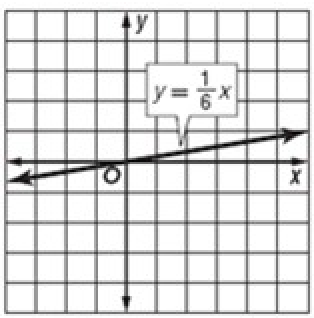
20. $y = \frac{1}{6}x$

SOLUTION:

The slope of $y = \frac{1}{6}x$ is $\frac{1}{6}$. Write the slope as $\frac{\text{rise}}{\text{run}}$.

$\frac{\text{rise}}{\text{run}} = \frac{1}{6}$

Graph (0, 0). From there, move up 1 unit and right 6 units to find another point. Then draw a line containing the points.



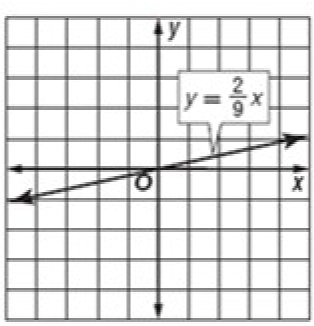
21. $y = \frac{2}{9}x$

SOLUTION:

The slope of $y = \frac{2}{9}xx$ is $\frac{2}{9}$. Write the slope as $\frac{\text{rise}}{\text{run}}$.

$\frac{\text{rise}}{\text{run}} = \frac{2}{9}$

Graph (0, 0). From there, move up 2 units and right 9 units to find another point. Then draw a line containing the points.



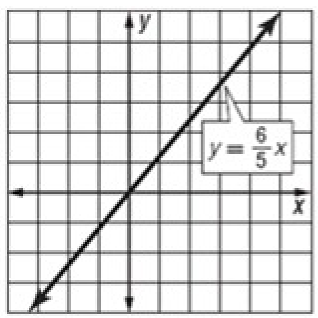
22. $y = \frac{6}{5}x$

SOLUTION:

The slope of $y = \frac{6}{5}x$ is $\frac{6}{5}$. Write the slope as $\frac{\text{rise}}{\text{run}}$.

$\frac{\text{rise}}{\text{run}} = \frac{6}{5}$

Graph (0, 0). From there, move up 6 units and right 5 units to find another point. Then draw a line containing the points.



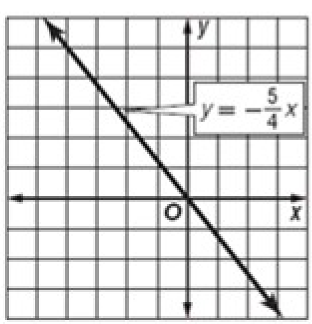
23. $y = -\frac{5}{4}x$

SOLUTION:

The slope of $y = -\frac{5}{4}x$ is $-\frac{5}{4}$. Write the slope as $\frac{\text{rise}}{\text{run}}$.

$\frac{\text{rise}}{\text{run}} = \frac{5}{-4}$

Graph (0, 0). From there, move up 5 units and left 4 units to find another point. Then draw a line containing the points.



Suppose y varies directly as x. Write a direct variation equation that relates x and y. Then solve.

24. If $y = 6$ when $x = 10$, find x when $y = 18$.

SOLUTION:

$y = kx$ Direct variation formula

$6 = k(10)$ Replace y with 6.

$\frac{6}{10} = \frac{k(10)}{10}$ Divide each side by 10.

$\frac{3}{5} = k$ Simplify.

So, the direct variation equation is $y = \frac{3}{5}x$. Substitute 18 for y and find x.

$y = \frac{3}{5}x$ Direct variation formula

$18 = \frac{3}{5}x$ Replace y with 18.

$\frac{5}{3}(18) = \frac{5}{3}\left(\frac{3}{5}x\right)$ Multiply each side by $\frac{5}{3}$.

$30 = x$ Simplify.

So, $x = 30$ when $y = 18$.

25. If $y = 22$ when $x = 8$, find y when $x = -16$.

SOLUTION:

$y = kx$ Direct variation formula

$22 = k(8)$ Replace y with 22 and x with 8.

$\frac{22}{8} = \frac{k(8)}{8}$ Divide each side by 8.

$\frac{11}{4} = k$ Simplify.

So, the direct variation equation is $y = \frac{11}{4}x$. Substitute -16 for x and find y.

$y = \frac{11}{4}x$ Direct variation formula

$y = \frac{11}{4}(-16)$ Replace x with -16.

$y = -44$ Simplify.

So, $y = -44$ when $x = -16$.

26. If $y = 4\frac{1}{4}$ when $x = \frac{3}{4}$, find y when $x = 4\frac{1}{2}$.

SOLUTION:

$$y = kx$$
$$4\frac{1}{4} = k(\frac{3}{4})$$
$$\frac{4}{3}\left(\frac{17}{4}\right) = \frac{4}{3}\left(\frac{3}{4}k\right)$$
$$\frac{17}{3} = k$$
$$5\frac{2}{3} = k$$

So, the direct variation equation is $y = 5\frac{2}{3}x$. Substitute $4\frac{1}{2}$ for x and find y.”

$$y = 5\frac{2}{3}x$$
$$y = 5\frac{2}{3}\left(4\frac{1}{2}\right)$$
$$y = \frac{17}{3}\left(\frac{9}{2}\right)$$
$$y = \frac{51}{2}$$
$$y = 25\frac{1}{2}$$

So, $y = 25\frac{1}{2}$ when $x = 4\frac{1}{2}$.

27. If $y = 12$ when $x = \frac{6}{7}$, find x when $y = 16$.

SOLUTION:

$$
\begin{aligned}
y &= kx \\
12 &= k\left(\frac{6}{7}\right) \\
\frac{7}{6}(12) &= \frac{7}{6}\left(\frac{6}{7}k\right) \\
14 &= k
\end{aligned}
$$

So, the direct variation equation is $y = 14x$. Substitute 16 for y and find x.

$$
\begin{aligned}
y &= 14x \\
16 &= 14x \\
\frac{16}{14} &= \frac{14x}{14} \\
\frac{8}{7} &= x \\
1\frac{1}{7} &= x
\end{aligned}
$$

So, $x = 1\frac{1}{7}$ when $y = 16$.

28. **SPORTS** The distance a golf ball travels at an altitude of 7000 feet varies directly with the distance the ball travels at sea level, as shown.
a. Write and graph an equation that relates the distance a golf ball travels at an altitude of 7000 feet y with the distance at sea level x.
b. What would be a person's average driving distance at 7000 feet if his average driving distance at sea level is 180 yards?

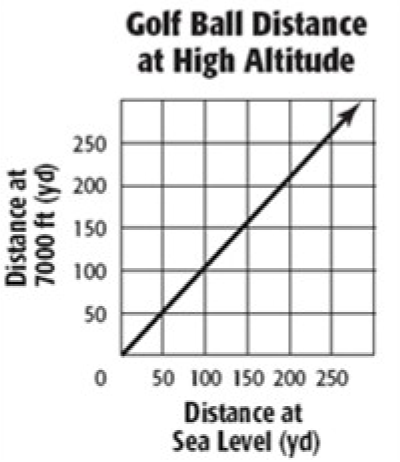
Hitting a Golf Ball		
Altitude (ft)	0 (sea level)	7000
Distance (yd)	200	210

SOLUTION:

a.

$$y = kx$$
$$210 = k(200)$$
$$\frac{210}{200} = \frac{200k}{200}$$
$$1.05 = k$$

So the equation is $y = 1.05x$.



b.

$$y = 1.05x$$
$$y = 1.05(180)$$
$$y = 189$$

So, the average driving distance at 7000 feet would be 189 yards.

29. **FINANCIAL LITERACY** Depreciation is the decline in a car's value over the course of time. The table below shows the values of a car with an average depreciation.
a. Write an equation that relates the age x of the car to the value y that it lost after each year.
b. Find the age of the car if the value is $300

Age of Car (years)	1	2	3	4	5
Value (dollars)	12,000	10,200	8400	6600	4800

SOLUTION:
a. The difference in y–values is 1800 times the difference of x–values. This suggests $y = 1800x$.

b. First find the values of the car when it was purchased. If after 1 year the car was worth 12,000 and it had decreased in value by 1800, its original worth was 13,800. Next, subtract its current value to find how much its value had decreased by, $13800 - 300 = 13500$. Substitute this value into the equation for y.

$$\begin{aligned} y &= 1800x \\ 13500 &= 1800x \\ \frac{13500}{1800} &= \frac{1800x}{1800} \\ x &= 7.5 \end{aligned}$$

So, this means that the car will be worth $300 in 7.5 years or 7 years and 6 months.

Suppose y varies directly as x. Write a direct variation equation that relates x and y. Then solve.
30. If $y = 3.2$ when $x = 1.6$, find y when $x = 19$.

SOLUTION:

$$\begin{aligned} y &= kx \\ 3.2 &= k(1.6) \\ \frac{3.2}{1.6} &= \frac{k(1.6)}{1.6} \\ 2 &= k \end{aligned}$$

So, the direct variation equation is $y = 2x$. Substitute 19 for x and find y.

$$\begin{aligned} y &= 2x \\ y &= 2(19) \\ y &= 38 \end{aligned}$$

So, $y = 38$ when $x = 19$.

31. If $y = 15$ when $x = \frac{3}{4}$, find x when $y = 25$.

SOLUTION:

$$\begin{aligned} y &= kx \\ 15 &= k\left(\frac{3}{4}\right) \\ \frac{4}{3}(15) &= \frac{4}{3}\left(\frac{3}{4}k\right) \\ 20 &= k \end{aligned}$$

So, the direct variation equation is $y = 20x$. Substitute 25 for y and find x.

$$\begin{aligned} y &= 20x \\ 25 &= 20x \\ \frac{25}{20} &= \frac{20x}{20} \\ \frac{5}{4} &= x \end{aligned}$$

So, $x = \frac{5}{4}$ when $y = 25$.

32. If $y = 4.5$ when $x = 2.5$, find y when $x = 12$.

SOLUTION:

$$\begin{aligned} y &= kx \\ 4.5 &= k(2.5) \\ \frac{4.5}{2.5} &= \frac{k(2.5)}{2.5} \\ 1.8 &= k \end{aligned}$$

So, the direct variation equation is $y = 1.8x$. Substitute 12 for x and find y.

$$\begin{aligned} y &= 1.8x \\ y &= 1.8(12) \\ y &= 21.6 \end{aligned}$$

So, $y = 21.6$ when $x = 12$.

33. If $y = –6$ when $x = 1.6$, find y when $x = 8$.

SOLUTION:

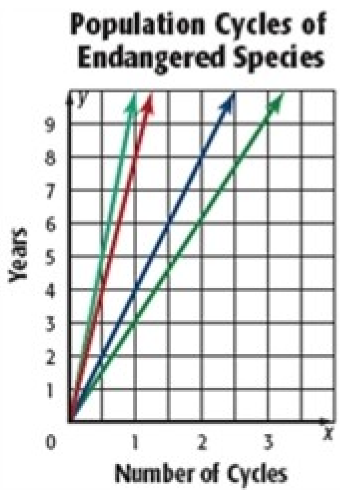
$$\begin{aligned} y &= kx \\ -6 &= k(1.6) \\ \frac{-6}{1.6} &= \frac{k(1.6)}{1.6} \\ -3.75 &= k \end{aligned}$$

So, the direct variation equation is $y = –3.75x$. Substitute 8 for x and find y.

$$\begin{aligned} y &= -3.75x \\ y &= -3.75(8) \\ y &= -30 \end{aligned}$$

So, $y = –30$ when $x = 8$.

CCSS SENSE-MAKING Certain endangered species experience cycles in their populations as shown in this graph. Match each animal to one of the colored lines in the graph.



34. red grouse, 8 years per cycle

SOLUTION:

Move right to cycle number 1 and then move up 8 years. The line that intersects with this point is red.

35. voles, 3 years per cycle

SOLUTION:

Move right to cycle number 1 and then move up 3 years. The line that intersects with this point is dark green.

36. lemmings, 4 years per cycle

SOLUTION:

Move right to cycle number 1 and then move up 4 years. The line that intersects with this point is blue.

37. lynx, 10 years per cycle.

SOLUTION:

Move right to cycle number 1 and then move up 10 years. The line that intersects with this point is lime green.

Write and graph a direct variation equation that relates the variables.

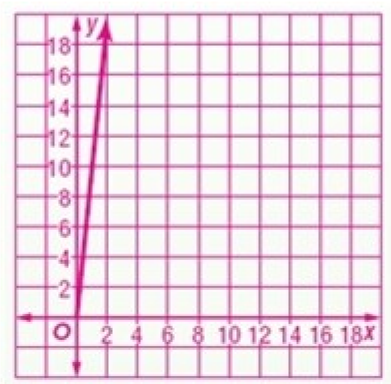
38. **PHYSICAL SCIENCE** The weight *W* of an object is 9.8 m/s^2 times the mass of the object *m*.

SOLUTION:

The slope of $W = 9.8m$ is 9.8. Write the slope as $\frac{\text{rise}}{\text{run}}$

$9.8 = \frac{9.8}{1}$

Graph (0, 0). From there, move up 9.8 units and right 1 unit to find another point. Then draw a line containing the points.



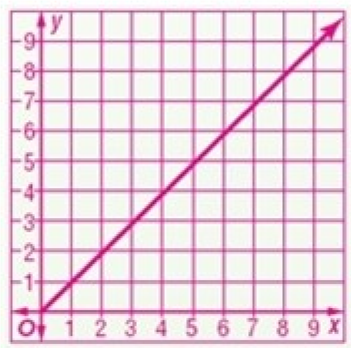
39. **MUSIC** Music downloads are $0.99 per song. The total cost of d songs is T.

SOLUTION:

The slope of $T = 0.99d$ is 0.99. Write the slope as $\frac{\text{rise}}{\text{run}}$.

$$0.99 = \frac{0.99}{1}$$

Graph (0, 0). From there, move up 0.99 units and right 1 unit to find another point. Then draw a line containing the points.



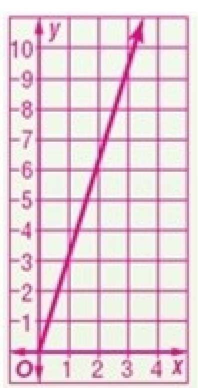
40. **GEOMETRY** The circumference of a circle C is approximately 3.14 times the diameter d.

SOLUTION:

The slope of $C = 3.14d$ is 3.14. Write the slope as $\frac{\text{rise}}{\text{run}}$.

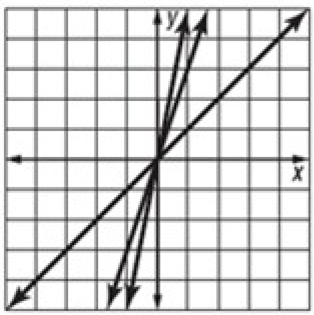
$$3.14 = \frac{3.14}{1}$$

Graph (0, 0). From there, move up 3.14 units and right 1 unit to find another point. Then draw a line containing the points.



41. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the family of direct variation functions.
a. GRAPHICAL Graph $y = x$, $y = 3x$, and $y = 5x$ on the same coordinate plane.
b. ALGEBRAIC Describe the relationship among the constant of variation, the slope of the line, and the rate of change of the graph.
c. VERBAL Make a conjecture about how you can determine without graphing which of two direct variation equations has steeper graph.

SOLUTION:



a.
b. The constant of variation, slope, and rate of change of a graph all have the same value.
c. For each equation, find the absolute value of k . The equatoin with the greater value of $|k|$ has the steeper graph.

42. **TRAVEL** A map of North Carolina is scaled so that 3 inches represents 93 miles. How far apart are Raleigh and Charlotte if they are 1.8 inches apart on the map?

SOLUTION:

$$y = kx$$
$$93 = k(3)$$
$$\frac{93}{3} = \frac{k(3)}{3}$$
$$31 = k$$

So, the direct variation equation is $y = 31x$. Substitute 1.8 for x and find y.

$$y = 31x$$
$$y = 31(1.8)$$
$$y = 55.8$$

So, y = 55.8 miles when x = 1.8 inches.

43. **INTERNET** A company will design and maintain a Web site for your company for \$9.95 per month. Write a direct variation equation to find the total cost C for having a Web page for n months.

SOLUTION:
The direct variation equation is $C = 9.95n$.

44. **BASEBALL** Before their first game, high school student Todd McCormick warmed all 5200 seats in a new minor league stadium. By literally sitting in every seat. He started at 11:50 a.m. and finished around 3 p.m.

a. Write a direct variation equation relating the number of seats to time. What is the meaning of the constant of variation in this situation?

b. About how many seats had Todd sat in by 1:00 p.m.?

c. How long would you expect it to take Todd to sit in all of the seats at a major league stadium with more than 40,000 seats?

SOLUTION:

a. Convert the time to minutes: 3 hours and 10 minutes = 190 minutes.

$$\begin{aligned} y &= kx \\ 5200 &= k(190) \\ \frac{5200}{190} &= \frac{190k}{190} \\ 27.3684 &= k \end{aligned}$$

So, the equation is $y = 27.3684t$. This means that Todd warms about 27 seats every minute.

b.

$$\begin{aligned} y &= 27.3684t \\ y &= 27.3684(70) \\ y &= 1915.788 \end{aligned}$$

So, Todd had warmed about 1915 seats.

c.

$$\begin{aligned} y &= 27.3684t \\ 40{,}000 &= 27.3684t \\ \frac{40{,}000}{27.3684} &= \frac{27.3684t}{27.3684} \\ 1461.54 &= t \end{aligned}$$

It would take Todd about 1461 minutes or 24 hours and 21 minutes.

45. **WHICH ONE DOESN'T BELONG?** Identify the equation that does not belong. Explain.

$9 = rt$	$9a = 0$	$z = \frac{1}{9}x$	$w = \frac{9}{t}$

SOLUTION:

$z = \frac{1}{9}x$ is a direct variation with the constant of variation of $\frac{1}{9}$.

$9 = rt$, is not a direct variation. When it is rewritten with variables on each side it is $t = \frac{9}{r}$, which is not linear.

$9a = 0$ only has one variable and thus can not be a direct variation.

$w = \frac{9}{t}$ is not a direct variation since it is not linear.

Thus $z = \frac{1}{9}x$ is the only equation that is a direct variation.

46. **REASONING** How are the constant of variation and the slope related in a direct variation equation? Explain your reasoning.

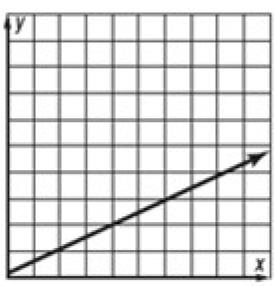
SOLUTION:

They are equal; In $y = kx$, the constant of variation is k. The graph passes through (0, 0) and (1, k), so its slope is k.

47. **OPEN ENDED** Model a real–world situation using a direct variation equation. Graph the equation and describe the rate of change.

SOLUTION:

Students' answers may vary. Sample answer: $y = 0.50x$ represents the cost of x apples. The rate of change, 0.50, is the cost per apple.



48. **CCSS STRUCTURE** Suppose y varies directly as x. If the value of x is doubled, then the value of y is also *always*, *sometimes* or *never* doubled. Explain your reasoning.

SOLUTION:

The statement If the value of x is doubled, then the value of y is also doubled is always true. Given the equation is $y = kx$ ($k \neq 0$), then the value of y when $x = a$ is ka, and the value of y when $x = 2a$ is $k(2a)$ or $2(ka)$.

49. **ERROR ANALYSIS** Eddy says the slope between any two points on the graph of a direct variation equation $y = kx$ is $\frac{1}{k}$. Adelle says the slope depends on the points chosen. Is either of them correct? Explain.

SOLUTION:
Neither of them is correct. The slope is constant, so it does not depend on which points you choose, so Adelle is incorrect. The slope of $y = kx$ is k, not $\frac{1}{k}$, so Eddy is incorrect also.

50. **WRITING IN MATH** Describe the graph of a direct variation equation.

SOLUTION:
The graph of a direct variation equation $y = kx$ is a line that always passes through the origin. The graph has a positive slope if k is positive and the graph has a negative slope if k is negative.

51. Patricia pays $1.19 each to download songs to her digital media player. If n is the number of downloaded songs, which equation represents the cost C in dollars?
A $C = 1.19n$
B $n = 1.19C$
C $C = 1.19 \div n$
D $C = n + 1.19$

SOLUTION:
If Patricia pays $1.19 for each song, this is a direct variation. The equation that represents this is $C = 1.19n$. So the correct choice is A.

52. Suppose that y varies directly as x, and $y = 8$ when $x = 6$. What is the value of y when $x = 8$?

F 6
G 12
H $10\frac{2}{3}$
J 16

SOLUTION:

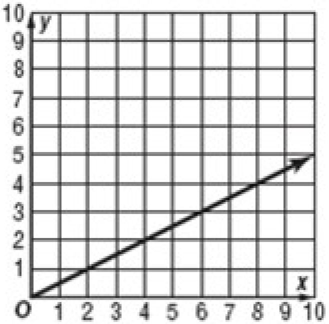
$$y = kx$$
$$8 = k(6)$$
$$\frac{8}{6} = \frac{k(6)}{6}$$
$$\frac{4}{3} = k$$

So, the direct variation equation is $y = \frac{4}{3}x$. Substitute 8 for x and find y.

$$y = \frac{4}{3}x$$
$$y = \frac{4}{3}(8)$$
$$y = \frac{32}{3}$$
$$y = 10\frac{2}{3}$$

$y = 10\frac{2}{3}$ when $x = 8$, so the correct choice is A.

53. What is the relationship between the input (x) and output (y)?



A The output is two more than the input.
B The output is two less than the input.
C The output is twice the input.
D The output is half the input.

SOLUTION:
Compare some points to check on the relationship. Look at the point (2, 1).

A The output is two more than the input.	$2+2=4$
B The output is two less than the input.	$2-2=0$
C The output is twice the input.	$2 \cdot 2=4$
D The output is half the input.	$2 \div 2=1$

The only choice that provides the correct choice is D.

54. **SHORT RESPONSE** A telephone company charges $40 per month plus $0.07 per minute. How much would a month of service cost a customer if the customer talked for 200 minutes?

SOLUTION:
Let m represent the number of minutes a customer talks.

$$\begin{aligned}40+0.07m &= 40+0.07(200)\\ &= 40+14\\ &= 54\end{aligned}$$

So, a customer would be charged $54.

55. **TELEVISION** The graph shows the average number of television channels American households receive. What was the annual rate of change from 2004 to 2008? Explain the meaning of the rate of change.

SOLUTION:

$$\text{rate of change} = \frac{\text{change in tv channels}}{\text{change in time}}$$
$$= \frac{118.6 - 92.6}{4}$$
$$= \frac{26}{4}$$
$$= 6.5$$

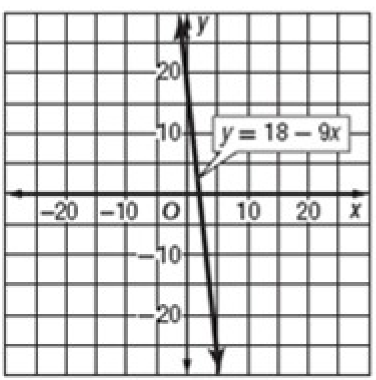
The average rate of change in the average number of television channels is about 6.5. This means that there was an average increase of 6.5 channels per year.

Solve each equation by graphing.

56. $0 = 18 - 9x$

SOLUTION:

The related function is $y = 18 - 9x$.

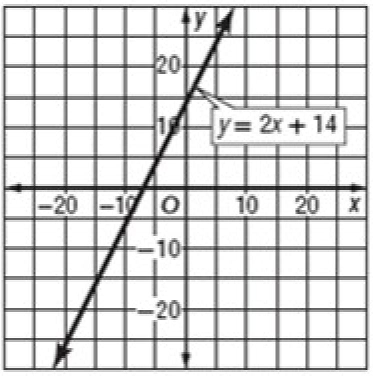


The graph intersects the x–axis at 2. So, the solution is 2.

57. $2x + 14 = 0$

SOLUTION:

The related function is $y = 2x + 14$.

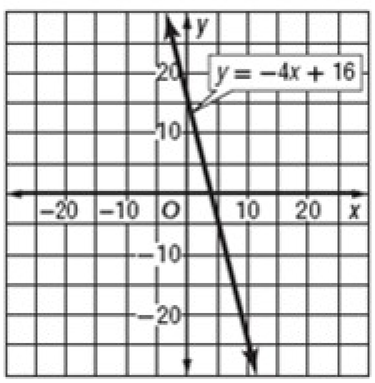


The graph intersects the x–axis at –7. So, the solution is –7.

58. $–4x + 16 = 0$

SOLUTION:

The related function is $y = –4x + 16$.

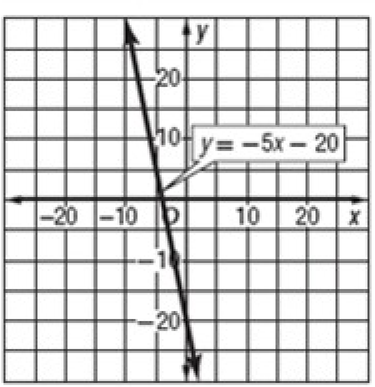


The graph intersects the x–axis at 4. So, the solution is 4.

59. $–5x – 20 = 0$

SOLUTION:

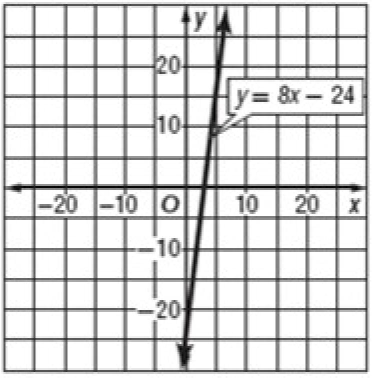
The related function is $y = –5x – 20$.



The graph intersects the x–axis at –4. So, the solution is –4.

60. $8x - 24 = 0$

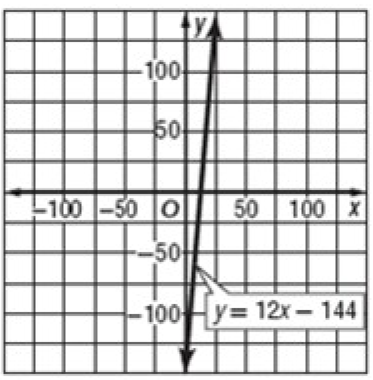
SOLUTION:
The related function is $y = 8x - 24$.



The graph intersects the x–axis at 3. So, the solution is 3.

61. $12x - 144 = 0$

SOLUTION:
The related function is $y = 12x - 144$.



The graph intersects the x–axis at 12. So, the solution is 12.

Evaluate each expression if $a = 4$, $b = -2$, and $c = -4$.

62. $|2a + c| + 1$

SOLUTION:
Replace a with 4 and c with −4.

$$\begin{aligned} |2a + c| + 1 &= |2(4) + -4| + 1 \\ &= |8 - 4| + 1 \\ &= |4| + 1 \\ &= 4 + 1 \\ &= 5 \end{aligned}$$

63. $4a - |3b + 2|$

SOLUTION:

Replace *a* with 4 and *b* with −2.

$$\begin{aligned} 4a - |3b + 2| &= 4(4) - |3(-2) + 2| \\ &= 16 - |-6 + 2| \\ &= 16 - |-4| \\ &= 16 - 4 \\ &= 12 \end{aligned}$$

64. $-|a + 1| + |3c|$

SOLUTION:

Replace *a* with 4 and *c* with −4.

$$\begin{aligned} -|a + 1| + |3c| &= -|4 + 1| + |3(-4)| \\ &= -|5| + |-12| \\ &= -5 + 12 \\ &= 7 \end{aligned}$$

65. $-a + |2 - a|$

SOLUTION:

Replace *a* with 4

$$\begin{aligned} -a + |2 - a| &= -4 + |2 - 4| \\ &= -4 + |-2| \\ &= -4 + 2 \\ &= -2 \end{aligned}$$

66. $|c - 2b| - 3$

SOLUTION:

Replace *b* with −2 and *c* with −4

$$\begin{aligned} |c - 2b| - 3 &= |-4 - 2(-2)| - 3 \\ &= |-4 + 4| - 3 \\ &= |0| - 3 \\ &= 0 - 3 \\ &= -3 \end{aligned}$$

67. $-2|3b-8|$

SOLUTION:

Replace b with −2.

$$\begin{aligned}-2|3b-8| &= -2|3(-2)-8|\\ &= -2|-6-8|\\ &= -2|-14|\\ &= -2(14)\\ &= -28\end{aligned}$$

Find each difference.

68. 13 – (–1)

SOLUTION:

$$\begin{aligned}13-(-1) &= 13+1\\ &= 14\end{aligned}$$

69. 4 – 16

SOLUTION:

$4-16=-12$

70. –3 – 3

SOLUTION:

$-3-3=-6$

71. –8 – (–2)

SOLUTION:

$$\begin{aligned}-8-(-2) &= -8+2\\ &= -6\end{aligned}$$

72. 16 – (–10)

SOLUTION:

$$\begin{aligned}16-(-10) &= 16+10\\ &= 26\end{aligned}$$

73. –8 – 4

SOLUTION:

$-8-4=-12$