English Spanish

3.8 Inverse Variation

STATE STANDARDS
MA.7.A.1.5

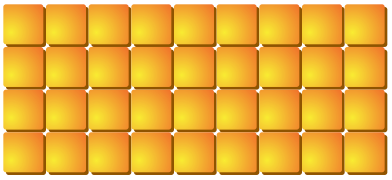
Essential Question How can you recognize when two variables are inversely proportional?

1 ACTIVITY: Comparing the Height and the Base

Work with a partner.

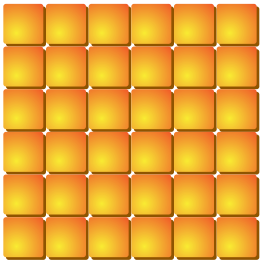
a. There are nine ways to arrange 36 square blocks to form a rectangle. Here are two ways. Find the other seven ways.

Height = 4



Base = 9

Height = 6



Base = 6

b. Order the nine ways according to height. Record your results in a table.

Height, h	Base, b	Area, A
4	9	$A = 9 \cdot 4 = 36$
6	6	$A = 6 \cdot 6 = 36$

c. Look at the first and second columns. Complete each sentence.

- When the height increases, the base _____.
- When the height decreases, the base _____.

In Activity 1, the relationship between the height and the base is an example of **inverse variation**. You can describe the relationship with an equation.

$$h = \frac{36}{b}$$ h and b are inversely proportional.

2 ACTIVITY: Comparing Direct and Inverse Variation

Work with a partner. Discuss each description. Tell whether the two variables are examples of *direct variation* or *inverse variation*. Use a table to explain your reasoning. Write an equation that relates the variables.

a. You bring 200 cookies to a party. Let n represent the number of people at the party and c represent the number of cookies each person receives.

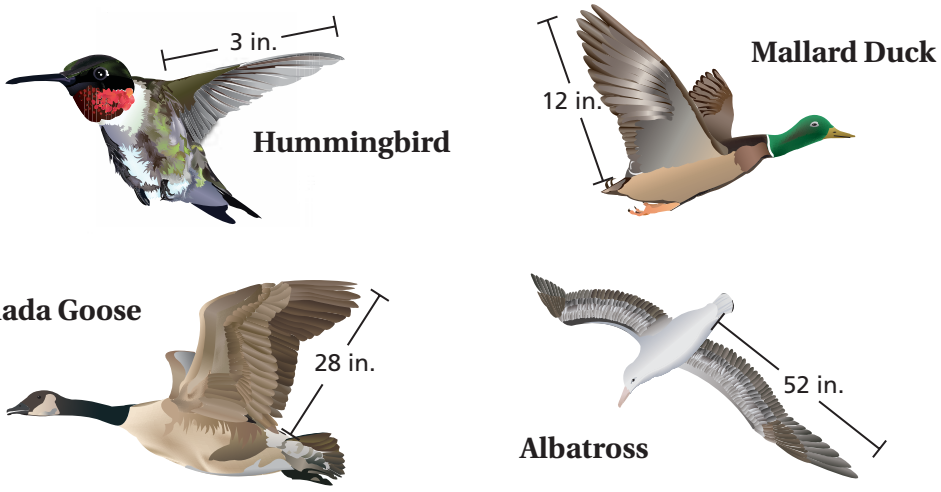
b. You work at a restaurant for 20 hours. Let r represent your hourly pay rate and p represent the total amount you earn.

c. You are going on a 240-mile trip. Let t represent the number of hours driving and s represent the speed of the car.

What Is Your Answer?

3. IN YOUR OWN WORDS How can you recognize when two variables are inversely proportional? Explain how a table can help you recognize inverse variation.

4. SCIENCE The *wing beat frequency* of a bird is the number of times per second the bird flaps its wings.



Which of the following seems true? Explain your reasoning.

- Wing length and wing beat frequency are directly proportional.
- Wing length and wing beat frequency are inversely proportional.
- Wing length and wing beat frequency are unrelated.

5. SCIENCE Think of an example in science where two variables are inversely proportional.

Practice Use what you learned about inverse variation to complete Exercises 4–7 on page 146.

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3.8 Lesson

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Key Vocabulary
inverse variation, *p. 144*

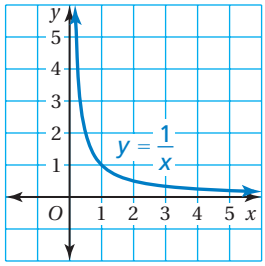
Key Idea

Inverse Variation

Words

Two quantities x and y show **inverse variation** when $y = \frac{k}{x}$, where k is a number and $k \neq 0$.

Graph



EXAMPLE 1 Identifying Direct and Inverse Variation

Tell whether x and y show *direct variation*, *inverse variation*, or *neither*. Explain your reasoning.

a. $5y = x$

$y = \frac{1}{5}x$ Solve for y.

∴ The equation can be written as $y = kx$. So, x and y show direct variation.

b. $\frac{1}{3}y = \frac{1}{x}$

$y = \frac{3}{x}$ Solve for y.

∴ The equation can be written as $y = \frac{k}{x}$. So, x and y show inverse variation.

c. $-x = y + 7$

$-x - 7 = y$ Solve for y.

∴ The equation cannot be written as $y = kx$ *or* $y = \frac{k}{x}$. So, x and y do *not* show direct or inverse variation.

Study Tip

Other ways to say that x and y show inverse variation are "y varies inversely with x" and "x and y are inversely proportional."

On Your Own

Now You're Ready
Exercises 4–15

Tell whether x and y show *direct variation*, *inverse variation*, or *neither*. Explain your reasoning.

1. $y - 1 = 2x$ **2.** $\frac{1}{5}y = x$ **3.** $2y = \frac{1}{x}$

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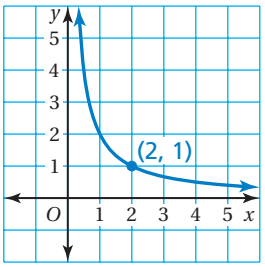
EXAMPLE 2 Standardized Test Practice

In the graph, x and y show inverse variation. Which equation relates x and y?



Ⓐ $y = -\frac{2}{x}$ Ⓑ $y = \frac{2}{x}$

Ⓒ $y = -2x$ Ⓓ $y = 2x$

The graph passes through (2, 1). Substitute to find k.

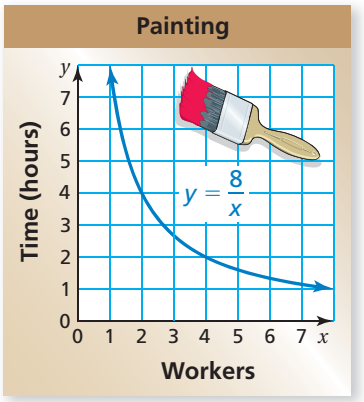
$y = \frac{k}{x}$ Write inverse variation equation.

$1 = \frac{k}{2}$ Substitute 2 for x and 1 for y.

$2 = k$ Solve for k.

∴ So, the equation $y = \frac{2}{x}$ relates x and y. The correct answer is Ⓑ.

EXAMPLE 3 Real-Life Application



The graph shows the number of hours y it takes x workers to paint a room. (a) How does y change as x increases? (b) Do x and y show direct or inverse variation? (c) How many hours does it take five workers to paint the room?

a. From the graph, you can see that y decreases as x increases. So, as the number of workers increases, the time to paint the room decreases.

b. The equation is written as $y = \frac{k}{x}$. So, x and y show inverse variation.

c. Use the equation to find y when $x = 5$.

$y = \frac{8}{x}$ Write equation.

$= \frac{8}{5} = 1.6$ Substitute. Then simplify.

∴ It takes 1.6 hours for five workers to paint the room.

On Your Own

Now You're Ready
Exercises 23 and 24

4. Suppose y varies inversely with x and $y = 3$ when $x = 1$. Write an equation that relates x and y.

5. WHAT IF? In Example 3, how many hours does it take three workers to paint the room?

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3.8 Exercises

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Vocabulary and Concept Check

1. **WRITING** What does it mean for x and y to vary inversely?
2. **NUMBER SENSE** When x increases from 1 to 10, does $\frac{1}{x}$ increase or decrease?
3. **OPEN-ENDED** Describe a real-life situation that shows inverse variation.

Practice and Problem Solving

Tell whether x and y show *direct variation*, *inverse variation*, or *neither*. Explain your reasoning.

4. $y = \frac{1}{x}$
5. $xy = 8$
6. $y - x = 0$
7. $\frac{1}{2}y = 2x$
8. $\frac{y}{3} = \frac{2}{x}$
9. $y - 2 = \frac{7}{x}$
10. $x = y + 9$
11. $x = 4y$
12. $y = \frac{5}{2x}$
13. $2y = \frac{6}{x}$
14. $\frac{5x}{3} = \frac{y}{4}$
15. $x = \frac{7 + y}{2}$

16. **ERROR ANALYSIS** Describe and correct the error in telling whether x and y show inverse variation.

✗ $\frac{y}{2} = \frac{8}{x}$

The equation does not show inverse variation because it is not of the form $y = \frac{k}{x}$.

Graph the data. Tell whether x and y show *direct variation* or *inverse variation*.

17.

x	−2	2	4	6
y	−1	1	2	3

18.

x	0.5	1	3	6
y	6	3	1	0.5

19.

x	2	5	8	20
y	10	4	2.5	1

20.

x	2	4	8	11
y	1.5	3	6	8.25

Tell whether x and y show *direct variation* or *inverse variation*. Explain.

21. **STADIUM** The time y it takes to empty a stadium and the number x of open exits are related by the equation $y = \frac{0.8}{x}$.
22. **TRAVEL** The number y of miles driven and the number x of gallons of gas used are related by the equation $y = 28.5x$.

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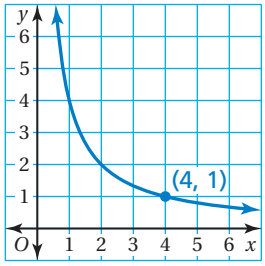
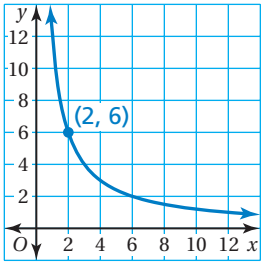
The variables x and y vary inversely. Write an equation relating x and y.

23.



24.

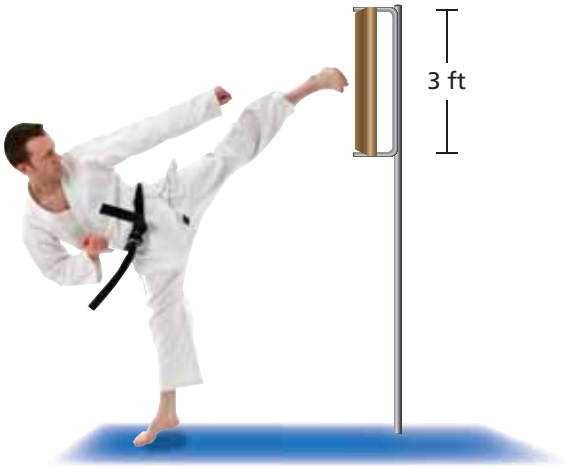


25. BICYCLING The table shows the times it takes to bicycle 12 miles at various speeds.

Speed (mi/h)	12	6	3	2
Time (h)	1	2	4	6

a. Does the time t vary inversely with the speed s? If so, write an equation relating t and s.

b. What time corresponds to a speed of 4 miles per hour?



26. MARTIAL ARTS It takes 3.6 pounds of force to break a 5-foot board.

a. You remember from science that force and board length vary directly or inversely, but you've forgotten which. How can you use reason to remember?

b. How much force does it take to break the board shown?

27. SALARY A salesperson has a fixed weekly salary. The person works twice as many hours this week as last week. What happens to the person's hourly rate?

28. Reasoning The price per person to rent a limousine varies inversely with the number of passengers. It costs \$90 each for five people. How many people are renting the limousine when the cost per person is \$56.25?

Fair Game Review What you learned in previous grades & lessons

Find the percent of the number. SKILLS REVIEW HANDBOOK

29. 40% of 220 **30.** 32% of 275 **31.** 84% of 75 **32.** 21% of 300

Tell whether the ratios form a proportion. SECTION 3.3

33. $\frac{9}{15}, \frac{18}{30}$ **34.** $\frac{21}{9}, \frac{18}{8}$ **35.** $\frac{42}{91}, \frac{24}{52}$ **36.** $\frac{24}{38}, \frac{36}{57}$

37. MULTIPLE CHOICE In the 2008 NBA Draft Lottery, the Miami Heat's ratio of ping-pong balls to the total number of balls was 1 : 4. There were 1000 balls in the lottery. How many belonged to Miami? SECTION 3.5

(A) 150 (B) 250 (C) 400 (D) 750