3. Carrier Modulation – Analog

Modulation is the process of using an information signal (such as voice or music) to alter some property of a higher frequency waveform which can then be efficiently radiated by reasonably small antenna. Waveforms at different "radio" frequencies can simultaneously "carry" a plurality of information signals. The information signal amplitude is used to vary either the carrier amplitude (A_c) or the carrier phase angle ($\omega_c t + \theta$). The unmodulated carrier is normally a sinusoid described by:

$c(t) = A_c \cos(\omega_c t + \theta)$ where:

A_c = peak amplitude of the carrier
$\omega_c = 2\pi f_c$ = carrier frequency
θ = carrier phase at $t = 0$

3.1 Amplitude Modulation

Consider a carrier sinusoid $c(t) = A_c \cos\omega_c t$ that is multiplied by a factor which varies between 0 and 1; a process that can be simply realized with a potentiometer. For rapidly fluctuating information signals, the potentiometer is replaced with an electronic multiplier that implements a gain coefficient (again constrained between 0 and 1). For this introduction, we assume the gain coefficient (or modulation) to be always positive and of the form $k(t) = A_0 + A_m \cos\omega_m t$ where $0 \le k(t) \le 1$. In broadcast AM radio, the modulation signal (or gain) is always positive as it is in Figure 3-1. We will see that the broadcast signal has transmitted carrier with double sideband (DSB-TC). Amplitude modulation is, in fact, a non-linear process and this is evidenced by the generation of new frequencies. However, it is often called *linear modulation* since the output carrier amplitude varies in one-to-one correspondence with the message signal.

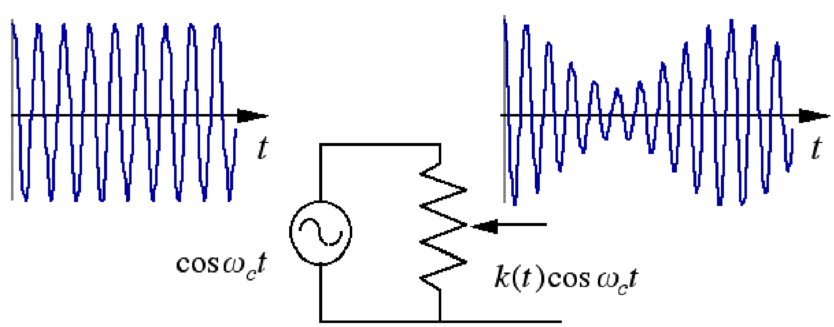


Figure 3-1 Multiplication of a unit amplitude carrier by the factor $k(t)$

3.1.1 Sinusoidal modulation of a carrier

We now put aside the requirement for a positive gain coefficient and focus on the product of two sinusoids. In this section, we consider modulating signals that have zero average value and, as a result, there is no carrier component in the product.

The output product has new component frequencies (ω_c - ω_m) and (ω_c + ω_m) that are different from either of the input frequencies. Signals in the modulation process are illustrated with "double sided" frequency spectra, a representation consistent with complex exponential representation of the signals. Using the Euler identity $2\cos\theta = e^{j\theta} + e^{-j\theta}$, the product $s(t) = A_m \cos\omega_m t \times A_c \cos\omega_c t$ is illustrated below in terms of exponential components. We can then restate the product *s*(*t*) in cosine form as $0.5 A_c A_m [\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t]$, a result consistent with the trigonometric identity $\cos x \cos y = 0.5[\cos(x+y) + \cos(x-y)]$.

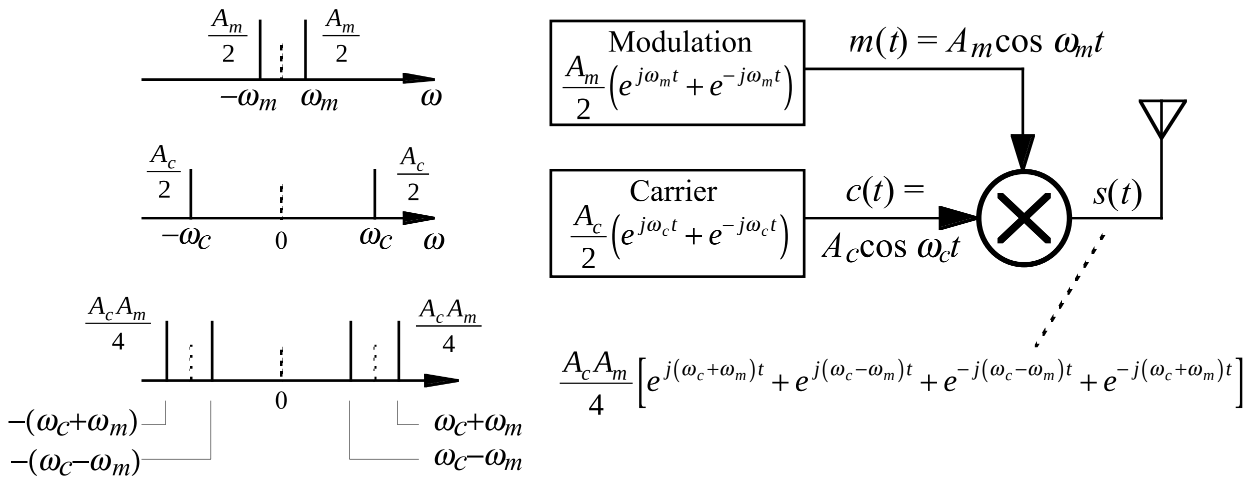
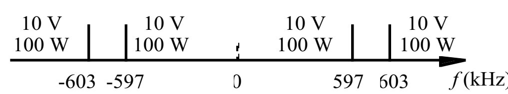


Figure 3-2 Multiplication of $A_m\cos\omega_m t$ and $A_c\cos\omega_c t$

Example 3.1 – Product of Sinusoids – A 600 kHz cosine waveform with peak amplitude 10 volts is multiplied by a 4 volt peak cosine waveform with frequency 3 kHz. Determine the frequency and amplitude of all product components and determine their normalized powers. Illustrate the amplitude spectrum and normalized power spectrum on a two-sided frequency axis.



$$\left(10\,\mathrm{V}\cos 2\pi 600\times 10^3 t\right)\times\left(4\,\mathrm{V}\cos 2\pi 3000 t\right)$$
$$= 20\,\mathrm{V}\cos 2\pi 597\times 10^3 t + 20\,\mathrm{V}\cos 2\pi 603\times 10^3 t$$

$\Rightarrow 597\ \text{kHz},\ V_p = 20\,\mathrm{V},\ P_N = 200\,\mathrm{W}$ $\Rightarrow 603\ \text{kHz},\ V_p = 20\,\mathrm{V},\ P_N = 200\,\mathrm{W}$

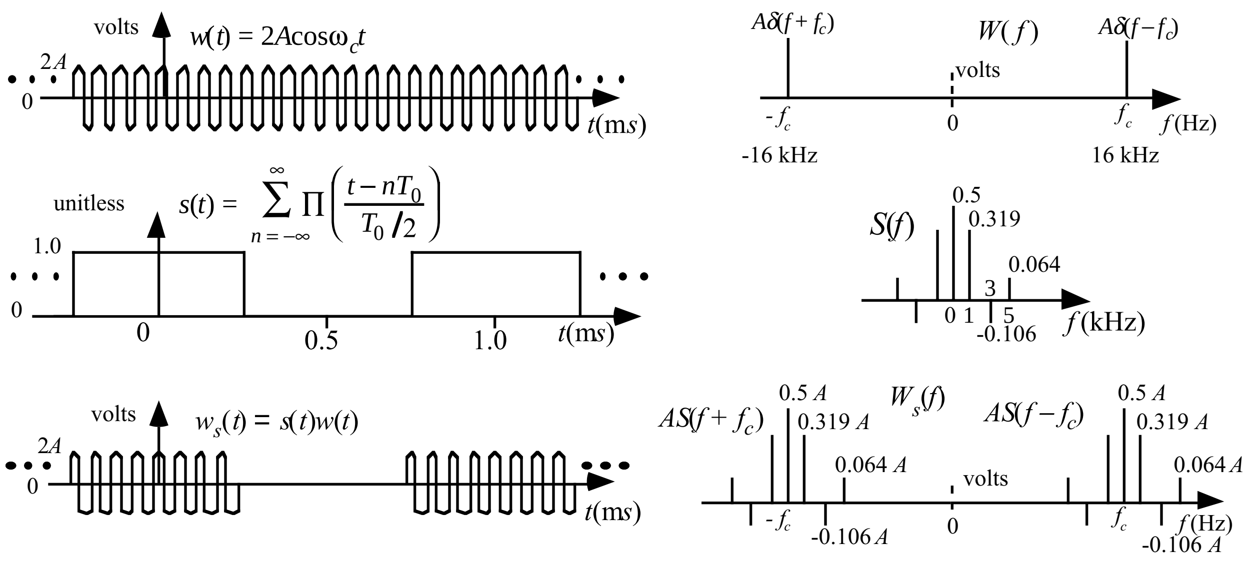
Drill Problem 3.1 - Amplitude Modulation - For a carrier signal *c(t)*=100Vcos2π20000*t* and the following modulation signals, determine the sinusoidal component amplitudes (in volts) and component frequencies (in kHz).

Modulation Signal	A1	F1	A2	F2	A3	F3	A4	F4
2cos 2π4000t								
4cos 2π11000t								
2 + 4cos2π23000t	200	3	200	20	200	43		
cos 2π4000t + cos 2π8000t								
Checksum	550	40	550	91	250	67	50	28

3.1.2 On/Off modulation of a carrier

Double-sided spectral representations facilitate rapid understanding of the spectra resulting from signal multiplication. The example below shows multiplication of a carrier sinusoid by an on/off gating signal. (Note that on/off modulation is used in optical transmission systems). The gating (or modulation) signal is represented by a low frequency "square" waveform with levels 0 and 1 and its spectrum has a cluster of components near zero frequency. Through the multiplication process, the "positive" frequency components of the squarewave translate to an "upper sideband" (USB) with frequencies greater than that of the carrier while "negative" frequency components translate to the "lower sideband" (LSB) with frequencies less than the carrier.

Example 3.2 – **On-Off Switched Sinusoid** - A 16 kHz sinusoid $2A\cos\omega_c t$ is multiplied by a 1 kHz repetitive on/off "square" waveform with 50% duty cycle. Illustrate the time waveform of this gated signal. Also illustrate the Fourier series for the two input signals, $W(f)$ and $S(f)$, and the output signal, $W_S(f)$.



Drill Problem 3.2 – On/Off Modulation - a) In the 1 kHz *baseband* on/off gating waveform of Example 3.2, what positive frequency has exponential component amplitude -0.106, b) In the output signal, $w_s(t)$, what are the highest positive component frequencies with amplitudes 0.064 A and 0.035 A, and c) if A = 40 V, what is the *magnitude* of the output component at –13 kHz?

a) _______ kHz b1) ______ kHz b2) ______ kHz c) ___________ V Checksum = 55.2

3.1.3 Spectral Translation & Convolution

Amplitude modulation of a sinusoidal carrier can be viewed as translating the baseband modulation signal to a higher frequency. In the output product spectrum, there is a copy of the two-sided baseband spectrum centered about the carrier frequency thus we have "frequency translated" the baseband signal. Each modulation frequency exponent has been increased by the carrier frequency.

When the carrier signal itself has several frequency components, translated copies of the baseband spectrum may overlap. Several combinations of input frequencies can result in the same output frequency. Upon multiplication, the frequency exponents add and thus the output spectrum may be calculated by convolution of input signal spectra. Convolution has been previously demonstrated in the calculation of PDF for the arithmetic sum of signal voltages.

3.2 RADIO BROADCAST AMPLITUDE MODULATION (AM-DSB-TC)

In AM broadcast radio, the carrier amplitude increases and decreases in proportion to the information signal voltage. The modulation voltage is always positive and thus there are no polarity reversals in the modulated carrier. This format is known as amplitude modulation with double sideband and transmitted carrier (AM-DSB-TC) or simply AM. A broadcast radio system needs simple, low cost, receivers (since there are many of them). By transmitting a large carrier component, low-cost envelope or diode detectors can be used in the receivers. So the extra cost in transmitting carrier power is balanced by reduced receiver cost.

In North America, AM broadcast carrier frequencies are spaced at 10 kHz intervals ranging from 540 kHz to 1700 kHz. Originally, modulation signals were restricted to less than 5 kHz resulting in 10 kHz transmission bandwidth in each channel. Recently adopted standards now allow signal frequencies up to 10 kHz with 20 kHz transmission bandwidth. The obvious interference between channels occupying the same or an adjacent frequency band is controlled by geographic separation between transmitters. The maximum allowed unmodulated transmitter power is 50 kW and the modulating waveform is constrained to be positive by limiting the (negative) modulation index to $\mu = 0.9$ (90%). The majority of AM radio stations are licensed for 10 kW or less and, in many cases, transmitter power is reduced at night.

Amplitude modulation is modeled by the multiplication of a constant carrier with a positively biased baseband information signal (it never becomes negative). In the illustration below, we assume an information signal of $u \cos\omega_m t$ where the modulation index $u = 0.5$.

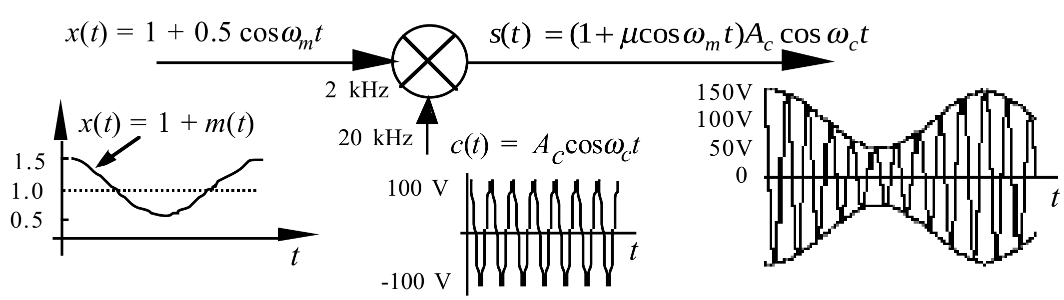


Figure 3-3 Amplitude modulation waveforms

3.2.1 AM signal spectra

We assume an information signal $m(t)$ with frequency, f_m (smaller than carrier frequency, f_c). The transmitted product waveform has amplitude variation proportional to modulating signal amplitude. For a sinusoidal information signal, we have $1.0 + \mu \cos \omega_m t$ and the modulation index, μ, must be less than unity. For other information signals, the index, μ, is defined in terms of the most negative excursion of the baseband signal i.e. $\mu = |m(t)_{\min}|$. Spectral analysis reveals a large carrier component plus an upper sideband (USB) tone and a lower sideband (LSB) tone. As μ is increased, sideband components increase while the carrier component remains constant. This leads to the description: amplitude modulation with double sideband and transmitted carrier (AM-DSB-TC). Sideband and carrier components are evident by expanding the product $s(t)$ as

$$\begin{aligned} s(t) &= (1+\mu\cos\omega_m t)A_c\cos\omega_c t \\ &= A_c\cos\omega_c t + \frac{\mu A_c}{2}\cos(\omega_c-\omega_m)t + \frac{\mu A_c}{2}\cos(\omega_c+\omega_m)t \quad . \end{aligned}$$

3.2.2 AM power efficiency

Only the sidebands carry information so the power efficiency of AM transmission is calculated as the ratio of sideband power to total transmitted power. This is expressed below for *sinusoidal* modulation and, at the upper limit, we have $\mu = 1$ resulting in 33% efficiency.

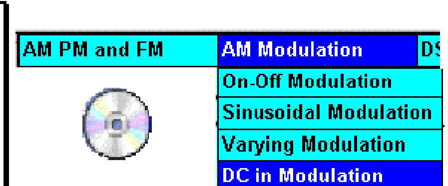
$$\text{Total Transmitter Power} = P_C + P_{USB} + P_{LSB} = \frac{A_c^2}{2} + \frac{(uA_c/2)^2}{2} + \frac{(uA_c/2)^2}{2}$$

$$\text{Power Efficiency} = \frac{P_{USB}+P_{LSB}}{P_C+P_{USB}+P_{LSB}} = \frac{u^2/8+u^2/8}{1/2+u^2/8+u^2/8} = \frac{u^2}{2+u^2} \quad .$$

Review "Virtual Laboratory" on amplitude modulation and observe 4 volt added dc component. Modulation voltage range is then 4 V ± 2.5 V.

a) Determine the largest positive peak of the output. ____
b) Calculate the modulation index. ____
c) The spectrum analyzer shows output carrier at 2.8 Vrms. Calculate rms sideband amplitude (compare with observation) ____

Checksum = 8.0



Drill Problem 3.3 – AM Power Efficiency - For a carrier signal $c(t) = V_p \cos 2\pi 600000 t$ and sinusoidal modulation, complete the following table. Refer to Example 3.3.

Carrier Vp	Pc (kW)	μ	Vp $s(t)$	Ptotal (kW)	Vp/rms	η(%)	PEP (kW)	Checksum
100 V	5	0.5	150	5.625	2.0	11.1	11.25	185.5
200 V			350					481.8
200 V			380	28.10	2.27	28.8		532.3
300 V	45	0.75					137.8	790.4
Checksum	**90**	**2.90**	**1405**	**117.0**	**8.64**	**83.8**	**282.5**	**1989.9**

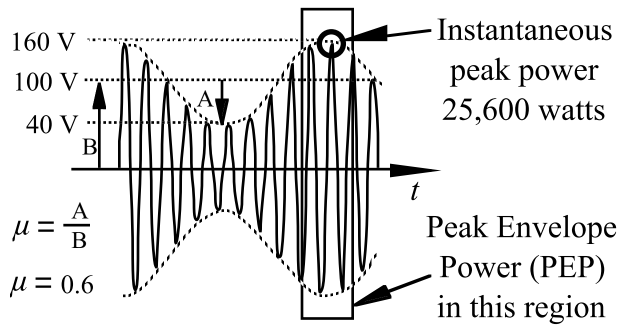
Example 3.3 – **Average Power, Peak Power, and Peak Envelope Power (PEP) –** A 100 V, 50 kHz carrier with normalized power 5 kW is amplitude modulated by a 5 kHz sinusoid using modulation index $\mu = 0.6$.

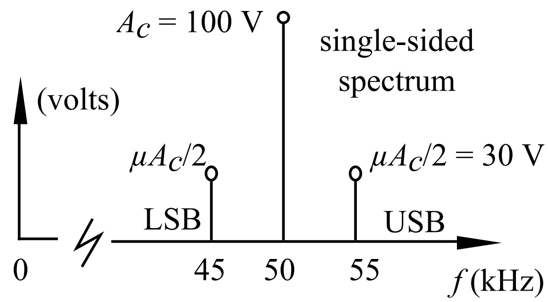
a) Illustrate the time waveform and, during the modulation cycle, illustrate the maximum carrier amplitude and calculate the instantaneous peak power.

b) Determine the sideband amplitudes, the power in the sidebands and the total average power.

c) Calculate average normalized power when the envelope is at its maximum (this is known as peak-envelope-power or PEP).

a)





b) LSB	$V_p = (0.6)100/2 = 30$ V	$P_N = V_p^2/2 =$ 450 watts		
Carrier	$V_p = 100$ V	$P_N = V_p^2/2 =$ 5,000 watts		
USB	$V_p = (0.6)100/2 = 30$ V	$P_N = V_p^2/2 =$ 450 watts	Total power = 5,900 watts	

c) PEP = $(160\text{ V})^2/2$ = 12,800 watts (normalized)

3.2.3 AM efficiency for realistic signals

Typical information signals contain many frequency components and the efficiency is much less for these compound signals. For the compound modulation signal, $m(t)$, the transmitted AM signal is expressed as: $s(t) = A_c \cos\omega_c t + m(t) A_c \cos\omega_c t$. The time averaged output power and sideband power efficiency are computed as follows

$$P_{ave} = \langle s^2(t) \rangle = \langle A_c^2 \cos^2 \omega_c t + 2m(t) A_c^2 \cos^2 \omega_c t + m^2(t) A_c^2 \cos^2 \omega_c t \rangle \qquad \text{and}$$

$$P_{ave} = \frac{A_c^2}{2}\left(1 + \langle m^2(t) \rangle\right) \qquad \text{if} \quad \omega_c >> \omega_m \qquad \text{and} \qquad \langle m(t) \rangle = 0$$

$$\text{Power Efficiency} = \frac{\text{sideband power}}{\text{average total power}} = \frac{\langle m^2(t) \rangle A_c^2 / 2}{\left(1 + \langle m^2(t) \rangle\right) A_c^2 / 2} = \frac{\langle m^2(t) \rangle}{1 + \langle m^2(t) \rangle} \quad .$$

For the specific case of sinusoidal modulation, $m(t) = \mu \cos \omega_m t$ and $\langle m^2(t) \rangle = \mu^2/2$. For a compound signal with peak/rms = 2.5, efficiency is less than 15% at the loudest volume (μ = 1). With normal AM broadcast modulation μ < 1 and efficiency may be considerably less than 10%.

Example 3.4: calculate the power efficiency when transmitting a compound signal using AM-DSB-TC with $m(t)_{min}$ = - 0.9 and the modulating signal peak/rms = 2.82.

$$\langle m^2(t) \rangle = \frac{(0.9)^2}{(2.82)^2} = \frac{0.81}{8} = 0.81(0.125)$$

$$\text{Efficiency} = \frac{\langle m^2(t) \rangle}{1+\langle m^2(t) \rangle} = \frac{0.81(0.125)}{1+0.81(0.125)} = 0.0919 = 9.2\ \%$$

3.2.4 AM Demodulators (Detectors)

Coherent Detector - Recovering the modulation signal $m(t)$ directly from the AM modulated signal, $s(t)$, using a synchronized oscillator is called synchronous detection, coherent detection, or homodyne (single frequency) detection. The receiver is known as a *zero-IF* or *direct conversion* receiver. Coherent detection requires a receiver local oscillator that is synchronized in frequency to the carrier used at the transmitter. The amplitude modulated signal, $s(t) = [1 + m(t)]A_c\cos(\omega_c t)$, is translated to baseband by multiplication with the local oscillator, $2\cos(\omega_c t + \theta)$, where θ is a small phase offset, ideally equal to zero. The multiplier output is

$$r_1(t) = [1 + m(t)]\, A_c \cos(\omega_c t)\, 2\cos(\omega_c t + \theta) = [1+m(t)]A_c[\cos\theta + \cos(2\omega_c t + \theta)]\ .$$

The term with twice the carrier frequency is removed by the receiver output low-pass filter, and the filtered output becomes $r_d(t) = [1 + m(t)]\, A_c \cos\theta$.

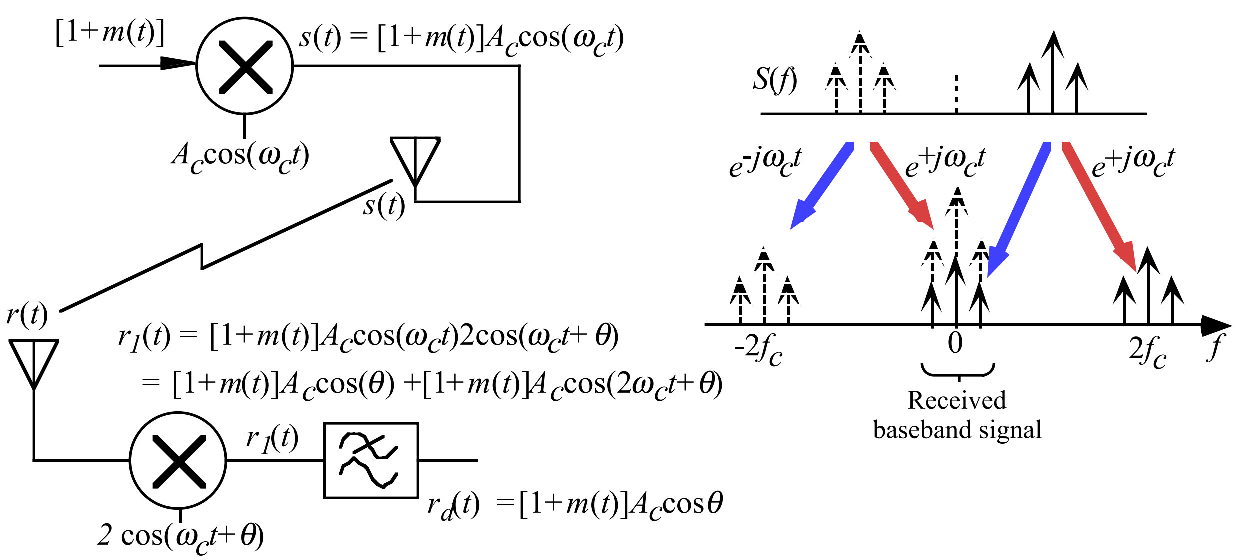


Figure 3-4 Coherent AM Detector

Diode Detector (Rectifier) - Demodulation (or detection) of AM signals is most economically accomplished by diode detectors or envelope detectors, however, these detectors require a large carrier component in the transmitted AM signal. Both methods are an alternative to synchronous (i.e. coherent) demodulation of the previous section. Diode and envelope detectors are sometimes referred to as self-homodyne or self-coherent detectors since the received carrier component is directly used in demodulation. If AM is applied to a diode and a resistor circuit the negative part of the AM wave will be eliminated and the output across the resistor, $r_2(t)$, is a rectified version of the AM signal. Rectification can be modeled as multiplication by a 1 and 0 "squarewave" signal, $k(t)$, at the same frequency and phase as the carrier. The rectified output is:

$$r_2(t) = \{[1+m(t)]A_c \cos\omega_c t\} \cdot k(t)$$

$$r_2(t) = [1+m(t)]A_c \cos\omega_c t \cdot \left[\frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - ....\right)\right]$$

$$r_2(t) = [1+m(t)]A_c\left[\frac{1}{2}\cos\omega_c t + \frac{1}{\pi}\left\{1+\left(1-\frac{1}{3}\right)\cos 2\omega_c t - \left(\frac{1}{3}-\frac{1}{5}\right)\cos 4\omega_c t + \left(\frac{1}{5}-\frac{1}{7}\right)\cos 6\omega_c t - ....\right\}\right]$$

When the rectified output $r_2(t)$ is applied to a lowpass filter with cutoff frequency $f_{bw} < \omega_c / 2\pi$, the filter output is $[1 + m(t)]A_c/\pi$, and all the other components with frequencies higher than f_{bw} are suppressed. The dc term is blocked by a capacitor to give the desired output $m(t)A_c/\pi$. Note that rectifier detection is, in effect, synchronous detection performed without using a local carrier. The large carrier content in the received signal makes this possible. If a full-wave rectifier is used, the equivalent multiplication is by +1 and –1 and the output amplitude is doubled as compared to a half wave rectifier.

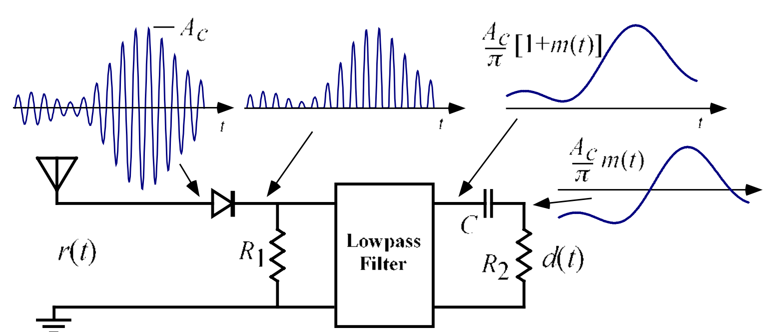
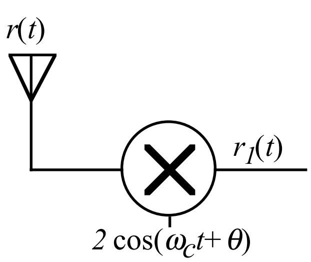
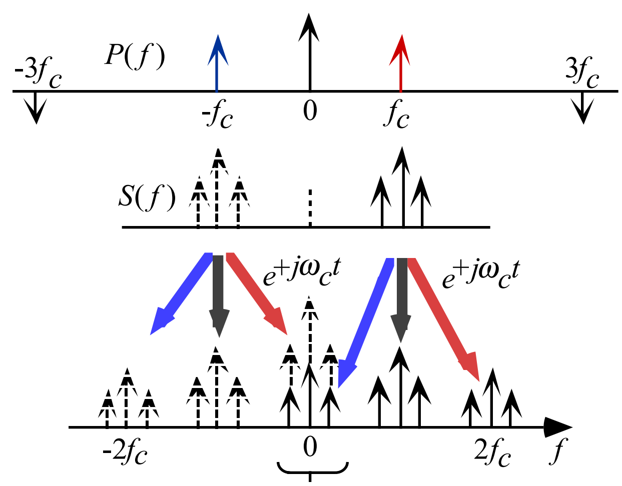


Figure 3-5 Rectifier Detector (adapted from Stremler)





Envelope Detector

The key point of interest in an AM signal is that the positive outer envelope of $s_{AM}(t)$ has the exact shape of $m(t)$. If we could construct a receiver that follows the envelope of the positive signal peaks, then $m(t)$ can be recovered. An envelope detector as shown in Figure 3-6 has higher output amplitude than the previously described diode detector. During a positive cycle of the AM wave, the diode is forward biased and capacitor charges up to the peak value. Ripple in the output occurs when the capacitor discharges through the resistor during the time period between peaks of the AM wave. The ripple can be decreased by increasing the carrier frequency or by increasing the RC time constant. The output lowpass filter can remove any remaining ripple. If the RC time constant is too large, the capacitor voltage cannot follow the rate of decrease in the envelope and there will be distortion in the shape of the demodulated waveform. If the modulating frequency is much less than the carrier frequency, the time constant should be selected as follows:

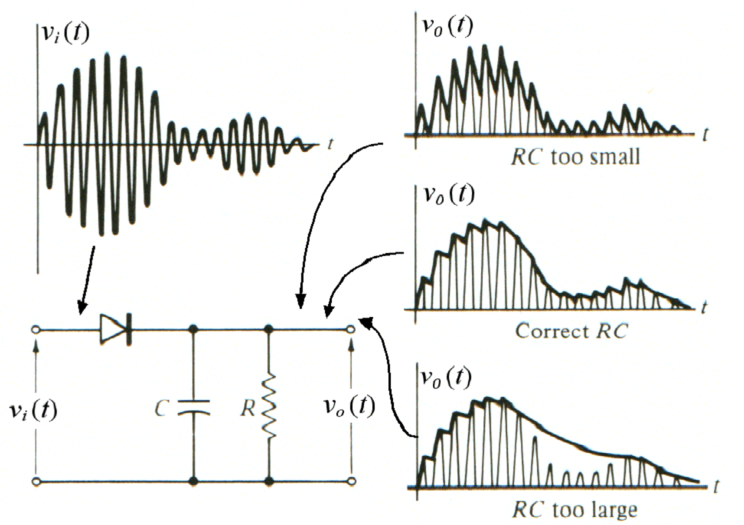
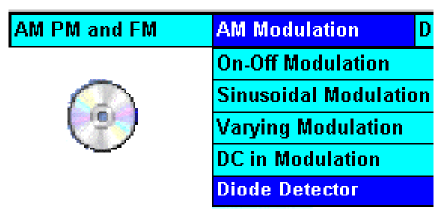


Figure 3-6 Envelope detector (adapted from Stremler)

$$f_{bw} << \frac{1}{2\pi RC} << f_c \qquad \text{where } f_{bw} = f_m(\max) \ .$$

Observe the diode detector in the "Virtual Experiments" CD.

a) What is the primary baseband output frequency when the modulation has no dc component (suppressed carrier)?
b) What is the primary baseband output frequency when the maximum dc component is applied at the transmitter?
c) Why are there only odd harmonics of the output baseband frequency if there is no dc component (suppressed carrier)?



3.2.5 Superheterodyne receiver

To separate radio stations, receivers were initially equipped with a *tunable* highly selective band-pass filter. In these tuned radio frequency (TRF) receivers, several resonant circuits were used to realize a band-pass filter with "flat" gain over the signal bandwidth but unfortunately it was difficult to *proportionately* tune all resonant circuits over the entire AM frequency band.

This problem was overcome in the super-hetero-dyne (Latin & Greek for "above-different-force/frequency") receiver, where a higher frequency local oscillator (LO) is multiplied with (mixed with or modulated by) the incoming desired signal. This produces an intermediate frequency (IF) signal that still contains the information signal and can be selected by a *fixed frequency* “flat” filter. The following diagram illustrates the spectral amplitudes at several points in a superheterodyne receiver (the vertical axis has units of $V/\sqrt{Hz}$).

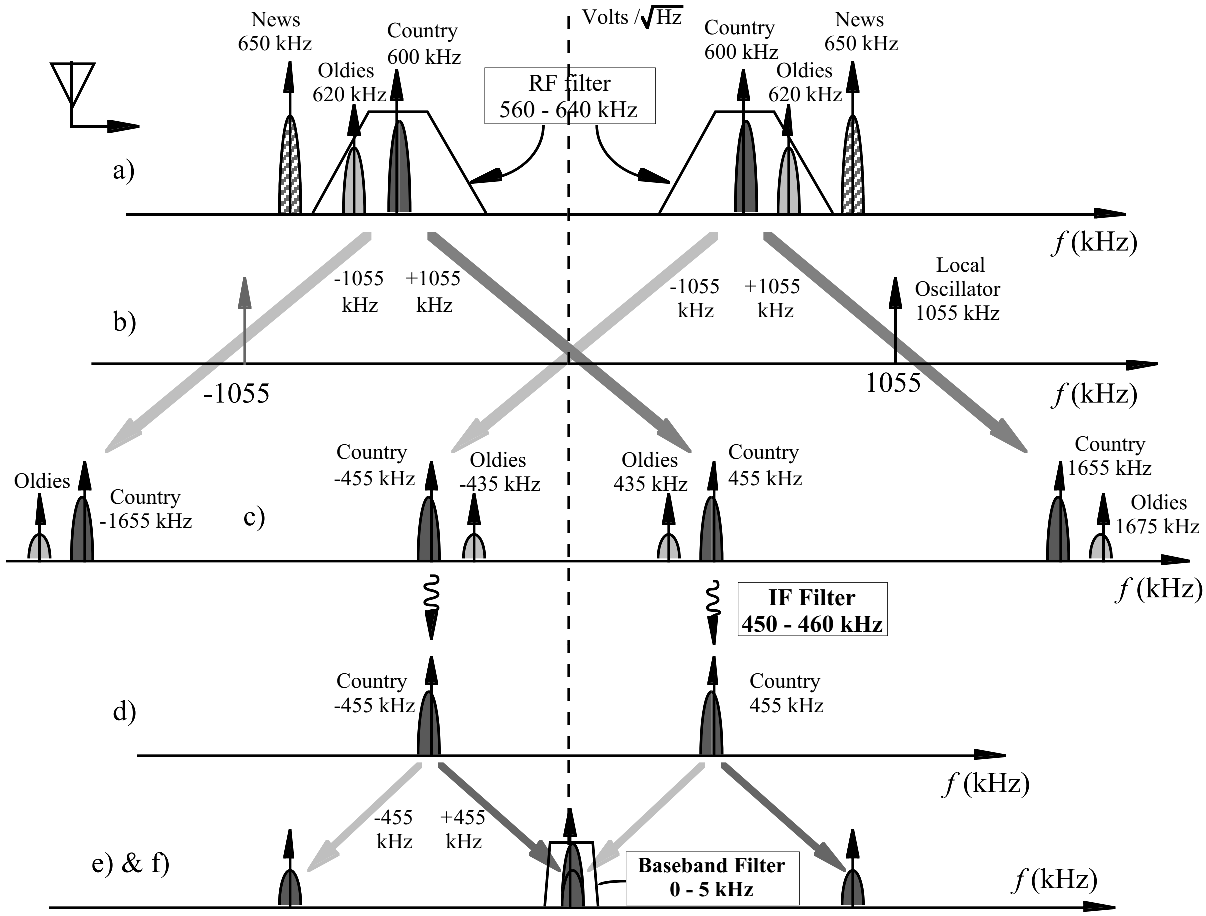


Figure 3-7 Signal spectra in a superheterodyne receiver.

The desired frequency relationship in the mixer is $f_{LO} - f_{RF} = f_{IF}$ where f_{LO} is the frequency of the local oscillator, f_{IF} is the intermediate frequency (IF), and f_{RF} is the carrier frequency of the desired incoming RF signal. By using a variable frequency LO, any desired RF signal can be translated to the IF frequency. All frequency components are translated by the same amount so the sidebands are not changed (except for reversal) by the translation. The advantage of this method is that the highly selective accurate band-pass filtering can be done at the intermediate frequency (IF). The accurate filter is fixed in frequency and there is no requirement to tune it.

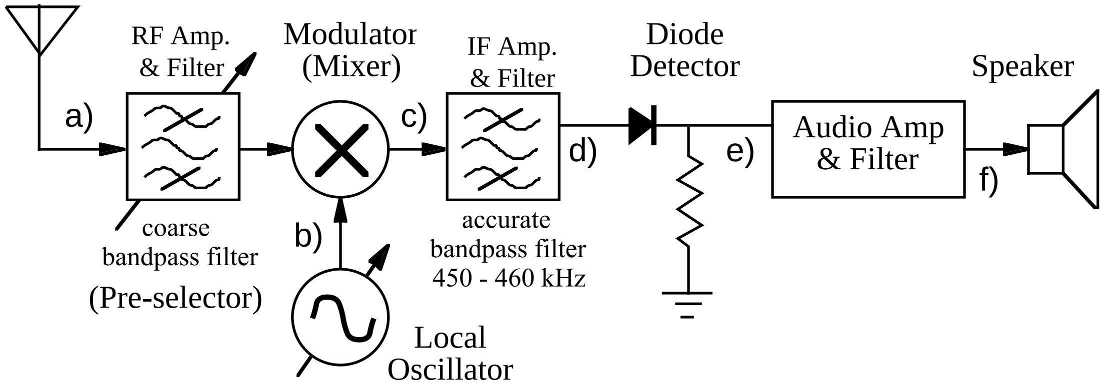


Figure 3-8 Superheterodyne receiver.

Example 3.5 – An envelope detector is used to demodulate a 455 kHz intermediate frequency DSB-TC signal to a baseband signal that has bandwidth 5 kHz. The envelope detector resistance is 10,000 ohms. What is a reasonable value for the capacitor?

Answer - To prevent excessive ripple in the output, the cut-off frequency (or bandwidth f_b) of the RC network should be much less than the IF frequency. To prevent excessive distortion, the bandwidth should be much greater than the highest modulating frequency.

$$f_m(\max) << f_b << f_{IF} \quad \Rightarrow \quad \text{Select geometric mean:} \quad f_b = \sqrt{f_{m,\max} \bullet f_{IF}} = 47.7\,kHz$$

$$\frac{1}{2\pi RC} = 47.7\,kHz \quad \Rightarrow \quad C = \frac{1}{2\pi \bullet 10^4 \bullet 47.7 \times 10^3} = \boxed{334\,\text{pF}}$$

3.2.6 The problem of image frequency in superheterodyne receivers

In a superheterodyne receiver, the LO frequency is greater than the desired RF frequency and the IF frequency is the difference between them. (i.e. $f_{IF} = f_{LO} - f_{RF}$) Unfortunately it is possible for an "image" signal (with frequency greater than the LO) to also result in a mixer output at the IF frequency. (i.e. $f_{IF} = f_{IMAGE} - f_{LO}$) The unwanted signal at the "image" frequency, after down conversion to IF, will overlay the desired signal and result in interference. It is therefore important to prevent image frequencies from reaching the mixer and this is the reason for the RF pre-selector filter. It is used to remove signals at f_{IMAGE} and allow signals at f_{RF}. The ratio of pre-selector filter gain at the two frequencies is known as the image rejection ratio.

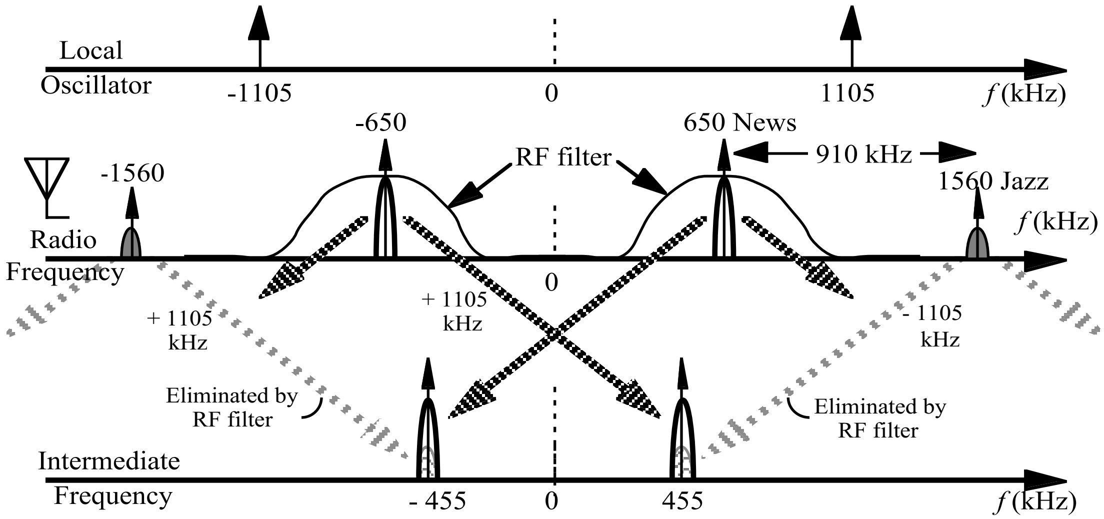


Figure 3-9 Removal of image frequency signals by the RF filter.

Example 3.6 – Image Frequency - a commercial AM radio station with carrier frequency 540 kHz is being received by a superheterodyne receiver with 455 kHz intermediate frequency. What is the local oscillator (LO) frequency and what is the image frequency? What is the significance of the image frequency?

Solution -
$$f_{RF} - f_{LO} = -f_{IF} \quad \Rightarrow \quad f_{LO} = 540 + 455 = 995\,kHz$$
$$f_{IM} - f_{LO} = f_{IF} \quad \Rightarrow \quad f_{IM} = 995 + 455 = 1450\,kHz$$

Any signal at the image frequency would be translated to the IF frequency and would interfere with reception of the desired signal. Image frequency signals must be removed by the RF filter before reaching the mixer. Here, the image frequency is 910 kHz (2 x 455) higher than the desired RF signal.

3.3 DOUBLE SIDEBAND SUPPRESSED CARRIER (DSB-SC)

A double sideband suppressed carrier (DSB-SC) signal is essentially an AM signal that has no discrete carrier component. When the baseband information signal with spectrum *M*(*f*) is multiplied by a unit amplitude sinusoid at carrier frequency, f_c, the following spectrum results.

$$S_{DSB}(f) = \frac{A_c}{2}\left[M(f + f_c) + M(f - f_c)\right]$$

DSB-SC wastes no power on transmitting a carrier component thus the modulation efficiency is 100%. For a given peak transmitter voltage, the sideband amplitudes can be twice as large (and sideband power 4 times larger) as for AM-DSB-TC. Despite this advantage, DSB-SC is not generally suitable for broadcast transmission since the exact carrier frequency and phase is required for demodulation and this presents a difficult problem.

3.3.1 Demodulation of DSB-SC

Direct demodulation of DSB-SC signals to recover *m*(*t*) requires coherent detection (synchronous detection); the receiver local oscillator must be exactly the same frequency (and phase) as the modulated carrier that would be received from the transmitter. This is not always possible (especially if the baseband signal has periods of silence). A receiver using direct coherent detection is also known as a “zero-IF” receiver.

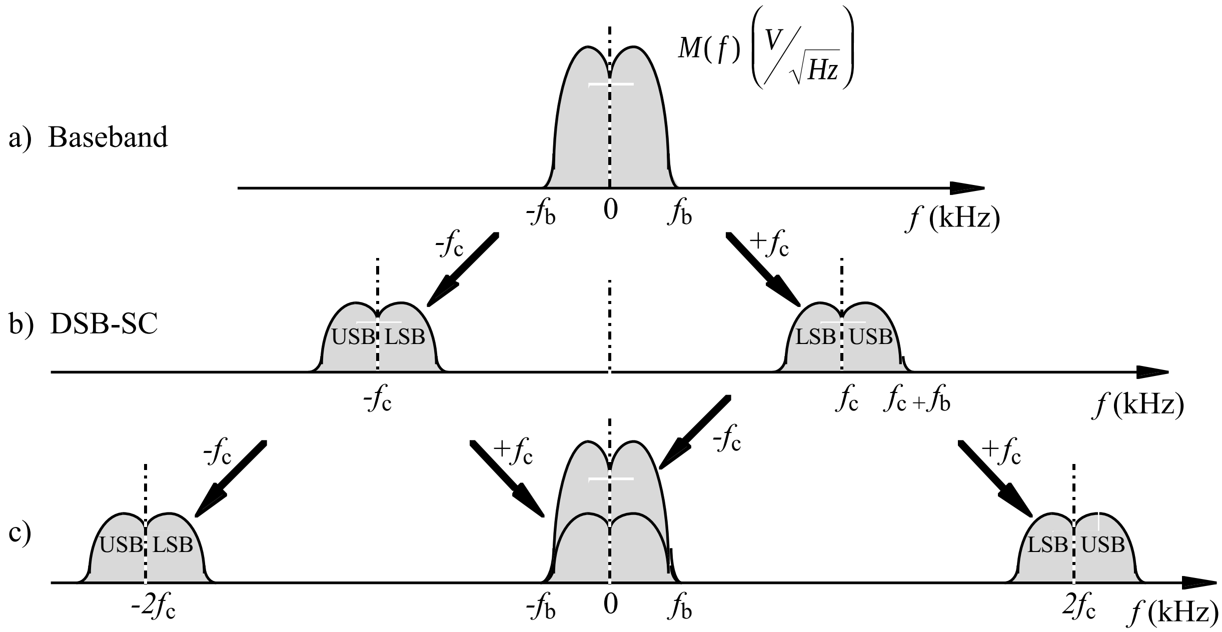


Figure 3-10 Spectra in modulation and coherent (“zero-IF”) demodulation of DSB-SC.

A homodyne, *direct conversion*, receiver has some advantages over the superheterodyne receiver. It eliminates the IF filter, the image rejection filter and the need for additional oscillators. It does, however, require the receiver to synthesize an accurate local oscillator with small phase error θ (ideally zero).

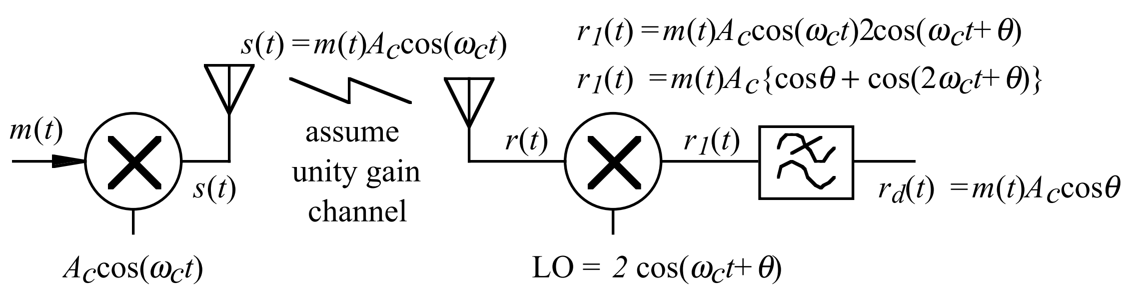


Figure 3-11 Zero-IF (homodyne) receiver for DSB-SC signals.

As the phase error θ increases, the output amplitude diminishes and, if θ is 90 degrees, the product demodulator output is *zero*! In order to ensure minimum phase error, the local oscillator (LO) signal can be developed using a phase locked loop incorporating the square of the received signal. This fails, however, during periods of silence when the sidebands become zero.

To permit the use of a low-cost diode detector, we can re-insert the locally generated carrier signal to form DSB-TC. This method avoids the relatively costly multiplier/mixer illustrated in Figure 3-11.

3.3.2 Application of DSB-SC within a FM stereo broadcast signal

An interesting application of DSB-SC is in commercial FM stereo broadcasting. Original frequency modulation systems were monaural and they carried one 20 Hz - 15 kHz audio signal.

Stereo broadcast in FM systems now uses a baseband signal with 20 Hz - 53 kHz frequency range to carry two audio signals. To be compatible the previous monaural system, the sum of the left and right (L+R) audio signals is broadcast in the 20 Hz – 15 kHz frequency range. The difference of the left and right audio signals (L-R) is placed at a higher (inaudible) frequency range in the baseband signal. The L-R signal amplitude modulates a 38 kHz sub-carrier producing a lower sideband in the frequency range 23 – 37.980 kHz and an upper sideband in the frequency range 38.020 – 53 kHz. The 38 kHz sub-carrier is not broadcast so that (almost) all of the combined signal voltage relates to the audio signals.

In effect, the L-R audio signal is relocated in the frequency domain by multiplying its time waveform by a pure sinusoid at f_c = 38 kHz. This is expressed as $S_{DSB}(t) = m_{L\text{-}R}(t)A_c\cos\omega_c t$. For example, a pure sinusoidal L-R modulation would result in frequencies $f_c \pm f_m$ and no component at the carrier frequency f_c. To avoid spectral overlap with the L+R baseband signal, the 38 kHz sub-carrier is selected such that $f_c > 2f_m$.

In addition to the main stereo program carried in the L+R and L-R signals, some broadcast signals include a background music program in the frequency range 59-75 kHz. This is shown in Figure 3-12 as subsidiary communications.

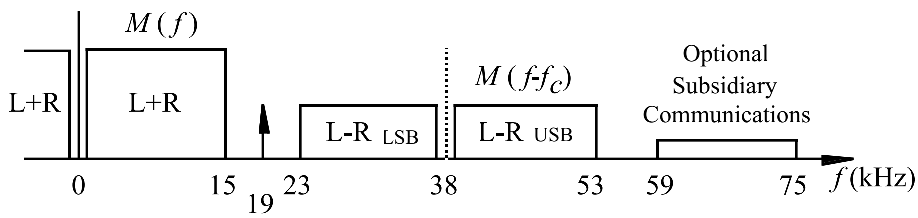


Figure 3-12 Spectrum of sub-carrier DSB-SC stereo signal.

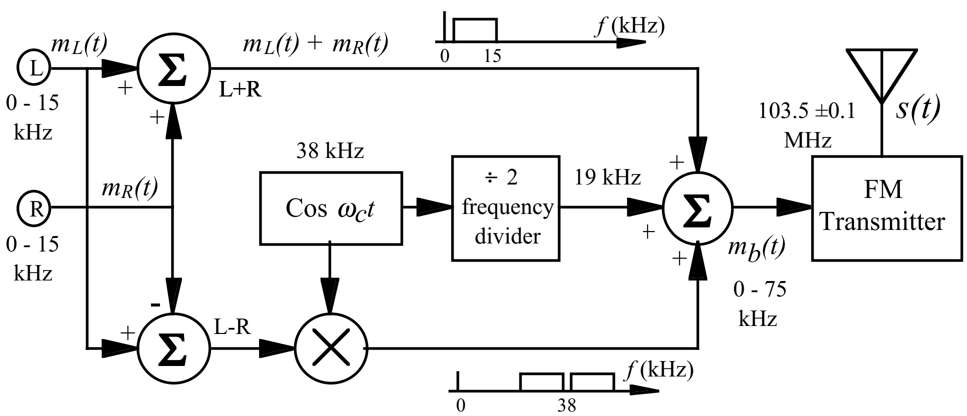


Figure 3-13 DSB-SC modulator for stereo broadcast on FM.

The broadcast frequency allocation, or in other words, the allowed bandwidth for commercial FM transmission is ± 100 kHz about the carrier. For example, a licensed transmitter might have an allowed transmission bandwidth of 103.1±0.1 MHz. The combined audio signal must not result in a frequency deviation that would cause this bandwidth to be exceeded. Since the receiver signal to noise ratio (SNR) improves with the degree of modulation, it is advantageous to remove any non-information component such as a baseband subcarrier.

The receiver, however, requires a large 38 kHz local oscillator to perform coherent demodulation. This is addressed by sending a small amplitude 19 kHz pilot tone that the receiver uses to synthesize the required 38 kHz. The 19 kHz pilot signal represents only a small portion of the baseband voltage used to modulate the FM transmitter so the SNR at the receiver is not significantly reduced.

At the receiver, the FM broadcast signal is initially demodulated to a signal with 75 kHz bandwidth. The 75 kHz signal is then separated by filters to form a L+R baseband signal and a L-R signal in DSB-SC format. A 38 kHz local oscillator signal is used to demodulate the L-R signal by synchronous demodulation as shown in Figure 3-14. Alternately the local oscillator (LO) output can be added to the L-R signal to form DSB-TC that can be subsequently demodulated by an envelope detector.

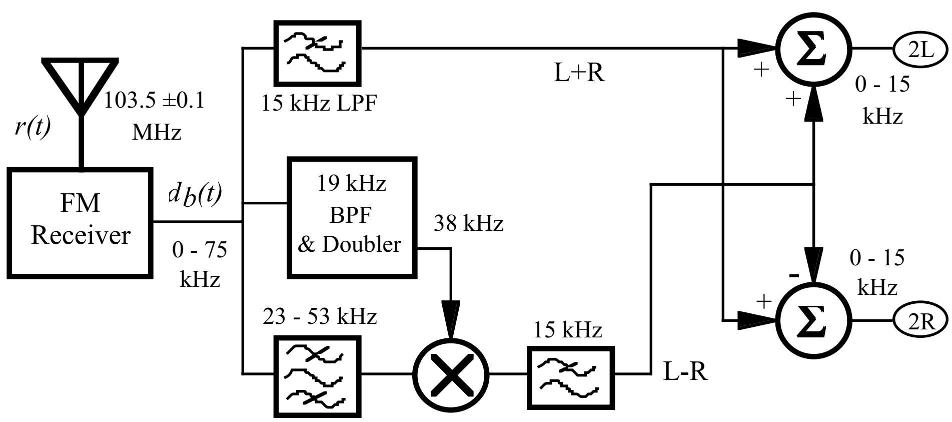
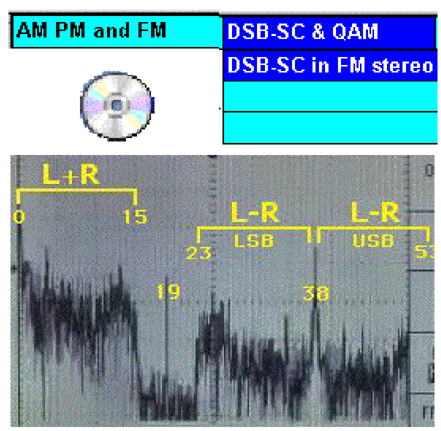


Figure 3-14 DSB-SC Demodulator for sub-carrier DSB-SC stereo signal

Using the "Virtual Experiments" CD, observe the spectrum analyzer display showing the demodulated spectrum of a FM stereo broadcast signal. This spectrum shows a baseband signal extending up to 15 kHz plus a DSB-SC signal containing L-R information.

a) Observe the pilot frequency and determine the bandwidth of the unused spectrum surrounding it.
b) Observe the receiver's 38 kHz carrier reference and determine it's amplitude in dB relative to the received pilot at 19 kHz.
c) Pause the movie and observe the similarity of the baseband spectrum to the sidebands about the 38 kHz carrier



3.4 SINGLE SIDEBAND AMPLITUDE MODULATION (SSB)

The most power and bandwidth efficient linear modulation scheme is single sideband amplitude modulation. Both sidebands in DSB-SC have equal amplitude and are symmetric in frequency about the carrier; the information contained in one sideband is also in the other. Transmitting only one sideband halves the transmission bandwidth and, when compared to AM-DSB-TC, power is saved by not transmitting the carrier and the redundant sideband.

3.4.1 Filter method of generating SSB

One method of generating a SSB signal is to first generate a DSB-SC signal and then remove one of the sidebands with a filter. Either the upper or lower sideband can be transmitted. The following diagram implies an ideal bandpass filter with perfectly sharp cutoff at f_c, however, real filters have a transition frequency range between stopband and passband. Since this transition is generally in proportion to the cutoff frequency, it is advantageous to filter at a relatively low intermediate frequency. For the example of voice signals with 300 – 3400 Hz bandwidth, the DSB-SC signal has 600 Hz separation between the sidebands and the SSB filter transition must occur over less than 600 Hz.

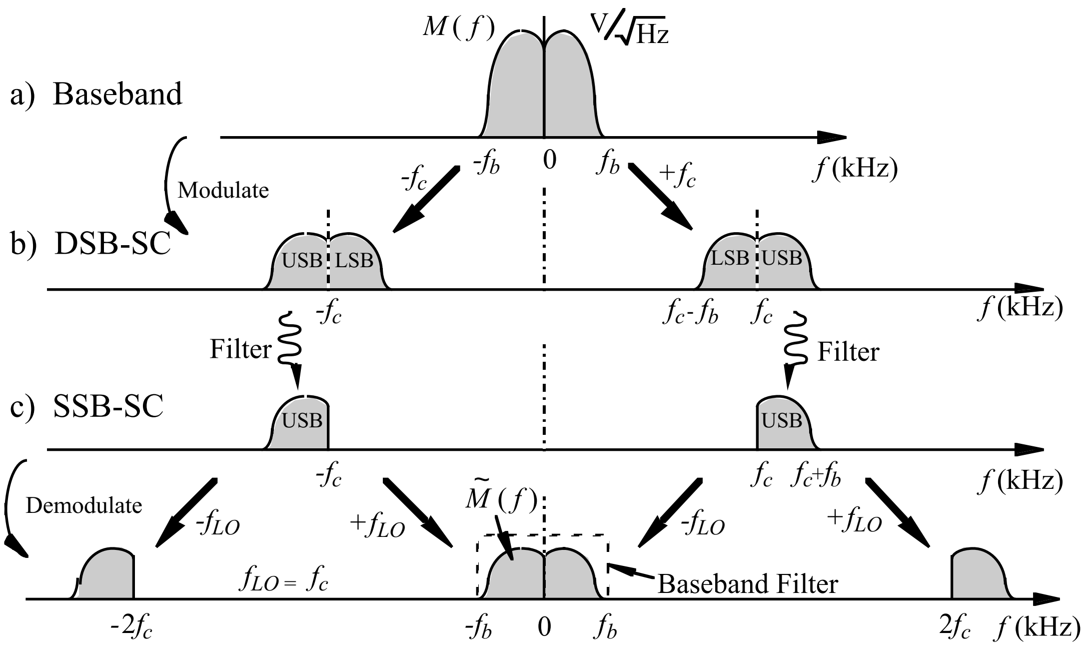


Figure 3-15 Spectra in modulation and coherent demodulation of SSB-SC.

3.4.2 Demodulation of SSB using a mixer

Demodulation of SSB requires an "almost-coherent" local oscillator at the receiver. Phase shift and even a *small amount of frequency error* can sometimes be tolerated. When the receiver's local oscillator has a small frequency offset relative to the transmitter frequency, the *demodulated information signal incurs a slight frequency shift*. This would not be tolerable for television signals, however a modest translation is acceptable in voice transmission systems such as in amateur radio, citizen band radio and in telephony. The 28.8 and 33.6 kb/s voiceband modems can tolerate up to 7 Hz of frequency translation. Early telephone systems used SSB for multichannel microwave transmission and a pilot tone enabled the receiver to synthesize a series of almost synchronous demodulation carriers for each SSB voice channel.

SSB demodulation is accomplished with a mixer (multiplier) in the same way as for DSB systems. Alternately, if replica of the transmitter carrier is inserted at the receiver, then an envelope or diode detector may be used in the same way as for broadcast AM.

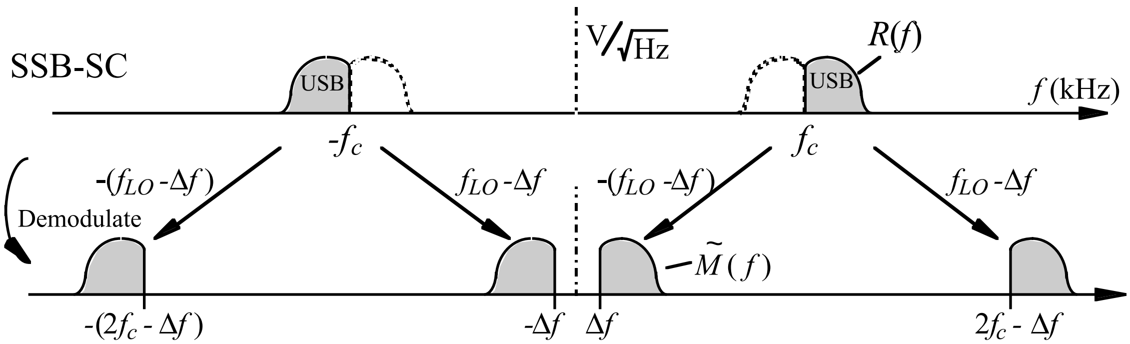


Figure 3-16 SSB demodulation with LO frequency error.

3.4.3 Envelope Detection of SSB

The most economical method of detecting a SSB-SC signal is to re-insert the missing carrier and then use a diode or envelope detector. As illustrated in Figure 3-17, a large carrier component is required to minimize the inherent distortion in $|g_1(t)|$. If the added carrier, represented by A_r , has large amplitude relative to the sideband amplitude, then $|g_1(t)| \approx x(t) = \mathrm{Re}\{g_r(t) + A_r\}$.

It is also possible to have a single sideband signal with *transmitted carrier*. Although this form is not power efficient, it allows the direct use of a self-homodyne diode detector while still conserving transmission bandwidth. The reduced bandwidth has advantages for optical fiber transmission where propagation delay varies with optical frequency.

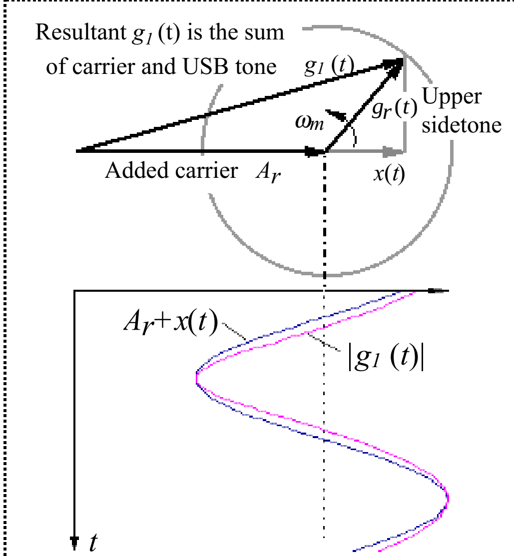


Figure 3-17 Phasor diagram showing carrier added to received upper sidetone

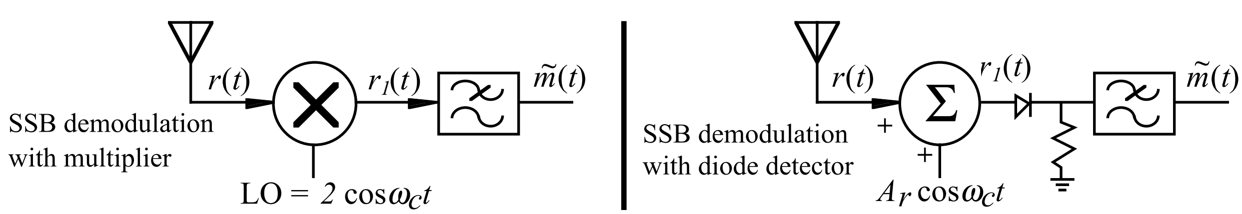
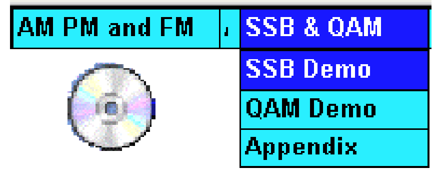


Figure 3-18 SSB demodulation with multiplier and with diode detection.

Using the "Virtual Experiments" CD, observe the phasor diagrams, spectrum analyzer and scope display for the SSB-SC demo as receiver carrier is added for the purpose of diode or envelope detection.

a) What envelope change do you observe as one sideband is removed.
b) Is this compensated by doubling the transmitted sideband amplitude?
c) For equivalent mod. index, any difference between DSB & SSB envelope?



3.4.4 SSB generation using the Phase Shift Method

The "phase shift" SSB modulator illustrated below produces a single sideband output without the use of filters to eliminate one sideband. The sine carrier $\hat{c}(t)$ is easily produced from the cosine carrier source $c(t)$ since, for *sinusoidal modulation*, a similar simple phase shift can be used. In the illustration, trigonometric identities verify the production of an upper side*tone* at $\omega_c + \omega_m$. (When both summation signs are positive, the lower sidetone is produced.) This modulation concept is useful in microwave or optical systems to frequency shift a relatively narrowband information signal.

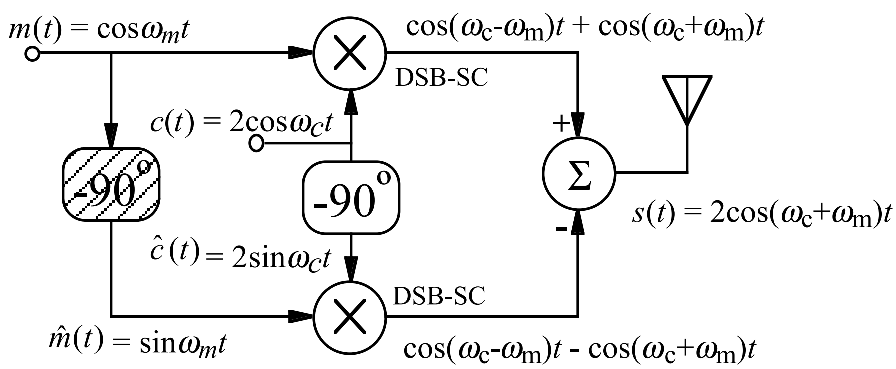


Figure 3-19 Phase shift method of generating SSB.

3.4.5 Analytic Signals

When the information signal $m(t)$ is compound (i.e. has many component frequencies), a ***Hilbert transform*** filter (shaded in Figure 3-19) is employed to phase-shift all frequency components of the signal by 90 degrees without changing the amplitudes. The information signal $m(t)$ and its Hilbert transform $\hat{m}(t)$, form the *analytic* signal $g(t) = m(t) + j\hat{m}(t)$. On a double-sided frequency scale, this analytic signal has only positive frequency components. In digital signal processing, both "in-phase" and "quadrature" signal components are retained and this analytic form facilitates digital signal processing for phase rotation or frequency shift.

Analytic signals are have both real and imaginary components (i.e. two signal lines are required). The analytic signal $x_p(t)$ is characterized by a spectrum that has only positive frequency components and is zero valued for all negative frequencies. Similarly, the signal $x_n(t)$ is zero valued for positive frequencies. An analytic signal is defined in terms of the real signal and its Hilbert transform.

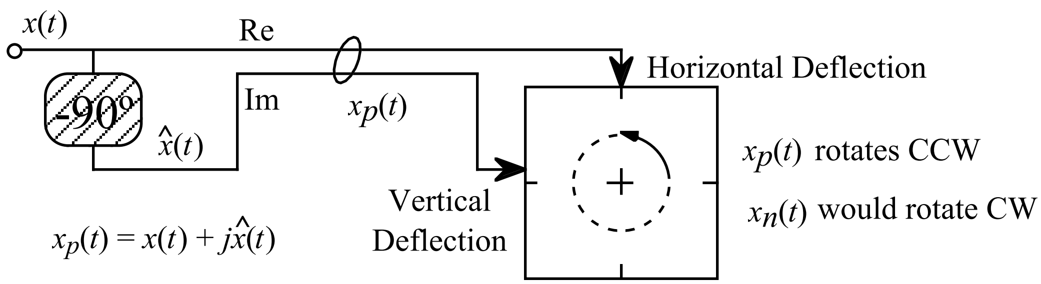


Figure 3-20 Components of an analytic signal and the X-Y oscilloscope display.

The analytic signal $x_p(t)$ can be viewed on an oscilloscope and, providing that the frequency is low enough, the counter-clockwise rotation can be observed. Similarly, the signal $x_n(t)$ which has only negative frequencies and will result in a clockwise rotation.

Example 3.7: Analytic signals – Given the real signal $x(t) = A\cos\omega_r t$ and its Hilbert transform $\hat{x}(t) = A\sin\omega_r t$ (for positive frequencies transform shifts by –90° or –*j*), write expressions for $x_p(t)$ and for $x_n(t)$.

Solution: $x_p(t) = \cos\omega_r t + j\sin\omega_r t = e^{j\omega_r t}$ and $x_n(t) = \cos\omega_r t - j\sin\omega_r t = e^{-j\omega_r t}$

3.4.6 Implementation of the Hilbert Transform*

The single sideband modulator requires a 90-degree phase shift and this might be implemented with an integrator or differentiator (with negative sign). Unfortunately, these circuits do not maintain constant gain when the frequency is changed. The required function is known as a Hilbert transform filter having the transfer function $H(f) = -j \ \mathrm{sgn}(f)$. Thus $H(f)$ = $-j$ for positive frequency, $H(f) = +j$ for negative frequency and $H(f) = 0$ for zero frequency. The impulse response for the Hilbert transform filter is given by

$$h(t) = \Im^{-1}[H(f)] = \Im^{-1}[-j \ \mathrm{sgn}(f)] = 1/\pi t.$$

If we assume the time function $x(t)$ with Fourier transform $X(f)$, the corresponding Hilbert transformed time function $\hat{x}(t)$ is given by:

$$\hat{x}(t) = \Im^{-1}[H(f)X(f)] = \Im^{-1}[-j \ \mathrm{sgn}(f)X(f)]$$

or $$\hat{x}(t) = x(t) * h(t) = x(t) * 1/\pi t$$

One approximation to a Hilbert transform filter uses a tapped delay line. It is reasonably accurate over a range of frequencies but fails at low frequencies when the half period of the input frequency exceeds the total time delay in the filter and it also fails at high frequencies when the half period is less than the time delay between the taps (see sampling theory). A tapped delay line Hilbert transform filter with 6 taps is illustrated below.

The tapped delay line can be viewed as a distributed differentiator. The two central taps contribute the difference $-x(t') + x(t'-T_d)$ where $x(t')$ is a delayed version of the input $x(t)$. The pair of taps outside the central pair, increase the gain for signals of long period and decrease the filter gain for signals with short period. Thus the gain, which would increase with frequency for the simple differentiator, becomes uniform over a wider range of frequencies as more taps are added.

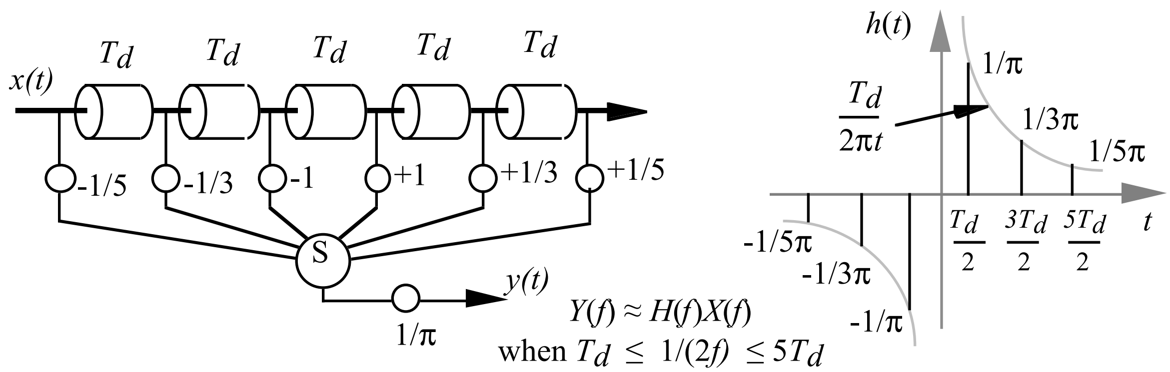


Figure 3-21 Approximation to Hilbert transform filter.

Another approximation to the Hilbert transform is the phase shift circuit illustrated below and which is used for SSB transmission of voice signals. While it does not provide the Hilbert transform of $x(t)$, it does provide two (differential) output signals that have a 90 degree phase relation over the voice frequency range 300 – 3300 Hz. The circuit contains 8 phase shifter stages, each affecting a different band of frequencies and in total covering the voice frequency range. Within the operating frequency range, the four outputs are equally spaced at 90 degree intervals. Although $m(t)$ is somewhat different than $x(t)$, the resulting SSB signal is quite satisfactory for transmission of voice and voiceband data.

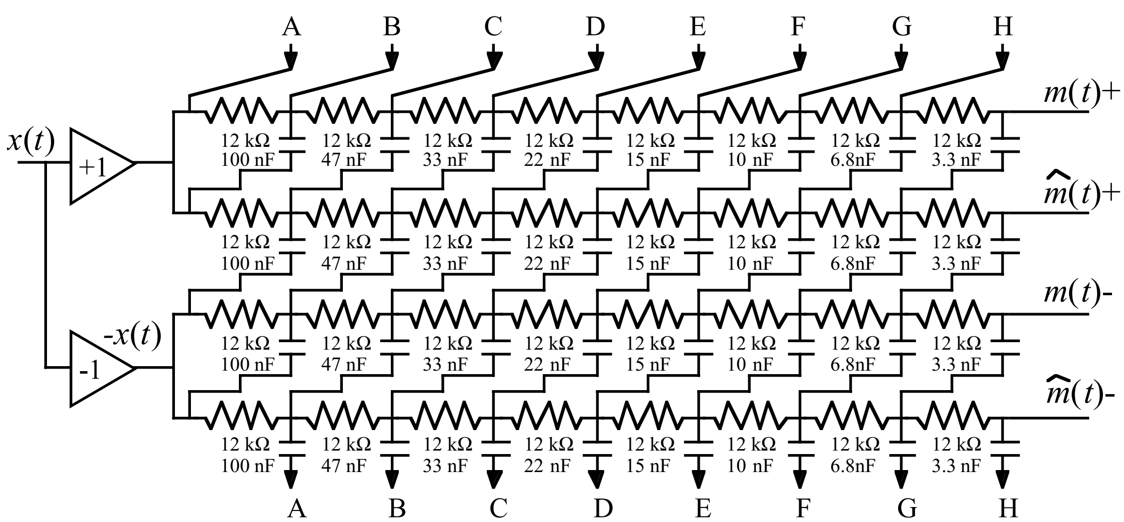


Figure 3-22 Phase Shifter with 90 degree outputs (Courtesy of Critical Telecom Inc.)

3.4.7 Frequency Translation using Analytic Signals*

Multiplication of the analytic modulating signal $m_p(t) = e^{j\omega_m t}$ by the analytic carrier signal $c_p(t) = e^{j\omega_c t}$ yields the output $s_p(t) = e^{j\omega_m t} \bullet e^{j\omega_c t} = e^{j(\omega_c + \omega_m)t}$ where all three signals have only positive frequency components. Thus the positive frequencies of the modulating signal are directly translated to a higher frequency as is required for SSB transmission.

In trigonometric form, we multiply $m_p(t) = \cos\omega_m t + j\sin\omega_m t$ by the carrier $c_p(t) = \cos\omega_c t + j\sin\omega_c t$ to get the product which is also an analytic signal $s_p(t) = \cos\omega_c t\cos\omega_m t - \sin\omega_c t\sin\omega_m t + j\sin\omega_c t\cos\omega_m t + j\cos\omega_c t\sin\omega_m t$. The first two terms form the real part of the product (the part that would be transmitted by an antenna) and this simplifies to $s(t) = 2\cos(\omega_c + \omega_m)t$ as illustrated in Figure 3-19.

With a multifrequency modulating signal, $m(t)$, each frequency component must be shifted by 90 degrees (this is where we need the Hilbert transform) to form $\hat{m}(t)$ and permit the "construction" of the analytic signal $m_p(t) = m(t) + j\hat{m}(t)$. The output $s_p(t) = s(t) + j\hat{s}(t)$, where $s(t) = m(t)c(t) - \hat{m}(t)\hat{c}(t)$ and $\hat{s}(t) = \hat{m}(t)c(t) + m(t)\hat{c}(t)$, is calculated by 4 multipliers as illustrated in Figure 3-23. The use of analytic signals is especially convenient in digital signal processing (DSP) receivers. Signals are maintained in complex form during the processing and filters are not required to eliminate image frequency, double frequency or sideband components.

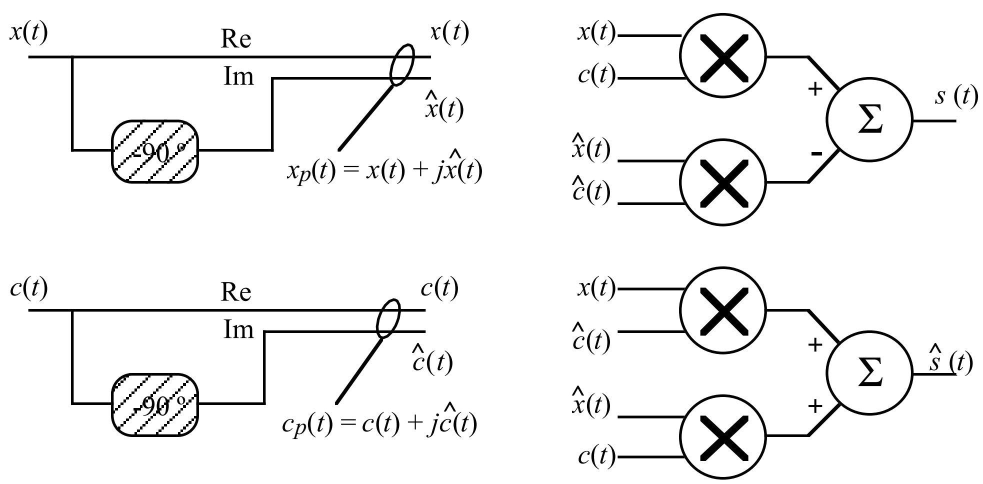


Figure 3-23 Two analytic signals and a complex multiplier

Drill Problem 3.4 – Placeholder - For a carrier signal *c(t)= Vp* cos2π600000*t* and sinusoidal modulation, complete the following table. Refer to Example 3.3.

Carrier Vp	Pc (kW)	μ	Vp *s(t)*	Ptotal (kW)	Vp/rms	η(%)	PEP (kW)	Checksum
100 V	5	0.5	150	5.625	2.0	11.1	11.25	185.5
200 V			350					481.8
200 V			380	28.10	2.27	28.8		532.3
300 V	45	0.75					137.8	790.4
Checksum	**90**	**2.90**	**1405**	**117.0**	**8.64**	**83.8**	**282.5**	**1989.9**

3.5 VESTIGIAL SIDEBAND MODULATION (VSB-TC)

Vestigial sideband transmission is a widely used standard for broadcast and cable television (TV). It is a compromise that combines the benefits of DSB-TC with the bandwidth conservation of SSB. Vestigial sideband (VSB) signals are relatively easy to generate and have only slightly greater (5-15%) bandwidth than an equivalent SSB signal. The broadcast television signal has one full sideband plus a small vestige of the other sideband. At low frequencies, where components from both sidebands are received, transmission is equivalent to DSB with excellent reproduction of the low frequency components that are essential for TV receiver synchronization. As with other forms of DSB transmission, the receiver requires a coherent local oscillator and thus broadcast television transmits a large carrier to allow envelope detection in the receiver.

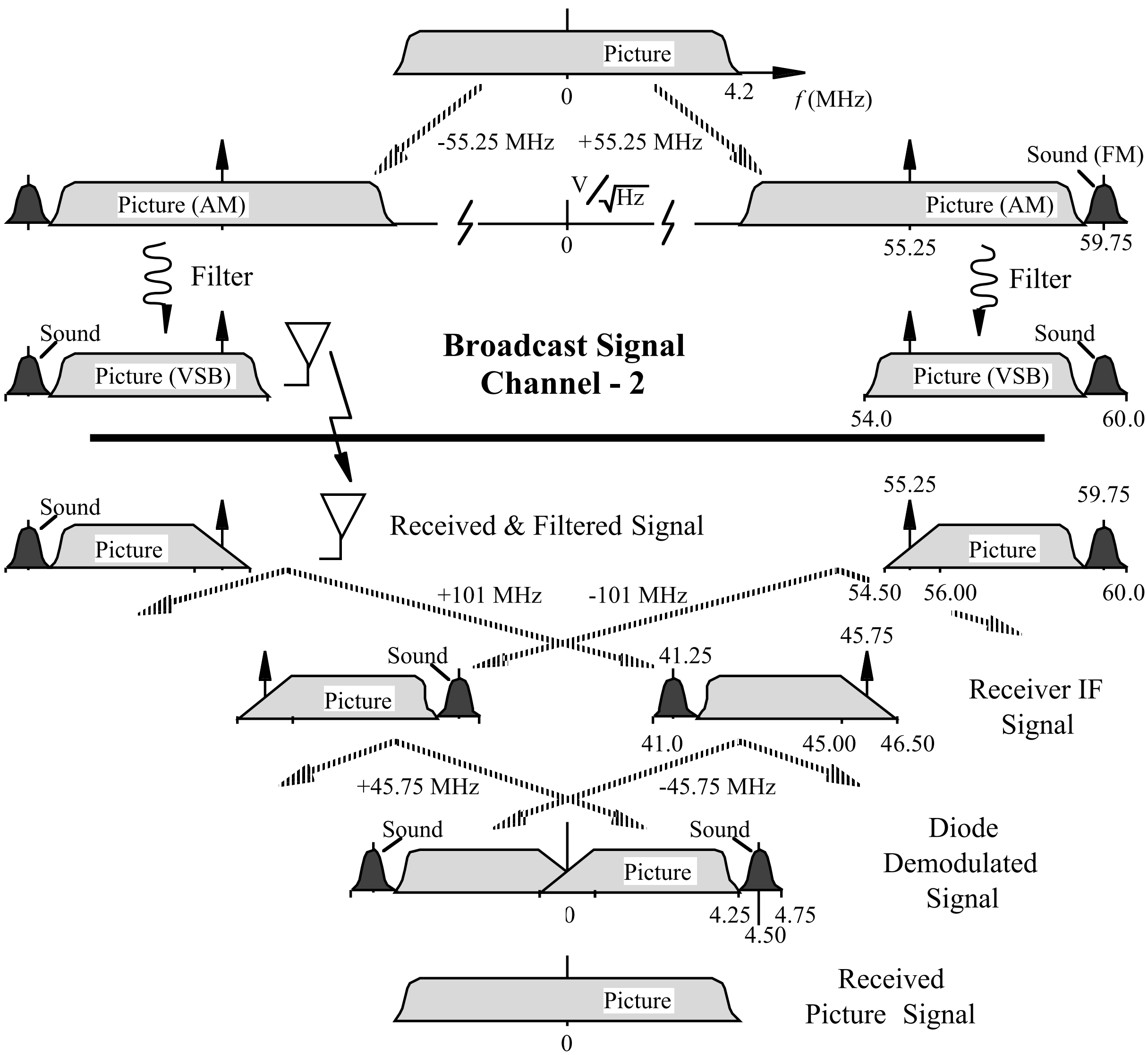


Figure 3-24 Modulation and demodulation in VSB broadcast television.

In Figure 3-24, a 0 - 4.2 MHz baseband television picture signal is used to DSB-TC modulate a channel 2 carrier of frequency 55.25 MHz. The lower sideband of the resulting signal is "truncated" by removing components less than 54 MHz. The vestige of the lower sideband is attenuated starting 0.75 MHz below the carrier and is completely attenuated at frequencies more than 1.25 MHz below the carrier. The associated television sound signal, which frequency modulates a carrier centered at 55.75 MHz, is added to the VSB picture signal prior to broadcast.

At the receiver, a filter is applied to spectrally shape the signal such that $M(f-f_c)+M(f+f_c)=M_{DSB}(f)$ where the spectral density of the equivalent DSB signal is represented by $M_{DSB}(f)$. With this shaping, the demodulated video signals overlap at baseband to provide uniform transmission gain throughout the frequency range.

3.6 QUADRATURE AMPLITUDE MODULATION (QAM)

By using quadrature amplitude modulation (QAM), two independent modulation signals can be transmitted in the same bandwidth that would be used by a single DSB transmission. The QAM scheme provides spectral efficiency equal to SSB but it does not require accurate filters or the use of the Hilbert transform.. For the majority of recently developed systems, QAM has been the choice. We assume unit amplitude information signals $x(t)$ and $y(t)$ and modulation indicies μ_i and μ_q such that $m_i(t)=\mu_i x(t)$ and $m_q(t)=\mu_q y(t)$. Two baseband signals are separated at the receiver through synchronous detection using two quadrature phase carriers. QAM modulation and demodulation are illustrated in Figure 3-25.

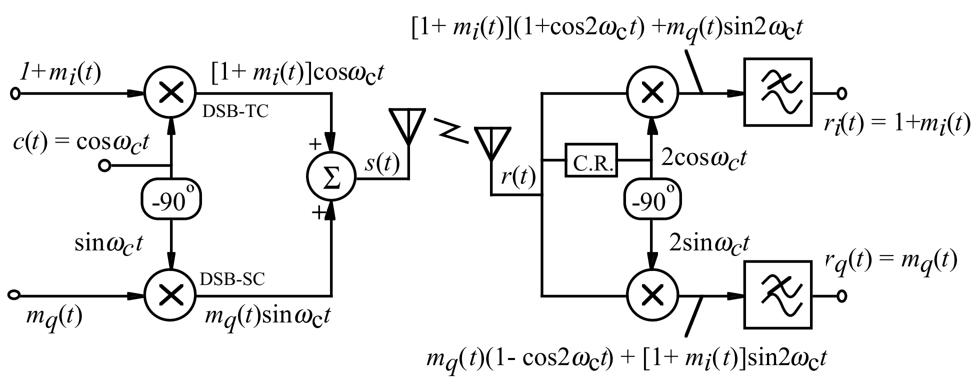


Figure 3-25 QAM Transmitter and Receiver.

At the receiver, a carrier recovery circuit develops a demodulator carrier that is ideally equal to the transmitter carrier in frequency and in phase (i.e. coherent). Phase error (i.e. $\theta \neq 0$) in the demodulator carrier (LO) will result in crosstalk between the two channels. We assume no channel attenuation and that the received signal is identical to the transmitted signal.

$$r(t)=s(t)=(1+m_i(t))\cos\omega_c t+m_q(t)\sin\omega_c t$$

In the upper (in-phase) portion of the receiver, the local oscillator (with phase error) is $2\cos(\omega_c t+\theta)=2(\cos\theta\cos\omega_c t-\sin\theta\sin\omega_c t)$. The product demodulator output then becomes

$$\left[(1+m_i(t))\cos\omega_c t+m_q(t)\sin\omega_c t\ \right]\ 2(\cos\theta\cos\omega_c t-\sin\theta\sin\omega_c t)\ .$$

This product has terms at twice the carrier frequency plus terms at baseband. After the low-pass filter, only baseband terms remain in the in-phase output $r_i(t)$.

$$r_i(t) = [1 + m_i(t)]\cos\theta - [m_q(t)]\sin\theta$$

In the lower or quadrature portion of the receiver, the local oscillator (with phase shift) is $2\sin(\omega_c t + \theta) = 2(\cos\theta \sin\omega_c t - \sin\theta\cos\omega_c t)$ and the product demodulator output becomes $[(1 + m_i(t))\cos\omega_c t + m_q(t)A_c \sin\omega_c t]\ 2(\cos\theta\sin\omega_c t + \sin\theta\cos\omega_c t)$ After the low-pass filter, this results in

$$r_q(t) = [m_q(t)]\cos\theta + [1 + m_i(t)]\sin\theta\ .$$

When $\theta = 0$, the two signals can be transmitted independently over the same bandwidth. When there is a phase error, the in-phase output includes some of the quadrature signal and vice versa.

Example 3.8: Assume a QAM system with receiver phase error $\phi = 0.1$ rad = 5.7° Calculate the crosstalk, *X*(dB), between channels. Note that $r_i(t) = [1 + m_i(t)]\cos\theta - [m_q(t)]\sin\theta$

Solution: $X(dB) = 20\log_{10}(|\sin\theta\ /\cos\theta|) = 20\log_{10}(|0.0993/0.995|) = -20.0dB$

A widespread example of QAM transmission is found in stereo AM broadcast. The block diagram of Figure 3-26 incorporates the same in-phase and quadrature modulators as in Figure 3-25. At the receiver, the local oscillator is derived from a phase locked loop (PLL) that extracts the carrier component from the received signal. To maintain compatibility with existing AM receivers, broadcast QAM utilizes two automatic gain control (AGC) elements. At the transmitter, the gain control loop scales back the resultant amplitude so that its envelope is equal to the in-phase modulation signal 1+L+R. Existing AM receivers can demodulate the signal (as usual) with an envelope detector. A stereo AM receiver envelope detects the AM signal then uses a feedback control loop to scale up the QAM signal prior to demodulation. The amount of downscaling and upscaling is dependent on the magnitude of the L-R signal and ideally the product of the two operations (αβ) is unity.

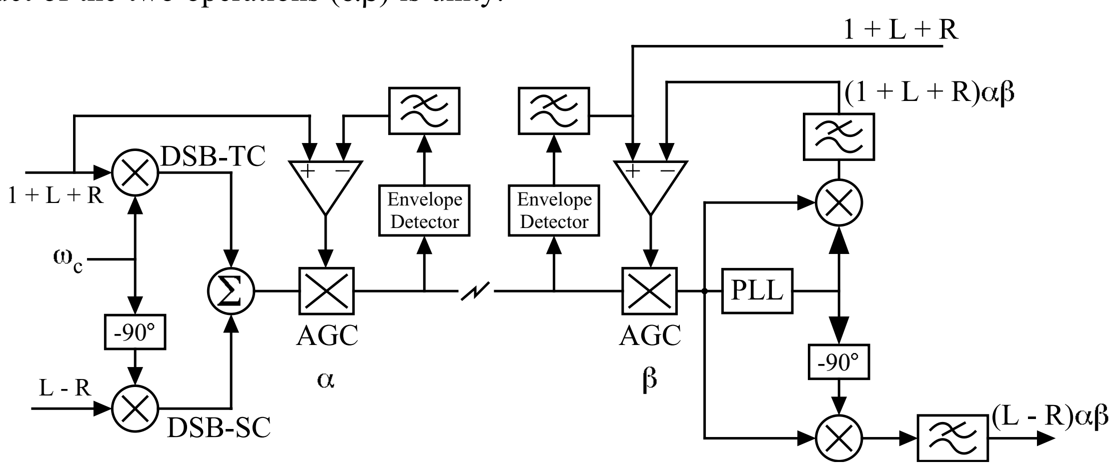


Figure 3-26 Stereo AM system.

3.7 PHASE MODULATION (PM)

Improved immunity to noise is the principle advantage of phase modulation (PM) or frequency modulation (FM). We begin with a study of phase modulation since this can be directly compared with noise immunity in AM. Phase modulation is widely used in digital modulation (such as QPSK), however, it is rarely used in analog systems because FM is easier to demodulate. A well-known example is broadcast FM which uses frequency modulation for signal frequencies up to 2.1 kHz, however, phase modulation is used for remaining signal frequencies up to 15 kHz since PM has superior performance. In PM and FM (together known as angle modulations), we increase transmission bandwidth and gain the benefit of improved noise performance - something that was not possible with AM.

With phase modulation, carrier phase varies directly with the baseband signal voltage $m(t)$; the carrier amplitude remains constant. We define a modulation coefficient k_p in units of radians/volt so carrier phase $\theta(t) = k_p m(t)$ and the transmitted signal $s(t)$ is then

$$s(t) = A_c \cos\left[\omega_c t + \theta(t)\right] = A_c \cos\left[\omega_c t + k_p m(t)\right] \quad .$$

We also define a modulation index β_p that indicates the peak phase deviation θ_p (in radians).

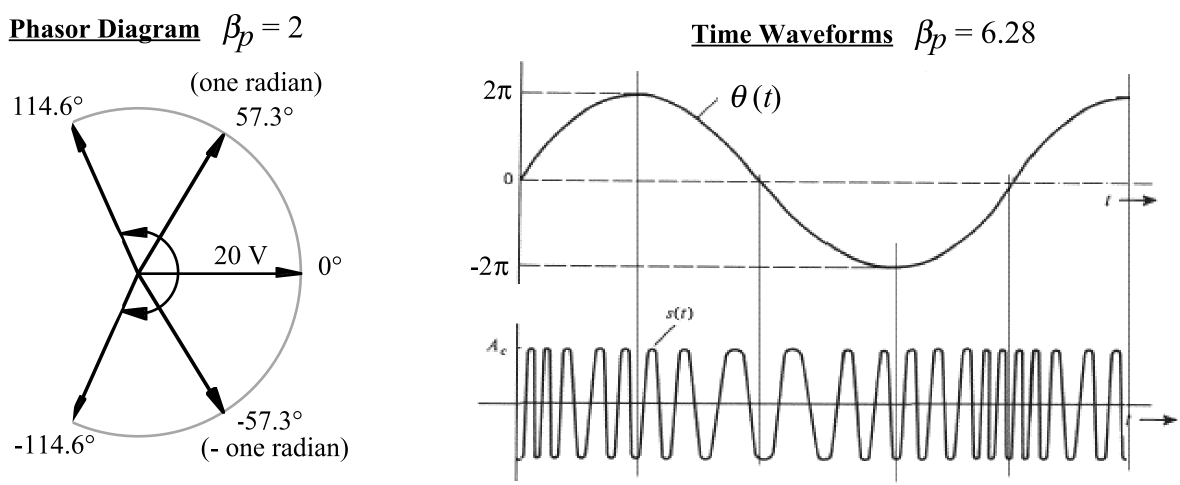


Figure 3-27 Two examples of wideband phase modulation (second is adapted from Couch-6).

Drill Problem 3.4 - Phase Modulation – In the waveforms illustrated in the above right-hand figure, assume that the carrier frequency is 1 MHz, the carrier amplitude is 25 volts and the peak modulation voltage is 1.5 volts. Answer the following questions with a precision of 2 decimal places.

a) Determine k_p, the gain coefficient of phase modulation. ________

b) Determine the phase advance (in radians) when the modulation voltage is +0.75 V. ________

c) What is the modulation frequency (in MHz)? ________

Checksum 7.39

3.7.1 Narrowband Phase Modulation

When the modulation index β is small compared to one radian, we have *narrowband* phase modulation. At these low levels of modulation, the PM spectrum is composed primarily of carrier plus two first order sidebands. The spectrum resembles that of amplitude modulation and, for small phase deviations, a good approximation to PM is obtained by summing an in-phase carrier with components generated by DSB-SC modulation of a quadrature carrier. Figure 3-28 illustrates the case when sinusoidal modulation is applied; the small amount of incidental amplitude variation in the output $s(t)$ is neglected.

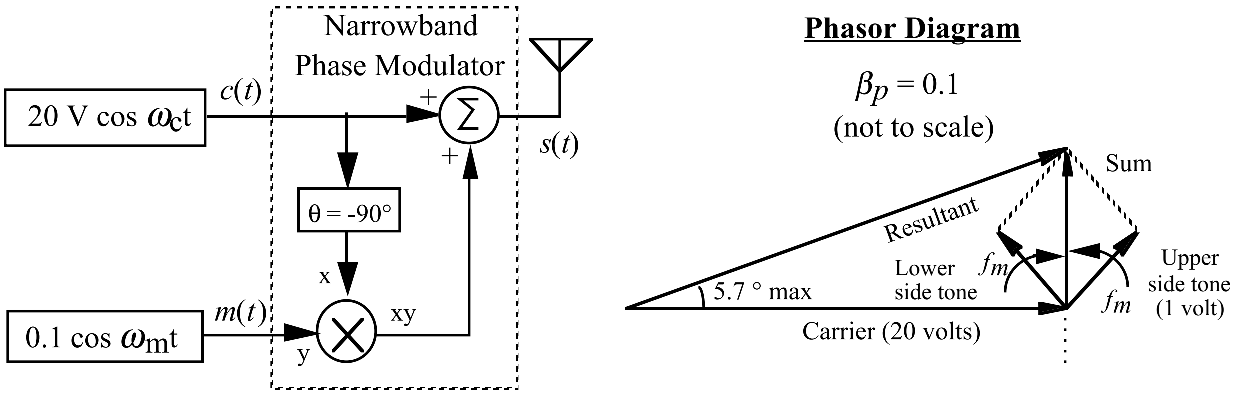


Figure 3-28 Narrowband phase modulation ($\beta = 0.1$).

In Figure 3-28 it is instructive to consider the alternate case where there is no phase shift and the sideband components add in-phase with the carrier resulting in amplitude modulation. In these two cases, the AM index μ is numerically equal to the PM index β_p.

Early angle modulation systems used narrowband angle modulation as a base and generated wideband modulation through frequency multiplication (i.e. using non-linear 3rd or 5th harmonic generation followed by filters). In this indirect method, the modulation index increased in proportion to the multiplication factor.

3.7.2 Wideband Phase Modulation

The appeal of phase modulation lies in its ability to improve SNR in noisy channels - but this occurs only for large modulation indices ($\beta_p > 1$) and this can greatly increase transmission bandwidth. We initially digress from this noise reduction objective to study PM spectral properties.

Phase modulation is a *non-linear modulation process* and the spectrum of a PM signal is not a simple replica of the modulating signal spectrum - particularly at higher levels of modulation. To study spectra of wideband phase modulation, we return to the mathematical expression $s(t) = A_c\cos[\omega_c t + k_p m(t)]$. This can also be expressed as $s(t) = \mathrm{Re}\{A_c e^{j\theta(t)} e^{j\omega_c t}\}$ with complex envelope $g(t) = A_c e^{j\theta(t)}$ where $\theta(t) = k_p m(t)$. As in amplitude modulation, the baseband spectrum of the complex envelope centers about ω_c and we can simply focus on spectral analysis of the complex envelope. The message signal directly varies the exponent in $e^{j\theta(t)}$ and Taylor series expansion shows many higher order spectral terms as $\theta(t)$ increases.

$$e^{j\theta} = \sum \frac{(j\theta)^n}{n!} = 1 + j\theta - \frac{\theta^2}{2} - j\frac{\theta^3}{6} + \frac{\theta^4}{24} + \frac{\theta^5}{120} - \frac{\theta^6}{720} + \bullet\bullet\bullet \quad .$$

Recall that sinusoidal modulation is useful for system analysis and testing (but of no interest as an information signal). Accordingly, we study spectral properties by assuming $\theta(t) = k_p A_m \cos\omega_m t$ and, after substituting into the above Taylor expansion, we have

$$e^{j\theta(t)} = 1 + jk_p A_m \cos\omega_m t - \frac{(k_p A_m)^2}{2}\cos^2\omega_m t - j\frac{(k_p A_m)^3}{6}\cos^3\omega_m t + \frac{(k_p A_m)^4}{24}\cos^4\omega_m t + \bullet\bullet\bullet \, .$$

It is clear that higher harmonics of the modulating frequency ω_C become more significant as $k_p A_m$ increases. To evaluate, we expand trigonometrically and collect similar frequency components from all terms. Assuming $A_c = 1$ and modulation index $\beta = \theta_{max} = k_p A_m$, the expansion becomes:

$$e^{j\theta(t)} = J_0(\beta) + j2J_1(\beta)\cos\omega_m t - 2J_2(\beta)\cos 2\omega_m t - j2J_3(\beta)\cos 3\omega_m t + \bullet\bullet\bullet \quad .$$

Collecting terms with $\omega = 0$ yields J_0, where J_0 is a Bessel function of the first kind of order zero and this represents the dc term of the complex envelope

$$J_0(\beta) = 1 - \frac{(\beta/2)^2}{1!\times 1!} + \frac{(\beta/2)^4}{2!\times 2!} - \frac{5(\beta/2)^6}{3!\times 3!} + \frac{25(\beta/2)^8}{4!\times 4!} - \bullet\bullet\bullet \quad .$$

Similar collecting of terms yields J_1, the amplitude of the fundamental frequency in the complex envelope, J_2 the amplitude of the second harmonic and so on. Tabulated values for J_0, J_1, J_2, etc. are widely available and the functions are known as Bessel functions of the first kind of order n. A series expansion for the Bessel function is

$$J_n(\beta) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\beta/2\right)^{n+2k}}{k!(k+n)!} \quad .$$

The zero frequency component amplitude J_0 becomes zero when $\beta = k_p A_m = 2.40$. The carrier vanishes leaving only sideband components thus providing a convenient observation for testing PM systems in the laboratory.

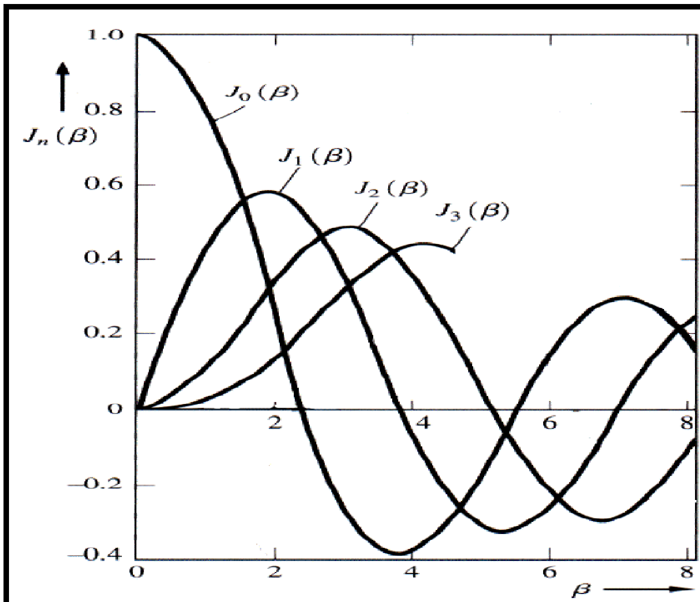


Figure 3-29 Bessel Functions

Example 3.9 – Use the series expansion to calculate Bessel functions $J_0(\beta)$ and $J_1(\beta)$ for index $\beta = 3.00$.

Answer:

$$J_0(3) = 1 - \frac{(\beta/2)^2}{1} + \frac{(\beta/2)^4}{2^2} - \frac{(\beta/2)^6}{6^2} + \frac{(\beta/2)^8}{24^2} - \frac{(\beta/2)^{10}}{120^2} + ...$$

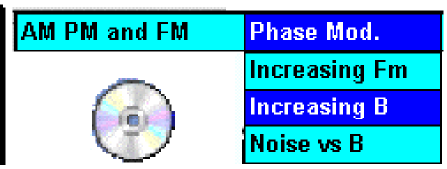
$$= 1.0 - 2.25 + 1.2656 - 0.3164 + 0.0445 - 0.0004 + ... = -0.2567$$

$$J_1(3) = \frac{(\beta/2)}{1} - \frac{(\beta/2)^3}{1(2)} + \frac{(\beta/2)^5}{2(6)} - \frac{(\beta/2)^7}{6(24)} + \frac{(\beta/2)^9}{24(120)} - \frac{(\beta/2)^{11}}{120(720)} + ...$$

$$= 1.5 - 1.6875 + 0.6328 - 0.1186 + 0.0133 - 0.0001 + ... = 0.3399$$

Observe the phase modulation demonstration in the "Virtual Experiments" CD. Here, a 30 kHz carrier is phase modulated by a 500 Hz signal.

a) Determine the demodulation coefficient in volts/rad.
b) Determine deviation (in degrees) when modulation index $\beta = 2.4$.
c) Roughly compare the spectral power for $\beta = 0.5$ and $\beta = 2.4$.

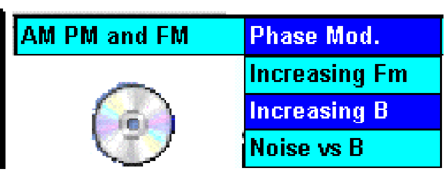


Drill Problem 3.5 – A 100 MHz carrier frequency is phase modulated by $m(t) = 5\,\text{V}\cos 2\pi 10^7 t$. The peak phase deviation is 4 radians. Determine the modulation index β_p and the gain coefficient k_p of the modulator. From the graph of Bessel functions, determine the modulation voltage V_m that will result in no spectral component at 110 MHz. With 200 MHz carrier frequency, what V_m will result in no spectral component at 210 MHz?

β_p = ______ k_p = _________ V_m = ___________ V_m = ___________ Checksum = 14.3 .

Observe the phase modulation demonstration in the "Virtual Experiments" CD. Here, a 30 kHz carrier is phase modulated by a 500 Hz signal.

a) Determine the demodulation coefficient in volts/rad.
b) Determine deviation (in degrees) when modulation index $\beta = 2.4$.
c) Roughly compare the spectral power for $\beta = 0.5$ and $\beta = 2.4$.



3.7.3 Graphical Illustration of Modulation Index and Spectra

With sinusoidal modulation and low modulation index, β=0.5, the spectrum is principally composed of the carrier and two first order sidebands. The imaginary portion of the complex envelope has a large sinusoidal component (first order quadrature sidebands) while the real portion has a large dc portion (carrier) with a very small double frequency component (second order, in-phase, sidebands).

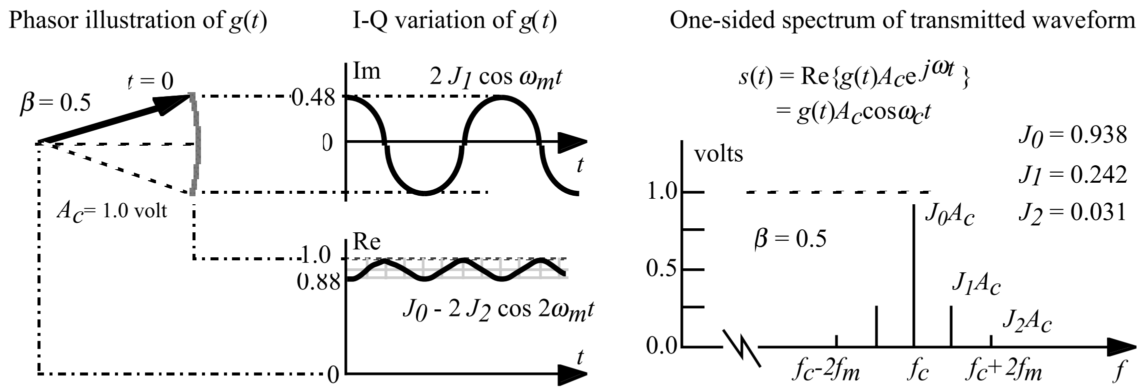


Figure 3-30 Complex envelope in PM with sinusoidal modulation (β = 0.5)

When the modulation index increases to β=1.5, the imaginary portion of the complex envelope is clearly non-sinusoidal and contains a substantial third harmonic component. The real part of the complex envelope contains a smaller dc component, larger second harmonic component and a small amount of 4^{th} harmonic. The transmitted carrier component decreases as the modulation index increases. When the modulation index increases to β =2.4, the dc component of the complex envelope reduces to zero and the carrier component of the transmitted waveform is completely eliminated. Total power in the spectrum remains constant.

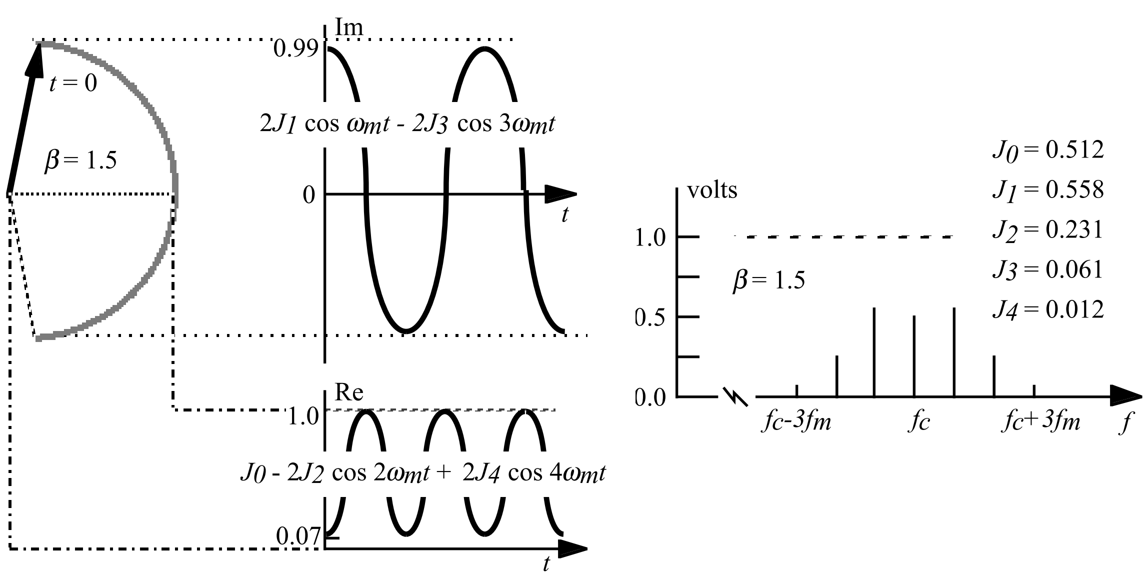


Figure 3-31 Complex envelope in PM with sinusoidal modulation (β = 1.5)

3.7.4 Noise reduction in phase modulation systems*

We now return to the study of noise and signal to noise ratio (SNR) in phase modulation systems. Typical noise resembles a continuum of equal amplitude sinusoids with infinitely small frequency spacing. We use the term "white" since this parallels the case of visible radiation with uniform spectral density. The sum of many sinusoidal voltages yields a Gaussian amplitude density with standard deviation equal to the rms voltage. Since noise voltage adds to the signal voltage, we use the term *additive white Gaussian noise* (AWGN). A portion of this noise is in-phase with the carrier while an equal portion is in quadrature to the carrier which results in a two-dimensional "fuzzy ball" in the phasor diagram.

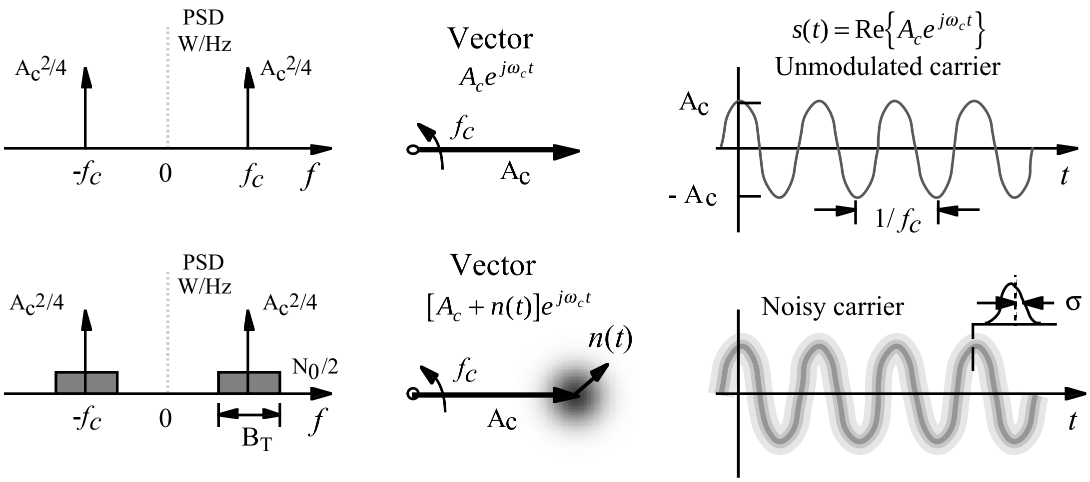


Figure 3-32 Bandlimited noise added to an unmodulated carrier.

We assume a receiver that accepts a limited transmission bandwidth, B_T, centered about the signal carrier. Channel noise is assumed to have white spectral density quantified as $N_0/2$ watts/Hz (two-sided density) thus the noise power entering the receiver equal to N_0B_T. The carrier power to noise power ratio is denoted *CNR* and is

$$C/N = SNR_{IN} = \frac{P_s}{P_n} = \frac{A_c^2/2}{N_0 B_T}$$

where $P_n = N_0 B_T = \langle n^2(t) \rangle = \sigma_n^2$ and where the transmission bandwidth $B_T \approx 2(\beta_p+2)B_m$ where B_m is the bandwidth of the message signal.

The *phase modulated carrier* is described with a complex envelope $g_S(t)$ as illustrated in the phasor diagram of Figure 3-33 where the amplitude A_c is constant and the phase $\theta_m(t)$ is modulated. When noise is added to the signal, we have $g_T(t) = g_S(t) + g_n(t)$ and the demodulated carrier phase has two portions, $\theta_m(t) = k_p m(t)$ and $\theta_n(t) \approx n_p(t)/A_c$ where $n_p(t)$ is the phase noise as measured along the circumference in the phasor diagram. We now have three components of interest: a) carrier power, b) message signal sideband power and c) noise power.

After demodulation with a phase detector, noise spectral components are translated to baseband where the spectral density becomes N_0 on a two-sided spectrum (twice the density when compared to RF). We assume a phase detector with unity gain (k_d = 1 volt/radian) to avoid complicating the following expression. The ratio of demodulated message signal power divided by the average noise power at the detector output is then

$$SNR_D = \frac{\left\langle \theta_m{}^2(t) \right\rangle}{\left\langle \theta_n{}^2(t) \right\rangle} = \frac{k_p{}^2\left\langle m^2(t) \right\rangle}{\left\langle n_p{}^2(t) \right\rangle / A_c{}^2} = \frac{A_c{}^2 k_p{}^2 \left\langle m^2(t) \right\rangle}{2N_0 B_T}$$

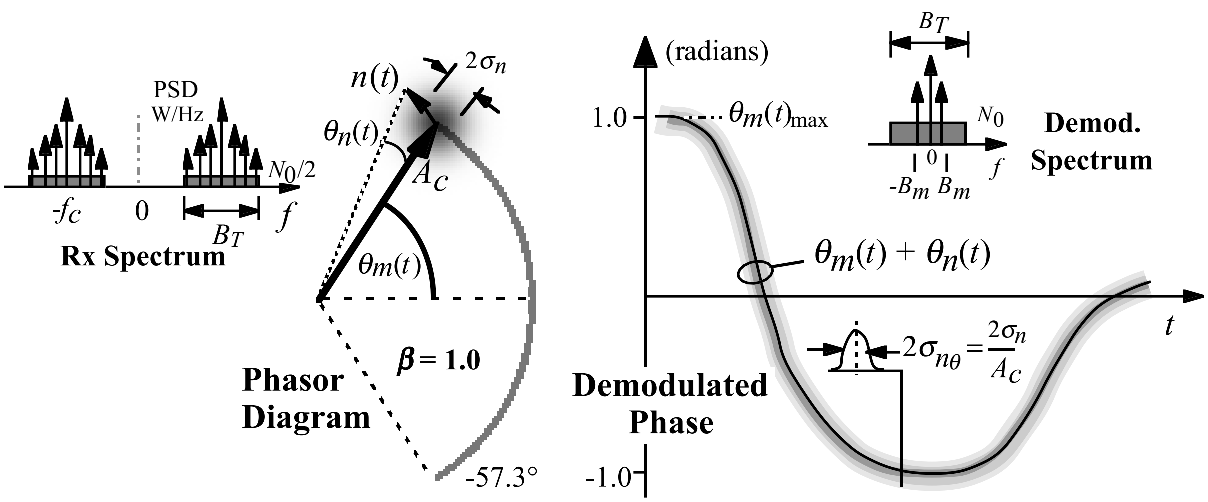


Figure 3-33 Phase demodulation with noise

The message signal spectrum, after detection, is collapsed to its original bandwidth and we can then limit the output signal to the bandwidth B_m. This eliminates excess noise received in the wider transmission bandwidth B_T and improves the SNR.

$$SNR_O = \frac{A_c{}^2 k_p{}^2 \left\langle m^2(t) \right\rangle}{2N_0 B_m} \quad .$$

Signal to noise ratio can be improved by increasing the modulation index β. For example, doubling the phase deviation, quadruples the modulation signal power and, since the demodulated noise voltage remains constant, this results in 6 dB increase in SNR.

Example 3.10: A phase modulation system with carrier 100 MHz and sinusoidal modulation at 1 kHz has modulation index β_p = 5, received carrier amplitude A_c = 10 volts and carrier to noise power ratio = 20 (13 dB). Receiver bandwidth is 100 MHz ±7 kHz so as to accommodate extended sidebands caused by the high modulation index. Determine the received noise power, the angular variance of the noise, and the demodulated SNR.

Solution:

$$P_n = \sigma_n{}^2 = \frac{P_c}{20} = \frac{(10\,\mathrm{V})^2/2}{20} = 2.5\,\mathrm{V}^2 \qquad P_{m\theta} = \sigma^2_{m\theta} = <(\beta\cos\omega_m t)^2> = \frac{(5\,\mathrm{rad})^2}{2} = 12.5\ \mathrm{rad}^2$$

$$\sigma^2_{n\theta} = \left(\frac{\sigma_n}{10}\right)^2 = \frac{2.5\ \mathrm{W}}{100\,\mathrm{V}^2} = 0.025\ \mathrm{rad}^2 \qquad SNR_d = \frac{P_m}{P_n} = \frac{\sigma^2_{m\theta}}{\sigma^2_{n\theta}} = \frac{12.5}{0.025} = 500\ \ (27\,\mathrm{dB})^*$$

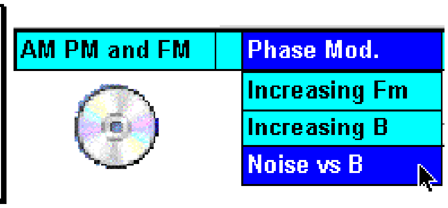
* this calculation does not include any improvement gained by post detection filtering.

Example 3.11: Consider the phase modulation system of the previous example. Determine the output SNR if the demodulated signal is lowpass filtered to remove noise power at frequencies greater than 1 kHz.

Solution: $$SNR_o = SNR_d\left(\frac{B_T}{B_m}\right) = 500\left(\frac{7\text{ kHz}}{1\text{ kHz}}\right) = 3500 \text{ (35.4 dB)}$$

Observe phase modulation of a 30 kHz carrier in the "Virtual Experiments" CD. The carrier is modulated by a 500 Hz sinusoid. The received carrier amplitude is approximately 0.9 volts. Click buttons for pop-ups

a) At the receiver, estimate σ_n in volts, Pc/Pn, and $\sigma_{n\theta}$ in rad.
b) Using the pop-up for $\beta = 1.0$, determine receiver sensitivity k_θ in V/rad.
c) Estimate demodulated Pm/Pn at $\beta = 0.5$, $\beta = 1.5$ and $\beta = 5.0$



3.7.5 Comparison with Noise performance in AM Systems

In phase modulation, the modulation index β has no limit and this allows for a substantial improvement in SNR. In AM-DSB-TC systems, the modulation index μ must be limited so the carrier phase will never reverse. The gain coefficient k_a in the amplitude modulator is chosen such that $\mu = |k_a m(t)_{\min}| < 1$. In AM-DSB-TC systems, the transmitted power is $0.5A_c^2[1+k_a^2 < m^2(t) >]$, the transmission bandwidth is $2B_m$ and the noise at the receiver input is $N_0 B_T$ where $B_T = 2B_m$. Assuming a lossless channel, the ratio of received signal power to noise power is

$$SNR_{IN} = \frac{\left[1+k_a^2\left\langle m^2(t)\right\rangle\right]A_c^2/2}{N_0 B_T} \quad .$$

The additive white Gaussian noise (AWGN) has in-phase and quatrature components $n(t) = n_i(t) + jn_q(t)$ and, after coherent product demodulation (see Figure 3-4), the output is $\tilde{m}(t) = A_c[1+k_a m(t)] + n_i(t)$. Noise power spectral density after demodulation is N_0 on a two-sided spectrum, the output bandwidth is B_m , the message signal power is $A_c^2 k_a^2\left\langle m^2(t)\right\rangle$ and the output SNR is then

$$SNR_O = \frac{A_c^2 k_a^2\left\langle m^2(t)\right\rangle}{2N_0 B_m} \quad .$$

The figure of merit for AM-DSB-TC is then the ratio of the output SNR to the input SNR.

$$\frac{SNR_O}{SNR_{IN}} = \frac{k_a^2\left\langle m^2(t)\right\rangle}{1+k_a^2\left\langle m^2(t)\right\rangle} \qquad \text{where: } k_a m(t)_{\min} > -1.0$$

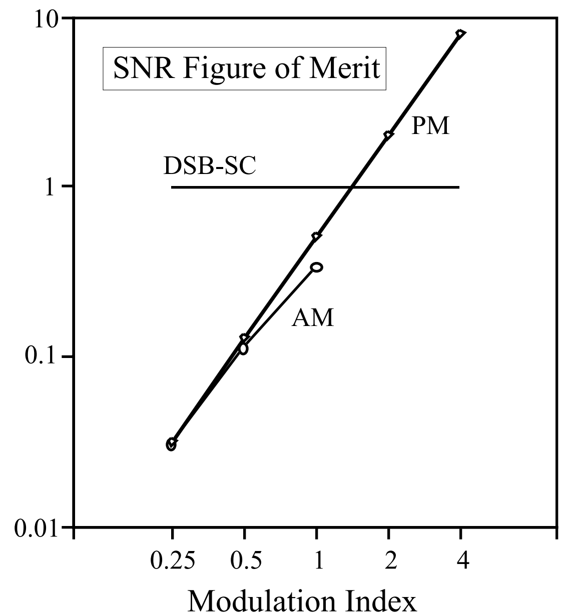
In DSB-SC (and for SSB), no carrier component is transmitted and figure of merit equals 1.0. For SSB, one initially expects 3 dB better performance since all signal power is transmitted in one sideband and only half the noise power is received. However, the two systems have the same figure of merit since, on demodulation, DSB uncorrelated noise sidebands add on a power basis (+3 dB) but the correlated signal sidebands add on a voltage basis (+6 dB).

In the case of PM, the transmission bandwidth is larger than for AM and a factor will be added to the figure-of-merit calculation to give all systems an equivalent bandwidth at radio frequency.

Example 3.11 – SNR Figure of Merit – Consider a transmission system with 250 mW unmodulated carrier power and 10 mW noise power at the receiver. Assume modulation by a unit amplitude sinusoid so that $\mu = k_A$ and $\beta = k_p$. Calculate the SNR figure of merit for DSB-TC and PM assuming sinusoidal modulation and μ = 0.25, 0.5, 1.0 and β = 0.25, 0.5, 1.0, 2.0, 3.0.

Solution:

Index	Demod. Power (normalized) $\sigma_A^2 = k_A^2 \sigma_m^2$	Tx Power (norm.) $1+\sigma_A^2$	Merit $\frac{\sigma_A^2}{1+\sigma_A^2}$
μ=.25	$(.25)^2(0.5) = 0.031$	1.031	0.030
μ=.5	$(0.5)^2(0.5) = 0.125$	1.125	0.111
μ=1.0	$(1.0)^2(0.5) = 0.500$	1.500	0.333
	$\sigma_\theta^2 = k_\theta^2 \sigma_m^2$		$\sigma_\theta^2 / 1.0$
β=.25	$(.25)^2(0.5) = 0.031$	1.00	0.031
β=.5	$(0.5)^2(0.5) = 0.125$	1.00	0.125
β=1.0	$(1.0)^2(0.5) = 0.500$	1.00	0.500
β=2.0	$(2.0)^2(0.5) = 2.00$	1.00	2.00
β=4.0	$(4.0)^2(0.5) = 8.00$	1.00	8.00



$P_{PM} = A_c^2/2 = 250$ mW $\qquad P_{AM} = \left(1+\sigma_A^2\right)\left(\frac{A_c^2}{2}\right)$

Note: for sinusoidal modulation $\sigma_A^2 = k_A^2 \sigma_m^2 = k_A^2 \langle m^2(t) \rangle = k_A^2 (0.5)$

3.7.6 Implementation of phase modulation

Electronic implementation of a phase modulator is generally difficult with the exception of optical systems where refractive index can be varied with an applied electrical field. Optical modulators can deviate phase by as much as 10 radians. Detection of PM is also difficult since a reference local oscillator must be synthesized in the receiver and available phase detectors cannot detect large phase deviations. Unfortunately, the SNR benefits of phase modulation are only realized at large phase shifts ($\beta > 2$).

In the next section, we introduce frequency modulation (FM) to easily generate large phase shifts and, at the receiver, frequency detection (differential detection of phase) is easily performed with frequency discriminators or phase locked loops. Through the use of pre-emphasis and de-emphasis circuits, FM systems emulate the good performance of a PM system.

In both PM and FM systems, information is carried in the carrier phase and not in its amplitude. In the receiver, after filters have selected the desired signal, it is therefore advantageous to "hard limit" the signal prior to demodulation. Limiting the signal amplitude largely eliminates impulsive noise introduced by lightning or electrical switching transients. It also mitigates the effect of non-linearity in the transmission system and variations in the received signal strength.

3.8 FREQUENCY MODULATION (FM)

Frequency modulation provides the benefits of phase modulation and has advantages where there is need for inexpensive receivers. The message signal varies the instantaneous frequency which is expressed as $f_i(t) = f_c + \Delta f(t)$ where: $\Delta f(t) = k_f m'(t)$, the modulation signal is $m'(t)$, and k_f is the modulation coefficient in units of hertz/volt. Since carrier angle is the time integral of instantaneous frequency, the relationship to phase modulation is simply

$$s(t) = A_c \cos[\omega_c t + \theta(t)] = A_c \cos[\omega_c t + \int_0^t \Delta\omega(t)dt] \quad .$$

From the equation we see that placing an integrator at the modulation input of a PM modulator results in a FM transmitter. Similarly, a differentiator at the output of a PM demodulator results in a FM demodulator. Today's FM transmitters can directly modulate a voltage controlled oscillator (VCO) and receivers economically perform FM demodulation by sensing the control voltage of a VCO within a phase locked loop (PLL).

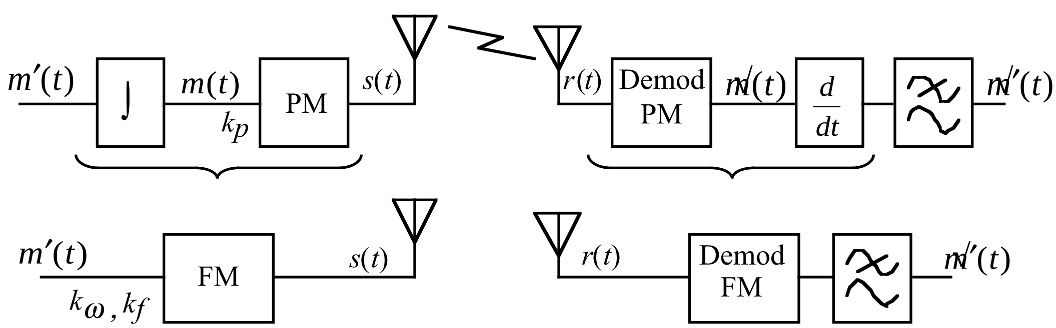


Figure 3-34 Relationship between PM and FM.

In the top part of Figure 3-34 we consider the specific case of *sinusoidal modulation* $m'(t) = A_m \cos\omega_m t$ which integrates to $m(t) = (A_m/\omega_m)\sin\omega_m t$ (volts) and results in carrier phase deviation $\theta(t) = (k_p A_m/\omega_m)\sin\omega_m t$ (radians). The peak phase deviation is $\Delta\theta_p = k_p A_m/\omega_m$ and the modulation index $\beta = \Delta\theta_p$ is defined in terms of this peak phase deviation (in radians). We differentiate $\theta(t) = (k_p A_m/\omega_m)\sin\omega_m t$ to find the carrier frequency deviation

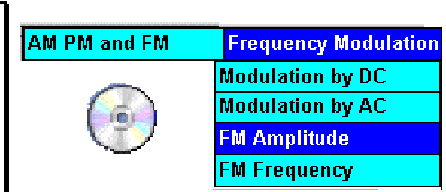
$$\Delta\omega(t) = \frac{d\theta(t)}{dt} = k_\omega A_m \cos\omega_m t$$

where k_ω is numerically equal to k_p but with units (radians/second/volt). We now define a frequency modulation index β_f for the specific case of modulation by a *single sinusoid*.

$$\beta_f = \Delta\theta_p = \frac{k_\omega A_m}{\omega_m} = \frac{\Delta\omega_p}{\omega_m} \quad \text{or} \quad \beta_f = \frac{\Delta f_p}{f_m} \quad . \quad \text{(single sinusoid only)}$$

Observe the **frequency modulation** demonstration in the "Virtual Experiments" CD. Here, a 25 kHz carrier is frequency modulated by a 5 kHz signal with variable amplitude. Note that for sinusoidal modulation, phase modulation and frequency modulation are equivalent and that the modulation indices are numerically equal.

a) Determine the modulation coefficient in kHz/volt.
b) Determine the peak frequency deviation and phase deviation when the modulation index $\beta = 2.4$.
c) Calculate and observe the Carson bandwidth when $\beta = 1.6$.



3.8.1 Carson's bandwidth rule

The bandwidth of an angle modulated signal depends both on β and on the highest frequency B of the information signal. For sinusoidal modulation, the spectrum can be exactly determined using the above Bessel functions. In Figure 3-35, a narrowband modulating signal is used to demonstrate the spectral spreading that occurs at higher harmonics. For useful modulating signals, the spectrum is very complex and we resort to a "rule-of-thumb" expression known as Carson's rule to evaluate the transmission bandwidth. It has been shown that 98% of the transmitted power is contained in the bandwidth B_T where

$$B_T = 2B(\beta + 1) \qquad \beta \le 2$$

and $$B_T = 2B(\beta + 2) \qquad \beta > 2 \quad .$$

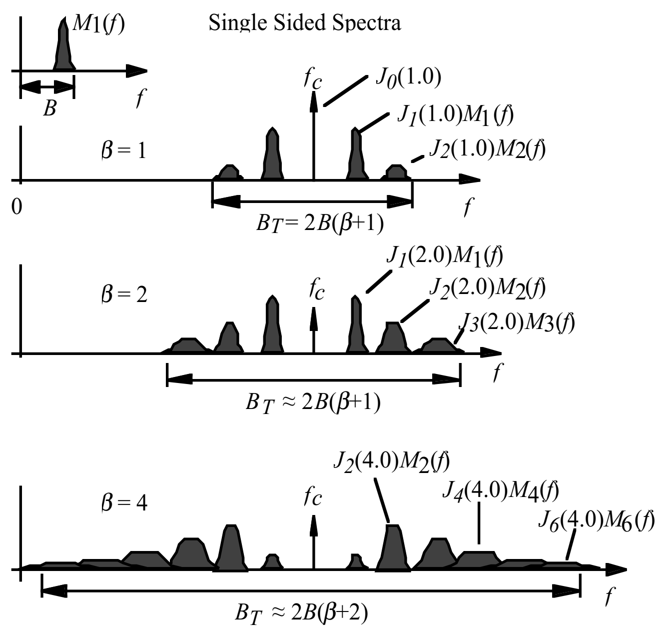


Figure 3-35 PM and FM Carson's Rule Bandwidth

To show the spectral effects of an increasing modulation index, we select a relatively narrowband information signal with highest frequency equal to B. At very low modulation index, the spectrum is very similar to AM where the information signal appears as 2 sidebands about the carrier. At modulation index $\beta = 1.0$, the spectrum has a carrier component with relative weight 0.77, a pair of first order sidebands each with relative weight 0.44 and a second order sideband pair (with two times the baseband bandwidth) each with relative weight 0.11. The Carson bandwidth contains more than 98% of the transmitted power and encompasses the second order sidebands. For modulation index β =4.0, the rule is changed to include the 5th and 6th order sidebands. The spectrum becomes more continuous as the modulation index is increased.

Example 3.12 – Frequency Modulation - In monaural FM broadcast, the carrier frequency is varied in the range $f_C \pm 75$ kHz (i.e. Δf = 75 kHz) and the audio program signal was limited to the frequency range 20Hz–15 kHz. What is the modulation index if the program signal is a 15 kHz sinusoid with amplitude such that Δf = 75 kHz?

Solution: $$\beta = \frac{\Delta f}{f_m} = \frac{75kHz}{15kHz} = 5$$

Note that this is an extreme case. Typical audio signals have most of their energy at frequencies near 1 kHz and thus the effective modulation index is much higher.

3.8.2 Noise in FM systems*

In PM and FM systems, the message signal $s(t)$ is transmitted by variations in the angle of the sinusoidal carrier wave. Transmission noise is considered to have zero average value and to be composed of a continuum of uniform amplitude frequency components with power spectral density denoted $N_0/2$ on a two-sided spectrum. The received signal includes the carrier frequency f_c, the information bearing sidebands and also the noise within the required receiver bandwidth B_T (which can be approximated by Carson's rule). When considered alone, the bandlimited noise, denoted $n(t)$, has a Rayleigh magnitude distribution and a random phase $\phi(t)$ uniformly distributed over 2π radians relative to the angle of $s(t)$. The bandlimited received signal $r_I(t)$ is the sum of the transmitted signal $s(t)$ and the narrowband noise $n(t)$ and it has a Rician magnitude distribution.

An angle modulation receiver is sensitive to received signal phase and is ideally insensitive to any variation in the amplitude. In this case, the detected noise has a Gaussian amplitude distribution. The noise output of the angle detector is essentially determined by the amplitude A_c of the "in-phase" unmodulated carrier and the quadrature component $n_q(t)$ of the narrowband noise $n(t)$. For a modulated carrier, we similarly define the phase noise $n_p(t)$ as the component of noise along the circumference in the phasor diagram of the complex envelope.

$$\theta_{n_p}(t) = \sin^{-1}\left(\frac{n_p(t)}{A_c}\right) \approx \left(\frac{n_p(t)}{A_c}\right) \quad \text{for } n(t) << s(t)\,.$$

We now illustrate amplitude spectral density for sinusoidal angle modulation (at 15 kHz) and develop the demodulated SNR for phase modulation and for frequency modulation.

Drill Problem 3.6 – Incomplete

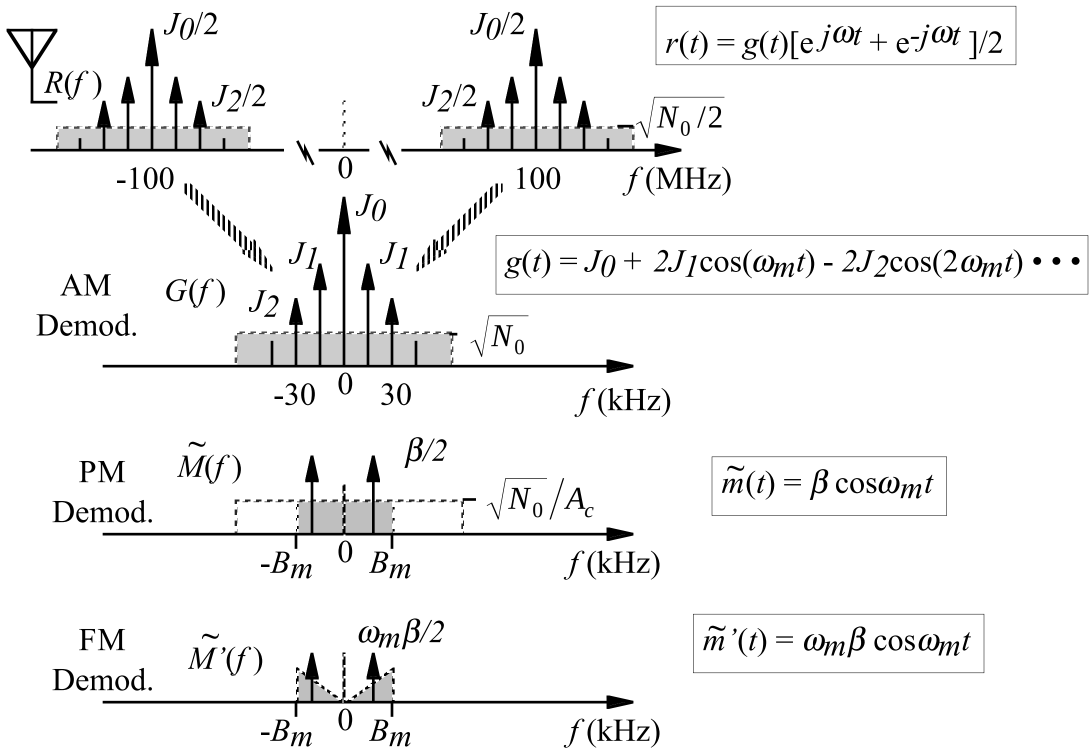


Figure 3-36 Sinusoidal modulation with and noise in angle modulated transmission

Power spectral density of the noise in the received RF signal is $N_0/2$ and, upon coherent demodulation with $2\cos\omega_c t$, noise from the upper sideband adds to noise from the lower sideband (on a power basis) and the resulting noise power spectral density is N_0 (watts/Hz). To study demodulation, it is convenient to illustrate amplitude spectral density and thus the noise density in the RF signal is shown as $\sqrt{N_0/2}\ (\mathrm{W}/\sqrt{\mathrm{Hz}})$. For coherent demodulation with $2\cos\omega_c t$, signal components add on a voltage basis and we recover, for analytical purposes only, the complex envelope $g(t)$.

With phase demodulation, we recover the modulation signal $\beta\cos\omega_m t$ together with noise having spectral density $\sqrt{N_0}/A_c$. Calculation of the demodulated SNR in phase modulation is simply the signal power divided by the noise power and this can be compared with the calculation of SNR_O with $\langle m^2(t)\rangle = 0.5$ as in Section 3.7.4.

$$SNR = \frac{P_S}{P_N} = \frac{\beta^2(1/2)}{2B_m N_0 / A_c{}^2} \quad \text{for phase demodulation.}$$

With frequency demodulation, we recover the modulation signal $\omega_m \beta \cos \omega_m t$ together with noise having spectral density $\omega_m \sqrt{N_0} / A_c$. Calculation of demodulated SNR requires integration of the noise power density over the output bandwidth.

$$SNR = \frac{P_S}{P_N} = \frac{\omega_m{}^2 \beta^2 (1/2)}{\int_{-B_m}^{B_m} \left(N_0 / A_c{}^2\right)(\omega)^2 df} \quad \text{for frequency demodulation.}$$

$$SNR = \frac{\omega_m{}^2 \beta^2 (1/2)}{(2\pi)^2 \left(N_0 / A_c{}^2\right)\left[f^3/3\right]_{-B_m}^{B_m}} = \frac{3 f_m{}^2 \beta^2 (1/2)}{2\left(N_0 / A_c{}^2\right) B_m{}^3}$$

Drill Problem 3.7 - Frequency Modulation – PLACEHOLDER signal *c(t)*=100Vcos2π20000*t* and the following modulation signals, determine the sinusoidal component amplitudes (in volts) and component frequenci

Xxxx Signal	A1	F1	A2	F2	A3	F3	A4	F4
2V cos2π4000t								
4V cos2π11000t								
2V + 4V cos2π23000t	200	3	200	20	200	43		
cos2π4000t + cos2π8000t								
Checksum	550	40	550	91	250	67	50	28

3.8.3 Broadcast FM (is Primarily Phase Modulation)*

Commercial FM broadcast systems use pre-emphasis and de-emphasis circuits to effectively transmit PM for frequencies greater than 2.1 kHz and to transmit FM for low frequencies. This combines the low cost detection afforded by FM plus the high performance of PM for the frequency range 2.1 – 15.0 kHz.

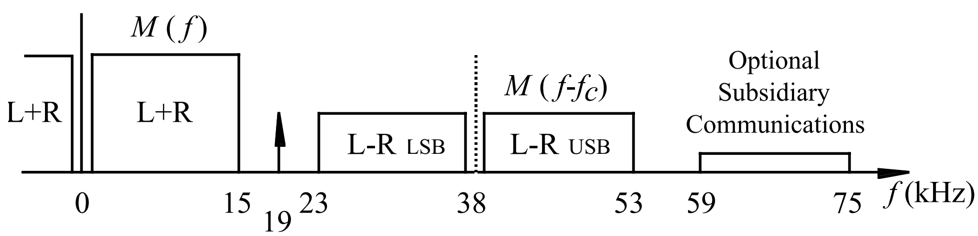


Figure 3-37 Sinusoidal modulation with and noise in angle modulated TX - PLACEHOLDER

Most treatments of FM consider the most optimistic calculation of SNR which assumes sinusoidal modulation at the highest possible frequency (giving the largest signal output after differentiation). In this specific case, FM is 3 times (4.8 dB) better than PM.

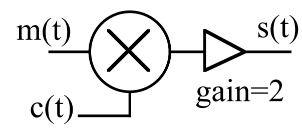
$$SNR = \frac{3 f_m{}^2 \beta^2 (1/2)}{2\left(N_0 / A_c{}^2\right) B_m{}^3} = \frac{3\beta^2 (1/2)}{2 B_m N_0 / A_c{}^2} \quad \text{for sinusoidal modulation and } f_m = B_m$$

To the contrary, practical signals such as music, voice and television, have large signal energies at low frequencies and this favors phase modulation.

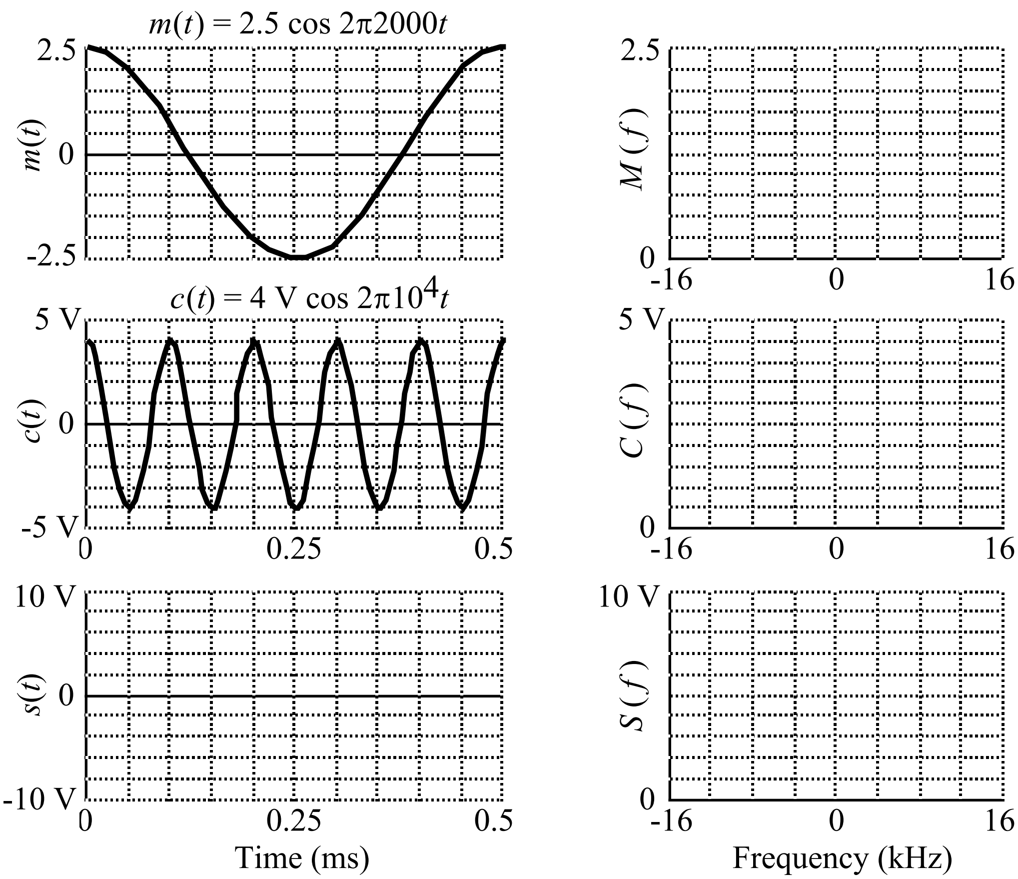
Problems:

*3.1 Two signals $m_1(t)$ and $m_2(t)$ are multiplied together to give the output $y(t)$. The signals are: $m_1(t)$ = 2V + 5V Cos $2\pi2000$t and $m_2(t)$ = 20V Cos $2\pi6000$t. Determine the output voltages and frequencies. Use a voltage vs frequency graph to illustrate the output spectrum.

*3.2 The modulation system shown includes a multiplier and an amplifier with voltage gain of two. The biased modulation signal is m(t) = 1.0 + 0.5 cos $2\pi1000t$ + 0.3 cos $2\pi2000t$ and the carrier c(t) = 10 cos $2\pi7000t$. Write an expression for the output product s(t) and provide a calibrated sketch of the two-sided spectrum.

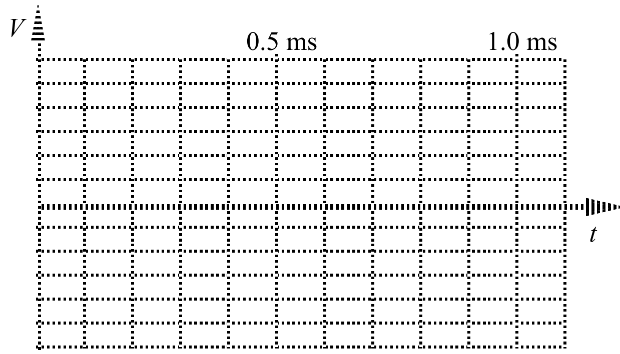


*3.3 An analog multiplier forms the product $s(t) = m(t) \bullet c(t)$. a) Accurately sketch the output signal and b) Sketch the double sided spectral components (magnitude and frequency) of the three periodic signals.

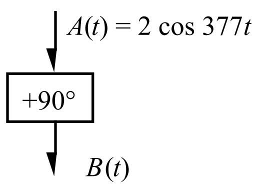


*3.3a An AM radio braodcast station has unmodulated transmitter power of 50,000 watts and uses an antenna with 10 ohm effecive resistance. Determine the transmitter carrier amplitude (in volts).

*3.4 Two signals are multiplied to form an AM-DSB-TC output waveform. The carrier signal is 12 V cos$2\pi10^4$t and the modulation signal is 1+ 0.75 cos $2\pi10^3$t. a) Sketch the first 1 ms of the output waveform making certain to calibrate the voltage and time axes. b) Calculate (normalized) peak envelope power and the (sideband) power efficiency.



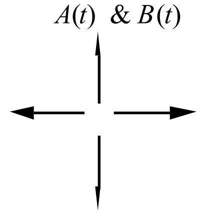
*3.5 Quadrature sinusoidal carriers differ by 90° and are used in several modern modulation methods. Complete the following three-part question.



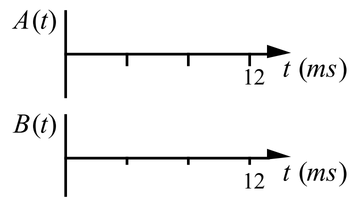
Select one
$B(t) =$

$-2 \cos 377t$
$2 \sin 377t$
$-2 \sin 377t$
$754 \sin 377t$
$-754 \sin 377t$

Designate phasors
$A(t)$ & $B(t)$



Sketch first 12 ms



*3.6 Sketch the first 1ms of AM-DSB output waveform with 150% modulation index. The modulation signal is $\cos 2\pi 10^3 t$ and carrier signal is $10V \cos 2\pi 10^4 t$. In other words, this question assumes an ideal multiplier with modulation input as $1 + 1.5 \cos 2\pi 10^3 t$. Calibrate the voltage and time axes in your solution.

*3.7 An amplitude modulated DSB-TC transmitter develops an unmodulated power output of 40 kW to a 50 ohm load. A sinusoidal test tone is applied to the modulator input and the power output increases to 50 kW. Determine:

a) the average power in each sideband
b) the modulation index

*3.8 An AM modulator operates with message signal $m(t) = 4\cos(1000\pi t)$ and carrier $c(t) = 10V\cos(20{,}000\pi t)$. A dc value is added to the message such that $x(t) = A_{dc} + m(t)$.

a) Find the modulation index, μ, when $A_{dc} = 5$.
b) Determine the power efficiency (sideband power vs. total power)
c) Describe the preferred demodulator to recover $m(t)$.
d) Describe the preferred demodulator when $A_{dc} = 2$.

*3.9 An AM broadcast transmitter with 20 kW carrier power is broadcasting a sinusoid with 60% modulation index. Calculate a) the power in the sidebands and b) the power efficiency of the transmission.

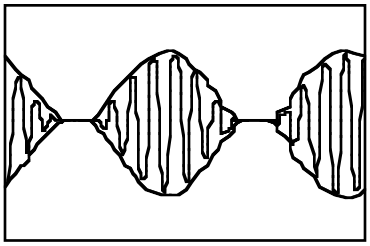
*3.10 A compound signal with peak/rms ratio of 4.0 has a peak amplitude which switches from 0.1 to 1.0 volts with equal probability. The compound signal has frequency range 300 - 3300 Hz. If this signal amplitude modulates a 600 kHz carrier with modulation index 1.0, what is the sideband power efficiency at the modulator output ?

*3.11 A double sideband transmitted carrier AM signal is generated by the product of the signals $m_I(t)$ and $c_I(t)$.

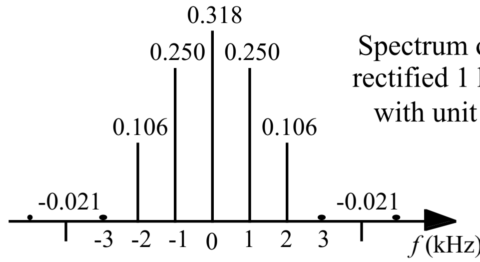
Determine the modulation index, the normalized power in the transmitted carrier and the normalized power in each sideband.

$m_I(t) = 1.0 + 0.5 \cos 2\pi\ 10^3\ t$ $\qquad$ $c_I(t) = 100 \cos 2\pi\ 10^6\ t$

*3.12 AM transmitters must not exceed 100% modulation otherwise the clipping effect illustrated below may result. When negative modulation voltage is applied, the modulator effectively multiplies by zero. The distorted output waveform has harmonic components that increase the bandwidth of the transmitted signal and cause interference in adjacent radio frequency bands. Illustrate the transmitted waveform and spectrum for an extreme case where the transmitted signal resembles that of modulation of a 10 kHz carrier (with amplitude 20 volts) by a half-wave rectified unit amplitude 1 kHz sinusoid. The spectrum of the half-wave sinusoid is reproduced for convenience.

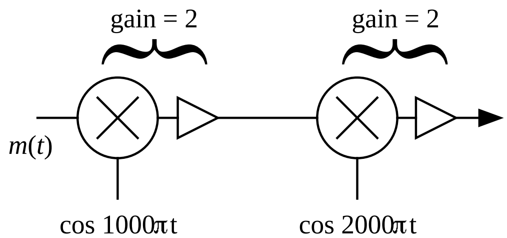


Signal Clipping with Slight Overmodulation



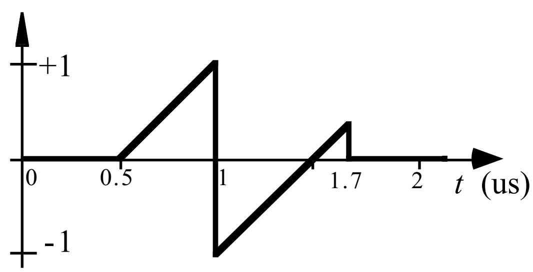
Spectrum of half-wave rectified 1 kHz sinusoid with unit amplitude.

*3.13 The DSB-SC modulation system shown below has an overall gain of 2 in each stage. The input signal $m(t)$ has power spectral density of 10 mW/Hz in the band 30 Hz to 150 Hz and zero elsewhere. Sketch the spectral density of the output using a 2-sided frequency axis.

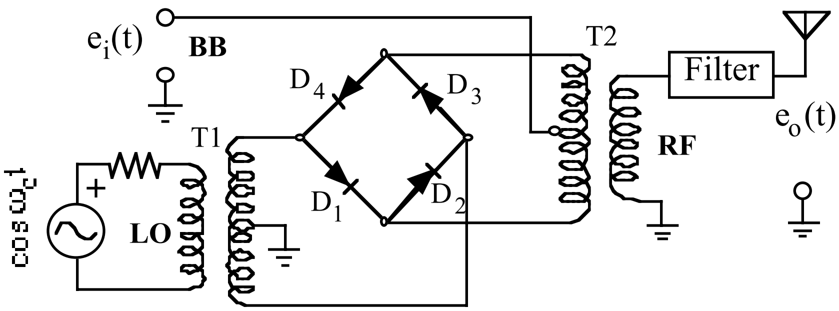


*3.14 The modulated waveform shown in Example 3.3 is demodulated using a) a synchronous detector and b) a diode detector. Illustrate the resulting spectrum in each case using a 2-sided frequency axis extending up to 200 kHz. Include amplitudes and frequency on your illustration and indicate any assumptions.

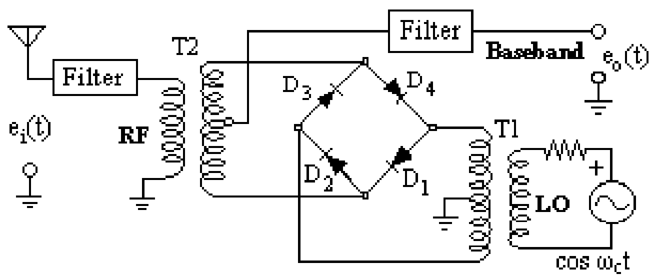
*3.15 The following ramp waveform $m(t)$ is to be transmitted over a bandpass channel. Sketch the resulting time waveform if $m(t)$ modulates a 10 MHz carrier using (a) DSB-TC Amplitude Modulation and (b) double sideband suppressed carrier (AM-DSB-SC) modulation



*3.16 A ring modulator is a cost-efficient circuit that provides, under specific conditions, the functtionality of a multiplier. In the circuit shown, the local oscillator, while at positive potential, causes current to flow in diodes D1 and D2. This effecively grounds the lower terminal on the center-tapped primary of transformer T2. When the local oscillator (LO) voltage is negative, diodes D3 and D4 conduct and the upper terminal of the primary winding is effectively grounded. If the transformer has turns ratio 1:1:1, this terminal switching then multiplies the baseband signal (BB) by +1 and –1 at the the rate of the local oscillator. The filter shown is a bandpass filter centered about the local oscillator frequency. Give an expression for the output RF frequency spectrum just prior to the filter.



*3.17 The ring demodulator shown uses two 1:1:1 center tapped transformers and it demodulates a DSB-SC signal with f_c = 30 kHz and spectral density ±5 kHz about the carrier frequency. The radio frequency (RF) bandpass filter has unity gain in the range $f_c \pm f_c/4$. When the local oscillator voltage is positive, the lower primary terminal of T2 is effectively grounded through the diodes. When the local oscillator voltage is negative, the upper primary terminal is grounded. The baseband lowpass filter has unity gain in the range 0 to $f_c/4$. Illustrate the signal spectra after the RF filter and before the baseband filter.



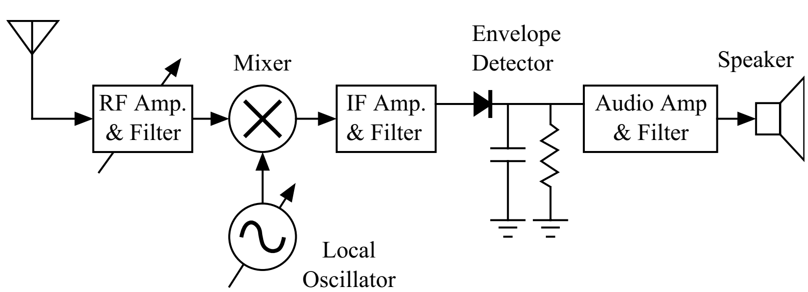
*3.18 A 400 kHz sinusoid is input to a ring modulator that multiplies by ±1 with frequency 1 MHz and "square" waveform. Recall the Fourier series of the "square" multiplying waveform and illustrate the output 2 sided discrete spectrum up to 6 MHz.

*3.19 A commercial AM radio transmitter with carrier frequency 540 kHz is being received by a superheterodyne receiver with 455 kHz intermediate frequency. What is the local oscillator (LO) frequency, what is the image frequency and what is the significance of the image frequency ?

*3.20 Assume a superheterodyne receiver with 455 kHz intermediate frequency is tuned to a 600 kHz AM station.

a) what is the meaning of the word "superheterodyne"?
b) what is the required LO frequency and what is the "image frequency" for the above case?
c) with the aid of diagrams, show that the sidebands are inverted at the IF frequency.

*3.21 The superheterodyne receiver illustrated below has intermediate frequency 156 kHz and is intended to receive the AM broadcast signals in the frequency range 535 - 1705 kHz.



(a) What is the required range of local oscillator frequency for this superheterodyne receiver?
(b) What is the image frequency when the desired signal frequency is 600 kHz?
(c) Sketch the IF filter frequency response required for “flat” baseband output in the range 0-10 kHz.
(d) Why might a **designer** select 156 kHz for IF rather than today’s 455 kHz? Would 156 kHz cost more?

*3.22 An AM superheterodyne receiver is tuned to a strong local station with frequency 1560 kHz. The receiver dial is set to 1560 kHz, the local oscillator (LO) frequency is 2015 kHz and the intermediate frequency (IF) is 455 kHz. Assuming that the receiver's coarse pre-selector filter (prior to the mixer) is not functioning, the same 1560 kHz local station could be received at a different dial setting. (This could be described as sub-heterodyne mode). Calculate the following:

Dial setting ______ kHz LO frequency _______ kHz IF frequency ______ kHz

*3.23 A heterodyne receiver is tuned to a station at 3.7 MHz. The local oscillator frequency is 7 MHz and the IF is 10.7 MHz.

(a) What is the image frequency?
(b) make a sketch showing the spectral translation.
(c) If the LO has appreciable second-harmonic content, what two additional frequencies are received?

*3.24 A 4.02 GHz satellite television signal enters an Earth station receiver with IF frequency 70 MHz. What is the LO frequency and the image frequency for the cases of:

(a) High-side injection (superheterodyne)?
(b) Low-side injection ("subheterodyne")?

*3.25 A heterodyne receiver is tuned to a station at 20 MHz. The local oscillator frequency is 80 MHz and the IF is 100 MHz.

(a) What is the image frequency?
(b) At the IF frequency, are the sidebands inverted? Yes or No
(c) If the LO has appreciable second-harmonic content, what two additional frequencies are received?

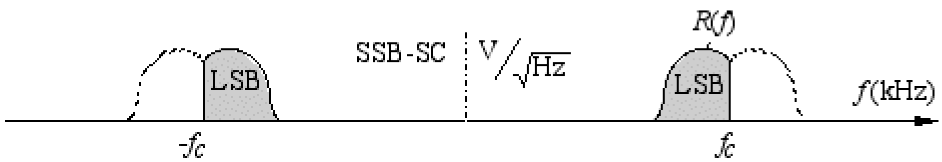
*3.26 A NTSC baseband television signal uses a FM modulated sound sub-carrier at 4.5 MHz, which, when demodulated, provides an "audio" transmission bandwidth of 110 kHz. Multichannel television sound (MTS) transmits L+R, L-R and SAP (second audio program) signals in a manner similar stereo sound in FM broadcast radio. For pilot tone and sub-carrier frequencies, the MTS system uses multiples of the video horizontal scan frequency f_h = 15.734 kHz. System designers chose to center the SAP channel at 5 f_h (although it might have been possible to center the secondary channel at 4 f_h). Sketch the magnitude spectrum of the demodulated FM signal showing the spectral bands used by the L+R, L-R and SAP signals. Also illustrate the pilot tone and indicate what audio bandwidth can be supported for MTS stereo transmission.

*3.27 Assume that two audio signals with constant amplitude spectral density from 20 Hz to 10 kHz are to be multiplexed into one composite signal using a method similar to FM and TV stereo. One of the signals remains at baseband while the other signal is sent with DSB-SC modulation using a sub-carrier at 28 kHz. Note that a pilot tone (at one half the sub-carrier frequency) is transmitted and used to synthesize a receiver sub-carrier for DSB-SC demodulation.

(a) Why is the pilot tone NOT sent at the DSB-SC sub-carrier frequency ?
(b) Assuming that the pilot frequency is 14 kHz, sketch the one sided composite signal spectrum and indicate all important frequencies.
(c) What would be the disadvantage of using a pilot frequency of 11 kHz (i.e. 22 kHz DSB carrier) ?
(d) What is the result of demodulation with 60 degree receiver sub-carrier phase error.
(e) What is the result of demodulation with 90 degree receiver sub-carrier phase error.

*3.28 A double sideband suppressed carrier (AM-DSB-SC) signal is represented by the sum of the components 10 cos 2π 37000 t and 10 cos 2π 39000 t. The signal is demodulated by the locally generated carrier 2 cos (2π 38000 t + $\pi/6$). Note that the receiver has established the correct frequency but there is a phase error. Write an expression for the demodulated baseband signal.

*3.29 The following illustration shows LSB spectra in SSB-SC transmission. Illustrate the receiver baseband spectrum when the receiver LO frequency is slightly less than the transmitter carrier.

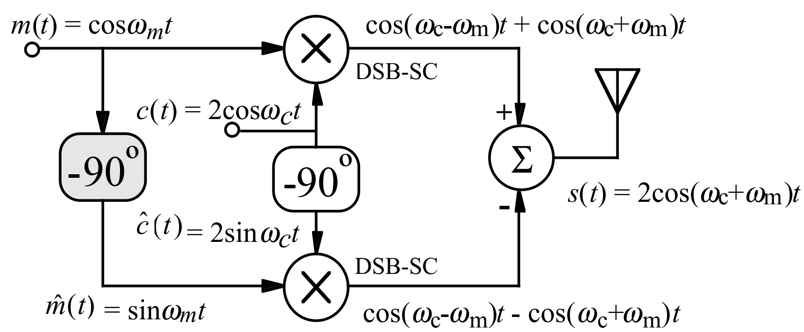


*3.30 Assume a Hilbert transform/phase shift modulator in a SSB-SC transmitter modulated with the sinusoid $m(t) = 5 \cos \omega_m t$. The carrier is $A_c \cos \omega_c t$ (where $A_c = 1$).

(a) Evaluate $m_h(t)$ - the Hilbert transform of m(t).
(b) Find the expression for a lower SSB signal.
(c) Find the rms value of the SSB signal.
(d) Find the peak value of the SSB signal.
(e) Find the normalized average power of the SSB signal.
(f) Find the normalized PEP of the SSB signal.

*3.31 Modern communication receivers use complex signal processing with "analytic" signals. An analytic signal designated $m+(t)$ has only positive frequency components and can be expressed as $m^+(t) = m(t) + j\hat{m}(t)$. When implementing analytic signal processing, two wires (or two sample streams) are required for each signal. This is illustrated below in the "phase shift" method of generating single sideband.

(a) Identify the analytic signals.
(b) Discuss how the –90 degree phase shift can be implemented for a sinusoid and for a realistic (wideband) signal such as voice.
(c) Expand the block diagram below so that it provides an analytic signal at the output.



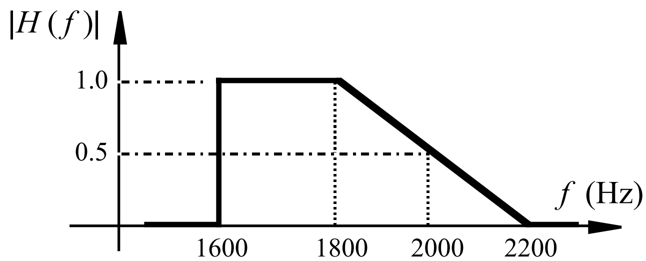
*3.32 A single sideband transmitter transmits the carrier signal 5 cos 2π27000 *t* and the modulation signals 3 cos 2π27001 *t* and 1 cos 2π27003 *t*. Note that a carrier *is transmitted*. Sketch a phasor diagram of the transmitted signal when t = 0.1 ms.

*3.33 A large added carrier (20cos 2π27000 *t*) and a diode detector are used to demodulate a SSB signal with single tone sinusoidal modulation (3 cos 2π27001 *t*). Describe the change in the demodulated signal if the added carrier is shifted by –90 degrees (20sin 2π27000 *t*).

*3.34 A single sideband transmitter with carrier frequency 27.000 MHz is used to transmit the modulation signal 6 cos 2π1200 t + 2 cos 2π3600 t. Lower sideband transmission is used (SSB-L) and the carrier is not transmitted.

i) Sketch the spectrum of the transmitted signal.
ii) Synchronous demodulation can be used to recover the baseband signal. If there is some frequency offset in the receiver carrier, there will be distortion in the recovered baseband signal. Sketch the baseband output spectrum if the receiver carrier is 27.001 MHz
iii) Discuss the use of an envelope detector in a SSB receiver when a LO signal is added to the received signal

*3.35 In a vestigial sideband (VSB) transmitter, the carrier signal $c(t) = 2 \cos(2\pi 2000t)$ is first amplitude modulated by the message signal $m(t) = 2 \cos(2\pi 100t) + 3 \cos(2\pi 300t)$. The vestigial sideband filter illustrated below is applied before transmission. Write an expression for the transmitted VSB signal $s(t)$.



*3.36 A television receiver uses the superheterodyne principle with video intermediate frequency (IF) at 45.75 MHz. If the receiver is tuned to Channel 2 (with RF carrierr at 55.25 MHz), what is the local oscillator (LO) frequency and what is the image frequency?.

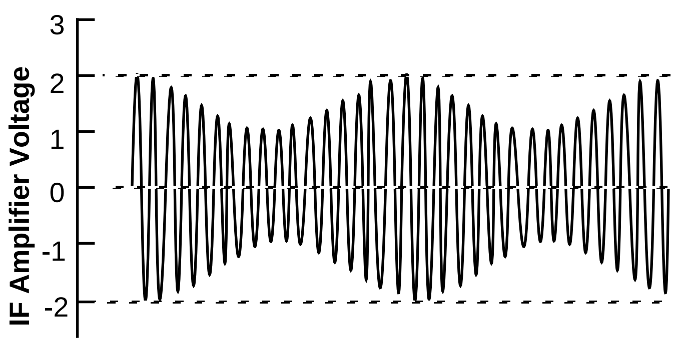
*3.37 A television receiver is tuned to channel 6 which has video carrier at 83.25 MHz and audio carrier at 87.75 MHz. The TV uses a superheterodyne receiver and produces video IF carrier at 45.75 MHz and sound IF carrier at 41.25 MHz.

i) What is the local oscillator frequency when tuned to channel 6 ?
ii) What are the video and sound image frequencies ?
iii) Repeat i) for a *sub*-heterodyne receiver with the same IF frequencies. Do you see any problems ?

*3.38 Each commercial broadcast format has specific advantages.

(a) what is the advantage of DSB-TC used in the AM radio standard ?
(b) what advantages does AM have over FM ?
(c) why was VSB transmission chosen for television ?
(d) is a carrier transmitted with the television VSB signal ? Explain why.

*3.39 The load resistance in the envelope detector is 10 kilohm, select capacitor(s) and diode(s) appropriate for the 51 kHz IF amplifier output signal shown below.



Capacitor

10 pF, 30 pF, 100 pF
300 pF, 1 nF, 3 nF, 10 nF

Diode (& cutin voltage)

Ge, Schottky, Si, GaAs
0.3, 0.4, 0.7, 1.4

*3.40 A Quadrature Amplitude Modulation (QAM) signal is formed from the product of quadrature modulation, $m_y(t)$, and quadrature carrier, $c_y(t)$, plus the AM signal generated by the in-phase product $m_x(t) \bullet c_x(t)$.

a) Determine the normalized power in the carrier and all sidebands.

$m_y(t) = 0.25 \cos 4\pi\, 10^3\, t$ $\qquad m_x(t) = 1.0 + 0.5 \cos 2\pi\, 10^3\, t$

$c_y(t) = 100 \sin 2\pi\, 10^6\, t$ $\qquad c_x(t) = 100 \cos 2\pi\, 10^6\, t$

b) Using the relation below for complex envelope representation, determine an expression for $g(t)$. The complex envelope $g(t)$ is time varying phasor. Sketch it's locus and identify some points in time.

$$v(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = x(t)\cos\omega_c t - y(t)\sin\omega_c t \qquad \text{where: } g(t) = x(t) + jy(t)$$

*3.41 Stereo AM broadcasts two signals in the same bandwidth as the traditional monaural AM broadcast system. Circle answers.

a) What modulation scheme is used ? DSB, SSB, QAM, VSB, or FM

b) What is the advantage of sending L+R and L-R information instead of L and R?

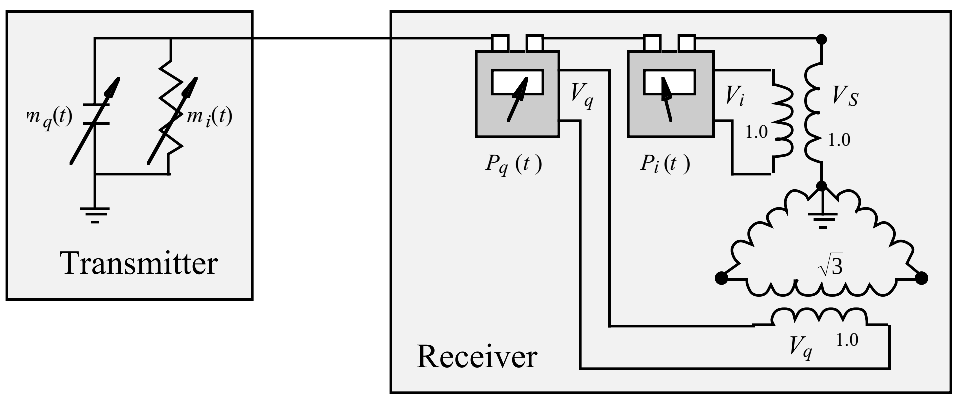
c) A diode detector can be used for which channel(s)? L+R, L-R, or Both

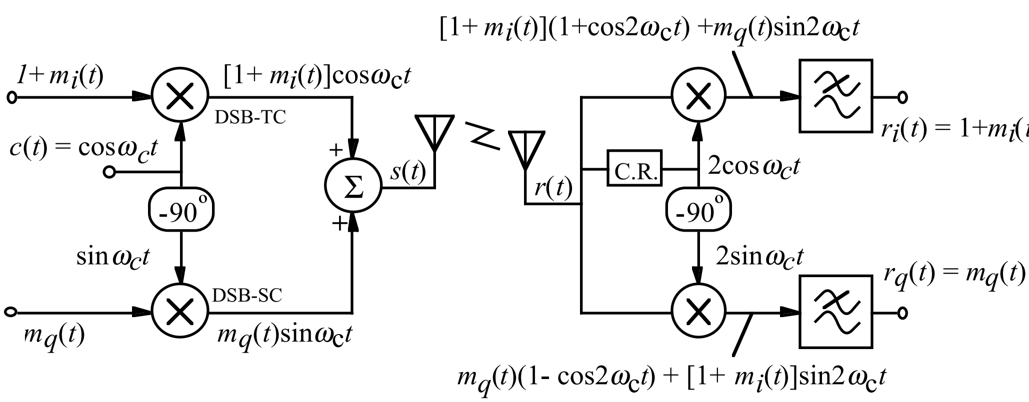
d) To be compatible with traditional AM, amplitude scaling is used in which channel ?
L+R, L-R, or Both

*3.42 Consider the illustration below where a resistor-capacitor load is supplied from phase A of a 3-phase voltage source. The two wattmeters are configured to measure in-phase power (real power) and quadrature power (VARs). The reference voltage V_i of the in-phase power meter has the same voltage and phase as the phase A voltage while the out-of-phase (quadrature) reactive power meter is supplied with a voltage that has the same magnitude as phase A but with 90 degree shift.

Consider the wattmeter circuit to be a QAM communication system where the variable resistor modulates in-phase current and the variable capacitance modulates quadrature current. The two signal currents are read independently on wattmeters in the receiver. Relate each element of the “standard” QAM block diagram below to an element or point in the “wattmeter communication system”. For example:

a) how does the multiplication and summation take place in the “transmitter”?
b) where are the multipliers in the “receiver” and where is the 90 degree phase shift?
c) where are the “mechanical” lowpass filters in the receiver to remove the double frequency output?





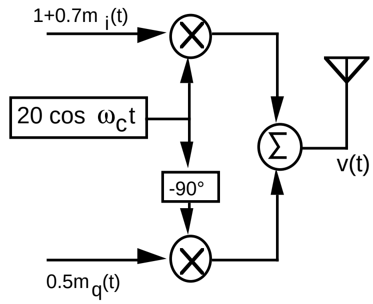
*3.43 A QAM signal is generated by the modulator illustrated below. The carrier frequency is 600 kHz and the sinusoidal input signals are: $m_x(t) = \cos 2\pi 125\, t$ and $m_y(t) = \sin 2\pi 125\, t$

a) Label the correct points on the diagram with DSB-SC and DSB-TC.

b) For the output QAM signal $v(t)$, determine $x(t)$, $y(t)$ and $g(t)$. Note the complex envelope relations.

$$v(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = x(t)\cos\omega_c t - y(t)\sin\omega_c t \qquad \text{where: } g(t) = x(t) + jy(t)$$

c) Sketch the complex envelope "phasor" on real and imaginary axes and a locus to show its variation with time. Identify points at t = 0, t = 1ms, t = 2ms, etc.



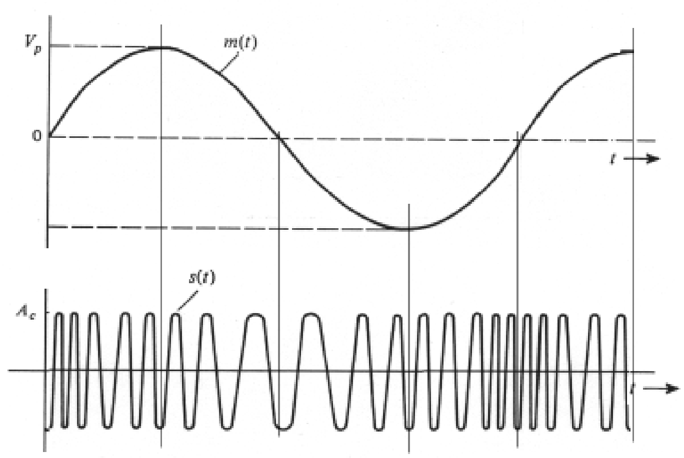
*3.44 A carrier is angle modulated by $m(t)$ and the result, $s(t)$, is described by $s(t)$ =20V cos ($\omega_c\, t + \theta\,(t)$). If the output is $s(t)$ =20V cos ($80000\pi\, t$ + 0.4(sin $4000\pi\, t$)), find the following:

a) normalized power of the modulated signal.	P_N	=	(W)
b) peak-to-peak phase deviation.	$\theta_{p\text{-}p}$	=	(rad)
c) maximum frequency deviation.	$f_{\Delta\ max}$	=	(Hz)
d) The Carson bandwidth of the modulated waveform.	f_{BW}	=	(Hz)

*3.45 A carrier is angle modulated by $m(t)$ and the result is described by $s(t)$ =20 cos ($\omega_c\, t + \theta\,(t)$). If the output is $s(t)$ =20 cos ($80000\pi\, t$ + 0.4 sin ($4000\pi\, t$)), find the following:

a) phase modulation coefficient if $m(t)$ = 2V sin $4000\pi\, t$.	k_p	=	(rad/volt)
b) frequency modulation coefficient if $m(t)$ = 2V cos $4000\pi\, t$.	k_f	=	(Hz/volt)

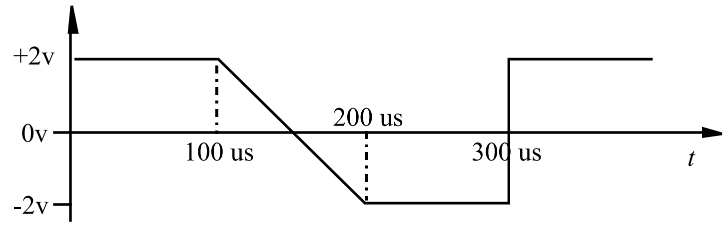
*3.46 In the angle modulated waveform illustrated below, the modulating frequency is 2 kHz and the carrier frequency is 32 kHz. The peak modulating voltage is V_p = 7 V and the peak carrier voltage is 100 V. Estimate the gain coefficient in the modulator, the peak deviation and the modulation index.



*3.47 A 3 Vp-p 1 kHz sinusoidal signal is transmitted using phase modulation. The maximum phase deviation is ±0.75 radians and the carrier phase (in radians) is expressed as $\theta(t) = 0.75 \sin 2\pi 1000t$.

a) Determine the modulation index β_p. (unitless)
b) Determine the sensitivity coefficient k_p of the phase modulator. (radians/volt)
c) Determine the maximum rate of change in carrier phase during the 1 kHz cycle. (radians/second)

*3.48 The following ramp signal is to be transmitted over a bandpass channel with center frequency 50 kHz. Sketch the resulting time waveform if (a) Phase Modulation (PM) is used with k_p = 3.14 rad/volt and (b) if Frequecy Modulation (FM) is used with k_f = 10 kHz/volt.



*3.49 An AM/FM/PM transmitter has an unmodulated carrier power equal to 50 watts (normalized).

a) Individually determine the power in the carrier and AM sidebands when the carrier is AM-DSB-TC modulated with a 1 kHz sinusoid with modulation index $\mu = 0.20$. (watts)
b) Determine the total transmitted signal power when the carrier is phase modulated (PM) with a 1 kHz sinusoid with modulation index $\beta = 0.20$. (Note that in phase modulation, the transmitted signal amplitude is constant)
c) Individually determine the power in the carrier and in the first order (i.e. fundamental) PM sidebands. (It will be useful to recall Parseval's theorem)

*3.50 The Advanced Mobile Phone System (AMPS) is a first generation cellular telephony system. It uses FM transmission with 30 kHz frequency allocation per channel. Which of the following attributes are false:

a) FM has fixed power consumption that does not depend on the message being sent.
b) FM suppresses interfering signals at the same frequency from users in other cells.
c) This FM system uses larger bandwidth than a comparable AM system.
d) FM signals are not degraded by nonlinear gain in the transmitter or receiver.
e) This system can communicate modulation signals up to 15 kHz.
f) Because there is transmitted carrier, the receiver can use an envelope or diode detector.

*3.51 Why is broadcast FM radio more like phase modulation than frequency modulation?

*3.52 Why can't we use the same modulator to generate WBFM as we use to generate NBFM?

*3.53 An angle-modulated signal with carrier frequency $f_c = 100$ kHz is described by the equation:

$$s(t) = 20\cos\left(2\pi f_c t + 3\cos 1000\pi t + 4\cos 2718\pi t\right)$$

a) What are the modulation frequencies (in Hz)?
b) Find the power of the modulated signal.
c) Find the phase deviation in radians.
d) Find the frequency deviation in hertz.
e) Estimate the bandwidth of the signal using Carson's Rule.

*3.54 A cellular telephone handset receives a carrier signal from the local basestation at 840 MHz. It also receives an *unmodulated* carrier from a distant basestation at the same frequency and 20 dB lower amplitude. Assume that the handset is moving at highway speed toward the local basestation (Dopper shift is +80 Hz) and away from the remote basestation (Doppler shift is –80 Hz). The received baseband signal at the handset is thus corrupted by 160 Hz interference caused by the remote basestation signal.

a) Calculate the amplitude ratio of local carrier to remote carrier (not in dB).

b) Assuming 1 kHz sinusoidal amplitude modulation of the local carrier with modulation index $\mu = 0.5$, calculate the demodulated interference SNR in dB.

c) Assuming 1 kHz sinusoidal phase modulation of the local carrier with modulation index $\beta = 2.0$, calculate the demodulated interference SNR in dB.

d) Assuming 1 kHz sinusoidal frequency modulation of the local carrier with modulation index $\beta = 2.0$, calculate the demodulated interference SNR in dB.

*3.60 **Virtual Laboratory -** Analysis of PM and FM systems is often done with sinusoidal modulation. Access the web page http://www.engr.usask.ca/classes/EE/352/expindex/am_fm_pm/amplitude.htm, and observe the spectrum when β = 2.0. Justify the component amplitudes with the Bessel function graph shown on page 52 of the notes. How the observed spectra relate to the following expression?

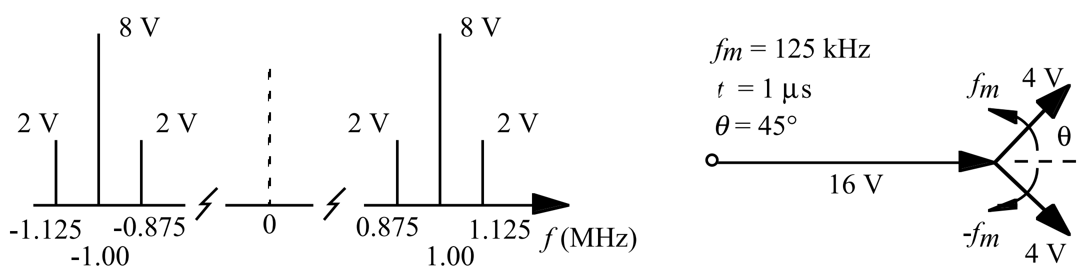
$$e^{j\theta(t)} = J_0(\beta) + j2J_1(\beta)\cos\omega_m t - 2J_2(\beta)\cos 2\omega_m t - j2J_3(\beta)\cos 3\omega_m t + \bullet\bullet\bullet$$

*3.61 **Design Problem -** It is required to check the emission spectrum of a local AM radio station. Available to you is the HP3580 spectrum analyzer, however, its maximum measurement frequency is only 50 kHz. Precise frequency signal generators are available. Design a modulation system that will frequency translate the radio signal so it can be observed using the spectrum analyzer. Standards relating to AM broadcast emission spectra are in Part 73.44 of the USA Code of Federal Regulations on telecommunications at http://www.access.gpo.gov/cgi-bin/cfrassemble.cgi?title=200147. The regulation shows limits on transmitted signals more than 10 kHz above and below the carrier frequency.

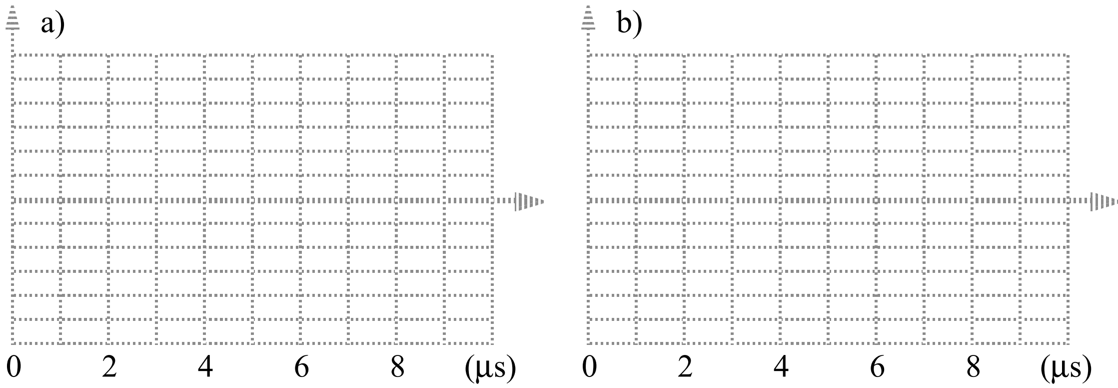
*3.62 A **design team** intends to increase their company's AM portable radio market share by introducing a low cost on/off gating circuit to replace the mixer/multiplier normally used to translate the radio frequency (RF) signal to 455 kHz intermediate frequency (IF). For example, in a radio receiver tuned to 650 kHz, the local oscillator (LO) would operate at 1105 kHz and it could drive an on/off gate circuit to effectively multiply the RF signal by 0 and 1. We assume 50% on/off duty cycle so the multiplying waveform has dc and 3^{rd} harmonic components in addition to the fundamental (i.e. 0 Hz, 1105 kHz, and 3315 kHz).

a) Since the on/off gating circuit effectively multiplies by more frequency components that does a mixer/multiplier, there are more image frequencies to consider. Assuming only the 0 Hz, 1105 kHz, and 3315 kHz components listed above, determine all the image frequencies that must be eliminated by the RF filter when the radio is tuned to receive 650 kHz.
b) The proposed cost reduction might be worthless if the RF filter is required to be more frequency selective. In this regard, show that the dc component in the multiplication process becomes especially problematic when the radio is tuned to 540 kHz. Suggest a modified gating circuit that can eliminate the dc component in the effective multiplying waveform

*3.70 A 1.0 MHz cosine carrier and two sidebands (USB and LSB) form an AM-DSB-TC signal. The two-sided spectrum is shown below along with a phasor diagram indicating the complex envelope at time t = 1 us.



a) Accurately sketch the voltage vs time waveform on the first grid below. Indicate voltage.
b) Illustrate the voltage waveform when the carrier is removed and both sidebands remain.
c) Illustrate the voltage waveform when only the LSB is removed.
d) Illustrate the waveform with all components present and the carrier shifted by +90°.



*3.xx The signal $m(t) = \text{sinc}(10t) = \sin(10\pi t)/10\pi t$ is to be multiplied by $c(t) = A_c\cos(\omega_c t)$ and transmitted (this is AM-DSB). Circle the properties of the transmitted signal, $m(t)c(t)$.

a) energy signal or a power signal

b) stochastic (random) or deterministic signal

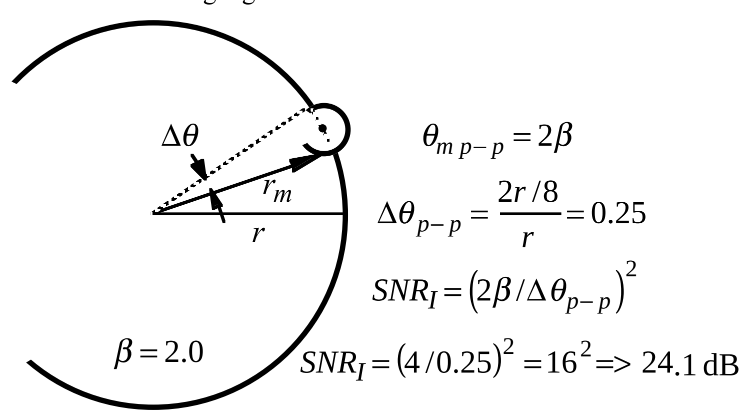
c) bandwidth when transmitted 5Hz, 10Hz, 16Hz, 20Hz, 31.4Hz or 62.8Hz.

3a. **AM-SSB** – (4 pts) The "ripply" energy signal $m(t)$ = 5 V $\sin(2\pi f_c t)/2\pi f_c t$ has a rectangular baseband spectrum in the frequency range $\pm f_c$ with amplitude $1/(2f_c)$. When this signal modulates a carrier, the resulting signal is $s(t)$ = 5 V $\cos 2\pi f_0 t\{\sin(2\pi f_c t)/2\pi f_c t\}$. Assuming f_0 = 10000 Hz and f_c = 1000 Hz, then

(a) Sketch the "rectangular" spectrum of the modulated signal.

(b) If the upper sideband (above 10000 Hz) is removed, mathematically describe the time waveform resulting from the inverse Fourier transform.

3b. **SNR in PM** – A cellular telephone handset receives a phase modulated 840 MHz carrier signal from the local basestation. The received signal is corrupted by a distant *modulated* carrier of the same frequency and received at 18.1 dB lower amplitude (one-eighth voltage). (this amplitude is consistent with theoretical calculations for a sectored, seven-frequency cell pattern). The illustration shows an interference calculation when the modulation index is β = 2.0 in both the local and interfering signals.



a) When β = 5 for both signals, determine the power ratio of demodulated signal to interference. ***SNR$_I$*** = ____ (dB)

b) When β = 0.5 for both signals, estimate the power ratio of demodulated signal to interference. ***SNR$_I$*** = ____ (dB)

