Quadratic Functions

MGSE9-12.N.CN.7 Solve quadratic equations with real coefficients that have complex solutions by (but not limited to) square roots, completing the square, and the quadratic formula.

MGSE9-12.A.REI.4 Solve quadratic equations in one variable.

MGSE9-12.A.REI.4b Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation (limit to real number solutions).

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What am I learning today?

The methods to solve quadratic functions

How will I show that I learned it?

Solve a quadratic using any method and write the answer in the correct format.

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Forms of Quadratics

1. Standard Form: $f(x) = ax^2 + bx + c$

2. Vertex Form: $f(x) = a(x - h)^2 + k$

3. Factored Form: $f(x) = a(x - r_1)(x - r_2)$

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Standard form of a Quadratic

$ax^2 + bx + c = 0$

Discriminant: $b^2 - 4ac$

Discriminant > 0: 2 real roots
Discriminant = 0: 1 real root
Discriminant < 0: 2 imaginary roots

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Solving a Quadratic

a. Factoring (only rational answers)
b. Taking a Square-Root
c. Quadratic Formula
d. Completing the Square

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Solving a Quadratic by Factoring

Example 1: $6x^2 = 2x$

$-2x \quad -2x$

$6x^2 - 2x = 0$

$2x(3x - 1) = 0$

$2x = 0$ $\quad$ $3x - 1 = 0$

$x = 0$ $\quad$ $x = \frac{1}{3}$

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Solving a Quadratic by Square-Root

Example 1: $3(x - 6)^2 - 7 = -43$

$3(x-6)^2 = -36$

$\sqrt{(x-6)^2} = \sqrt{-12}$

$x - 6 = \pm 2i\sqrt{3}$

$+6 \quad +6$

$x = 6 \pm 2i\sqrt{3}$

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Solving by Quadratic Formula

1. Put in standard form and set equal to 0.

2. Label a, b, and c.

3. Plug into the formula and simplify.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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Solving by Quadratic Formula

Example 1: $6x^2 - 2x + 12 = 2x^2 - 2x + 3$

$-2x^2+2x-3 \quad -2x^2+2x-3$

$4x^2+9=0$

$b^2-4ac = (0)^2-4(4)(9)$

$= -144$

$\frac{-(0)\pm\sqrt{-144}}{2(4)} = \frac{\pm 12i}{8}$

$= \boxed{\pm\frac{3}{2}i}$

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Solving by Quadratic Formula

Example 2: $2x^2 - 4x + 11 = x^2 + 2x$

$-x^2-2x \quad -x^2-2x$

$x^2-6x+11=0$

$(-6)^2-4(1)(11) = -8$

$\frac{-(-6)\pm\sqrt{-8}}{2(1)} = \frac{6\pm 2i\sqrt{2}}{2}$

$= \boxed{3\pm i\sqrt{2}}$

$\frac{6\pm 2i\sqrt{2}}{4} = \frac{3}{2}\pm\frac{i\sqrt{2}}{2}$

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Solving a Quadratic by Completing the Square

1. Move the x^2 and x terms to one side.
2. Move the constant to the opposite side.
3. Divide by a.
4. Add $\left(\frac{b}{2}\right)^2$ to each side of the equation to make a perfect square trinomial.
5. Factor the perfect square.
6. Solve by square-roots.

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Solving a Quadratic by Completing the Square

Example 1: $2x^2 - 8x + 8 = 4$

$-8 \quad -8$

$\frac{2x^2}{2} - \frac{8x}{2} = \frac{-4}{2}$

$x^2 - 4x = -2$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = 4$

$+4 \quad +4$

$x^2 - 4x + 4 = 2$

$(x-2)(x-2) = 2$

$\sqrt{(x-2)^2} = \sqrt{2}$

$x - 2 = \pm\sqrt{2}$

$+2 \quad +2$

$x = 2 \pm \sqrt{2}$

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Solving a Quadratic by Completing the Square

Example 2: $6x^2 + 3 = 11x$

$6x^2 - 11x + 3 = 0$

$\left(\frac{-11}{12}\right)^2 = \frac{121}{144}$

$\frac{6x^2}{6} - \frac{11x}{6} = \frac{-3}{6}$

$x^2 - \frac{11}{6}x = -\frac{1}{2}$

$+\frac{121}{144} = +\frac{121}{144}$

$\sqrt{\left(x - \frac{11}{12}\right)^2} = \sqrt{\frac{49}{144}}$

$121 - 72 = 49$

$x - \frac{11}{12} = \pm\frac{7}{12}$

$x = \frac{11}{12} \pm \frac{7}{12}$

$x = \frac{18}{12} = \frac{3}{2}$

$x = \frac{4}{12} = \frac{1}{3}$

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Solving a Quadratic Inequality

1. Turn into an equation and solve using any method.
2. Plot solutions on a numberline paying attention to whether the points are part of the solution. Pick a point in between the answers to test in the original inequality.
3. If the point makes the inequality true, the solutions are between the roots. If false, the solutions are outside of the roots.
4. Shade the solution set and write the solution set in interval notation.

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Solving a Quadratic Inequality

Example 1: $2x^2 + 2x + 15 \geq 15x$

$-15x \quad -15x$

$2x^2 - 13x + 15 \geq 0$

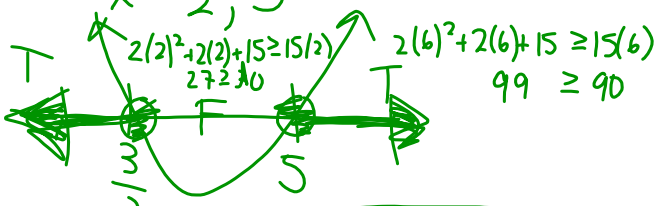
30, -10×-3, -13

$(2x^2 - 10x) - 3x + 15) = 0$

$2x(x-5) - 3(x-5) = 0$

$(2x-3)(x-5) = 0$

$x = \frac{3}{2}, 5$



$(-\infty, \frac{3}{2}] \cup [5, \infty)$

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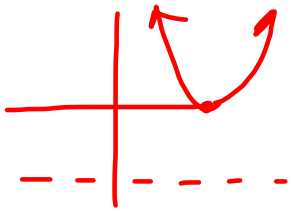
Solving a Quadratic Inequality

Example 2: $2(x - 3)^2 < -8$



$(x-3)^2 = -4$

$x - 3 = \pm 2i$

$x = 3 \pm 2i$

no solution

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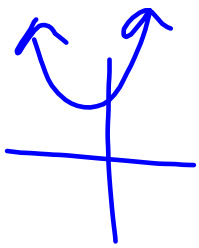
Solving a Quadratic Inequality

Example 3: $3x^2 + 12 > 2x^2 - 6x$

$-2x^2+6x \quad -2x^2+6x$

$x^2+6x+12>0$



$\frac{-6\pm\sqrt{-12}}{2(1)} = \frac{-6\pm 2i\sqrt{3}}{2}$

$= -3\pm i\sqrt{3}$

$(-\infty,\infty)$

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Practice

Solve by Factoring:

1. $2x^2 - 6x = x^2 + 27$

$-x^2-27 \quad -x^2-27$

$x^2-6x-27=0$

$(x-9)(x+3)=0$

$x=9,-3$

2. $5x^3 + 6x^2 = -9x^2 - 10x$

$+9x^2+10x \quad +9x^2+10x$

$5x^3+15x^2+10x=0$

$5x(x^2+3x+2)=0$

$5x(x+1)(x+2)=0$

$x=0,-1,-2$

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Practice

Solve by Quadratic Formula:

1. $x^2 - 8x = -16$

$+16 \quad +16$

$x^2 - 8x + 16 = 0$

$(-8)^2 - 4(1)(16) = 0$

$\frac{-(-8) \pm \sqrt{0}}{2(1)} = \boxed{4}$

2. $6x^2 = -9x^2 - 10$

$-6x^2 \quad -6x^2$

$0 = -15x^2 - 10$

$(0)^2 - 4(-15)(-10) = -600$

$\frac{-(0) \pm \sqrt{-600}}{2(-15)}$

$= \frac{\pm 10i\sqrt{6}}{-30}$

$= \boxed{\frac{\pm i\sqrt{6}}{3}}$

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Practice

Solve by Completing the Square:

1. $x^2 - 6x = -16$

$+9 \quad +9$

$x^2 - 6x + 9 = -7$

$(x-3)^2 = -7$

$x - 3 = \pm i\sqrt{7}$

$\boxed{x = 3 \pm i\sqrt{7}}$

2. $6x^2 = -9x^2 - 10$

$+9x^2 \quad +9x^2$

$15x^2 = -10$

$x^2 = -\frac{2}{3}$

$x = \frac{\pm i\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\boxed{x = \frac{\pm i\sqrt{6}}{3}}$

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Practice

Solve the Inequality.

1. $x^2 - 2x > 2x + 44$

$-2x-44 \quad -2x-44$

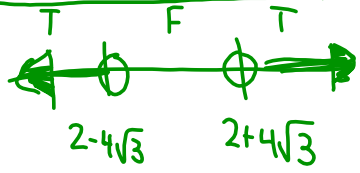
$x^2-4x-44>0$

$(-4)^2-4(1)(-44)=192$

$\frac{-(-4)\pm\sqrt{192}}{2(1)} = \frac{4\pm 8\sqrt{3}}{2}$

$= 2\pm 4\sqrt{3}$

$(-\infty, 2-4\sqrt{3}) \cup (2+4\sqrt{3}, \infty)$

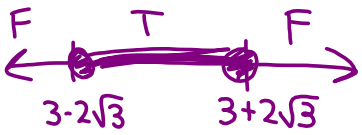


2. $(x - 3)^2 \le 12$

$x-3 = \pm 2\sqrt{3}$

$x = 3\pm 2\sqrt{3}$

$[3-2\sqrt{3}, 3+2\sqrt{3}]$



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Practice

$2(x-2)^2-7=0$

$(x-2)^2=\frac{7}{2}$

$x-2=\pm\frac{\sqrt{7}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{14}}{2}$

$x=2\pm\frac{\sqrt{14}}{2}$

Graph the Quadratic and Find all its Characteristics.

$f(x) = 2(x - 2)^2 - 7$

Vertex: $(2,-7)$ AoS: $x=2$

Domain: $(-\infty,\infty)$ Range: $[-7,\infty)$

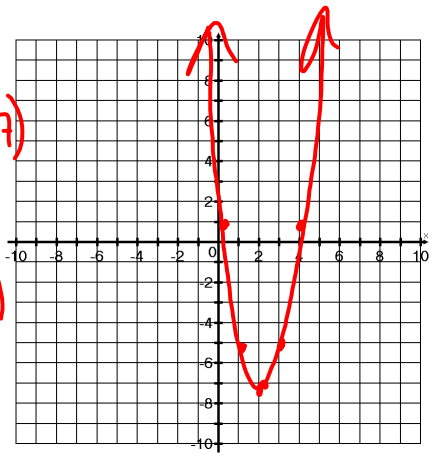
Extrema: min of $y=-7$ at $(2,-7)$

Root(s): $x=2\pm\frac{\sqrt{14}}{2}$ Y-Int: $(0,1)$

Int. of Inc: $(2,\infty)$ Int of Dec: $(-\infty,2)$

End Behavior: As $x\to-\infty$, $f(x)\to\infty$

As $x\to\infty$, $f(x)\to\infty$



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