3. ONE-DIMENSIONAL FLOW OF WATER THROUGH SOILS

From the discussions in the previous chapter, we have seen that water changes the soil states in fine-grained soils; the greater the water content the weaker it is. Soils are porous materials due to the presence of interconnected void spaces between the solid grains. Hence, particle sizes and the structural arrangement of the particles influence the rate of flow.

Water can cause instability and many geotechnical structures such as roads, bridges, dams and excavations have failed due to instability induced by flow of water. It is therefore necessary to estimate the quantity of underground seepage under various hydraulic conditions, for investigating problems involving the pumping of water for underground construction, and for making stability analyses of earth dams and earth-retaining structures that are subject to seepage forces.

The key physical property that governs flow of water in soils is permeability. Prior to discussing permeability in detail, we should first note the following key terms:

- **Groundwater** is water under the influence of gravity that fills the soil particles.
- **Head** is the mechanical energy per unit weight.
- **Hydraulic conductivity** (otherwise referred to as the **coefficient of permeability**) is a proportionality constant to determine the flow velocity of water through soils.

GROUNDWATER

If we dig a hole into a soil mass that has all the voids filled with water, we will observe water filling the hole upto a certain level. This water level is called groundwater level or groundwater table and exists under a hydrostatic condition. A hydrostatic condition occurs when there is no flow; i.e. the flow is zero. The top of the groundwater level is under atmospheric pressure. We will denote the ground water level by the symbol▼.

HEAD

From basic fluid mechanics, we know that, according to Bernoulli’s equation, the total head at a point in water under motion can be given by the sum of the pressure, velocity, and elevation heads, or

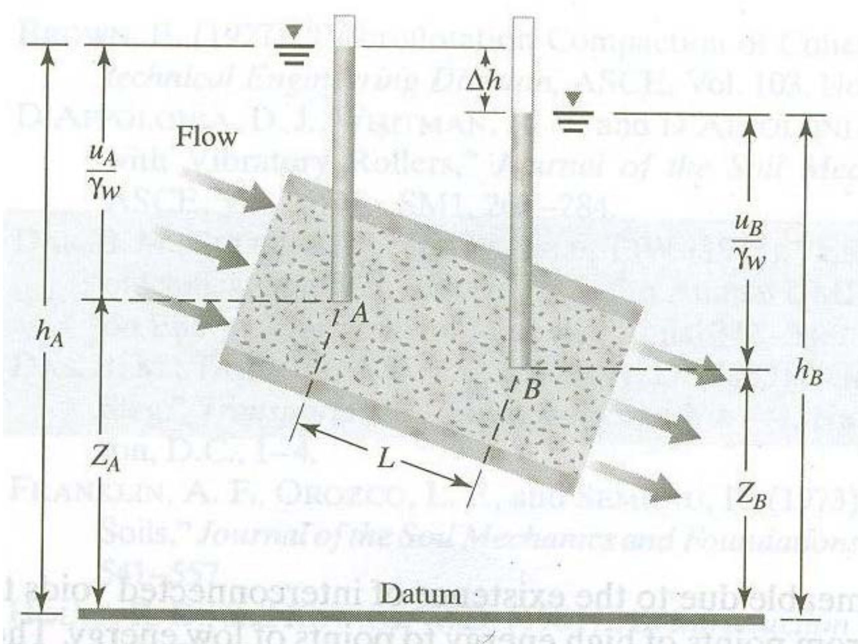
$$h = \frac{u}{\gamma_w} + \frac{v^2}{2g} + Z$$

where h = total head
u = water pressure
v = velocity
Z = elevation head or vertical distance of a given point above or below a datum plane

For flow of water through soil, the seepage velocity is very small and can be neglected. The total head at any point can thus be adequately represented by

$$h = \frac{u}{\gamma_w} + Z$$

The figure below shows the relationship among pressure, elevation, and total heads for the flow of water through soil. Open stand pipes known as ***piezometers*** are installed at the two points. The levels to which water rises in the piezometer tubes situated at the two points are known as the ***piezometric level*** of their respective point. The pressure head at a point is the height of the vertical column of water in the piezometer installed at that point.



As water flows thorough a soil media as depicted in the figure, there will be a head loss between the two points. This head loss can be given by

$$\Delta h = h_A - h_B = \left(\frac{u_A}{\gamma_w} + Z_A\right) - \left(\frac{u_B}{\gamma_w} + Z_B\right)$$

HYDRAULIC CONDUCTIVITY

Darcy (1856) proposed that average flow velocity through soils is proportional to the gradient of the total head. The flow in any direction, j, is

$$v_j = k_j \frac{dH}{dx_j}$$

Where *v* is the average flow velocity, *k* is a coefficient of proportionality called the hydraulic conductivity (sometimes called the coefficient of permeability), and *dH* is the change in total head over a distance *dx*. The unit of measurement for *k* is length/time, and it is usually expressed in cm/sec or m/sec in SI units. With reference to the previous figure, Darcy's law becomes

$$v = k\frac{\Delta h}{L} = ki$$

Where $i = \Delta h/L$ is the hydraulic gradient. Darcy's law is valid for all soils if the flow is laminar, i.e. where Reynold's number is less than 2000. Turbulent flow conditions may exist in very coarse sands and gravels, and Darcy's law may not be valid for these materials. However, uner a low hydraulic gradient, laminar flow conditions usually exist.

The average velocity calculated from the previous equation is for the cross-sectional area normal to the direction of flow. Flow through soils, however, occurs only through the interconnected voids. The velocity through the void spaces is called the seepage velocity (v_s) and is obtained by dividing the average velocity by the porosity of the soil:

$$v_s = \frac{k_j}{n} i$$

The volume rate of flow, q, is the product of the average velocity and the cross-sectional area:

$$q = vA = Aki$$

The hydraulic conductivity of soils depends on several factors: fluid viscosity, pore-size distribution, grain-size distribution, void ratio, roughness of mineral particles, and degree of saturation. The value of hydraulic conductivity varies widely for various soils. Some typical values for saturated soils are given in the table below. The hydraulic conductivity of unsaturated soils is lower and increases rapidly with the degree of saturation.

Soil type	k (cm/sec)
Clean gravel	$10^{2} – 1.0$
Coarse sand	$1.0 – 10^{-2}$
Fine sand	$10^{-2} – 10^{-3}$
Silty clay	$10^{-3} – 10^{-5}$
Clay	$<10^{-6}$

Various researchers have proposed several empirical equations for estimating the hydraulic conductivity of soils.

For fairly uniform sand, i.e. for sand with a small uniformity coefficient, Hazen (1930) proposed and empirical relationship for hydraulic conductivity in the form

$$k = c\, D^2_{10}$$

where k = coefficient of permeability in cm/sec

c = a constant varying from 1.0 to 1.5 (usually taken to be 1.0)

D_{10} = effective size in mm

This equation is based on Hazen's observations of loose, clean, filter sands. A small quantity of silts and clays, when present in a sandy soil, may change the hydraulic conductivity substantially.

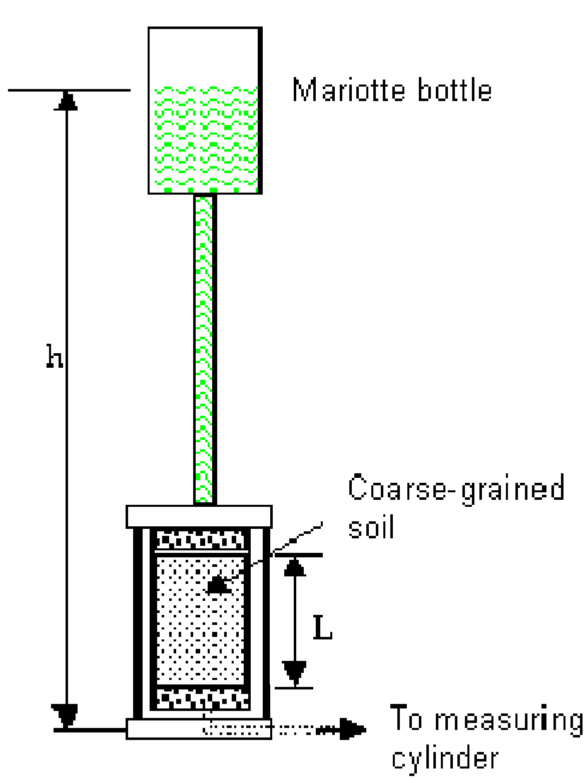
Other empirical relationships have also been suggested by various individuals and institutions. One has to be extremely cautious in using empirical relationships for *k* because it is very sensitive to changes in void ratio, pore size, and homogeneity of the actual soil mass.

LABORATORY DETERMINATION OF HYDRAULIC CONDUCTIVITY

Two standard laboratory tests are used to determine the hydraulic conductivity of soil – the ***constant-head test*** and the ***falling-head test***.

CONSTANT-HEAD TEST

The constant head test is used to determine the hydraulic conductivity of coarse-grained soils. A typical constant-head test arrangement is shown below. In this test, water supply at the inlet is adjusted in such a way that the difference of head between the inlet and the outlet remains constant during the test period. After a constant flow rate is established, water is collected in a graduated cylinder for a known duration.



The total volume of water collected may be expressed as

$$Q = Avt = A(ki)t$$

where Q = volume of water collected

A = area of cross-section of the soil specimen

t = duration of water collection

And since $i = \Delta h/L = h/L$ for this test, where L is the length of the specimen (height),

$$Q = Akht/L$$

or

$$k = \frac{QL}{Aht}$$

The viscosity of the fluid, which is a function of temperature, influences the value of k. It is customary to express the value of k at a temperature of 20ºC. The experimental value ($k_{T°C}$) is corrected to this baseline temperature of 20ºC using the following relationship.

$$k_{20°C} = \left(\frac{\eta_{T°C}}{\eta_{20°C}}\right) k_{T°C}$$

where $k_{T°C}$ = hydraulic conductivity at the test temperature

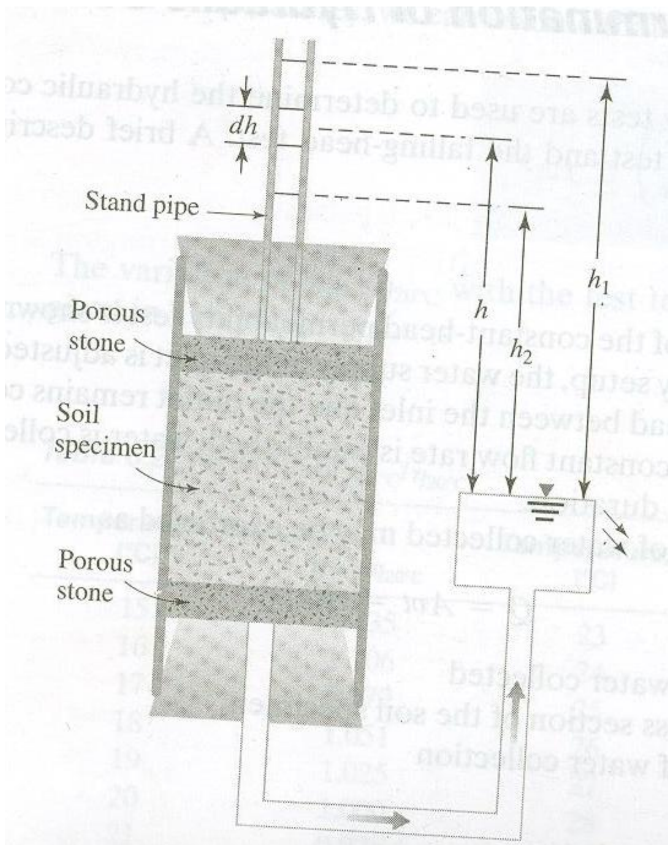
$\eta_{20°C}$ *and* $\eta_{T°C}$ = viscosity water at 20ºC and the test temperature respectively

The ratio $\eta_{T°C} / \eta_{20°C}$ can be calculated from,

$\eta_{T°C} / \eta_{20°C} = 2.42 - 0.475 \ln (T)$

FALLING-HEAD TEST

Due to low hydraulic conductivity of fine-grained soils, it will take a considerable time to obtain reasonable discharge volume using the constant-head test. It is therefore customary to use the falling-head test for such materials. A typical arrangement of the falling-head permeability test is shown below.



During this test water from the standpipe flows through the soil. The head of water (h) changes with time as flow occurs through the soil. At different times the head of water is recorded. Let *dh* be the drop in head over a time period *dt*. The velocity or rate of head loss in the tube is

$$v = -\frac{dh}{dt}$$

The rate of flow of water through the specimen at any time to can be given by

$$q = k\frac{h}{L}A = -a\frac{dh}{dt}$$

where q = flow rate

a = cross-sectional area of the standpipe

A = cross-sectional area of the soil specimen

Rearranging the equation gives,

$$dt = \frac{aL}{Ak}\left(-\frac{dh}{h}\right)$$

Integration of the left side of this equation with limits of time from 0 to t and the right side with limits of head difference from h_1 to h_2 gives

$$t = \frac{aL}{Ak}\ln\left(\frac{h_1}{h_2}\right).$$

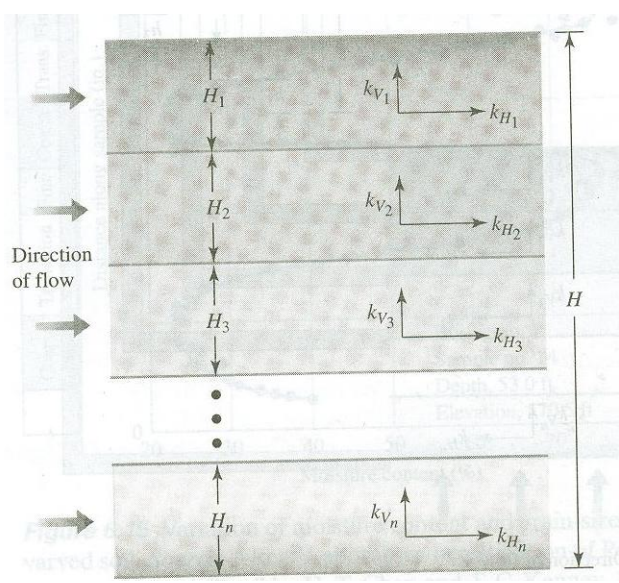
or

$$k = 2.303\frac{aL}{At}\log_{10}\left(\frac{h_1}{h_2}\right)$$

EQUIVALENT HYDRAULIC CONDUCTIVITY IN STRATIFIED SOIL

In stratified soil deposits where the hydraulic conductivity for flow in a given direction changes from layer to layer, an equivalent hydraulic conductivity can be computed to simplify calculations. Two cases of flow shall be considered here.

i. FLOW PARALLEL TO STRATIFICATION



When the flow is parallel to the soil layer, the hydraulic gradient is the same at all points. The flow through the soil mass as a whole is equal to the sum of the flow through each of the layers. There is a similarity here with the flow of electricity through resistors in parallel. If we consider a unit width perpendicular of flow, then flow rate is given by:

$$q = v \cdot 1 \cdot H$$
$$= v_1 \cdot 1 \cdot H_1 + v_2 \cdot 1 \cdot H_2 + v_3 \cdot 1 \cdot H_3 + \cdots + v_n \cdot 1 \cdot H_n$$

where v = average discharge velocity

$v_1, v_2, v_3, \cdots, v_n$ = discharge velocities of flow in layers

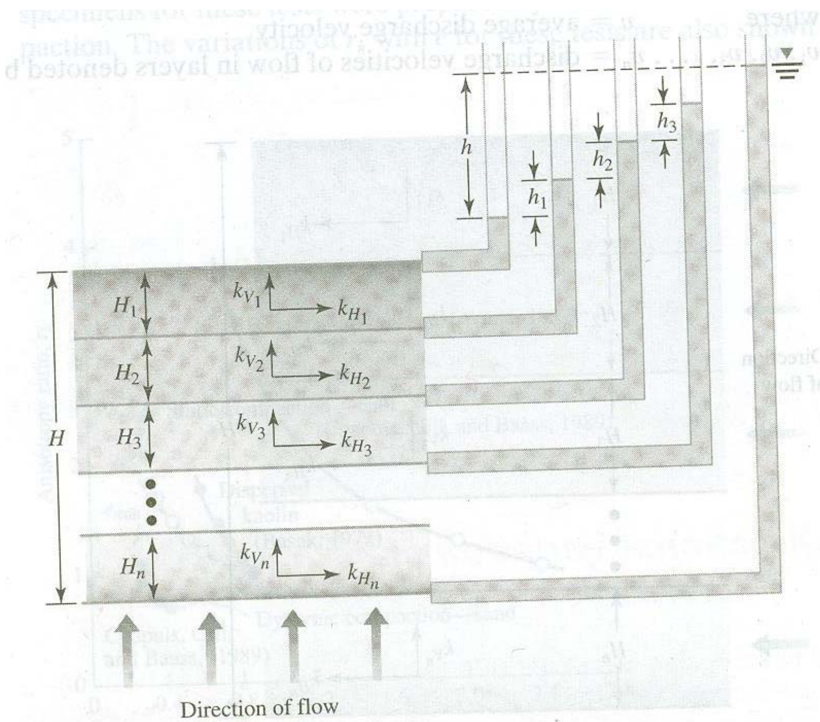
It thus follows that

$$k_{H(eq)} i_{eq} H = k_{H_1} i_1 H_1 + k_{H_2} i_2 H_2 + k_{H_3} i_3 H_3 + \cdot \quad \cdot \quad \cdot + k_{H_n} i_n H_n$$

Since $i_{eq} = i_1 = i_2 = i_3 = \cdots = i_n$,

$$k_{H(eq)} = \frac{1}{H}\left(k_{H_1} H_1 + k_{H_2} H_2 + k_{H_3} H_3 + \cdot \quad \cdot \quad \cdot + k_{H_n} H_n\right)$$

ii. FLOW PARALLEL TO STRATIFICATION



When the flow is perpendicular to stratification, the head loss in the soil mass is the sum of the head losses in each of the layers. The velocity of flow through all the layers is the same. The analogy to electricity is flow of current through resistors in series.

$v = v_1 = v_2 = v_3 = \cdots + v_n$

$h = h_1 + h_2 + h_3 + \cdots + h_n$

Using Darcy's law these equations can be rewritten as,

$$k_{V(eq)}\left(\frac{h}{H}\right) = k_{V_1} i_1 = k_{V_2} i_2 = k_{V_3} i_3 = \cdot \quad \cdot \quad \cdot = k_{V_n} i_n$$

$$h = H_1 i_1 + H_2 i_2 + H_3 i_3 + \cdot \quad \cdot \quad \cdot + H_n i_n$$

Solving these two equations for the equivalent hydraulic conductivity,

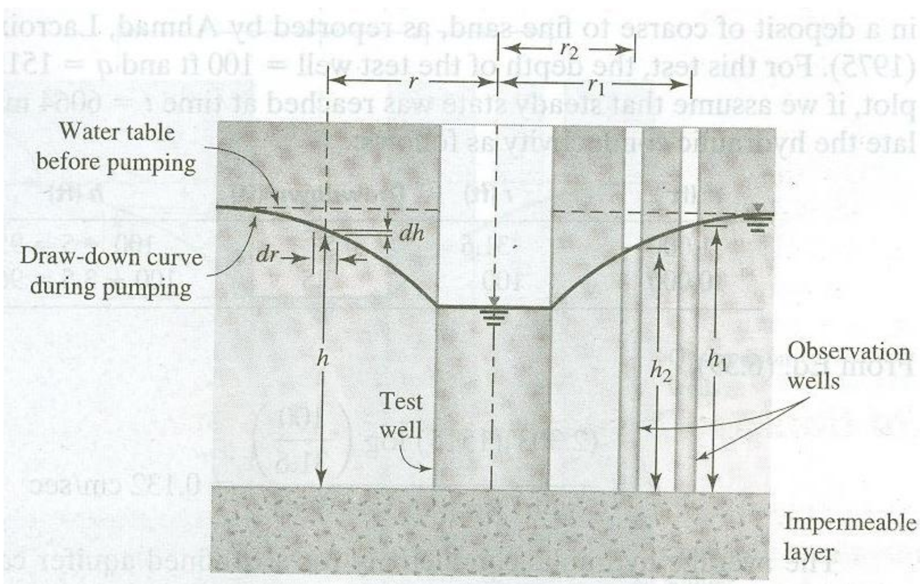
$$k_{V(eq)} = \frac{H}{\left(\frac{H_1}{k_{V_1}}\right) + \left(\frac{H_2}{k_{V_2}}\right) + \left(\frac{H_3}{k_{V_3}}\right) + \cdot \quad \cdot \quad \cdot + \left(\frac{H_n}{k_{V_n}}\right)}$$

PERMEABILITY TEST IN THE FIELD BY PUMPING FROM WELLS

In the field the average hydraulic conductivity of a soil deposit in the direction of flow can be determined by performing pumping tests from wells. This process involves pumping water at a constant flow rate from a test well and measuring the decrease in groundwater level at observation wells. We shall discuss the case of both unconfined and confined aquifers.

a. UNCONFINED AQUIFER

The figure below shows a case where the top permeable layer, whose hydraulic conductivity has to be determined, is unconfined and underlain by an impermeable layer. During the test, water is pumped out at a constant rate from a test well that has a perforated casing. Several observation wells at various radial distances are made around the test well.



Continuous observations of the water level in the test well and in the observation well are made after the start of pumping, until a steady state is reached. The steady state is established when the water level in the test and observation wells becomes constant.

Let *dh* be the drop in the total head over a distance *dr*. Then the hydraulic gradient is then
$i = dh/dr$

The area of flow at a radial distance r from the center of the pumping well is
$A = 2\pi rh$

Where h is the thickness of an elemental volume of the pervious layer.
From Darcy's law, the flow rate is
$q = 2\pi rh\, k\, (dh/dr)$

We need to rearrange the above equation and integrate it between the limits r_1 and r_2, and h_1 and h_2:

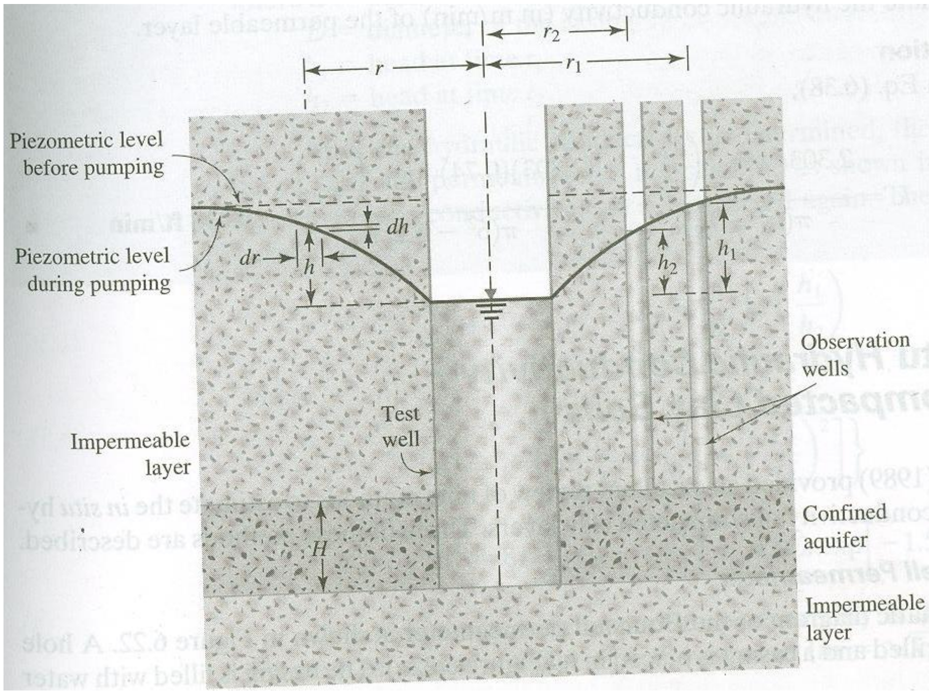
$$\int_{r_2}^{r_1} \frac{dr}{r} = \left(\frac{2\pi k}{q}\right)\int_{h_2}^{h_1} h\,dh$$

Thus,

$$k = \frac{2.303q \log_{10}\left(\dfrac{r_1}{r_2}\right)}{\pi\left(h_1^2 - h_2^2\right)}$$

b. CONFINED AQUIFER

The average hydraulic conductivity for a confined aquifer can also be determined by conducting a pumping test from a well with perforated casing that penetrates the full depth of the aquifer and by observing the piezometer level in a number of observation wells at various radial distances. Pumping is continued at a uniform rate q until a steady state is reached.



Because water can enter the test well only from the aquifer of thickness H, the steady state of discharge is

$q = 2\pi rH\, k\, (dh/dr)$

or

$$\int_{r_2}^{r_1} \frac{dr}{r} = \int_{h_2}^{h_1} \frac{2\pi kH}{q} dh$$

This gives the hydraulic conductivity in the direction of flow as

$$k = \frac{q \log_{10}\left(\frac{r_1}{r_2}\right)}{2.727H(h_1 - h_2)}$$