ARMA 10-185

3D geomechanics in UGS projects. A comprehensive study in northern Italy

Castelletto, N., Ferronato, M., Gambolati, G., Janna, C. and Teatini, P.
Dept. of Mathematical Methods and Models for Scientific Applications, University of Padova, Padova, Italy
Marzorati, D. and Cairo, E.
Stogit S.p.A., Crema, Italy
Colombo, D. and Ferretti, A.
Tele-Rilevamento Europa S.r.l. (TRE), Milano, Italy
Bagliani, A. and Mantica, S.
Eni S.p.A. - Divisione E&P, San Donato Milanese, Italy

Copyright 2010 ARMA, American Rock Mechanics Association

This paper was prepared for presentation at the 44th US Rock Mechanics Symposium and 5th U.S.-Canada Rock Mechanics Symposium, held in Salt Lake City, UT June 27–30, 2010.

This paper was selected for presentation at the symposium by an ARMA Technical Program Committee based on a technical and critical review of the paper by a minimum of two technical reviewers. The material, as presented, does not necessarily reflect any position of ARMA, its officers, or members. Electronic reproduction, distribution, or storage of any part of this paper for commercial purposes without the written consent of ARMA is prohibited. Permission to reproduce in print is restricted to an abstract of not more than 300 words; illustrations may not be copied. The abstract must contain conspicuous acknowledgement of where and by whom the paper was presented.

ABSTRACT: The 3D geomechanical response to seasonal gas storage is investigated for a gas field managed by Stogit. The1200 m deep reservoir is located in the Po basin, Italy, and a UGS program is under way since 1986 following a 5-yr primary production life. The use of i) a basin-scale geomechanical characterization of the Po basin, ii) a detailed knowledge of the subsurface geology made available by 3D seismic surveys, iii) almost 30-yr measurements related to the gas field activities, and iv) an advanced PSInSAR analysis providing the vertical and horizontal West-East displacements of the ground surface above the field from 2003 to 2007 has allowed for the development, set up, and calibration of a representative 3D fluid-dynamical model and a transversally isotropic nonlinear geomechanical model. The latter successfully reproduces the largest vertical and horizontal seasonal land displacements, on the range of 8-10 mm and 6-8 mm, respectively, as observed above the gas reservoir. The model is then used to investigate the ground surface displacements in connection with UGS future programs where the maximum overpressure achieved in the field is planned to be raised to 107% p_i and 120% p_i, with p_i the original in-situ pore pressure.

1. INTRODUCTION

Due to the growing importance of natural gas for energy production in the present society, the interest to develop underground gas storage (UGS) projects is continuously increasing worldwide. The first successful underground storage of natural gas was made in 1916 in the depleted Zoar gas field, south of Buffalo, New York (USA) [1], where the injection facilities are still operational. Worldwide, there are currently about 630 functioning UGS projects, with a working gas volume, i.e. the maximum volume of gas available for withdrawal during the normal activity of the storage facility, of approximately 350×10^9 Sm^3 in the reference year 2009. This amount is more than 300% the one available in 1970. Most of the gas volume is installed in East Europe (39%), North America (36%), and West Europe (24%) [2].

In response to summer gas injection and winter gas withdrawal, the three-dimensional (3D) surface motion in response to stress changes in both the gas disposal reservoir and the surrounding formations can be reliably predicted with the aid of advanced fluid-dynamical and geomechanical models. Land moves vertically down and horizontally toward the area of maximum pressure decline (usually the reservoir gravity center where the storage wells are concentrated) when gas is withdrawn, and vertically up and horizontally away from the injection area when gas is stored. Magnitude of the displacement and size of the area involved by this sort of “earth breathing” depend primarily on the burial depth and geometry (thickness and area extent) of the field, the geomechanical properties of the reservoir and surrounding structural-geological formations, and the pore pressure changes induced by gas injection/production.

The present paper describes the results of a multidisciplinary effort aimed at detecting the full displacement due to the UGS in the Lombardia field, Po River basin, northern Italy (Fig. 1). The reservoir was

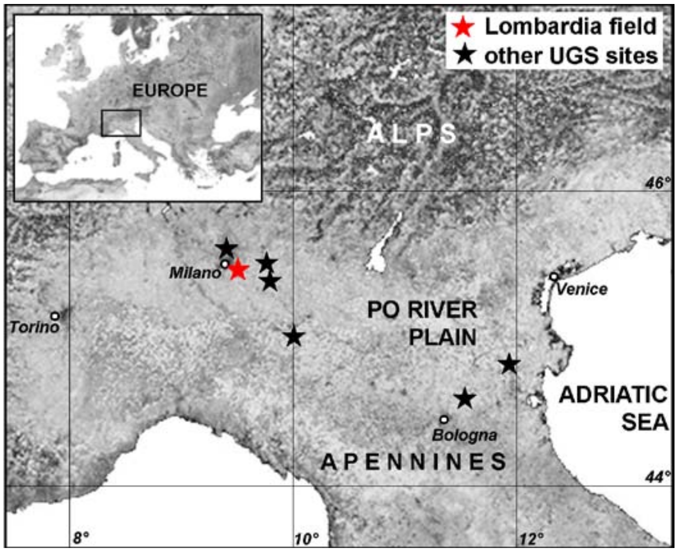

Fig. 1. Map of the Po River plain with the location of the Lombardia field and the other UGS projects implemented by Stogit in the area.

developed from 1981 to 1986 and later used for gas storage by Stogit. Methane is injected from April to October and extracted from November to March each year. A 3D seismic survey [3], together with almost 30-yr long records of removed/stored gas volumes and fluid pore pressure, have allowed for the construction of a detailed static geologic model of the gas field and neighbouring formations, and the accurate calibration of a dynamic multiphase flow and pressure model.

The pattern, magnitude, and timing of land displacement above the field both in the vertical and in the East-West directions have been obtained from an advanced Permanent Scatterer InSAR (PSInSAR^{TM}) analysis [4] using ascending and descending RADARSAT-1 images acquired from 2003 to 2007. This information is implemented into a geomechanical transversally isotropic finite element (FE) model of the porous medium [5,6], which will be shown to successfully match the full 3D ground surface displacements measured above the reservoir.

Finally, we discuss the results of the geomechanical prediction for two scenarios of a future enhanced UGS program where the largest storage fluid pore pressure is increased from the present 103%p_i, p_i being the original in-situ gas pressure, to 107% p_i and 120% p_i, thus allowing for an increase of the stored gas volume by approximately 65% and 180% relative to the current stored value.

2. GEOLOGICAL AND GEOMECHANICAL SETTING OF THE PO RIVER BASIN

The Po River plain is located in northern Italy between latitude 44°-46° North and longitude 8°-12° East (Fig. 1) and is bounded by the Alps to the West and North, the Apennines to the South and the Adriatic Sea to the East.

To start from the early 1950s Eni E&P, the Italian Oil Company, has discovered several gas/oil fields in the Po River basin. Most of the hydrocarbons detected in the area are Pliocene and Pleistocene biogenetic/diagenetic gas. The Pliocene and Quaternary reservoirs are located in thrusted anticlines, simple drape structures, and stratigraphic traps.

The Lombardia field is one of these reservoirs. It is located at the northern margin of the pre-Alpine monocline and is made of sandy sediments corresponding to the lateral end of Pliocene turbidites onlapping the partly folded Messianic basin. Being the field placed in the zone of steeper dip, the gas traps are of stratigraphic type with the permeable units pinching out northward against the impermeable clays of the Santerno Formation. The reservoir is located at a burial depth of 1050-1350 m below m.s.l. and consists of the three gas-bearing pools, A, B, and C that are vertically separated by 20-30 m thick clay layers. The major pool is C which is composed of 3 sub-units (Fig. 2). A large aquifer connected to the reservoir is developed in the southward direction only, since northward the sandy layers pinch out against the Santerno Formation.

The geomechanical properties of the Po River basin are obtained from a number of in-situ deformation measurements carried out in the off-shore portion of the area by the radioactive marker technique [7]. The data have been statistically processed and used to derive a basin-scale relationship that provides the vertical uniaxial compressibility c_M as an exponential function of the vertical effective stress σ_z [8]:

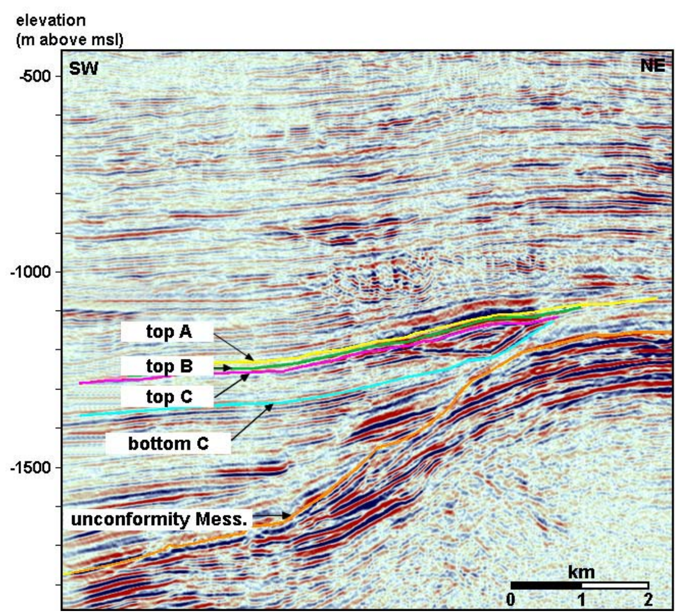

Fig. 2. Vertical seismic section through the Lombardia gas field along a South-West - North-East direction. The geological units of major interest are shown.

$$c_M = 1.3696 \times 10^{-2} \sigma_z^{-1.1347} \quad (1)$$

where the units of c_M and σ_z are [bar^{-1}] and [bar], respectively. Eq. 1 holds for rock compression in virgin loading conditions (I loading cycle), while rock expansion is controlled by c_M in unloading/reloading conditions (II loading cycle). From in-situ marker data and oedometer tests, the value of the ratio s between loading and unloading/reloading c_M has been estimeted to range between 1.8 and 3.5 for $100<\sigma_z<600$ bar (i.e., for a depth range between 1000 and 6000 m in normally pressurized conditions), with s decreasing as σ_z rises [9].

3. FLUID-DYNAMIC MODEL OF THE LOMBARDIA FIELD

The Lombardia gas field is oparated by Stogit. Primary production started from the C unit in 1981. About 2.7×10^9 Sm^3 were withdrawn from 1981 to 1986. The pore pressure decline achieved its maximum value of 35 bar in 1986 at the end of the primary gas production life (Fig. 3). UGS started soon after and superposed to the natural pressure recovery due to the surrounding active aquifer. Starting from 2002, the UGS program is pushing the maximal gas pore overpressure to 103%p_i. This value, together with a cyclic pressure drop/recovery of about 30-35 bar, has allowed to yearly manage a working gas volume of ~1.2×10^9 Sm^3 (Fig. 3).

The gas/water dynamics within the Lombardia reservoir is simulated by the multi-phase flow Eclipse™ simulator using the Carter-Tracy analytical aquifer as boundary condition. The model is calibrated with a "history match" procedure based on pressure and production measurements collected over the time interval 1981-2007.

Because of the importance of the actual aquifer dynamics for a most correct prediction of the geomechanical effects, in particular the land surface displacement, during the field production life [10,11], we "couple" the Eclipse model to a 3D finite element (FE) model that solves the flow equation within the aquifer. The inner domain of the groundwater flow model is bounded by the outer boundary of the reservoir production model, where the fluid pore pressure matched within the field is prescribed as a known boundary condition. The coupling procedure is described in detail by [12]. The aquifer model is strictly consistent with the geomechanical model in the sense that the FE flow grid represents a subset of the global 3D FE structural grid, with the actual medium compressibility variable in time and space as it is in the basin-scale geomechanical model.

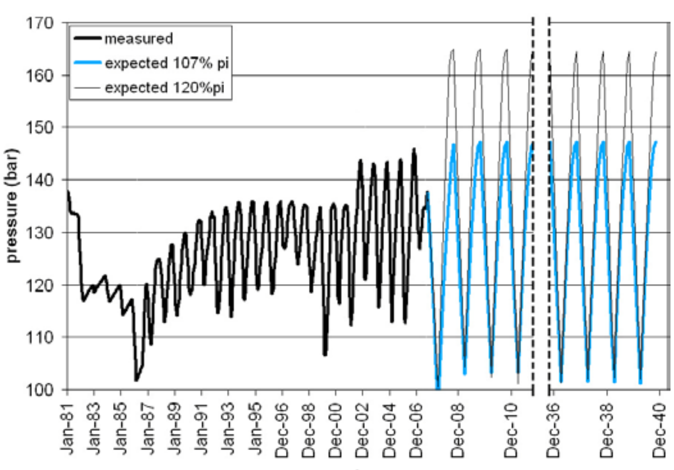

Fig. 3. Time behavior of the average gas pore pressure as measured in the C pool from the inception of the gas production (1981) to date (2007) and as provided by the production model for the two planned UGS scenarios over the period 2008-2040.

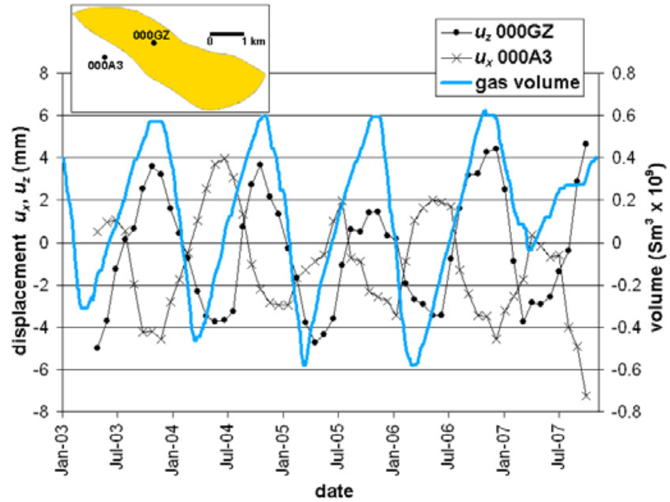

Fig. 4. Seasonal behavior of the vertical (u_z) and West-East horizontal (u_x) displacements as measured by PSInSAR for a couple of PS. The pore gas volume stored in the field over the same period is also provided. The trace of the Lombardia field and the location of the two PS are shown in the inset.

4. PSINSAR DATA OVER THE LOMBARDIA FIELD

The pattern, magnitude, and timing of land displacement above the field both in the vertical and in the East-West directions have been obtained from an advanced Permanent Scatterer InSAR (PSInSAR™) analysis [4] using ascending and descending images acquired by the Canadian RADARSAT-1 satellite. PSInSAR is a methodology where long temporal series of Synthetic Aperture Radar (SAR) scenes acquired over the same area at different times provide a radar phase information that allows for the detection and measurement of millimetric-scale movements of many "radar benchmarks", or permanent scatterers (PS), due to their favorable reflectivity conditions. PS correspond to persistent, bright radar reflectors, such as buildings, poles, metallic structures, exposed rocks or other similar objects.

PS data are 1D measurements related to the projection along the satellite Line Of Sight (LOS) of the 3D displacement vector affecting the radar target. Nonetheless, the combination of the Earth's rotation and satellite motion makes it possible for any area of interest to be illuminated by the satellite radar sensor along two different acquisition geometries: one having the satellite flying from South to North (called ascending mode) and the other from North to South (called descending mode). Whenever two datasets of SAR images are available, acquired over the same area and during the same time frame along ascending and descending orbits, the PSInSAR results can be used successfully to estimate the vertical and East-West components of the local displacement field [13]. In fact, since satellite orbits are almost parallel to the Earth meridians, the sensitivity to possible target motion along the North-South direction is usually low.

Satellite radar images acquired by RADARSAT-1 from March 2003 to October 2007 are employed in this study. The extent of the area of interest above the gas storage reservoir under study is about 30 km^2. Fig. 4 shows the vertical, u_z, and West-East, u_x, components of the movement of two PS overlying and close to the field together with the gas volume V_g stored in the reservoir, respectively, versus time. Fig. 5 provides the u_z and u_x pattern over the November 2005 - April 2006 and April 2006 - November 2006 production and injection periods, respectively. Note in Fig. 4 the good correspondence between the behavior of V_g and both u_x and u_z in connection with the injection and the removal phase. The vertical displacement follows closely the gas volume stored in the reservoir, i.e. land surface goes up when gas is injected and goes down when it is withdrawn. The horizontal movements depend on the actual measurement position relative to the reservoir gravity center: the PS located to the West (e.g., 000A3 in Fig. 4) moves westward and eastward, hence u_x decreases and increases, when the gas is injected and pumped out, respectively. The opposite behavior characterizes the PS to the East of the reservoir center of gravity. Based on the PSInSAR measurements, the UGS activities are responsible for an overall seasonal land movement of up to 8-10 mm and 6-8 mm in the vertical and Easting direction, respectively.

5. GEOMECHANICAL MODEL

Poroelasticity theory [14] predicts the 3D surface motion in response to stress changes in the gas storage reservoir and surrounding formations. The equilibrium equations governing the deformation of a mechanically isotropic medium read:

$$G\nabla^2\mathbf{u} + (G+\lambda)\nabla\mathrm{div}\mathbf{u} = \alpha\nabla p + \mathbf{b} \qquad (2)$$

where ∇^2 and ∇ are the Laplace and gradient operator, respectively, G and λ the shear modulus and Lamé constant of the medium generally dependent on the

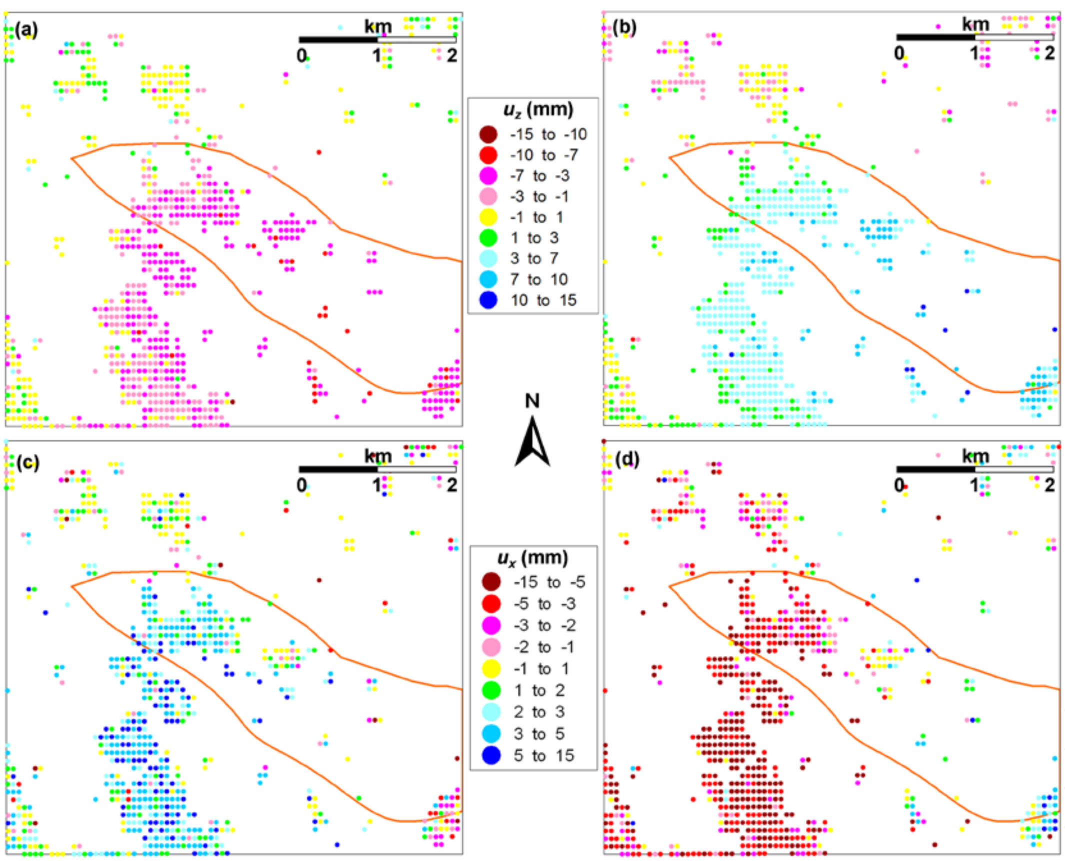

Fig. 5. (a,b) Vertical and (c,d) West-East horizontal displacements as measured by PSInSAR above the Lombardia field from November 2005 to April 2006 and from April 2006 to November 2006, respectively. Positive values mean uplift and eastward movement. The trace of the reservoir is shown.

stress path, **u** is the displacement vector, α the Biot coefficient, p the in-situ pore pressure variation provided by the fluid-dynamic model, and **b** the vector of the body forces. Eqs. (2) are solved in a 3D setting by linear FE following the infinite pore pressure gradient approach developed in [15] for a heterogeneous porous medium with nonlinear and hysteretic mechanical properties.

An isotropic stress-strain relationship is the most common assumption in reservoir geomechanics, also because only the vertical component of the land displacement or in-situ deformation is usually available for the model calibration. The present study addresses for the first time the calibration of a 3D geomechanical model using both the vertical and the horizontal components of the land surface motion due to fluid injection/production into/from a real reservoir in its complex geologic environment. To this aim, the medium is assumed to be transversely isotropic. This can be viewed as a generalization of a mechanically isotropic soil where the medium elastic properties in a horizontal plane (subindex h) differ from those in a vertical plane (subindex z). The constitutive matrix D relating the effective stress tensor $\boldsymbol{\sigma}$ to the displacement **u** via the strain tensor $\boldsymbol{\varepsilon}$ is given by:

$$D = \begin{bmatrix} \frac{1}{E_h} & -\frac{\nu_h}{E_h} & -\frac{\nu_z}{E_h} & 0 & 0 & 0 \\ -\frac{\nu_h}{E_h} & \frac{1}{E_h} & -\frac{\nu_z}{E_h} & 0 & 0 & 0 \\ -\frac{\nu_z}{E_h} & -\frac{\nu_z}{E_h} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu_h)}{E_h} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_z} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_z} \end{bmatrix}^{-1} \qquad (3)$$

with E_h and E_z, ν_h and ν_z, the Young and the Poisson moduli in a horizontal and vertical plane, respectively, and G_z the vertical shear modulus. The shear modulus G_h is dependent on E_h and ν_h via the usual relation $G_h=E_h(1+\nu_h)/2$. The mechanical parameters E and ν are related to c_M through the relationship [5]:

$$c_M = \frac{1}{E_z}\left(1 - \frac{2\nu_z^2}{1-\nu_h}\frac{E_h}{E_z}\right). \qquad (4)$$

Introducing the adimensional parameters $\beta=E_h/E_z$ and $\gamma=G_h/G_z$ and using eq. (1) the elastic matrix D for a transversally mechanical isotropic medium in unloading/reloading conditions depends on the 5 independent parameters β, γ, ν_h, ν_z, and s.

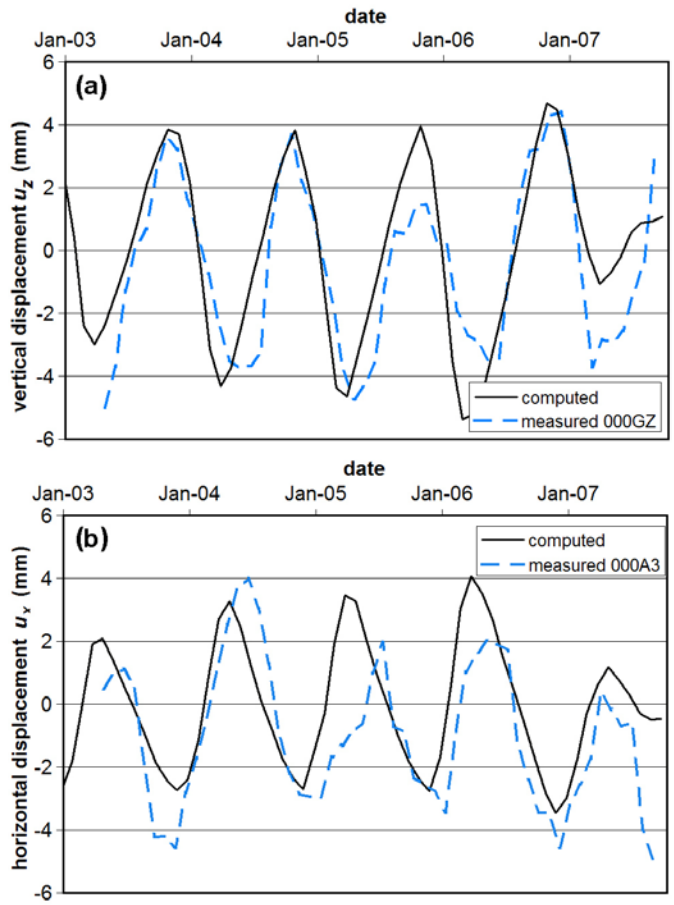

Fig. 6. (a) Seasonal behavior of the calculated and measured vertical land displacement for a PS located as shown in Fig. 7(a); (b) the same as (a) for the easting movement of the PS as shown in Fig. 7(c).

Eqs (2) is solved numerically by FE. For details about the numerical implementation of the transversal isotropy into the 3D FE model, see [6]. The model domain has an areal extent of 60×50 km centered on the reservoir and is confined above by the ground surface and a rigid basement 10 km deep below. The domain is discretized into 1568466 tetrahedra with 268558 nodes, with the mesh generated based on information on top and bottom of the Lombardia pools and surrounding aquifers.

Since no land displacement measurements are available in the study area before 2003, the geomechanical model has been calibrated over the UGS cycles from April 2003 to November 2007 using the PSInSAR measurements. We have elected to use eq. (1) to evaluate the geomechanical response of the field during the primary production life, i.e., in the virgin loading phase, and to calibrate the 5 parameters which the model response depends upon in the unloading/re-loading conditions only.

Comparison with the recorded displacements shows an excellent agreement of both u_z and u_x as predicted with s=4, i.e. a value that is consistent with the available geomechanical data, and the physically plausible values β=3, ν_h=0.15, ν_h=0.25, and γ=1. Fig. 6 shows an

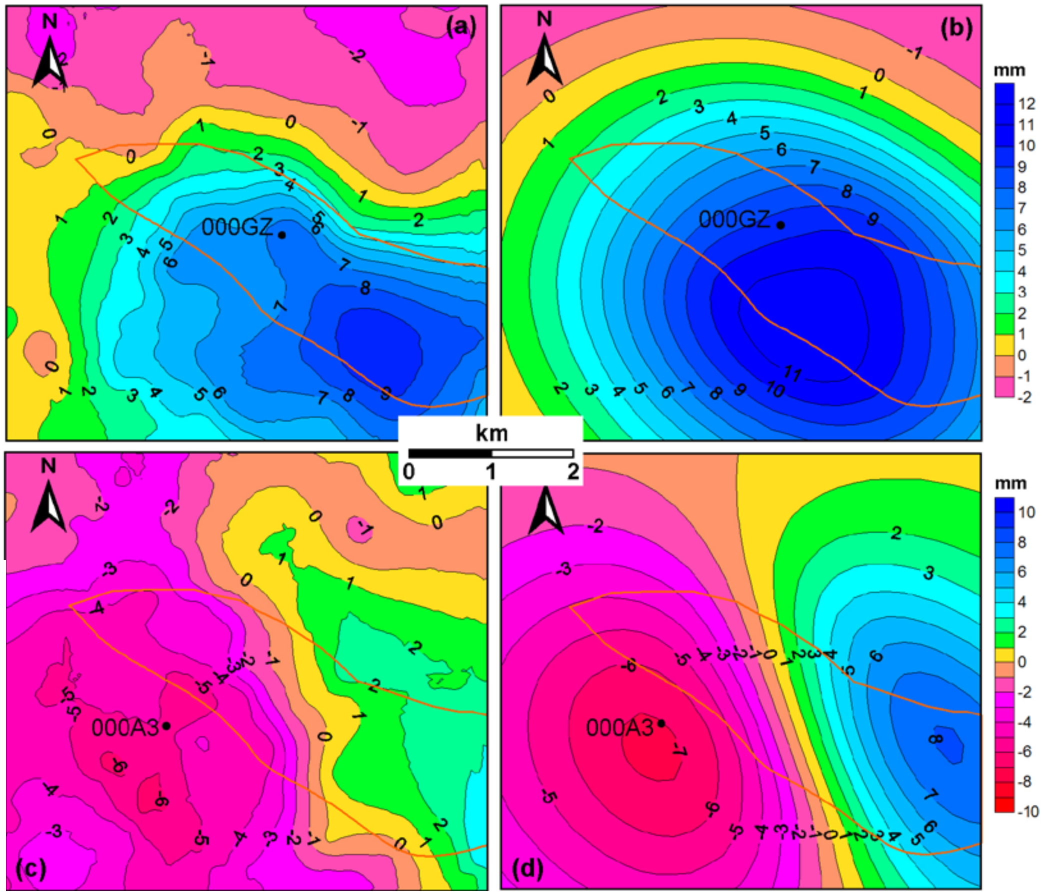

Fig. 7. Spatial land uplift (mm) from April 2006 to November 2006 as provided by the PS interpolation (a) and the model (b); (c) and (d) the same as (a) and (b) for the horizontal West-East displacement. The trace of the gas storage field is shown. Negative values mean downward and westward displacements.

example for the two PS provided in the inset of Fig. 4 and in Fig. 7. Fig. 7 shows the predicted land uplift and horizontal movement from April 2006 to November 2006 and the similar maps obtained by interpolation of the PSInSAR measurements. Interpolation has been made by the use of the kriging technique based on an analytical variogram with a nugget effect to filter out the local-scale measurement variabilities that are not related to the gas storage/production. On the other hand the above smoothing produces an attenuation of the

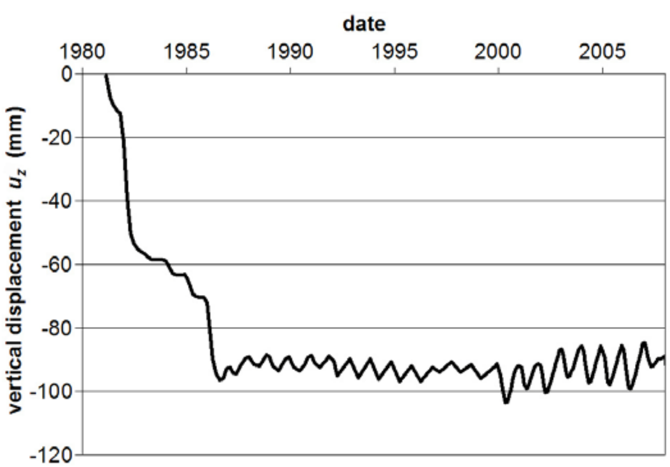

Fig. 8. Maximum vertical land displacement versus time as provided by the geomechanical model between 1981 and 2007.

observed largest displacements.

An interesting modeling result deals with the past history of the field production life and is concerned with the vertical displacement occurred from the inception of gas production to date. It is to be pointed out that the stress prediction at the end of primary production is of paramount importance to evaluate the geomechanical properties of the reservoir and the aquifer at the starting time for the geomechanical model calibration. The computed subsidence versus time during the primary gas production and the subsequent UGS activities is given in Fig. (8). Land settlement mainly developed before 1987 with no noticeable increase afterwards.

6. FUTURE GAS STORAGE SCENARIOS

The calibrated model is used to predict the expected geomechanical response of the Lombardia field to future UGS programs. In particular, two scenarios are addressed, a first where the gas pore overpressure is pushed to 107% of the original pressure prior to the field development and a second where the peak is further increased to 120%p_i. A constant amount of CH_4 is planned to be injected/produced each year from 2008 to 2040 (Fig. 3). These scenarios are of the greatest importance because they allow to increase the stored gas volume by approximately 65% and 180% relative to the current maximum storage value.

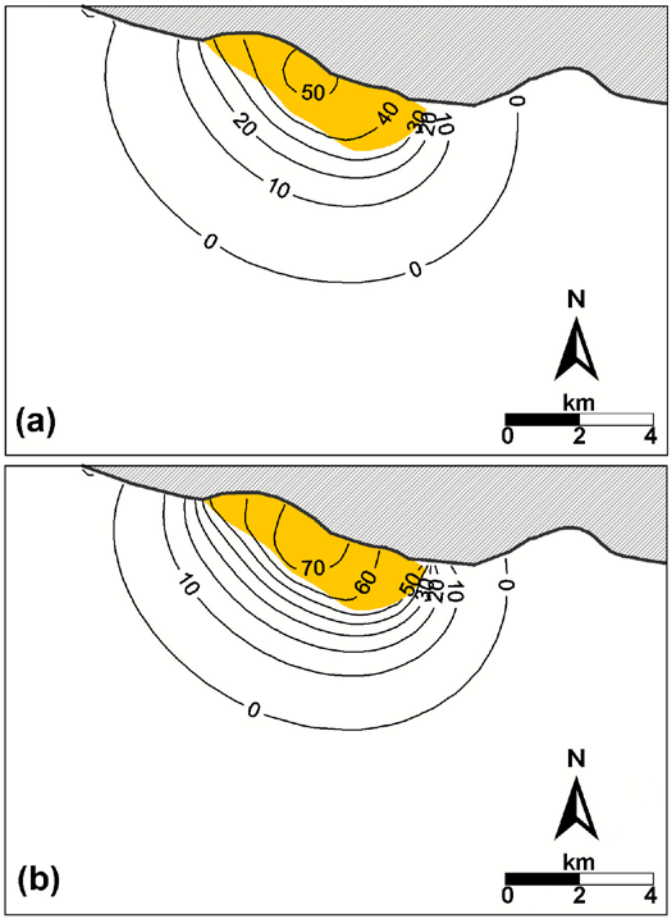

Fig. 9. Predicted overall pressure change (bar) in the C unit for the two scenarios (a) 107%p_i and (b) 120%p_i, respectively. The trace of the gas pool is shown in yellow. The grey area represents the northward closure of the aquifer against the Santerno formation.

The overall pore pressure change for each single UGS phase, i.e. the difference between the maximum and minimum p experienced by the C unit each year, is provided in Fig. 9.

The u_z modeling prediction in Fig. 10. The predicted overall vertical land displacement during each injection/production cycle is expected to increase to 17 mm and 27 mm for the two UGS scenarios addressed by the study.

7. CONCLUSIVE REMARKS

Simulation of seasonal gas storage in underground reservoirs is a complex multidisciplinary effort that involves both fluid-dynamical and geomechanical simulations that may prove helpful to evaluate the working gas volume, the safety of the disposal, and the impact of land motion on the surface structures-infrastructures and the environmental ecosystem.

The geomechanical response to cyclic gas storage has been investigated for a depleted gas field located in the Po River basin, Italy. The use of i) the geomechanical dataset from previous studies of the Po River basin, ii) a detailed knowledge of the subsurface geology made available by 3D seismic surveys, iii) almost 30-yr measurements related to the gas field activities, and iv) an advanced PSInSAR analysis providing the vertical and horizontal West-East displacements of the ground surface above the field from 2003 to 2007 has allowed for the development and calibration of an ad hoc transversally isotropic nonlinear FE geomechanical model that has proven successful in reproducing both the observed land subsidence/uplift and West-East displacements due to the UGS activities. Two UGS scenarios where the gas pore overpressure is pushed from the current 103%p_i to 107%p_i and 120%p_i are addressed.

For the first time the present study demonstrates i) the capability of a PSInSAR analysis to reveal the temporal long-term vertical and West-East horizontal displacements as a response to (seasonal) changes of gas pore pressure in storage reservoirs; and ii) the possibility to match the full geomechanical response over a depleted reservoir subject to fluid injection/production in a 3D setting with a transversally isotropic

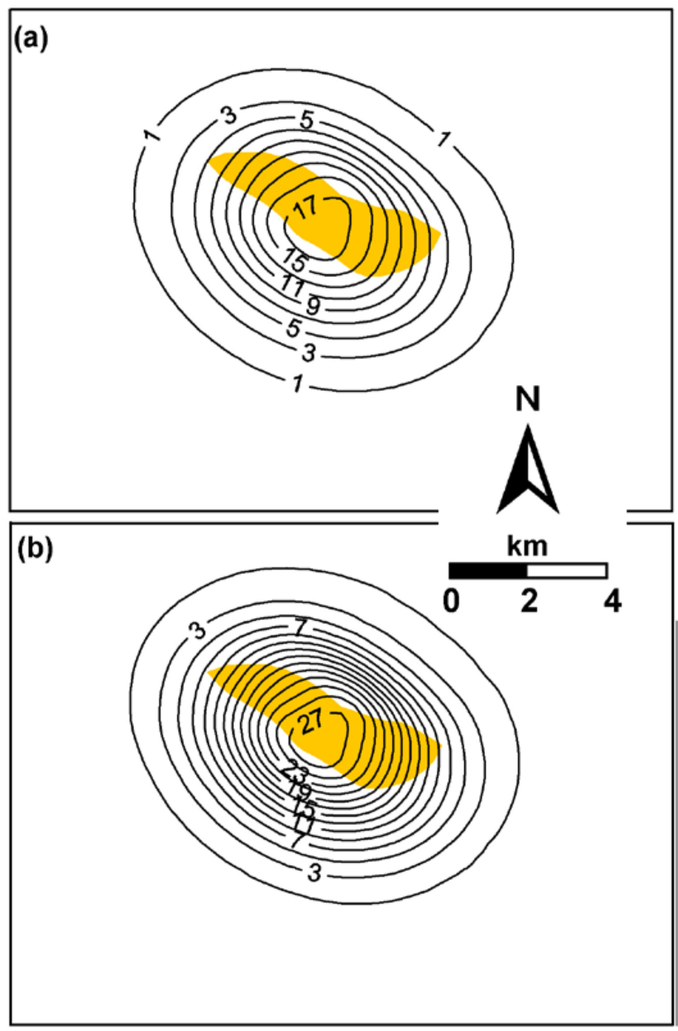

Fig. 10. Absolute value of the predicted vertical ground displacements (mm) during each injection/production cycle for the UGS scenarios with an overpressure at (a) 107%p_i and (b) 120%p_i.

geomechanical model.

For the Lombardia field case, the result emphasize that the UGS scenarios investigated in the present study do not produce significant land movements above the storage field and hence are not expected to impact seriously on the surface man-made structures-infrastructures and the natural environment. The maximum gradient of the predicted ground displacements is 3.8×10^{-6} and 5.9×10^{-6} for the 107%p_i and 120%p_i scenario, respectively. These values are two orders of magnitude smaller than the most severe limits required for the safety of masonry buildings with more than one floor, for which the relevant literature indicates 5×10^{-4} [16].

REFERENCES

1. Buschback, T.C., and D.C. Bond. 1967. *Underground storage of natural gas in Illinois – 1967*. Urbana (IL): Illinois State Geological Survey Illinois Petroleum 86.
2. International Gas Union. 2009. Underground Storage of Gas, Report of Working Committee 2, Triennium 2006-2009. In *Proceedings of the 24rd World Gas Conference, Buenos Aires (Argentina), 5 – 9 October 2009*, 55 pp.
3. De Tomasi, V., L. Kovacic, and E. Visgalli. 2002. *Concessione "Lombardia" Stoccaggio - Interpretazione sismica 3D*. ENI Report, June 2002, Vol. 2c.
4. Ferretti, A., C. Prati, and F. Rocca. 2001. Permanent scatterers in SAR interferometry. *IEEE T. Geosci. Remote Sensing* 39: 8–20.
5. Gambolati, G., P. Gatto, and G. Ricceri. 1986. Land subsidence due to gas-oil removal in layered anisotropic soils by finite elements. In *Land Subsidence*, eds A.I. Johson et al, 29–41. IAHS Publ. n. 151.
6. Janna, C., M. Ferronato, P. Teatini, and G. Gambolati. 2010. A geomechanical transversally isotropic FE model for the Po River basin, Italy, using PSInSAR-derived horizontal land displacements. *Int. J. Solids Struct*. submitted.
7. Ferronato, M., G. Gambolati, P. Teatini, and D. Baù. 2003. Interpretation of radioactive marker measurements in the Northern Adriatic gas fields. *SPE Reserv. Eval. Eng.*, December: 401–412.
8. Baù D., M. Ferronato, G. Gambolati, and P. Teatini. 2002. Basin-scale compressibility of the Northern Adriatic by the radioactive marker technique. *Géotechnique*, 52: 605–616.
9. Ferronato, M., G. Gambolati, P. Teatini, and D. Baù. 2003. Unloading-reloading uniaxial compressibility of deep reservoirs by marker measurements. In Proc. 11th Int. Symp. on Deformation Measurements, eds S.C. Stiros and S. Pytharouli, 341–346. Patras University Publ.
10. Teatini, P., D. Baù, and G. Gambolati. 2000. Water-gas dynamics and coastal land subsidence over Chioggia Mare field, northern Adriatic Sea. *Hydrogeol. J.*, 8: 462–469.
11. Stright Jr., D.H., A. Settari, D.A. Walters, and K. Aziz. 2005. Characterization of the Pliocene gas reservoir aquifers for predicting subsidence on the Ravenna coast. In *Land Subsidence - Multi-disciplinary Assessment of Subsidence Phenomena in the Ravenna Area*, eds F.B.J. Barends et al, 19–33. Millpress Science Publ.
12. Baù, D., G. Gambolati, and P. Teatini. 1999. Residual land subsidence over depleted gas fields in the Northern Adriatic basin. *Environ. Eng. Geosci.*, V: 389–405.
13. Ferretti, A., G. Savio, R. Barzaghi, A. Borghi, S. Musazzi, F. Novali, C. Prati, and F. Rocca. 2007. Sub-millimeter accuracy of InSAR time series: Experimental validation. *IEEE T. Geosci. Remote*, 45: 1,142–1,153.
14. Biot, M. A. 1941. A general theory of three-dimensional consolidation. *J. Appl. Phys.*, 12: 155–164.
15. Gambolati, G., M. Ferronato, P. Teatini, R. Deidda, and G. Lecca. 2001. Finite element analysis of land subsidence above depleted reservoirs with pore pressure gradient and total stress formulations. *Int. J. Numer. Anal. Meth. Geomech.*, 25: 307–327.
16. Skempton, A.W., and D.H. Mc Donald. 1956. Allowable settlement of buildings. *Proc. Inst. Civil. Eng.*, 5: 727–768.