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3D truss topology optimization for automated robotic spatial extrusion

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Abstract

This paper contributes new knowledge and results to the fields of topology optimization and robotic spatial extrusion through the consideration of how these methods can be used together. Specifically, a new topology optimization formulation is presented that accounts for the manufacturing constraints of uniform cross section and average member length. In addition, a new robotic assembly planning framework is demonstrated, which allows the complex but structurally efficient results of the topology optimization to be materialized in a reasonable amount of time. Three novel case studies produced by the proposed topology optimization framework are presented to demonstrate how automated assembly planning and robotic extrusion can enable a direct and efficient means to materialize a 3D topology-optimized truss.

Keywords: topology optimization, robotic extrusion, assembly planning

1. Introduction

Topology optimization of continuum structures has received widespread adoption in the design of innovative and efficient structural systems, devices and material architectures [1]. The increased interest in this rigorous, freeform design method has partially been fueled by the development of additive manufacturing technologies, which has made fabrication of the typically complex designs possible. In contrast, while the theoretical development of truss-based topology optimization has preceded continuum-based methods, truss-based optimization has not received extensive adoption in industry because of the lack of a suitable construction method. Typical digital fabrication techniques such as layer-based additive manufacturing (or 3D printing) are not appropriate for topology-optimized structures, since the anisotropy incurred by the material deposition process undermines the theoretical material savings [2]. This paper demonstrates how a newer additive manufacturing process called robotic spatial extrusion can be used to materialize topology-optimized structures. While this fabrication method has been used in recent years to make standard and geometrically morphed lattice structures, this paper is the first to show how it can also be used for nonstandard and optimized topologies.

This paper represents the first attempt in the field to incorporate an automated robotic assembly planning system into an integrated digital design, optimization, and fabrication process. The proposed digital process incorporates two main contributions: (1) the incorporation of robotic extrusion constraints in the existing truss-based topology optimization workflow, and (2) an automated robotic assembly planning system. The presented work emphasizes the mutual constraining relationship between topology optimization and assembly planning, and presents three novel case studies to exemplify the power of this integrated computational design-build framework.

2. Literature Review

The aim of this section is to identify the gap between the exisiting development of 3D truss topology optimization and robotic extrusion, and demonstrate why an integration between the two fields is needed to push the boundary of freedom and efficiency that can be achieved.

2.1. Truss topology optimization

A popular method for truss topology optimization is the ground structure approach [3]. In the ground structure approach, the designer defines a design domain with applied loads and boundary conditions. This domain is then populated with nodes and potential bars in a ground structure mesh. By defining a formal optimization problem and using a mathematical program, the potential bars in the truss are sized. The minimum cross-sectional area is allowed to approach zero and the resulting connectivity of thicker members define the optimized truss topology. Figure 1 illustrates the process for a 2D example.

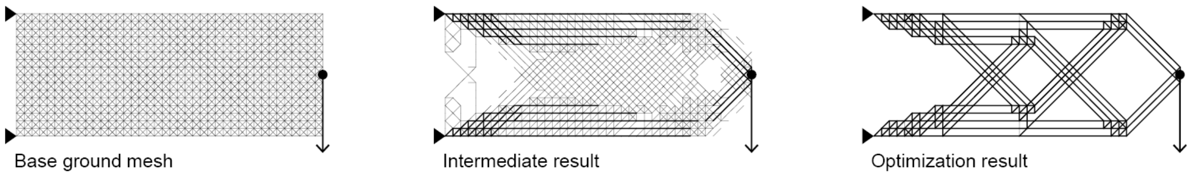

Figure 1: topology optimization process for a 2D cantilever setup

Several researchers have developed ground-structure-based truss topology optimization frameworks. Most work has only been demonstrated on planar 2D examples since the methodologies in theory extendable to 3D [4]. Some work has been suggested to improve the stability and constructability of topology-optimized trusses. Achtziger and Stolpe [5] have suggested a global optimization algorithm for discrete area constraints. Incorporation of local stability constraints have been suggested by several researchers [6]–[9] and algorithms that enforce a global stability constraint to ensure the design of a space truss have been proposed [10]. One challenge with local constraints in existing work is that the formulation often results in prohibitively slow convergence that precludes practical use of the algorithm in design iteration processes. Local buckling is an example of a local constraint that can be solved in theory but typically results in extremely slow runtimes. Finally, there are few examples of case study or benchmark problems for truss topology optimization in 3D in published literature.

2.2. Robotic extrusion

Robotic extrusion, sometimes called spatial 3D printing, involves extruding and solidifying thermoplastic along prescribed linear paths in space, typically to form spatial meshes or lattices. This fabrication technique takes advantage of robot's precision and speed and has been proposed as a compelling alternative to layer-based additive manufacturing for materializing discrete spatial structures [11], [12]. Most of the existing work has been focused on utilizing the industrial robot's flexibility in facilitating shape's complexity and size (as opposed to topology), morphing grids with standard topology to achieve structural efficiency [13] or visual variantion [14], [15]. In most of this exisiting work, the robot's end effector has fixed or limited orientations during the entire printing process. This means that these multi-axis machines are still used as a 3-axis gantry machine and its dexterity and flexibility are far from fully ultilzed.

This underultilization is caused by the computational challenge of robotic assembly planning. In contrast to simple Cartesian instructions for a 3-axis CNC machine, the dexterity that originates from the industrial robot's multiple axes requires careful planning of the robot's joint trajectory. The planning problem is exacerbated by the need for finding an assembly sequence that allows enough workspace for the robot to operate during each extrusion step. Most of the exisiting work involves manual planning of the assembly sequence. Specifically, Søndergaard et al. [16], [17] used a robot to assemble topology-optimized spatial trusses using manually planned sequence and trajectory. While this manual planning method works for designs with standard topology and/or a small number of elements, the assembly sequence and robotic motion planning is much more nuanced for designs with denser material distribution and non-standard topology, especially for the results generated from topology optimization.

Recently, there has been some success in tackling this extrusion planning problem using computational methods. Huang et al., Wu et al. and Gelber et al. [18]–[20] have printed irregular topologies in which only the outer surface of a shape is materialized. However, none of this previous work considers the planning of entire robotic trajectories. Recently, the authors of this paper introduced an assembly planning framework called Choreo that integrates assembly sequence and robotic motion planning, and applied it to robotic extrusion of architectural structures with nonstandard topology [21]. The current paper serves to show the new fabrication possibilities that are enabled by this new assembly planning technique in the context of topology optimization and is dedicated to showing the new constraints associated with robotic extrusion that must be formulated in the topology optimization framework.

3. Methodology

This section presents the original contributions of the paper: first, a topology optimization formulation that accounts for manufacturability in robotic extrusion, and second, a robotic motion planning method that enables topology-optimized structures to be fabricated.

3.1. Topology optimization

The general topology optimization problem used in this work is formulated as a minimum weight problem subject to a user-specified compliance constraint C_{max}:

$$\begin{aligned} \min_{\rho^e} \quad & f = \sum \rho^e A^e l^e \\ s.t. \quad & \mathbf{K}(\rho^e)\mathbf{d} = \mathbf{F} \\ & \mathbf{F}^{\mathbf{T}}\mathbf{d} \leq C_{max} \\ & 0 \leq \rho^e \leq 1 \qquad \text{for all } e \in \Omega \end{aligned} \tag{1}$$

where ρ^e are the design variables that determine if an element e is active in the design. The objective function f is taken as the sum of the volume of all active elements within the design domain Ω. The equilibrium condition is solved by $\mathbf{K}(\rho^e)\mathbf{d} = \mathbf{F}$ where $\mathbf{K}(\rho^e)$ is the global stiffness matrix, $\mathbf{d}$ is the global displacement vector, and $\mathbf{F}$ is the vector with applied forces. The discrete constraint on the design variables is relaxed to allow for the use of gradient-based optimizers and the design variables are instead bound by the limits 0 and 1. All work herein uses MMA [22] as the gradient-based optimizer and all sensitivities are calculated using the adjoint method.

To improve the manufacturability for robotic extrusion, additional fabrication considerations are incorporated into the problem defined in Eq. 1. All structures herein have been designed with a local connectivity in the ground structure mesh, implicitly controlling the maximum numbers of bars connected at each node. In addition, a uniform cross section is enforced on all members. Since only existence or non-existence of a potential bar is considered, the Solid Isotropic Material Penalization (SIMP) [23] scheme is used to guide the design variable toward a 0-1 solution. SIMP is commonly used in many continuum topology optimization algorithms and relates the element density to its stiffness $\mathbf{K_0}^e(\rho^e)$ as follows:

$$\mathbf{K}^e(\rho^e) = ((\rho^e)^\eta + \rho_{min})\, \mathbf{K}_\mathbf{0}^e \tag{2}$$

where $\mathbf{K_0}^e$ is the stiffness of a solid element and η is the user-specified SIMP penalization factor. To ensure positive-definiteness of the global stiffness matrix, ρ_{min} is chosen as a small positive number. $\rho_{min} = 10^{-4}$ and $\eta = 3$ are used for all the work in this paper.

The final topology might have several shorter members that are connected in chains. These elements are merged in a post-processing step, commonly referred to as hinge cancellation. For design problems with local buckling constraints, Achtziger [8], [9] suggested a model that modifies the lengths of elements in chains in each iteration. However, since variations in the cross-sectional area of the bars are not allowed, a buckling constraint might allow long members with very low force. Therefore, the chain model from

[8], [9] is used to define a constraint on the maximum length of an element chain. To ensure a fast convergence of the algorithm, a globalized constraint using a P-norm formulation is proposed:

$$c = \left(\sum \rho^e (l^e_{chain})^P\right)^{1/P} - \left(\sum \rho^e\right)^{1/P} l_{avg} \leq 0 \tag{3}$$

where l^e_{chain} is the length of the entire chain and l_{avg} is a user-specified maximum value for the average element length. In this work, P-norm value $P = 2$ is used for all designs. The sensitivity of Eq. 3 is found by differentiation:

$$\frac{\partial c}{\partial \rho^e} = \frac{1}{P}\left(\left(\sum \rho^e (l^e_{chain})^P\right)^{\frac{1}{P}-1} (l^e_{chain})^P - \left(\sum \rho^e\right)^{\frac{1}{P}-1} l_{avg} \right) \tag{3}$$

The above constraint is formulated for restricting members that are too long to be printed, but can also be seen as an informal restriction on local buckling. As noted in Section 2, a full local buckling check is computationally expensive because the number of members in the ground structure tends to be quite large. Solving local buckling in a more rigorous but quick way is left for future work.

3.2. Robotic extrusion planning

The efficient structures generated by topology optimization often come with non-trivial topology that give no clues on finding a feasible extrusion sequence, which requires computing a chronological construction sequence of motions to extrude each element as well as moving through free space to connect adjacent extrusion processes. In contrast to previous work where the robot's end effector is fixed throughout the entire process, the computational complexity involved in this extrusion planning problem is propagated along multiple levels of a branched search tree: extrusion sequence, end effector pose and robot joint configuration and transition trajectory.

To solve this combined task and motion planning problem, a new assembly planning framework, called Choreo, is proposed and implemented to harness this computational complexity. As shown in Figure 1, there are four broad steps used in this workflow. First, a spatial truss model that consists of linear elements is generated from a topology optimization. The second and third step is carried out in Choreo, without the need for human intervention: a feasible extrusion sequence is automatically generated, and the robot's path and instructions are automatically planned. The planning output is tagged with metadata so that users can easily weave in hardware control commands and fabrication-informed micro path modifications using any programming platform to synchronize the robot's trajectory and the end effector's behavior. Finally, using an existing robot-brand-specific post-processor, an executable robot instruction can be harvested and uploaded to a robot controller for execution.

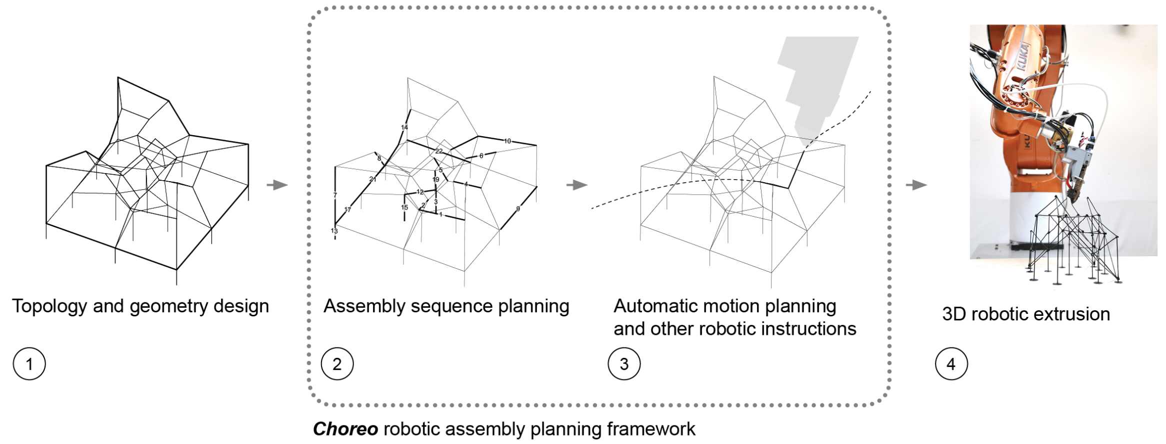

Figure 2: Overview of the robotic extrusion workflow including the automated planning system

Choreo is powered by a three-layer hierarchical assembly planner. First, a constraint-based sequence planner is introduced to search the extrusion sequence while guaranteeing the intermediate construction's stiffness and stability as well as enough workspace for a collision-free robot kinematic solution at each extrusion step. Then, a sampling-based semi-constrained Cartesian planner is proposed to resolve redundant degrees of freedom and to compute the robot's joint configuration during each extrusion step. Finally, a state-of-the-art, open-source motion planning algorithm is called to compute the robot's trajectory to navigate through free space and connect adjacent extrusions.

The assembly planning platform Choreo is implemented in C++ on Robot Operating System (ROS) [24]. Choreo's system architecture is designed to be modularized and adaptable, which offers users and researchers the flexibility to plug in and experiment with their customized sequence or motion planner to adapt to their own robotic assembly application without changing the entire system's codebase. Choreo can be easily configured to support 6- or 7-axis industrial robots of any brand with any user-defined end effector. Readers are referred to an in-progress paper [25] for a more detailed description on Choreo's algorithms and implementation.

4. Case studies

This section presents three case studies for canonical structural conditions, demonstrating that topology-optimized spatial trusses can be designed and materialized in realistic timescales for a design environment. While the scale of these structures is small in this paper (as shown in Table 1), there is potential to use this design and fabrication method for full-scale structures in the future.

The resulting designs – shown in Figures 2, 3, and 4 – appear intuitively to be efficient structural designs, although they are nevertheless different from what the authors would have designed by hand. The large member and node counts might be rationalized and reduced if the structures were to be built conventionally with individual members and joints, but the robotic extrusion process eliminates this concern, allowing for a more intricate and delicate structural vocabulary.

Table 1: Statistics of the case studies. All computational experiments were performed on a Linux virtual machine with 4 processors and 16 GB setup on desktop PC with a quad-core Intel Xeon CPU.

Model	Element count	Topology optimization time [s]	Choreo planning time [min]	Fabrication time [hr]	Size [mm]
Vault	112	24	25	1.25	150 x 150 x 150
3D cantilever	145	182	31	2.25	105 x 105 x 105
Simply-supported beam	271	37	72	3.40	400 x 100 x 100

5. Conclusion

This paper contributes new knowledge and results to the fields of topology optimization and robotic spatial extrusion through the consideration of how these methods can be used together. Specifically, a new topology optimization formulation is presented that accounts for the manufacturing constraints of uniform cross section and average member length. In addition, a new robotic assembly planning framework is demonstrated, which allows the complex but structurally efficient results of the topology optimization to be materialized in a reasonable amount of time.

Limitations include the small physical scale of the current work as well as the need to incorporate local buckling constraints more directly into the algorithm. Future work can include demonstrating this design and fabrication method on a larger scale approaching that of the built environment, validating the results through physical load testing, and refining the optimization formulation to consider buckling.

In conclusion, this research demonstrates that new manufacturing and fabrication methods for architectural construction can transform how existing design methods are used. While topology optimization theory has existed since the 1960s and has been used conceptually to inform design, robotic fabrication offers a new means to realize the potential of this powerful technique directly.

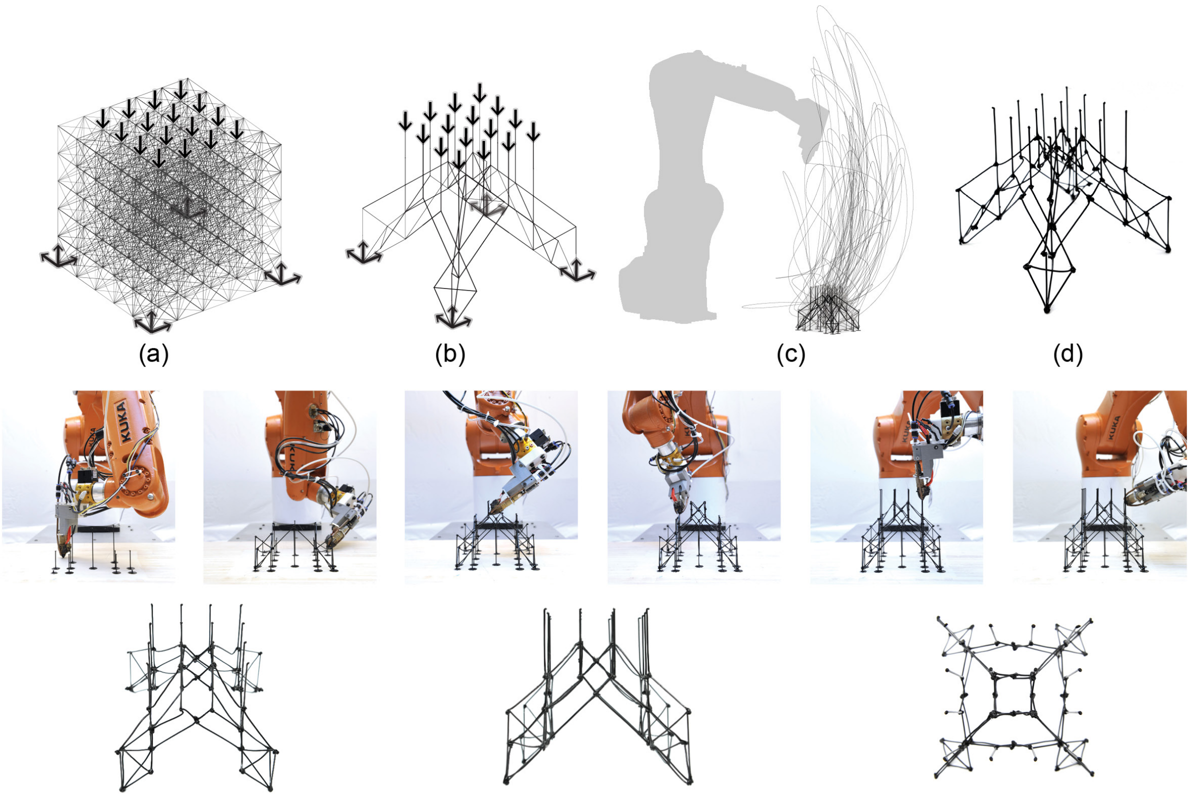

Figure 2: *Vault* case study, with (a-b) topology optimization input and result, (c) robot trajectories, and (d) final extruded result

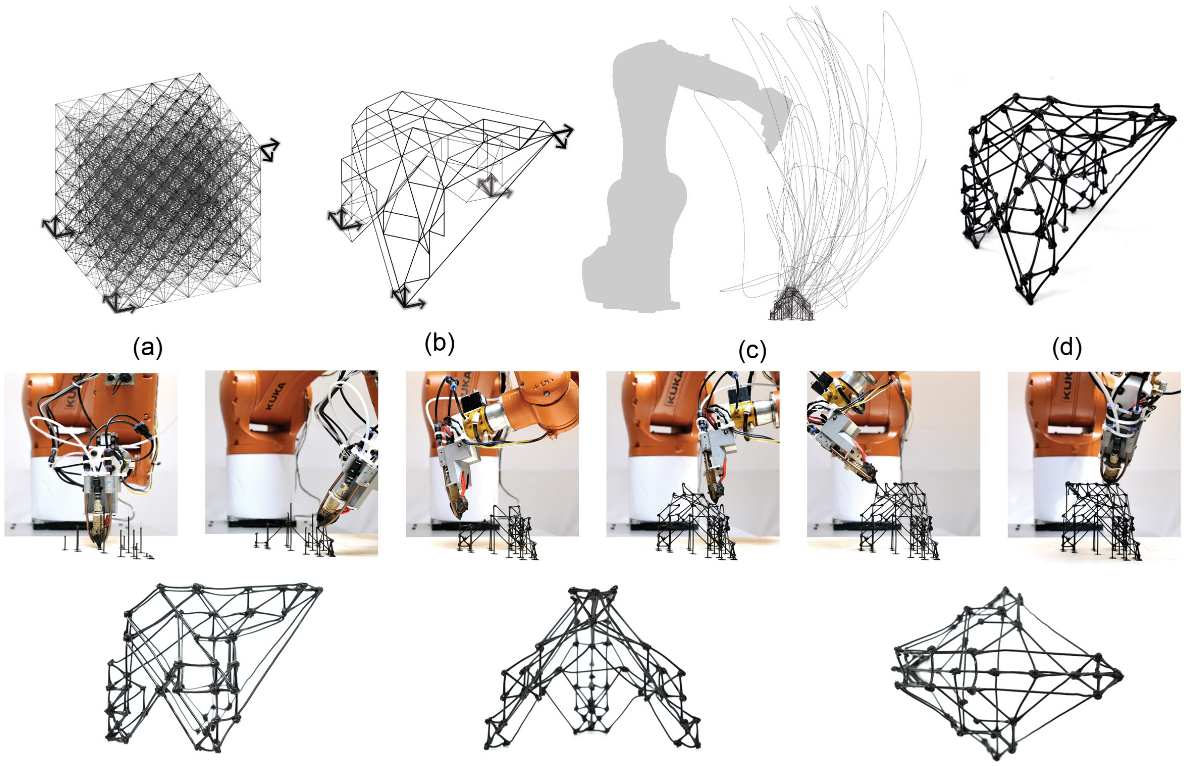

Figure 3: *3D cantilever* case study, with (a-b) topology optimization input and result, (c) robot trajectories, and (d) final extruded result

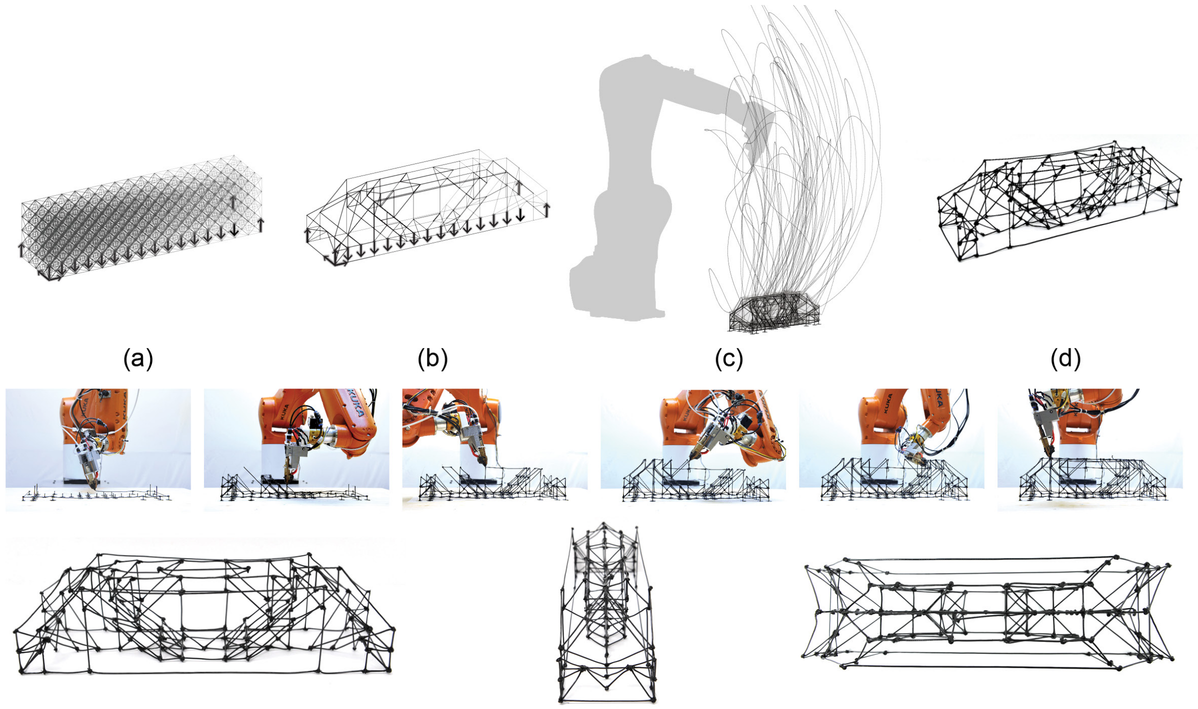

Figure 4: *Simply supported beam* case study, with (a-b) topology optimization input and result, (c) robot trajectories, and (d) final extruded result

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