4-1 Identifying Linear Functions

Warm Up

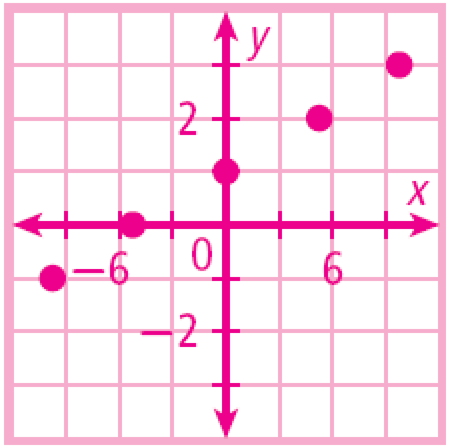
Lesson Presentation

Lesson Quiz

Warm Up

1. Solve 2x – 3y = 12 for y. $y = \frac{2}{3}x - 4$

2. Graph $y = \frac{1}{5}x + 1$ for D: {–10, –5, 0, 5, 10}.



Objectives

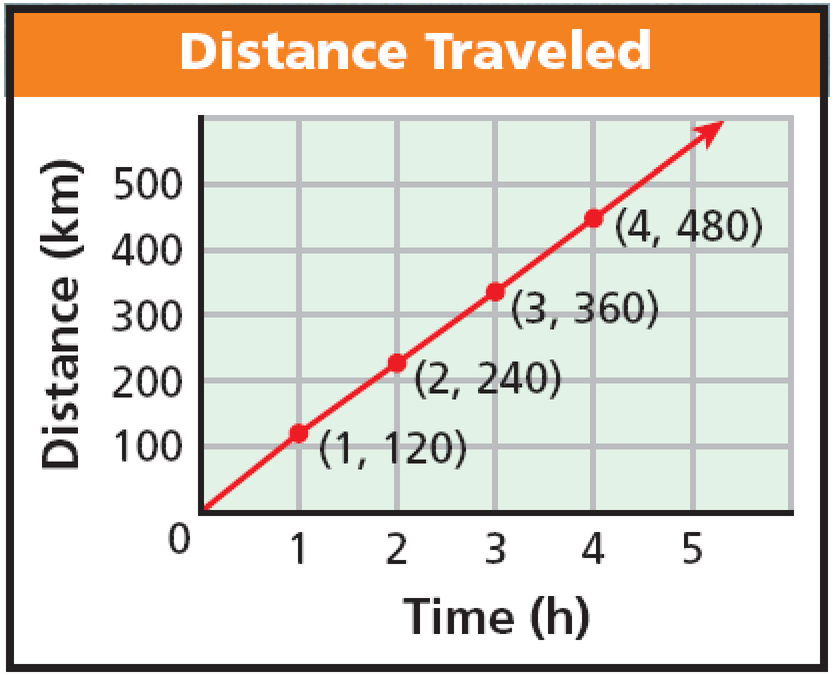
Identify linear functions and linear equations.

Graph linear functions that represent real-world situations and give their domain and range.

Vocabulary

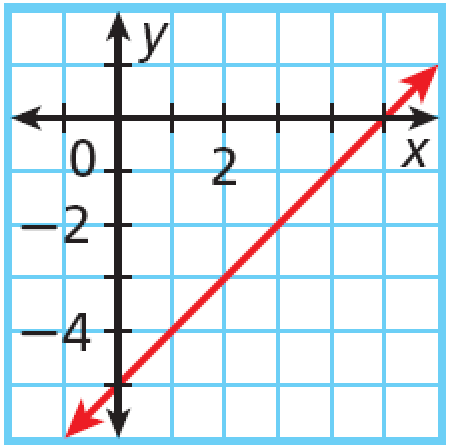
linear function
linear equation

The graph represents a function because each domain value (*x*-value) is paired with exactly one range value (*y*-value). Notice that the graph is a straight line. A function whose graph forms a straight line is called a **linear function**.



Example 1A: Identifying a Linear Function by Its Graph

Identify whether the graph represents a function. Explain. If the graph does represent a function, is the function linear?

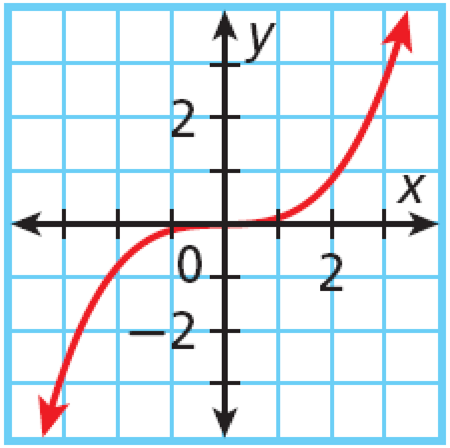


Each domain value is paired with exactly one range value. The graph forms a line.

linear function

Example 1B: Identifying a Linear Function by Its Graph

Identify whether the graph represents a function. Explain. If the graph does represent a function, is the function linear?

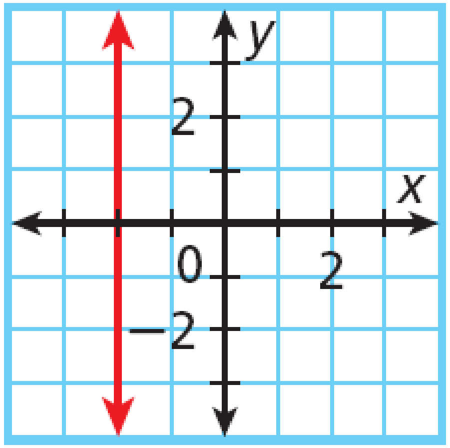


Each domain value is paired with exactly one range value. The graph is not a line.

not a linear function

Example 1C: Identifying a Linear Function by Its Graph

Identify whether the graph represents a function. Explain. If the graph does represent a function, is the function linear?

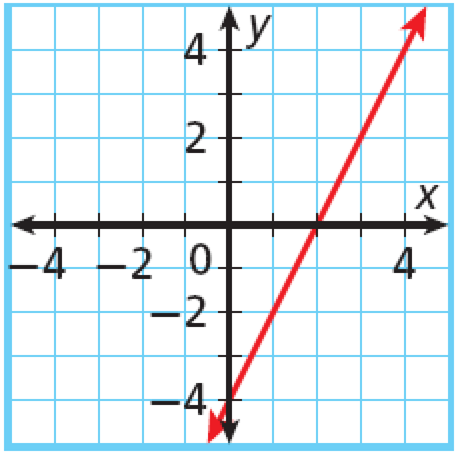


The only domain value, –2, is paired with many different range values.

not a function

Check It Out! Example 1a

Identify whether the graph represents a function. Explain. If the graph does represent a function, is the function linear?

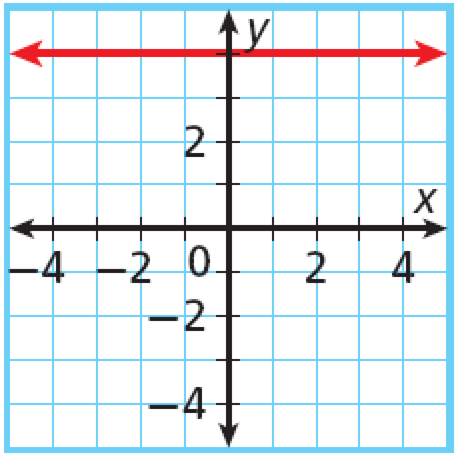


Each domain value is paired with exactly one range value. The graph forms a line.

linear function

Check It Out! Example 1b

Identify whether the graph represents a function. Explain. If the graph does represent a function, is the function linear?

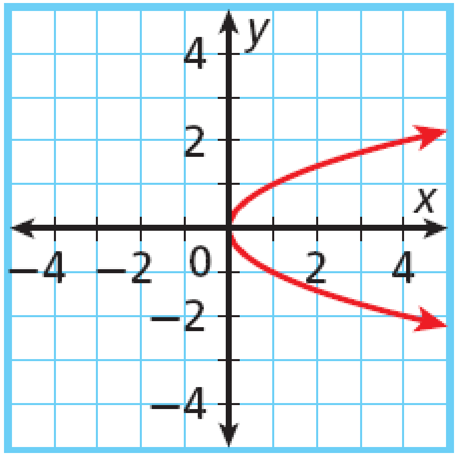


Each domain value is paired with exactly one range value. The graph forms a line.

linear function

Check It Out! Example 1c

Identify whether the graph represents a function. Explain. If the graph does represent a function, is the function linear?

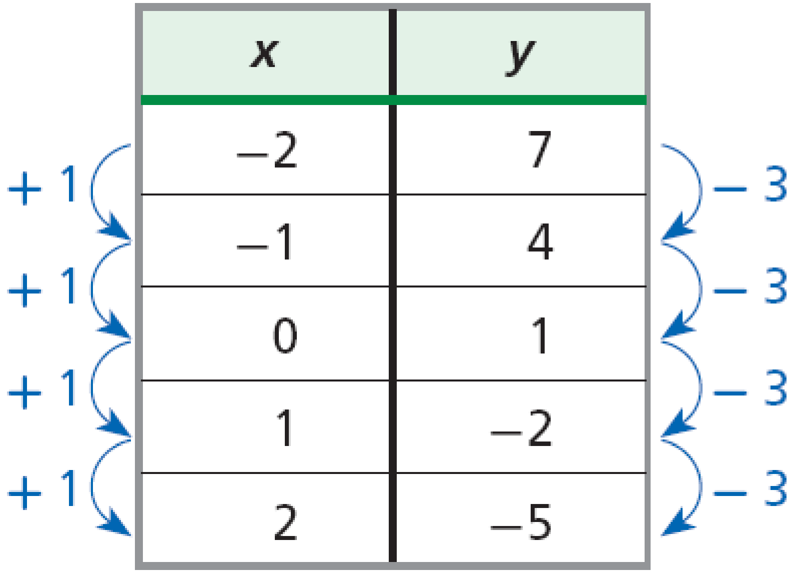


Each domain value is not paired with exactly one range value.

not a function

You can sometimes identify a linear function by looking at a table or a list of ordered pairs. In a linear function, a constant change in x corresponds to a constant change in y.

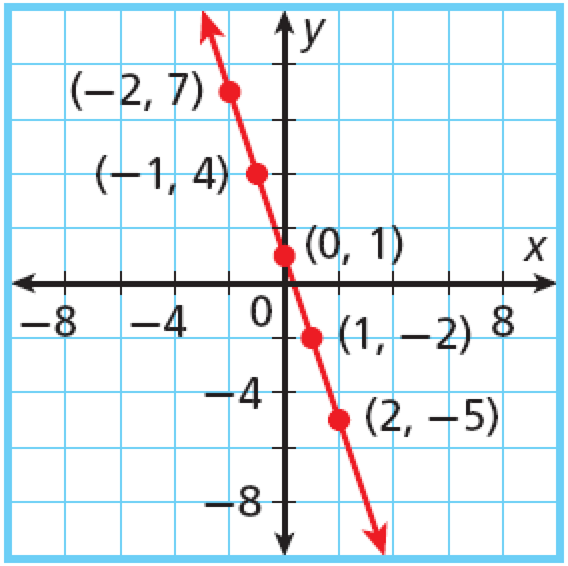
4-1 Identifying Linear Functions



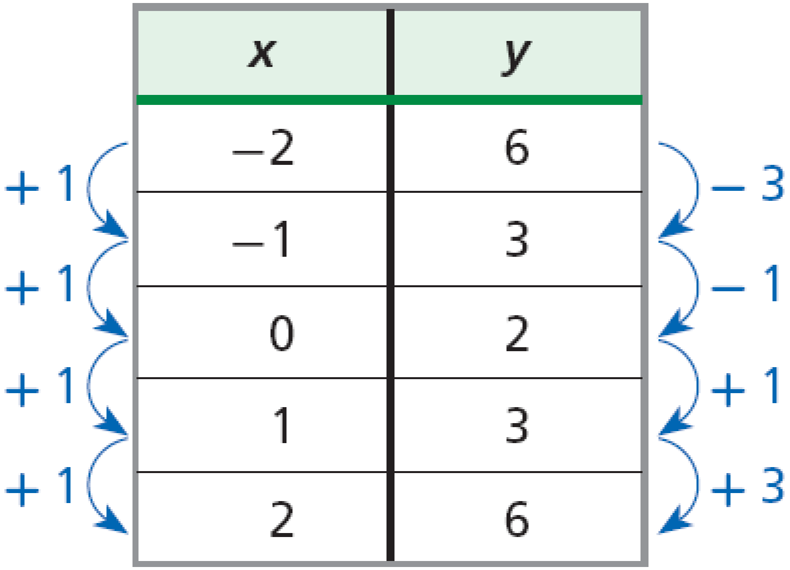
x	y
–2	7
–1	4
0	1
1	–2
2	–5

The points from this table lie on a line.

In this table, a constant change of +1 in *x* corresponds to constant change of –3 in *y*. These points satisfy a linear function.

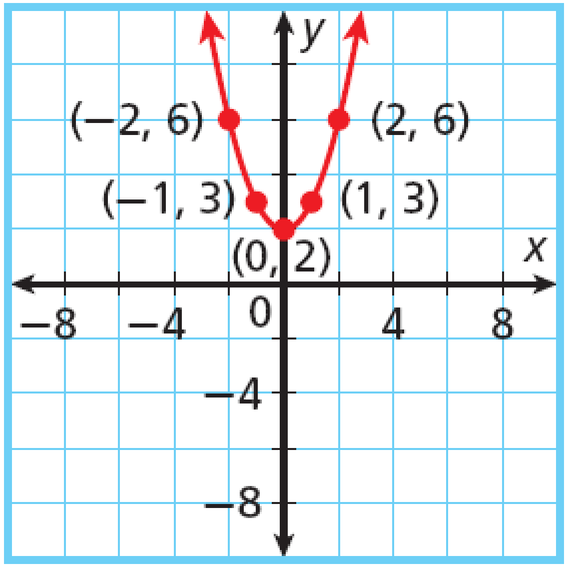


4-1 Identifying Linear Functions



x	y
−2	6
−1	3
0	2
1	3
2	6

The points from this table do not lie on a line.

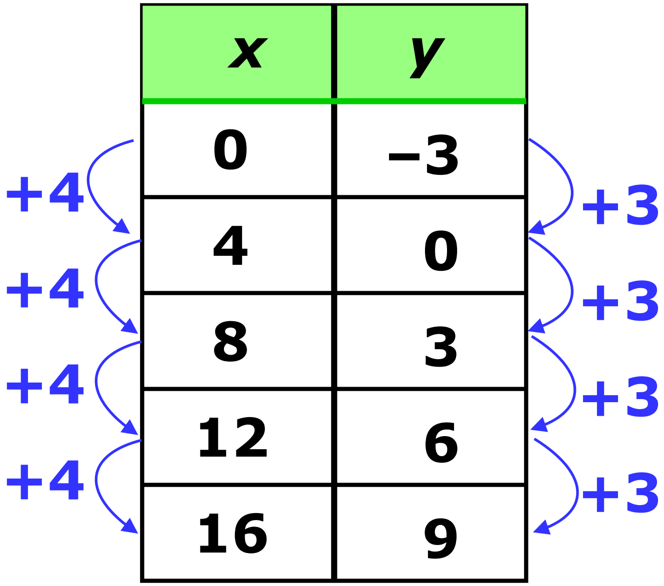


In this table, a constant change of +1 in *x* does *not* correspond to a constant change in *y*. These points do *not* satisfy a linear function.

Example 2A: Identifying a Linear Function by Using Ordered Pairs

Tell whether the set of ordered pairs satisfies a linear function. Explain.

$\{(0, -3), (4, 0), (8, 3), (12, 6), (16, 9)\}$



x	y
0	–3
4	0
8	3
12	6
16	9

Write the ordered pairs in a table.

Look for a pattern.

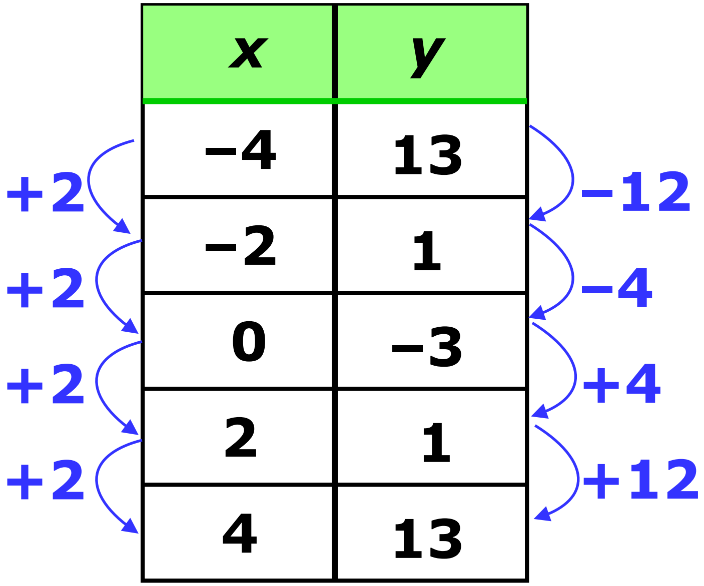
A constant change of +4 in x corresponds to a constant change of +3 in y.

These points satisfy a linear function.

Example 2B: Identifying a Linear Function by Using Ordered Pairs

Tell whether the set of ordered pairs satisfies a linear function. Explain.

{(–4, 13), (–2, 1), (0, –3), (2, 1), (4, 13)}



x	*y*
–4	13
–2	1
0	–3
2	1
4	13

Write the ordered pairs in a table.

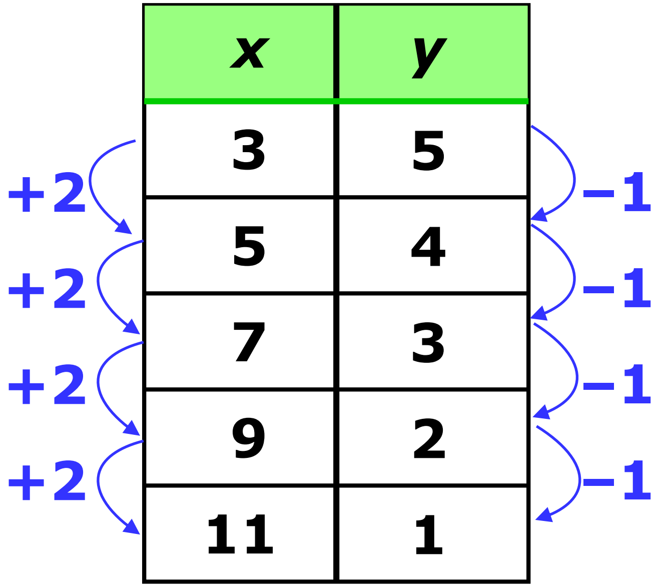
Look for a pattern.

A constant change of 2 in x corresponds to different changes in y.

These points do not satisfy a linear function.

Check It Out! Example 2

Tell whether the set of ordered pairs {(3, 5), (5, 4), (7, 3), (9, 2), (11, 1)} satisfies a linear function. Explain.



x	*y*
3	5
5	4
7	3
9	2
11	1

Write the ordered pairs in a table.

Look for a pattern.

A constant change of +2 in x corresponds to a constant change of –1 in y.

These points satisfy a linear function.

Another way to determine whether a function is linear is to look at its equation. A function is linear if it is described by a *linear equation.* A **linear equation** is any equation that can be written in the *standard form* shown below.

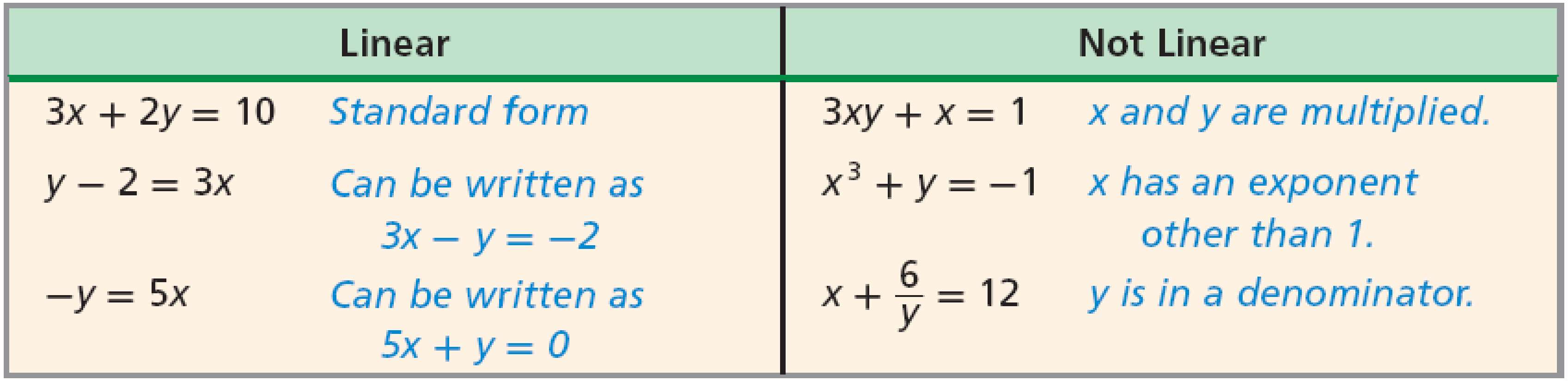
Standard Form of a Linear Equation

$Ax + By = C$ where A, B, and C are real numbers and A and B are not both 0

Notice that when a linear equation is written in standard form

- x and y both have exponents of 1.
- x and y are not multiplied together.
- x and y do not appear in denominators, exponents, or radical signs.

4-1 Identifying Linear Functions



Linear		Not Linear	
$3x + 2y = 10$	*Standard form*	$3xy + x = 1$	*x and y are multiplied.*
$y - 2 = 3x$	*Can be written as* $3x - y = -2$	$x^3 + y = -1$	*x has an exponent other than 1.*
$-y = 5x$	*Can be written as* $5x + y = 0$	$x + \frac{6}{y} = 12$	*y is in a denominator.*

For any two points, there is exactly one line that contains them both. This means you need only two ordered pairs to graph a line.

Example 3A: Graphing Linear Functions

Tell whether the function is linear. If so, graph the function.

$x = 2y + 4$

Write the equation in standard form. Try to get both variables on the same side. Subtract 2y from both sides.

$$\begin{array}{rcl} x & = & 2y + 4 \\ \underline{-2y} & & \underline{-2y\quad\quad} \\ x - 2y & = & 4 \end{array}$$

The equation is in standard form (A = 1, B = –2, C = 4).

The equation can be written in standard form, so the function is linear.

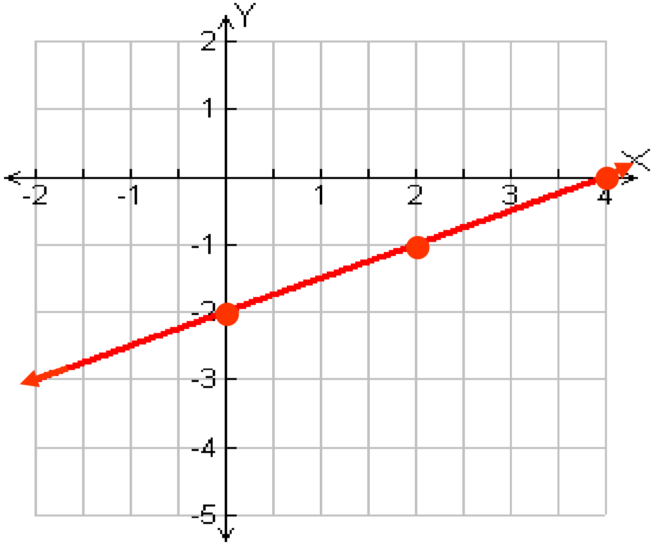
Example 3A Continued

$x = 2y + 4$

To graph, choose three values of *y*, and use them to generate ordered pairs. (You only need two, but graphing three points is a good check.)

y	$x = 2y + 4$	(x, y)
0	$x = 2(0) + 4 = 4$	$(4, 0)$
−1	$x = 2(-1) + 4 = 2$	$(2, -1)$
−2	$x = 2(-2) + 4 = 0$	$(0, -2)$

Plot the points and connect them with a straight line.



Example 3B: Graphing Linear Functions

Tell whether the function is linear. If so, graph the function.

$xy = 4$

This is not linear, because x and y are multiplied. It is not in standard form.

Check It Out! Example 3a

Tell whether the function is linear. If so, graph the function.

$$\boldsymbol{y = 5x - 9}$$

Write the equation in standard form. Try to get both variables on the same side. Subtract 5x from both sides.

$$\begin{aligned} y &= 5x - 9 \\ \underline{-5x} \quad & \underline{-5x \qquad\quad} \\ -5x + y &= \qquad - 9 \end{aligned}$$

The equation is in standard form (A = –5, B = 1, C = –9).

The equation can be written in standard form, so the function is linear.

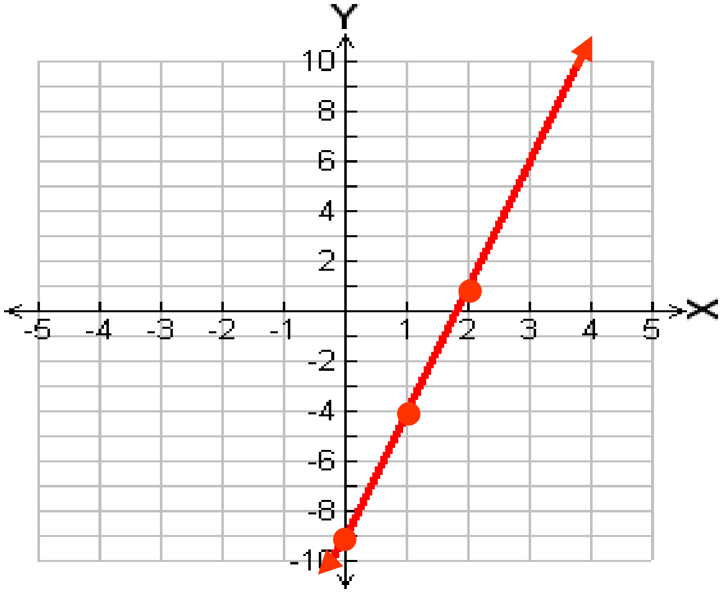
Check It Out! Example 3a Continued

$y = 5x - 9$

To graph, choose three values of x, and use them to generate ordered pairs. (You only need two, but graphing three points is a good check.)

Plot the points and connect them with a straight line.

x	$y = 5x - 9$	(x, y)
0	$y = 5(0) - 9 = -9$	$(0, -9)$
1	$y = 5(1) - 9 = -4$	$(1, -4)$
2	$y = 5(2) - 9 = 1$	$(2, 1)$



Check It Out! Example 3b

Tell whether the function is linear. If so, graph the function.

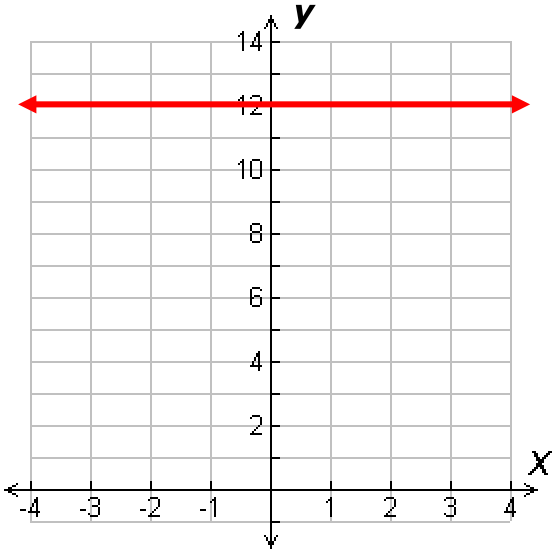
$y = 12$

The equation is in standard form ($A = 0$, $B = 1$, $C = 12$).

The equation can be written in standard form, so the function is linear.

Check It Out! Example 3b Continued

$y = 12$



Check It Out! Example 3c

Tell whether the function is linear. If so, graph the function.

$y = 2^x$

This is not linear, because x is an exponent.

For linear functions whose graphs are not horizontal, the domain and range are all real numbers. However, in many real-world situations, the domain and range must be restricted. For example, some quantities cannot be negative, such as time.

Sometimes domain and range are restricted even further to a set of points. For example, a quantity such as number of people can only be whole numbers. When this happens, the graph is not actually connected because every point on the line is not a solution. However, you may see these graphs shown connected to indicate that the linear pattern, or trend, continues.

Example 4: *Application*

An approximate relationship between human years and dog years is given by the function $y = 7x$, where x is the number of human years. Graph this function and give its domain and range.

Choose several values of x and make a table of ordered pairs.

x	$f(x) = 7x$
1	$f(1) = 7(1) = 7$
2	$f(2) = 7(2) = 14$
3	$f(3) = 7(3) = 21$

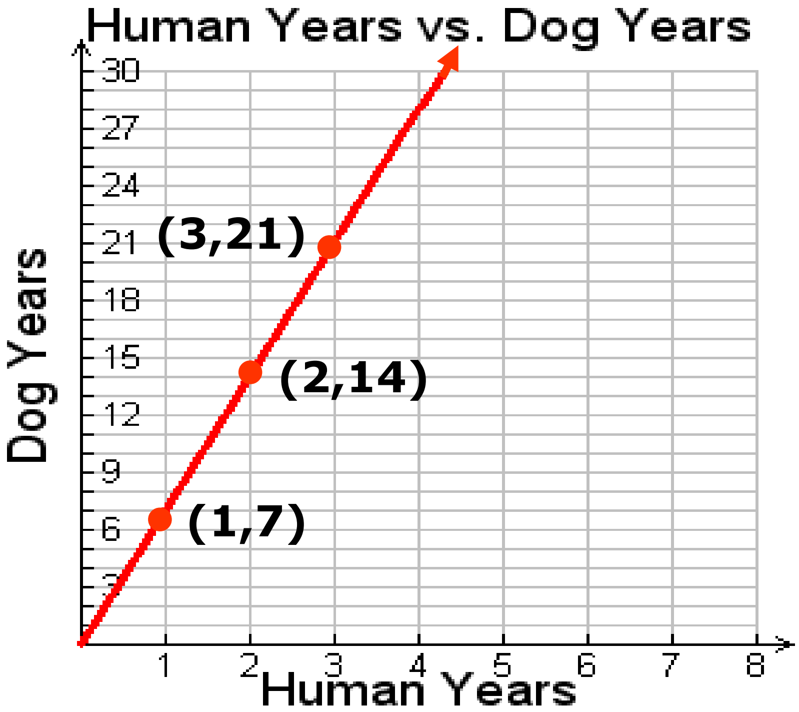
The number of human years must be positive, so the domain is $\{x \geq 0\}$ and the range is $\{y \geq 0\}$.

Example 4 Continued

An approximate relationship between human years and dog years is given by the function $y = 7x$, where x is the number of human years. Graph this function and give its domain and range.

Graph the ordered pairs.

x	$f(x) = 7x$
1	$f(1) = 7(1) = 7$
2	$f(2) = 7(2) = 14$
3	$f(3) = 7(3) = 21$



Check It Out! Example 4

What if…? At a salon, Sue can rent a station for $10.00 per day plus $3.00 per manicure. The amount she would pay each day is given by $f(x) = 3x + 10$, where x is the number of manicures. Graph this function and give its domain and range.

Check It Out! Example 4 Continued

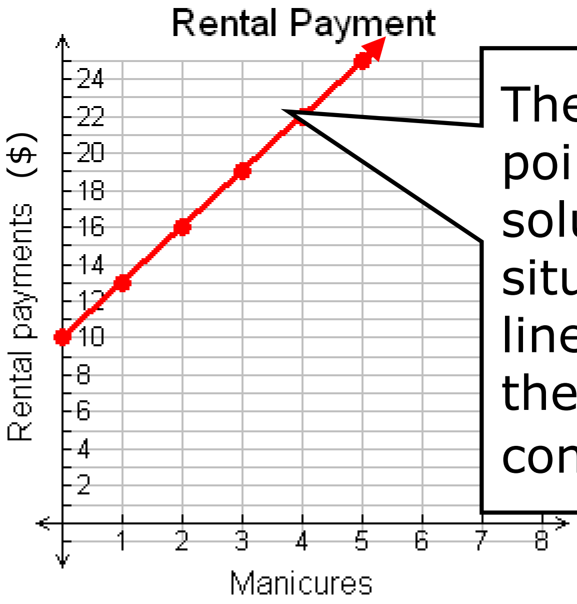
Choose several values of x and make a table of ordered pairs.

x	$f(x) = 3x + 10$
0	$f(0) = 3(0) + 10 = 10$
1	$f(1) = 3(1) + 10 = 13$
2	$f(2) = 3(2) + 10 = 16$
3	$f(3) = 3(3) + 10 = 19$
4	$f(4) = 3(4) + 10 = 22$
5	$f(5) = 3(5) + 10 = 25$

The number of manicures must be a whole number, so the domain is {0, 1, 2, 3, …}. The range is {10.00, 13.00, 16.00, 19.00, …}.

Check It Out! Example 4 Continued

Graph the ordered pairs.



The individual points are solutions in this situation. The line shows that the trend continues.

Lesson Quiz: Part I

Tell whether each set of ordered pairs satisfies a linear function. Explain.

1. {(–3, 10), (–1, 9), (1, 7), (3, 4), (5, 0)}

No; a constant change of +2 in *x* corresponds to different changes in *y*.

2. {(3, 4), (5, 7), (7, 10), (9, 13), (11, 16)}

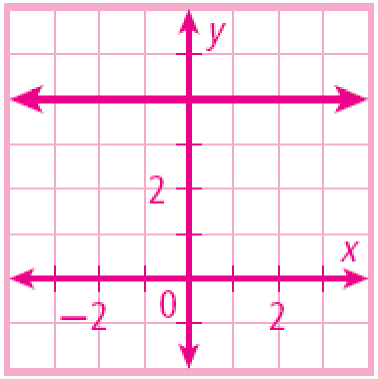
Yes; a constant change of +2 in *x* corresponds to a constant change of +3 in *y*.

Lesson Quiz: Part II

Tell whether each function is linear. If so, graph the function.

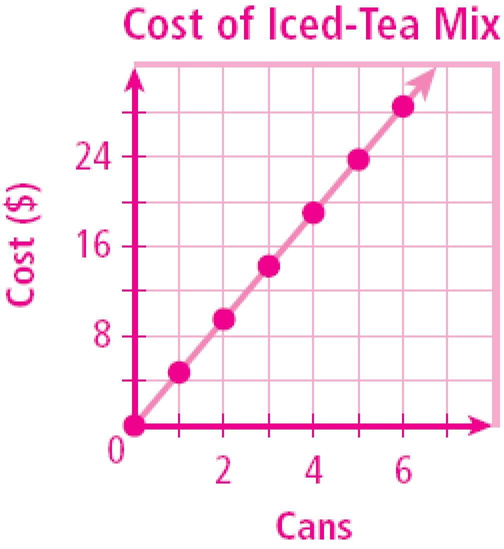
3. $y = 3 - 2^x$ no

4. $3y = 12$ yes



Lesson Quiz: Part III

5. The cost of a can of iced-tea mix at Save More Grocery is $4.75. The function $f(x) = 4.75x$ gives the cost of x cans of iced-tea mix. Graph this function and give its domain and range.



D: {0, 1, 2, 3, …}

R: {0, 4.75, 9.50, 14.25,…}