



4-3 Triangle Congruence by ASA and AAS

TEKS FOCUS

TEKS (6)(B) Prove two triangles are congruent by applying the Side-Angle-Side, Angle-Side-Angle, Side-Side-Side, Angle-Angle-Side, and Hypotenuse-Leg congruence conditions.

TEKS (1)(F) Analyze mathematical relationships to connect and communicate mathematical ideas.

Additional TEKS (1)(A), (1)(D), (1)(G)

VOCABULARY

- **Analyze** – closely examine objects, ideas, or relationships to learn more about their nature.

ESSENTIAL UNDERSTANDING

You can prove that two triangles are congruent without having to show that *all* corresponding parts are congruent. In this lesson, you will prove triangles congruent by using one pair of corresponding sides and two pairs of corresponding angles.

take note

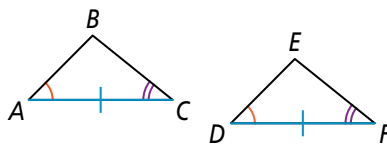
Postulate 4-3 Angle-Side-Angle (ASA) Postulate

Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

If ...

$$\angle A \cong \angle D, \overline{AC} \cong \overline{DF}, \angle C \cong \angle F$$



Then ...

$$\triangle ABC \cong \triangle DEF$$

take note

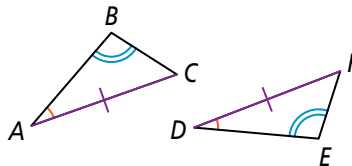
Theorem 4-2 Angle-Angle-Side (AAS) Theorem

Theorem

If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the triangles are congruent.

If ...

$$\angle A \cong \angle D, \angle B \cong \angle E, \overline{AC} \cong \overline{DF}$$



Then ...

$$\triangle ABC \cong \triangle DEF$$

You will prove Theorem 4-2 in Exercise 15.



Problem 1

TEKS Process Standard (1)(F)

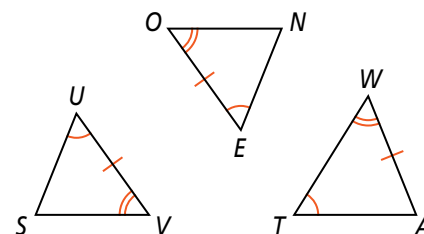
Using ASA

Which two triangles are congruent by ASA? Explain.

Know

From the diagram you know

- $\angle U \cong \angle E \cong \angle T$
- $\angle V \cong \angle O \cong \angle W$
- $\overline{UV} \cong \overline{EO} \cong \overline{AW}$



Need

To use ASA, you need two pairs of congruent angles and a pair of included congruent sides.

Plan

You already have pairs of congruent angles. So, identify the included side for each triangle and see whether it has a congruence marking.

In $\triangle SUV$, \overline{UV} is included between $\angle U$ and $\angle V$ and has a congruence marking. In $\triangle NEO$, \overline{EO} is included between $\angle E$ and $\angle O$ and has a congruence marking. In $\triangle ATW$, \overline{TW} is included between $\angle T$ and $\angle W$ but does *not* have a congruence marking.

Since $\angle U \cong \angle E$, $\overline{UV} \cong \overline{EO}$, and $\angle V \cong \angle O$, $\triangle SUV \cong \triangle NEO$.



Problem 2

TEKS Process Standard (1)(A)

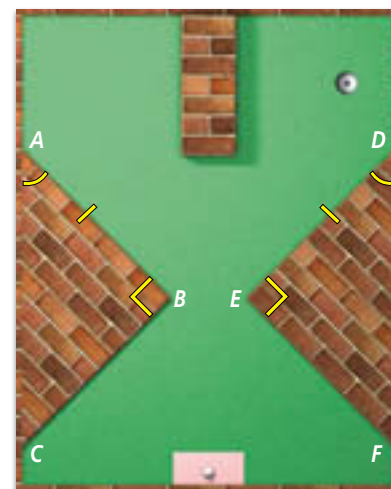
Proof Writing a Proof Using ASA

Recreation Members of a teen organization are building a miniature golf course at your town's youth center. The design plan calls for the first hole to have two congruent triangular bumpers. Prove that the bumpers on the first hole, shown at the right, meet the conditions of the plan.

Given: $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, $\angle B$ and $\angle E$ are right angles

Prove: $\triangle ABC \cong \triangle DEF$

Proof: $\angle B \cong \angle E$ because all right angles are congruent, and you are given that $\angle A \cong \angle D$. \overline{AB} and \overline{DE} are included sides between the two pairs of congruent angles. You are given that $\overline{AB} \cong \overline{DE}$. Thus, $\triangle ABC \cong \triangle DEF$ by ASA.



Plan

Can you use a plan similar to the plan in Problem 1?

Yes. Use the diagram to identify the included side for the marked angles in each triangle.



Problem 3

Plan

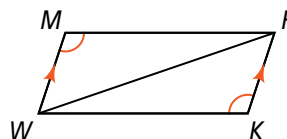
How does information about parallel sides help?

You will need another pair of congruent angles to use AAS. Think back to what you learned in Topic 3. \overline{WR} is a transversal here.

Proof Writing a Proof Using AAS

Given: $\angle M \cong \angle K$, $\overline{WM} \parallel \overline{RK}$

Prove: $\triangle WMR \cong \triangle RKW$



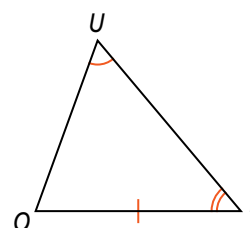
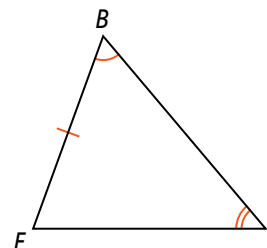
Statements	Reasons
1) $\angle M \cong \angle K$	1) Given
2) $\overline{WM} \parallel \overline{RK}$	2) Given
3) $\angle MWR \cong \angle KRW$	3) If lines are \parallel , then alternate interior \angle s are \cong .
4) $\overline{WR} \cong \overline{WR}$	4) Reflexive Property of Congruence
5) $\triangle WMR \cong \triangle RKW$	5) AAS

Problem 4

Determining Whether Triangles Are Congruent

Multiple Choice Use the diagram at the right. Which of the following statements best represents the answer and justification to the question, "Is $\triangle BIF \cong \triangle UTO$?"

- (A) Yes, the triangles are congruent by ASA.
- (B) No, \overline{FB} and \overline{OT} are not corresponding sides.
- (C) Yes, the triangles are congruent by AAS.
- (D) No, $\angle B$ and $\angle U$ are not corresponding angles.



The diagram shows that two pairs of angles and one pair of sides are congruent. The third pair of angles is congruent by the Third Angles Theorem. To prove these triangles congruent, you need to satisfy ASA or AAS.

ASA and AAS both fail because \overline{FB} and \overline{TO} are not included between the same pair of congruent corresponding angles, so they are not corresponding sides. The triangles are not necessarily congruent. The correct answer is B.

Think

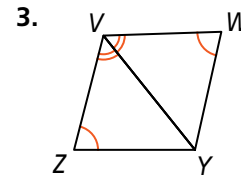
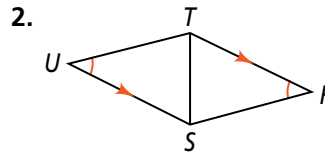
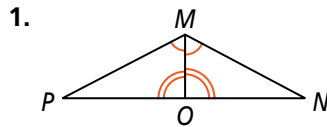
Can you eliminate any of the choices?

Yes. If $\triangle BIF \cong \triangle UTO$ then $\angle B$ and $\angle U$ would be corresponding angles. You can eliminate choice D.

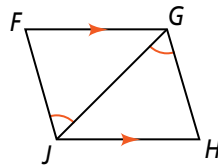


For additional support when completing your homework, go to PearsonTEXAS.com.

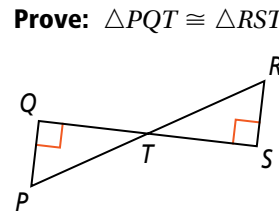
Determine whether the triangles must be congruent. If so, name the postulate or theorem that justifies your answer. If not, explain.



4. **Given:** $\angle FJG \cong \angle HGJ$, $\overline{FG} \parallel \overline{JH}$
Proof **Prove:** $\triangle FGJ \cong \triangle HJG$



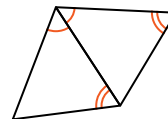
5. **Given:** $\overline{PQ} \perp \overline{QS}$, $\overline{RS} \perp \overline{SQ}$,
Proof T is the midpoint of \overline{PR}



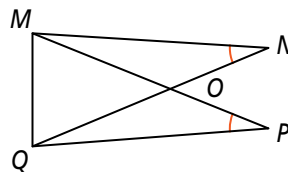
6. **Evaluate Reasonableness (1)(B)** While helping your family clean out the attic, you find the piece of paper shown at the right. The paper contains clues to locate a time capsule buried in your backyard. The maple tree is due east of the oak tree in your backyard. Will the clues always lead you to the correct spot? Explain.

7. **Connect Mathematical Ideas (1)(F)** Anita says that you can rewrite any proof that uses the AAS Theorem as a proof that uses the ASA Postulate. Do you agree with Anita? Explain.

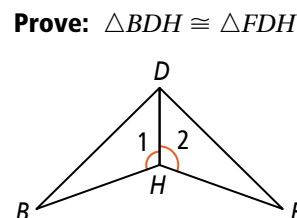
8. **Justify Mathematical Arguments (1)(G)** Can you prove that the triangles at the right are congruent? Justify your answer.



9. **Given:** $\angle N \cong \angle P$, $\overline{MO} \cong \overline{QO}$
Proof **Prove:** $\triangle MON \cong \triangle QOP$



10. **Given:** $\angle 1 \cong \angle 2$, and
Proof \overline{DH} bisects $\angle BDF$



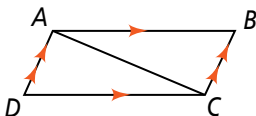
Mark a line on the ground from the oak tree to the maple tree. From the oak tree, walk along a path that forms a 70° angle with the marked line, keeping the maple tree to your right. From the maple tree, walk along a path that forms a 40° angle with the marked line. The time capsule is buried where the paths meet.



11. Given: $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$

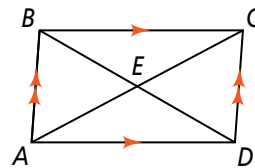
Proof

Prove: $\triangle ABC \cong \triangle CDA$



12. **Create Representations to Communicate Mathematical Ideas (1)(E)** Draw two noncongruent triangles that have two pairs of congruent angles and one pair of congruent sides.

13. Given $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{DC}$, name as many pairs of congruent triangles as you can.

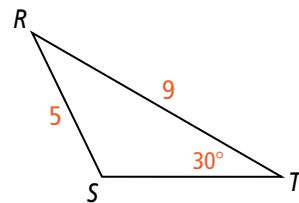


14. **Create Representations to Communicate Mathematical Ideas (1)(E)**

Use a straightedge to draw a triangle. Label it $\triangle JKL$. Construct $\triangle MNP \cong \triangle JKL$ so that the triangles are congruent by ASA.

15. Prove the Angle-Angle-Side Theorem (Theorem 4-2). Use the diagram next to it on page 158.

16. In $\triangle RST$ at the right, $RS = 5$, $RT = 9$, and $m\angle T = 30^\circ$. Show that there is no SSA congruence rule by constructing $\triangle UVW$ with $UV = RS$, $UW = RT$, and $m\angle W = m\angle T$, but with $\triangle UVW \not\cong \triangle RST$.



TEXAS Test Practice

17. Suppose $\overline{RT} \cong \overline{ND}$ and $\angle R \cong \angle N$. What additional information do you need to prove that $\triangle RTJ \cong \triangle NDF$ by ASA?

A. $\angle T \cong \angle D$

C. $\angle J \cong \angle D$

B. $\angle J \cong \angle F$

D. $\angle T \cong \angle F$

18. You plan to make a 2 ft-by-3 ft rectangular poster of class trip photos. Each photo is a 4 in.-by-6 in. rectangle. If the photos do not overlap, what is the greatest number of photos you can fit on your poster?

F. 4

H. 32

G. 24

J. 36

19. Write the converse of the true conditional statement below. Then determine whether the converse is true or false.

If you are less than 18 years old, then you are too young to vote in the United States.