### 4.4 Conversion, Obversion, \& Contraposition

In this lecture, we will learn how to swap various components of the four kinds of categorical proposition around, and see how this swapping changes the truth or falsity of each proposition.

1. Conversion: The first sort of swap we'll look at is called "conversion."

Conversion: Swapping the subject with the predicate.

For instance, consider some cases of conversion:

- "No birds are bumblebees." CONVERSION $\rightarrow$ "No bumblebees are birds."
- "Some birds are flying creatures." CONVERSION $\rightarrow$ "Some flying creatures are birds."

Below is a chart depicting how each of the four types of categorical proposition are converted. The statements about dogs and cats provide specific examples:

## Given statement form

Converse


All mammals are dogs.
All $B$ are $A$.


No dogs are cats.
No $A$ are $B$.


No cats are dogs.
No $B$ are $A$.


Some cats are hairless.
Some $A$ are $B$.


Some dogs are not trained.
Some A are not $B$.


Some trained things are not dogs.

Some $B$ are not $A$.


Note: (1) The conversion of the middle two examples above yields a Venn diagram that is exactly the same. (2) The conversion of the top and the bottom examples yields a Venn diagram that is NOT the same.

Conversion of (E) and (I) preserves truth value: We say that the converted proposition of the middle two examples above "preserves the truth value" of the original proposition. This happens whenever the conversion of a proposition yields a Venn diagram that is exactly the same as the converted proposition. In other words, if the original proposition is TRUE, then the conversion of it will ALSO be true (and, similarly, if the original proposition is false, then the conversion of it will also be false).

Example \#1: Since it is TRUE that "No dogs are cats" then it is automatically ALSO true that "No cats are dogs."

Example \#2: Since it is FALSE that "No dogs are mammals" then it is automatically ALSO false that "No mammals are dogs."

Conversion of $(\mathbf{A})$ and $(\mathbf{O})$ does not preserve truth value: But, if the conversion of a proposition does NOT yield a Venn diagram that is exactly the same as the converted proposition, then we cannot know for sure whether or not the converted proposition has the same truth value as the original proposition.

Example \#1: If "All dogs are mammals" is true, it is NOT automatically true that "All mammals are dogs."

Example \#2: Similarly, if "Some dogs are not trained creatures" is true, it is NOT automatically true that "Some trained creatures are not dogs."

If we DID claim that the converted proposition preserved the truth value of the original proposition in these two cases, we would be making a mistake known as "the fallacy of illicit conversion."

Here is a chart that summarizes what we have just said about conversion:

| Proposition | Converse | Preserves Truth Value? |
| :--- | :--- | :---: |
| (A) All S are P. | All P are S. | No |
| (E) No S are P. | No P are S. | Yes |
| (I) Some S are P. | Some P are S. | Yes |
| (O) Some S are not P. | Some P are not S. | No |

2. Obversion: Let's move on to the second sort of swap.

Obversion: Changing the quality of a proposition and then replacing the predicate with its complement.

Step 1: Change the quality: We learned how to change the quality of a proposition in section 4.2. Remember, quality refers to whether the proposition is "affirmative" or "negative." For instance, changing the quality of "All $S$ are $P^{\prime}$ would give us "No S are P."

Step 2: Replace predicate with its complement: The complement of a predicate class is the set of everything that is NOT a member of the predicate class. So, for instance, if our predicate class is "dogs", then the COMPLEMENT class includes everything from cats to chairs to clouds to stars-in short, everything that is not a dog (all "non-dogs").

Example: To illustrate, let's obvert "All dogs are mammals."

Step 1: First, change the quality. The proposition becomes "No dogs are mammals."

Step 2: Next, replace the predicate with its complement. The complement of "mammals" is the set of all things that are NOT mammals-in other words, "non-mammals"-so the proposition becomes, "No dogs are non-mammals."
"All dogs are mammals" tells us that ALL of the dogs are INSIDE of the mammal class. But, "No dogs are non-mammals" tells us that NONE of the dogs are OUTSIDE of the mammal class. These are both saying the same thing. So, it appears that the obverted proposition has preserved the truth value of the original proposition.

As it turns out, obversion ALWAYS preserves the truth value for all four kinds of categorical proposition. Below is an illustration of obversion for each of these four kinds, with specific examples:

Note about graphing complements: Consider the complement of "All A are B", which is "No A are non-B." We can interpret this as telling us that none of the A's are outside of the set of B's. For example, "No dogs are non-mammals" tells us that none of the dogs are outside of the set of mammals. So, to graph this in a Venn diagram, we simply shade the entire portion of the $A$-circle that lies outside of the $B$-circle (i.e., the entire nonoverlapping part).

All dogs are mammals.
All $A$ are $B$.


No $A$ are non- $B$.


No dogs are cats.
No $A$ are $B$.


All cats are non-dogs.
All $A$ are non- $B$.


Some cats are hairless.

Some $A$ are $B$.


All cats are not non-hairless.

Some $A$ are not non-B.


Some dogs are not trained.
Some $A$ are not $B$.


Some dogs are non-trained.
Some $A$ are non-B.

3. Contraposition: Last but not least, the third sort of swap.

Contraposition: Performing an conversion on a proposition (i.e., swapping the subject with the predicate) and then replacing both the subject and the predicate terms with their complements.

Example: Let's try one: "All dogs are mammals."
Step 1: Obversion: First, we obvert it. That is, we replace the subject and the predicate to get, "All mammals are dogs."

Step 2: Replace subject and predicate with complements: Next, we replace both terms (subject and predicate) with their complements. So, "mammals" becomes "non-mammals", while "dogs" becomes "non-dogs." The end result is this: "All non-mammals are non-dogs."

Here is a graph to illustrate the four types:
Given statement form
Contrapositive

All dogs are mammals.
All $A$ are $B$.


All non-mammals are non-dogs.

All non- $B$ are non- $A$.


No dogs are cats.
No $A$ are $B$.


Some cats are hairless.
Some $A$ are $B$.


Some non-hairless (hairy) things are non-cats.

Some non- $B$ are non- $A$.


Some non-trained things are not non-dogs (i.e., they ARE dogs).

Some non- $B$ are not non- $A$.


Graphing multiple complements: It can be a bit confusing to diagram a proposition which has multiple complements. Let's graph some examples:

Example \#1: "All non-B are non-A." Step 1: Subject Term: To graph this, first think only about the first part of the statement: "All non-B..." We know that this is telling us something about EVERYTHING outside of the B-circle. Step 2: Predicate Term: But, WHAT are we being told about all those things outside of the B-circle? Well, let's consider the next part of the statement ("...are non-A"). What this is telling us about the entire world of things outside of the $B$-circle is that they are all non-A's. In short, for everything outside of the B-circle (subject), none of them are A's (predicate). So, we shade in the part of the A-circle that does not overlap with the B -circle, to show that it is empty (as pictured above).

Example \#2: "No non-B are non-A." To graph it, let's take it one part at a time. Think about the first part of the statement: "No non-B..." What this is telling us is that, out of EVERYTHING outside of the B-circle, none of them... (we have to think about the second part now) ...none of them "are non-A." So, out of EVERYTHING outside of the B-circle (subject), none of them are also outside of the A-circle (predicate). So, apparently, nothing exists outside of the $A$ and $B$ circles! Now we shade in the region that is both outside of the B-circle AND outside of the Acircle, to show that it is empty (as pictured above).

Note: (1) The contraposition of the top and the bottom examples yields a Venn diagram that is exactly the same. (2) The conversion of the middle two examples yields a Venn diagram that is NOT the same.
(A) and (O) preserve truth value. (E) and (I) do not: So, this time, the top and bottom (A \& O) preserve the truth value, while the middle two (E \& I) do not.

As with obversion, a valid inference CAN immediately be derived for two of the four types of categorical proposition, but NOT for the other two. For instance, if you have an (A) or an (O) proposition as a true premise, you can contrapose it and automatically conclude that it is also true. On the other hand, if you have a true (E) or an (I) proposition, and you contrapose it, and then claim that contraposed proposition is also true, you are making a mistake known as "the fallacy of illicit contraposition."

Here is a chart that summarizes what we have just said about contraposition:

Proposition
(A) All S are P .
(E) No $S$ are $P$.
(I) Some $S$ are P.
(O) Some $S$ are not $P$.

## Contraposition

All non-P are non-S.
No non-P are non-S.
Some non-P are non-S.
Some non-P are not non-S.

Preserves Truth Value?
Yes
No
No
Yes

Note: Do homework for section 4.4 at this time.

