

4.4

Similar Polygons

Focus on...

After this lesson, you will be able to...

- identify similar polygons and explain why they are similar
- draw similar polygons
- solve problems using the properties of similar polygons



Did You Know?

The star quilt is a traditional pattern used by many different cultures including the Sioux, Lakota, Dakota, and Europeans. The star pattern is derived from designs for early buffalo robes. When the buffalo herds disappeared, the star quilt replaced the buffalo robe in Aboriginal traditions. Today, star quilts given as gifts are highly valued by recipients. They are often made for special events, such as memorial feasts, naming ceremonies, marriages, and celebrations.

The single star in a Lakota star quilt is made from fabric cut into diamond shapes and pieced together in eight sections. When the sections are joined together, an eight-pointed star is formed.

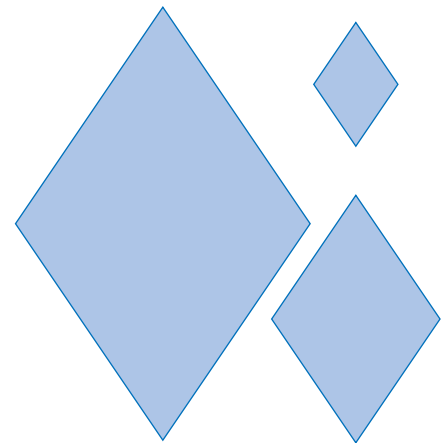
Are the different-sized diamonds formed on the quilt similar? What strategies might you use to find out?

Materials

- tracing paper
- protractor
- ruler

Explore How to Identify Similar Polygons

1. Trace each diamond on separate pieces of tracing paper.
2. a) Organize your data about corresponding angles and corresponding sides.
b) What do you observe about the corresponding angles?
c) What do you observe about the ratios of the corresponding sides?



polygon

- a two-dimensional closed figure made of three or more line segments



Reflect and Check

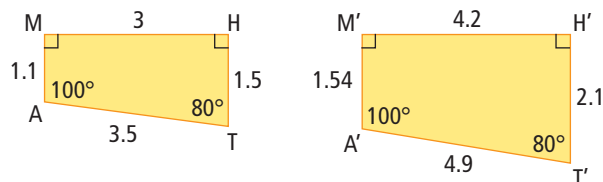
3. What conclusions can you make about the three diamonds?
4. a) What conditions do you think are necessary in order for two **polygons** to be similar?
b) Test the conditions on a different set of two polygons. Are the polygons similar? Discuss with a classmate why you think the polygons are, or are not, similar.

Link the Ideas

Similar polygons have corresponding angles that are equal and corresponding side lengths that are proportional.

Example 1: Identify Similar Polygons

The two quadrilaterals look similar. Is $M'A'T'H'$ a true enlargement of $MATH$? Explain.



Solution

Compare corresponding angles:

$$\angle M = 90^\circ \text{ and } \angle M' = 90^\circ$$

$$\angle A = 100^\circ \text{ and } \angle A' = 100^\circ$$

$$\angle T = 80^\circ \text{ and } \angle T' = 80^\circ$$

$$\angle H = 90^\circ \text{ and } \angle H' = 90^\circ$$

Compare corresponding sides:

$$\frac{M'A'}{MA} = \frac{1.54}{1.1} \qquad \frac{A'T'}{AT} = \frac{4.9}{3.5}$$

$$= 1.4 \qquad \qquad \qquad = 1.4$$

$$\frac{H'T'}{HT} = \frac{2.1}{1.5} \qquad \frac{M'H'}{MH} = \frac{4.2}{3}$$

$$= 1.4 \qquad \qquad \qquad = 1.4$$

The corresponding side lengths are proportional with a scale factor of 1.4.

$M'A'T'H'$ is a true enlargement of $MATH$ by a scale factor of 1.4.

What is the sum of the interior angles in a quadrilateral?

Literacy Link

M' is read "M prime."

Did You Know?

Polygons can be divided into non-overlapping triangles. The sum of the interior angles in a triangle is 180° . You can determine the sum of the interior angles in a polygon by multiplying the number of triangles by 180.

To draw the triangles, start with any vertex of the polygon, and from there draw a line to connect to each of the other vertices. The pentagon can be divided into three triangles.

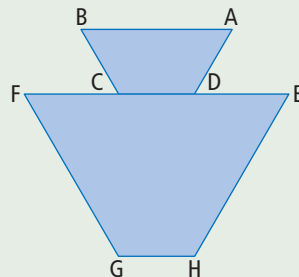


$$3 \times 180^\circ = 540^\circ$$

The sum of the interior angles in a pentagon is 540° .

Show You Know

Determine if the two trapezoids are similar. Explain how you know.

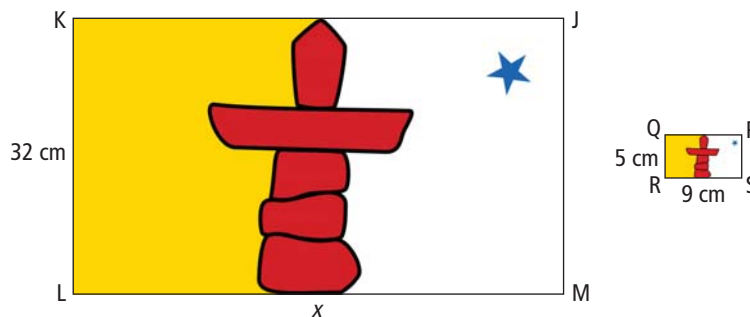


Web Link

To learn more about the properties of similar polygons, go to www.mathlinks9.ca and follow the links.

Example 2: Determine a Missing Side Length

Jason wants to make an enlargement of the flag of Nunavut. He knows that the two rectangles JKLM and PQRS are similar. What is the missing side length of rectangle JKLM?



Solution

Since the rectangles are similar, the side lengths are proportional. Use corresponding sides to set up a proportion.

$$\frac{KL}{QR} = \frac{LM}{RS}$$

$$\frac{32}{5} = \frac{x}{9}$$

$$6.4 = \frac{x}{9}$$

$$x = 57.6$$

The missing side length is 57.6 cm.

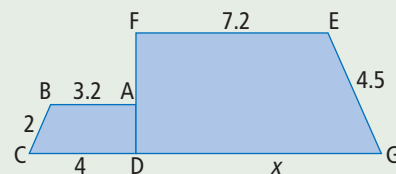
What different method could you use to solve the problem? Try it.

Strategies

Solve an Equation

Show You Know

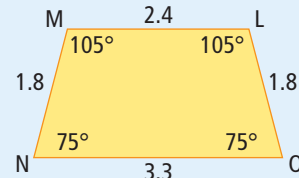
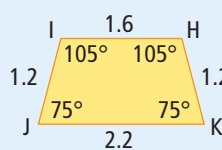
The two trapezoids shown are similar. Determine the missing side length. Show your work.



Key Ideas

- Polygons are similar if both of the following conditions hold true:
 - corresponding angles are equal in measure
 - corresponding side lengths are proportional
- You can use similar polygons to determine unknown side lengths or angle measures.

The trapezoids HIJK and LMNO are similar.



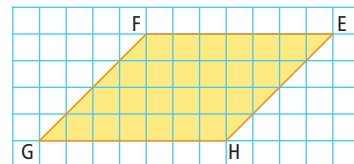
$$\angle H = \angle L, \angle I = \angle M, \\ \angle J = \angle N, \angle K = \angle O$$

Each side length of LMNO is 1.5 times the corresponding side length of HIJK.

Check Your Understanding

Communicate the Ideas

- Develop an example and a solution to help explain how to determine a missing side length in a pair of similar polygons.
- Use grid paper to draw a parallelogram that is similar to the one shown. Explain how you know the two are similar.
 - Compare your similar parallelogram with the one of a classmate. Are your parallelograms similar to each other? Explain.

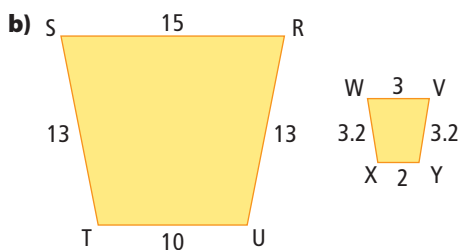
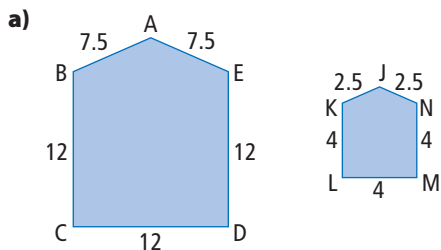


Practise

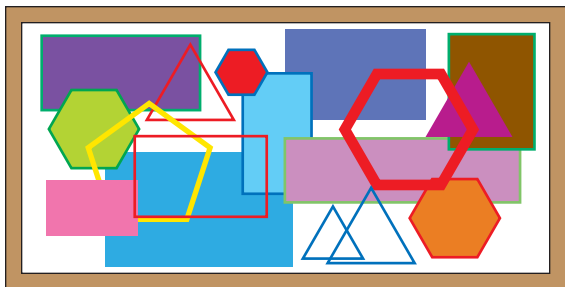
For help with #3 and #4, refer to Example 1 on page 155.

For help with #5 and #6, refer to Example 2 on page 156.

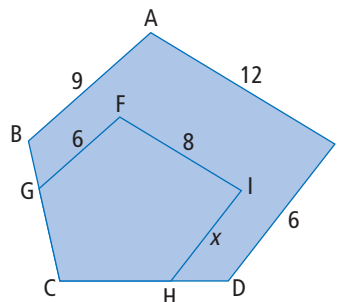
- Decide if each pair of polygons is similar. Show your reasoning.



- Identify all sets of similar polygons in this collage. You might trace the image and colour code sets of similar polygons.



- Use the two similar pentagons to help determine the missing side length. Show your work.



- The sides of rectangle A measure 22.4 m and 14.7 m. One side of a similar rectangle B measures 4.3 m. The measure for the other side is missing. Rectangle A is an enlargement of rectangle B with a scale factor of 5.2. What is the missing measurement, to the nearest tenth?

Apply

- William made the statement, “All quadrilaterals with sides the same length are similar.” Is he correct? Explain.

WWW Web Link

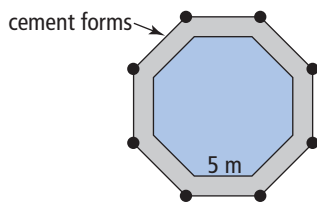
To explore the changes when you manipulate two similar figures and vary the scale factor, go to www.mathlinks9.ca and follow the links.

8. Chicken wire is often used for building fences. It is made of flexible wire with gaps that are shaped like hexagons.
- Use grid paper to draw and label:
 - two hexagons that are similar to one shown in the picture
 - two hexagons that are not similar to one shown
 - For each pair of hexagons, explain how you know they are similar or not similar.



9. Michelle plans to make a game board that is a reduction of an actual baseball diamond. A baseball diamond is a square with sides that measure 27.4 m (2740 cm). Draw Michelle's game board using a scale of 1:182.5.

10. a) Rachel's family is making a cement deck around a pool that is a regular octagon. They want the cement deck to keep the same shape as the pool, but with sides 1.5 times as long as the pool. What do the lengths of the cement forms along the sides of the outer octagon need to be in order to pour the cement?



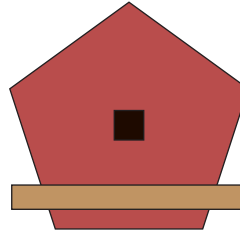
- b) What is the sum of the interior angles in an octagon? Show how you know.

Literacy Link

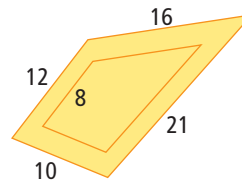
A regular polygon, has all sides equal and all angles equal.

11. The pattern shows the front of a birdhouse. Chris enlarged the pattern using a scale factor of 3. He needs to make it twice as large as that.

- Draw the correct size.
- Explain how you know the enlargement is similar to the original pattern.

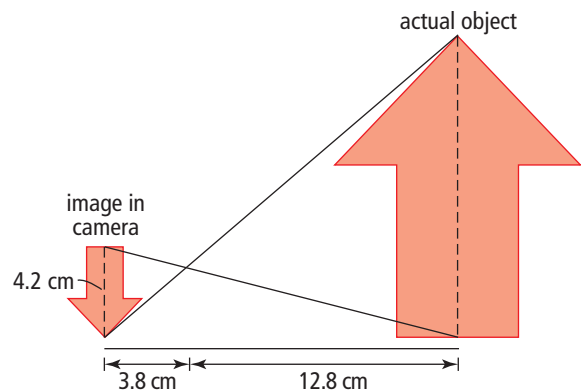


12. A piece of cardboard is cut showing the inner and outer boundaries of a pair of similar quadrilaterals. Calculate the perimeter of the smaller quadrilateral.



Extend

13. In a camera, similar figures occur as shown. Calculate the actual height of the arrow.



Did You Know?

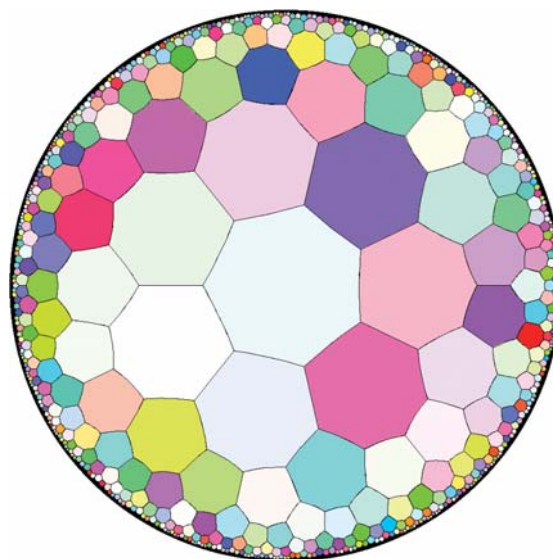
In the past, some cameras showed the image upside down in the viewfinder.

- 14.** Eliza is building a model of the canvas tent her family uses in Behchoko, NWT. The model will have a peak height of 12 cm. The actual tent floor measures 2.4 m by 3 m. The walls are 1.5 m high and the peak height is 2.4 m.
- What scale factor will Eliza need to use for her model?
 - The front of the tent is a pentagon. Calculate the dimensions of this polygon on the model.
 - Calculate the other dimensions of the tent model.



- 15.** An old rectangular tank with length 0.3 m could hold 154 L of water. A new similar tank has a length of 1.5 m. What is the capacity of the new tank?

- 16.** How do the ratios of areas compare to the ratios of corresponding side lengths in similar polygons? Use pairs of similar polygons to help explain your answer.
- 17.** Develop an argument showing that if two prisms have corresponding side lengths in the ratio of 3:1, then their volumes are in the ratio of 27:1.
- 18. a)** Identify the similar polygons shown in the tessellation.
- b)** Describe the pattern verbally. Use your description to create your own tessellation that features similar polygons.
- c)** Sketch each different set of similar polygons in your tessellation. Label the dimensions of each set.



Math Link

For your design project, include a polygon.

- Use a polygon that is similar to one shown here.
- Use an appropriate scale factor and draw a scale diagram of the polygon to fit on your design project. Identify the scale factor used on your design.

