Congruence in Right Triangles

What You'll Learn

• To prove triangles congruent using the HL Theorem

... And Why

To show that one pattern can be used to cut the fabric for the two entrance flaps of a tent, as in Example 1

(Victorial Check Skills You'll Need)



Tell whether the abbreviation identifies a congruence statement.

1. SSS ves

2. SAS ves

3. SSA no

4. ASA ves

5. AAS ves

6. AAA no

Can you conclude that the two triangles are congruent? Explain.

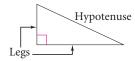




New Vocabulary • hypotenuse • legs of a right triangle

The Hypotenuse-Leg Theorem

In a right triangle, the side opposite the right angle is the longest side and is called the **hypotenuse**. The other two sides are called **legs**.



Right triangles provide a special case for which there is an SSA congruence rule. (See Lesson 4-3, Exercise 32.)

It occurs when hypotenuses are congruent and one pair of legs are congruent.



Key Concepts

Theorem 4-6

Hypotenuse-Leg (HL) Theorem

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

Proof

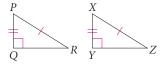
Proof of the HL Theorem

Given: $\triangle PQR$ and $\triangle XYZ$ are right triangles,

with right angles Q and Y respectively.

 $\overline{PR} \cong \overline{XZ}$, and $\overline{PQ} \cong \overline{XY}$.

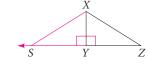
Prove: $\triangle PQR \cong \triangle XYZ$



Proof: On $\triangle XYZ$ at the right, draw \overline{ZY} . Mark point S as shown so that YS = QR. Then,

 $\triangle PQR \cong \triangle XYS$ by SAS. By CPCTC, $\overline{PR} \cong \overline{XS}$.

It is given that $\overline{PR} \cong \overline{XZ}$, so $\overline{XS} \cong \overline{XZ}$ by the Transitive Property of Congruence.



By the Isosceles Triangle Theorem, $\angle S \cong \angle Z$, so $\triangle XYS \cong \triangle XYZ$ by AAS. Therefore, $\triangle PQR \cong \triangle XYZ$ by the Transitive Property of Congruence.

Lesson 4-6 Congruence in Right Triangles

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Differentiated Instruction Solutions for All Learners

Special Needs [1]

Point out that although only two letters are used to name the HL Theorem, there are three conditions: two right angles, one pair of congruent hypotenuses, and one pair of congruent legs.

Below Level 12

Have students use the diagram in the proof of the HL Theorem to explain why the HL Theorem is not a special case of the SAS Postulate.

learning style: verbal

learning style: verbal

1. Plan

Objectives

To prove triangles congruent using the HL Theorem

Examples

- **Real-World Connection** 1
- Using the HL Theorem
- Using the HL Theorem



Math Background

The HL Theorem is an example of using a SSA relationship to prove triangles congruent. This is not generally possible. The HL Theorem also can be proved by first proving the Pythagorean Theorem and then applying it to establish SSS congruence.

More Math Background: p. 196D

Lesson Planning and Resources

See p. 196E for a list of the resources that support this lesson.



♂ Check Skills You'll Need

For intervention, direct students to:

Using the SSS and SAS Postulates Lesson 4-2: Examples 1 and 2

Extra Skills, Word Problems, Proof Practice, Ch. 4

Using the ASA Postulate

Lesson 4-3: Example 1 Extra Skills, Word Problems, Proof Practice, Ch. 4

2. Teach

Guided Instruction

Teaching Tip

Before reading the proof of the HL Theorem, discuss a Plan for Proof with the class. Draw $\triangle XYZ$. and discuss why you might want to extend \overline{ZY} to form another right angle. Make sure that the class understands that point S can be located on \overrightarrow{ZY} so that YS = QR. This may seem like an arbitrary construction, but careful consideration of the subsequent triangle congruence statements will help students appreciate its usefulness.

1 EXAMPLE

Math Tip

Point out that applying the Transitive Property of Congruence to triangles is an extension of the same property for segments and angles.

2 EXAMPLE Visual Learners

Highlight how three statements come together in the conclusion of the flow proof. Discuss how this is similar to the way triangles are proved congruent using SSS, SAS, ASA, or AAS. Point out that the flow proof uses the three bulleted statements just before Example 2.

Quick Check

2. $\overline{CB} \cong \overline{EB}$ and $m \angle CBD =$ $m \angle EBA$ because \overline{AD} is the \perp of *CE*. It is given that $CD \cong \overline{EA}$. $\triangle CBD \cong \triangle EBA$ by HL.

To use the HL Theorem, you must show that three conditions are met.

- There are two right triangles.
- The triangles have congruent hypotenuses.
- There is one pair of congruent legs.

EXAMPLE

Real-World (Connection

Tent Design On the tent, $\angle CPA$ and $\angle MPA$ are right angles and $\overline{CA} \cong \overline{MA}$. Can you use one pattern to cut fabric for both flaps of the tent? Explain.

Check whether the two right triangles meet the three conditions for the HL Theorem.

- You are given that $\angle CPA$ and $\angle MPA$ are right angles. $\triangle CPA$ and $\triangle MPA$ are right triangles.
- The hypotenuses of the triangles are \overline{CA} and \overline{MA} . You are given that $\overline{CA} \cong \overline{MA}$.
- \overline{PA} is a leg of both $\triangle CPA$ and $\triangle MPA. \overline{PA} \cong \overline{PA}$ by the Reflexive Property of Congruence.

 $\triangle CPA \cong \triangle MPA$ by the HL Theorem. The triangles are the same shape and size. You can use

one pattern for both flaps.





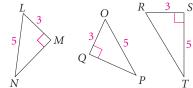
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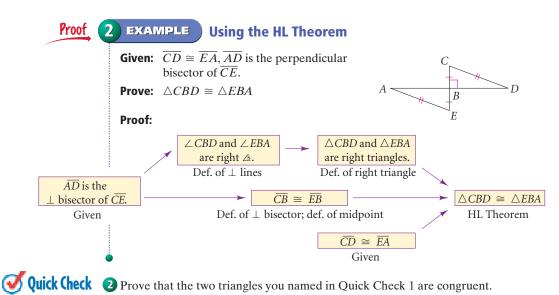
active math

For: Right Triangles Activity

Use: Interactive Textbook, 4-6

HL Theorem? Write a correct congruence statement. $\triangle LMN \cong \triangle OQP$





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Differentiated Instruction Solutions for All Learners

Advanced Learners L4

After completing Example 3, have students prove that WZKJ must contain four right angles.

English Language Learners ELL

Some students think right triangles can only use the HL Theorem. Clarify that they can also apply the SSS, SAS, and ASA Postulates and the AAS Theorem to right triangles.

learning style: verbal

See margin.

learning style: verbal

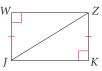
Proof

EXAMPLE

Using the HL Theorem

Given: $\overline{WJ} \cong \overline{KZ}$, $\angle W$ and $\angle K$ are right angles.

Prove: $\triangle JWZ \cong \triangle ZKJ$

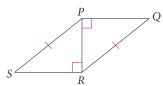


Statements	Reasons
1. $\angle W$ and $\angle K$ are right angles.	1. Given
2. $\triangle JWZ$ and $\triangle ZKJ$ are right triangles.	2. Definition of right triangle
3. $\overline{JZ} \cong \overline{JZ}$	3. Reflexive Property of Congruence
4. $\overline{WJ} \cong \overline{KZ}$	4. Given
5. $\triangle JWZ \cong \triangle ZKJ$	5. HL Theorem



Quick (heck 3 Given: $\angle PRS$ and $\angle RPQ$ are right angles, $\overline{SP} \cong \overline{OR}$.

Prove: $\triangle PRS \cong \triangle RPQ$ **See back of book.**



EXERCISES

For more exercises, see Extra Skill, Word Problem, and Proof Practice.

Practice and Problem Solving



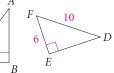
Practice by Example

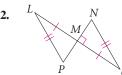
1-2. See left. Write a short paragraph to explain why the two triangles are congruent.

Example 1 (page 236)





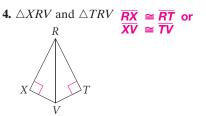




What additional information do you need to prove the triangles congruent by HL?

1. $\triangle ABC \cong \triangle DEF$ by HL. Both \triangle are rt. \triangle , \overline{AC} $\cong \overline{DF}$, and $\overline{CB} \cong \overline{FE}$.

2. $\triangle LMP \cong \triangle OMN$ by HL. Both A are rt. A because vert. *△* are ≅; $\overline{LP} \cong \overline{NO}$, and $\overline{LM} \cong \overline{OM}$. ∠*T* and ∠Q are rt. ⅍.

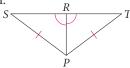


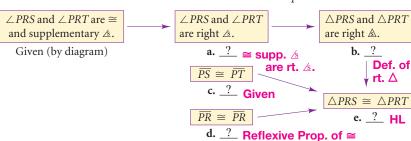
5. Developing Proof Complete the flow proof. Example 2 (page 236)

3. $\triangle BLT$ and $\triangle RKQ$

Given: $\overline{PS} \cong \overline{PT}, \angle PRS \cong \angle PRT$

Prove: $\triangle PRS \cong \triangle PRT$





Lesson 4-6 Congruence in Right Triangles 237

3 EXAMPLE Teaching Tip

As students read the proof, ask: Why is step 2 included in the proof? It establishes a needed condition for the HL Thm. to apply.

Additional Examples

1 In Example 1, one student wrote " $\triangle CPA \cong \triangle MPA$ by SAS." Is the student correct? Explain. No; the congruent angles are not included angles.

 $\triangle XYZ$ is isosceles. From vertex X, a perpendicular is drawn to \overline{YZ} , intersecting \overline{YZ} at point M. Explain why $\triangle XMY \cong \triangle XMZ$. $\triangle XMY$ and $\triangle XMZ$ are right triangles, $\overline{XY} = \overline{XZ}$ by def. of isosceles, and $\overline{XM} \cong \overline{XM}$ by Reflexive Prop., so $\triangle XMY \cong \triangle XMZ$ by HL Thm.

3 Write a two-column proof.

Given: ∠ABC and ∠DCB are right

angles, $\overline{AC} \cong \overline{DB}$. **Prove:** $\triangle ABC \cong \triangle DCB$



- 1. ∠ABC and ∠DCB are rt. angles. (Given)
- 2. $\triangle ABC$ and $\triangle DCB$ are rt. triangles. (Def. of rt. triangle)
- 3. $\overline{AC} \cong \overline{DB}$ (Given)
- 4. $\overline{BC} \cong \overline{CB}$ (Reflexive Prop.
- 5. $\triangle ABC \cong \triangle DCB$ (HL Thm.)

Resources

- Daily Notetaking Guide 4-6
- Daily Notetaking Guide 4-6-L1 Adapted Instruction

Closure

How are SAS and HL alike, and how are they different? Both prove triangles congruent using two pairs of sides and one pair of angles. SAS is a postulate, and the angle is an included angle. HL is a theorem, the triangle must be right, and the angle is not an included angle.

3. Practice

Assignment Guide

V A B 1-24

C Challenge 25-26

Test Prep 27-30 Mixed Review 31-39

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 4, 7, 10, 14, 22.

Connection to Algebra

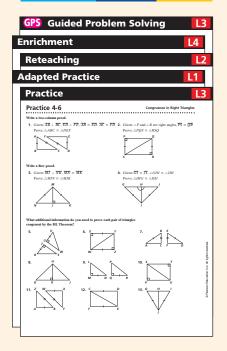
Exercises 10, 11 Students must solve a system of two equations. If necessary, have them reread the Algebra Review on page 234.

Visual Learners

Exercise 14 Students may need to copy the diagram and extend \overline{PM} to see that it is a transversal for the parallel lines in the diagram.

Exercises 16-19 Students will need compasses and straightedges. Have students demonstrate and explain their constructions to partners.

Differentiated Instruction Resources



<u>Proof</u> 6. Given: $\overline{AD} \cong \overline{CB}$, $\angle D$ and $\angle B$ are right angles.

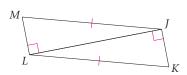
Prove: $\triangle ADC \cong \triangle CBA$ **See back of book.**



Example 3 (page 237) **7. Developing Proof** Complete the two-column proof.

Given: $\overline{JL} \perp \overline{LM}$, $\overline{LJ} \perp \overline{JK}$, $\overline{MJ} \cong \overline{KL}$

Prove: $\triangle JLM \cong \triangle LJK$



Statements

1. $\overline{JL} \perp \overline{LM}$ and $\overline{LJ} \perp \overline{JK}$

2. $\angle JLM$ and $\angle LJK$ are right angles.

c. ? $\triangle MLJ$ and $\triangle KJL$ are rt. \triangle .

4. $\overline{MJ} \cong \overline{KL}$

e. ? $\overline{LJ} \cong \overline{LJ}$

6. $\triangle JLM \cong \triangle LJK$

Proof 8. Given: $\overline{HV} \perp \overline{GT}, \overline{GH} \cong \overline{TV},$ I is the midpoint of \overline{HV} .

> **Prove:** $\triangle IGH \cong \triangle ITV$ See back of book.



a. ? Given

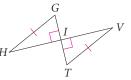
b. $\underline{?}$ Def. of \bot

3. Definition of a right triangle

d. ? Given

5. Reflexive Property of Congruence

f. ? HL



Apply Your Skills



Real-World (Connection

Interest in antiques and shifts

in fashion have stabilized the

need for dial-clock repair skills.

by the isosc. \triangle thm.

Visit: PHSchool.com

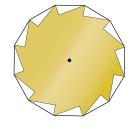
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 $\angle PRS \cong \angle PRT.$ $\triangle PRS \cong \triangle PRT$ by

nline Homework Help

AAS.

9. Antiques To repair an antique clock, a 12-toothed wheel has to be made by cutting right triangles out of a regular polygon that has twelve 4-cm sides. The hypotenuse of each triangle is a side of the regular polygon, and the shorter leg is 1 cm long. Explain why the 12 triangles must be congruent. HL; each rt. \triangle has a \cong hyp. and side.



 x^2 Algebra In Exercises 10 and 11, for what values of x and y are the triangles congruent by HL?

10. œs _x



12. Critical Thinking While working for a landscape architect, you are told to lay out a flower bed in the shape of a right triangle with sides of 3 yd and 7 yd. Explain what else you need to know in order to make the flower bed. whether the 7-yd side is the hyp. or a leg

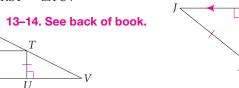
<u>Proof</u> 13. Given: $\overline{RS} \cong \overline{TU}, \overline{RS} \perp \overline{ST},$ 15. $\overline{PS} \cong \overline{PT}$ so $\angle S \cong \angle T$

 $\overline{TU} \perp \overline{UV}$. T is the midpoint of \overline{RV} .

14. Given: $\overline{JM} \cong \overline{WP}, \overline{JP} \parallel \overline{MW}.$ $\overline{JP} \perp \overline{PM}$

Prove: $\triangle JMP \cong \triangle WPM$

Prove: $\triangle RST \cong \triangle TUV$



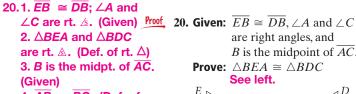
Proof 15. Study Exercise 5. There is a different set of steps that will prove $\triangle PRS \cong \triangle PRT$. Decide what they are. Then write a proof using these steps.

238 **Chapter 4** Congruent Triangles **Constructions** Copy the triangle and construct a triangle congruent to it using the method stated.

16. by SAS **16–19.**

17. by HL

18. by ASA of book.



4. $\overline{AB} \cong \overline{BC}$ (Def. of midpt.)

5. $\triangle BEA \cong \triangle BDC$ (HL)



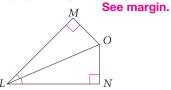
are right angles, and B is the midpoint of \overline{AC} .

> **Prove:** $\triangle BEA \cong \triangle BDC$ See left.



21. Given: \overline{LO} bisects $\angle MLN$, $\overline{OM} \perp \overline{LM}$, and $\overline{ON} \perp \overline{LN}$.

Prove: $\triangle LMO \cong \triangle LNO$





Exercise 22

22. Open-Ended You are the DJ for the school dance. To set up, you have placed one speaker in the corner of the platform. What measurement(s) could you make with a tape measure to make sure that a matching speaker is in the other corner at exactly the same angle? Explain why your method works. See margin.

23. a. Coordinate Geometry Use grid paper. Graph the points E(-1, -1), F(-2, -6), G(-4, -4), and D(-6, -2). Connect the points with segments.

b. Find the slope for each of \overline{DG} , \overline{GF} , and \overline{GE} . **a-c. See back of book.**

c. Use your answer to part (b) to describe $\angle EGD$ and $\angle EGF$.

d. Use the Distance Formula to find *DE* and *FE*. **DE** = $\sqrt{26}$; **FE** = $\sqrt{26}$

e. Write a paragraph to prove that $\triangle EGD \cong \triangle EGF$. See back of book.

24. Critical Thinking "A HA!" exclaims Francis. "There is an HA Theorem ..., something like the HL Theorem!" Explain what Francis is saying and why he is correct or incorrect. An HA Thm. is the same as AAS with AAS corr. to the rt. \angle , an acute \angle , and the hyp.

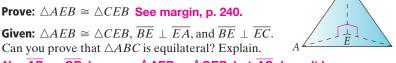


Geometry in 3 Dimensions Use the figure at the right for Exercises 25 and 26.

Proof 25. Given: $\overline{BE} \perp \overline{EA}, \overline{BE} \perp \overline{EC}, \triangle ABC$ is equilateral.



Can you prove that $\triangle ABC$ is equilateral? Explain. No; $\overline{AB} \cong \overline{CB}$ because $\triangle AEB \cong \triangle CEB$, but \overline{AC} doesn't have





Test Prep

Multiple Choice

In Exercises 27 and 28, which additional congruence statement could you use to prove that $\triangle BJK \cong \triangle CFH$ by HL?

27. Given: $\overline{BJ} \cong \overline{CF}$ A

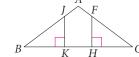
to be \cong to \overrightarrow{AB} or to \overrightarrow{CB} .

A. $\overline{JK} \cong \overline{FH}$

B. $\angle B \cong \angle C$

C. $\overline{AI} \cong \overline{AF}$

D. $\angle BJK \cong \angle CFH$



В

28. Given: $\overline{BK} \cong \overline{CH}$ H

F. $\overline{JK} \cong \overline{FH}$

G. $\angle B \cong \angle C$

H. $\overline{JB} \cong \overline{FC}$

J. $\angle BJK \cong \angle CFH$



Lesson 4-6 Congruence in Right Triangles

- 21. 1. *LO* bisects ∠*MLN*. $\overline{OM} \perp \overline{LM}, \overline{ON} \perp \overline{LN},$ (Given)
 - 2. $\angle M$ and $\angle N$ are rt. \triangle (Def. of \perp)
- 3. $\angle MLO \cong \angle NLO$ (Def. of \angle bis.)
- 4. $\angle M \cong \angle N$ (All rt. \triangle are ≅.)
- 5. $\overline{LO} \cong \overline{LO}$ (Reflexive Prop. of \cong)
- 6. $\triangle LMO \cong \triangle LNO$ (AAS)

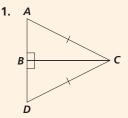
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22. Answers may vary. Sample: Measure 2 sides of the Δ formed by the amp. and the platform's

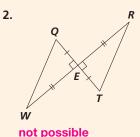
4. Assess & Reteach



For Exercises 1 and 2, tell whether the HL Theorem can be used to prove the triangles congruent. If so, explain. If not, write not possible.

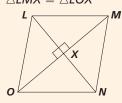


Yes; use congruent hypotenuses and leg \overline{BC} to prove $\triangle ABC \cong \triangle DBC$.



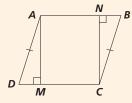
For Exercises 3 and 4, what additional information do you need to prove the triangles congruent by the HL Theorem?

3. $\triangle LMX \cong \triangle LOX$



 $\overline{LM} \cong \overline{LO}$

4. $\triangle AMD \cong \triangle CNB$



 $\overline{AM} \cong \overline{CN} \text{ or } \overline{MD} \cong \overline{NB}$

corner. Since the A will be \cong by HL or SAS, the sare the same.

Alternative Assessment

Have each student write a paragraph to explain why the following statement is true or false: To prove triangles congruent, you usually need 3 pairs of congruent parts. With HL, you need only 2 pairs of congruent parts.

Test Prep



Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 253
- Test-Taking Strategies, p. 248
- Test-Taking Strategies with Transparencies



Use this Checkpoint Quiz to check students' understanding of the skills and concepts of Lessons 4-4 through 4-6.

Resources

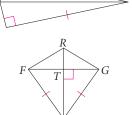
Grab & Go

• Checkpoint Quiz 2

- 29. Which congruence statement can be used to prove that the two triangles are congruent? D
 - A. SAS
- B. SSS
- C. ASA
- D. HL



- 30. a. Use the diagram at the right to name all the pairs of triangles you could prove congruent by using the HL Theorem. a-b. See margin.
 - **b.** Suppose you need to prove $\triangle RFW \cong \triangle RGW$. What specifically do you need to prove before you can use the HL Theorem?



Mixed Review



Lesson 4-5

For Exercises 31 and 32, what type of triangle must $\triangle XYZ$ be?



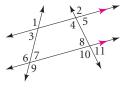
- 31. $\triangle XYZ \cong \triangle ZYX$ isosceles
- **32.** $\triangle XYZ \cong \triangle ZXY$ equilateral

- Lesson 3-7
- **33.** Connect A(3,3), B(5,5), C(9,1), and D(9,-3) in order. Are any sides of thefigure parallel? Are any sides perpendicular? Explain. See back of book.

Lesson 3-1 State the postulate or theorem that justifies each statement.

- **34.** ∠5 ≅ ∠8
- **35.** $m \angle 4 + m \angle 8 = 180$ **34–39.**
- **36.** ∠6 ≅ ∠9
- **37.** ∠4 \cong ∠10
- See back of book.

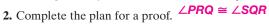
- **38.** ∠1 ≅ ∠6
- **39.** $\angle 6$ and $\angle 3$ are supplementary.

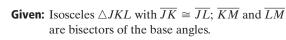


Checkpoint Quiz 2

Lessons 4-4 Through 4-6

1. In the diagram at the right, $\triangle PQR \cong \triangle SRQ$ by SAS. What other pairs of sides and angles can you conclude are congruent by CPCTC? $\overline{PR} \cong \overline{SQ}$; $\angle P \cong \angle S$;





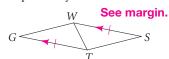
Prove: $\triangle KML$ is isosceles.

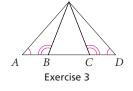
Plan: Since $\triangle JKL$ is isosceles, $\angle JKL \cong \angle JLK$ by the a. ? Theorem. Since \overline{KM} and \overline{LM} are angle Isosc. \triangle bisectors, $\angle MKL$ **b.** ? $\angle MLK$. Therefore, $\triangle KML \cong$

is isosceles by the c. ? Theorem. Converse of the Isosc. \triangle Thm.



- **4.** Why are these triangles congruent?
- **5.** Explain why $\overline{GW} \cong \overline{ST}$.





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- 30. [2] a. $\triangle TFW \cong \triangle TGW$
 - b. $\angle RFW$ and $\angle RGW$ are rt. \(\lambda\).
 - [1] one part correct

Checkpoint Quiz 2

- 3. $\triangle AED$: $\angle EAB \cong \angle EDC$ (Given) $\triangle EBC$; $\angle EBC \cong \angle ECB$ (Suppl. of $\cong \triangle$ are \cong .)
- 5. $\triangle GTW \cong \triangle SWT$ by SAS since $\overline{WT} \cong \overline{WT}$, $\angle WTG$ $\cong \angle TWS$, and $\overline{GT} \cong \overline{SW}$. So $\overline{GW} \cong \overline{ST}$ by CPCTC.

25. Since $\overline{BE} \perp \overline{EA}$ and $\overline{BE} \perp \overline{EC}$, $\triangle AEB$ and $\triangle CEB$ are both rt. > . $\overline{AB} \cong \overline{BC}$ because \triangle ABC is equilateral, and $BE \cong BE. \triangle AEB \cong$ 240 $\triangle CEB$ by HL.