## Congruence in Right Triangles

## What You'll Learn

- To prove triangles congruent using the HL Theorem


## ... And Why

To show that one pattern can be used to cut the fabric for the two entrance flaps of a tent, as in Example 1

## Check Skills You'll Need



Tell whether the abbreviation identifies a congruence statement.

1. SSS yes
2. SAS yes
3. SSA no
4. ASA yes
5. AAS yes
6. AAA no

Can you conclude that the two triangles are congruent? Explain.
7.

8.


New Vocabulary • hypotenuse • legs of a right triangle

The Hypotenuse-Leg Theorem
In a right triangle, the side opposite the right angle is the longest side and is called the hypotenuse. The other two sides are called legs.

Right triangles provide a special case for which there
 is an SSA congruence rule. (See Lesson 4-3, Exercise 32.)
It occurs when hypotenuses are congruent and one pair of legs are congruent.

## Theorem 4-6 Hypotenuse-Leg (HL) Theorem

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

## Proof of the HL Theorem

Given: $\triangle P Q R$ and $\triangle X Y Z$ are right triangles, with right angles $Q$ and $Y$ respectively. $\overline{P R} \cong \overline{X Z}$, and $\overline{P Q} \cong \overline{X Y}$.
Prove: $\triangle P Q R \cong \triangle X Y Z$
Proof: On $\triangle X Y Z$ at the right, draw $\overrightarrow{Z Y}$.
Mark point $S$ as shown so that $Y S=Q R$. Then, $\triangle P Q R \cong \triangle X Y S$ by SAS. By CPCTC,$\overline{P R} \cong \overline{X S}$.
It is given that $\overline{P R} \cong \overline{X Z}$, so $\overline{X S} \cong \overline{X Z}$ by
 the Transitive Property of Congruence.


By the Isosceles Triangle Theorem, $\angle S \cong \angle Z$, so $\triangle X Y S \cong \triangle X Y Z$ by AAS. Therefore, $\triangle P Q R \cong \triangle X Y Z$ by the Transitive Property of Congruence.

Lesson 4-6 Congruence in Right Triangles

## Differentiated Instruction Solutions for All Learners

## Special Needs <br> L1

Point out that although only two letters are used to name the HL Theorem, there are three conditions: two right angles, one pair of congruent hypotenuses, and one pair of congruent legs.

## Below Level L2

Have students use the diagram in the proof of the HL Theorem to explain why the HL Theorem is not a special case of the SAS Postulate.

## 2. Teach

## Guided Instruction

## Teaching Tip

Before reading the proof of the HL Theorem, discuss a Plan for Proof with the class. Draw $\triangle X Y Z$, and discuss why you might want to extend $\overline{Z Y}$ to form another right angle. Make sure that the class understands that point $S$ can be located on $\overrightarrow{Z Y}$ so that $Y S=Q R$. This may seem like an arbitrary construction, but careful consideration of the subsequent triangle congruence statements will help students appreciate its usefulness.

## EXAMPLE Math Tip

Point out that applying the Transitive Property of Congruence to triangles is an extension of the same property for segments and angles.

## (2) ЕхаNPLE Visual Learners

Highlight how three statements come together in the conclusion of the flow proof. Discuss how this is similar to the way triangles are proved congruent using SSS, SAS, ASA, or AAS. Point out that the flow proof uses the three bulleted statements just before Example 2.

## Quick Check

2. $\overline{C B} \cong \overline{E B}$ and $m \angle C B D=$ $m \angle E B A$ because $\overline{A D}$ is the $\perp$ of $\overline{C E}$. It is given that $\overline{C D} \cong \overline{E A} . \triangle C B D \cong \triangle E B A$ by HL.


For: Right Triangles Activity Use: Interactive Textbook, 4-6

To use the HL Theorem, you must show that three conditions are met.

- There are two right triangles.
- The triangles have congruent hypotenuses.
- There is one pair of congruent legs.


## (1) EXADMPLE Real-World Connection

Tent Design On the tent, $\angle C P A$ and $\angle M P A$ are right angles and $\overline{C A} \cong \overline{M A}$.
Can you use one pattern to cut fabric for both flaps of the tent? Explain.
Check whether the two right triangles meet the three conditions for the HL Theorem.

- You are given that $\angle C P A$ and $\angle M P A$ are right angles. $\triangle C P A$ and $\triangle M P A$ are right triangles.
- The hypotenuses of the triangles are $\overline{C A}$ and $\overline{M A}$. You are given that $\overline{C A} \cong \overline{M A}$.
- $\overline{P A}$ is a leg of both $\triangle C P A$ and $\triangle M P A \cdot \overline{P A} \cong \overline{P A}$ by the Reflexive Property of Congruence.
$\triangle C P A \cong \triangle M P A$ by the HL
Theorem. The triangles are the same shape and size. You can use
one pattern for both flaps.


Which two triangles are congruent by the HL Theorem? Write a correct congruence statement. $\triangle L M N \cong \triangle O Q P$


Quick Check 2
Prove that the two triangles you named in Quick Check 1 are congruent. See margin.

## Differentiated Instruction solutions for All Learners

## Advanced Learners L4

After completing Example 3, have students prove that WZKJ must contain four right angles.

## English Language Learners ELL

Some students think right triangles can only use the HL Theorem. Clarify that they can also apply the SSS, SAS, and ASA Postulates and the AAS Theorem to right triangles.

Proof 3 ExADIPLE Using the HL Theorem
Given: $\overline{W J} \cong \overline{K Z}, \angle W$ and $\angle K$ are right angles.
Prove: $\triangle J W Z \cong \triangle Z K J$

Statements

1. $\angle W$ and $\angle K$ are right angles.
2. $\triangle J W Z$ and $\triangle Z K J$ are right triangles.
3. $\frac{\overline{J Z} \cong \overline{J Z}}{\overline{W Z}}$
4. $\triangle J W Z \cong \triangle Z K J$

Quick Check (3) Given: $\angle P R S$ and $\angle R P Q$ are right angles, $\overline{S P} \cong \overline{Q R}$
Prove: $\triangle P R S \cong \triangle R P Q$ See back of book.

## 1. Given

2. Definition of right triangle
3. Reflexive Property of Congruence
4. Given
5. HL Theorem
$\forall$


## EXERCISES

For more exercises, see Extra Skill, Word Problem, and Proof Practice.

## Practice and Problem Solving

A Practice by Example
Example 1


1. $\triangle A B C \cong \triangle D E F$ by HL. Both $\triangle$ are rt. $\triangle, \overline{A C}$ $\cong \overline{D F}$, and $\overline{C B} \cong \overline{F E}$.
2. $\triangle L M P \cong \triangle O M N$ by $H L$.

Both \& are rt. ©
because vert. $\stackrel{s}{ }$ are $\cong$; $\overline{L P} \cong \overline{N O}$, and $\overline{L M} \cong \overline{O M}$.

Example 2 (page 236)

Write a short paragraph to explain why the two triangles are congruent
1.


2.


What additional information do you need to prove the triangles congruent by HL?
3. $\triangle B L T$ and $\triangle R K Q$

4. $\triangle X R V$ and $\triangle T R V \overline{R X} \cong \overline{R T}$ or

5. Developing Proof Complete the flow proof.

Given: $\overline{P S} \cong \overline{P T}, \angle P R S \cong \angle P R T$
Prove: $\triangle P R S \cong \triangle P R T$


Lesson 4-6 Congruence in Right Triangles

As students read the proof, ask: Why is step 2 included in the proof? It establishes a needed condition for the HL Thm. to apply.

## Additional Examples

1 In Example 1, one student wrote " $\triangle C P A \cong \triangle M P A$ by SAS." Is the student correct? Explain. No; the congruent angles are not included angles.
2. $\triangle X Y Z$ is isosceles. From vertex $X$, a perpendicular is drawn to $\overline{Y Z}$, intersecting $\overline{Y Z}$ at point $M$. Explain why $\triangle X M Y \cong \triangle X M Z . \triangle X M Y$ and $\triangle X M Z$ are right triangles, $\overline{X Y}=\overline{X Z}$ by def. of isosceles, and $\overline{X M} \cong \overline{X M}$ by Reflexive Prop. so $\triangle X M Y \cong \triangle X M Z$ by HL Thm.

Write a two-column proof.
Given: $\angle A B C$ and $\angle D C B$ are right angles, $\overline{A C} \cong \overline{D B}$.
Prove: $\triangle A B C \cong \triangle D C B$


1. $\angle A B C$ and $\angle D C B$ are rt. angles. (Given)
2. $\triangle A B C$ and $\triangle D C B$ are rt. triangles. (Def. of rt. triangle)
3. $\overline{A C} \cong \overline{D B}$ (Given)
4. $\overline{B C} \cong \overline{C B}$ (Reflexive Prop.
of $\cong$ )
5. $\triangle A B C \cong \triangle D C B$ (HL Thm.)

## Resources

- Daily Notetaking Guide 4-6 L3
- Daily Notetaking Guide 4-6Adapted Instruction


## Closure

How are SAS and HL alike, and how are they different? Both prove triangles congruent using two pairs of sides and one pair of angles. SAS is a postulate, and the angle is an included angle. HL is a theorem, the triangle must be right, and the angle is not an included angle.

## 3. Practice

## Assignment Guide

1 A B 1-24
C Challenge 25-26
Test Prep 27-30
Mixed Review
31-39

## Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 4, 7, 10, 14, 22.

## Connection to Algebra

Exercises 10, 11 Students must solve a system of two equations. If necessary, have them reread the Algebra Review on page 234.

## Visual Learners

Exercise 14 Students may need to copy the diagram and extend $\overline{P M}$ to see that it is a transversal for the parallel lines in the diagram.

Exercises 16-19 Students will need compasses and straightedges. Have students demonstrate and explain their constructions to partners.


Example 3
(page 237)

Proof 6. Given: $\overline{A D} \cong \overline{C B}, \angle D$ and $\angle B$ are right angles.
Prove: $\triangle A D C \cong \triangle C B A$ See back of book.
7. Developing Proof Complete the two-column proof.
Given: $\overline{J L} \perp \overline{L M}, \overline{L J} \perp \overline{J K}, \overline{M J} \cong \overline{K L}$
Prove: $\triangle J L M \cong \triangle L J K$
Statements

1. $\overline{J L} \perp \overline{L M}$ and $\overline{L J} \perp \overline{J K}$
2. $\angle J L M$ and $\angle L J K$ are right angles.
c. ? $\triangle M L J$ and $\triangle K J L$ are rt. © .
3. $\overline{M J} \cong \overline{K L}$
e. ? $\overline{L J} \cong \overline{L J}$
4. $\triangle J L M \cong \triangle L J K$


## Reasons

a. ? Given
b. ? Def. of $\perp$
3. Definition of a right triangle
d. ? Given
5. Reflexive Property of

Congruence
f. ? HL

Proof 8. Given: $\overline{H V} \perp \overline{G T}, \overline{G H} \cong \overline{T V}$,
$I$ is the midpoint of $\overline{H V}$.
Prove: $\triangle I G H \cong \triangle I T V$
See back of book.


Apply Your Skills


## Real-World Connection

Interest in antiques and shifts in fashion have stabilized the need for dial-clock repair skills.
15. $\overline{P S} \cong \overline{P T}$ so $\angle S \cong \angle T$
by the isosc. $\Delta$ thm.
$\angle P R S \cong \angle P R T$.
$\triangle P R S \cong \triangle P R T$ by AAS.
nline Homework Help
Visit: PHSchool.com Web Code: aue-0406

Chapter 4 Congruent Triangles
9. Antiques To repair an antique clock, a 12 -toothed wheel has to be made by cutting right triangles out of a regular polygon that has twelve $4-\mathrm{cm}$ sides. The hypotenuse of each triangle is a side of the regular polygon, and the shorter leg is 1 cm long. Explain why the 12 triangles must be congruent. HL ; each rt. $\Delta$ has a $\cong$ hyp. and side.
Algebra In Exercises 10 and 11, for what values of
 $x$ and $y$ are the triangles congruent by HL?

12. Critical Thinking While working for a landscape architect, you are told to lay out a flower bed in the shape of a right triangle with sides of 3 yd and 7 yd . Explain what else you need to know in order to make the flower bed. whether the 7-yd side is the hyp. or a leg
Proof 13. Given: $\overline{R S} \cong \overline{T U}, \overline{R S} \perp \overline{S T}, \quad$ 14. Given: $\overline{J M} \cong \overline{W P}, \overline{J P} \| \overline{M W}$,
$\overline{T U} \perp \overline{U V}, T$ is the
midpoint of $\overline{R V}$.
Prove: $\triangle R S T \cong \triangle T U V$


Prove: $\triangle J M P \cong \triangle W P M$


Proof 15. Study Exercise 5. There is a different set of steps that will prove $\triangle P R S \cong \triangle P R T$. Decide what they are. Then write a proof using these steps. See left.

Constructions Copy the triangle and construct a triangle congruent to it using the method stated.
16. by SAS
16-19.
17. by HL
18. by ASA See back of book.
19. by SSS

20.1. $\overline{E B} \cong \overline{D B} ; \angle A$ and
$\angle C$ are rt. $\angle$. (Given) Proof 20. Given: $\overline{E B} \cong \overline{D B}, \angle A$ and $\angle C$
2. $\triangle B E A$ and $\triangle B D C$ are rt. ©. (Def. of rt. $\Delta$ ) 3. $B$ is the midpt. of $\overline{A C}$. (Given)
4. $\overline{A B} \cong \overline{B C}$ (Def. of midpt.)
5. $\triangle B E A \cong \triangle B D C(H L)$


Exercise 22
(32) 22. Open-Ended You are the DJ for the school dance. To set up, you have placed one speaker in the corner of the platform. What measurement(s) could you make with a tape measure to make sure that a matching speaker is in the other corner at exactly the same angle? Explain why your method works. See margin.
23. a. Coordinate Geometry Use grid paper. Graph the points $E(-1,-1)$, $F(-2,-6), G(-4,-4)$, and $D(-6,-2)$. Connect the points with segments.
b. Find the slope for each of $\overline{D G}, \overline{G F}$, and $\overline{G E}$. a-c. See back of book.
c. Use your answer to part (b) to describe $\angle E G D$ and $\angle E G F$.
d. Use the Distance Formula to find $D E$ and $F E . D E=\sqrt{26} ; F E=\sqrt{26}$
e. Write a paragraph to prove that $\triangle E G D \cong \triangle E G F$. See back of book.
24. Critical Thinking "A HA!" exclaims Francis. "There is an HA Theorem . . ., something like the HL Theorem!" Explain what Francis is saying and why he is correct or incorrect. An HA Thm. is the same as AAS with AAS corr. to the rt. $\angle$, an acute $\angle$, and the hyp.
C Challenge
Geometry in 3 Dimensions Use the figure at the right for
Exercises 25 and 26.
Proof 25. Given: $\overline{B E} \perp \overline{E A}, \overline{B E} \perp \overline{E C}, \triangle A B C$ is equilateral. Prove: $\triangle A E B \cong \triangle C E B$ See margin, p. 240.
26. Given: $\triangle A E B \cong \triangle C E B, \overline{B E} \perp \overline{E A}$, and $\overline{B E} \perp \overline{E C}$. Can you prove that $\triangle A B C$ is equilateral? Explain.


Multiple Choice
In Exercises 27 and 28, which additional congruence statement could you use to prove that $\triangle B J K \cong \triangle C F H$ by HL?
27. Given: $\overline{B J} \cong \overline{C F} \mathrm{~A}$
A. $\overline{J K} \cong \overline{F H}$
B. $\angle B \cong \angle C$
C. $\overline{A J} \cong \overline{A F}$
D. $\angle B J K \cong \angle C F H$

28. Given: $\overline{B K} \cong \overline{C H} \quad H$
F. $\overline{J K} \cong \overline{F H}$
G. $\angle B \cong \angle C$
H. $\overline{J B} \cong \overline{F C}$
J. $\angle B J K \cong \angle C F H$
nline lesson quiz, PHSchool.com, Web Code: aua-0406
Lesson 4-6 Congruence in Right Triangles

## 21. 1. $\overline{L O}$ bisects $\angle M L N$, $\overline{O M} \perp \overline{L M}, \overline{O N} \perp \overline{L N}$, (Given)

2. $\angle M$ and $\angle N$ are rt. $\angle s$ (Def. of $\perp$ )

> 3. $\angle M L O \cong \angle N L O$ (Def. of $\angle$ bis.)
4. $\angle M \cong \angle N$ (All rt. $\angle \mathrm{s}$ are $\cong$.
5. $\overline{L O} \cong \overline{L O}$ (Reflexive Prop. of $\cong$ )
6. $\triangle L M O \cong \triangle L N O$ (AAS)
22. Answers may vary. Sample: Measure 2 sides of the $\Delta$ formed by the amp. and the platform's

## 4. Assess \& Reteach

## Powerpoint <br> Lesson Quiz

For Exercises 1 and 2, tell whether the HL Theorem can be used to prove the triangles congruent. If so, explain. If not, write not possible.
1.


Yes; use congruent hypotenuses and leg $\overline{B C}$ to prove $\triangle A B C \cong \triangle D B C$.
2.

not possible
For Exercises 3 and 4, what additional information do you need to prove the triangles congruent by the HL Theorem?
3. $\triangle L M X \cong \triangle L O X$

$\overline{L M} \cong \overline{L O}$
4. $\triangle A M D \cong \triangle C N B$

$\overline{A M} \cong \overline{C N}$ or $\overline{M D} \cong \overline{N B}$
corner. Since the $\triangle$ will be $\cong$ by HL or SAS, the $\measuredangle$ are the same.

## Alternative Assessment

Have each student write a paragraph to explain why the following statement is true or false: To prove triangles congruent, you usually need 3 pairs of congruent parts. With HL, you need only 2 pairs of congruent parts.

## Test Prep

## Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 253
- Test-Taking Strategies, p. 248
- Test-Taking Strategies with Transparencies


## Checkpoint Quiz

Use this Checkpoint Quiz to check students' understanding of the skills and concepts of Lessons 4-4 through 4-6.

## Resources

## Grab \& Go

- Checkpoint Quiz 2

25. Since $\overline{B E} \perp \overline{E A}$ and $\overline{B E} \perp \overline{E C}, \triangle A E B$ and $\triangle C E B$ are both rt. > . $\overline{A B} \cong \overline{B C}$ because $\triangle A B C$ is equilateral, and $\overline{B E} \cong \overline{B E} . \triangle A E B \cong$ $\triangle C E B$ by HL.

Short Response
congruence statement can be used to prove that the two triangles are congruent? D
A. SAS
B. SSS
C. ASA
D. HL

30. a. Use the diagram at the right to name all the pairs of triangles you could prove congruent by using the HL Theorem. a-b. See margin.
b. Suppose you need to prove $\triangle R F W \cong \triangle R G W$ What specifically do you need to prove before you can use the HL Theorem?


## Mixed Review

Lesson 4-5
For Exercises 31 and 32, what type of triangle must $\triangle X Y Z$ be?
31. $\triangle X Y Z \cong \triangle Z Y X$ isosceles
32. $\triangle X Y Z \cong \triangle Z X Y$ equilateral

Lesson 3-7
33. Connect $A(3,3), B(5,5), C(9,1)$, and $D(9,-3)$ in order. Are any sides of the figure parallel? Are any sides perpendicular? Explain. See back of book.

Lesson 3-1 State the postulate or theorem that justifies each statement.
34. $\angle 5 \cong \angle 8$
36. $\angle 6 \cong \angle 9$
35. $m \angle 4+m \angle 8=180$ 34-39.
37. $\angle 4 \cong \angle 10$

See back
of book.
38. $\angle 1 \cong \angle 6$
39. $\angle 6$ and $\angle 3$ are supplementary.


Lessons 4-4 Through 4-6

1. In the diagram at the right, $\triangle P Q R \cong \triangle S R Q$ by SAS. What other pairs of sides and angles can you conclude are congruent by CPCTC? $\overline{P R} \cong \overline{S Q} ; \angle P \cong \angle S$;
2. Complete the plan for a proof. $\angle P R Q \cong \angle S Q R$


Given: Isosceles $\triangle J K L$ with $\overline{J K} \cong \overline{J L} ; \overline{K M}$ and $\overline{L M}$ are bisectors of the base angles.
Prove: $\triangle K M L$ is isosceles.
Plan: Since $\triangle J K L$ is isosceles, $\angle J K L \cong \angle J L K$ by
 the a. ? Theorem. Since $\overline{K M}$ and $\overline{L M}$ are angle Isosc. $\triangle$ bisectors, $\angle M K L$ b. ? $\angle M L K$. Therefore, $\triangle K M L \cong$ is isosceles by the c. ? Theorem. Converse of the Isosc. $\Delta$ Thm.
3. Six triangles are pictured in the diagram at the left. Which of the triangles are isosceles? Explain. See margin.
4. Why are these triangles congruent?

5. Explain why $\overline{G W} \cong \overline{S T}$.


Chapter 4 Congruent Triangles
30. [2] a. $\triangle T F W \cong \triangle T G W$
b. $\angle R F W$ and $\angle R G W$ are rt. $\llcorner$.
[1] one part correct

## Checkpoint Quiz 2

3. $\triangle A E D ; \angle E A B \cong \angle E D C$ (Given)
$\triangle E B C ; \angle E B C \cong \angle E C B$
(Suppl. of $\cong \measuredangle$ are $\cong$.)
4. $\triangle G T W \cong \triangle S W T$ by SAS since $\overline{W T} \cong \overline{W T}, \angle W T G$ $\cong \angle T W S$, and $\overline{G T} \cong \overline{S W}$. So $\overline{G W} \cong \overline{S T}$ by СРСТС.
