4-9 Slopes of Parallel and Perpendicular Lines

Objectives

Identify and graph parallel and perpendicular lines.

Write equations to describe lines parallel or perpendicular to a given line.

Vocabulary

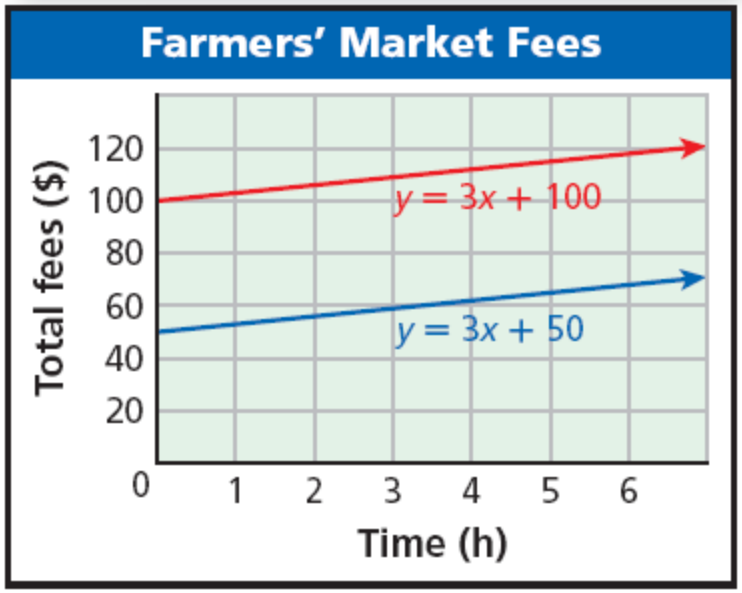
parallel lines

perpendicular lines

Slopes of Parallel and Perpendicular Lines

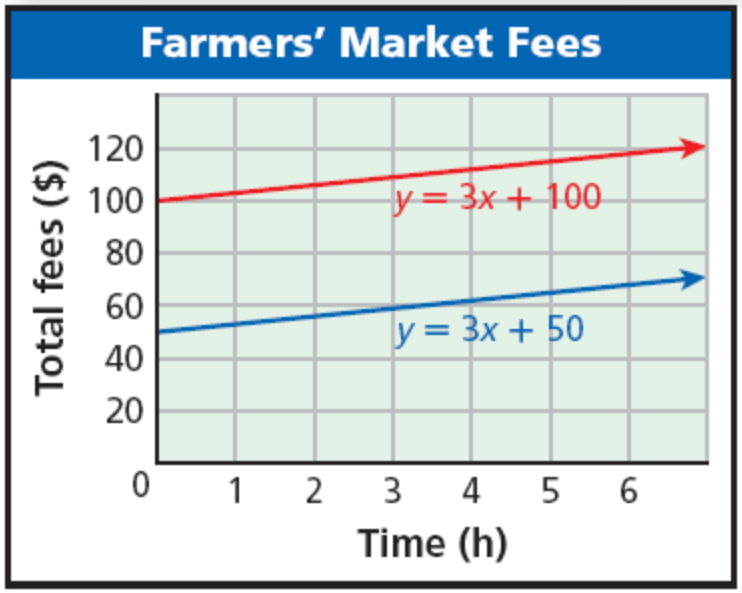
To sell at a particular farmers' market for a year, there is a $100 membership fee. Then you pay $3 for each hour that you sell at the market. However, if you were a member the previous year, the membership fee is reduced to $50.

- The red line shows the total cost if you are a new member.
- The blue line shows the total cost if you are a returning member.



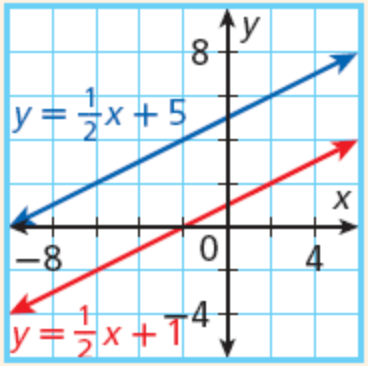
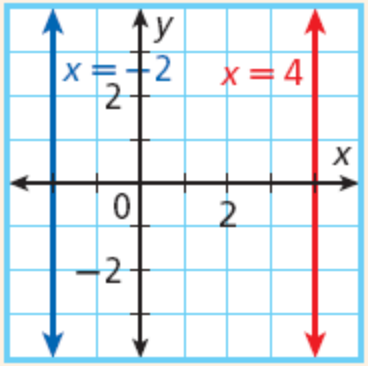
Slopes of Parallel and Perpendicular Lines

These two lines are *parallel.* **Parallel lines** are lines in the same plane that have no points in common. In other words, they do not intersect and they have the same slope.



Slopes of Parallel and Perpendicular Lines

Parallel Lines

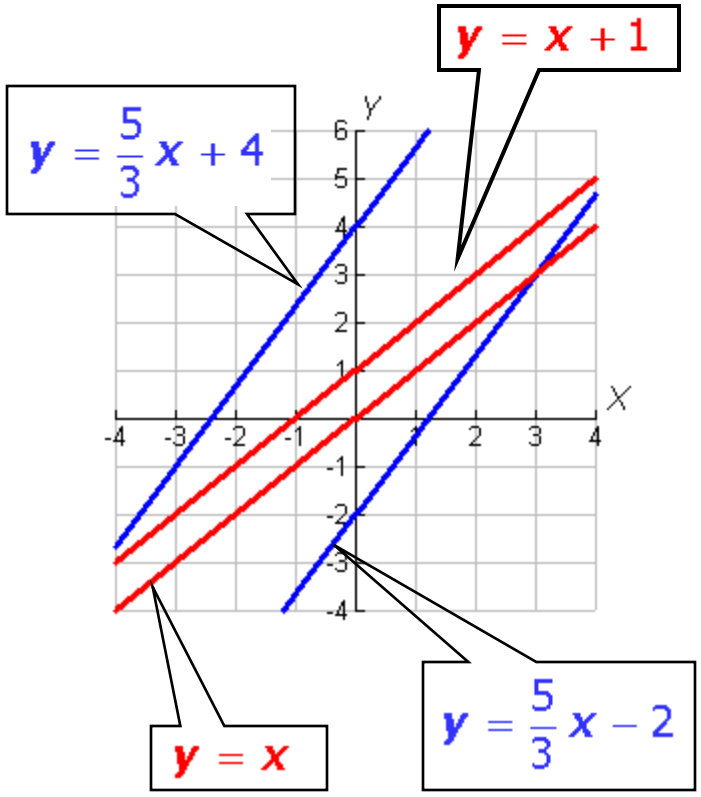
WORDS	Two different nonvertical lines are parallel if and only if they have the same slope.	All different vertical lines are parallel.
GRAPH		

Example 1A: Identifying Parallel Lines

Identify which lines are parallel.

$$y = \frac{5}{3}x - 2;\ y = x;\ y = \frac{5}{3}x + 4;\ y = x + 1$$

The lines described by $y = \frac{5}{3}x - 2$ and $y = \frac{5}{3}x + 4$ both have slope $\frac{5}{3}$. These lines are parallel. The lines described by $y = x$ and $y = x + 1$ both have slope 1. These lines are parallel.



Example 1B: Identifying Parallel Lines

Identify which lines are parallel.

$y = 2x - 3;\ y = -\frac{2}{3}x + 3;\ 2x + 3y = 8;\ y + 1 = 3(x - 3)$

Write all equations in slope-intercept form to determine the slope.

$y = 2x - 3$ slope-intercept form ✓

$y = -\frac{2}{3}x + 3$ slope-intercept form ✓

Example 1B Continued

Identify which lines are parallel.

$y = 2x - 3;\ y = -\frac{2}{3}x + 3;\ 2x + 3y = 8;\ y + 1 = 3(x - 3)$

Write all equations in slope-intercept form to determine the slope.

$2x + 3y = 8$

$-2x \qquad -2x$

$3y = -2x + 8$

$\frac{3y}{3} = \frac{-2x + 8}{3}$

$y = -\frac{2}{3}x + \frac{8}{3}$

$y + 1 = 3(x - 3)$

$y + 1 = 3x - 9$

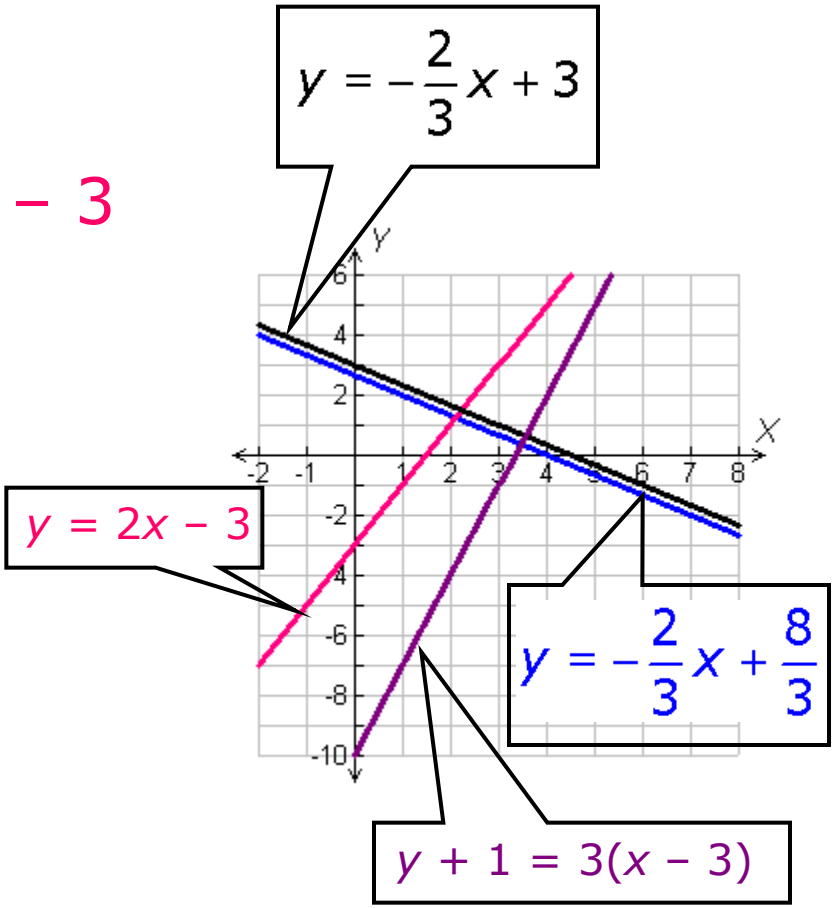
$-1 \qquad -1$

$y = 3x - 10$

Example 1B Continued

The lines described by $y = 2x - 3$ and $y + 1 = 3(x - 3)$ are not parallel with any of the lines.

The lines described by $y = -\frac{2}{3}x + \frac{8}{3}$ and $y = -\frac{2}{3}x + 3$ represent parallel lines. They each have the slope $-\frac{2}{3}$.



Example 1c

Identify which lines are parallel.

$$y = \frac{3}{4}x + 8;\ -3x + 4y = 32;\ y = 3x;\ y - 1 = 3(x + 2)$$

Write all equations in slope-intercept form to determine the slope.

$y = \frac{3}{4}x + 8$

Slope-intercept form ✓

$y = 3x$

Slope-intercept form ✓

Example 1c

Identify which lines are parallel.

$$y = \frac{3}{4}x + 8;\ -3x + 4y = 32;\ y = 3x;\ y - 1 = 3(x + 2)$$

Write all equations in slope-intercept form to determine the slope.

$-3x + 4y = 32$

$+3x \qquad +3x$

$4y = 3x + 32$

$\frac{4y}{4} = \frac{3x + 32}{4}$

$y = \frac{3}{4}x + 8$

$y - 1 = 3(x + 2)$

$y - 1 = 3x + 6$

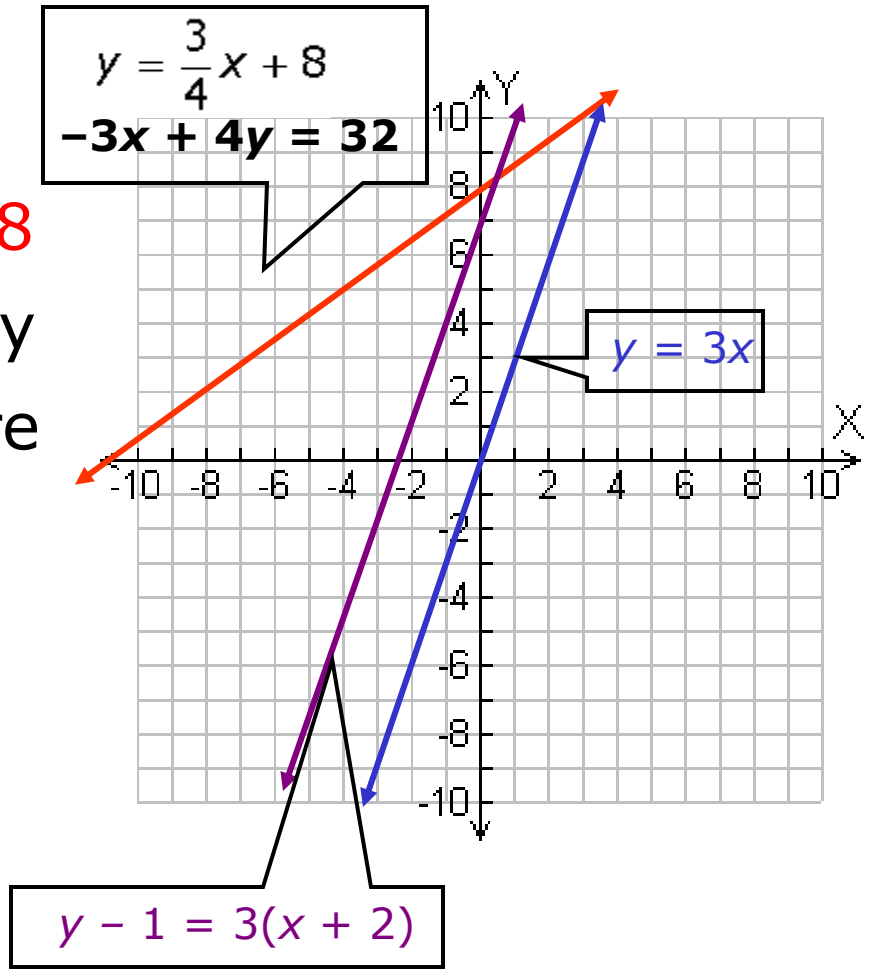
$+ 1 \qquad + 1$

$y = 3x + 7$

Example 1c

The lines described by $-3x + 4y = 32$ and $y = \frac{3}{4}x + 8$ have the same slope, but they are not parallel lines. They are the same line.

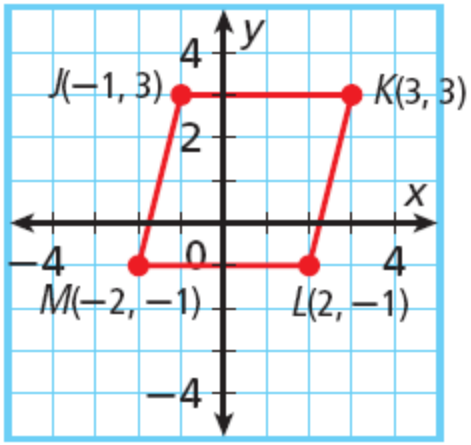
The lines described by $y = 3x$ and $y - 1 = 3(x + 2)$ represent parallel lines. They each have slope 3.



Example 2: *Geometry Application*

Show that *JKLM* is a parallelogram.

Use the ordered pairs and the slope formula to find the slopes of $\overline{MJ}$ and $\overline{KL}$.



$$\text{slope } \overline{MJ} = \frac{3-(-1)}{-1-(-2)} = \frac{4}{1} = 4$$

$$\text{slope } \overline{KL} = \frac{-1-3}{2-3} = \frac{-4}{-1} = 4$$

$\overline{MJ}$ is parallel to $\overline{KL}$ because they have the same slope.

$\overline{JK}$ is parallel to $\overline{ML}$ because they are both horizontal.

Since opposite sides are parallel, *JKLM* is a parallelogram.

Slopes of Parallel and Perpendicular Lines

Perpendicular lines are lines that intersect to form right angles (90°). These lines also have opposite reciprocal slopes.

Ex. 3, -1/3 5, ? -7, ? 4/3, ? -3/8, ?

Slopes of Parallel and Perpendicular Lines

Perpendicular Lines

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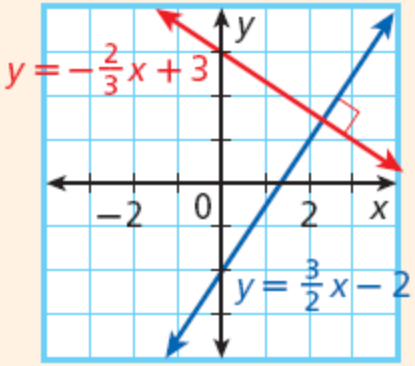
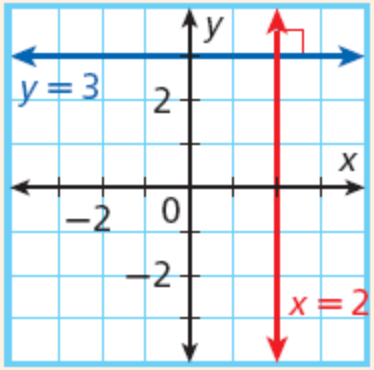
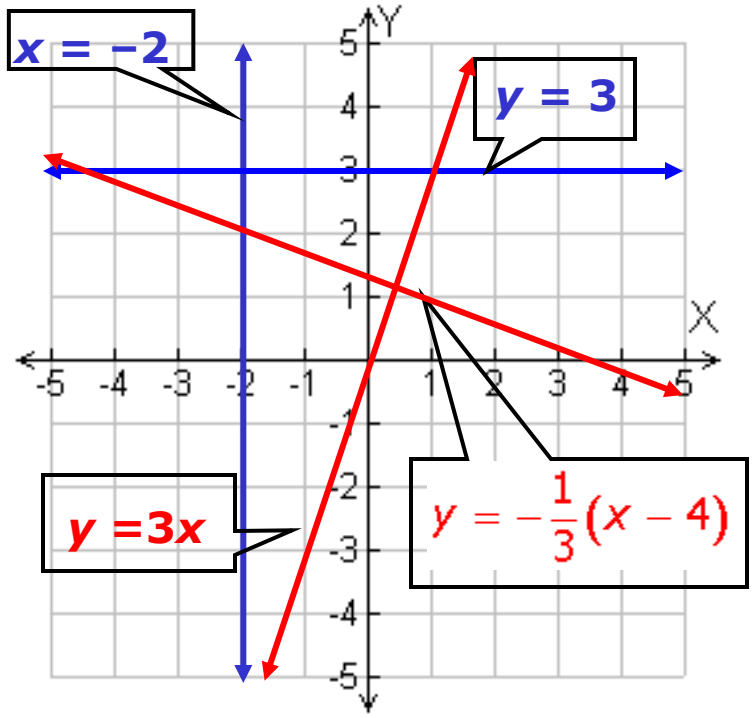
GRAPH		

Example 3A: Identifying Perpendicular Lines

Identify which lines are perpendicular: $y = 3$; $x = -2$; $y = 3x$; $y = -\frac{1}{3}(x - 4)$.

The graph described by $y = 3$ is a horizontal line, and the graph described by $x = -2$ is a vertical line. These lines are perpendicular.

The slope of the line described by $y = 3x$ is 3. The slope of the line described by $y = -\frac{1}{3}(x - 4)$ is $-\frac{1}{3}$.

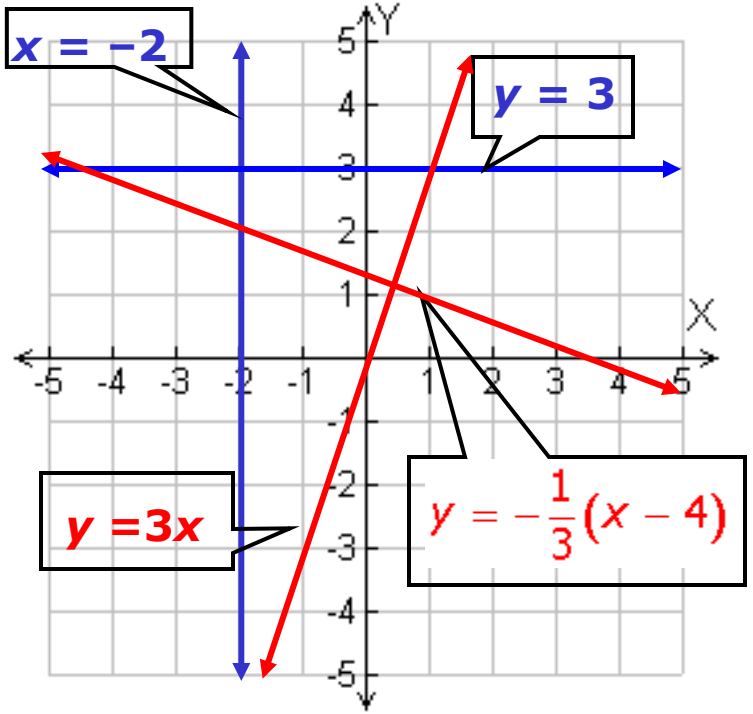


Example 3A Continued

Identify which lines are perpendicular: $y = 3$; $x = -2$; $y = 3x$; $y = -\frac{1}{3}(x - 4)$.

$$(3)\left(-\frac{1}{3}\right) = 1$$

These lines are perpendicular because the product of their slopes is –1.

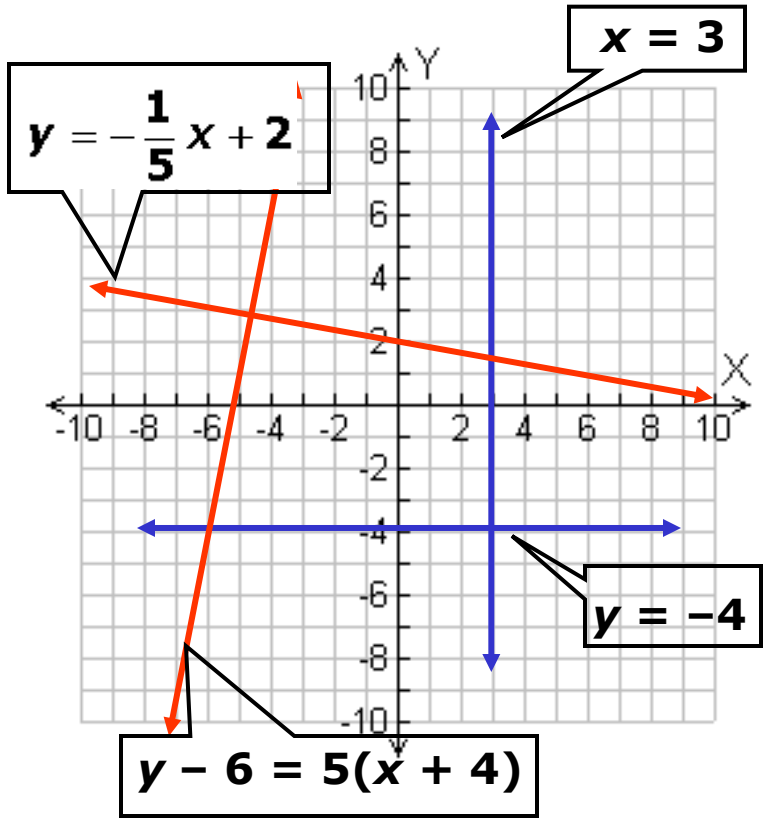


Example 3B

Identify which lines are perpendicular: $y = -4$; $y - 6 = 5(x + 4)$; $x = 3$; $y = -\frac{1}{5}x + 2$.

The graph described by $x = 3$ is a vertical line, and the graph described by $y = -4$ is a horizontal line. These lines are perpendicular.

The slope of the line described by $y - 6 = 5(x + 4)$ is 5. The slope of the line described by $y = -\frac{1}{5}x + 2$ is $-\frac{1}{5}$.

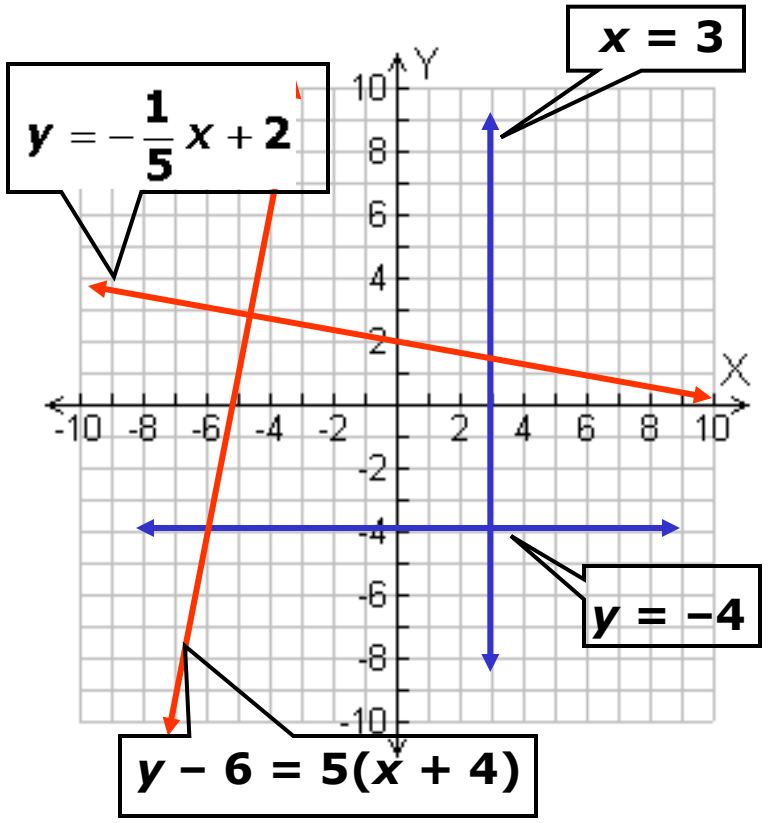


Example 3B Continued

Identify which lines are perpendicular: $y = -4$; $y - 6 = 5(x + 4)$; $x = 3$; $y = -\frac{1}{5}x + 2$.

$$5\left(-\frac{1}{5}\right) = -1$$

These lines are perpendicular because the product of their slopes is −1.



Example 4A: Writing Equations of Parallel and Perpendicular Lines

Write an equation in slope-intercept form for the line that passes through (4, 10) and is parallel to the line described by $y = 3x + 8$.

Step 1 Find the slope of the line.

$y = 3x + 8$ *The slope is 3.*

The parallel line also has a slope of 3.

Step 2 Write the equation in point-slope form.

$y - y_1 = m(x - x_1)$ *Use the point-slope form.*

$y - 10 = 3(x - 4)$ *Substitute 3 for m, 4 for x_1, and 10 for y_1.*

Example 4A Continued

Write an equation in slope-intercept form for the line that passes through (4, 10) and is parallel to the line described by $y = 3x + 8$.

Step 3 Write the equation in slope-intercept form.

$y - 10 = 3(x - 4)$

$y - 10 = 3x - 12$ *Distributive Property*

$y = 3x - 2$ *Addition Property of Equality*

Example 4B: Writing Equations of Parallel and Perpendicular Lines

Write an equation in slope-intercept form for the line that passes through (2, –1) and is perpendicular to the line described by $y = 2x - 5$.

Step 1 Find the slope of the line.

$y = 2x - 5$ *The slope is 2.*

The perpendicular line has a slope of $-\frac{1}{2}$ because $2\left(-\frac{1}{2}\right) = -1$.

Step 2 Write the equation in point-slope form.

$y - y_1 = m(x - x_1)$ *Use the point-slope form.*

$y + 1 = -\frac{1}{2}(x - 2)$ *Substitute $-\frac{1}{2}$ for m, –1 for y_1, and 2 for x_1.*

Example 4B Continued

Write an equation in slope-intercept form for the line that passes through (2, –1) and is perpendicular to the line described by $y = 2x - 5$.

Step 3 Write the equation in slope-intercept form.

$$y + 1 = -\frac{1}{2}(x - 2)$$

$$y + 1 = -\frac{1}{2}x + 1$$

Distributive Property

$$y = -\frac{1}{2}x$$

Addition Property of Equality.

Slopes of Parallel and Perpendicular Lines

Helpful Hint

If you know the slope of a line, the slope of a perpendicular line will be the "opposite reciprocal."

$$\frac{2}{3} \rightarrow -\frac{3}{2}$$

$$\frac{1}{5} \rightarrow -5$$

$$-7 \rightarrow \frac{1}{7}$$

Example 5a

Write an equation in slope-intercept form for the line that passes through (5, 7) and is parallel to the line described by $y = \frac{4}{5}x - 6$.

Step 1 Find the slope of the line.

$y = \frac{4}{5}x - 6$ *The slope is* $\frac{4}{5}$.

The parallel line also has a slope of $\frac{4}{5}$.

Step 2 Write the equation in point-slope form.

$y - y_1 = m(x - x_1)$ *Use the point-slope form.*

$y - 7 = \frac{4}{5}(x - 5)$ *Substitute* $\frac{4}{5}$ *for m, 5 for* x_1, *and 7 for* y_1

Example 5a Continued

Write an equation in slope-intercept form for the line that passes through (5, 7) and is parallel to the line described by $y = \frac{4}{5}x - 6$.

Step 3 Write the equation in slope-intercept form.

$$y - 7 = \frac{4}{5}(x - 5)$$

$$y - 7 = \frac{4}{5}x - 4 \qquad \textit{Distributive Property}$$

$$y = \frac{4}{5}x + 3 \qquad \textit{Addition Property of Equality}$$

Example 5b

Write an equation in slope-intercept form for the line that passes through (–5, 3) and is perpendicular to the line described by $y = 5x$.

Step 1 Find the slope of the line.

$y = 5x$ *The slope is 5.*

The perpendicular line has a slope of $-\frac{1}{5}$ because $5\left(-\frac{1}{5}\right) = -1$.

Step 2 Write the equation in point-slope form.

$y - y_1 = m(x - x_1)$ *Use the point-slope form.*

$y - 3 = -\frac{1}{5}(x + 5)$ *Substitute $-\frac{1}{5}$ for m, -5 for x_1 and 3 for y_1*

Example 5b Continued

Write an equation in slope-intercept form for the line that passes through (–5, 3) and is perpendicular to the line described by $y = 5x$.

Step 3 Write the equation in slope-intercept form.

$$y - 3 = -\frac{1}{5}(x + 5)$$

$$y - 3 = -\frac{1}{5}x - 1$$ *Distributive Property*

$$y = -\frac{1}{5}x + 2$$ *Addition Property*

Lesson Quiz: Part I

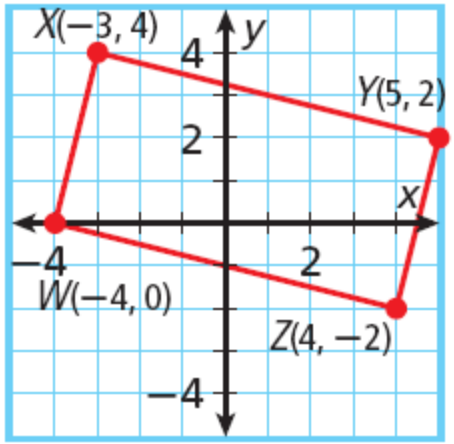
Write an equation is slope-intercept form for the line described.

1. contains the point (8, –12) and is parallel to $y = -\frac{3}{4}x - 9$ $y = -\frac{3}{4}x - 6$

2. contains the point (4, –3) and is perpendicular to $y = 4x + 5$ $y = -\frac{1}{4}x - 2$

Lesson Quiz: Part II

3. Show that *WXYZ* is a rectangle.



slope of $\overline{XY} = -\frac{1}{4}$

slope of $\overline{YZ} = 4$

slope of $\overline{WZ} = -\frac{1}{4}$

slope of $\overline{XW} = 4$

The product of the slopes of adjacent sides is −1. Therefore, all angles are right angles, and *WXYZ* is a rectangle.