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A Method to Calculate Uncertainties of Drag Coefficient Wind Tunnel Data

AIRCRAFT COMPATIBILITY BRANCH
MUNITIONS DIVISION

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A method to calculate wind tunnel drag coefficient uncertainty for a point on a fitted lift-drag curve is developed. The total uncertainty is shown to consist of two parts; that associated with the measurement process and that due to the curve fit. Equations to combine the two sources of uncertainty are presented, and a general method is given to calculate the part caused by the curve fit process. Because measurement uncertainties depend on the test facility and instrumentation, only general guidelines are presented to account

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Item 20 concluded: for this contribution. A computer program to calculate drag uncertainty is included. Program equations for measurement uncertainty are based on AEDC Tunnel 4T.

PREFACE

This study was conducted by the Aircraft Compatibility Branch of the Munitions Division of the Air Force Armament Laboratory, Armament Division, Eglin Air Force Base, Florida.

This report has been reviewed by the Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

BARNES E. HOLDER, JR., Colonel, USAF
Chief, Munitions Division

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LIST OF SYMBOLS

A	AIRCRAFT MODEL REFERENCE AREA
a_o, a_1, a_2	REGRESSION COEFFICIENTS USED TO FIT LIFT - DRAG DATA
C_D	DRAG COEFFICIENT
C_L	LIFT COEFFICIENT
FA	MEASURED AXIAL FORCE
FN	MEASURED NORMAL FORCE
IM	ANGLE BETWEEN AIRCRAFT MODEL WATERLINE AND BALANCE AXIS
M	MACH NUMBER
Q	DYNAMIC PRESSURE
S	ESTIMATED SAMPLE STANDARD DEVIATION
S ( )	PRECISION INDEX
t_n	nth PRECENTILE POINT FOR THE TWO TAILED, STUDENT "t" DISTRIBUTION
U (C_D)	UNCERTAINTY IN DRAG COEFFICIENT
α	AIRCRAFT ANGLE OF ATTACK
β	AIRCRAFT ANGLE OF SIDESLIP
μ	TRUE MEAN VALUE OF A PARAMETER
ν	NUMBER OF DEGREES OF FREEDOM FOR A STATISTICAL TEST
σ	STANDARD DEVIATION

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SECTION I
INTRODUCTION

The effects of external store carriage on aircraft performance can be significant, especially for fighter aircraft carrying fuel tanks or air to ground weapons. Degradation in top speed and range must be accurately estimated to determine if a particular store configuration is practical from a mission standpoint. Flight testing can provide answers, but it is extremely expensive and time consuming. For this reason, only selected aircraft loadings are usually tested.

To estimate performance parameters for a greater number of configurations, wind tunnel test data are often used. A scale model of the clean aircraft and one with the aircraft loaded with the store configuration of interest is tested. Curves of lift versus drag coefficient are constructed, and the drag increment due to external stores is calculated for a particular flight condition, i.e., lift coefficient.

Because of the nature of any measurement process, an uncertainty is associated with the wind tunnel data. The total data uncertainty is a combination of uncertainties in tunnel test conditions, model positioning, and model instrumentation. Since the lift coefficient value for the flight condition of interest rarely appears as a test point, a fitted curve must be constructed from the data. This curve fit process is another source of possible uncertainty. This report investigates the uncertainty associated with wind tunnel drag data and presents a method that can be used to calculate it for a specific flight condition of interest.

SECTION II

GENERAL METHOD OF UNCERTAINTY CALCULATION

Uncertainty associated with C_D at a given condition is caused by uncertainties in the measurement process and those induced by the curve fit procedure. This requires that the uncertainties associated with each factor be propagated to arrive at a final result. The method chosen here is the Taylor series method of error propagation.

A derivation of the Taylor series method can be found in Reference 1. Some of the assumptions used in the derivation are:

1. Response, Z, is defined as a function of the measured variables $X_1, X_2, \ldots, X_n$.

2. Z is continuous in the neighborhood of $\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}$. $\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}$ are the mean values associated with $x_1, x_2, \ldots, x_n$ which all have error distributions about the point of interest.

3. Z has continuous partial derivatives in the vicinity of $\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}$.

4. $x_1, x_2, \ldots, x_n$ are independent of each other.

5. $(\mu_{x_1} - x_1), (\mu_{x_2} - x_2), \ldots, (\mu_{x_n} - x_n)$ are small or $\frac{\partial^2 z}{\partial x_1^2}, \frac{\partial^2 z}{\partial x_2^2}, \ldots, \frac{\partial^2 z}{\partial x_n^2}$ are small or zero.

The assumptions will be satisfied if the functions considered are restricted to smooth curves near the point of interest with no discontinuities and with higher order derivatives either small or zero.

The results of the derivation show that:

If

$$Z = f(x_1, x_2, \ldots, x_n) \tag{1}$$

then

$$S(Z) = \{ [\frac{\partial z}{\partial x_1} S(x_1)]^2 + [\frac{\partial z}{\partial x_2} S(x_2)]^2 + \ldots + [\frac{\partial z}{\partial x_n} S(x_n)]^2 \}^{\frac{1}{2}} \tag{2}$$

where $S(Z)$, $S(X_1)$, $S(X_2)$, ..., $S(X_n)$ are the precision indices of the response, Z, and the variables X_1, X_2, ..., X_n. The precision index is the computed standard deviation of the measurements (i.e., random error). It is defined as

$$S = \left[\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right]^{\frac{1}{2}} \tag{3}$$

X_i and $\bar{x}$ are, of course, the value of x of a particular point and the mean value of x, respectively. If the sample size is large, the precision index is approximately equal to the actual population standard deviation (σ) associated with the random variable Z.

Depending on the confidence level attached to the response, the uncertainty is simply a function of the precision index or standard deviation. This assumes that no bias or systematic errors are present. For drag increments caused by external stores, this is a reasonable assumption. The drag increments are calculated as differences between data taken during one test with the same model and instrumentation. While bias errors may be present, they are approximately equal for different model configurations. When the increment is determined, the bias errors should drop out. This assumption is of critical importance in the following discussions. It would not be true if raw data from separate tests were compared or if different instrumentation was used during one test.

For the case of interest, C_L and C_D are both random variables since both are measured during testing. With the assumption that measured values of C_L and C_D are normally distributed with mean values μ_{C_L} and μ_{C_D}, the uncertainty associated with C_D on the fitted C_L - C_D curve is found as follows.

Figure 1 shows a number of data points. From aerodynamic considerations, C_D is considered a second order polynomial function of C_L. That is:

$$C_D = a_o + a_1 C_L + a_2 C_L^2 \tag{4}$$

Using the regression coefficients a_o, a_1, and a_2, the values of C_D at the data point (C_{L_1}) and the flight condition of interest (C_{L_2}) can be found.

$$C_{D_1} = a_o + a_1 C_{L_1} + a_2 C_{L_1}^2 \tag{5}$$

$$C_{D_2} = a_o + a_1 C_{L_2} + a_2 C_{L_2}^2 \tag{6}$$

The difference between the values of C_D is:

$$C_{D_2} - C_{D_1} = \Delta C_D = a_1 (C_{L_2} - C_{L_1}) + a_2(C_{L_2}^2 - C_{L_1}^2) \tag{7}$$

C_{D_2} can be redefined in terms of C_{D_1} and ΔC_D.

$$C_{D_2} = C_{D_1} + \Delta C_D = C_{D_1} + a_1 (C_{L_2} - C_{L_1}) + a_2 (C_{L_2}^2 - C_{L_1}^2) \tag{8}$$

Now assume that

$$C_{D_2} = f (C_{D_1}, C_{L_1}, C_{L_2}) \tag{9}$$

Using the Taylor series method of error propagation:

$$S(C_{D_2}) = \left\{\left[\frac{\partial C_{D_2}}{\partial C_{D_1}} S(C_{D_1})\right]^2 + \left[\frac{\partial C_{D2}}{\partial C_{L_1}} S(C_{L_1})\right]^2 + \left[\frac{\partial C_{D2}}{\partial C_{L_2}} S(C_{L_2})\right]^2\right\}^{1/2} \tag{10}$$

The partial derivatives in Equation (10) can easily be evaluated from the definition of C_{D_2} in Equation (8).

They are:

$$\frac{\partial C_{D_2}}{\partial C_{D_1}} = 1 \tag{11a}$$

$$\frac{\partial C_{D_2}}{\partial C_{L_1}} = - (a_1 + 2a_2 C_{L_1}) \tag{11b}$$

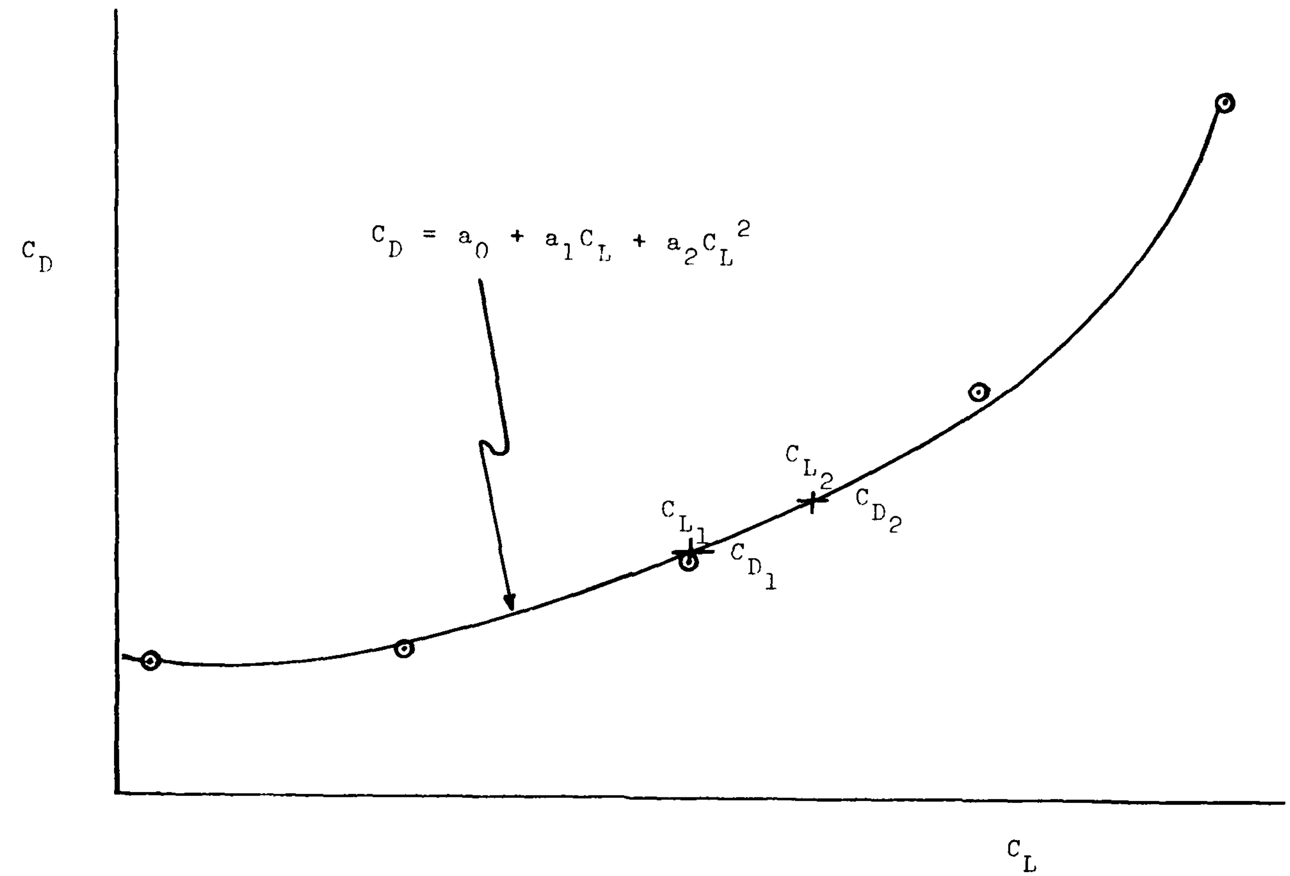

Figure 1. Example of C_L Versus C_D Data

$$\frac{\partial C_{D_2}}{\partial C_{L_2}} = a_1 + 2a_2 \, C_{L_2} \qquad (11c)$$

The terms $S(C_{D_1})$, $S(C_{L_1})$, and $S(C_{L_2})$ in Equation (10) require interpretation. They are the precision indices of C_D and C_L at point 1 and that of C_L at point 2. The C_L at point 2 is simply the desired flight condition. It can be thought of as a mathematical and not a random variable and can be exactly defined. Therefore:

$$S(C_{L_2}) = 0 \qquad (12)$$

C_{L_1} is a measured data point, and because of this, it is a random variable with a mean and standard deviation. $S(C_{L_1})$ can be thought of as the uncertainty associated with the measurement process. If $S(C_L)$ is constant or it changes slowly in the vicinity of the flight condition of interest, $S(C_{L_1})$ should be a close approximation to the value that $S(C_L)$ would have if C_{L_2} were a measured data point. Figure 2 indicates that $S(C_L)$ does not vary much over the angle of attack range of interest ($\alpha = 2 - 6^{o}$). Because of this, $S(C_{L_1})$, where C_{L_1} is the data point nearest the flight condition of interest, can be used to indicate the uncertainty associated with the measurement process. Calculation of $S(C_L)$ is discussed in Section III.

Recall that the value of C_{D_1} used in Equation (8) to define C_{D_2} was not the measured value at C_{L_1}; it was the value obtained from the curve fit equation. In other words:

$$C_{D_1} = a_o + a_1 \, C_{L_1} + a_2 \, C_{L_1}^2 \qquad (13)$$

Therefore, $S(C_{D_1})$ is actually the precision index of C_{D_1}, S^2, introduced by the curve fit process. A method for calculating $S(C_{D_1})$ is found in Section IV.

Given the values of $S(C_{L_1})$ and $S(C_{D_1})$, the final calculation of the uncertainty in C_D at a given flight condition can be performed as follows:

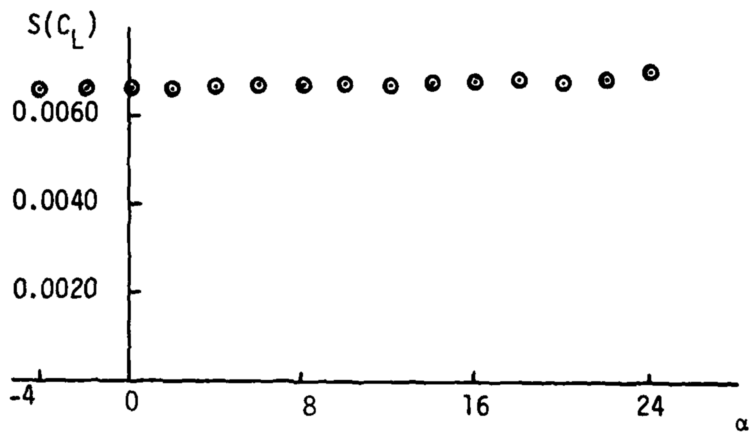

Figure 2. Precision Index of Lift Coefficient versus Angle of Attack
M = 0.8

$$U\,(C_D) = t_n\,S \tag{14}$$

t_n is the n^{th} percentile point for the two tailed, student "t" distribution. Various percentiles or confidence levels can be chosen. The following discussion is based on a 95-percent confidence level. The ninety-fifth percentile point, $t_{0.95}$, depends on the number of degrees of freedom, which is a measure of the sample size used to determine the precision index. For $S(C_{L_1})$, $t_{0.95}$ is usually taken equal to 2 since $S(C_L)$ is based on a large sample size. This value is not appropriate to use for $S(C_D)$. The Aircraft Compatibility Branch uses a five-point curve fit around the appropriate C_L to find C_D. For this case, the degrees of freedom are:

$$\nu = n - k - 1 = 2 \tag{15}$$

where:

n - number of points used = 5

k - number of variables in regression equation = 2 (i.e., C_L, $C_L{}^2$)

For two degrees of freedom:

$$t_{0.95} = 4.303$$

The equation for uncertainty can be rewritten as:

$$U(C_D) = \{[t_{0.95_{C_D}}\,S(C_{D_1})]^2 + [t_{0.95_{C_L}}\,(a_1 + 2a_2\,C_{L_1})\,S(C_{L_1})]^2\}^{1/2} \tag{16a}$$

$$= \{4.303^2\,S(C_{D_1})^2 + 2^2[(a_1 + 2a_2\,C_{L_1})\,S(C_{L_1})]^2\}^{1/2} \tag{16b}$$

The value $U(C_D)$ is the uncertainty in drag coefficient at a given flight condition for a given configuration. More important is the uncertainty in the drag increment between the clean aircraft and the aircraft with the external stores. The increment is defined as:

$$\Delta C_D = C_{D_{Stores}} - C_{D_{Clean}} \tag{17}$$

From Reference 1, the uncertainty is:

$$U(\Delta C_D) = \{U(C_{D_{Stores}})^2 + U(C_{D_{Clean}})^2\}^{1/2} \tag{18}$$

SECTION III

DATA POINT PRECISION INDEX

Section II described a method to calculate drag data uncertainties for a point on a fitted C_L-C_D curve. Because of the random nature of the assumed independent variable, C_L, the final calculation for $U(C_D)$ requires a value for the precision index of C_L. This can be done in the following manner:

C_L can be defined as:

$$C_L = \frac{1}{QA} [FN (\cos \alpha \cos IM + \sin \alpha \sin IM) + FA (\cos \alpha \sin IM - \sin \alpha \cos IM)] \quad (19)$$

If

$$C_L = f(x_1, x_2, \ldots, x_n) \quad (20)$$

then the Taylor series method of error propagation yields:

$$S(C_L) = \{[\frac{\partial C_L}{\partial x_1} S(x_1)]^2 + [\frac{\partial C_L}{\partial x_2} S(x_2)]^2 + \ldots + [\frac{\partial C_L}{\partial x_n} S(x_n)]^2\}^{\frac{1}{2}} \quad (21)$$

It is apparent that the equations become more and more complex as the number of independent parameters increases. In addition, these variables must be independent or nearly so of each other for Equation (21) to hold. Assumptions need to be made on the nature of the measurements. These are:

(1) The uncertainties associated with some of the independent parameters are sufficiently small that their effects on the uncertainty of the dependent parameter are negligible. This is assumed to be the case for IM and A.

(2) The measured forces, FN and FA, are independent of one another.

Using these two assumptions, it can be said:

$$C_L = f (FN, \alpha, FA, Q) \quad (22)$$

Therefore:

$$S(C_L) = \{[\frac{\partial C_L}{\partial FN} S(FN)]^2 + [\frac{\partial C_L}{\partial \alpha} S(\alpha)]^2 + [\frac{\partial C_L}{\partial FA} S(FA)]^2 + [\frac{\partial C_L}{\partial Q} S(Q)]^2\}^{\frac{1}{2}} \quad (23)$$

After evaluating the partial derivatives, Equation (23) may be rewritten as:

$$S(C_L) = \{[(\cos\alpha \cos IM + \sin\alpha \sin IM) \frac{S(FN)}{QA}]^2 + [C_D S(\alpha)]^2$$
$$+ [(\cos\alpha \sin IM - \sin\alpha \cos IM) \frac{S(FA)}{QA}]^2 + [C_L \frac{S(Q)}{Q}]^2\}^{1/2} \qquad (24)$$

For a given test point, α, IM, Q, A, C_D, and C_L are defined. S(FN), S(FA), S(α), and S(Q) are functions of test conditions and instrumentation. Equations for these parameters can be developed in a manner similar to that used for Equation (24). S(α) and S(Q) depend on the wind tunnel where the data are taken. S(FN) and S(FA) are dependent on the tunnel and model instrumentation. Equations to calculate the quantities for a specific case of interest should be obtained from the applicable test facility. The computer program described in Section V uses equations that apply to the Aerodynanmic Wind Tunnel (PWT/4T) at Arnold Engineering Development Center (AEDC).

SECTION IV

CURVE FIT PRECISION INDEX

A method to determine the precision index of C_L for use in Equation (16) was just described in Section III. The other input required to solve for $U(C_D)$ is $S(C_{D_1})$. Recall that it was defined as the precision index of the curve fit process. To calculate this quantity, certain assumptions on the nature of the C_D data are made. These are:

(1) C_D is a random variable.

(2) At a given C_L, possible values of C_D are approximately normally distributed.

(3) Over the C_L range where the curve fit is applied, $S(C_D)$ is nearly constant. This allows a simplified curve fit procedure to be used (see pp 106-108, Reference 2).

Since it is possible to calculate $S(C_D)$ for a given data point in a manner similar to that used for $S(C_L)$, the validity of the third assumption can be verified. Figure 3 shows a typical example. The angle of attack region of interest is about 2 to 6 degrees. As the figure indicates, $S(C_D)$ does not vary significantly over this range.

Recall from Section II, the C_L-C_D curve is assumed to be of the form

$$C_D = a_o + a_1 C_L + a_2 C_L^2 \tag{25}$$

For a second order curve fit, the normal equations to solve for a_o, a_1, a_2 are, in matrix form:

$$[B]\ [a] = [g] \tag{26}$$

where

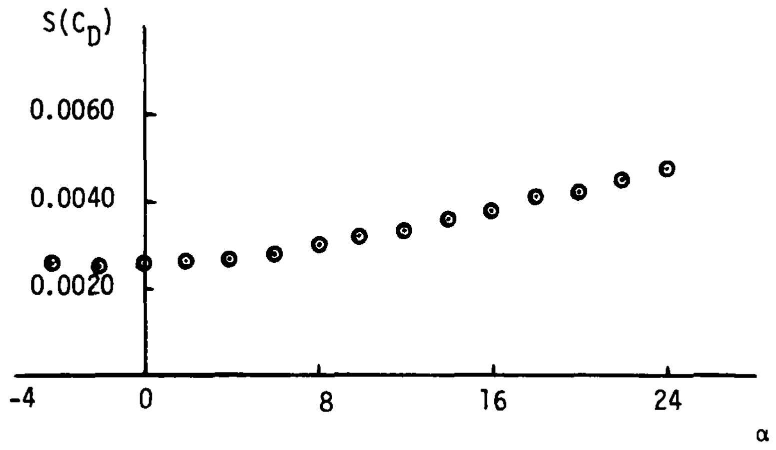

Figure 3. Precision Index of Drag Coefficient versus Angle of Attack
M = 0.8

$$[B] = \begin{bmatrix} n & \sum_{i=1}^{n} C_{L_i} & \sum_{i=1}^{n} C_{L_i}^{2} \\ \sum_{i=1}^{n} C_{L_i} & \sum_{i=1}^{n} C_{L_i}^{2} & \sum_{i=1}^{n} C_{L_i}^{3} \\ \sum_{i=1}^{n} C_{L_i}^{2} & \sum_{i=1}^{n} C_{L_i}^{3} & \sum_{i=1}^{n} C_{L_i}^{4} \end{bmatrix} \quad (27)$$

$$[a] = \begin{bmatrix} a_o \\ a_1 \\ a_2 \end{bmatrix} \quad (28)$$

$$[g] = \begin{bmatrix} \sum_{i=1}^{n} C_{D_i} \\ \sum_{i=1}^{n} C_{L_i} C_{D_i} \\ \sum_{i=1}^{n} C_{L_i}^{2} C_{D_i} \end{bmatrix} \quad (29)$$

Solution for the regression coefficients is given by:

$$[a] = [B]^{-1} [g]$$

Where $[B]^{-1}$ is the inverse of [B]. For a five-point curve fit, n is equal to five.

In this study, the value of $S(C_D)$ is based on the mean response $\mu_{C_D | C_L, C_{L^2}}$. The term $\mu_{C_D | C_L, C_{L^2}}$ may be thought of as the true value of C_D for a given C_L. In other words, if many values of C_D were measured at a given C_L, the mean of C_D would tend to $\mu_{C_D | C_L, C_{L^2}}$ as the sample size increased to infinity. Based on this, $S(C_D)$ is defined as:

$$S(C_D) = s\sqrt{[x_o'] \, [B]^{-1} \, [x_o]} \quad (31)$$

where

s = estimated sample standard deviation

$$= \left[\frac{\sum_{i=1}^{n} [C_{D_i} - (a_o + a_1 C_{L_i} + a_2 C_{L_i}^{2})]^2}{n-k-1} \right]^{1/2} \qquad (32)$$

$$x_o' = [1 \quad C_{L_{FC}} \quad C_{L_{FC}}^{2}] \qquad (33)$$

$$x_o = \begin{bmatrix} 1 \\ C_{L_{FC}} \\ C_{L_{FC}}^{2} \end{bmatrix} \qquad (34)$$

The FC subscript indicates the flight condition of interest where the calculation is made. For a more detailed discussion of the method used to calculate $S(C_D)$, see a statistics text such as Reference 3.

SECTION V

COMPUTER PROGRAM AND SAMPLE CALCULATION

Appendix A contains a computer program developed in the Aircraft Compatibility Branch to calculate uncertainties in drag coefficient where C_D is found from a fitted C_L-C_D curve. Required inputs are explained in the program. The uncertainty can be determined for confidence levels of 99, 95, 90, and 80 percent, i.e., x percent of measured data points should lie within the uncertainty band above and below the fitted curve. The confidence level is changed by using different t values as explained in Section II.

As noted earlier, the equations used to define $S(C_L)$ are applicable to the four-foot transonic wind tunnel at AEDC. Changes would be required for other tunnels.

Assumptions are made to simplify the equations for $S(C_L)$. These include:

(1) Only aircraft pitch excursions are considered, i.e., $\beta \cong 0$.

(2) Uncertainty in angle of attack is constant and equal to 0.1 degree.

(3) $S(\alpha)$ equals one half the uncertainty in α (0.05 degree).

(4) A small sting roll angle is assumed (0.04 degree).

(5) Model weight does not vary between configurations.

(6) To a first approximation, model angle of attack is equal to sting pitch angle.

(7) Balance uncertainties [S(FNB), S(FAB)] are functions of normal and axial forces only (i.e., no side load or rolling moment interactions are present).

(8) There is no model roll angle relative to the balance.

(9) The precision indices of model weight measured by axial and normal force gages are equal to the precision indices of the balance.

(10) Tunnel total pressure is less than 1500 lbs/ft^2.

(11) S(M) is constant and equal to 0.002 which is 40 percent of the uncertainty.

The assumptions are reasonable for the data of interest. Only pitch excursions are considered because the performance problem concerns mainly flight at a constant angle of attack.

Appendix A contains sample calculations of $U(C_D)$ for a specific case. The example is for a clean aircraft at M = 0.8 and $C_L = 0.3$. If the equation for $U(C_D)$ is examined using the values of $S(C_D)$ and $S(C_L)$ from the example, it shows that most of the uncertainty in C_D is a result of the curve fit. Note all the confidence levels are shown in the example, and $U(C_D)$ decreases as the confidence level is lowered. To be more precise, the values S(FN), S(FA), $S(\alpha)$, and S(Q) used to find $S(C_L)$ should be redefined for each confidence level instead of using the 95-percent values. Since the contribution of $S(C_L)$ to $U(C_D)$ is not large, this is not significant.

SECTION VI

CONCLUSIONS AND RECOMMENDATIONS

A method has been presented that shows how drag coefficient uncertainties can be calculated for wind tunnel data. Because of the need to examine C_D as a function of C_L, the uncertainties in C_D are due both to the uncertainties in C_L and those of the curve fit. While the method of calculating the precision index of the curve fit is general, that used to determine $S(C_L)$ will depend on the wind tunnel and particular test instrumentation. Consultation with the applicable test facility will be necessary to work out suitable equations for $S(C_L)$.

The example given in Appendix A indicates that the uncertainty can be fairly large for a high confidence level. Reducing the confidence level results in a considerably smaller uncertainty. It would be very useful if data uncertainties associated with flight tests were given a thorough analysis. Determination of these uncertainties would increase the confidence in performance estimates based on flight test data.

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APPENDIX A

COMPUTER PROGRAM FOR UNCERTAINTY CALCULATION

```
      PROGRAM UNCER(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C
C     THIS PROGRAM CALCULATES THE UNCERTAINTY IN DRAG COEFFICIENT
C     FOR A SPECIFIC FLIGHT CONDITION OF INTEREST ON A FITTED
C     CL-CD CURVE. THE UNCERTAINTY IN CD IS MADE UP OF A
C     COMBINATION OF UNCERTAINTY IN LIFT COEFFICIENT AND THE
C     UNCERTAINTY INTRODUCED BY CURVE FITTING CL-CD DATA.
C
C     THE FOLLOWING CONSTANTS ARE STORED IN BLOCKDATA.
C
C       PHII-ASSUMED STING ROLL ANGLE DURING A PITCH SWEEP (DEG)
C       SPHII-STANDARD DEVIATION OF STING ROLL ANGLE (DEG)
C       SL-CONSTANT DEPENDING ON PARTICULAR WIND TUNNEL
C       W-AIRCRAFT MODEL WEIGHT (LBS)
C       A5,A6-CONSTANTS USED TO OBTAIN STANDARD DEVIATION OF TUNNEL
C         TOTAL PRESSURE. DEPENDS ON TUNNEL OF INTEREST.
C       SM-STANDARD DEVIATION OF MACH NUMBER
C       SALPHA-STANDARD DEVIATION OF ANGLE OF ATTACK (RAD)
C       AIM-ANGLE BETWEEN AIRCRAFT WATER LINE AND BALANCE AXIS(DEG)
C       A-AIRCRAFT MODEL REFERENCE AREA (SQ/FT)
C       A17,A18-CONSTANTS USED TO DETERMINE STANDARD DEVIATION OF
C         TUNNEL STING PITCH ANGLE. DEPENDS ON WIND TUNNEL.
C       X1,X2,X3,X9-BALANCE CONSTANTS
C       AK1,AK15,AK29-BALANCE CONSTANTS (CALLED K1,K15,K29)
C       SFAN2,SFAA2-BALANCE AXIAL FORCE GAGE UNCERTAINTIES WITH
C         APPLIED NORMAL AND AXIAL LOAD RESPECTIVELY.
C       SFNN2,SFNA2-BALANCE NORMAL FORCE GAGE UNCERTAINTIES WITH
C         APPLIED NORMAL AND AXIAL FORCE.
C
C
C     THE FOLLOWING VALUES ARE CALCULATED FROM THE INPUT
C
C       SALPI-STANDARD DEVIATION OF STING PITCH ANGLE
C       SFA-       "          "       " AXIAL FORCE
C       SFN-       "          "       " NORMAL FORCE
C       SQ-        "          "       " DYNAMIC PRESSURE
C       SCLTS-     "          "       " LIFT COEFFICIENT
C
C
C     THE FOLLOWING INPUTS ARE FOR THE TEST POINT CLOSEST TO THE
C       FLIGHT CONDITION OF INTEREST.
C
C       ALPHA-AIRCRAFT ANGLE OF ATTACK
C       AMACH-MACH NUMBER
C       PT-TUNNEL TOTAL PRESSURE
C       Q-TUNNEL DYNAMIC PRESSURE
C       CLTS-LIFT COEFFICIENT
C       CDTS-DRAG COEFFICIENT
C       POI-VALUE OF LIFT COEFFICIENT AT THE FLIGHT CONDITION
C         OF INTEREST.
C
C
C     CONFI IS THE CONFIDENCE LEVEL SELECTED.
C       VALUES OF .99, .95, .9, AND .8 ARE ALLOWED.
C
C     THE FIVE CL AND CD VALUES ARE THE ONES USED FOR THE CURVE FIT
C
C     XO,XOP,B,G, AND AC ARE MATRICES USED TO OBTAIN REGRESSION
C       COEFFICIENTS AND CALCULATE THE STANDARD DEVIATION DUE TO FIT
C
C     A0,A1,A2 ARE THE REGRESSION COEFFICIENTS CALCULATED FOR THE
C       FITTED CURVE WHERE IT IS OF THE FORM
C       CD = A0 + A1*CL + A2*(CL**2)
C
C     SFIT2 IS THE SQUARE OF THE STANDARD DEVIATION DUE TO CURVE FIT
C
C
C
      COMMON/A/PHII,SPHII,SL,W,A5,A6,SM,SALPHA,AIM,A,A17,A18
      COMMON/B/X9,AK15,SFAN2,SFAA2
      COMMON/C/X1,X2,X3,AK1,AK29,SFNN2,SFNA2
      COMMON/D/SALPI,SFA,SFN,SQ,SCLTS
      COMMON/E/ALPHA,AMACH,PT,Q,CLTS,CDTS,POI
      COMMON/F/CL(5),CD(5),XO(3),XOP(3),B(3,3),G(3),AC(3)
```

```
      COMMON/G/A0,A1,A2,SFIT2
      COMMON/H/AD(1),AL(3),AM(3),Z(3),X(1)
   10 READ(5,1)ALPHA,AMACH,PT,Q,CLTS,CDTS,POI
    1 FORMAT(7F10.5)
      IF(EOF(5))99,3
    3 READ(5,8) CONFI
    8 FORMAT(F10.5)
      READ(5,2) (CL(I),I=1,5)
      READ(5,2) (CD(I),I=1,5)
    2 FORMAT(5F10.5)
      CALL SOCL
      CALL SDFIT
      WRITE(6,11) AMACH
   11 FORMAT(1H1,13HMACH NUMBER =,F10.3)
      WRITE(6,12)
   12 FORMAT(///3X,23HWIND TUNNEL DATA POINTS)
      WRITE(6,13) (CL(I),I=1,5)
   13 FORMAT(/5X,26HVALUES OF LIFT COEFFICIENT/1H0,5F12.4)
      WRITE(6,14) (CD(I),I=1,5)
   14 FORMAT(/5X,26HVALUES OF DRAG COEFFICIENT/1H0,5F12.4)
      WRITE(6,15) POI
   15 FORMAT(///3X,30HLIFT COEFFICIENT OF INTEREST =,F10.4)
      WRITE(6,7) A0,A1,A2
    7 FORMAT(///,1X,22HCURVE FIT COEFFICIENTS,/,1H0,4X,4HA0 =,F12.5,/
     1,5X,4HA1 =,F12.5,/,5X,4HA2 =,F12.5)
      WRITE(6,4) SCLTS
    4 FORMAT(//1X,26HSTANDARD DEVIATION IN CL =,F12.4)
      SFIT=SQRT(SFIT2)
      WRITE(6,5) SFIT
    5 FORMAT(//1X,27HSTANDARD DEVIATION IN FIT =,F12.4)
      CONF=CONFI*100.
      WRITE(6,16) CONF
   16 FORMAT(//,1X,21HCONFIDENCE INTERVAL =,F5.0,2H %)
      IF(CONFI.EQ.0.99) GO TO 100
      IF(CONFI.EQ.0.95) GO TO 101
      IF(CONFI.EQ.0.9) GO TO 102
      IF(CONFI.EQ.0.8) GO TO 103
  100 TVAL1=9.925
      TVAL2=2.58
      GO TO 200
  101 TVAL1=4.303
      TVAL2=2.
      GO TO 200
  102 TVAL1=2.92
      TVAL2=1.65
      GO TO 200
  103 TVAL1=1.886
      TVAL2=1.28
  200 VAR1=(TVAL1**2.)*SFIT2
      VAR2=(TVAL2**2.)*((A1+2.*A2*CLTS)**2.)*(SCLTS**2.)
      UCD=SQRT(VAR1+VAR2)
      WRITE(6,6) UCD
    6 FORMAT(///1X,37HTHE UNCERTAINTY IN DRAG COEFFICIENT =,F12.4)
      GO TO 10
   99 STOP
      END
```

```
C
C
C
      SUBROUTINE SOCL
C
C     THIS SUBROUTINE CALCULATES SCLTS, THE STANDARD DEVIATION IN
C       LIFT COEFFICIENT FOR THE DATA POINT NEAREST THE FLIGHT
C       CONDITION OF INTEREST.
C
      COMMON/A/PHII,SPHII,SL,W,A5,A6,SM,SALPHA,AIM,A,A17,A18
      COMMON/B/X9,AK15,SFAN2,SFAA2
      COMMON/C/X1,X2,X3,AK1,AK29,SFNN2,SFNA2
      COMMON/D/SALPI,SFA,SFN,SQ,SCLTS
      COMMON/E/ALPHA,AMACH,PT,Q,CLTS,CDTS,POI
      SALPI=A17+A18*ALPHA
      SFAB=.5*((SFAN2**2.+SFAA2**2.)**.5)
      SWA=SFAB
      SFAG2=(SL*X9*AK15)**2.+SFAB**2.
      SFAST2=(W*COSD(ALPHA)*SALPI)**2.+(SIND(ALPHA)*SWA)**2.
      SFA=(SFAG2+SFAST2)**.5
      SALPHA=.000873
      SFNB=.5*((SFNN2**2.+SFNA2**2.)**.5)
      SFNG2=((SL/X1)**2.)*((X2*AK1)**2.+(X3*AK29)**2.)
     1  +SFNB**2.
      SWN=SFNB
      VAR1N=(W*SIND(ALPHA)*COSD(PHII)*SALPI)**2.
      VAR2N=(COSD(ALPHA)*COSD(PHII)*SWN)**2.
      VAR3N=(W*COSD(ALPHA)*SIND(PHII)*SPHII)**2.
      SFNST2=VAR1N+VAR2N+VAR3N
      SFN=(SFNG2+SFNST2)**.5
      VAR1P=(((2.*Q/AMACH)*(1.-1.2*(AMACH**2.)))/
     1  (1.+.2*(AMACH**2.)))*SM
      SPT=A5+A6*PT
      VAR2P=(Q/PT)*SPT
      SQ=(VAR1P**2.+VAR2P**2.)**.5
      VAR1T=((COSD(ALPHA)*COSD(AIM)+SIND(ALPHA)*SIND(AIM))
     1  *SFN/(Q*A))**2.
      VAR2T=(CDTS*SALPHA)**2.
      VAR3T=((COSD(ALPHA)*SIND(AIM)-SIND(ALPHA)*COSD(AIM))
     1  *SFA/(Q*A))**2.
      VAR4T=(CLTS*(SQ/Q))**2.
      SCLTS=SQRT(VAR1T+VAR2T+VAR3T+VAR4T)
      RETURN
      END
```

```
C
C
C
      SUBROUTINE SDFIT
C
C  THIS SUBROUTINE CALCULATES THE REGRESSION COEFFICIENTS TO FIT
C     THE CL-CD DATA AND DETERMINES THE STANDARD DEVIATION OF
C     THE CURVE FIT.
C
      COMMON/E/ALPHA,AMACH,PT,Q,CLTS,CDTS,POI
      COMMON/F/CL(5),CD(5),X0(3),X0P(3),B(3,3),G(3),AC(3)
      COMMON/G/A0,A1,A2,SFIT2
      COMMON/H/AD(1),AL(3),AM(3),Z(3),X(1)
      SUM11=0.
      SUM21=0.
C
      SUM31=0.
      SUM41=0.
      DO 10 I=1,5
      SUM11=SUM11+CL(I)
      SUM21=SUM21+CL(I)**2.
      SUM31=SUM31+CL(I)**3.
      SUM41=SUM41+CL(I)**4.
   10 CONTINUE
      B(1,1)=5.
      B(1,2)=SUM11
      B(1,3)=SUM21
      B(2,1)=SUM11
      B(2,2)=SUM21
      B(2,3)=SUM31
      B(3,1)=SUM21
      B(3,2)=SUM31
      B(3,3)=SUM41
      SUM12=0.
      SUM22=0.
      SUM32=0.
      DO 20 I=1,5
      SUM12=SUM12+CD(I)
      SUM22=SUM22+CL(I)*CD(I)
      SUM32=SUM32+CL(I)**2.*CD(I)
   20 CONTINUE
      G(1)=SUM12
      G(2)=SUM22
      G(3)=SUM32
      CALL MINV(B,3,AD,AL,AM)
C
C  CALL TO SYSTEM ROUTINE TO INVERT MATRIX B
C
      CALL GMPRD(B,G,AC,3,3,1)
C
C  MULTIPLY INVERSE OF B BY G TO OBTAIN MATRIX AC
      A0=AC(1)
      A1=AC(2)
      A2=AC(3)
      SUM3=0.
      DO 30 I=1,5
      SUM3=SUM3+(CD(I)-A0-A1*CL(I)-A2*(CL(I)**2.))
     1 **2.
   30 CONTINUE
      SSD=SUM3/2.
      X0P(1)=1.
      X0P(2)=POI
      X0P(3)=POI**2.
      DO 40 I=1,3
      X0(I)=X0P(I)
   40 CONTINUE
      CALL GMPRD(X0P,B,Z,1,3,3)
C
C  MULTIPLY X0P BY INVERSE OF B TO OBTAIN Z
C
      CALL GMPRD(Z,X0,X,1,3,1)
C
C  MULTIPLY Z BY X0 TO OBTAIN X. X IS A ONE COLUMN ONE ROW
C     MATRIX. THE VALUE IS MULTIPLIED BY THE SQUARE OF THE SAMPLE
C     STANDARD DEVIATION TO OBTAIN THE SQUARE OF THE STANDARD
```

SUBROUTINE SDFIT 74/74 OPT=0 TRACE FTN 4.8+518

```
      C    DEVIATION DUE TO CURVE FIT.
      C
           X1=X(1)
           SFIT2=SSO*X1
   80      RETURN
           END
```

```
      BLOCKDATA
      COMMON/A/PHII,SPHII,SL,W,A5,A6,SM,SALPHA,AIM,A,A17,A18
      COMMON/B/X9,AK15,SFAN2,SFAA2
      COMMON/C/X1,X2,X3,AK1,AK29,SFNN2,SFNA2
      DATA PHII,SPHII,SL,W,A5,A6,SM,SALPHA,AIM,A,A17,A18/
     1   .04,.04,1.8,33.,.15,.0004,.002,.000873,1.0,
     1   .75,.004,.00014/
      DATA X9,AK15,SFAN2,SFAA2/1.,.0359,.4,.45/
      DATA X1,X2,X3,AK1,AK29,SFNN2,SFNA2/3.912,-1.,1.,
     1   .393,.3994,.89,.76/
      END
```

MACH NUMBER = .799

WIND TUNNEL DATA POINTS

VALUES OF LIFT COEFFICIENT

.0231	.1770	.3324	.4925	.6374

VALUES OF DRAG COEFFICIENT

.0152	.0169	.0272	.0468	.0803

LIFT COEFFICIENT OF INTEREST = .3000

CURVE FIT COEFFICIENTS

A0 = .01684
A1 = -.04606
A2 = .22597

STANDARD DEVIATION IN CL = .0033

STANDARD DEVIATION IN FIT = .0014

CONFIDENCE INTERVAL = 99. %

THE UNCERTAINTY IN DRAG COEFFICIENT = .0139

MACH NUMBER = .799

WIND TUNNEL DATA POINTS

VALUES OF LIFT COEFFICIFNT

.0231	.1770	.3324	.4925	.6374

VALUES OF DRAG COEFFICIENT

.0152	.0169	.0272	.0468	.0803

LIFT COEFFICIENT OF INTEREST = .3000

CURVE FIT COEFFICIENTS

A0 = .01684
A1 = -.04606
A2 = .22597

STANDARD DEVIATION IN CL = .0033

STANDARD DEVIATION IN FIT = .0014

CONFIDENCE INTERVAL = 95. %

THE UNCERTAINTY IN DRAG COEFFICIENT = .0061

MACH NUMBER = .799

WIND TUNNEL DATA POINTS

VALUES OF LIFT COEFFICIENT

.0231	.1770	.3324	.4925	.6374

VALUES OF DRAG COEFFICIENT

.0152	.0169	.0272	.0468	.0803

LIFT COEFFICIENT OF INTEREST = .3000

CURVE FIT COEFFICIENTS

A0 = .01684
A1 = -.04606
A2 = .22597

STANDARD DEVIATION IN CL = .0033

STANDARD DEVIATION IN FIT = .0014

CONFIDENCE INTERVAL = 90. %

THE UNCERTAINTY IN DRAG COEFFICIENT = .0041

MACH NUMBER = .799

WIND TUNNEL DATA POINTS

VALUES OF LIFT COEFFICIENT

.0231	.1770	.3324	.4925	.6374

VALUES OF DRAG COEFFICIENT

.0152	.0169	.0272	.0468	.0803

LIFT COEFFICIENT OF INTEREST = .3000

CURVE FIT COEFFICIENTS

A0 = .01684
A1 = -.04636
A2 = .22597

STANDARD DEVIATION IN CL = .0033

STANDARD DEVIATION IN FIT = .0014

CONFIDENCE INTERVAL = 80. %

THE UNCERTAINTY IN DRAG COEFFICIENT = .0027

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