## MATHCOUNTS.

2004
National Competition Sprint Round Problems 1-30

Name $\qquad$
School $\qquad$

State $\qquad$

## DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of 30 problems. You will have 40 minutes to complete. You are not allowed to use calculators, books or any other aids during this round. Calculator wrist watches should be given to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only your final answer in the answer blanks. If you complete the problems before time is called, use the remaining time to check your answers.

| Total Correct | Scorer's Initials |
| :---: | :---: |
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[^0]1. John thought of a number. He added three to the number and then multiplied this sum by two. He then divided this product by three. If the final result is now fourteen, what was John's original number?
2. 



In order to color the 10 numbered squares in the diagram so that any two of the squares that share more than a single point are different colors, what is the least number of colors you need to use? (Notice that square 1 can be the same color as square 4 , but not the same color as square 2.)
3. Some numbers of this tree are given, and only the top five rows of the tree are shown. If each number in a circle is the sum of the two numbers it is attached to in the row below it, what number will appear in circle A?

4. Three-fourths of the students in Mr. Shearer's class have brown hair and six-sevenths of his students are right-handed. If Mr. Shearer's class has 28 students, what is the smallest possible number of students that could be both right-handed and have brown hair?
5. How many triangles are in the figure to the right?

3. $\qquad$
4. $\qquad$
5. $\qquad$
6. When a number is divided by 3 , the result is 50 more than if the number had been divided by 4 . What is the number?
7. Which of the following points is not on the same line as the other three: $\mathrm{A}(-3,-7), \mathrm{B}(15,4), \mathrm{C}(3,-3)$ or $\mathrm{D}(12,3)$ ?
8. The length of a rectangle is $3 x+10$ feet and the width is $x+12$ feet. If the perimeter of the rectangle is 76 feet, how many square feet are in the area of the rectangle?
9. What percent of square $A B C D$ is shaded? All angles in the diagram are right angles.

10. Jorge randomly chooses two chips from the six chips shown, without replacement. What is the probability that the sum of the numbers on the two chips is greater than 18? Express your answer as a common fraction.

7. $\qquad$
8. $\qquad$
9. $\qquad$
10. $\qquad$
11. Suppose $a$ and $b$ are positive integers and $\frac{a}{4}+\frac{b}{3}=\frac{13}{12}$. What is
the value of $a^{2}+b^{2}$ ?
12. Reid took seven tests. On the first five tests that he took, he averaged 86 points. On the last three tests, he averaged 95 points. If he averaged 88 points on all seven tests, how many points did he average on the last two tests?
13. Jar A has exactly four red buttons and eight blue buttons. Carla then removes the same number of red buttons as blue buttons from Jar A and places them in an empty Jar B. Jar A now has $\frac{2}{3}$ of its original number of buttons. If Carla were now to randomly choose a button from Jar A and a button from Jar B, what is the probability that the two chosen buttons will both be red? Express your answer as a common fraction.
14. The letters $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D represent four distinct integers from 0 through 9. When $A$ is added to $B$ the result is $C$. When $B$ is subtracted from A the result is D . How many possible ways are there to assign values to $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D ?
15. The side lengths of a triangle are 15 units, 47 units and $\frac{x}{2}$ units. How many integer values of $x$ are possible?
11. $\qquad$
12. $\qquad$
13. $\qquad$
14. $\qquad$
15. $\qquad$
16. Two years ago there were 20 trailer homes on Elm Street with an average age of 18 years. At that time, a group of brand new trailer homes was then added to Elm Street. Today, the average age of all the trailer homes on Elm Street is 14 years. How many new trailer homes were added two years ago?
17. A rectangular block of candy 10 inches by 10 inches by 5 inches is coated on all faces with a very thin layer of chocolate. The block of candy is then cut into cubes measuring 1 inch by 1 inch by 1 inch. What percent of the cubes have no chocolate on them? Express your answer to the nearest tenth.
18. Parallelogram ABCD with $\mathrm{A}(2,5)$, $\mathrm{B}(4,9), \mathrm{C}(6,5)$ and $\mathrm{D}(4,1)$ is reflected across the $x$-axis to $A^{\prime} \mathrm{B}^{\prime} \mathrm{C}^{\prime} \mathrm{D}^{\prime}$ and then $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ is reflected across the line $y=x+1$ to $\mathrm{A}^{\prime \prime} \mathrm{B}^{\prime \prime} \mathrm{C}^{\prime \prime} \mathrm{D}^{\prime}$. This is done such that $\mathrm{D}^{\prime}$ is the image of D , and $D^{\prime \prime}$ is the image of $D^{\prime}$. What is the ordered pair of $\mathrm{D}^{\prime \prime}$ in the coordinate plane?
19. How many distinct, non-equilateral triangles with a perimeter of 60 units have integer side lengths $a, b$ and $c$ such that $a, b, c$ is an arithmetic sequence?
20. The shaded region shown consists of 11 unit squares and rests along the $x$-axis and $y$-axis. The shaded region is rotated about the $x$-axis to form a solid. In cubic units, what is the volume of the resulting solid? Express your answer in simplest form in terms of $\pi$.

$\qquad$
17. $\qquad$ 020 -
21.


This L-shape piece of land is made of 32 unit squares. By constructing a fence, only along drawn segments in the figure, the one large piece of land can be divided into two congruent smaller pieces of land, each with area 16 square units. What is the number of units in the perimeter of one of these smaller pieces of land?
22. The intersection of the graphic solutions of all three inequalities listed below form an enclosed region in the coordinate plane. What is the area of this region? Express your answer in terms of $\pi$.

$$
\left\{\begin{array}{l}
(x-4)^{2}+y^{2} \leq 16 \\
y \geq x-4 \\
y \geq-\frac{1}{3} x
\end{array}\right.
$$

23. Three positive integers $a, b$ and $c$ satisfy $a \cdot b \cdot c=8$ ! and $a<b<c$. What is the smallest possible value of $c-a$ ?
24. Henri stands in the rectangle marked "Start." Teresa flips a fair coin. If the coin lands Heads, Henri moves one rectangle upward; if it lands
 Tails, Henri moves one rectangle to the right. This process is continued until Henri moves into a rectangle with a star in it, at which time the game ends. If this final rectangle with the star is shaded, Teresa wins; if it is not shaded, Henri wins. What is the probability that Teresa wins? Express your answer as a common fraction.
25. If $a+b=7$ and $a^{3}+b^{3}=42$, what is the value of the sum $\frac{1}{a}+\frac{1}{b}$ ? Express your answer as a common fraction.
26. The product $2^{2} \cdot 3^{3} \cdot 5^{7}$ can be rewritten in the form $a^{2} \cdot b^{3} \cdot c$ with 26. positive integers $a, b$ and $c$. What is the greatest possible value of $a \cdot b-c$ ?
27. $\qquad$
28. $\qquad$
29. $\qquad$
30. $\qquad$
1
31. In the set of four equations below, each distinct letter represents
32. $\qquad$ a different non-zero digit. For example, "AHJ" represents the three-digit number with a hundreds digit of $A$, a tens digit of $H$ and a units digit of $J$.

$$
A B \times F=J G \quad A D^{E}=A H J \quad C^{E}=E C \quad B^{D}=H B
$$

What is the value of $\mathrm{G}^{\mathrm{E}}$ when expressed as an integer?
28. Jeff will pick a card at random from ten cards numbered 1 through 10. The number on this card will indicate his starting point on the number line shown below. He will then spin the fair spinner shown below (which has three congruent sectors) and follow the instruction indicated by his spin. From this new point he will spin the spinner again and follow the resulting instruction. What is the probability that he ends up at a multiple of 3 on the number line? Express your answer as a common fraction.

29. From a circular piece of paper with radius BC, Jeff removes the unshaded sector shown. Using the larger shaded sector, he joins edge BC to edge BA (without overlap) to form a cone of radius 12 centimeters and of volume $432 \pi$ cubic centimeters. What is the number of degrees in the measure of angle ABC of the sector that is not used?

30. A 2 by 2003 rectangle consists of unit squares as shown below. The middle unit square of each row is shaded. If a rectangle from the figure is chosen at random, what is the probability that the rectangle does not include a shaded square? Express your answer as a common fraction.
29. $\qquad$
30. $\qquad$
28. $\qquad$


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