Republic of the Philippines
Department of Education
Regional Office IX, Zamboanga Peninsula

8

Zest for Progress
Zeal of Partnership

MATHEMATICS

4th QUARTER – Module 3:
PROVING INEQUALITIES IN A TRIANGLE

Name of Learner: ___________________________

Grade & Section: ___________________________

Name of School: ___________________________

Mathematics – Grade 8
Alternative Delivery Mode
Quarter 4 - Module 3: Proving Inequalities in a Triangle
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Secretary: Leonor Magtolis Briones
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Development Team of the Module

Writer:	**Merry-Joy L. Caluyo**
Editors:	**Viola I. Quiniquito**
	Loraine S. Enguio
Layout Artist:	**Abdurauf J. Baldomero**
Reviewers: EPS, Mathematics	**Vilma A. Brown, Ed. D.**
Principal	**Garry Sta. Ana**
Management Team: SDS	**Roy C. Tuballa, EMD, JD, CESO VI**
ASDS	**Jay S. Montealto, CESO VI**
ASDS	**Norma T. Francisco, DM, CESE**
EPS Mathematics	**Vilma A. Brown, Ed. D.**
EPS LRMS	**Aida F. Coyme, Ed. D.**

Printed in the Philippines
Department of Education – Region IX, Zamboanga Peninsula
Office Address: Tiguma, Airport Road, Pagadian City
Telefax: (062) – 215 – 3751; 991 – 5975
E-mail Address: region9@deped.gov.ph

Introductory Message

This Self – Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you can proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. Read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

The main goal of this module is to help students learn and understand the concepts of proving inequalities in a triangle and polish your reasoning skills. This was designed to address the needs of the student in learning the Most Essential Learning Competencies for Grade 8 Mathematics. This module was written with detailed definition of the important terms followed by illustrative examples and activities.

After going through this module, you are expected to prove inequalities in a triangle (M8GE-IVc-1).

What I Know

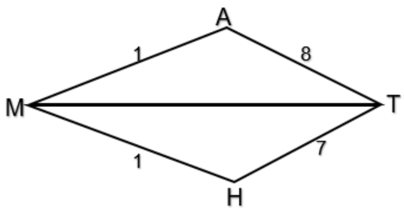
Directions: Choose the letter of the correct answer and write on a separate sheet.

1. The measure of an exterior angle of a triangle is always ________.
   A. greater than its adjacent interior angle.
   B. greater than either of the remote interior angles.
   C. less than its adjacent interior angle.
   D. less than either of the remote interior angles.

2. Joseph form triangles by bending a 16-inch wire. Which of the following sets of wire lengths successfully form a triangle?
   I. 5 in, 5 in, 6 in III. 4 in, 5 in, 7 in
   II. 4 in, 4 in, 8 in IV. 3 in, 4 in, 9 in

   A. I,III B. II,IV C. I,II D. III,IV

3. From the inequalities in the triangle shown below, a conclusion can be reached using the converse of hinge theorem. Which of the following is the last statement?



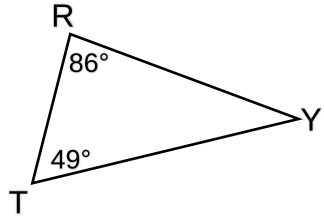
   A. $\overline{MT} \cong \overline{TM}$
   B. $m\angle AMT > m\angle HMT$
   C. $\overline{MA} \cong \overline{MH}$
   D. $m\angle HMT > m\angle AMT$

4. In ΔCAT, if $\overline{CA} = 3$, $\overline{AT} = 5$, $\overline{CT} = 2$, which statement is true?
   A. $m\angle C > m\angle A$ B. $m\angle A > m\angle T$ C. $m\angle A > m\angle C$ D. $m\angle T > m\angle C$

5. Which theorem justifies the statement in No. 4?
   A. Exterior Angle Inequality theorem
   B. Triangle Inequality Theorem 1(Ss→Aa)
   C. Triangle Inequality Theorem 2(Aa→Ss)
   D. Triangle Inequality Theorem 3($S_1 + S_2 > S_3$)

What's In

Directions: Given ΔTRY, where $m\angle R > m\angle$ T, which of the following statement is true? Explain your answer.



a. $TY = TR$
b. $\overline{TY} \cong \overline{TR}$
c. $\overline{TY} > \overline{TR}$
d. $\overline{TY} < \overline{TR}$

What's New

Directions: Determine what is asked in the problem. Write your answer in a separate sheet of paper.

Ana, Joan, May, and Mark were given a 20-cm stick each. They were instructed to create a triangle using the stick. Each cut the stick in their own chosen lengths as shown in the table below.

STUDENTS	Ana	Joan	May	Mark
CUTS OF STICK	5 cm, 6 cm, 9 cm	6 cm, 6 cm, 8 cm	5 cm, 7 cm, 8 cm	3 cm,4 cm,13 cm

Who among them was not able to make a triangle? **Explain your answer.**

LESSON 1 Inequalities in One Triangle

What is It

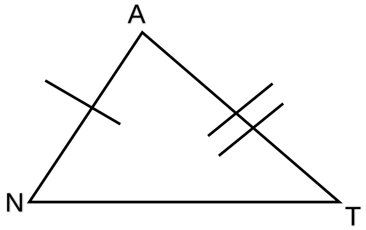
Triangle Inequality Theorem 1(Ss→Aa)

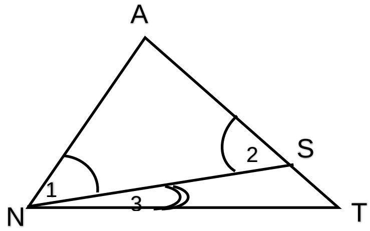
If one side of a triangle is longer than a second side, then the angle opposite the first side is larger than the angle opposite the second side.

EXAMPLE

Given: ΔANT; $\text{AT} > \text{AN}$

Prove: $m\angle\text{ANT} > m\angle\text{T}$





Proof:

STATEMENTS	REASONS
1. $\overline{AN} \cong \overline{AS}$	By construction
2. ΔANS is isosceles	Definition of Isosceles Triangle.
3. $\angle 1 \cong \angle 2$	Base angles of an isosceles triangle is congruent.
4. $\angle\text{ANT} = \angle 1 + \angle 3$	Angle Addition Postulate
5. $\angle\text{ANT} > \angle 1$	Comparison Property of Inequality
6. $\angle\text{ANT} > \angle 2$	Substitution Property of Inequality
7. $\angle 2 + \angle\text{NST} = 180$	Linear Pair Postulate
8. $\angle\text{NST} + \angle\text{T} + \angle 3 = 180$	The sum of the interior angles of a triangle is 180.
9. $\angle 2 + \angle\text{NST} = \angle\text{NST} + \angle\text{T} + \angle 3$	Substitution/Transitive Property
10. $\angle 2 = \angle\text{T} + \angle 3$	Subtraction Property of Equality
11. $\angle 2 > \angle\text{T}$	Comparison Property of Inequality
12. $\angle\text{ANT} > \angle\text{T}$	Transitive Property

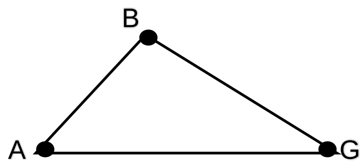
Triangle Inequality Theorem 2 (Aa→Ss)

If one angle of a triangle is larger than a second angle, then the side opposite the first angle is longer than the side opposite the second angle.

EXAMPLE

Given: ΔBAG; $m\angle\text{B} > m\angle\text{G}$

Prove: $\text{AG} > BA$



Let us use an indirect proof for this example. (Note: In an ***indirect proof***, instead of showing that the conclusion to be provided is true, you must show that all of the alternatives are false. To do this, you must assume the negation of the statement to be proved.

Assume that $\mathbf{AG} < \boldsymbol{BA}$.

Proof:

STATEMENTS	REASONS
1. $\text{AG} < \text{BA}$ such that $\text{AG} = \text{BA}$ or $\text{AG} < \text{BA}$	Temporary assumption
2. Considering $\text{AG} = \text{BA}$: If $\text{AG} = \text{BA}$, then ΔBAG is an isosceles triangle.	Definition of Isosceles Triangle.

3. Consequently, $\angle B = \angle G$	Base angles of isosceles triangles are congruent.
4. The assumption that $AG = BA$ is false.	The conclusion that $\angle B \cong \angle G$ contradicts the given ($m\angle B > m\angle G$)
5. Considering $AG = BA$: If $AG < BA$, then $m\angle B < m\angle G$.	Triangle Inequality Theorem 1 (Ss→Aa)
6. The assumption that $AG < BA$ is false.	The conclusion that $m\angle B < m\angle G$ contradicts the given ($m\angle B > m\angle G$)
7. Therefore, $AG > BA$ must be true.	The assumption that contradicts the known fact that $m\angle B > m\angle G$.

Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

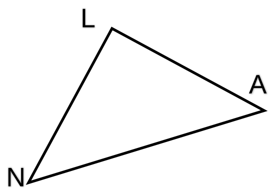
EXAMPLE 1.

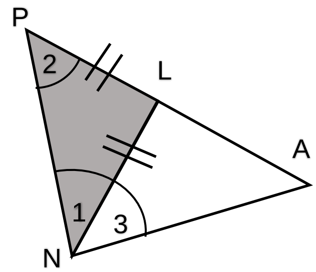
Given: ΔLAN *where* $\overline{LA} < \overline{LN} < \overline{AN}$

Prove: $\overline{AN} + \overline{LN} > \overline{LA}$

$\overline{AN} + \overline{LA} > \overline{LN}$

$\overline{LA} + \overline{LN} > \overline{AN}$





Proof:

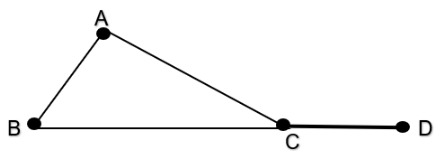
- ➔ Since $\overline{AN} > \overline{LN}$ and that $\overline{AN} > \overline{LA}$, then $\overline{AN} + \overline{LA} > \overline{LN}$ and $\overline{AN} + \overline{LN} > \overline{LA}$ are true.
- ➔ Hence, what remains to be proven is the third statement: $\overline{LA} + \overline{LN} > \overline{AN}$. Let us construct LP as an extension of LA such that L is between A and P, $LP \cong LN$ and ΔLNP is formed.

STATEMENTS	REASONS
1. LP=LN	By construction
2. ΔLNP is an isosceles triangle	Definition of isosceles triangle
3. $\angle LNP \cong \angle LPN$	Base angles of isosceles triangle are congruent
4. $\angle ANP = \angle LNA + \angle LNP$	Angle addition postulate
5. $\angle ANP = \angle LNA + \angle APN$	Substitution Property of Equality
6. $\angle ANP > \angle APN$	Comparison Property of Inequality
7. AP>AN	Triangle Inequality Theorem 2
8. LA+LP=AP	Segment addition postulate
9. LA+LP>AN	Substitution property of Inequality
10. LA+LN>AN	Substitution property of Inequality

EXAMPLE

Given: ∆ABC with exterior angle ∠ACD

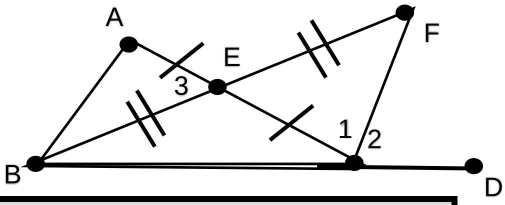
Prove: ∠ACD > ∠BAC



Proof:

To prove that ∠ACD > ∠BAC, we need to construct the following

1. midpoint of E on $\overline{AC}$ such that $\overline{AE} \cong \overline{CE}$
2. $\overline{BF}$ through E such that $\overline{BE} \cong \overline{EF}$



STATEMENTS	REASONS
1. $\overline{AE} \cong \overline{CE}$; $\overline{BE} \cong \overline{EF}$	By construction
2. $\angle 3 \cong \angle 4$	Vertical angles are congruent.
3. $\Delta AEB \cong \Delta CEF$	SAS Triangle Congruence Postulate
4. $\angle BAC \cong \angle 1$	Corresponding parts of congruent triangle are congruent. (CPCTC)
5. $\angle ACD = \angle 1 + \angle 2$	Angle Addition Postulate
6. $\angle ACD > \angle 1$	Comparison Property of Inequality
7. $\angle ACD > \angle BAC$	Substitution Property of Inequality

LESSON 2 Inequalities in Two Triangles

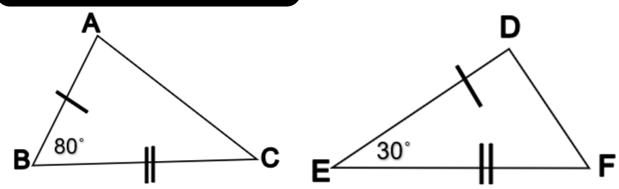
What is It

Hinge Theorem (SAS Triangle Inequality Theorem)

If two sides of one triangle are congruent to two sides of another triangle but the included angle of the first triangle is larger than the included angle of the second, then the third side of the first triangle is longer than the third side of the second triangle.

If $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $m\angle B > m\angle E$, then $AC > DF$.

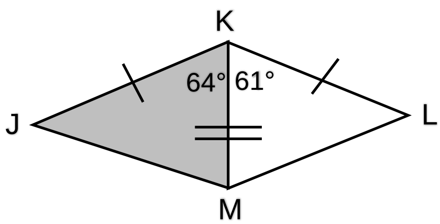
ILLUSTRATION



EXAMPLE

Given: $\overline{JK} \cong \overline{LK}$

Prove: $\overline{JM} > \overline{LM}$



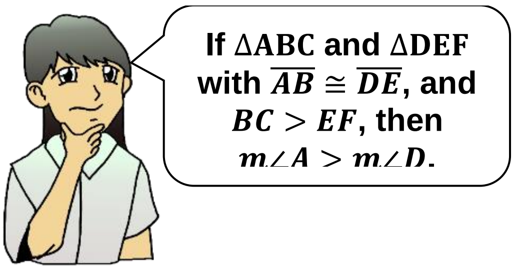
Proof:

You are given that $\overline{\mathbf{JK}} \cong \overline{\mathbf{LK}}$, and you know that $\overline{\mathbf{KM}} \cong \overline{\mathbf{KM}}$ by the reflexive property of congruence . $64° > 61°$, $\mathbf{m\angle JKM} > \mathbf{m\angle LKM}$. So, two sides of ΔJKM are congruent to two sides of ΔLKM, and the included angle in ΔJKM is larger. By the HINGE THEOREM, $\overline{JM} > \overline{LM}$.

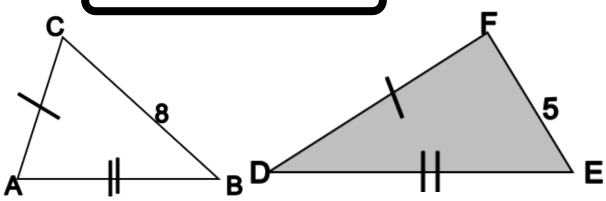
STATEMENTS	REASONS
$\overline{JK} \cong \overline{LK}$	Given
$\overline{KM} \cong \overline{KM}$	Reflexive property of congruence
$JM > LM$	Hinge Theorem

Converse of Hinge Theorem (SSS Triangle Inequality Theorem)

If two sides of one triangle are congruent to two sides of another triangle but the third side of the first triangle is longer than the third side of the second, then included angle of the first triangle is larger than the included angle of the



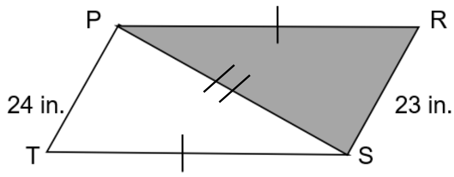
ILLUSTRATION



EXAMPLE

Given: $\overline{ST} \cong \overline{PR}$

Prove: $m\angle PST > m\angle SPR$



Proof:

You are given that $\overline{\mathbf{ST}} \cong \overline{\mathbf{PR}}$, and you know that $\overline{\mathbf{PS}} \cong \overline{\mathbf{PS}}$ by the Reflexive Property of Congruence. $24 \text{ inches} > 23 \text{ inches}$, $PT > SR$. So, two sides of $\Delta\mathbf{STP}$ are congruent to two sides of $\Delta\mathbf{PRS}$ and the third side of $\Delta\mathbf{STP}$ is longer. By the Converse of the Hinge Theorem, $\mathbf{m\angle PST} > m\angle\mathbf{SPR}$.

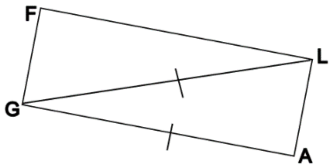
STATEMENTS	REASONS
$\overline{ST} \cong \overline{PR}$	Given
$\overline{PS} \cong \overline{PS}$	Reflexive property of congruence
$m\angle PST > m\angle SPR$	Converse of the Hinge Theorem

What's More

A. Directions: Complete the proof table by picking the statement or reasons from the given below.

$\overline{AG} \cong \overline{GL}$	Definition of congruent segments	Triangle equality Theorem
Substitution Property	$\overline{FL} \cong \overline{AL}$	Triangle Inequality Theorem

Given: $\overline{AG} \cong \overline{GL}$
Prove: $FL + AG > FG$



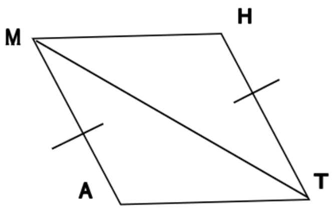
Proof:

STATEMENTS	REASONS
1.	Given
$AG = GL$	2.
$GL + FL > FG$	3.
$FL + AG > FG$	4.

B. Directions: Complete the proof table by picking the statement or reasons from the given below.

Given	Reflexive Property of Equality	Hinge Theorem
Converse of the Hinge Theorem	Given	

Given: $\overline{MA} \cong \overline{HT}, m\angle AMT > m\angle HTM$



Prove: $AT > HM$

STATEMENTS	REASONS
$\overline{MA} \cong \overline{HT}$	1.
$\overline{MT} \cong \overline{MT}$	2.
$\angle AMT > \angle HTM$	3.
$\overline{AT} \cong \overline{HM}$	4.

What I Have Learned

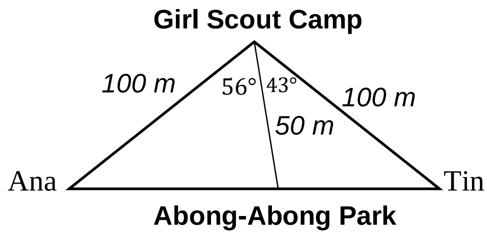
To test what you have learned from this lesson, answer the following statements below and fill up the blank of what you think the definition or statement all about.

1. The sum of the lengths of any two sides is greater than the length of the third side.
2. The measure of an exterior angle of a triangle is greater than the measure of either of its remote interior angles.
3. The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
4. In this type of proof, instead of showing that the conclusion to be provided is true, you must show that all the alternatives are false.
5. If one angle of a triangle is larger than a second angle, then the side opposite the first angle is longer than the side opposite the second angle.

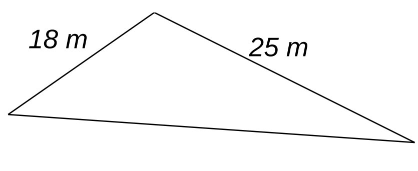
What I Can Do

Directions: Read and understand the given problem and write your answer on a separate sheet**.**

1. Ana and Tin want to meet at Abong – abong Park for recreation. Both of them are 100 meters away from the Girls Scout Camp in a different direction. The Girl Scout Camp is directly 50 meters away from the Abong-abong Park. At a constant speed, who will reach the destination first? Why?



2. Mr X wants to enclose his triangular garden using a barb- wire for protection. When he bought the barb- wire he only remembers the measures of the two sides of his garden 18 meters and 25 meters. If he decides to buy 50 meters of barbwire, do you think it is enough to enclose his garden? Why or why not?

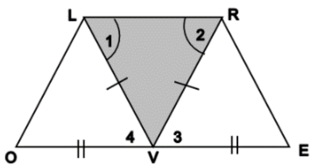


Assessment

A. Directions. Complete the following proof by choosing from the statements or reasons given below and unlock the secret message. Write your answers on a separate sheet.

(M) Converse of Hinge Theorem	(M) Hinge Theorem	(R) Definition of a midpoint
(T) $\angle 1 \cong \angle 2$	(H) Definition of an Isosceles Triangle	(O) V is the midpoint of OE
(E) Legs of isosceles triangles are congruent	(E) $m\angle 3 > m\angle 4$	

Given: V is the midpoint of $\overline{OE}$, $\angle 1 \cong \angle 2$, $m\angle 3 > m\angle 4$
Prove: $\overline{RE} > \overline{LO}$



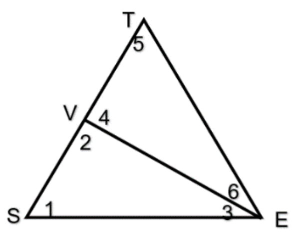
Proof:

STATEMENTS	REASONS
1.	Given
ΔLVR is isosceles	2.
$\overline{LV} \cong \overline{RV}$	3.
4.	Given
$\overline{OV} \cong \overline{EV}$	5.
6.	Given
$\overline{RE} > \overline{LO}$	7.

What is the secret message? __ __ __ __ __ __ __.

B. Directions: Complete the following proof by matching the statements to its possible reasons use arrow to connect it.

Given: $\Delta SET, \overline{VE}$ *bisects* $\angle SET$
Prove: $ET > TV$



Proof:

STATEMENTS	REASONS
1. $\Delta SET, \overline{VE}$ *bisects* $\angle SET$	A. Given
2. $\angle 3 \cong \angle 6$	B. Transitive Property
3. $m\angle 4 = m\angle 1 + m\angle 3$	C. The whole is greater than its parts.
4. $m\angle 4 > m\angle 3$	D. The exterior angle of a triangle is equal to the sum of the two nonadjacent interior angles.
5. $m\angle 4 > m\angle 6$	E. In a triangle, the longest side is opposite the largest angle.
6. ET>TV	F. An angle bisector divides an angle into two congruent parts.

Answer Key

What I Know:

1. B 3. B 5. B
2. A 4. A

What's In:

C, the opposite of the bigger angle is the longest side.

What's New:

Answer: *Mark, because the sum of the two shorter lengths 3 cm and 4 cm is not greater than the third side which is 13 cm.*

What's More:

Activity 1

Statements	Reasons
1. $\overline{AG} \cong \overline{GL}$	Given
$AG = GL$	2.Definition of congruent segments
$GL + FL > FG$	3.Triangle Inequality theorem
$FL + AG > FG$	4. Substitution Property

Activity 2

Statements	Reasons
$\overline{MA} \cong \overline{HT}$	1.Given
$\overline{MT} \cong \overline{MT}$	2.Reflexive Property of Equality
$\angle AMT > \angle HTM$	3.Given
$\overline{AT} \cong \overline{HM}$	4.Hinge Theorem

What I Have Learned:

1. *Triangle Inequality theorem*
2. *Exterior Angle Inequality theorem*
3. Triangle Inequality theorem
4. Indirect proof
5. Triangle Inequality theorem 2

Assessment:

A.

STATEMENTS	REASONS
$\angle 1 \cong \angle 2$	
ΔLVR is isosceles	Definition of an Isosceles Triangle
$\overline{LV} \cong \overline{RV}$	Legs of isosceles triangles are congruent
V is the midpoint of OE	
$\overline{OV} \cong \overline{EV}$	Definition of a midpoint
$m\angle 3 > m\angle 4$	
$\therefore \overline{RE} > \overline{LO}$	Hinge Theorem

secret message: THEOREM

B.

1. A 2. F 3. D 4. C 5. B 6. E

What I Can Do:

1. Applying hinge theorem$56° > 43°$ means Ana is farther than Tin. Therefore at a constant rate, Tin will reach the destination first.
2. Given the measures of two side of a triangle 18 and 25, the least measure of the third side should be 8 meters and the greatest should be 42 meters. So, Mr X must have at least 51 meters of barb-wire to enclose his garden. Therefore Mr X does not have enough barb wire to enclose his garden.

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Abuzo, E., et al, Mathematics Learner's Module (First Edition), Pasig; DepEd-IMCS,2013, pp. 408-413, 425-426.

Oronce, O., & Mendoza, M., E-math Worktext in Mathematics (8- Revised Edition), Manila; REX Bookstore Inc.,2015, pp. 451-454, 458-459.

I AM A FILIPINO
by Carlos P. Romulo

I am a Filipino – inheritor of a glorious past, hostage to the uncertain future. As such, I must prove equal to a two-fold task – the task of meeting my responsibility to the past, and the task of performing my obligation to the future.

I am sprung from a hardy race – child many generations removed of ancient Malayan pioneers. Across the centuries, the memory comes rushing back to me: of brown-skinned men putting out to sea in ships that were as frail as their hearts were stout. Over the sea I see them come, borne upon the billowing wave and the whistling wind, carried upon the mighty swell of hope – hope in the free abundance of the new land that was to be their home and their children's forever.

This is the land they sought and found. Every inch of shore that their eyes first set upon, every hill and mountain that beckoned to them with a green and purple invitation, every mile of rolling plain that their view encompassed, every river and lake that promised a plentiful living and the fruitfulness of commerce, is a hollowed spot to me.

By the strength of their hearts and hands, by every right of law, human and divine, this land and all the appurtenances thereof – the black and fertile soil, the seas and lakes and rivers teeming with fish, the forests with their inexhaustible wealth in wild and timber, the mountains with their bowels swollen with minerals – the whole of this rich and happy land has been for centuries without number, the land of my fathers. This land I received in trust from them, and in trust will pass it to my children, and so on until the world is no more.

I am a Filipino. In my blood runs the immortal seed of heroes – seed that flowered down the centuries in deeds of courage and defiance. In my veins yet pulses the same hot blood that sent Lapulapu to battle against the alien foe, that drove Diego Silang and Dagohoy into rebellion against the foreign oppressor.

That seed is immortal. It is the self-same seed that flowered in the heart of Jose Rizal that morning in Bagumbayan when a volley of shots put an end to all that was mortal of him and made his spirit deathless forever; the same that flowered in the hearts of Bonifacio in Balintawak, of Gregorio del Pilar at Tirad Pass, of Antonio Luna at Calumpit, that bloomed in flowers of frustration in the sad heart of Emilio Aguinaldo at Palanan, and yet burst forth royally again in the proud heart of Manuel L. Quezon when he stood at last on the threshold of ancient Malacanang Palace, in the symbolic act of possession and racial vindication. The seed I bear within me is an immortal seed.

It is the mark of my manhood, the symbol of my dignity as a human being. Like the seeds that were once buried in the tomb of Tutankhamen many thousands of years ago, it shall grow and flower and bear fruit again. It is the insigne of my race, and my generation is but a stage in the unending search of my people for freedom and happiness.

I am a Filipino, child of the marriage of the East and the West. The East, with its languor and mysticism, its passivity and endurance, was my mother, and my sire was the West that came thundering across the seas with the Cross and Sword and the Machine. I am of the East, an eager participant in its struggles for liberation from the imperialist yoke. But I know also that the East must awake from its centuried sleep, shake off the lethargy that has bound its limbs, and start moving where destiny awaits.

For I, too, am of the West, and the vigorous peoples of the West have destroyed forever the peace and quiet that once were ours. I can no longer live, a being apart from those whose world now trembles to the roar of bomb and cannon shot. For no man and no nation is an island, but a part of the main, and there is no longer any East and West – only individuals and nations making those momentous choices that are the hinges upon which history revolves. At the vanguard of progress in this part of the world I stand – a forlorn figure in the eyes of some, but not one defeated and lost. For through the thick, interlacing branches of habit and custom above me I have seen the light of the sun, and I know that it is good. I have seen the light of justice and equality and freedom, my heart has been lifted by the vision of democracy, and I shall not rest until my land and my people shall have been blessed by these, beyond the power of any man or nation to subvert or destroy.

I am a Filipino, and this is my inheritance. What pledge shall I give that I may prove worthy of my inheritance? I shall give the pledge that has come ringing down the corridors of the centuries, and it shall be compounded of the joyous cries of my Malayan forebears when first they saw the contours of this land loom before their eyes, of the battle cries that have resounded in every field of combat from Mactan to Tirad Pass, of the voices of my people when they sing:

"I am a Filipino born to freedom, and I shall not rest until freedom shall have been added unto my inheritance—for myself and my children and my children's children—forever."