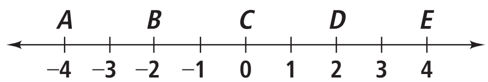
# 5-1 Midsegments of Triangles

## Vocabulary

### Review

**Use the number line at the right for Exercises 1–3.**

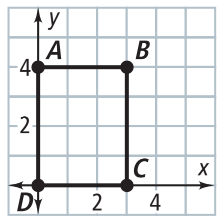



1. Point **C** is the *midpoint* of $\overline{AE}$.

2. Point **D** is the *midpoint* of $\overline{CE}$.

3. Point **B** is the *midpoint* of $\overline{AC}$.

**Use the graph at the right for Exercises 4–6. Name each *segment*.**




4. a *segment* that lies on the *x*-axis

   $\overline{DC}$ or $\overline{CD}$

5. a *segment* that contains the point (0, 4)

   $\overline{DA}$, $\overline{AD}$, $\overline{AB}$, or $\overline{BA}$

6. a *segment* whose endpoints both have *x*-coordinate 3

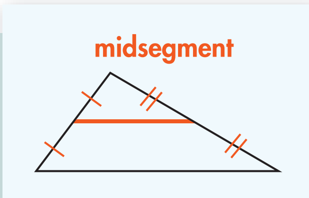
   $\overline{BC}$ or $\overline{CB}$

### Vocabulary Builder

**midsegment** (noun) MID **seg munt**

**Related Words:** midpoint, segment

**Definition:** A **midsegment** of a triangle is a segment connecting the midpoints of two sides of the triangle.




### Use Your Vocabulary

**Circle the correct statement in each pair.**

7. A *midsegment* connects the midpoints of two sides of a triangle.

   A *midsegment* connects a vertex of a triangle to the midpoint of the opposite side.

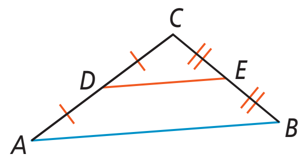
8. A triangle has exactly one *midsegment*. A triangle has three *midsegments.*

take note

## Theorem 5-1 Triangle Midsegment Theorem

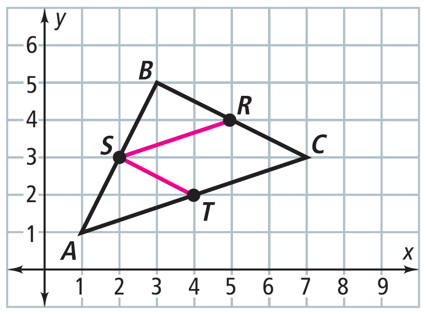
If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side and is half as long.

**9.** Use the triangle at the right to complete the table below.




| If | Then |
|---|---|
| *D* is the midpoint of $\overline{CA}$ and | $\overline{DE} \parallel \overline{AB}$ |
| *E* is the midpoint of $\overline{CB}$ | $DE = \frac{1}{2}AB$ |

**Use the graph at the right for Exercises 10–11.**




**10.** Draw $\overline{RS}$. Then underline the correct word or number to complete each sentence below.

$\overline{RS}$ is <u>a midsegment of</u> / parallel to $\triangle ABC$.

$\overline{RS}$ is a midsegment of / <u>parallel to</u> $\overline{AC}$.

**11.** Use the Triangle Midsegment Theorem to complete.

$RS = \frac{1}{2} AC$

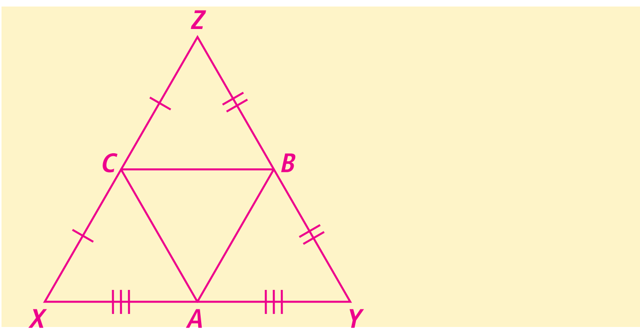
**12.** Draw $\overline{ST}$. What do you know about $\overline{ST}$?

Sample: It is a midsegment of $\triangle ABC$; it is parallel to and half the length of $\overline{BC}$.

## Problem 1 Identifying Parallel Segments

**Got It?** **In $\triangle XYZ$, *A* is the midpoint of $\overline{XY}$, *B* is the midpoint of $\overline{YZ}$, and *C* is the midpoint of $\overline{ZX}$. What are the three pairs of parallel segments?**

**13.** Draw a diagram to illustrate the problem.

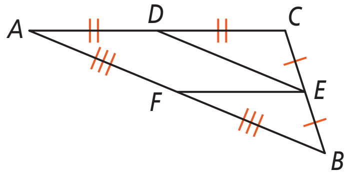



**14.** Write the segment parallel to each given segment.

$\overline{AB} \parallel \overline{ZX}$ $\quad$ $\overline{CB} \parallel \overline{XY}$ $\quad$ $\overline{CA} \parallel \overline{YZ}$

## Problem 2 Finding Lengths

**Got It?** **In the figure below, $AD = 6$ and $DE = 7.5$. What are the lengths of $\overline{DC}$, $\overline{AC}$, $\overline{EF}$, and $\overline{AB}$?**




**15.** Complete the problem-solving model below.

| Know | Need | Plan |
|---|---|---|
| $AD = 6$ and $DE = 7.5$. $CE = EB$, $AD = DC$, $BF = \boxed{FA}$ | The lengths of $\overline{DC}$, $\overline{AC}$, $\overline{EF}$, and $\overline{AB}$ | Use the Triangle Midsegment Theorem to find $DC$, $AC$, $EF$, and $\boxed{AB}$. |

**16.** The diagram shows that $\overline{EF}$ and $\overline{DE}$ join the midpoints of two sides of $\triangle$ **ABC**.

By the Triangle Midsegment Theorem, $EF = \frac{1}{2} \cdot$ **AC** and $DE = \frac{1}{2} \cdot$ **AB**.

**Complete each statement.**

**17.** $DC = AD =$ **6**

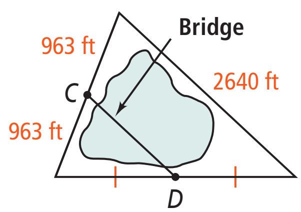
**18.** $AC = AD +$ **DC** $=$ **6** $+$ **6** $=$ **12**

**19.** $EF =$ **$\frac{1}{2}$** $\cdot AC =$ **$\frac{1}{2}$** $\cdot$ **12** $= 6$

**20.** $CB =$ **2** $\cdot DE =$ **2** $\cdot$ **7.5** $= 15$

## Problem 3 Using the Midsegment of a Triangle

**Got It?** **$\overline{CD}$ is a bridge being built over a lake, as shown in the figure at the right. What is the length of the bridge?**




**21.** Complete the flow chart to find the length of the bridge.

$\overline{CD}$ joins the _?_ of two sides of a triangle. **midpoints**

$\overline{CD}$ is parallel to a side that is **2640** ft.

Use the Triangle _?_ Theorem. **Midsegment**

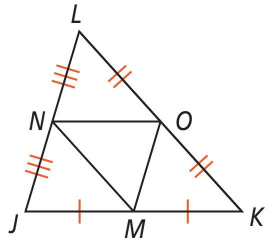
$CD = \frac{1}{2} \cdot$ **2640**

$CD =$ **1320**

**22.** The length of the bridge is **1320** ft.

## Lesson Check • Do you know HOW?

**If $JK = 5x + 20$ and $NO = 20$, what is the value of $x$?**




**Complete each statement.**

**23.** N is the midpoint of $\overline{LJ}$.

**24.** O is the midpoint of $\overline{LK}$.

**25.** $\overline{NO}$ is a _?_ of $\triangle JKL$, so $NO = \frac{1}{2}JK$. midsegment

**26.** Substitute the given information into the equation in Exercise 25 and solve for $x$.

$NO = \frac{1}{2}JK$

$20 = \frac{1}{2}(2x + 20)$

$40 = (2x + 20)$

$20 = 5x$

$x = 4$

## Lesson Check • Do you UNDERSTAND?

**Reasoning** **If two noncollinear segments in the coordinate plane have slope 3, what can you conclude?**

**27.** Place a ✓ in the box if the response is correct. Place an ✗ if it is incorrect.

✓ If two segments in a plane are parallel, then they have the same slope.

✗ If two segments lie on the same line, they are parallel.

**28.** Now answer the question. Answers may vary. Sample:

The segments do not lie on the same line,

so they are parallel lines.

## Math Success

**Check off the vocabulary words that you understand.**

☐ midsegment ☐ midpoint ☐ segment

**Rate how well you can *use properties of midsegments.***

| Need to review | 0 | 2 | 4 | 6 | 8 | 10 | Now I get it! |
|---|---|---|---|---|---|---|---|

# 5-2 Perpendicular and Angle Bisectors

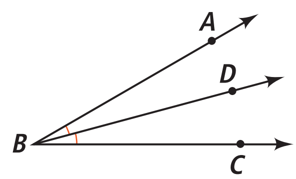
## Vocabulary

### Review

**Complete each statement with *bisector* or *bisects*.**

1. $\overrightarrow{BD}$ is the ? of $\angle ABC$. bisector

2. $BD$ ? $\angle ABC$. bisects



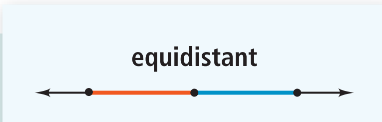

**Write T for *true* or F for *false*.**

T 3. Two *perpendicular* segments intersect to form four right angles.

F 4. You can draw more than one line *perpendicular* to a given line through a point not on the line.

### Vocabulary Builder

**equidistant** (adjective) **ee kwih DIS tunt**

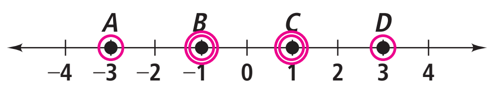



**Related Words:** equal, distance

**Definition:** **Equidistant** means at an equal distance from a single point or object.

### Use Your Vocabulary

**Use to the number line at the right for Exercises 5 and 6.**

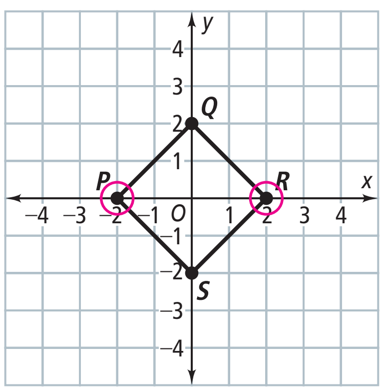



5. Circle two points *equidistant* from zero.

6. Name points that are *equidistant* from point *C*.

   B and D

**Use to the diagram at the right for Exercises 7 and 8.**




7. Circle two points *equidistant* from point *Q*.

8. Name four segments that are *equidistant* from the origin.

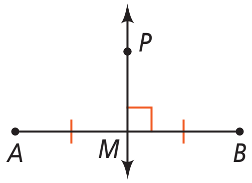
   $\overline{PQ}$ $\overline{QR}$ $\overline{RS}$ $\overline{SP}$

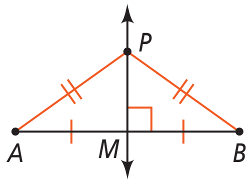
take note

## Theorem 5-2 Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

**9.** Use the diagrams below to complete the hypothesis and the conclusion.




If
$\overleftrightarrow{PM} \perp \overline{AB}$ and $AM =$ **MB**

Then
$PA =$ **PB**

take note

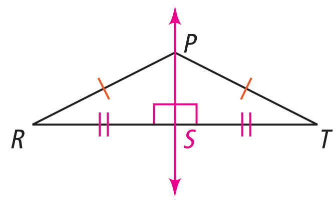
## Theorem 5-3 Converse of the Perpendicular Bisector Theorem

**10.** Complete the converse of Theorem 5-2.

If a point is equidistant from the endpoints of a segment, then it is on the _?_ of the segment.

**perpendicular bisector**

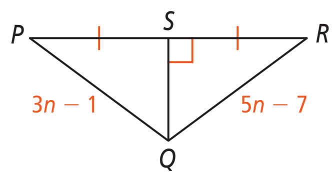
**11.** Complete the diagram at the right to illustrate Theorem 5-3.






## Problem 1 Using the Perpendicular Bisector

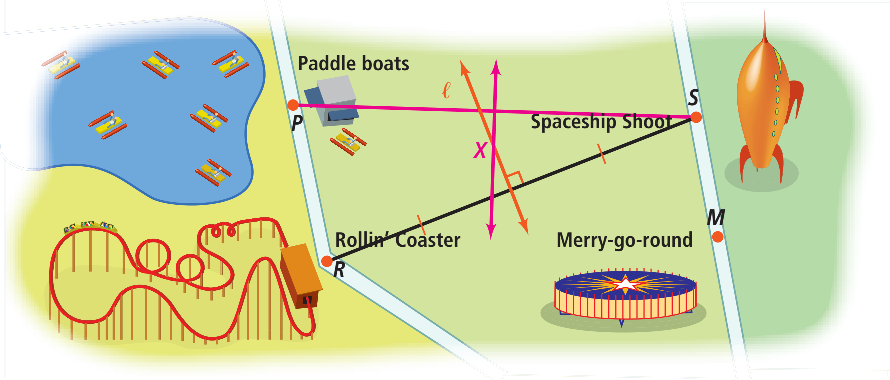
**Got It?** **Use the diagram at the right. What is the length of $\overline{QR}$?**




**12.** Complete the reasoning model below.

| Think | Write |
|---|---|
| $\overline{QS}$ is the perpendicular bisector of $\overline{PR}$, so $Q$ is equidistant from $P$ and $R$ by the Perpendicular Bisector Theorem. | $PQ =$ **QR**<br>$3n - 1 =$ **5n − 7** |
| I need to solve for $n$. | $3n + 6 =$ **5n**<br>$6 =$ **2n**<br>**3** $= n$ |
| Now I can substitute for $n$ to find $QR$. | $QR = 5n - 7$<br>$= 5($**3**$) - 7 =$ **8** |

## Problem 2 Using a Perpendicular Bisector




**Got It?** **If the director of the park at the right wants a T-shirt stand built at a point equidistant from the Spaceship Shoot and the Rollin' Coaster, by the Perpendicular Bisector Theorem he can place the stand anywhere along line $\ell$. Suppose the park director wants the T-shirt stand to be equidistant from the paddle boats and the Spaceship Shoot. What are the possible locations?**

**13.** On the diagram, draw $\overline{PS}$.

**14.** On the diagram, sketch the points that are equidistant from the paddle boats and the Spaceship Shoot. Describe these points.

the points on the perpendicular bisector of $\overline{PS}$

**Got It?** **Reasoning** **Can you place the T-shirt stand so that it is equidistant from the paddle boats, the Spaceship Shoot, and the Rollin' Coaster? Explain.**

**15.** Does the line you drew in Exercise 14 intersect line $\ell$? Yes / No

**16.** Where should the T-shirt stand be placed so that it is equidistant from the paddle boats, the Spaceship Shoot, and the Rollin Coaster? Explain. Answers may vary.

Sample: Place the stand at the intersection point $X$ of the perpendicular bisectors of $\overline{RS}$ and $\overline{PS}$. By the Perpendicular Bisector Theorem, $XR = XS$ and $XS = XP$, so $XR = XS = XP$ by the Transitive Property.

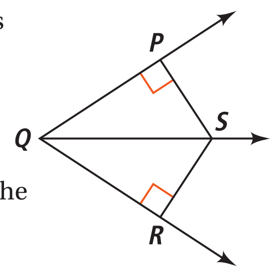
The **distance from a point to a line** is the length of the perpendicular segment from the point to the line. This distance is also the length of the shortest segment from the point to the line.

take note

### Theorems 5-4 and 5-5

**Angle Bisector Theorem**
If a point is on the bisector of an angle, then the point is equisdistant from the sides of the angle.

**17.** If point $S$ is on the angle bisector of $\angle$ PQR, then $SP =$ SR.



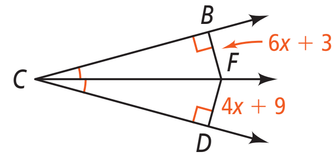

**Converse of the Angle Bisector Theorem**
If a point in the interior of an angle is equidistant from the sides of the angle, then the point is on the angle bisector.

**18.** Point $S$ is in the interior of $\angle$ PQR.

**19.** If $SP = SR$, then $S$ is on the _?_ of $\angle PQR$. angle bisector

## Problem 3 Using the Angle Bisector Theorem

**Got It?** What is the length of $\overline{FB}$?

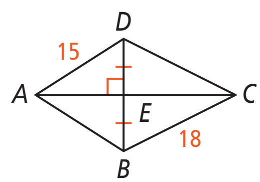



**20.** The problem is solved below. Justify each step.

| | |
|---|---|
| $FB = FD$ | Angle Bisector Theorem |
| $6x + 3 = 4x + 9$ | Substitute. |
| $6x = 4x + 6$ | Subtract 3 from each side. |
| $2x = 6$ | Subtract 4*x* from each side. |
| $x = 3$ | Divide each side by 2. |
| $FB = 6x + 3$ | Given |
| $= 6(3) + 3 = 21$ | Substitute 3 for *x* and simplify. |

## Lesson Check • Do you know HOW?

**Use the figure at the right. What is the relationship between $\overline{AC}$ and $\overline{BD}$?**




**21.** Underline the correct word or symbol to complete each sentence.

$\overline{AC}$ is parallel / perpendicular to $\overline{BD}$.

$\overline{AC}$ divides $\overline{BD}$ into two congruent / noncongruent segments.

$\overline{BD}$ divides $\overline{AC}$ into two congruent / noncongruent segments.

$\overline{AC}$ / $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$ / $\overline{BD}$.

## Math Success

**Check off the vocabulary words that you understand.**

☐ perpendicular bisector ☐ equidistant ☐ distance from a point to a line

**Rate how well you can *understand bisectors.***

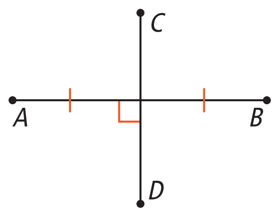
Need to review 0 2 4 6 8 10 Now I get it!

# 5-3 Bisectors in Triangles

## Vocabulary

### Review

**Use the figure at the right. Write T for *true* or F for *false*.**




F **1.** $\overline{AB}$ is the *perpendicular bisector* of $\overline{CD}$.

T **2.** $\overline{CD}$ is a *perpendicular bisector*, so it intersects $\overline{AB}$ at its midpoint.

T **3.** Any point on $\overline{CD}$ is *equidistant* from points $A$ and $B$.

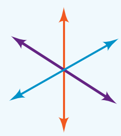
### Vocabulary Builder

**concurrent** (adjective) **kun KUR unt**

**Main Idea:** **Concurrent** means occurring or existing at the same time.

**Math Usage:** When three or more lines intersect in one point, they are **concurrent.**

concurrent lines



### Use Your Vocabulary

**Complete each statement with *concurrency, concurrent,* or *concurrently.***

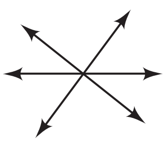
**4.** Two classes are _?_ when they meet at the same time. concurrent

**5.** The point of _?_ of three streets is the intersections of the streets. concurrency

**6.** A person may go to school and hold a job _?_. concurrently

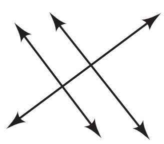
**Label each diagram below *concurrent* or *not concurrent.***

**7.**



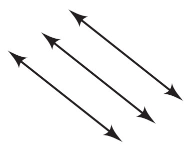
concurrent

**8.**



not concurrent

**9.**



not concurrent

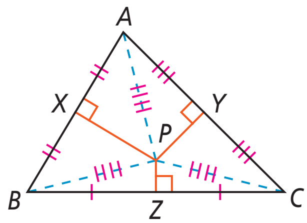
take note

## Theorem 5-6 Concurrency of Perpendicular Bisectors Theorem

The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices.

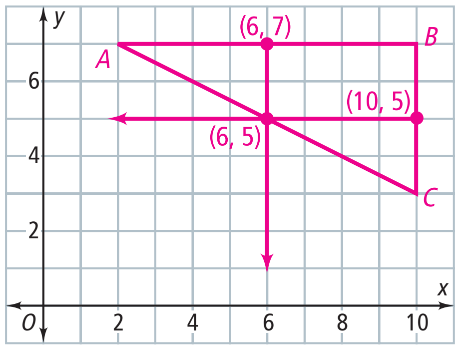
Perpendicular bisectors $\overline{PX}$, $\overline{PY}$ and $\overline{PZ}$ are concurrent at $P$.

**10.** Mark $\triangle ABC$ to show all congruent segments.




## Problem 1 Finding the Circumcenter of a Triangle

**Got It?** **What are the coordinates of the circumcenter of the triangle with vertices $A(2, 7)$, $B(10, 7)$, and $C(10, 3)$?**




**11.** Draw $\triangle ABC$ on the coordinate plane.

**12.** Label the coordinates the midpoint of $\overline{AB}$ and the midpoint of $\overline{BC}$.

**13.** Draw the perpendicular bisector of $\overline{AB}$.

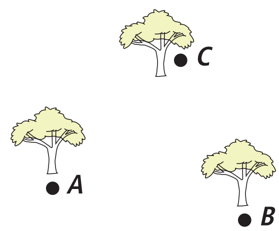
**14.** Draw the perpendicular bisector of $\overline{BC}$ .

**15.** Label the coordinates of the point of intersection of the bisectors.

**16.** The circumcenter of $\triangle ABC$ is ( 6 , 5 ).

## Problem 2 Using a Circumcenter

**Got It?** **A town planner wants to place a bench equidistant from the three trees in the park. Where should he place the bench?**




**17.** Complete the problem-solving model below.

**Know**

The trees form the ? of a triangle.

vertices

**Need**

Find the point of concurrency of the ? of the sides.

perpendicular

bisectors

**Plan**

Find the ? of the triangle, which is equidistant from the three trees.

circumcenter

**18.** How can the town planner determine where to place the bench? Explain.

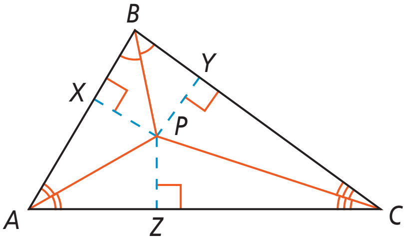
Explanations may vary. Sample: The town planner can place the bench at the circumcenter of the triangle formed by the three trees.

take note

## Theorem 5-7 Concurrency of Angle Bisectors Theorem

The bisectors of the angles of a triangle are concurrent at a point equidistant from the sides of the triangle.

Angle bisectors $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are concurrent at $P$.



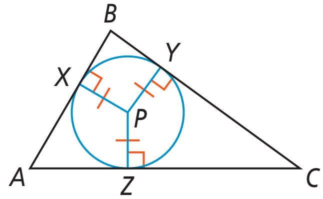

**19.** $PX$ = PY = PZ

**Complete each sentence with the appropriate word from the list.**

incenter      inscribed      inside

**20.** The point of concurrency of the angle bisectors of a triangle is the _?_ of the triangle.   incenter

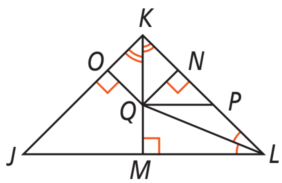
**21.** The point of concurrency of the angle bisectors of a triangle is always _?_ the triangle.   inside




**22.** The circle is _?_ in $\triangle ABC$.   inscribed

## Problem 3 Identifying and Using the Incenter

**Got It?** $QN = 5x + 36$ and $QM = 2x + 51$. What is $QO$?



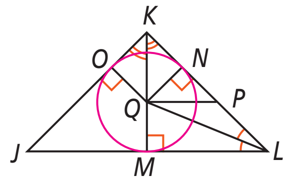

**23.** Complete the reasoning model below.

| Think | Write |
|---|---|
| I know that $Q$ is the point of concurrency of the angle bisectors. | $Q$ is the incenter / midpoint of $\triangle JKL$. |
| And I know that | the distance from $Q$ to each side of $\triangle JKL$ is equal / unequal. |
| I can write an equation and solve for $x$. | $QO = QM$<br>$5x + 36 = 2x + 51$<br>$5x = 2x + 15$<br>$3x = 15$<br>$x = 5$ |

**24.** Use your answer to Exercise 23 to find $QO$.

$QO = 5x + 36$
$QO = 5(5) + 36$
$QO = 25 + 36$
$QO = 61$

**Got It? Reasoning** **Is it possible for $QP$ to equal 50? Explain.**




**25.** Drawn an inscribed circle in the diagram at the right.

**26.** $QN$ and $QM$ are two segments that have the same length as $\overline{QO}$.

**27.** Circle the correct relationship between $QO$ and $QP$.

$QO < QP$ $\qquad$ $QO = QP$ $\qquad$ $QO > QP$

**28.** Given your answer to Exercise 27, is it possible for $QP$ to equal 50? Explain. Answers may vary. Sample:

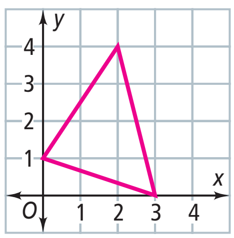
The radii measure 61, since $\overline{QO}$ is one of the radii. $\overline{QP}$ is longer than the radius of the circle. Therefore, its length cannot be 50.

## Lesson Check • Do you UNDERSTAND?

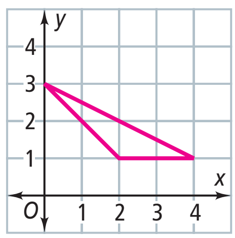
**Vocabulary** **A triangle's circumcenter is outside the triangle. What type of triangle is it?**

**29.** Draw an example of each type of triangle on a coordinate plane below. Answers may vary. Samples are given.

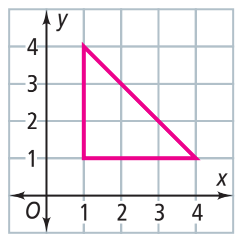
acute




obtuse




right




**30.** Circle the phrase that describes the circumcenter of a triangle.

the point of concurrency of the angle bisectors $\qquad$ the point of concurrency of the perpendicular bisectors of the sides

**31.** Underline the correct word to complete the sentence.

When a triangle's circumcenter is outside the triangle, the triangle is acute / obtuse / right .

## Math Success

**Check off the vocabulary words that you understand.**

☐ concurrent ☐ circumscribed about ☐ incenter ☐ inscribed in ☐ bisector

**Rate how well you can *use bisectors in triangles.***

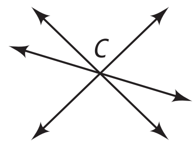
Need to review 0 2 4 6 8 10 Now I get it!

# 5-4 Medians and Altitudes

## Vocabulary

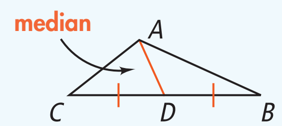
### Review

1. Are three diameters of a circle *concurrent*? Yes / No
2. Are two diagonals of a rectangle *concurrent*? Yes / No
3. Is point *C* at the right a point of *concurrency*? Yes / No




### Vocabulary Builder

**median** (noun) MEE **dee un**




**Related Words:** median (adjective), middle (noun), midpoint (noun)

**Definition:** A **median** of a triangle is a segment whose endpoints are a vertex and the midpoint of the opposite side.

### Use Your Vocabulary

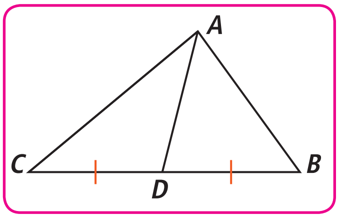
**Write T for *true* or F for *false*.**

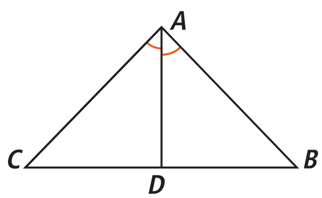
F 4. The *median* of a triangle is a segment that connects the midpoint of one side to the midpoint of an adjacent side.

T 5. The point of concurrency of the *medians* of a triangle is where they intersect.

F 6. A triangle has one median.

7. Circle the drawing that shows *median* $\overline{AD}$ of $\triangle ABC$.



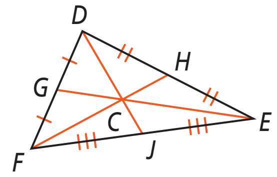



take note

## Theorem 5-8 Concurrency of Medians Theorem

The medians of a triangle are concurrent at a point (the centroid of the triangle) that is two thirds the distance from each vertex to the midpoint of the opposite side.

For any triangle, the centroid is always inside the triangle.

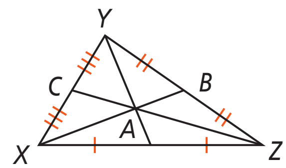



**8.** Complete each equation.

$DC = \frac{2}{3}$ **DJ** $\quad EC = \frac{2}{3}$ **EG** $\quad FC =$ $\frac{2}{3}$**FH**

## Problem 1 Finding the Length of a Median

**Got It?** **In the diagram at the right, $ZA = 9$. What is the length of $\overline{ZC}$?**




**9.** Point **A** is the centroid of $\triangle XYZ$.

**10.** Use the justifications at the right to solve for $ZC$.

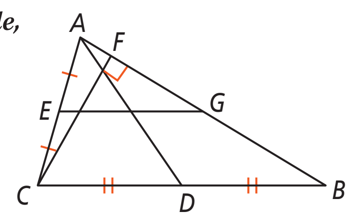
| | |
|---|---|
| $ZA = \frac{2}{3} \cdot ZC$ | Concurrency of Medians Theorem |
| $9 = \frac{2}{3} \cdot ZC$ | Substitute for $ZA$. |
| $9\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)\frac{2}{3} \cdot ZC$ | Multiply each side by $\frac{3}{2}$. |
| $\frac{27}{2} = ZC$ | Simplify. |

**11.** $ZC$ is $\frac{27}{2}$, or $13\frac{1}{2}$.

An *altitude* of a triangle is the perpendicular segment from a vertex of the triangle to the line containing the opposite side.

## Problem 2 Identifying Medians and Altitudes

**Got It?** **For $\triangle ABC$, is each segment, $\overline{AD}$, $\overline{EG}$, and $\overline{CF}$, a *median*, an *altitude*, or *neither*? Explain.**




**12.** Read each statement. Then cross out the words that do NOT describe $\overline{AD}$.

$\overline{AD}$ is a segment that extends from vertex $A$ to $\overline{CB}$, which is opposite $A$.

$\overline{AD}$ meets $\overline{CB}$ at point $D$, which is the midpoint of $\overline{CB}$ since $\overline{CD} \cong \overline{DB}$.

$\overline{AD}$ is not perpendicular to $\overline{CB}$.

~~altitude~~ median ~~neither altitude nor median~~

**13.** Circle the correct statement below.

$\overline{AD}$ is a median. $\quad$ $\overline{AD}$ is an altitude. $\quad$ $\overline{AD}$ is neither a median nor an altitude.

**14.** Read the statement. Then circle the correct description of $\overline{EG}$.

$\overline{EG}$ does not extend from a vertex.

$\overline{EG}$ is a median. $\quad$ $\overline{EG}$ is an altitude. $\quad$ $\overline{EG}$ is neither a median nor an altitude.

15. Read each statement. Then circle the correct description of $\overline{CF}$.

$\overline{CF}$ is a segment that extends from vertex $C$ to $\overline{AB}$, which opposite $C$.

$\overline{CF} \perp \overline{AB}$

$\overline{CF}$ is median. $\quad$ $\overline{CF}$ is an altitude. $\quad$ $\overline{CF}$ is neither a median nor an altitude.

take note

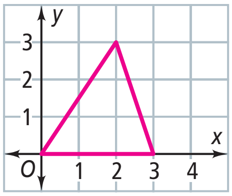
## Theorem 5-9 Concurrency of Altitudes Theorem

The lines that contain the altitudes of a triangle are concurrent.

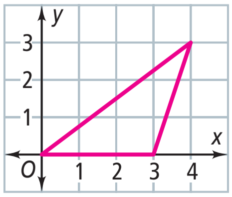
The point of concurrency is the *orthocenter of the triangle*. The orthocenter of a triangle can be inside, on, or outside the triangle.

16. Draw an example of each type of triangle on a coordinate plane below. **Answers may vary. Samples are given.**

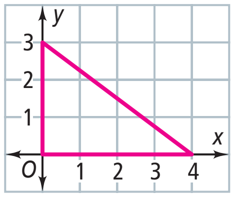
acute




obtuse




right



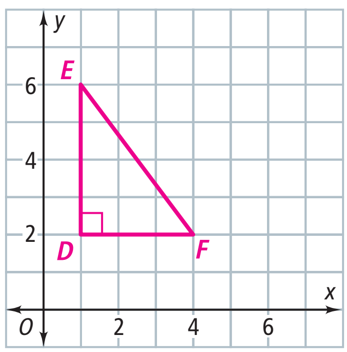

**Draw a line from the type of triangle in Column A to the location of its orthocenter in Column B.**

| Column A | Column B |
|---|---|
| 17. acute | outside the triangle |
| 18. right | inside the triangle |
| 19. obtuse | at a vertex of the triangle |

## Problem 3 Finding the Orthocenter

**Got It?** **$\triangle DEF$ has vertices $D(1, 2)$, $E(1, 6)$, and $F(4, 2)$. What are the coordinates of the orthocenter of $\triangle DEF$?**

20. Graph $\triangle DEF$ on the coordinate plane.




**Underline the correct word to complete each sentence.**

21. $\triangle DEF$ is a(n) acute / <u>right</u> triangle, so the orthocenter is at vertex $D$.

22. The altitude to $\overline{DF}$ is horizontal / <u>vertical</u>.

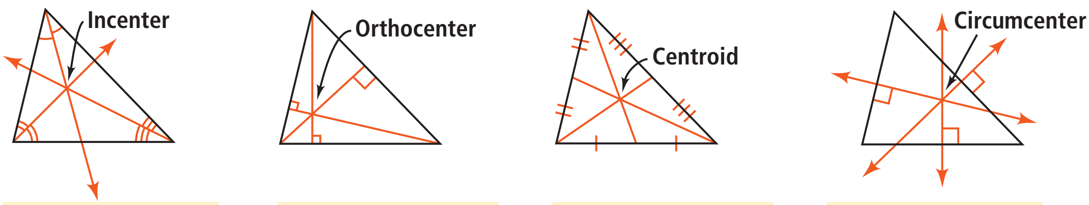
23. The altitude to $\overline{DE}$ is <u>horizontal</u> / vertical.

24. The coordinates of the orthocenter of $\triangle DEF$ are ( 1 , 2 ).

take note

## Concept Summary Special Segments and Lines in Triangles

**25.** Use the words *altitudes, angle bisectors, medians,* and *perpendicular bisectors* to describe the intersecting lines in each triangle below.




| Incenter | Orthocenter | Centroid | Circumcenter |
|---|---|---|---|
| angle bisectors | altitudes | medians | perpendicular bisectors |

## Lesson Check • Do you UNDERSTAND?

**Reasoning The orthocenter of $\triangle ABC$ lies at vertex $A$. What can you conclude about $\overline{BA}$ and $\overline{AC}$? Explain.**

**26.** Circle the type of triangle whose orthocenter is located at a vertex.

acute right obtuse

**27.** $\overline{BA}$ and $\overline{AC}$ are sides of $\angle$ A .

**28.** Write your conclusion about $\overline{BA}$ and $\overline{AC}$. Justify your reasoning.

$\overline{BA}$ is perpendicular to $\overline{AC}$. Explanations may vary. Sample: $\triangle ABC$ is a right triangle and vertex $A$ is a right angle.

## Math Success

**Check off the vocabulary words that you understand.**

- ☐ median of a triangle
- ☐ altitude of a triangle
- ☐ orthocenter of a triangle

**Rate how well you can *understand medians and altitudes.***

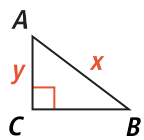
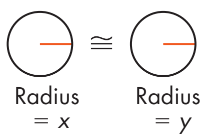
Need to review 0 2 4 6 8 10 Now I get it!

# 5-5 Indirect Proof

## Vocabulary

### Review

**Draw a line from each statement in Column A to one or more pictures that contradict it in Column B.**

| Column A | Column B |
|---|---|
| **1.** $x < y$ |  |
| **2.** $x = y$ |  |
| **3.** $x > y$ | Ordered pair (3, 3) |





### Vocabulary Builder

**indirect** (adjective) in duh REKT

**Definition:** **Indirect** means not direct in course or action, taking a roundabout route to get to a point or idea.

**Math Usage:** In **indirect** reasoning, all possibilities are considered and then all but one are proved false. The remaining possibility must be true.

### Use Your Vocabulary

**Write *indirect* or *indirectly* to complete each sentence.**

**4.** The _?_ way home from school takes a lot more time. indirect

**5.** By finding the negation of a statement false, you _?_ prove the statement true. indirectly

take note

## Key Concept Writing an Indirect Proof

**Step 1**
State as a temporary assumption the opposite (negation) of what you want to prove.

**Step 2**
Show that this temporary assumption leads to a contradiction.

**Step 3**
Conclude that the temporary assumption must be false and what you want to prove must be true.

## Problem 1 Writing the First Step of an Indirect Proof

**Got It?** **Suppose you want to write an indirect proof of the statement. As the first step of the proof, what would you assume?**

**$\triangle BOX$ is not acute.**

**6.** What do you want to prove?

$\triangle BOX$ is not acute.

**7.** What is the opposite of what you want to prove?

$\triangle BOX$ is acute.

**8.** The first step in the indirect proof is to write the following:

Assume temporarily that $\triangle BOX$ is _?_.

acute

**Got It?** **Suppose you want to write an indirect proof of the statement. As the first step of the proof, what would you assume?**

**At least one of the items costs more than $25.**

**9.** What do you want to prove?

At least one of the items costs more than $25.

**For Exercises 10–11, use $<$, $>$, $\leq$, $\geq$, or $=$ to complete each statement. Let $n =$ the cost of at least one of the items.**

**10.** What do you want to prove?

$n$ $>$ 25

**11.** What is the opposite of what you want to prove?

$n$ $\leq$ 25

**12.** The first step in the indirect proof is to write the following:

Assume temporarily that at least one of the items costs _?_ $25.

at most

**Write the first step of the indirect proof of each statement.**

**13.** Prove: $AB = CD$

Assume temporarily that $AB \neq CD$.

**14.** Prove: The sun is shining.

Assume temporarily that the sun is not shining.

## Problem 2 Identifying Contradictions

**Got It?** **Which two statements contradict each other?**

**I.** $\triangle XYZ$ **is acute.** **II.** $\triangle XYZ$ **is scalene** **III.** $\triangle XYZ$ **is equiangular**

**15.** Use the words in the box at the right to complete the flow chart below.

| |
|---|
| 90 |
| congruent |
| equal |
| unequal |

**I.** $\triangle XYZ$ **is acute.** → All angles measure less than 90 .

**II.** $\triangle XYZ$ **is scalene.** → All sides have _?_ measure. unequal

**III.** $\triangle XYZ$ **is equiangular.** → All angles are _?_ . congruent → All sides have _?_ measure. equal

**16.** In the first row of the flow chart above, circle the two statements that contradict one another.

## Problem 3 Writing an Indirect Proof

**Got It?** **Given:** $7(x + y) = 70$ **and** $x \neq 4$.

**Prove:** $y \neq 6$

**17.** Give the reason for each statement of the proof. **Answers may vary. Samples are given.**

| Statements | Reasons |
|---|---|
| 1) Assume $y = 6$. | 1) Assume the opposite of what you want to prove. |
| 2) $7(x + y) = 70$ | 2) Given |
| 3) $7(x + 6) = 70$ | 3) Substitute 6 for y. |
| 4) $7x + 42 = 70$ | 4) The Distributive Property. |
| 5) $7x = 28$ | 5) Subtract 42 from each side. |
| 6) $x = 4$ | 6) Divide each side by 7. |
| 7) $x \neq 4$ | 7) Given |
| 8) $y \neq 6$ | 8) Statements (6) and (7) contradict each other. Reaching a contradiction means the assumption was wrong. |

## Lesson Check • Do you know HOW?

**Suppose you want to write an indirect proof of the following statement. As the first step of the proof, what would you assume?**

**Quadrilateral *ABCD* has four right angles.**

**18.** Place a ✓ if the statement is the correct assumption to make as the first step in the indirect proof. Place an ✗ if it is not.

✗ Quadrilateral *ABCD* is a rectangle.

✗ Quadrilateral *ABCD* has four non-right angles.

✓ Quadrilateral *ABCD* does *not* have four right angles.

## Lesson Check • Do you UNDERSTAND?

**Error Analysis** **A classmate began an indirect proof as shown at the right. Explain and correct your classmate's error.**

~~Given: $\triangle ABC$~~
~~Prove: $\angle A$ is obtuse.~~
~~Assume temporarily that $\angle A$ is acute.~~

**19.** Complete the flow chart.

Statement: $\angle A$ is obtuse. → Negation: $\angle A$ is _?_ obtuse. not

$\angle A$ is acute. OR $\angle A$ is right.

**20.** Underline the correct words to complete the sentence.

The indirect proof has an incorrect conclusion / <u>assumption</u> because the opposite of "$\angle A$ is obtuse" is "$\angle A$ is acute / <u>not obtuse</u> / right."

## Math Success

**Check off the vocabulary words that you understand.**

☐ indirect reasoning ☐ indirect proof ☐ contradiction

**Rate how well you can *use indirect reasoning*.**

Need to review 0 2 4 6 8 10 Now I get it!

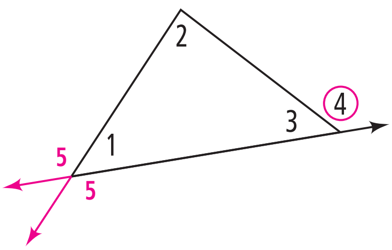
# 5-6 Inequalities in One Triangle

## Vocabulary

### Review

1. Circle the labeled *exterior angle*.
2. Write the *Exterior Angle* Theorem as it relates to the diagram.

   $m\angle$ 4 $= m\angle$ 1 $+ m\angle$ 2
3. Draw an *exterior angle* adjacent to $\angle 1$ and label it $\angle 5$.




**Circle the statement that represents an *inequality* in each pair below.**

4. $x \neq 32$

   $x = 32$

5. The number of votes is equal to 10,000.

   The number of votes is greater than 10,000.

**Complete each statement with an inequality symbol.**

6. $y$ is less than or equal to $z$.

   $y \leq z$

7. The temperature $t$ is at least 80 degrees.

   $t \geq 80°$

### Vocabulary Builder

There are more letters in the word *comparison* than in the word *compare*.

**compare** (verb) kum PEHR

**Other Word Form:** comparison (noun)

**Definition:** To **compare** is to examine two or more items, noting similarities and differences.

**Math Usage:** Use inequalities to **compare** amounts.

### Use Your Vocabulary

8. Complete each statement with the appropriate form of the word *compare*.

| | | |
|---|---|---|
| NOUN | By _?_, a spider has more legs than a beetle. | comparison |
| VERB | You can _?_ products before deciding which to buy. | compare |
| VERB | To _?_ quantities, you can write an equation or an inequality. | compare |

take note

## Property Comparison Property of Inequality

If $a = b + c$ and $c > 0$, then $a > b$.

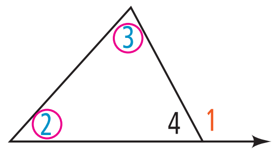
**9.** Circle the group of values that satisfies the Comparison Property of Inequality.

$a = 5, b = 5$, and $c = 0$ $a = 5, b = 2$, and $c = 3$ $a = 8, b = 6$, and $c = 1$

take note

## Corollary Corollary to the Triangle Exterior Angle Theorem

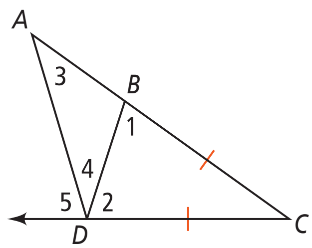
The measure of an exterior angle of a triangle is greater than the measure of each of its remote interior angles.




**10.** Circle the angles whose measures are always less than the measure of $\angle 1$.

## Problem 1 Applying the Corollary

**Got It?** **Use the figure at the right. Why is $m\angle 5 > m\angle C$?**




**Write the justification for each statement.**

**11.** $\angle 5$ is an exterior angle of $\triangle ADC$.

Definition of an exterior angle

**12.** $m\angle 5 > m\angle C$

Corollary to the Triangle Exterior Angle Theorem

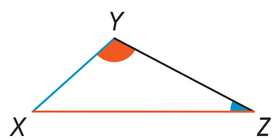
You can use the Corollary to the Triangle Exterior Angle Theorem to prove the following theorem.

take note

## Theorem 5-10 and Theorem 5-11

**Theorem 5-10**

If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.




If $XZ > XY$, then $m\angle Y > m\angle Z$.

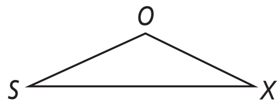
**13.** Theorem 5-11 is related to Theorem 5-10. Write the text of Theorem 5-11 by exchanging the words "larger angle" and "longer side."

**Theorem 5-11** If two sides of a triangle are not congruent, then

the longer side lies opposite the larger angle.

## Problem 3 Using Theorem 5-11

**Got It? Reasoning In the figure at the right, $m\angle S = 24$ and $m\angle O = 130$. Which side of $\triangle SOX$ is the shortest side? Explain your reasoning.**




**14.** By the Triangle Angle-Sum Theorem, $m\angle S + m\angle O + m\angle X = 180$, so $m\angle X =$ 180 $- m\angle S - m\angle O$.

**15.** Use the given angle measures and the equation you wrote in Exercise 14 to find $m\angle X$.

$m\angle X =$ 180 $-$ 24 $-$ 130 $=$ 26

**16.** Complete the table below.

| angle | $\angle O$ | $\angle X$ | $\angle S$ |
|---|---|---|---|
| angle measure | 130 | 26 | 24 |
| opposite side | $\overline{SX}$ | $\overline{SO}$ | $\overline{OX}$ |

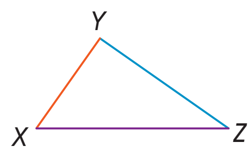
**17.** Which is the shortest side? Explain.

The shortest side is $\overline{OX}$ because it is opposite the smallest angle, $\angle$ $S$.

take note

### Theorem 5-12 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.




**18.** Complete each inequality.

$XY + YZ >$ $XZ$ $\qquad$ $YZ + ZX >$ $YX$ $\qquad$ $ZX + XY >$ $ZY$

## Problem 4 Using the Triangle Inequality Theorem

**Got It? Can a triangle have sides with lengths 2 m, 6 m, and 9 m? Explain.**

**19.** Complete the reasoning model below.

| Think | Write |
|---|---|
| The sum of the lengths of any two sides must be greater than the length of the third side. I need to write three sums and three inequalities. | $2 + 6 = 8$ $\quad$ $6 + 9 = 15$ $\quad$ $2 + 9 = 11$ <br> $8 < 9$ $\quad$ $15 > 2$ $\quad$ $11 > 6$ |
| One of those sums is greater / <u>not greater</u> than the length of the third side. | It is / <u>is not</u> possible for a triangle to have sides with lengths 2 m, 6 m, and 9 m. |

## Problem 5 Finding Possible Side Lengths

**Got It?** **A triangle has side lengths of 4 in. and 7 in. What is the range of possible lengths for the third side?**

**20.** Let $x$ = the length of the third side. Use the Triangle Inequality Theorem to write and solve three inequalities.

| | | |
|---|---|---|
| $x + 4 > 7$ | $x + 7 > 4$ | $7 + 4 > x$ |
| $x > 3$ | $x > -3$ | $11 > x$ |

**21.** Underline the correct word to complete each sentence.

Length is always / sometimes / never positive.

The first / second / third inequality pair is invalid in this situation.

**22.** Write the remaining inequalities as the compound inequality $3 < x < 11$.

**23.** The third side must be longer than 3 in. and shorter than 11 in.

## Lesson Check • Do you UNDERSTAND?

**Error Analysis** **A friend tells you that she drew a triangle with perimeter 16 and one side of length 8. How do you know she made an error in her drawing?**

**24.** If one side length is 8 and the perimeter is 16, then the sum of the lengths of the two remaining sides must be $16 - 8 = 8$.

**25.** Underline the correct words or number to complete each sentence.

By the Triangle Inequality Theorem, the sum of the lengths of two sides of a triangle must be equal to / greater than / less than the length of the third side.

By the Triangle Inequality Theorem, the sum of the lengths of the two unknown sides must be equal to / greater than / less than the length 8 / 16.

But 8 is *not* equal to / greater than 8, so there must be an error in the drawing.

## Math Success

**Check off the vocabulary words that you understand.**

☐ exterior angle ☐ comparison property of inequality

**Rate how well you can *use the Triangle Inequality Theorem.***

Need to review 0 2 4 6 8 10 Now I get it!

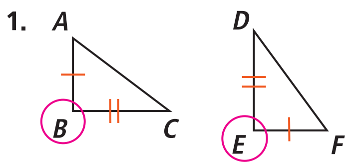
# 5-7 Inequalities in Two Triangles

## Vocabulary

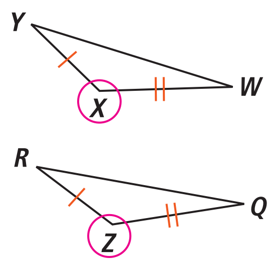
### Review

**Circle the *included angles* in each diagram.**

**1.**




**2.**




**In Exercises 3–5, cross out the group of values that does not satisfy the *Comparison Property of Inequality*.**

**3.** $\cancel{a = 3, b = 3, c = 0}$

$a = 6, b = 4, c = 2$

**4.** $a = 11, b = 3, c = 8$

$\cancel{a = 1, b = 2, c = 3}$

**5.** $a = 8, b = 3, c = 5$

$\cancel{a = 8, b = 5, c = 4}$

**Write a number so that each group satisfies the *Comparison Property of Inequality*.**

**6.** $a =$ **2** $, b = 0, c = 2$

**7.** $a = 9, b =$ **8** $, c = 1$

**8.** $a = 3, b =$ **1** $, c = 2$

### Vocabulary Builder

**hinge** (noun, verb) hinj

**Definition (noun):** A **hinge** is a device on which something else depends or turns.

**Definition (verb):** To **hinge** upon means to depend on.

### Use Your Vocabulary

**Circle the correct form of the word *hinge*.**

**9.** Everything *hinges* on his decision. Noun / Verb

**10.** The *hinge* on a gate allows it to swing open or closed. Noun / Verb

**11.** Your plan *hinges* on your teacher's approval. Noun / Verb

**12.** The lid was attached to the jewelry box by two *hinges*. Noun / Verb

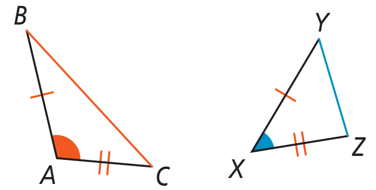
take note

## Theorems 5-13 and 5-14 The Hinge Theorem and its Converse

| If . . . | and . . . | then . . . | |
|---|---|---|---|
| two sides of one triangle are congruent to two sides of another triangle | the included angles are not congruent, | the longer third side is opposite the larger included angle. | Theorem 5-13 Hinge Theorem (SAS Inequality) |
| two sides of one triangle are congruent to two sides of another triangle | the third sides are not congruent, | the larger included angle is opposite the longer third side. | Theorem 5-14 Converse of the Hinge Theorem (SSS Inequality) |

**13.** Use the triangles at the right to complete the table.

| Theorem | If... | Then... |
|---|---|---|
| 5-13: Hinge Theorem | $m\angle A > m\angle X$ | $BC$ **>** $YZ$ |
| 5-14: Converse of the Hinge Theorem | $BC >$ **$YZ$** | $m\angle A >$ **$m\angle X$** |



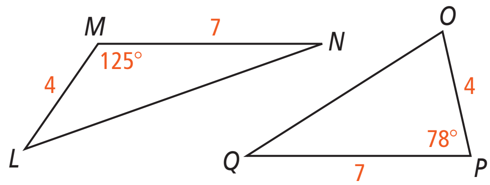

**14.** Explain why Theorems 5–13 and 5–14 are also called the SAS and SSS Inequality Theorems. **Answers may vary. Sample:**

**Theorem 5-13 compares two pairs of sides and the pair of included angles (SAS).**

**Theorem 5-14 compares three pairs of sides (SSS).**

## Problem 1 Using the Hinge Theorem

**Got It?** **What inequality relates *LN* and *OQ* in the figure at the right?**




**15.** Use information in the diagram to complete each statement.

The included angle in $\triangle LMN$ is $\angle$ **M**.

The included angle in $\triangle OPQ$ is $\angle$ **P**.

**16.** Circle the side opposite the included angle in $\triangle LMN$. Underline the side opposite the included angle in $\triangle OPQ$.

$\overline{LM}$ $\qquad$ $\overline{LN}$ $\qquad$ $\overline{MN}$

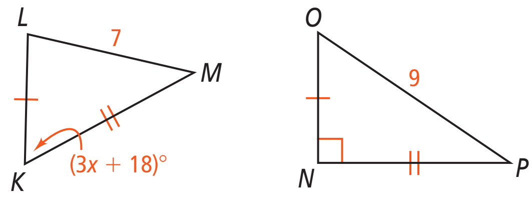
$\overline{QO}$ $\qquad$ $\overline{QP}$ $\qquad$ $\overline{OP}$

**17.** Use the Hinge Theorem to complete the statement below

$m\angle$ **M** $> m\angle$ **P**, so $LN >$ **OQ**.

## Problem 3 Using the Converse of the Hinge Theorem

**Got It?** **What is the range of possible values for $x$ in the figure at the right?**




**18.** From the diagram you know that the triangles have two pairs of congruent corresponding sides, that $LM <$ **OP**, and that $m\angle N =$ **90**.

**Complete the steps and justifications to find upper and lower limits on $x$.**

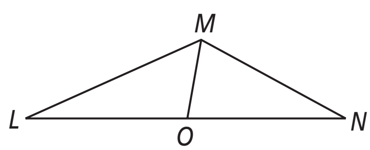
| | | |
|---|---|---|
| **19.** | $m\angle K <$ **$m\angle N$** | Converse of the Hinge Theorem |
| | $3x + 18 <$ **90** | Substitute. |
| | $3x <$ **72** | Subtract **18** from each side. |
| | $x <$ **24** | Divide each side by **3**. |
| **20.** | $m\angle K >$ **0** | The measure of an angle of a triangle is greater than 0. |
| | $3x + 18 >$ **0** | Substitute. |
| | $3x >$ **−18** | Subtract **18** from each side. |
| | $x >$ **−6** | Divide each side by **3**. |

**21.** Write the two inequalities as the compound inequality **−6** $< x <$ **24**.

## Problem 4 Proving Relationships in Triangles

**Got It?** **Given:** $m\angle MON = 80$; $O$ is the midpoint of $\overline{LN}$.

**Prove:** $LM > MN$

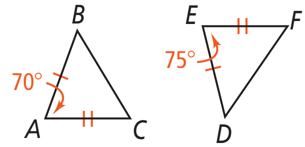



**22.** Write a justification for each statement.

| Statements | Reasons |
|---|---|
| 1) $m\angle MON = 80$ | 1) **Given** |
| 2) $m\angle MON + m\angle MOL = 180$ | 2) **Supplementary Angles** |
| 3) $80 + m\angle MOL = 180$ | 3) **Substitute 80 for $m\angle MON$.** |
| 4) $m\angle MOL = 100$ | 4) **Subtract 80 from each side.** |
| 5) $\overline{LO} \cong \overline{ON}$ | 5) **$O$ is the midpoint of $\overline{LN}$.** |
| 6) $\overline{MO} \cong \overline{MO}$ | 6) **Reflexive Property of Congruence** |
| 7) $m\angle MOL > m\angle MON$ | 7) **100 > 80** |
| 8) $LM > MN$ | 8) **Hinge Theorem** |

## Lesson Check • Do you know HOW?

**Write an inequality relating *FD* and *BC*.**




**In Exercises 23–26, circle the correct statement in each pair.**

**23.** $\overline{AC} \cong \overline{EF}$ $AC > EF$

**24.** $AB > ED$ $\overline{AB} \cong \overline{ED}$

**25.** $m\angle BAC > m\angle FED$ $m\angle BAC < m\angle FED$

**26.** By the Hinge Theorem, you can relate *FD* and *BC*.

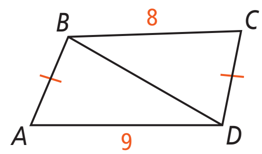
By the Converse of Hinge Theorem, you can relate *FD* and *BC*.

**27.** Write an inequality relating *FD* and *BC*.

$FD$ **>** $BC$

## Lesson Check • Do you UNDERSTAND?

**Error Analysis From the figure at the right, your friend concludes that $m\angle BAD > m\angle BCD$. How would you correct your friend's mistake?**




**Write T for *true* or F for *false*.**

**T** **28.** $AB = CD$

**F** **29.** $AD = CB$

**T** **30.** $BD = BD$

**31.** Your friend should compare **AD** and **CB**.

**32.** The longer of the two sides your friend should compare is **AD**.

**33.** How would you correct your friend's mistake? Explain.

**Answers may vary. Sample: $AD > CB$, so use the Converse of the Hinge Theorem to conclude that $m\angle ABD > m\angle CDB$.**

## Math Success

**Check off the vocabulary words that you understand.**

☐ exterior angle ☐ comparison property of inequality ☐ Hinge Theorem

**Rate how well you can *use triangle inequalities.***

Need to review 0 2 4 6 8 10 Now I get it!