

## 5-1

Rate of Change  
and Slope

## Vocabulary

## ● Review

1. Circle the *rate* that matches this situation: Ron reads 5 books every 2 weeks.

$\frac{5 \text{ weeks}}{2 \text{ books}}$

$\frac{2 \text{ books}}{5 \text{ weeks}}$

$\frac{5 \text{ books}}{2 \text{ weeks}}$

2. Write *always*, *sometimes*, or *never*.

A *rate* is ? a ratio.

always

A ratio is ? a *rate*.

sometimes

3. Underline the correct word to complete each sentence.

A *rate* compares two quantities by division / multiplication.

A *rate* compares quantities in different / the same unit(s).

## ● Vocabulary Builder

**slope** (noun) slohp

**Definition:** **Slope** is the ratio of the vertical change (or rise) to the horizontal change (or run) between two points on a line. **Slope** is also called the rate of change.

**Main Idea:** **Slope** describes the steepness of a line in the coordinate plane.

**Examples:** You can measure the **slope** of a hill, mountain, road, or roof.

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}$$

## ● Use Your Vocabulary

4. How does the *slope* of a road affect a person's driving?

Answers may vary. Sample: A person would drive slower on a road that has a steep slope.

5. What kind of ski *slope* would a beginner skier use?

Answers may vary. Sample: A beginner skier would use a slope that is not very steep.



## Problem 1 Finding Rate of Change Using a Table

**Got It?** The table at the right shows the distance a band marches over time. The rate of change from one row of the table to the next is 260 feet per minute. Do you get the rate of change of 260 feet per minute if you use nonconsecutive rows of the table? Explain.

Distance Marched

Time (min)	Distance (ft)
1	260
2	520
3	780
4	1040

6. Use the values from the second and fourth rows to find the rate of change.

$$\begin{aligned} \text{rate of change} &= \frac{\text{change in distance}}{\text{change in time}} \\ &= \frac{1040 - 520}{4 - 2} \\ &= \frac{520}{2} \\ &= \frac{260}{1} \end{aligned}$$

When you use nonconsecutive rows, the rate of change is 260 ft per min.

7. Is the rate of change you found in Exercise 6 the same as if you had used two consecutive rows? Explain why or why not.

Yes. Answers will vary. Sample: The rate of change is the same

because the band marches at a constant speed.



## Problem 2 Finding Slope Using a Graph

**Got It?** What is the slope of the line?

8. Label each point on the graph with its coordinates.

9. Draw a vertical arrow to represent the rise.

$$\text{rise} = 2$$

10. Draw a horizontal arrow to represent the run.

$$\text{run} = 5$$

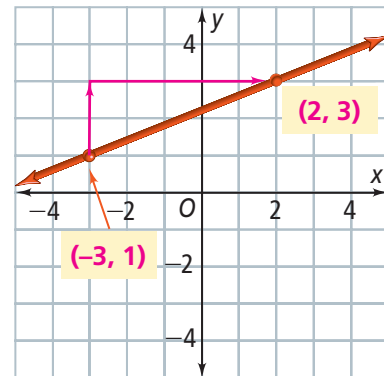
11. Underline the correct word to complete the sentence.

Because the points are on the same line, the rate of change from point to point

is constant / differs.

12. Write the slope of the line.

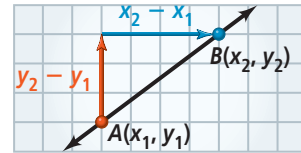
$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = \frac{2}{5}$$



### Key Concept The Slope Formula

In the diagram,  $(x_1, y_1)$  are the coordinates of point  $A$ , and  $(x_2, y_2)$  are the coordinates of point  $B$ . To find the slope of  $\overleftrightarrow{AB}$ , you can use the *slope formula*.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_2 - x_1 \neq 0$$



When using the *slope formula*, the  $x$ -coordinate you use first in the denominator must belong to the same ordered pair as the  $y$ -coordinate you use first in the numerator.

13. To find the change in  $x$ - or  $y$ -coordinates, do you add or subtract?

**You subtract to find the change in the coordinates.**

14. What number will you get in the denominator if the  $x$ -coordinates are the same? Explain how that will affect the answer you find for the slope.

**Zero. Sample: Division by 0 is undefined. The slope will be undefined.**



### Problem 3 Finding Slope Using Points

**Got It?** What is the slope of the line through  $(1, 3)$  and  $(4, -1)$ ?

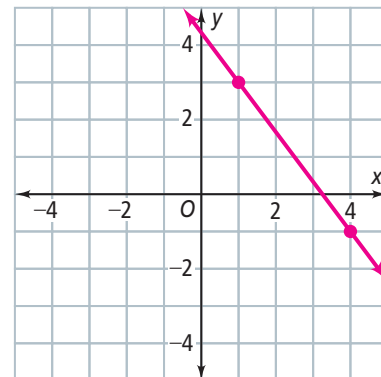
15. You can use either pair for  $(x_2, y_2)$  and complete the equation.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 3}{4 - 1} = \frac{-4}{3}$$

16. **Reasoning** Plot the points and draw a line through them. Does the slope of the line look as you expected it to? Explain.

**Explanations may vary. Sample: Yes. The line goes**

**down from left to right because the slope is negative.**



### Problem 4 Finding Slopes of Horizontal and Vertical Lines

**Got It?** What is the slope of the line through  $(4, -3)$  and  $(4, 2)$ ?

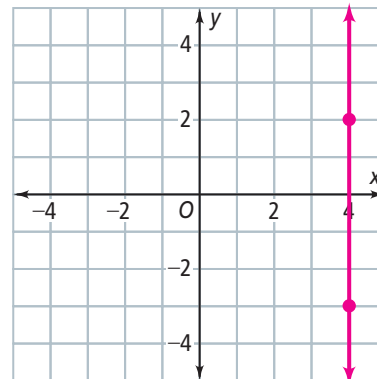
17. Graph the points  $(4, -3)$  and  $(4, 2)$  and draw the line that goes through the points.

18. Is the line that you drew *horizontal* or *vertical*?

**vertical**

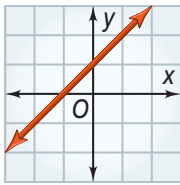
19. What is the slope of the line through  $(4, -3)$  and  $(4, 2)$ ?

**The slope of the line is undefined.**

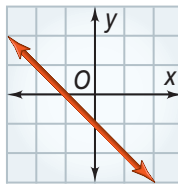


### Concept Summary Slopes of Lines

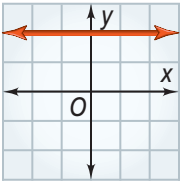
20. Label each graph with one of the descriptions in the box at the right.



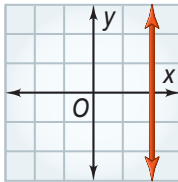
positive slope



negative slope



slope of 0



undefined slope

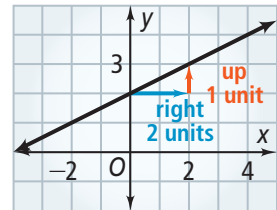
negative slope  
positive slope  
slope of 0  
undefined slope



### Lesson Check • Do you UNDERSTAND?

**Error Analysis** A student calculated the slope of the line at the right to be 2. Explain the mistake. What is the correct slope?

21. The **rise** of the graphed line is **1**.
22. The **run** of the graphed line is **2**.
23. What mistake did the student make by calculating the slope to be 2? Explain how to find the correct slope.



Answers may vary. Sample: To find the slope, the student found the ratio of  $\frac{\text{run}}{\text{rise}}$  instead of  $\frac{\text{rise}}{\text{run}}$ . The correct slope is  $\frac{1}{2}$ .



### Math Success

Check off the vocabulary words that you understand.

- rate of change                       slope

Rate how well you can *find the slope of a line*.



## 5-2

## Direct Variation



## Vocabulary

## ● Review

1. Cross out the expression below that does NOT show a formula for *slope*.

~~horizontal change~~  
~~vertical change~~

$$\frac{y_2 - y_1}{x_2 - x_1}$$

rise  
run

2. Underline the correct word in each sentence about *slope*.

The *slope* of a horizontal line is undefined / zero.

The *slope* of a vertical line is undefined / zero.

## ● Vocabulary Builder

**direct** (adjective) duh REKT

**Definition:** Direct means straightforward in language or action.

**Other Word Forms:** directly (adverb), direction(s) (noun)

**Math Usage:** If the ratio of two variables is constant, then the variables form a **direct** variation.

**What It Means:** In a **direct** variation, one variable *directly* affects another by multiplying it by a constant value.

**Both variables increase:** The more expensive the car, the more sales tax you pay.

**One variable increases, the other variable decreases:** As a candle burns longer, its height gets smaller.

$y = kx$ , where  $k \neq 0$ , is a **direct variation**.

In the above,  $k$  is called the *constant of variation*.

## ● Use Your Vocabulary

Choose the correct word from the list to complete each sentence.

directly

direct

directions

3. Renee gave the visitor ? to the museum.

**directions**

4. The fans went ? to their seats.

**directly**

5. There is a ? connection between the outside temperature and the number of people at the beach.

**direct**

A function in the form  $y = kx$ , where  $k \neq 0$ , represents a **direct variation**. The **constant of variation**  $k$  is the coefficient of  $x$ .

To determine whether an equation represents a direct variation, solve it for  $y$ . If you can write the equation in the form  $y = kx$ , where  $k \neq 0$ , it represents a direct variation.



### Problem 1 Identifying a Direct Variation

**Got It?** Does  $4x + 5y = 0$  represent a direct variation? If so, find the constant of variation.

6. Circle the equation that shows direct variation.

$$y = \frac{k}{x}$$

$$y = kx$$

$$yx = k$$

7. Complete the steps to solve  $4x + 5y = 0$  for  $y$ .

$$4x + 5y = 0$$

Write the original equation.

$$5y = 0 - 4x$$

Subtract  $4x$  from each side.

$$y = -\frac{4}{5}x$$

Divide each side by  $5$ .

8. Does  $4x + 5y = 0$  represent a direct variation? Explain. **Answers may vary.**

**Yes. Sample: The equation  $4x + 5y = 0$  represents a direct variation. It**

**can be represented by a function in the form  $y = kx$ , where  $k = -\frac{4}{5}$ .**

9. In the equation  $4x + 5y = 0$ ,  $-\frac{4}{5}$  is the constant of variation.



### Problem 2 Writing a Direct Variation Equation

**Got It?** Suppose  $y$  varies directly with  $x$ , and  $y = 10$  when  $x = -2$ . What direct variation equation relates  $x$  and  $y$ ? What is the value of  $y$  when  $x = -15$ ?

10. Complete the reasoning model below.

Think	Write
I start with the function form of direct variation.	$y = k \cdot x$
Then I substitute 10 for $y$ and $-2$ for $x$ .	$10 = k \cdot (-2)$
Now I divide each side by $-2$ to solve for $k$ .	$-5 = k$
Next, I write an equation by substituting $-5$ for $k$ .	$y = -5 \cdot x$
Finally, I determine the value of $y$ when $x = -15$ .	$y = -5 \cdot -15 = 75$



### Problem 3 Graphing a Direct Variation

**Got It?** Weight on the moon  $y$  varies directly with weight on Earth  $x$ . A person who weighs 100 lb on Earth weighs 16.6 lb on the moon. What is an equation that relates weight on Earth  $x$  and weight on the moon  $y$ ? What is the graph of this equation?

11. Find the value of  $k$ . Round  $k$  to the nearest hundredth if necessary.

$$y = kx$$

$$16.6 = k \cdot 100$$

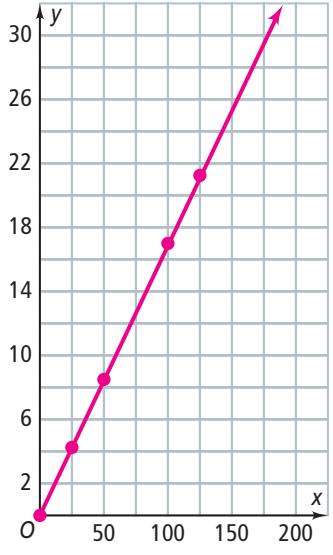
$$0.166 = k$$

12. To the nearest hundredth,  $k = 0.17$ . So,  $y \approx 0.17 \cdot x$ .

13. Make a table of values.

$x$	$y = 0.17x$
0	$y = 0.17 \cdot 0 = 0$
25	$y = 0.17 \cdot 25 = 4.25$
50	$y = 0.17 \cdot 50 = 8.5$
100	$y = 0.17 \cdot 100 = 17$
125	$y = 0.17 \cdot 125 = 21.25$

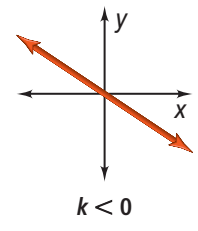
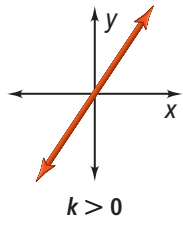
14. Graph the values from the table.



### Concept Summary Graphs of Direct Variations

The graph of a direct variation equation  $y = kx$  is a line with the following properties.

- The line passes through  $(0, 0)$ .
- The slope of the line is  $k$ .



15. Substitute  $x = 0$  and  $y = 0$  in the equation  $-2x + y = 3$ .

$$-2x + y = 3$$

$$-2 \cdot 0 + 0 = 3$$

$$0 + 0 = 3$$

$$0 \neq 3$$

16. Because the graph of  $-2x + y = 3$  passes / does not pass through  $(0, 0)$ , the equation is / is not a direct variation.



### Problem 4 Writing a Direct Variation From a Table

x	y
-3	2.25
1	-0.75
4	-3

**Got It?** For the data in the table at the right, does  $y$  vary directly with  $x$ ? If it does, write an equation for the direct variation.

18. Write each ordered pair as the ratio of the  $y$ -coordinate to the  $x$ -coordinate. Then write the ratio of  $y$  to  $x$  as a decimal.

$(-3, 2.25)$

$$\frac{2.25}{-3} = -0.75$$

$(1, -0.75)$

$$\frac{-0.75}{1} = -0.75$$

$(4, -3)$

$$\frac{-3}{4} = -0.75$$

19. For the data in the table, does  $y$  vary directly with  $x$ ?

Yes/No

20. The equation for the direct variation shown is  $y = -0.75 \cdot x$ .



### Lesson Check • Do you UNDERSTAND?

**Vocabulary** Determine whether each statement is *always*, *sometimes*, or *never* true.

The ordered pair  $(0, 0)$  is a solution of the direct variation equation  $y = kx$ .

21. Substitute  $(0, 0)$  into  $y = kx$ .

$$0 \stackrel{?}{=} k \cdot 0$$

22. The statement is ? true.

always

You can write a direct variation in the form  $y = k + x$ , where  $k \neq 0$ .

23. Is  $y = k + x$  of the form  $y = kx$ ?

Yes/No

24. The statement is ? true.

never

The constant of variation for a direct variation represented by  $y = kx$  is  $\frac{y}{x}$ .

25. When you divide each side of  $y = kx$  by  $x$ , you obtain  $k = \frac{y}{x}$ .

26. Because you cannot divide by 0, the statement is ? true.

sometimes



### Math Success

Check off the vocabulary words that you understand.

direct variation

constant of variation for a direct variation

Rate how well you can *work with direct variation*.

Need to review

0 2 4 6 8 10

Now I get it!



## 5-3

## Slope-Intercept Form



## Vocabulary

## ● Review

1. **Multiple Choice** Which equation is NOT a *linear* equation?

(A)  $y = -3x + 4$

(B)  $y = x$

(C)  $y = \frac{x}{5} - 7$

(D)  $y = 5^x$

2. Place a ✓ in the box if the statement applies to the graph of a *linear* equation.  
Place an ✗ if it does NOT apply to the graph of a *linear* equation.

✗ The graph of a linear equation is always a horizontal line.

✓ The graph of a linear equation is always a straight line.

✗ The graph of a linear equation may be shaped like a “U.”

## ● Vocabulary Builder

**intercept** (noun) IN tur sept

**Other Word Forms:** intercepted (verb), interception (noun)

**Definition:** An **intercept** is a point where someone or something is stopped along its way from one place to another.

**Main Idea:** You can find the **intercept(s)** of a graph by finding the point(s) where the graph crosses a coordinate axis.

**Related Words:** *x*-intercept; *y*-intercept

A **y-intercept** is the *y*-coordinate of a point where a graph crosses the *y*-axis.

## ● Use Your Vocabulary

Choose the correct word from the list to complete each sentence.

intercept

intercepted

interception

3. During a football game, the home team's quarterback threw an ?.

interception

4. The *y*-coordinate of a point where a graph crosses the *y*-axis is a *y*-?.

intercept

5. The teacher ? the message Charlie was passing to his friend.

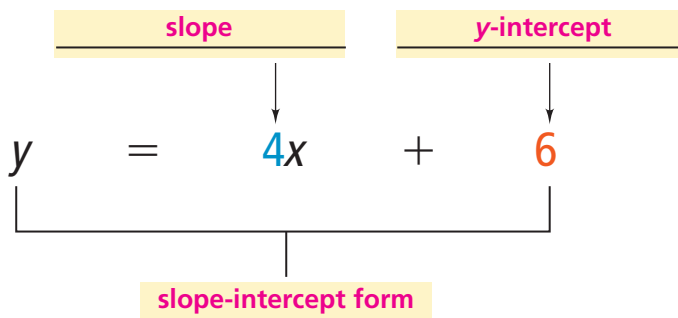
intercepted

### Key Concept Slope-Intercept Form of a Linear Equation

The **slope-intercept** form of a linear equation of a nonvertical line is  $y = mx + b$ .

The slope of the line is  $m$ . The  $y$ -intercept is  $b$ .

6. Use the words *slope*, *y-intercept*, and *slope-intercept form* to complete the diagram at the right.



### Problem 2 Writing an Equation in Slope-Intercept Form

**Got It?** What is an equation of the line with slope  $\frac{3}{2}$  and  $y$ -intercept  $-1$ ?

7. Write the numbers  $\frac{3}{2}$  and  $-1$  in the correct boxes below.

$$y = m \cdot x + b$$

$$y = \frac{3}{2} \cdot x + -1$$

8. An equation of the line in slope-intercept form is  $y = \frac{3}{2}x - 1$ .

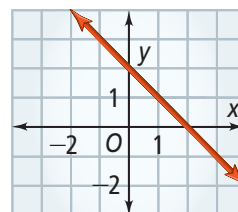


### Problem 3 Writing an Equation From a Graph

**Got It?** What is an equation of the line shown at the right?

9. What do you expect the slope of the line to be from looking at the graph? Explain.

Explanations may vary. Sample: I expect the slope to be a negative number because it is decreasing from left to right.



10. Choose two points and find the slope of the line.  $\frac{\text{slope}}{\text{slope}} = \frac{0 - 2}{2 - 0} = \frac{-2}{2} = -1$
11. The slope of the line is  $-1$ .
12. Use the graph to find the  $y$ -intercept. The  $y$ -intercept is  $2$ .

13. Write the slope-intercept form of the equation.

$$y = -x + 2$$

14. **Reasoning** Does the equation of the line depend on the points you use to find the slope? Explain.

Explanations may vary. Sample: No; you can choose any two points on a line to find the slope.



### Problem 4 Writing an Equation From Two Points

**Got It?** What equation in slope-intercept form represents the line that passes through the points  $(3, -2)$  and  $(1, -3)$ ?

15. Use the points  $(3, -2)$  and  $(1, -3)$  to find the slope of the line.

$$m = \frac{-2 - (-3)}{3 - 1} = \frac{1}{2}$$

16. Next, find the  $y$ -intercept. Substitute the slope for  $m$  and the coordinates of one of the points for  $x$  and  $y$ . Then solve for  $b$ .

$$y = m \cdot x + b$$

17. Write the equation of the line in slope-intercept form. Substitute the slope for  $m$  and the  $y$ -intercept for  $b$ .

$$y = \frac{1}{2} \cdot x + -3\frac{1}{2}$$

$$\begin{aligned} \text{Sample: } -3 &= \frac{1}{2} \cdot 1 + b \\ -3 &= \frac{1}{2} + b \\ -3\frac{1}{2} &= b \end{aligned}$$



### Problem 5 Graphing a Linear Equation

**Got It?** What is the graph of  $y = -3x + 4$ ?

18. The ordered pair for the  $y$ -intercept, 4, is  $(0, 4)$ .

19. Explain how you will use the slope to find another point on the line.

Answers may vary. Sample: I will use the rise and the run to plot another point.

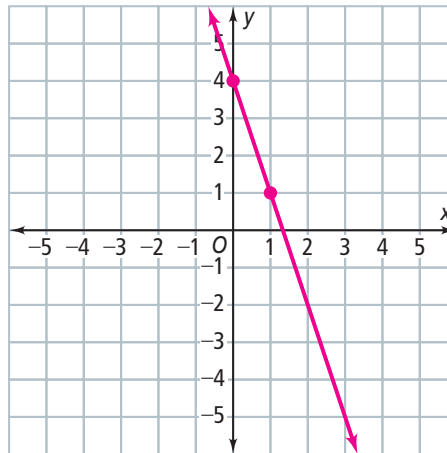
20. Use the slope,  $-3$ , to find another point on the line.

Answers may vary. Sample: I will start at the  $y$ -intercept, or the point  $(0, 4)$ . The rise is  $-3$ . The run is 1.

$$\begin{aligned} 4 + -3 &= 1 \\ 0 + 1 &= 1 \end{aligned}$$

Another point is  $(1, 1)$ .

21. Use the points you found in Exercises 18 and 20. What is the graph of  $y = -3x + 4$ ?



### Problem 6 Modeling a Function

**Got It?** A plumber charges a \$65 fee for a repair plus \$35 per hour. Write an equation to model the total cost  $y$  of a repair that takes  $x$  hours. What graph models the total cost?

22. Let  $x$  = the number of hours the plumber works. Let  $y$  = the total cost of a repair.

When  $x = 0$ ,  $y = 65$ . So the  $y$ -intercept is 65.

23. The slope is the amount of change each hour.

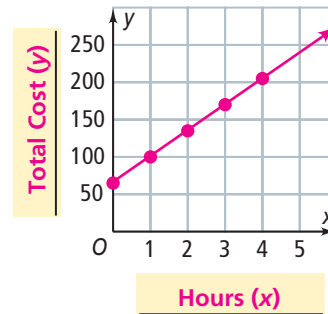
So, the slope is **35**.

24. Write an equation to model the cost of a repair.

$$y = 35x + 65$$

25. Complete the table for your equation.

Hours ( $x$ )	0	1	2	3	4
Total Cost ( $y$ )	65	100	135	170	205



26. Graph the data from the table to model the total cost. Be sure to label the axes.



### Lesson Check • Do you UNDERSTAND?

**Vocabulary** Is  $y = 5$  a linear equation? Explain.

27. Does  $y = 5$  have a slope? Explain.

**Yes. Explanations may vary. Sample: It has a slope of 0.**

28. Find three points that satisfy  $y = 5$ . **Answers may vary. Sample  $x$ -values are given.**

(**-1**, **5**)

(**0**, **5**)

(**3**, **5**)

29. Is  $y = 5$  a linear equation? Explain. **Explanations may vary.**

**Yes. Sample: The points of a linear equation all lie on a line. Since  $y = 5$  has a slope of 0, it is a horizontal line. Therefore, it is a linear equation.**



### Math Success

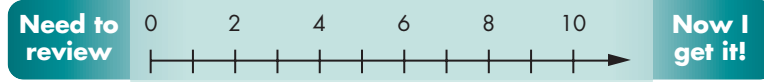
Check off the vocabulary words that you understand.

linear function

$y$ -intercept

slope-intercept form

Rate how well you can *find the slope-intercept form of a linear equation*.



# 5-4

## Point-Slope Form



### Vocabulary

#### Review

1. Circle the equation that has a *y*-intercept of 3.

$y = 3x + 4$

$y = 4x - 3$

$y = 5x + 3$

$y = -3x + 2$

2. Circle the equation that is in *slope-intercept* form.

$2x - y = 10$

$x + 3y + 11 = 0$

$y - 4 = \frac{2}{3}(x + 7)$

$y = 2x + 6$

3. Circle the statement that is true about the *y*-intercept of any graph.

occurs where  $y = 0$   
on the graph

occurs where  $x = 0$   
on the graph

occurs where graph  
touches the  $x$ -axis

#### Vocabulary Builder

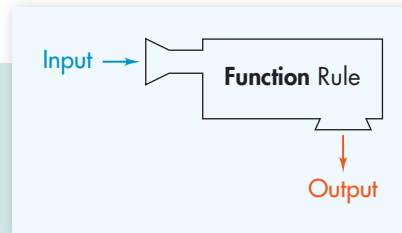
**function** (noun) FUNGK shun

**Related Words:** input, output, function rule

**Definition:** A **function** is a relationship that assigns exactly one output value to each input value.

**Main Idea:** A **function** is used to describe how one value depends on another.

**Example:** The function machine above shows that the **function** assigns an **output** to every **input** according to a specified rule.



#### Use Your Vocabulary

Complete each sentence with the appropriate word from the list.

price

sun

time

4. The length of a shadow is a *function* of the angle of the   ?.

sun

5. The amount of water that has leaked from a leaky faucet is a *function* of   ?.

time

6. The amount of sales tax you pay is a *function* of the item's   ?.

price

### Key Concept Point-Slope Form of a Linear Equation

The **point-slope form** of an equation of a nonvertical line with slope  $m$  and through point  $(x_1, y_1)$  is  $y - y_1 = m(x - x_1)$ .

7. In the above, what does  $(x_1, y_1)$  represent?

a point on the line

8. What does  $m$  represent?

the slope on the line



### Problem 1 Writing an Equation in Point-Slope Form

**Got It?** A line passes through  $(8, -4)$  and has slope  $\frac{2}{3}$ . What is an equation in point-slope form of the line?

9. Use the point-slope form of an equation. For a line that passes through  $(8, -4)$  and has slope  $\frac{2}{3}$ , circle  $x_1$  and underline  $y_1$ .

$$y - \underline{-4} = \frac{2}{3}(x - \textcircled{8})$$

10. Now substitute into point-slope form.

$$y - y_1 = m \cdot (x - x_1)$$

$$y - \underline{-4} = \frac{2}{3} \cdot (x - \underline{8})$$

11. An equation of the line is  $y + 4 = \frac{2}{3}(x - 8)$ .



### Problem 2 Graphing Using Point-Slope Form

**Got It?** What is the graph of the equation  $y + 7 = -\frac{4}{5}(x - 4)$ ?

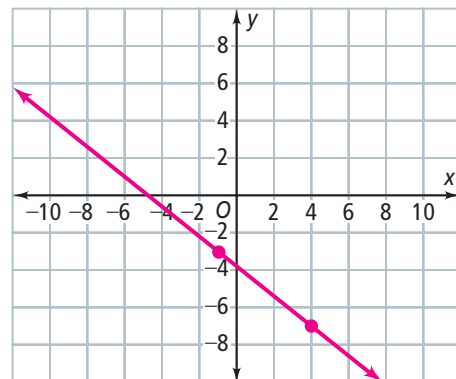
12. Circle the ordered pair of a point on the graph of  $y + 7 = -\frac{4}{5}(x - 4)$ .

$$(7, 4) \quad (4, 7) \quad (-4, -7) \quad \textcircled{(4, -7)}$$

13. Circle the correct description of the slope.

Go up 4 units and left 5 units    Go down 4 units and left 5 units    Go up 4 units and right 5 units

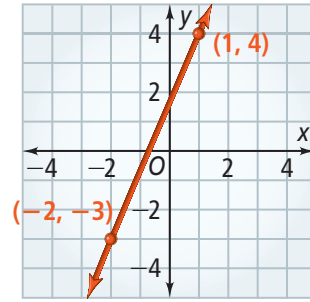
14. Use your answers to Exercises 12 and 13 to graph the line.





### Problem 3 Using Two Points to Write an Equation

**Got It?** Use the point  $(-2, -3)$  to write an equation of the line shown.



15. Follow the steps to write the equation of the line shown.

1

Find the slope of the line. Use two points and the

slope formula,  $m = \frac{y_2 - y_1}{x_2 - x_1}$ .

$$\frac{-3 - 4}{-2 - 1} = \frac{-7}{-3} = \frac{7}{3}$$

Use the slope and the point  $(-2, -3)$  to write an equation of the line in point-slope form.

2

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = \frac{7}{3} \cdot (x - (-2))$$

An equation of the line is  $y + 3 = \frac{7}{3}(x + 2)$ .



### Problem 4 Using a Table to Write an Equation

**Got It?** The table shows the number of gallons of water  $y$  in a tank after  $x$  hours. The relationship is linear. What is an equation in point-slope form that models the data? What does the slope represent?

Volume of Water in Tank

Time, $x$ (h)	Water, $y$ (gal)
2	3320
3	4570
5	7070
8	10,820

16. Complete the reasoning model below.

Think	Write
I can use any two points from the table to find the slope. <b>Sample points are used.</b>	$m = \frac{4570 - 3320}{3 - 2} = \frac{1250}{1}$
Then I can substitute one point and the slope into the point-slope equation. <b>Sample point is used.</b>	$y - 7070 = 1250 \cdot (x - 5)$
Finally, I can tell what the slope represents.	The slope represents a rate of <u>?</u> . <b>gallons of water added per hour</b>

**Got It? Reasoning** Write the equation from Exercise 16 in slope-intercept form. What does the  $y$ -intercept represent?

17. Write the equation in point-slope form from Exercise 16. Use it to write the equation in slope-intercept form.

$$y - 7070 = 1250(x - 5)$$

$$y - 7070 = 1250x - 6250$$

$$y = 1250x + 820$$

18. What does the  $y$ -intercept in your answer to Exercise 17 represent?

Answers may vary. Sample: The  $y$ -intercept represents the number of gallons of water there were in the tank at the beginning of measuring.



### Lesson Check • Do you UNDERSTAND?

**Reasoning** Can any equation in point-slope form also be written in slope-intercept form? Give an example to explain.

19. Use point-slope form,  $y - y_1 = m(x - x_1)$ , and any point and slope to write an equation in point-slope form.

$$y - 3 = 2(x - 1)$$

20. Now write your equation in slope-intercept form.

$$y - 3 = 2(x - 1) \qquad y = 2x - 2 + 3$$

$$y = 2(x - 1) + 3 \qquad y = 2x + 1$$

Answers may vary. Students should use the equation  $y - y_1 = m(x - x_1)$  and any value  $m$ ,  $y_1$ , and  $x_1$ . A sample is given.

Answers may vary depending on equation written in Exercise 19.

21. Can any equation in point-slope form also be written in slope-intercept form?

Yes/No

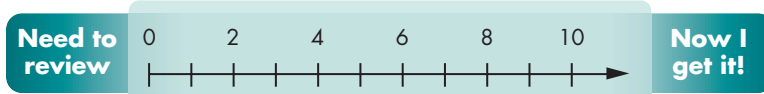


### Math Success

Check off the vocabulary words that you understand.

point-slope form       equation       graph

Rate how well you can *write equations in point-slope form*.





## 5-5

## Standard Form

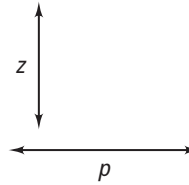


## Vocabulary

## ● Review

Underline the correct word to complete each sentence.

- Line  $z$  is a horizontal / vertical line.
- Line  $p$  is a horizontal / vertical line.
- A line with a slope of 0 is horizontal / vertical.
- A line with an undefined slope is horizontal / vertical.



## ● Vocabulary Builder

**standard** (adjective) STAN durd

**Other Word Forms:** standards (plural noun), standardized (adjective)

**Main Idea:** Something that is **standard** is well known and widely used.

**Example:** The **standard** measure of weight used in the U.S. is the pound.

**Math Usage:** The **standard** form of a linear equation is  $Ax + By = C$ , where  $A$ ,  $B$ , and  $C$  are real numbers, and  $A$  and  $B$  are not both zero.

**Opposites:** different, irregular

## ● Use Your Vocabulary

Underline the correct word(s) to complete each sentence.

- In gymnastics, judges use a set of standards / standardized to award a score.
- Most English words have a standard / standardized pronunciation.
- Many states use standard / standardized tests to assess their students' performance.
- Multiple Choice** Which linear equation is in *standard* form?

(A)  $y = -6x + 4$

(C)  $3x - 7y = 42$

(B)  $y = -7x - 3$

(D)  $y - 6 = 2(x + 7)$



## Problem 1 Finding $x$ - and $y$ -Intercepts

**Got It?** What are the  $x$ - and  $y$ -intercepts of the graph of  $5x - 6y = 60$ ?

Complete each sentence.

9. To find the  $x$ -intercept, let  $y = 0$ .

10. To find the  $y$ -intercept, let  $x = 0$ .

11. Find the  $x$ -intercept.

$$5x - 6 \cdot 0 = 60$$

$$5x - 0 = 60$$

$$5x = 60$$

$$\frac{5x}{5} = \frac{60}{5}$$

$$x = 12$$

12. Find the  $y$ -intercept.

$$5 \cdot 0 - 6y = 60$$

$$0 - 6y = 60$$

$$-6y = 60$$

$$\frac{-6y}{-6} = \frac{60}{-6}$$

$$y = -10$$

**Got It?** What are the  $x$ - and  $y$ -intercepts of the graph of  $3x + 8y = 12$ ?

13. Find the  $x$ -intercept.

$$3x + 8 \cdot 0 = 12$$

$$3x + 0 = 12$$

$$3x = 12$$

$$x = 4$$

14. Find the  $y$ -intercept.

$$3 \cdot 0 + 8y = 12$$

$$0 + 8y = 12$$

$$8y = 12$$

$$y = \frac{3}{2}$$



## Problem 2 Graphing a Line Using Intercepts

**Got It?** What is the graph of  $2x + 5y = 20$ ?

15. Circle the  $x$ -intercept of  $2x + 5y = 20$ .

$x = 1$

$x = 10$

$x = 20$

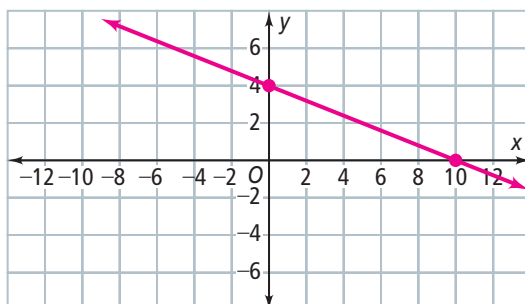
16. Circle the  $y$ -intercept of  $2x + 5y = 20$ .

$y = -5$

$y = -4$

$y = 4$

17. Use the intercepts to graph the line  $2x + 5y = 20$ .

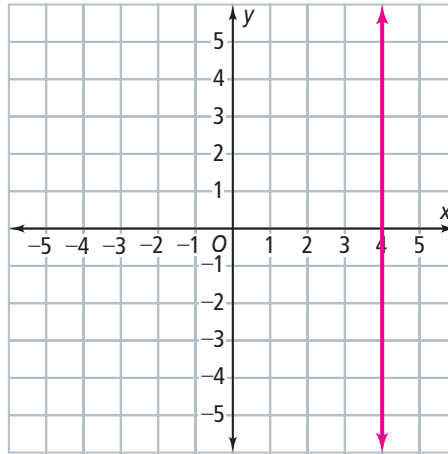




### Problem 3 Graphing Horizontal and Vertical Lines

**Got It?** What is the graph of the equation  $x = 4$ ?

18. The equation  $x = 4$  means that for all values of  $y$ , the value of  $x$  is **4**.
19. For the reason given above, the graph of  $x = 4$  is a **horizontal / vertical** line.
20. Graph the equation  $x = 4$ .



### Problem 4 Transforming to Standard Form

**Got It?** Write  $y - 2 = -\frac{1}{3}(x + 6)$  in standard form using integers.

21. Circle the first step to put  $y - 2 = -\frac{1}{3}(x + 6)$  in standard form.

Solve for  $y$ .      **Multiply both sides by  $-3$ .**      Add  $x$  to both sides.

22. Now find the standard form of the equation using integers. **Answers may vary. Sample:**

$$\begin{aligned}
 y - 2 &= -\frac{1}{3}(x + 6) \\
 -3 \cdot (y - 2) &= -3 \cdot \left(-\frac{1}{3}\right)(x + 6) \\
 -3y + 6 &= (x + 6) \\
 x + 3y &= 0
 \end{aligned}$$

23. The standard form of the equation is **1**  $\cdot x +$  **3**  $\cdot y = 0$ .



### Problem 5 Using Standard Form as a Model

**Got It?** A media download store sells songs for \$1 each and movies for \$15 each. You have \$60 to spend. Write and graph an equation that describes the numbers of songs and movies you can purchase for \$60.

24. You cannot buy a fraction of a song or movie. Describe how you will use the graph of the equation to find solutions that make sense. **Answers may vary.**

**Sample: I will use only points with integer coordinates for solutions.**

25. Use the model to help you complete the equation.

Relate  $\text{cost of a song} \cdot \text{number of songs} + \text{cost of a movie} \cdot \text{number of movies}$  is \$60

Define Let  $x$  = the number of songs purchased.

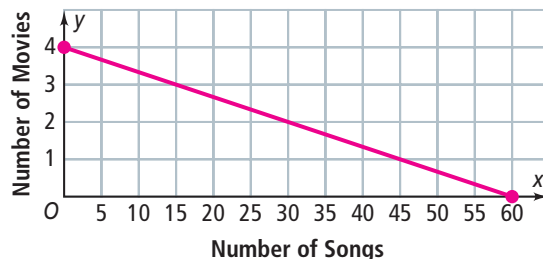
Let  $y$  = the number of movies purchased.

Write  $1 \cdot x + 15 \cdot y = 60$

26. Find the intercepts of the equation.

Solutions may vary. Sample given.  
 $1x + 15(0) = 60$        $1(0) + 15y = 60$   
 $x = 60$                        $15y = 60$   
 $y = 4$

27. Use the intercepts to graph the equation.



### Lesson Check • Do you UNDERSTAND?

**Vocabulary** Tell whether each linear equation is in *slope-intercept form*, *point-slope form*, or *standard form*.

$$y + 5 = -(x - 2)$$

$$y = -2x + 5$$

$$y - 10 = -2(x - 1)$$

$$2x + 4y = 12$$

28. Draw a line from each equation in Column A to the form of the equation in Column B.

Column A

Column B

$$y + 5 = -(x - 2)$$

$$y = mx + b \text{ (Slope-Intercept Form)}$$

$$y = -2x + 5$$

$$y - y_1 = m(x - x_1) \text{ (Point-Slope Form)}$$

$$y - 10 = -2(x - 1)$$

$$Ax + By = C \text{ (Standard Form)}$$

$$2x + 4y = 12$$



### Math Success

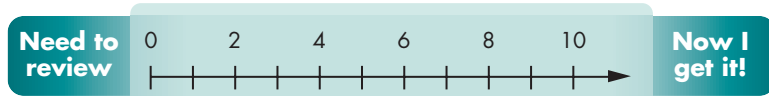
Check off the vocabulary words that you understand.

linear equation

x-intercept

standard form

Rate how well you can graph a linear equation using intercepts.



# 5-6

## Parallel and Perpendicular Lines



### Vocabulary

#### Review

1. Circle the product of a number and its *reciprocal*.

100

1

0

-1

2. Circle the pairs of numbers that are *reciprocals*.

7 and  $\frac{1}{7}$

1 and -1

0 and  $\frac{0}{12}$

$\frac{3}{4}$  and  $-\frac{4}{3}$

#### Vocabulary Builder

**parallel** (adjective) PA ruh lel

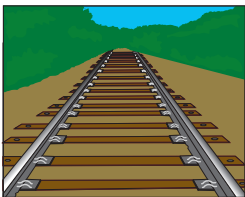
**Related Word:** perpendicular (adjective)

**Math Usage:** Lines that are **parallel** are lines in the same plane that never intersect.

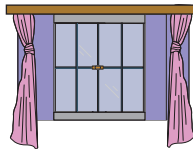
**Using symbols:**  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$  means line *AB* is parallel to line *CD*.

**Example:** The stripes on the American flag are **parallel**.

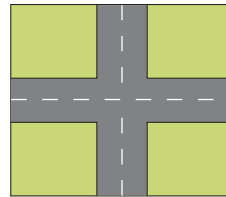
#### Use Your Vocabulary



Picture A



Picture B



Picture C

Complete each sentence with *parallel* or *perpendicular*.

3. The railroad tracks in Picture A are ?.

parallel

4. The window bars in Picture B that do NOT meet are ?.

parallel

5. The roads in Picture C are ?.

perpendicular

Take note

### Key Concept Slopes of Parallel Lines

Nonvertical lines are parallel if they have the same slope and different  $y$ -intercepts. Vertical lines are parallel if they have different  $x$ -intercepts.

6. Draw a line from each equation in Column A to an equation whose graph is parallel in Column B.

Column A

$$y = 2x + 4$$

$$y = \frac{1}{3}x - 2$$

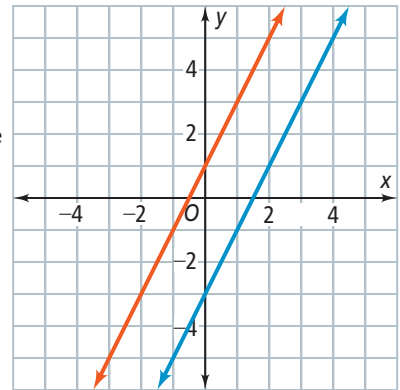
$$x = 3$$

Column B

$$x = 2$$

$$y = 2x - 4$$

$$y = \frac{1}{3}x - 1$$



### Problem 1 Writing an Equation of a Parallel Line

**Got It?** A line passes through  $(-3, -1)$  and is parallel to the graph of  $y = 2x + 3$ . What equation represents the line in slope-intercept form?

7. The slope of the graph of  $y = 2x + 3$  is **2**.
8. The slope of any line parallel to the graph of  $y = 2x + 3$  is **2**.
9. Use point-slope form to find an equation of the line that passes through  $(-3, -1)$  and uses the slope from Exercise 8.

$$y - y_1 = m(x - x_1)$$

$$\begin{aligned} y - (-1) &= 2(x - (-3)) \\ y + 1 &= 2(x + 3) \\ y + 1 &= 2x + 6 \\ y &= 2x + 5 \end{aligned}$$

Take note

### Key Concept Slopes of Perpendicular Lines

Two nonvertical lines are perpendicular if the product of their slopes is  $-1$ . Two numbers whose product is  $-1$  are called **opposite reciprocals**. A vertical line and a horizontal line are also perpendicular.

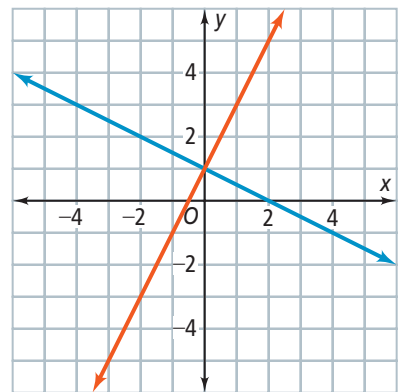
10. **Multiple Choice** The slope of a line is 2. What is the slope of a line perpendicular to that line?

(A) 2

(C)  $-2$

(B)  $\frac{1}{2}$

(D)  $-\frac{1}{2}$





## Problem 2 Classifying Lines

**Got It?** Are the graphs of the equations  $y = \frac{3}{4}x + 7$  and  $4x - 3y = 9$  *parallel*, *perpendicular*, or *neither*? Explain.

11. Write the equation  $4x - 3y = 9$  in slope-intercept form. **Answers may vary. Sample given.**

$$\begin{aligned}
 4x - 3y &= 9 \\
 -3y &= -4x + 9 \\
 \frac{-3}{-3}y &= \frac{-4x + 9}{-3} \\
 y &= \frac{4}{3}x - 3
 \end{aligned}$$

12. The slope of the line  $y = \frac{3}{4}x + 7$  is  $\frac{3}{4}$ .      13. The slope of the line  $4x - 3y = 9$  is  $\frac{4}{3}$ .
14. Are the lines *parallel*, *perpendicular*, or *neither*? Explain.

The lines are neither parallel nor perpendicular. Explanations may vary. Sample: Their slopes are reciprocals, but do not have opposite signs, so the lines are not perpendicular. Their slopes are not the same, so the lines are not parallel.



## Problem 3 Writing an Equation of a Perpendicular Line

**Got It?** A line passes through  $(1, 8)$  and is perpendicular to the graph of  $y = 2x + 1$ . What equation represents the line in slope-intercept form?

15. Find the slope of the perpendicular line.

Slope of the given line =  $2$        $-\frac{1}{2}$  = opposite reciprocal (slope of perpendicular line)

$$2 \times -\frac{1}{2} = -1$$

16. The slope of the perpendicular line is  $-\frac{1}{2}$ .
17. Use point-slope form and the point  $(1, 8)$  to write an equation of the perpendicular line.

$$y - y_1 = m \cdot (x - x_1)$$

$$\begin{aligned}
 y - 8 &= -\frac{1}{2} \cdot (x - 1) \\
 y - 8 &= -\frac{1}{2} \cdot x + \frac{1}{2} \\
 y &= -\frac{1}{2} \cdot x + \frac{17}{2}
 \end{aligned}$$



## Problem 4 Solving a Real-World Problem

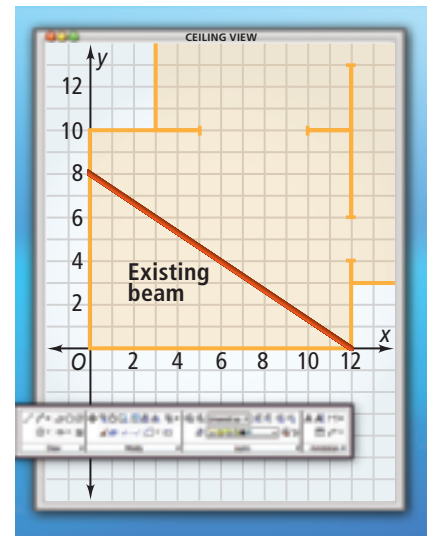
**Got It?** An architect uses software to design a ceiling. The architect needs to enter an equation that represents a new beam. The new beam will be parallel to the existing beam shown in the graph. The new beam will pass through the corner at (0, 10). What is an equation in slope-intercept form that represents the new beam?

18. Use the slope formula to find the slope of the beam shown in the graph.

$$m = \frac{4 - 6}{6 - 3} = -\frac{2}{3}$$

19. In order for the new beam to be parallel to the existing beam, their slopes should be the same / opposite reciprocals.
20. Now find the equation of the line that will be parallel to the existing line and will pass through (0, 10).

$$y = -\frac{2}{3}x + 10$$



## Lesson Check • Do you UNDERSTAND?

**Compare and Contrast** How is determining if two lines are parallel similar to determining if they are perpendicular? How are the processes different?

Answers may vary.  
Samples are given.

21. To determine if two lines are parallel, what do you need to do?

Find the slope of both lines and see whether they are the same.

22. To determine if two lines are perpendicular, what do you need to do?

Find the slope of both lines and see whether they are opposite reciprocals.

23. How are the processes similar?

For each process, you need to find the slope of both lines.

24. How are the processes different?

For each process, the requirement for the slopes is different.



## Math Success

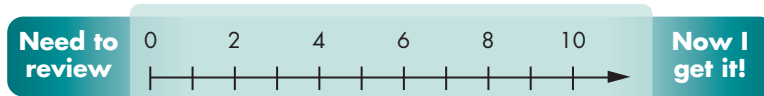
Check off the vocabulary words that you understand.

parallel lines

perpendicular lines

opposite reciprocals

Rate how well you can *write equations of parallel and perpendicular lines*.





# 5-7

## Scatter Plots and Trend Lines

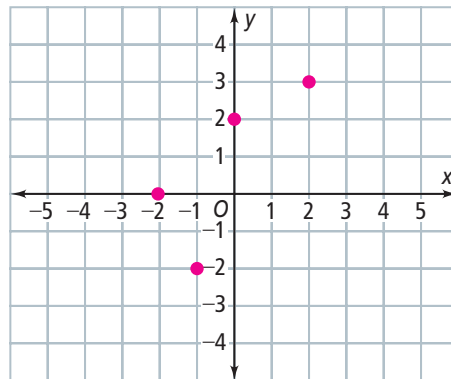


### Vocabulary

#### Review

A *scatter plot* is a graph that relates two sets of data. Plot each *ordered pair* on the graph at the right to make a scatter plot.

1. (2, 3)
2. (-1, -2)
3. (0, 2)
4. (-2, 0)



#### Vocabulary Builder

**correlation** (noun) kawr uh LAY shun

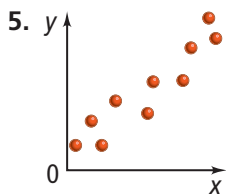
**Related Words:** correlate (verb), relationship (noun), relate (verb), scatter plot (noun)

**Definition:** A **correlation** is a measure of the strength of a relationship between two quantities.

**Example:** The more a student studies, the higher the student's grades tend to be. So, there is a **correlation** between time spent studying and grades.

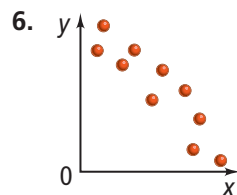
#### Use Your Vocabulary

Label each scatter plot *positive correlation*, *negative correlation*, or *no correlation*.



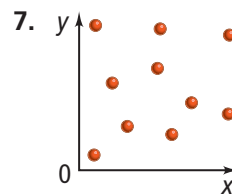
y increases as x increases

**positive correlation**



y decreases as x increases

**negative correlation**



x and y are not related

**no correlation**



## Problem 1 Making a Scatter Plot and Describing Its Correlation

**Got It?** Make a scatter plot of the data in the table. What type of relationship does the scatter plot show?

Gasoline Purchases								
Dollars Spent	10	11	9	10	13	5	8	4
Gallons Bought	2.5	2.8	2.3	2.6	3.3	1.3	2.2	1.1

8. Let  $x$  = dollars spent.

Let  $y$  = gallons bought.

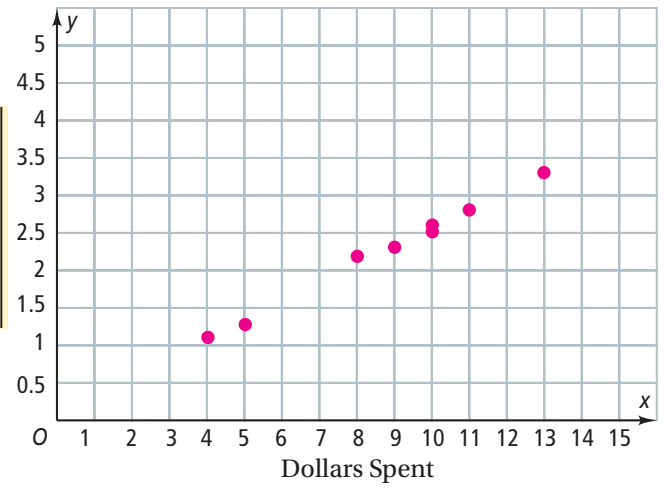
9. Use the data to make a scatter plot.

10. Underline the correct word or words to complete each sentence.

The number of gallons bought tends to increase / decrease as the number of dollars spent increases / decreases.

The two sets of data have a positive / negative correlation.

Gallons Bought



**Got It? Reasoning** Consider the population of a city and the number of letters in the name of the city. Would you expect a *positive correlation*, a *negative correlation*, or *no correlation* between the two sets of data? Explain your reasoning.

11. As an example, think of the city or town that you live in. How many letters are in the name of your city and approximately how many people live there?

Answers vary depending on each student's hometown.

12. Now think of another city of a very different size than the one you chose for Exercise 11. How many letters are in the name of this city and approximately how many people live there?

Answers vary depending on each student's hometown.

13. Is the size of either city dependent on the number of letters in its name? Yes / No

14. What kind of correlation would you expect between the two sets of data? Explain.

No correlation. Explanations may vary. Sample: The size of a city is not necessarily related to the number of letters in the name of that city. So a graph of these two quantities would be very scattered, which means there is no correlation.

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A **trend line** is a line on a scatter plot, drawn near the points, that shows a correlation. There should be about the same number of points above the line as below it.

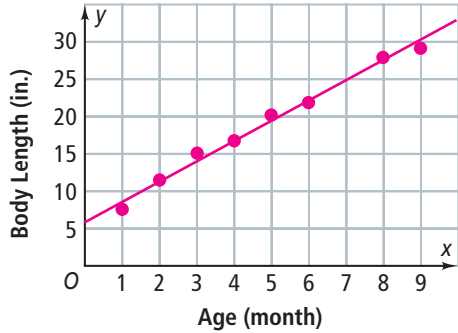


## Problem 2 Writing an Equation of a Trend Line

**Got It?** Make a scatter plot of the data. Draw a trend line and write its equation. What is the approximate body length of a 7-month-old panda?

Body Length of a Panda								
Age (month)	1	2	3	4	5	6	8	9
Body Length (in.)	8.0	11.75	15.5	16.7	20.1	22.2	26.5	29.0

15. Make a scatter plot and draw a trend line.



16. Write the equation of the trend line that you drew. **Answers may vary.**

**Sample:** Two points on the trend line are (4, 16.7) and (8, 26.5).

$$m = \frac{26.5 - 16.7}{8 - 4} = \frac{9.8}{4} = 2.45$$

$$y - y_1 = m(x - x_1)$$

$$y - 26.5 = 2.45(x - 8)$$

$$y = 2.45x + 6.9$$

17. Use the equation of your trend line to estimate the body length of a 7-month-old panda.

**Sample:**

$$y = 2.45x + 6.9$$

$$y = 2.45(7) + 6.9$$

$$y = 24.05$$

18. A 7-month-old panda would be approximately **24** inches in length.



## Problem 3 Finding the Line of Best Fit

**Got It?** For data of tuition and fees charged at public four-year colleges, the equation of the line of best fit is  $y = 409.43x - 815,446.71$ , where  $x$  = the year at the beginning of the academic year and  $y$  = cost. Predict the cost of attending a public four-year college in the 2016–2017 academic year.

19. Let  $x =$  **2016**.

20. Complete the steps to find the estimated cost.

$$y = 409.43 \cdot \mathbf{2016} - 815,446.71$$

$$y = \mathbf{825,410.88} - 815,446.71$$

$$y \approx \mathbf{9964}$$

21. The cost of attending a public four-year college in the 2016–2017 academic year will be about \$ **9964**.

**Causation** is when a change in one quantity causes a change in a second quantity. A correlation between quantities does not always imply causation.



## Problem 4 Identifying Whether Relationships Are Causal

**Got It?** Consider the cost of a family's vacation and the size of their house. Is there likely to be a correlation? If so, does the correlation reflect a causal relationship? Explain.

22. Is there likely to be a correlation between the cost of a family's vacation and the size of their house? Explain. **Explanations may vary.**

**Yes. Families that spend more on vacations may also spend more on their houses.**

23. If there is a correlation, does the correlation reflect a causal relationship? Explain.

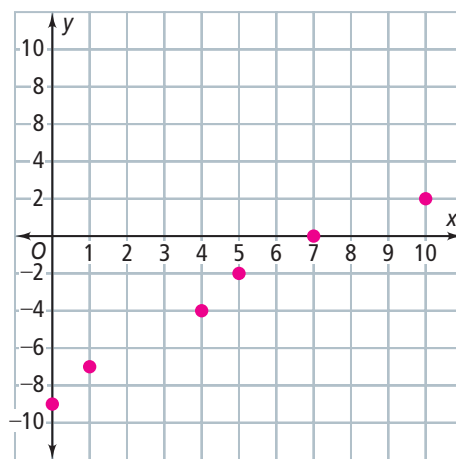
**No. Sample: Spending more on a vacation will not cause a house to increase in size.**



## Lesson Check • Do you UNDERSTAND?

**Error Analysis** Refer to the table below. A student says that the data have a negative correlation because as  $x$  decreases,  $y$  also decreases. What is the student's error?

$x$	10	7	5	4	1	0
$y$	1	0	-2	-4	-7	-9



24. Make a scatter plot of the data.
25. The scatter plot shows a positive / negative correlation.
26. Explain the student's error. **Answers may vary.**

**Sample: The student may have thought that when both  $x$  and  $y$  decrease, there is a negative correlation. This relationship shows a positive correlation.**



## Math Success

Check off the vocabulary words that you understand.

scatter plot       correlation       trend line       causation

Rate how well you can *make a scatter plot and determine the type of correlation*.

Need to review

0      2      4      6      8      10



Now I get it!

## 5-8

## Graphing Absolute Value Functions



## Vocabulary

## ● Review

Compare the *absolute values*. Write  $<$ ,  $>$ , or  $=$ .

1.  $|3|$   $=$   $|-3|$       2.  $|-3|$   $>$   $|-1|$       3.  $|9|$   $<$   $|-10|$       4.  $|-9|$   $>$   $|8|$

Write T for *true* or F for *false*.

- T** 5. The *absolute value* of a number is a measure of the distance from 0 to that number on the number line.
- F** 6. The *absolute value* of a number is always the opposite of that number.

## ● Vocabulary Builder

**translation** (noun) trans LAY shun

**Related Words:** translate or slide (verb)

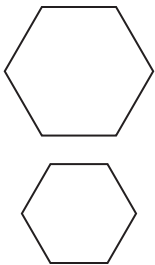
**Math Usage:** A **translation** (or a *slide*) is a shift of a graph (or shape) horizontally, vertically, or both. The graph (or shape) ends up being the same size and shape, but in a different place. This happens without rotation or reflection.

**Word Origin:** The word comes from the Latin word “translatum” or “transferre.” **Trans** means “across”; **ferre** means “to carry.”

## ● Use Your Vocabulary

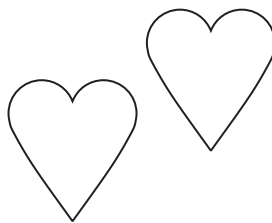
Identify each pair of figures as a *translation* or *NOT a translation*.

7.



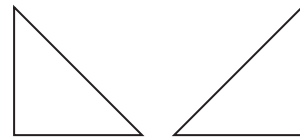
not a translation

8.



translation

9.

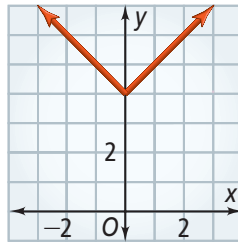
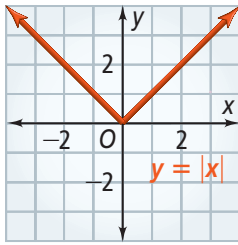


not a translation



## Problem 1 Describing Translations

**Got It?** An absolute value function has a V-shaped graph that opens up or down. Below is the graph of  $y = |x|$  and the graph of another absolute value function. How are the graphs related?



Underline the correct word to complete each sentence.

10. The two graphs have the same / different shape(s).
11. The second graph is translated up / down from  $y = |x|$ .
12. Complete the equation using the number of units the graph is translated.

$$y = |x| + \underline{4}$$

**Got It? Reasoning** What are the domain and range of each function above?

13. For  $y = |x|$ , are there any real numbers whose absolute value you could not find? Yes /  No
14. **Multiple Choice** What is the domain of  $y = |x|$ ?
- (A) all positive real numbers                      (C)  all real numbers
- (B) all positive integers                          (D) all integers
15. For  $y = |x|$ , will every  $y$ -value be positive or 0? Yes /  No
16. **Multiple Choice** What is the range of  $y = |x|$ ?
- (A)  all nonnegative real numbers                      (C) all real numbers
- (B) all nonnegative integers                      (D) all integers

Answer each question below about the function you wrote in Exercise 12.

17. The graph for this function was a vertical / horizontal translation of  $y = |x|$ .  
So the domain / range of this function will change.
18. What is the domain of this function? Explain your reasoning.

**All real numbers. Explanations may vary. Sample: The domain does not change because any real number can still be substituted for  $x$ .**

19. What is the range of this function? Explain your reasoning.

**All real numbers greater than or equal to 4. Explanations may vary.**

**Sample: The range of  $y = |x|$  is moved up 4 units.**



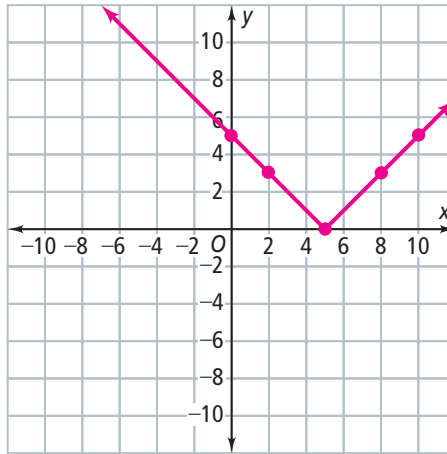
### Problem 3 Graphing a Horizontal Translation

**Got It?** What is the graph of  $y = |x - 5|$ ?

20. Complete the table of values.

$x$	$y =  x - 5 $
0	$y =  0 - 5  = 5$
2	$y =  2 - 5  = 3$
5	$y =  5 - 5  = 0$
8	$y =  8 - 5  = 3$
10	$y =  10 - 5  = 5$

21. Use the table of values to plot five points. Connect the points to graph the absolute value function.



### Problem 4 Graphing a Step Function

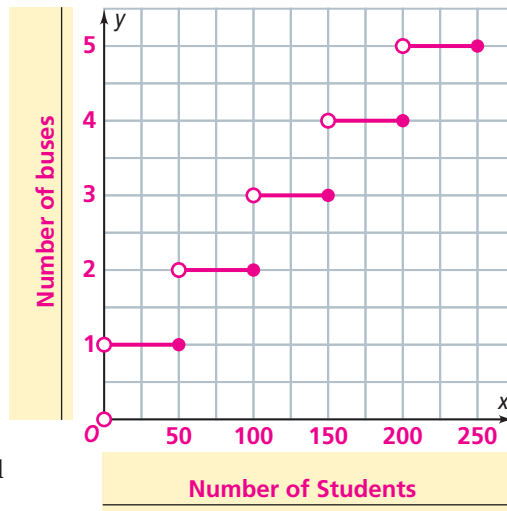
**Got It?** Make a graph that shows the relationship between the number of students  $x$  that go to the football game by bus and the number of buses  $y$  that are needed if each bus holds a maximum of 50 students.

**Think**

What numbers will work best for labeling the  $x$ -axis? Which for the  $y$ -axis?

**Write**

Label the axes accordingly.



22. What does the point  $(0, 1)$  mean in terms of students and buses? Will that point be open or closed?

If there are no students, then we do not need a bus; open circle.

23. What is the last  $x$ -value where there will only be one bus going to the game? What point is that on the graph? Will it be open or closed?

$x = 50$ , the point is  $(50, 1)$ , the point will be closed.

24. Will the point  $(50, 2)$  be open or closed? Explain.

Open because if there are 50 students you only need 1 bus,

so the point  $(50, 1)$  is closed.

25. Continue graphing until 236 students fit in the buses. How many buses are going to the game? **5 buses**



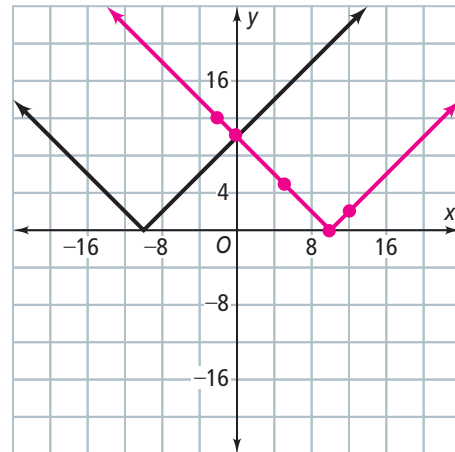
## Lesson Check • Do you UNDERSTAND?

**Error Analysis** A student is graphing the equation  $y = |x - 10|$  and translates the graph of  $y = |x|$  ten units left. Describe the student's error.

26. Complete the table to find values of the correct graph.

$x$	$y =  x - 10 $
0	$y =  0 - 10  = 10$
5	$y =  5 - 10  = 5$
10	$y =  10 - 10  = 0$
12	$y =  12 - 10  = 2$
-2	$y =  -2 - 10  = 12$

27. The graph below shows the student's incorrect translation. Now plot the points from the table and compare the graphs.



28. Describe the student's error. **Answers may vary. Sample:**

The student translated the graph 10 units to the left instead of 10 units to the right.

The student mistook the  $-10$  to mean "translate the graph to the left."



## Math Success

Check off the vocabulary words that you understand.

- absolute value function       translation

Rate how well you can graph *absolute value functions*.

