5.2.1 Adding Integers

Learning Objective(s)

1 Add two or more integers with the same sign.

2 Add two or more integers with different signs.

Introduction

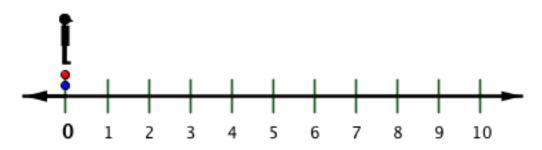
On an extremely cold day, the temperature may be -10. If the temperature rises 8 degrees, how will you find the new temperature? Knowing how to add integers is important here and in much of algebra.

Adding Integers with the Same Signs

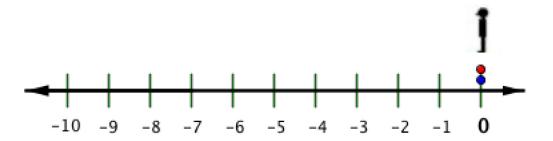
Objective 1

Since positive **integers** are the same as natural numbers, adding two positive integers is the same as adding two natural numbers.

To add integers on the number line, you move forward, and you face right (the positive direction) when you add a positive number.



As with positive numbers, to add negative integers on the number line, you move forward, but you face left (the negative direction) when you add a negative number.



In both cases, the total number of units moved is the total distance moved. Since the distance of a number from 0 is the absolute value of that number, then the absolute value of the sum of the integers is the sum of the absolute values of the **addends**.

When both numbers are negative, you move left in a negative direction, and the sum is negative. When both numbers are positive, you move right in a positive direction, and the sum is positive.

To add two numbers with the same sign (both positive or both negative):

• Add their absolute values and give the sum the same sign.

	Example		
Problem	Find -23 + (-16).		
	Both addends have the same sign (negative). So, add their absolute values: $ -23 = 23$ and $ -16 = 16$. The sum of those numbers is 23 + 16 = 39.		
	Since both addends are negative, the sum is negative.		
Answer	-23 + (-16) = -39		

With more than two addends that have the same sign, use the same process with all addends.

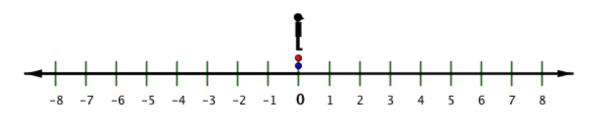
Example			
Problem	em Find −27 + (−138) + (−55).		
	All addends have the same sign (negative). So, add their absolute values: −27 = 27, −138 = 138, and −55 = 55.		
	The sum of those numbers is $27 + 138 + 55 = 220$.		
	Since all addends are negative, the sum is negative.		
Answer	-27 + (-138) + (-55) = -220		

Self Check A Find -32 + (-14).

Adding Integers with Different Signs

Consider what happens when the addends have different signs, like in the temperature problem in the introduction. If it's -10 degrees, and then the temperature rises 8 degrees, the new temperature is -10 + 8. How can you calculate the new temperature?

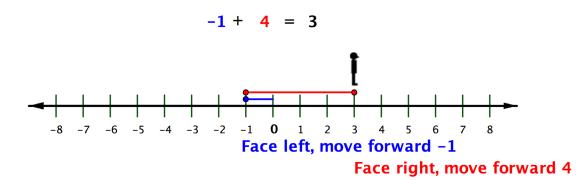
Using the number line below, you move forward to add, just as before. Face and move in a positive direction (right) to add a positive number, and move forward in a negative direction (left) to add a negative number.



See if you can find a rule for adding numbers without using the number line. Notice that when you add a positive integer and a negative integer, you move forward in the positive (right) direction to the first number, and then move forward in the negative (left) direction to add the negative integer.

Since the distances overlap, the absolute value of the sum is the difference of their distances. So to add a positive number and a negative number, you subtract their absolute values (their distances from 0.)

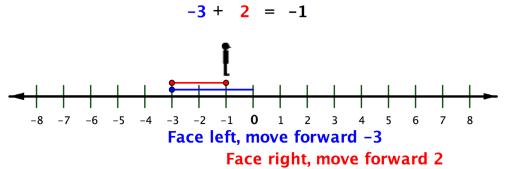
What is the sign of the sum? It's pretty easy to figure out. If you moved further to the right than you did to the left, you ended to the right of 0, and the answer is positive; and if you move further to the left, the answer is negative. Let's look at the illustration below and determine the sign of the sum.



If you didn't have the number line to refer to, you can find the sum of -1 + 4 by

- subtracting the distances from zero (the absolute values) 4 1 = 3 and then
- applying the sign of the one furthest from zero (the largest absolute value). In this case, 4 is further from 0 than -1, so the answer is positive: -1 + 4 = 3

Look at the illustration below.



If you didn't have the number line to refer to, you can find the sum of -3 + 2 by

- subtracting the distances from zero (the absolute values) 3 2 = 1 and then
- applying the sign of the one furthest from zero (the largest absolute value). In this case, |-3| > |2|, so the answer is negative: -3 + 2 = -1

To add two numbers with different signs (one positive and one negative):

- Find the *difference* of their absolute values.
- Give the sum the same sign as the number with the greater absolute value.

Note that when you find the difference of the absolute values, you always subtract the lesser absolute value from the greater one. The example below shows you how to solve the temperature question that you considered earlier.

Example		
Problem	Find 8 + (–10).	
	The addends have different signs. So find the difference of their absolute values. -10 = 10 and $ 8 = 8$.	
	The difference of the absolute values is $10 - 8 = 2$.	
	Since $10 > 8$, the sum has the same sign as -10 .	
Answer	8 + (-10) = -2	

	Example		
Problem	Find −22 + 37 when <i>x</i> = −22.		
	−22 = 22 and 37 = 37 37 – 22 = 15	The addends have different signs. So find the difference of their absolute values.	
		Since $ 37 > -22 $, the sum has the same sign as 37.	
Answer	-22 + 37 = 15		

With more than two addends, you can add the first two, then the next one, and so on.

	Example		
Problem	Find −27 + (−138) + 55.		
		Add two at a time, starting with −27 + (−138).	
	−27 = 27 and −138 = 138 27 + 138 = 165 −27 + −138 = −165	Since they have the same signs, you add their absolute values and use the same sign.	
	−165 + 55 −165 = 165 and 55 = 55 165 – 55 = 110	Now add $-165 + 55$. Since -165 and 55 have different signs, you add them by <i>subtracting</i> their absolute values.	
	-165 + 55 = -110	Since 165 > 55, the sign of the final sum is the same as the sign of -165 .	
Answer	-27 + (-138) + 55 = -110		

Self Check B	
Find 32 + (-14).	

Summary

There are two cases to consider when adding integers. When the signs are the same, you add the absolute values of the addends and use the same sign. When the signs are different, you find the difference of the absolute values and use the same sign as the addend with the greater absolute value.

5.2.1 Self Check Solutions

Self Check A Find -32 + (-14).

-46

The sum is found by first adding the absolute values of the addends: |-32| + |-14| = 32 + 14 = 46. Then you must give the sum the same sign as the two addends, so the answer is -46.

Self Check B Find 32 + (-14).

18

Since the addends have different signs, you must find the difference of the absolute values. |32| = 32 and |-14| = 14. The difference is 32 - 14 = 18. The sign of the sum is the same as the addend with the greater absolute value. Since |32| > |-14|, the sum is positive.

5.2.2 Adding Real Numbers

Learning Objective(s)

1 Add two or more real numbers with the same sign.

- 2 Add two or more real numbers with different signs.
- **3** Simplify by using the identity property of 0.
- 4 Solve application problems requiring the addition of real numbers.

Introduction

Adding real numbers follows the same rules as adding integers. The number 0 has some special attributes that are very important in algebra. Knowing how to add these numbers can be helpful in real-world situations as well as algebraic situations.

Rules for Adding Real Numbers

Objective 1, 2

The rules for adding integers apply to other real numbers, including rational numbers.

To add two numbers with the same sign (both positive or both negative)

- Add their absolute values.
- Give the sum the same sign.

To add two numbers with different signs (one positive and one negative)

- Find the *difference* of their absolute values. (Note that when you find the difference of the absolute values, you always subtract the lesser absolute value from the greater one.)
- Give the sum the same sign as the number with the greater absolute value.

Remember—to add fractions, you need them to have the same denominator. This is still true when one or more of the fractions are negative.

Example		
Problem Find $-\frac{3}{7} + \left(-\frac{6}{7}\right) + \frac{2}{7}$		
$\left -\frac{3}{7}\right = \frac{3}{7} \text{ and } \left -\frac{6}{7}\right = \frac{6}{7}$	This problem has three addends . Add the first two, and then add the third.	
$\frac{3}{7} + \frac{6}{7} = \frac{9}{7}$	Since the signs of the first two are the same, find the sum of the absolute values of the fractions	
$-\frac{3}{7} + \left(-\frac{6}{7}\right) = -\frac{9}{7}$	Since both addends are negative, the sum is negative.	

	$\left -\frac{9}{7}\right = \frac{9}{7}$ and $\left \frac{2}{7}\right = \frac{2}{7}$	Now add the third addend. The signs are different, so find the <i>difference</i> of their absolute values.
	$\frac{9}{7} - \frac{2}{7} = \frac{7}{7}$	
	$-\frac{9}{7}+\frac{2}{7}=-\frac{7}{7}$	Since $\left -\frac{9}{7}\right > \left \frac{2}{7}\right $, the sign of the final
		sum is the same as the sign of $-\frac{9}{7}$.
Answer	$-\frac{3}{7} + \left(-\frac{6}{7}\right) + \frac{2}{7} = -\frac{7}{7}$	

	Example		
Problem	Find $-2\frac{3}{4} + \frac{7}{8}$		
	$\left -2\frac{3}{4}\right = 2\frac{3}{4} \text{ and } \left \frac{7}{8}\right = \frac{7}{8}$	The signs are different, so find the <i>difference</i> of their absolute values.	
	$2\frac{3}{4}-\frac{7}{8}$		
	23 2(4)+3 11	First rewrite $2\frac{3}{4}$ as an improper	
	$2\frac{3}{4} = \frac{2(4)+3}{4} = \frac{11}{4}$	fraction, then rewrite the fraction using a common denominator.	
	$\frac{11}{4} = \frac{11 \cdot 2}{4 \cdot 2} = \frac{22}{8}$		
	$\frac{22}{8} - \frac{7}{8}$	Now substitute the rewritten fraction in the problem.	
	$\frac{22}{8} - \frac{7}{8} = \frac{15}{8}$	Subtract the numerators and keep the same denominator. Simplify to lowest terms, if possible.	
Answer	$-2\frac{3}{4}+\frac{7}{8}=-\frac{15}{8}$	Since $\left -2\frac{3}{4}\right > \left \frac{7}{8}\right $, the sign of the final	
	4 8 8	sum is the same as the sign of $-2\frac{3}{4}$.	

When you add decimals, remember to line up the decimal points so you are adding tenths to tenths, hundredths to hundredths, and so on.

Example			
Problem	Find 27.832 + (−3.06).		
	Since the addends have different signs, subtract their absolute values.		
	27.832		
	<u>- 3.06</u> -3.06 = 3.06		
	24.772		
	The sum has the same sign as 27.832 whose absolute value is greater.		
Answer	27.832 + (-3.06) = 24.772		

Self Check A Find -32.22 + 124.3.

Applications of Addition	Objective 3, 4
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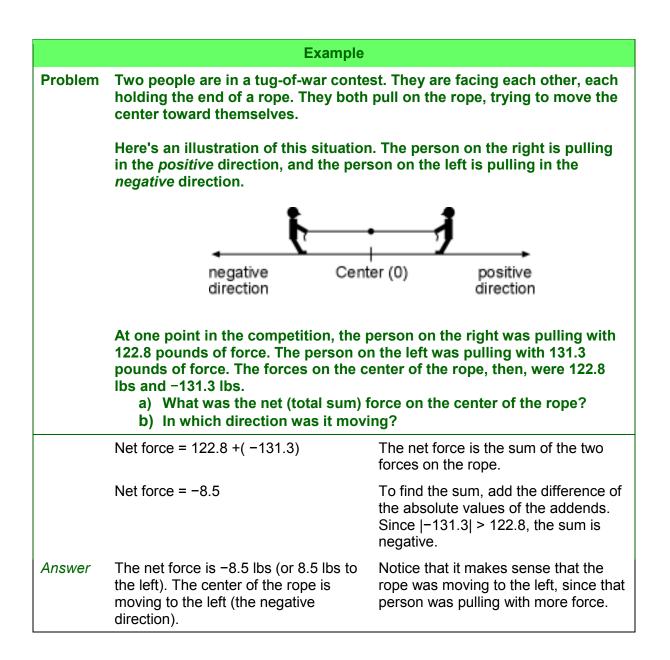
There are many situations that use negative numbers. For example, temperatures colder than 0° are usually described using negative numbers. In golf tournaments, players' scores are often reported as a number over or under par, instead of the total number of strokes it takes to hit the ball into the hole. (*Par* is the expected number of strokes needed to complete a hole.) A number under par is negative, and a number over par is positive.

The following examples show how addition of real numbers, including negative numbers, can be useful.

	Example		
Problem	Boston is, on average, 7 degrees warmer than Bangor, Maine. The low temperature on one cold winter day in Bangor was −13° F. About what low temperature would you expect Boston to have on that day?		
		The phrase "7 degrees warmer" means you add 7 degrees to Bangor's temperature to estimate Boston's temperature.	
	Boston's temperature is −13 + 7	On that day, Bangor's low was −13°, so you add 7° to −13°	
	-13 + 7 = -6	Add the integers. Since one is positive and the other is negative, you find the difference of $ -13 $ and $ 7 $, which is 6. Since $ -13 > 7 $, the final sum is negative.	
Answer	<i>ver</i> You would expect Boston to have a temperature of -6 degrees.		

Example		
Problem	Before Joanne could deposit her paycheck of \$802.83, she overdrew her checking account. The balance was −\$201.35. What was her balance after she deposited the paycheck?	
	-201.35 + 802.83 -201.35 + 802.83 = 601.83	By depositing her paycheck, Joanne is adding money to her account. The new balance is the sum of the old (−201.35) and the paycheck amount.
		Since the numbers have different signs, find the difference of -201.35 . Since $ 802.83 > -201.35 $, the sum is positive.
Answer	The new balance is \$601.48.	

When forces or objects are working in opposite directions, sometimes it's helpful to assign a negative value to one and a positive value to the other. This is done often in physics and engineering, but it could also be done in other contexts, such as football or a tug-of-war.



Self Check B

After Bangor reached a low temperature of -13° , the temperature rose only 4 degrees higher for the rest of the day. What was the high temperature that day?

Summary

As with integers, adding real numbers is done following two rules. When the signs are the same, you add the absolute values of the addends and use the same sign. When the signs are different, you subtract the absolute values and use the same sign as the addend with the greater absolute value.

5.2.2 Self Check Solutions

Self Check A Find -32.22 + 124.3. 92.08 Correct. Since the addends have different signs, you must subtract their absolute values.

124.3 - 32.22 is 92.08. Since |124.3| > |-32.22|, the sum is positive.

Self Check B

After Bangor reached a low temperature of -13° , the temperature rose only 4 degrees higher for the rest of the day. What was the high temperature that day?

The temperature rose (added) 4 degrees from -13, so the high temperature is -13 + 4. Since the addends have different signs, you must find the difference of the absolute values. |-13| = 13 and |4| = 4. The difference is 13 - 4 = 9. The sign of the sum is the same as the addend with the greater absolute value. Since |-13| > |4|, the sum is -9.

5.2.3 Subtracting Real Numbers

Learning Objective(s)

- 1 Subtract two or more real numbers.
- 2 Simplify combinations that require both addition and subtraction of real numbers.
- 3 Solve application problems requiring subtraction of real numbers.

Introduction

Subtraction and addition are closely related. They are called **inverse operations**, because one "undoes" the other. So, just as with integers, you can rewrite subtraction as addition to subtract real numbers.

Additive Inverses

Inverse operations, such as addition and subtraction, are a key idea in algebra. Suppose you have \$10 and you loan a friend \$5. An hour later, she pays you back the \$5 she borrowed. You are back to having \$10. You could represent the transaction like this:

$$10 - 5 + 5 = 10$$
.

This works because a number minus itself is 0.

3-3=0 63.5-63.5=0 39,283-39,283=0

So, adding a number and then subtracting the same number is like adding 0.

Thinking about this idea in terms of **opposite** numbers, you can also say that a number plus its opposite is also 0. Notice that each example below consists of a positive and a negative number pair added together.

3 + (-3) = 0 -63.5 + 63.5 = 0 39,283 + (-39,283) = 0

Two numbers are **additive inverses** if their sum is 0. Since this means the numbers are opposites (same absolute value but different signs), "additive inverse" is another, more formal term for the opposite of a number. (Note that 0 is its own additive inverse.)

Subtracting Real Numbers

Objective 1

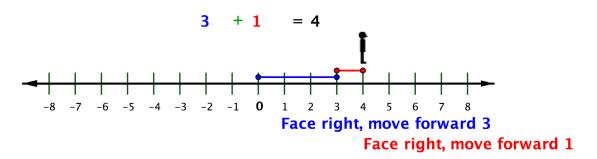
You can use the additive inverses or opposites to rewrite subtraction as addition. If you are adding two numbers with different signs, you find the difference between their absolute values and keep the sign of the number with the greater absolute value.

When the greater number is positive, it's easy to see the connection.

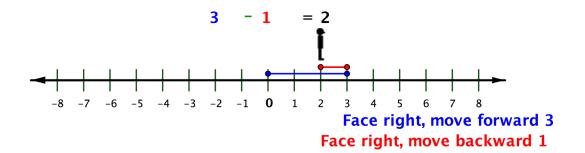
$$13 + (-7) = 13 - 7$$

Both equal 6.

Let's see how this works. When you add positive numbers, you are moving forward, facing in a positive direction.

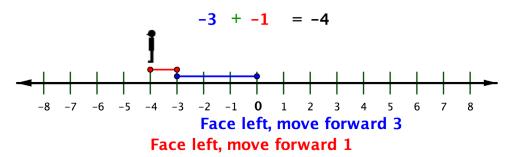


When you subtract positive numbers, you can imagine moving *backward*, but still facing in a positive direction.

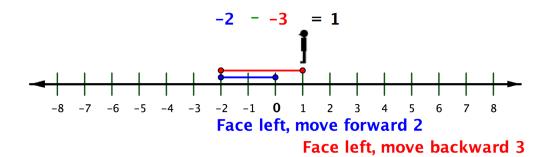


Now let's see what this means when one or more of the numbers is negative.

Recall that when you add a negative number, you move forward, but face in a negative direction (to the left).



How do you subtract a negative number? First face and move forward in a negative direction to the first number, -2. Then continue facing in a negative direction (to the left), but move *backward* to subtract -3.



But isn't this the same result as if you had added positive 3 to -2? -2 + 3 = 1.

In each addition problem, you face one direction and move some distance forward. In the paired subtraction problem, you face the *opposite* direction and then move the same distance *backward*. Each gives the same result!

To subtract a real number, you can rewrite the problem as adding the opposite (additive inverse).

Note, that while this always works, whole number subtraction is still the same. You can subtract 38 - 23 just as you have always done. Or, you could also rewrite it as 38 + (-23). Both ways you will get the same answer.

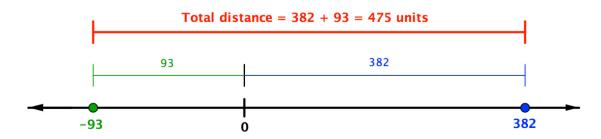
$$38 - 23 = 38 + (-23) = 15.$$

It's your choice in these cases.

Example		
Problem	Find 23 – 73.	
		You can't use your usual method of subtraction, because 73 is greater than 23.
	23 + (-73)	Rewrite the subtraction as adding the opposite.
	23 = 23 and −73 = 73 73 – 23 = 50	The addends have different signs, so find the difference of their absolute values.
Answer	23 - 73 = -50	Since −73 > 23 , the final answer is negative.

Example		
Problem	Find 382 – (−93).	
	382 + 93	Rewrite the subtraction as adding the opposite. The opposite of -93 is 93.
	382 + 93 = 475	So, this becomes a simple addition problem.
Answer	382 – (-93) = 475	

Another way to think about subtracting is to think about the distance between the two numbers on the number line. In the example above, 382 is to the *right* of 0 by 382 units, and -93 is to the *left* of 0 by 93 units. The distance between them is the sum of their distances to 0: 382 + 93.



	Example	
Problem	Find $22\frac{1}{3} - \left(-\frac{3}{5}\right)$.	
	$22\frac{1}{3} + \frac{3}{5}$	Rewrite the subtraction as adding the opposite. The opposite of $-\frac{3}{5}$ is $\frac{3}{5}$.
	$22\frac{1\cdot 5}{3\cdot 5} + \frac{3\cdot 3}{5\cdot 3} = 22\frac{5}{15} + \frac{9}{15}$	This is now just adding two rational numbers. Remember to find a common denominator when adding fractions. 3 and 5 have a common
	$22\frac{5}{15} + \frac{9}{15} = 22\frac{14}{15}$	multiple of 15; change denominators of both fractions to 15 (and make the necessary changes in the numerator!) before adding.
Answer	22 <mark>14</mark> 15	

Self Check A Find -32.3 – (-16.3).

When you have more than two **real numbers** to add or subtract, work from left to right as you would when adding more than two whole numbers. Be sure to change subtraction to addition of the opposite when needed.

	Example	
Problem	Find −23 + 16 – (−32) – 4 + 6.	
		Start with $-23 + 16$. The addends have different signs, so find the difference and use the sign of the addend with the greater absolute value. $-23 + 16 = -7$.
		Now you have $-7 - (-32)$. Rewrite this subtraction as addition of the opposite. The opposite of -32 is 32, so this becomes $-7 + 32$, which equals 25.
	25 – 4 + 6	You now have 25 – 4. You <i>could</i> rewrite this as an addition problem, but you don't need to.
	21 + 6	Complete the final addition of 21 + 6.
Answer	-23 + 16 - (-32) - 4 + 6 = 27	

Self Check B Find 32 - (-14) - 2 + (-82).

Situations that use negative numbers can require subtraction as well as addition. As you saw above, sometimes subtracting two positive numbers can give a negative result. You should be sure that a negative number makes sense in the problem.

Objective 3

	Example		
Problem	Boston is, on average, 7 degrees warmer than Bangor, Maine. The low temperature on one cold winter day in Boston was 3°F. About what low temperature would you expect Bangor to have on that day?		
		The phrase "7 degrees warmer" means you can subtract 7 degrees from Boston's temperature to estimate Bangor's temperature. (Note that you can also add 7 degrees to Bangor's temperature to estimate Boston's temperature. Be careful about which should have the greater number!)	
	Bangor's temperature is 3 – 7	On that day, Boston's low was 3°, so you subtract 7° from 3°.	
	3 – 7 = 3 + (-7)	Since 3 < 7, rewrite the subtraction problem as addition of the opposite. Add the numbers. Since one is positive and the other is negative, you find the difference of $ -7 $ and $ 3 $, which is 4. Since $ -7 > 3 $, the final sum is negative.	
Answer	You would expect the low temperature in Bangor, Maine to be -4° F.		

Example

ProblemEverett paid several bills without balancing his checkbook first! When
the last check he wrote was still to be deducted from his balance,
Everett's account was already overdrawn. The balance was -\$201.35. The
final check was for \$72.66, and another \$25 will be subtracted as an
overdraft charge. What will Everett's account balance be after that last
check and the overdraft charge are deducted?-201.35 - 72.66 - 25The new balance will be the existing balance of
-\$201.35, minus the check's amount and the
overdraft charge.

		Start with the first subtraction, $-201.35 - 72.66$. Rewrite it as the addition of the opposite of 72.66.
	-274.01 – 25	Since the addends have the same signs, the sum is the sum of their absolute values (201.35 + 72.66) with the same sign (negative).
	-274.01 + (-25)	Again, rewrite the subtraction as the addition of the opposite.
	-274.01 + (-25) = -299.01	Add, by adding the sum of their absolute values and use the same sign as both addends.
Answer	Everett's account balance will be \$-299.01.	

	Example	
Problem	One winter, Phil flew from Syracuse, NY to Orlando, FL. The temperature in Syracuse was −20°F. The temperature in Orlando was 75°F. What was the difference in temperatures between Syracuse and Orlando?	
	75 – (–20)	To find the difference between the temperatures, you need to subtract. We subtract the ending temperature from the beginning temperature to get the change in temperature.
	75 + 20	Rewrite the subtraction as adding the opposite. The opposite of −20 is 20.
	75 + 20 = 95	There is a 95 degree difference between 75° and −20°.
Answer	The difference in temperatures is 95 degrees.	

Self Check C

Louise noticed that her bank balance was -\$33.72 before her paycheck was deposited. After the check had been deposited, the balance was \$822.98. No other deductions or deposits were made. How much money was she paid?

Summary

Subtracting a number is the same as adding its opposite (also called its additive inverse). To subtract you can rewrite the subtraction as adding the opposite and then use the rules for the addition of real numbers.

5.2.3 Self Check Solutions

Self Check A Find -32.3 – (-16.3).

To subtract, change the problem to adding the opposite of -16.3, which gives -32.3 + 16.3. Then use the rules for adding two numbers with different signs. Since the difference between 32.3 and 16.3 is 16, and |-32.3| > |16.3|, the correct answer is -16.

Self Check B

Find 32 - (-14) - 2 + (-82).

To subtract 32 - (-14), write the subtraction as addition of the opposite, giving 32 + 14 = 46. Then subtract 2 to get 44, and add -82 to get -38.

Self Check C

Louise noticed that her bank balance was -\$33.72 before her paycheck was deposited. After the check had been deposited, the balance was \$822.98. No other deductions or deposits were made. How much money was she paid?

\$856.70. The amount she was paid is the difference between the two balances: 822.98 -(-33.72). This is the same as 822.98 + 33.72, or 856.70.