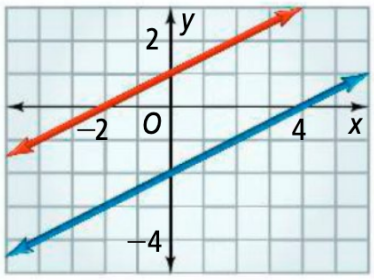
5-6 Parallel and Perpendicular Lines

Two distinct lines in a coordinate plane either intersect or are *parallel*. ***Parallel lines*** *are lines in the same plane that never intersect.* You can determine the relationship between two lines by comparing their slopes and y-intercepts.

KEY CONCEPT: SLOPES OF PARALLEL LINES

Nonvertical lines are parallel if they have the ***same*** *slope and* ***different*** *y-intercepts. Vertical lines are parallel if they have different x-intercepts.*

The graphs of $y = \frac{1}{2}x + 1$ *and* $y = \frac{1}{2}x - 2$ *are lines that have the same slope, ½, and different y-intercepts. Therefore, these lines are parallel.*



You can use the fact that the slopes of parallel lines are the same to write the equation of a line parallel to a given line.

PROBLEM 1: WRITING AN EQUATION OF A PARALLEL LINE

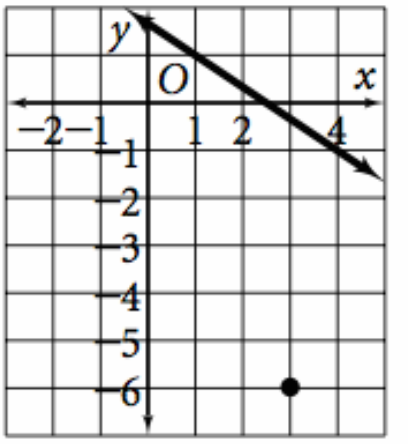
Write an equation in slope-intercept form of the line that passes through the given point and is parallel to the graph of the given equation.

1. $(12,5); y = \frac{2}{3}x - 1$
2. $(-3,-1); y = 2x + 3$
3. $(1,-3); y + 2 = 4(x - 1)$

4. $(4,2); x = -3$
5. $(1,5); y = -2$

6. *Write an equation for the line that is parallel to the given line and that passes through the given point.*

a)



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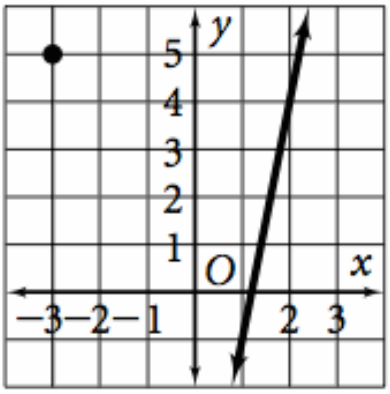
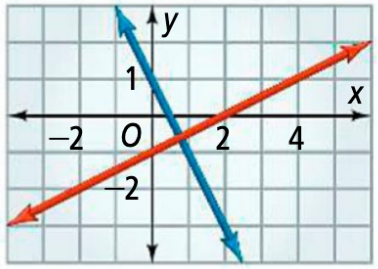


You can use slope to determine whether two lines are *perpendicular*. ***Perpendicular lines*** *are lines that intersect to form right angles.*

KEY CONCEPT: SLOPES OF PERPENDICULAR LINES

Two nonvertical lines are perpendicular if the product of their slopes is -1. A vertical line and a horizontal line are also perpendicular.



The graph of $y = \frac{1}{2}x - 1$ *has a slope of ½ . The graph of* $y = -2x + 1$ *has a slope of -2. Since* $\frac{1}{2}(-2) = -1$*, the lines are perpendicular.*

Two numbers whose product is -1 are ***opposite reciprocals***. So, the slopes of perpendicular lines are opposite reciprocals of each other. To find the opposite reciprocal of $-\frac{3}{4}$, for example, first find the reciprocal, $-\frac{4}{3}$. Then write its opposite, $\frac{4}{3}$. Since $-\frac{3}{4}\left(\frac{4}{3}\right) = -1$, $\frac{4}{3}$ is the opposite reciprocal of $-\frac{3}{4}$.

PROBLEM 2: WRITING THE EQUATION OF A PERPENDICULAR LINE

Write an equation in slope-intercept form of the line that passes through the given point and is perpendicular to the graph of the given equation.

7. $(2,4); y = \frac{1}{3}x - 1$

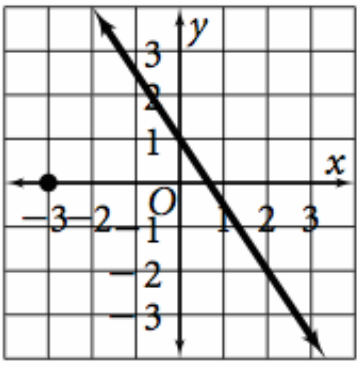
8. $(1,8); y = 2x + 1$

9. $(5,0); y + 1 = -3(x - 4)$

10. $(-3,2); 2x + 3y = 7$ 11. $(1,2); x = 4$

12. *Write an equation for the line that is perpendicular to the given line and that passes through the given point*.

a)



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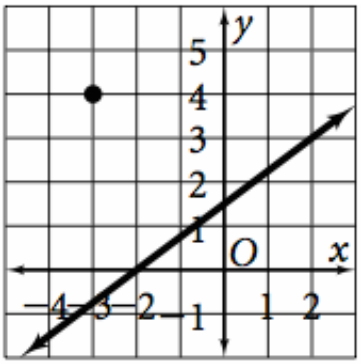


PROBLEM 3: CLASSIFYING LINES

Determine whether the graphs of the given equations are <u>parallel</u>, <u>perpendicular</u>, or <u>neither</u>.

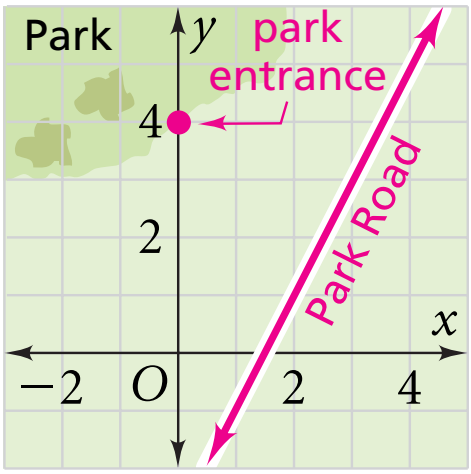
13. $4y = -5x + 12$ and $y = \frac{4}{5}x - 8$

14. $y = \frac{3}{4}x + 7$ and $4x - 3y = 9$

15. $x + 6y = 6$ and $y = -\frac{1}{6}x + 6$

PROBLEM 4: SOLVING A REAL-WORLD PROBLEM

16. A path for a new city park will connect the park entrance to Park Road. The path should be perpendicular to Park Road. What is an equation that represents the path?



17. A bike path is being planned for the park. The bike path will be parallel to Park Road and will pass through the park entrance. What is an equation of the line that represents the bike path?

Name _____ Period _____

5-6 Practice Worksheet

Find the slope of a line parallel to the graph of each equation.

1. $7x - y = 5$

2. $y = 5$

Are the graphs of the lines in each pair parallel?

3. $y = \frac{1}{3}x + 3$
   $x - 3y = 6$

4. $y = 4x + 12$
   $-4x + 3y = 21$

5. ***Write an equation for the line that is parallel to*** $y = -\frac{2}{3}x + 12$ ***and passes through*** $(5, -3)$.

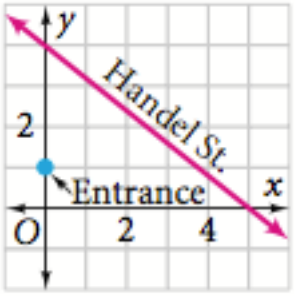
Find the slope of a line perpendicular to the graph of each equation.

6. $y = -\frac{x}{5} - 7$

7. $y = -8$

8. ***Write an equation for the line that is perpendicular to*** $-10x + 8y = 3$ ***and passes through*** $(15, 12)$.

9. ***A city's civil engineer is planning a new parking garage and a new street. The new street will go from the entrance of the parking garage to Handel St. It will be perpendicular to Handel St. What is the equation of the line representing the new street?***



Tell whether the lines for each pair of equations are <u>parallel</u>, <u>perpendicular</u>, or <u>neither</u>.

10. $y = 4x + \frac{3}{4}$
$y = -\frac{1}{4}x + 4$

11. $3x - 5y = 3$
$-5x + 3y = 8$

12. $y = \frac{x}{3} - 4$
$y = \frac{1}{3}x + 2$

13. $ax - by = 5$
$-ax + by = 2$

14. $ax + by = 8$
$bx - ay = 1$

15. ***For what value of*** k ***are the graphs of*** $3x + 12y = 8$ ***and*** $6y = kx - 5$ ***parallel?***

16. ***For what value of*** k ***are the graphs of*** $3x + 12y = 8$ ***and*** $6y = kx - 5$ ***perpendicular?***

17. ***A parallelogram has vertices A(0,2), B(2,-1), C(6,3), and D(p,q). Which of the following ordered pairs has possible values for (p,q)?***
(a) (0,6) (b) (6,1) (c) (4,6) (d) (6,4)

18. ***Suppose the line through points (x,6) and (1,2) is parallel to the graph of*** $2x + y = 3$. ***Find the value of x. Show your work***