5.7 Factoring by Special Products

OBJECTIVES

1. Factor a Perfect Square Trinomial.
2. Factor the Difference of Two Squares.
3. Factor the Sum or Difference of Two Cubes.

OBJECTIVE 1

Factoring a Perfect Square Trinomial

In the previous section, we considered a variety of ways to factor trinomials of the form $ax^2 + bx + c$. In Example 8, we factored $16x^2 + 24xy + 9y^2$ as

$$16x^2 + 24xy + 9y^2 = (4x + 3y)^2$$

Recall that $16x^2 + 24xy + 9y^2$ is a perfect square trinomial because its factors are two identical binomials. A perfect square trinomial can be factored quickly if you recognize the trinomial as a perfect square.

A trinomial is a perfect square trinomial if it can be written so that its first term is the square of some quantity a, its last term is the square of some quantity b, and its middle term is twice the product of the quantities a and b.

The following special formulas can be used to factor perfect square trinomials.

Perfect Square Trinomials

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

Notice that these formulas above are the same special products from Section 5.4 for the square of a binomial.

From

$$a^2 + 2ab + b^2 = (a + b)^2,$$

we see that

$$16x^2 + 24xy + 9y^2 = (4x)^2 + 2(4x)(3y) + (3y)^2 = (4x + 3y)^2$$

EXAMPLE 1 Factor $m^2 + 10m + 25$.

Solution Notice that the first term is a square: $m^2 = (m)^2$, the last term is a square: $25 = 5^2$; and $10m = 2 \cdot 5 \cdot m$.

Thus,

$$m^2 + 10m + 25 = m^2 + 2(m)(5) + 5^2 = (m + 5)^2$$ □

PRACTICE 1 Factor $b^2 + 16b + 64$.

EXAMPLE 2 Factor $12a^2x - 12abx + 3b^2x$.

Solution The terms of this trinomial have a GCF of $3x$, which we factor out first.

$$12a^2x - 12abx + 3b^2x = 3x(4a^2 - 4ab + b^2)$$

Now, the polynomial $4a^2 - 4ab + b^2$ is a perfect square trinomial. Notice that the first term is a square: $4a^2 = (2a)^2$; the last term is a square: $b^2 = (b)^2$; and $4ab = 2(2a)(b)$. The factoring can now be completed as

$$3x(4a^2 - 4ab + b^2) = 3x(2a - b)^2$$ □

PRACTICE 2 Factor $45x^2b - 30xb + 5b$.

Helpful Hint

If you recognize a trinomial as a perfect square trinomial, use the special formulas to factor. However, methods for factoring trinomials in general from Section 5.6 will also result in the correct factored form.

OBJECTIVE 2

Factoring the Difference of Two Squares

We now factor special types of binomials, beginning with the **difference of two squares.** The special product pattern presented in Section 5.4 for the product of a sum and a difference of two terms is used again here. However, the emphasis is now on factoring rather than on multiplying.

Difference of Two Squares

$$a^2 - b^2 = (a + b)(a - b)$$

Notice that a binomial is a difference of two squares when it is the difference of the square of some quantity a and the square of some quantity b.

EXAMPLE 3 Factor the following.

a. $x^2 - 9$ **b.** $16y^2 - 9$ **c.** $50 - 8y^2$ **d.** $x^2 - \frac{1}{4}$

Solution

a. $x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$

b. $16y^2 - 9 = (4y)^2 - 3^2 = (4y + 3)(4y - 3)$

c. First factor out the common factor of 2.

$$50 - 8y^2 = 2(25 - 4y^2) = 2(5 + 2y)(5 - 2y)$$

d. $x^2 - \frac{1}{4} = x^2 - \left(\frac{1}{2}\right)^2 = \left(x + \frac{1}{2}\right)\left(x - \frac{1}{2}\right)$ □

PRACTICE 3 Factor the following.

a. $x^2 - 16$

b. $25b^2 - 49$

c. $45 - 20x^2$

d. $y^2 - \frac{1}{81}$

The binomial $x^2 + 9$ is a **sum of two squares** and cannot be factored by using real numbers. **In general, except for factoring out a GCF, the sum of two squares usually cannot be factored by using real numbers.**

Helpful Hint

The sum of two squares whose GCF is 1 usually cannot be factored by using real numbers. For example, $x^2 + 9$ is a prime polynomial.

EXAMPLE 4 Factor the following.

a. $p^4 - 16$ **b.** $(x + 3)^2 - 36$

Solution

a. $p^4 - 16 = (p^2)^2 - 4^2 = (p^2 + 4)(p^2 - 4)$

The binomial factor $p^2 + 4$ cannot be factored by using real numbers, but the binomial factor $p^2 - 4$ is a difference of squares.

$$(p^2 + 4)\underbrace{(p^2 - 4)} = (p^2 + 4)\underbrace{(p + 2)(p - 2)}$$

b. Factor $(x + 3)^2 - 36$ as the difference of squares.

$$\begin{aligned}(x + 3)^2 - 36 &= (x + 3)^2 - 6^2 \\ &= [(x + 3) + 6][(x + 3) - 6] && \text{Factor.} \\ &= [x + 3 + 6][x + 3 - 6] && \text{Remove parentheses.} \\ &= (x + 9)(x - 3) && \text{Simplify.}\end{aligned}$$

PRACTICE
4 Factor the following.

a. $x^4 - 10{,}000$ **b.** $(x + 2)^2 - 49$

✓CONCEPT CHECK

Is $(x - 4)(y^2 - 9)$ completely factored? Why or why not?

EXAMPLE 5 Factor $x^2 + 4x + 4 - y^2$.

Solution Factoring by grouping comes to mind since the sum of the first three terms of this polynomial is a perfect square trinomial.

$$\begin{aligned}x^2 + 4x + 4 - y^2 &= (x^2 + 4x + 4) - y^2 && \text{Group the first three terms.} \\ &= (x + 2)^2 - y^2 && \text{Factor the perfect square trinomial.}\end{aligned}$$

This is not factored yet since we have a *difference,* not a *product*. Since $(x + 2)^2 - y^2$ is a difference of squares, we have

$$\begin{aligned}(x + 2)^2 - y^2 &= [(x + 2) + y][(x + 2) - y] \\ &= (x + 2 + y)(x + 2 - y)\end{aligned}$$

PRACTICE
5 Factor $m^2 + 6m + 9 - n^2$.

OBJECTIVE
3

Factoring the Sum or Difference of Two Cubes

Although the sum of two squares usually cannot be factored, the sum of two cubes, as well as the difference of two cubes, can be factored as follows.

Sum and Difference of Two Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

To check the first pattern, let's find the product of $(a + b)$ and $(a^2 - ab + b^2)$.

$$\begin{aligned}(a + b)(a^2 - ab + b^2) &= a(a^2 - ab + b^2) + b(a^2 - ab + b^2) \\ &= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3 \\ &= a^3 + b^3\end{aligned}$$

EXAMPLE 6 Factor $x^3 + 8$.

Solution First we write the binomial in the form $a^3 + b^3$. Then we use the formula

$$a^3 + b^3 = (a + b)(a^2 - a \cdot b + b^2), \quad \text{where } a \text{ is } x \text{ and } b \text{ is } 2.$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$x^3 + 8 = x^3 + 2^3 = (x + 2)(x^2 - x \cdot 2 + 2^2)$$

Thus, $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$

PRACTICE
6 Factor $x^3 + 64$.

Answer to Concept Check:
no; $(y^2 - 9)$ can be factored

EXAMPLE 7 Factor $p^3 + 27q^3$.

Solution

$$\begin{aligned} p^3 + 27q^3 &= p^3 + (3q)^3 \\ &= (p + 3q)[p^2 - (p)(3q) + (3q)^2] \\ &= (p + 3q)(p^2 - 3pq + 9q^2) \end{aligned}$$ □

PRACTICE 7 Factor $a^3 + 8b^3$.

EXAMPLE 8 Factor $y^3 - 64$.

Solution This is a difference of cubes since $y^3 - 64 = y^3 - 4^3$.

From

$$\begin{array}{ccccccccccccccccc} a^3 & - & b^3 & = & (& a & - & b &)(& a^2 & + & a & \cdot & b & + & b^2 &) \\ \downarrow & & \downarrow & & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & \\ y^3 & - & 4^3 & = & (& y & - & 4 &)(& y^2 & + & y & \cdot & 4 & + & 4^2 &) \end{array}$$

$$= (y - 4)(y^2 + 4y + 16)$$ □

PRACTICE 8 Factor $27 - y^3$.

Helpful Hint

When factoring sums or differences of cubes, be sure to notice the sign patterns.

Same sign

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

Opposite sign

Always positive

Same sign

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Opposite sign

EXAMPLE 9 Factor $125q^2 - n^3q^2$.

Solution First we factor out a common factor of q^2.

$$\begin{aligned} 125q^2 - n^3q^2 &= q^2(125 - n^3) \\ &= q^2(5^3 - n^3) \end{aligned}$$

Opposite sign Positive

$$\begin{aligned} &= q^2(5 - n)[5^2 + (5)(n) + (n^2)] \\ &= q^2(5 - n)(25 + 5n + n^2) \end{aligned}$$

Thus, $125q^2 - n^3q^2 = q^2(5 - n)(25 + 5n + n^2)$. The trinomial $25 + 5n + n^2$ cannot be factored further. □

PRACTICE 9 Factor $b^3x^2 - 8x^2$.

Vocabulary, Readiness & Video Check

Write each term as a square. For example, $25x^2$ as a square is $(5x)^2$.

1. $81y^2$ **2.** $4z^2$ **3.** $64x^6$ **4.** $49y^6$

Write each term as a cube.

5. $8x^3$ **6.** $27y^3$ **7.** $64x^6$ **8.** x^3y^6

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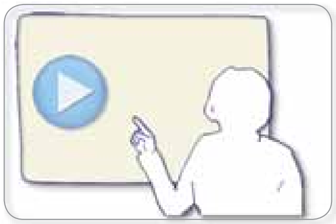

See Video 5.7

Watch the section lecture video and answer the following questions.

OBJECTIVE 1

9. From Example 1, what is the first step to see if you have a perfect square trinomial? How do you then finish determining that you do indeed have a perfect square trinomial?

OBJECTIVE 2

10. In Example 2, the original binomial is rewritten to write each term as a square. Give two reasons why this is helpful.

OBJECTIVE 3

11. In Examples 4 and 5, what tips are given to remember how to factor the sum or difference of two cubes rather than memorizing the formulas?

5.7 Exercise Set

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Factor. See Examples 1 and 2.

1. $x^2 + 6x + 9$

2. $x^2 - 10x + 25$

3. $4x^2 - 12x + 9$

4. $9a^2 - 30a + 25$

5. $25x^2 + 10x + 1$

6. $4a^2 + 12a + 9$

7. $3x^2 - 24x + 48$

8. $2x^2 + 28x + 98$

9. $9y^2x^2 + 12yx^2 + 4x^2$

10. $4x^2y^3 - 4xy^3 + y^3$

11. $16x^2 - 56xy + 49y^2$

12. $81x^2 + 36xy + 4y^2$

Factor. See Examples 3 through 5.

13. $x^2 - 25$

14. $y^2 - 100$

15. $\frac{1}{9} - 4z^2$

16. $\frac{1}{16} - y^2$

17. $(y + 2)^2 - 49$

18. $(x - 1)^2 - z^2$

19. $64x^2 - 100$

20. $4x^2 - 36$

21. $(x + 2y)^2 - 9$

22. $(3x + y)^2 - 25$

23. $x^2 + 6x + 9 - y^2$

24. $x^2 + 12x + 36 - y^2$

25. $x^2 + 16x + 64 - x^4$

26. $x^2 + 20x + 100 - x^4$

Factor. See Examples 6 through 9.

27. $x^3 + 27$

28. $y^3 + 1$

29. $z^3 - 1$

30. $x^3 - 8$

31. $m^3 + n^3$

32. $p^3 + 125q^3$

33. $27y^2 - x^3y^2$

34. $64q^2 - q^2p^3$

35. $8ab^3 + 27a^4$

36. $a^3b + 8b^4$

37. $250y^3 - 16x^3$

38. $54y^3 - 128$

MIXED PRACTICE

Factor completely. See Examples 1 through 9.

39. $x^2 - 12x + 36$

40. $x^2 - 18x + 81$

41. $18x^2y - 2y$

42. $12xy^2 - 108x$

43. $9x^2 - 49$

44. $25x^2 - 4$

45. $x^4 - 1$

46. $x^4 - 256$

47. $x^6 - y^3$

48. $x^3 - y^6$

49. $8x^3 + 27y^3$

50. $125x^3 + 8y^3$

51. $4x^2 + 4x + 1 - z^2$

52. $9y^2 + 12y + 4 - x^2$

53. $3x^6y^2 + 81y^2$

54. $x^2y^9 + x^2y^3$

55. $n^3 - \frac{1}{27}$

56. $p^3 + \frac{1}{125}$

57. $-16y^2 + 64$

58. $-12y^2 + 108$

59. $x^2 - 10x + 25 - y^2$

60. $x^2 - 18x + 81 - y^2$

61. $a^3b^3 + 125$

62. $x^3y^3 + 216$

63. $\dfrac{x^2}{25} - \dfrac{y^2}{9}$

64. $\dfrac{a^2}{4} - \dfrac{b^2}{49}$

65. $(x + y)^3 + 125$

66. $(r + s)^3 + 27$

REVIEW AND PREVIEW

Solve the following equations. See Section 2.1.

67. $x - 5 = 0$

68. $x + 7 = 0$

69. $3x + 1 = 0$

70. $5x - 15 = 0$

71. $-2x = 0$

72. $3x = 0$

73. $-5x + 25 = 0$

74. $-4x - 16 = 0$

CONCEPT EXTENSIONS

Determine whether each polynomial is factored completely. See the Concept Check in this section.

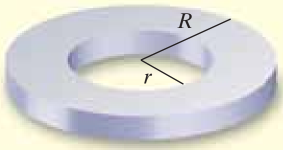
75. $5x(x^2 - 4)$

76. $x^2y^2(x^3 - y^3)$

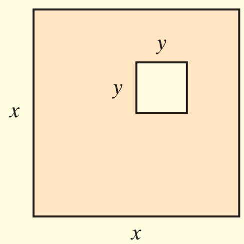
77. $7y(a^2 + a + 1)$

78. $9z(x^2 + 4)$

△ **79.** A manufacturer of metal washers needs to determine the cross-sectional area of each washer. If the outer radius of the washer is R and the radius of the hole is r, express the area of the washer as a polynomial. Factor this polynomial completely.

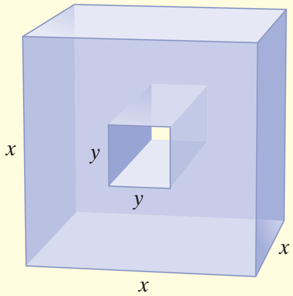


△ **80.** Express the area of the shaded region as a polynomial. Factor the polynomial completely.

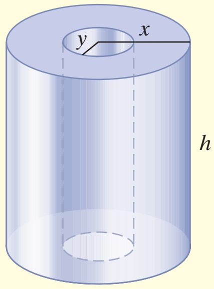


Express the volume of each solid as a polynomial. To do so, subtract the volume of the "hole" from the volume of the larger solid. Then factor the resulting polynomial.

△ **81.**



△ **82.**



Find the value of c that makes each trinomial a perfect square trinomial.

83. $x^2 + 6x + c$

84. $y^2 + 10y + c$

85. $m^2 - 14m + c$

86. $n^2 - 2n + c$

87. $x^2 + cx + 16$

88. $x^2 + cx + 36$

89. Factor $x^6 - 1$ completely, using the following methods from this chapter.

a. Factor the expression by treating it as the difference of two squares, $(x^3)^2 - 1^2$.

b. Factor the expression, treating it as the difference of two cubes, $(x^2)^3 - 1^3$.

c. Are the answers to parts (a) and (b) the same? Why or why not?

90. Factor $x^{12} - 1$ completely, using the following methods from this chapter:

a. Factor the expression by treating it as the difference of two squares, $(x^3)^4 - 1^4$.

b. Factor the expression by treating it as the difference of two cubes, $(x^4)^3 - 1^3$.

c. Are the answers to parts (a) and (b) the same? Why or why not?

Factor. Assume that variables used as exponents represent positive integers.

91. $x^{2n} - 25$

92. $x^{2n} - 36$

93. $36x^{2n} - 49$

94. $25x^{2n} - 81$

95. $x^{4n} - 16$

96. $x^{4n} - 625$

Integrated Review OPERATIONS ON POLYNOMIALS AND FACTORING STRATEGIES

Sections 5.1–5.7

Operations on Polynomials

Perform each indicated operation.

1. $(-y^2 + 6y - 1) + (3y^2 - 4y - 10)$

2. $(5z^4 - 6z^2 + z + 1) - (7z^4 - 2z + 1)$

3. Subtract $(x - 5)$ from $(x^2 - 6x + 2)$.

4. $(2x^2 + 6x - 5) + (5x^2 - 10x)$

5. $(5x - 3)^2$

6. $(5x^2 - 14x - 3) - (5x + 1)$

7. $\dfrac{2x^4}{x} - \dfrac{3x^2}{x} + \dfrac{5x}{x} - \dfrac{2}{2}$

8. $(4x - 1)(x^2 - 3x - 2)$

Factoring Strategies

The key to proficiency in factoring polynomials is to practice until you are comfortable with each technique. A strategy for factoring polynomials completely is given next.

Factoring a Polynomial

Step 1. Are there any common factors? If so, factor out the greatest common factor.

Step 2. How many terms are in the polynomial?

a. If there are *two* terms, decide if one of the following formulas may be applied:

i. Difference of two squares: $a^2 - b^2 = (a - b)(a + b)$

ii. Difference of two cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

iii. Sum of two cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

b. If there are *three* terms, try one of the following:

i. Perfect square trinomial: $a^2 + 2ab + b^2 = (a + b)^2$
$a^2 - 2ab + b^2 = (a - b)^2$

ii. If not a perfect square trinomial, factor by using the methods presented in Sections 5.5 and 5.6.

c. If there are *four* or more terms, try factoring by grouping.

Step 3. See whether any factors in the factored polynomial can be factored further.

A few examples are worked for you below.

EXAMPLE 1 Factor each polynomial completely.

a. $8a^2b - 4ab$ **b.** $36x^2 - 9$ **c.** $2x^2 - 5x - 7$

d. $5p^2 + 5 + qp^2 + q$ **e.** $9x^2 + 24x + 16$ **f.** $y^2 + 25$

Solution

a. Step 1. The terms have a common factor of $4ab$, which we factor out.

$$8a^2b - 4ab = 4ab(2a - 1)$$

Step 2. There are two terms, but the binomial $2a - 1$ is not the difference of two squares or the sum or difference of two cubes.

Step 3. The factor $2a - 1$ cannot be factored further.

b. Step 1. Factor out a common factor of 9.

$$36x^2 - 9 = 9(4x^2 - 1)$$

Step 2. The factor $4x^2 - 1$ has two terms, and it is the difference of two squares.

$$9(4x^2 - 1) = 9(2x + 1)(2x - 1)$$

Step 3. No factor with more than one term can be factored further.

c. Step 1. The terms of $2x^2 - 5x - 7$ contain no common factor other than 1 or -1.

Step 2. There are three terms. The trinomial is not a perfect square, so we factor by methods from Section 5.6.

$$2x^2 - 5x - 7 = (2x - 7)(x + 1)$$

Step 3. No factor with more than one term can be factored further.

d. Step 1. There is no common factor of all terms of $5p^2 + 5 + qp^2 + q$.

Step 2. The polynomial has four terms, so try factoring by grouping.

$$\begin{aligned} 5p^2 + 5 + qp^2 + q &= (5p^2 + 5) + (qp^2 + q) \quad \text{Group the terms.} \\ &= 5(p^2 + 1) + q(p^2 + 1) \\ &= (p^2 + 1)(5 + q) \end{aligned}$$

Step 3. No factor can be factored further.

e. Step 1. The terms of $9x^2 + 24x + 16$ contain no common factor other than 1 or -1.

Step 2. The trinomial $9x^2 + 24x + 16$ is a perfect square trinomial, and $9x^2 + 24x + 16 = (3x + 4)^2$.

Step 3. No factor can be factored further.

f. Step 1. There is no common factor of $y^2 + 25$ other than 1.

Step 2. This binomial is the sum of two squares and is prime.

Step 3. The binomial $y^2 + 25$ cannot be factored further. □

PRACTICE 1 Factor each polynomial completely.

a. $12x^2y - 3xy$ **b.** $49x^2 - 4$

c. $5x^2 + 2x - 3$ **d.** $3x^2 + 6 + x^3 + 2x$

e. $4x^2 + 20x + 25$ **f.** $b^2 + 100$

EXAMPLE 2 Factor each polynomial completely.

a. $27a^3 - b^3$ **b.** $3n^2m^4 - 48m^6$ **c.** $2x^2 - 12x + 18 - 2z^2$

d. $8x^4y^2 + 125xy^2$ **e.** $(x - 5)^2 - 49y^2$

Solution

a. This binomial is the difference of two cubes.

$$\begin{aligned} 27a^3 - b^3 &= (3a)^3 - b^3 \\ &= (3a - b)[(3a)^2 + (3a)(b) + b^2] \\ &= (3a - b)(9a^2 + 3ab + b^2) \end{aligned}$$

b. $3n^2m^4 - 48m^6 = 3m^4(n^2 - 16m^2)$ Factor out the GCF, $3m^4$.

$= 3m^4(n + 4m)(n - 4m)$ Factor the difference of squares.

c. $2x^2 - 12x + 18 - 2z^2 = 2(x^2 - 6x + 9 - z^2)$ The GCF is 2.

$= 2[(x^2 - 6x + 9) - z^2]$ Group the first three terms together.

$= 2[(x - 3)^2 - z^2]$ Factor the perfect square trinomial.

$= 2[(x - 3) + z][(x - 3) - z]$ Factor the difference of squares.

$= 2(x - 3 + z)(x - 3 - z)$

d. $8x^4y^2 + 125xy^2 = xy^2(8x^3 + 125)$ The GCF is xy^2.

$= xy^2[(2x)^3 + 5^3]$

$= xy^2(2x + 5)[(2x)^2 - (2x)(5) + 5^2]$ Factor the sum of cubes.

$= xy^2(2x + 5)(4x^2 - 10x + 25)$

e. This binomial is the difference of squares.

$$\begin{aligned} (x - 5)^2 - 49y^2 &= (x - 5)^2 - (7y)^2 \\ &= [(x - 5) + 7y][(x - 5) - 7y] \\ &= (x - 5 + 7y)(x - 5 - 7y) \end{aligned}$$

□

PRACTICE 2 Factor each polynomial completely.

a. $64x^3 + y^3$
b. $7x^2y^2 - 63y^4$
c. $3x^2 + 12x + 12 - 3b^2$
d. $x^5y^4 + 27x^2y$
e. $(x + 7)^2 - 81y^2$

Factor completely.

9. $x^2 - 8x + 16 - y^2$
10. $12x^2 - 22x - 20$
11. $x^4 - x$
12. $(2x + 1)^2 - 3(2x + 1) + 2$
13. $14x^2y - 2xy$
14. $24ab^2 - 6ab$
15. $4x^2 - 16$
16. $9x^2 - 81$
17. $3x^2 - 8x - 11$
18. $5x^2 - 2x - 3$
19. $4x^2 + 8x - 12$
20. $6x^2 - 6x - 12$
21. $4x^2 + 36x + 81$
22. $25x^2 + 40x + 16$
23. $8x^3 + 125y^3$
24. $27x^3 - 64y^3$
25. $64x^2y^3 - 8x^2$
26. $27x^5y^4 - 216x^2y$
27. $(x + 5)^3 + y^3$
28. $(y - 1)^3 + 27x^3$
29. $(5a - 3)^2 - 6(5a - 3) + 9$
30. $(4r + 1)^2 + 8(4r + 1) + 16$
31. $7x^2 - 63x$
32. $20x^2 + 23x + 6$
33. $ab - 6a + 7b - 42$
34. $20x^2 - 220x + 600$
35. $x^4 - 1$
36. $15x^2 - 20x$
37. $10x^2 - 7x - 33$
38. $45m^3n^3 - 27m^2n^2$
39. $5a^3b^3 - 50a^3b$
40. $x^4 + x$
41. $16x^2 + 25$
42. $20x^3 + 20y^3$
43. $10x^3 - 210x^2 + 1100x$
44. $9y^2 - 42y + 49$
45. $64a^3b^4 - 27a^3b$
46. $y^4 - 16$
47. $2x^3 - 54$
48. $2sr + 10s - r - 5$
49. $3y^5 - 5y^4 + 6y - 10$
50. $64a^2 + b^2$
51. $100z^3 + 100$
52. $250x^4 - 16x$
53. $4b^2 - 36b + 81$
54. $2a^5 - a^4 + 6a - 3$
55. $(y - 6)^2 + 3(y - 6) + 2$
56. $(c + 2)^2 - 6(c + 2) + 5$
△ **57.** Express the area of the shaded region as a polynomial. Factor the polynomial completely.