5. Chain Reactions

Nuclear explosions, cancer, the population growth, lightning, the spread of viruses (both biological and computer), and snow and rock avalanches all have something in common: they are based on the principle of the *chain reaction*. There are other related phenomena: Moore's Law (which has governed the computer revolution), compound interest, and PCR – the polymerase chain reaction which was used to prove that many men scheduled for execution were, in fact, innocent. You'll see in this chapter how understanding any one of these phenomena gives insights into understanding all the others. We begin with the story of the game of chess.

Chess

According to legend, the game of chess was invented by Grand Vizier Sissa Ben Dahir, who presented it to King Shirham of India as a gift.[1] In gratitude, the king offered the Grand Visir any reward requested – provided, of course, that it sounded reasonable. The Grand Vizier asked for only:

> "One grain of wheat, representing the first square of a chessboard. Two grains for the second square. Four grains for the next. Then eight, sixteen, thirty two … doubling for each successive square until the 64th and last square is counted."

The king was impressed with the apparent modesty of the request, and he immediately granted it. He took his chess board, removed the pieces, and asked for a bag of wheat to be brought in. But, to his surprise, by the bag was emptied by the 20th square. The king had another bag brought in, but then realized that the entire second bag was needed for just the next square. In fact, in 20 more squares, he would need as many bags as there were grains of wheat in the first bag! And that was only up to square 40. Legend does not record what the king then did to his Grand Vizier.

The number of grains for the last square can be calculated by multiplying 2 times itself 63 times. That doesn't take very long on a pocket calculator. But it is faster if you use the power law key (if you have a scientific calculator), marked y^x. Two multiplied by itself 63 times is 2^{63}. So put in 63 as x, and 2 as y. The answer is $9223372036854775808 \approx 0.922 \times 10^{19} \approx 10^{19}$. If you include the grains on the first 63 squares, the sum is about twice as large[2], $2 \times 2^{63} = 2^{64}$. The calculation can also be done easily on a spreadsheet.[3]

---

[1] In fact, the ancient game of chess was quite different from the modern game. Until about 500 years ago, the Queen could move only one square at a time, and the Bishop could jump over pieces in the manner of a knight.

[2] But *only* twice as large. Do you see why? Try it will smaller chessboards, maybe one with only 4 squares. You'll see that you always put one more on the last square than on

If all the grains for the last square were stacked into a cube, the cube would have about 2 million grains on each edge.[4] If each grain were 1 mm in size, then each edge of the giant cube would be 2,000,000 mm = 2,000 meters ≈ 2 km. The cube would be 2 km long, 2 km wide, and 2 km high. It would take a very large chessboard to hold this cube. If the grains had to fit on a square that was 2 cm × 2 cm, its height would be 20 billion km. At that height, it would take almost a day for light from the top of the pile to reach the bottom. This amount of wheat on the board would exceed the total world production for over a thousand years.

The amazing feature of this problem is that with just 63 steps, each one quite modest (you are only doubling) you get a really huge number. This type of rapid growth is called *exponential growth*. It is called that because you raise a number (in this case, 2) to a power (63) called the exponent. Exponential growth is the secret behind all the phenomena we discuss in this section.

We had something very similar phenomenon in the chapter on radioactivity, except there it worked backwards. Each half life, the number of remaining atoms was cut in half. That is the opposite of exponential growth, and is called *exponential decay*.

Nuclear Bomb

When a neutron hits a U-235 nucleus[5], it has a high probability of triggering the breakup of that nucleus into two large pieces, a process called "fission" (named in analogy with the fission of a biological cell). The two large pieces are called "fission fragments" (we discussed these in Chapter 4), and they are arguably the most dangerous kind of radiation that comes from nuclear bombs. In addition to the two large fragments, when U-235

---

all the preceding squares added together. If you are mathematically inclined, you might try to prove this as a theorem.

[3] For example, in Microsoft Excel, put the number 2 in location A1. In location A2, put the equation "=A1*2". Then use the spreading function to extend this to cells down to cell 64. The cell A20 will then contain the value of 2^{20}; the cell A64 will contain the value of 2^{64}.

There is an even simpler way to do this in Excel. Just put the symbols “=2^20” into any box. The carat ^ means “to the power of”, so 2^20 = 2^{20}. Excel will then evaluate this value.

[4] That’s because 2 million cubed $= (2\,\text{million})^3 = (2\times 10^6)^3 = 8\times 10^{18} \approx 10^{19}$.

[5] U-235 stands for uranium-235. Uranium is the heaviest atom that is found in relative abundance on the natural Earth. Uranium has atomic number 92, i.e. it has 92 positively charged protons, and 92 negatively charged electrons. U-235 also has 143 neutrons. The atomic "weight" is the sum of 92 protons + 143 neutrons = 235 heavy particles in the nucleus.

fissions there are an additional two neutrons usually released. These two neutrons make the chain reaction possible. If there are other U-235 nuclei around, then these neutrons might hit them, and cause additional fissions. That doubling process can continue until very large numbers of nuclei are split.

In the first fission (the first "generation"), one atom is split into two. In the second generation, two are split -- then four, eight, etc. In the 64^{th} generation, the number of atoms fissioned will be 10^{19} -- the same number as we found for the chess problem. The total number of atoms fissioned (including the ones in prior generations) is about twice this: 2×10^{19}.

How many atoms are there in, say, 10 kg of uranium? (The International Atomic Energy Agency calls that a "significant amount" – an amount that could be used in a nuclear weapon.) The answer[6] is 2.6×10^{25}, much larger than the number we fissioned in 64 generations. How many generations will it take to reach this number? To find the answer, you could keep on multiplying 2s until you get there. It won't take that long. In an additional 20 generations (84 total), you'll reach 2×10^{25}. In other words, after 84 successive doublings, every atom in 10 kg of uranium will be fissioned. How much energy is released? Each fission of a uranium nucleus releases about 30 million times as much as a molecule of TNT. So the 10 kg will be equivalent to about 30 million $\times$ 10 kg of TNT = 300 million kg = 300 kilotons of TNT. That is the fundamental idea behind an atomic bomb. (Many people prefer to use the term "nuclear bomb" today, since it is the nucleus of the atoms that fissions, and it is the nuclear energy that is released.) The first nuclear bomb released the energy of about 20 kilotons of TNT, less than what we just calculated, and that shows that not every atom was fissioned before the bomb blew itself apart.

Neutron-induced fission also takes place with plutonium-239, abbreviated Pu-239.[7] But in plutonium, typically 3 neutrons are released per stage. How many stages does it take to reach 2×10^{25}? To find the answer, multiply 3 by itself until you get this number. The answer (so you can check yourself) is in the footnote.[8] We'll talk more about nuclear chain reactions at the end of this chapter.

---

[6] If you have studied chemistry, then you may know how to calculate this number. Since the atomic weight of U-235 is 235, that implies that 235 grams of U-235 will contain one mole. One mole is 6×10^{23} atoms. (That number is known as "Avogadro's Number".) Ten kilograms of U-235 will contain $10{,}000/235 = 42.6$ times as many atoms as 235 grams, i.e. it will contain $42.6 \times \left(6 \times 10^{23}\right) = 2.6 \times 10^{25}$ atoms.

[7] Plutonium is an artificial element, created in nuclear reactors. It has atomic number 94, i.e. it has 94 protons in the nucleus, and 94 electrons orbiting the nucleus. The Pu-239 nucleus also contains 145 neutrons. The atomic weight 239 is the sum of 94 protons and 145 neutrons.

[8] The answer is 53 generations. That's even less then the number of squares on a chessboard!

We used a lot of numbers in this section, so let me summarize which are the important ones: 10 kg of U-235 is enough for a nuclear weapon. If the chain reaction proceeded by doubling, it would all be split in only 84 generations. Plutonium takes fewer generations since each fission releases 3 neutrons.

The Fetus: a chain reaction in the womb

You began life as a single cell, the result of the fusion of your father's sperm and your mother's egg. That cell then divided, in a process called fission. Then the two resulting pairs divided again. How many times did the cells have to divide to make up a complete human body? There are about 10^{11} cells in a human. As you can check, $10^{11} = 2^{37}$, so the answer is 37 stages of doubling. Even if each division stage took a day, the whole process would be over in 37 days.

So why does it take 9 months to be born? The answer is that the cells can't keep dividing that rapidly. They have to grow in between divisions, and that takes nutrients. Soon after the process begins, the growth is limited by the body's ability to bring nutrition to the dividing cells. If you did keep doubling in size with each successive generation, then the baby would double its weight in the last day. My wife claims that is exactly what being pregnant felt like.

Cancer: an unwanted chain reaction

When a baby reaches full size, the body "turns off" the chain reaction that is responsible for growth. It appears to have several mechanisms to do this. Many cells maintain the ability to turn the chain reaction back on, if it is needed. For example, if you are wounded or your skin is cut, then the local cells start reproducing again. They can fill the wound with remarkable speed, since they follow the doubling rule of the chain reaction. The potential danger of unrestricted splitting is so great, however, that our cells have several mechanisms to prevent this from happening when it isn’t needed. If all else fails, the cell can be signaled to kill itself, a process known as *apoptosis*.

If your cells are unlucky enough to receive several specific mutations, they lose this ability to commit suicide. If that happens, then the cells can grow unregulated, continuing to double and double and double. When that happens, we say that the cells have become a *cancer*. If the cells stay in one region of the body, they eventually may be limited by their ability to get nutrients, and the collection of cells is a *benign* cancer. However, if the cells are the type that can break off and drift in the blood or other body fluids to a different part of the body, then the cancer is termed *malignant*. Cancers that can spread in this way reach areas where nutrition is abundant, and they continue to grow in an unlimited chain reaction. Eventually their growth can interfere with vital body functions, and the victim dies.

The reason that cancer can be so devastating, and that people can die from it so quickly, is because it takes advantage of the chain reaction to grow with great rapidity.

The Population Bomb

In 1798, Thomas Rohr Malthus wrote his 'Essay on the Principle of Population'. It is hard to find a more influential essay in the history of public thought. In this he argued that population growth obeyed the rules of a chain reaction.[9] If each parent has, on the average, 2 children (per parent), then there will be a doubling every generation. If each parent has, on average, 1.4 children, then there will be a doubling every two generations (since $1.4 \times 1.4 \approx 2$). The doubling would take longer, but it would still lead to a chain reaction.

Malthus argued that the available food supply will not show a similar exponential growth. It will grow much more slowly, because it is limited by available land, water, and other resources. As a result, the growth of population will always outrun the growth of food. Malthus thought that the only thing that would stop this population bomb was disease and famine. Based on this observation, others argued that famine was not only inevitable, but it served an important purpose. That was a very bleak outlook on life. Malthus' analysis has been so influential that some call economics "The Dismal Science" based on this pessimistic outlook.

Many people today still think this population bomb is the ultimate disaster working on the human population. But there is an alternative to starvation: birth control. It is interesting to note that Malthus thought the bomb was imminent in 1798. We have managed quite a few doublings since then, and we are still well fed.[10] In 1968 Paul Erlich wrote "The Population Bomb" in which he predicted that massive starvation would hit the world in the 1970s. Now he maintains it will happen in the early 2000s.
But there is reason to be more optimistic. According to recent studies, world population is departing from exponential growth. The United Nations estimates made in 2003 are that the world population growth will slow, and the total population will not exceed 10 or 12 billion before it starts to decline. The reason for the slowing growth is not understood, but it is possible that humans tend to have fewer children when they become well-fed and secure. If this is true, then the secret to the end of the population bomb would then be a truly delightful one: make everyone in the world wealthy. To avoid polluting the world

---

[9] He said that population would grow in "geometric ratio." By the terminology of the time (still in use in some math classes) that means that it grows by the same factor with each generation. That factor could be 2; then we call it the doubling rule. But any factor greater than 1, e.g. 1.4, will still result in an unlimited chain reaction. With the factor 1.4, the population still doubles; it just takes 2 generations. (That's since $1.4^2 \approx 2$.)

[10] As of 2003, there appears to be enough food to feed everybody alive today. Starvation and hunger is present in the world not because of lack of food, but because available food is not reaching everybody.

(when everyone can afford SUVs), it may be necessary to increase energy conservation to match the wealth. This appears to be possible.

Mass Extinction Recovery

Sixty five million years ago, the dinosaurs were destroyed, yet the mammals survived. Why? Well, the extinctions were not as simple as many people think. It wasn't true that all the mammals survived. In fact, it was likely that 99.99% of all mammals were killed at the same time. For recovery, all it takes is for one or more "breeding pairs" to survive. Imagine, for example, that two rats made it through the bad period. Assume that rats take about a year for rats to grow and to breed. After the extinctions, the number of rats can double every year. After just 56 years (an instant in geologic time) there would be so many rats that they would completely cover the surface of the Earth like a carpet. [11]

Of course, that kind of massive rat bomb never happened. The growth of rats was limited by the availability of food, by disease, and by competition with other animals. But the example shows that it is difficult to spot great catastrophes in geologic history unless the extinctions are so great that they actually eliminate entire species, i.e. they leave no breeding pairs. The rats recovered; the dinosaurs didn't. It turns out that 65 million years ago all large animals disappeared. That may be because such animals are rare, and require lots of territory to stay alive. When 99.99% are killed, they are less likely to be able to find mates than are the little guys.

A population explosion sometimes occurs when a new species is introduced into an environment in which there are no natural predators. Twenty four rabbits were released in Australia in 1859. Seven years later (1866), 14,253 rabbits were shot for sport on the property of Thomas Austin, the person who released the original two dozens.[12]
By 1869, Austin had killed over two million on his property and he realized what a major mistake he had made. Wild rabbits are still a major pest throughout Australia.

Nobody knows what led to the plague of rats that gave rise to the story of the Pied Piper of Hamelin. If you aren't familiar with Robert Browning's poem, I recommend it highly. It is online at http://www.indiana.edu/~librcsd/etext/piper/text.html.

[11] The area of the Earth is $5\times10^{8}\,\mathrm{cm}^2$. After 56 generations, the number of rats would be $2^{56}=7\times10^{16}$. That allows an area of $10\,\mathrm{cm}\times10\,\mathrm{cm}$ for each rat, including the oceans.

[12] See http://rubens.anu.edu.au/student.projects/rabbits/history.html

DNA "fingerprinting":
the Polymerase Chain Reaction (PCR)

In every cell of your body, there is a collection of molecules called DNA[13] that contain the information that runs your body. For different people, the molecules are virtually identical. The DNA contains the "genetic code" that tells the cells how to reproduce, how to respire, how to function. But there is a tiny component that is different for different individuals, such as the parts that determine eye color. Half of your DNA came from your father and half from your mother, so your DNA is very similar to theirs, but not identical. (If it were identical, you would be either an "identical twin" or a "clone".) DNA fingerprinting consists of looking at the parts of the DNA that vary among different people. If you pick enough of these regions, you have a unique identification. Moreover, these regions will consist of parts inherited from your parents, so the fact that you are close biological relatives will be indicated by the presence of some segments that are identical to those of your father mixed with other segments that are identical to those of your mother.

The difficulty in DNA fingerprinting is that methods to "read" the code, i.e. determine the exact sequence of molecules in the relevant regions of the DNA, do not work with just a few molecules; they requires many billions of copies.

That's where the chain reaction comes it. DNA is a molecule that is designed to reproduce; it does that before the cell is split so that replicas can go in each half. Kary Mullis, a San Diego biologist and surfer (water, not web) realized the potential value of this fact, during a drive in the California mountains. His invention, called PCR (for polymerase chain reaction), a method that has transformed biology, and won him a Nobel Prize in Chemistry in 1993.[14]

Mullis realized that if he had even one DNA molecule, he could use the chain reaction to get billions of copies. The procedure involves using chemicals that will trigger duplication of the segment of the DNA molecule that is different for different people. The stages in the chain reaction are achieved by temperature cycling of the fluid containing the DNA. When the temperature is cool, the desired part of the DNA makes what is called a "complementary strand" which stays attached to the original DNA. The mixture is then heated (to near boiling) and the two strands separate. When cooled, both the original DNA and the complementary strand duplicate; these are separated by heating, and the cycle is repeated. After 32 cycles (taking less than an hour) there will be $2^{32} = 4.3 \times 10^9$ copies. This gives the scientist enough material that he can determine the exact genetic code of the fragments.

DNA fingerprinting is the method used to identify people based only a few cells from their body. The method was used to identify the remains of people at the World Trade

---

[13] DNA stands for deoxyribonucleic acid, if that helps.

[14] His Nobel Lecture is at http://www.nobel.se/chemistry/laureates/1993/mullis-lecture.html.

Center attack, and the Space Shuttle Columbia disaster. It was used to free innocent men who had been awaiting execution for crimes they didn't commit.[15] (Bits of blood left at the scene of the crime had matched their blood type, but this far more sensitive method showed the blood was not theirs.) It was used to identify rapists, beyond a reasonable doubt. It is a very reliable way to prove paternity, by the match between the father and the child. It has even been used 200 years after the father was dead -- to provide evidence that descendants of Sally Hemmings, the slave owned by President Thomas Jefferson, were also the descendants of Jefferson.

Illness and Epidemics – chain reactions of viruses and bacteria

A virus or bacteria duplicating in your body uses the chain reaction to reach enormous numbers. If your body has to devote major parts of its resources to killing the germ, then you feel sick. If it can't succeed in stopping the exponential growth, you die.

The math of a chain reaction also describes the spread of an epidemic. Consider a single person infected by the smallpox virus. That person can spread it to someone else by contact or by saliva droplets spread from the breath. If one person infects two people, and they each infect an additional two people, and that pattern continues, then it takes only 33 such stages to infect the entire world (since $2^{33} = 8.6$ billion > world population). Worse, suppose the first person infects 10 people, and they each infect 10 others. Then, after only 10 such stages, the number infected will be 10^{10}, and that is greater than the entire world population. Such spread, in the past, has been limited by the fact that people didn't travel widely, and so an epidemic could be localized. But today, with airplane travel, an infected person could infect thousands.

Note that not all illnesses are chain reactions. When humans are infected with anthrax (as in the incidents in 2001 when anthrax was sent by mail), the disease was not spread from one person to another. Those infected become ill, and some died, but the disease did not spread like a chain reaction.

Computer Viruses – an electronic chain reaction

Computer viruses obey the same laws as other chain reactions. A virus in your computer system can spread to other systems by the copying or sharing of infected programs. Email can spread like a chain reaction if it allows a message to be automatically forwarded, for example, to everyone on your mailing list, or if it contains an attached program that fools you into opening it.

---

[15] In 2003, Governor George Ryan of Illinois commuted the sentence of every prisoner, all 156 of them, scheduled to be executed.

Such computer viruses can spread with remarkable speed, in part, because the multiplication number at each stage can be large. If, for example, an infected computer infects 100 others, then the virus could spread around the world in fewer than 4 stages (since $100^4 = 10^8$, and that is greater than the total number of computers in the world), at least in principle. Of course, the infection would not spread to people who were on nobody's email list, or who run anti-virus programs that intercept and "kill" the infection.

Avalanches – a rock or snow chain reaction

A stone falls from a ledge, and knocks out two more stones. Each of them knocks out two, and so on. This is an avalanche. The doubling rule applies.

If each rock knocks out less than one additional rock, then the avalanche will fade away and die. Suppose, for example, that each rock knocks out, on the average, 0.5 additional rocks. Then after 4 stages, the number being knocked out is the number that started times $0.5^4 = \frac{1}{2^4} = \frac{1}{16}$. This typically happens when the avalanche reaches a part of the slope that is no longer steep, so the rocks are more strongly planted in the ground and harder to knock loose.

A snow avalanche can be similar, although the snow doesn't usually exist in discrete objects like rocks. And, like a rock avalanche, it will die when the slope is no longer steep.

Lightning – a chain reaction of electrons

Sparks, and their bigger relatives called lightning, are also examples of chain reactions. In fact, they are very similar to rock avalanches. Sparks occur when an electron has such a high electric voltage (see Chapter 6), that it breaks off whatever holds it and accelerates through air. If it picks up enough energy (by its repulsion from other electrons left behind), then it can break another electron off a molecule of air, doubling the number of moving electrons. The number of electrons increases exponentially, and that is the spark (or the lightning). In lightning, collisions of the electrons with the air molecules heat the air, causing it to expand rapidly (making thunder) and to glow (making the visible lightning stroke).

Compound Interest – seen as a chain reaction

Compound interest refers to the fact that you can earn interest on your interest. If you invest money at an annual rate of, say, 5%, then that means that after a year, you have 1.05 times as much as you started with. After two years, the amount is $1.05 \times 1.05 = (1.05)^2$ times greater, and after 14 years, the amount is $(1.05)^{14} \approx 2$ times the original amount. Your money will continue to double every additional 14 years. After 28 years it will have grown by a factor of 4, and after three doublings (42 years) it will have grown by a factor of 8.

Compound interest is a form of a chain reaction. The doubling creates two amounts each equal to the original, and each of these will continue to double. That's why the math is the same.

Suppose you start with \$1000, and would like to become a billionaire. All it takes is a factor of a million $\approx 2^{20}$. From this math, you can see that a factor of a million takes 20 doublings. At 14 years per doubling period, it would take 280 years, and your billion dollars would be worth a lot only if there was negligible inflation. To really become a self-made billionaire, you have to have a doubling time of no more than one or two years.

Moore's Law of Computers – Exponential Growth

The same doubling rule that we see in chain reactions occurs in other phenomena. One of the most famous ones is in computer technology. In 1965, Gordon Moore, one of the founders of the integrated circuit industry, noticed that the number of basic components that could be put on a chip had doubled every year for the previous six years. From what he knew of the technology, he expected the trend to continue, at least until 1975. By that time, instead of 50 components per chip, he predicted there would be 65,000! Below is the plot from his original paper (which was once available in pdf format at http://www.intel.com/research/silicon/moorespaper.pdf).

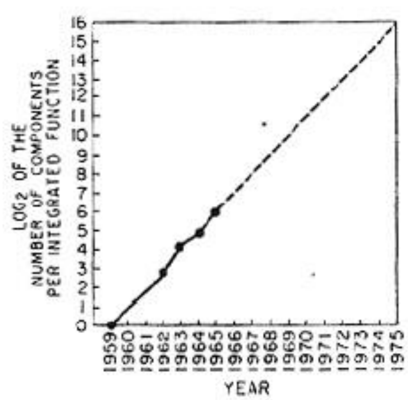


Figure: Moore's Law

Moore's prediction sounded so ludicrous that cartoonists made fun of it by taking his prediction to its extreme, and showing that it implied that someday consumers would buy their own hand-held computers, and even be able to buy them in a department store. Such a cartoon appeared in the original paper, and is reproduced below. It is hard today to imagine how ludicrous this cartoon must have appeared to most people back in 1965.

Figure: The ridiculous future if Moore's Law holds

As Moore's predictions began to come true, the newspapers picked up on it, and called the phenomena "Moore's Law." It seemed to apply to other aspects of computers besides the density of components, including processor speed and magnetic disk memory. The average doubling period, spread out over the last 35 years of the 20th century, turned out to be about 18 months. Thus, the explosion of computers that has taken place in the last 35 years is really analogous to a nuclear explosion. No wonder it has dazzled so many people.

The Nobel Prize in Physics in 2000 was given to Jack Kilby, who along with Noyce, invented the integrated circuit that allowed this expansion to take place. But we don't really understand the reason for Moore's Law. Every year – for the past two decades -- there are articles in magazines explaining why Moore's Law is soon going to fail. So far, these articles have always had "good" reasons, and they have always turned out to be wrong. I believe that Moore's Law will continue to hold for at least another decade, but I can't predict beyond that. We are about to reach the limit of smallness (since a circuit cannot be smaller than the size of an atom), but we have yet to truly exploit the third dimension (i.e. putting circuits not only alongside each other, but also on top of each other.)

Folding Paper and Damascus Swords

A particularly easy way to study the doubling rule is by folding paper. Suppose you take a sheet and fold it in half. There are then two layers. Fold again for 4 layers, and again for 8 layers. After 7 folds, there will be $2^7 = 128$ layers. An old trick is to challenge someone to fold paper 8 times. It turns out to be essentially impossible to do, because the thickness is so great. Take a newspaper sheet and try it.

An ancient but secret method for increasing the strength of metal used in swords was to fold the metal, hammer it back to the original thickness, and then repeat. This led to great strength, probably because the microscopic separation of the metal into thin layers prevented cracks from propagating through the material. The master sword makers would not do this indefinitely, but only about 20 times. What would the thickness of an individual layer be after the metal had been folded and beaten in this way? There would be $2^{20} = 10^6$ layers at that time. If the sword thickness was 1 mm = 0.1 cm, then the thickness of an individual layer would be a million times thinner, about 10^{-7} cm. That is only slightly larger than the size of an atom! It is possible that the reason that they stopped folding and doubling, is that you can't make layers any thinner, so additional folding didn't create new layers, and therefore didn't strengthen the sword. Even though the ancients didn't know the size of atoms, they may have inadvertently determined the size in this indirect way, if only they knew how to interpret their limit. (Maybe some of them did. People who work in secret sometimes make discoveries, but if they don't pass them on publicly, they may get no credit.)

Trees

Here is one last example illustrating how the doubling rule can lead, in a small number of steps, to a large number of objects. Suppose a tree has a trunk that divides into three large branches, and each of these divides into three more branches. Suppose the branches continue to divide another 6 times, and then you arrive at three leaves. How many leaves are there on the tree?[16] Do you suppose that Nature uses a trick like this to simplify the code required in the design of trees? Suppose, in addition to the doubling rule, it put in a random process. So, for example, the probability of two branches might be 50%, the probability of branching into three new branches might be 30%, and the probability of dividing into four is 20%. That would make for a more interesting tree. Take a look at actual trees and see what you think.

Next we'll combine our facts on radioactivity with the math of doubling to discuss the most famous of the chain reactions.

Nuclear Weapons Basics

As soon as it was discovered that a neutron-induced fission creates more neutrons, it was clear that there was a potential method for releasing enormous amounts of nuclear energy. The concept of the nuclear chain reaction had actually been patented in England by the nuclear physicist Leo Szilard in 1932. The first actual chain reaction was achieved by a team led by Enrico Fermi, at the University of Chicago, in 1942.

As discussed earlier in this chapter, the chain reaction makes used of the fact that more than one neutron comes out per uranium fission. If those neutrons can be made to hit other uranium nuclei, then soon the doubling rule will result in the fission of nearly every nucleus. It takes only 80 doublings. The key to achieving this is the concept of critical mass.

---

[16] $3^9 = 19{,}683$.

Critical Mass

If the uranium chain reaction is to work, there must be enough material so that the emitted neutron hits another uranium nucleus, instead of escaping in between the nuclei and out of the bomb. If enough uranium surrounds the initial fission that the neutrons will not escape, then we say that we have a "critical mass" of uranium. For many years, the value of this critical mass was highly classified. This was because many people thought it was larger than it turned out to be. The critical mass for a bomb based on uranium fission is different than that for plutonium fission. Part of this is due to the fact that more neutrons are emitted when plutonium fissions.

To make a critical mass, there must be enough material so that after each fission, more than one of the neutrons that are emitted must hit another nucleus, to keep the chain reaction going. A simple calculation indicates that this requires a sphere of uranium 13.5 cm in radius, weighing 200 kg = 440 lb.[17] There was no hope during World War II that so much U-235 could be obtained, and that may be the reason why the Germans (under the direction of the famous physicist Werner Heisenberg) abandoned the effort. But the US (under J. Robert Oppenheimer) invented ways to reduce the amount needed. According to the book “The Los Alamos Primer” (written by R. Serber during the US effort) the most important of these was to add a neutron reflector at the surface. According to this book, the critical mass can be reduced to 15 kg for U-235, and 5 kg for Pu-239. That much plutonium would fit in a cup.[18]

The term "critical mass" has worked its way from physics into our everyday language as a metaphor. One or two people, working on a problem, may not be enough. But if you assemble a "critical mass" of people, the progress can be explosive.

Uranium Bomb

The nuclear bomb that destroyed Hiroshima was a “gun” type bomb. By this I mean that a piece of U-235 was shot by a cannon at another piece of U-235; the combination was above the critical mass. The entire bomb, including cannon, weighed 4 tons. The energy release from the chain reaction was 13 kilotons of TNT equivalent. The day after Hiroshima was destroyed, President Harry Truman mistakenly announced the yield was 20 kilotons. This was the first uranium device ever exploded. It had not been tested. (The Alamogordo test was of a plutonium bomb.) The design was so simple that a test was decided to be a waste of uranium. After the bomb was dropped, there was not yet enough new uranium to make a new one, although the Oak Ridge plants were producing enough that a new bomb could be ready soon.

---

[17] This calculation is performed in the book “The Los Alamos Primer,” by Robert Serber (ISBN 0-520-07576-5) on page 27.

[18] The density of plutonium is 20 g/cm^3, so 5 kg = 5000 g would fit in 250 cm^3, which is about the volume of a standard cup.

Plutonium bombs are more difficult (see next section). For that reason, a bomb that uses uranium is the material of choice for a terrorist, since the design is so simple. But such a bomb requires highly enriched U-235, and that is not easy to make. When you dig uranium from the ground, it is 99.3% U-238, and only 0.7% U-235. It is only the rare isotope U-235 that can be used for a bomb. Separating this isotope is extremely difficult to do.

When the United States defeated Iraq in 1991, one of the conditions that Saddam Hussein agreed to was inspections of his nuclear facilities. The U.S. discovered that he had developed devices to separate U-235 from natural uranium. But these devices, instead of being the modern centrifuge or laser systems that we had anticipated, were Calutrons. This is short for "California utron", and it was the slow but sure method invented by Ernest Lawrence. Lawrence had invented this method during World War II, and his system had separated virtually all the U-235 that was used in the attack on Hiroshima.

Prior to the Hiroshima attack, a nuclear weapon had been tested in Alamogordo, New Mexico. That was the first nuclear explosion. But the Alamogordo test did not use U-235. It used an isotope of plutonium, Pu-239.

Plutonium Bomb

The bomb tested at Alamogordo, and the one dropped on Nagasaki, were both plutonium bombs, using Pu-239. Plutonium is relatively easy to get: it is produced in most nuclear reactors, including those intended to produce electric power, and then it can be separated using chemistry. However it normally has a high component of Pu-240, which is highly radioactive. This radioactivity tends to pre-detonate the bomb, i.e. make it explode before the chain reaction is complete. As a result, a special design had to be used: implosion. This is extremely difficult to design and engineer and build, and probably could not be built by a small organization such as a terrorist group. The resources of a full country (Pakistan, North Korea) are probably necessary.

The bomb dropped on Nagasaki yielded 20 kilotons of explosion. It used only 6 kg of plutonium (about 13.5 pounds). That much plutonium could easily fit in a coffee cup. The higher yield per gram (compared to uranium) results from the fact that plutonium emits more neutrons in fission than does uranium, so the reaction goes faster, and we get a more complete chain reaction before the plutonium is blown apart.

The plutonium is often arranged as a hollow shell, with explosives on the outside. The explosives drive the shell into a little blob, and compress it (even though it is solid). The compression pushes the atoms close enough together that neutrons produced in the chain reaction are unlikely to be able to leak in between them. Thus compressed plutonium has a smaller critical mass than uncompressed plutonium.

The explosives often use a special kind of explosive "lens" (a special shape in the explosive that tends to make the explosion converge on a point).

According to the chief nuclear weapons designer of Saddam Hussein, a U.S. trained physicist named Khidhir Hamza, the Iraqi bomb was not going to be a gun-style design. Instead, they would use uranium, but do an implosion in order to reduce the critical mass.[19]

Thermonuclear Weapon or "Hydrogen Bomb"

In the hydrogen bomb, deuterium and tritium are heated by a plutonium or uranium fission bomb, to the point where they overcome their natural repulsion (the nuclei of both are positively charged) and fuse. This releases energy, and neutrons. The high energy neutrons cause fission in a uranium case (usually just U-238), and that releases even more energy. The biggest hydrogen bomb ever tested (they have never been used in war) released an energy over 50 million tons of TNT. That is million, not thousand!
The "secret" of the hydrogen bomb, kept highly classified until just a few years ago, is that the plutonium bomb emits enough x-rays that they can be used, after bouncing off the uranium cases, to compress and ignite the tritium/deuterium combination. There is a second secret, although this has been public for a longer period. Instead of using tritium, the bomb can contain a stable (not radioactive) isotope of lithium called Li-6. This is a solid, which means that the material is stored at high density. The neutrons from the fission weapon break up the Li-6 to make the tritium. Thus the fuel is created in the same microsecond that the bomb is exploding. The fusion fuel is usually lithium combined with deuterium, called lithium deuteride.

Boosted Fission Weapon

You can increase the energy of a fission bomb by added a small container with tritium/deuterium gas. In the heat of the explosion, the tritium and deuterium fuse, releasing more energy and more neutrons. The additional neutrons mean that the fission bomb chain reaction becomes more complete, and that increases the yield of the bomb. A boosted fission weapon uses fusion, but it uses the fusion for neutrons to split plutonium, rather than for energy production, so it usually isn't consider a fusion bomb.

Terminology: Atomic Bombs, Hydrogen Bombs, and all that

A bomb that uses the energy of the nucleus to release energy can safely be called a "nuclear bomb." Some prominent people (starting with President Eisenhower, and lastly President George W. Bush) pronounced nuclear as "nukular" – but that is generally

[19] Hamza eventually defected, and now lives in the United States. He told his story in the book, "Saddam's Bombmaker." (ISBN 0-684-87386-9)

considered incorrect. President Harry Truman referred to the bombs dropped over Japan as “atomic bombs.” This name is still used.

The bomb based on fusion of hydrogen is often referred to as a “hydrogen bomb.” A name typically used by scientists is “thermonuclear bomb.” The word thermonuclear refers to the fact that the fusion takes place because of the high temperature (that’s the thermo part).

Fallout

Much of the danger from nuclear weapons comes from the nuclear fallout. This consists of the fission fragments from the uranium or plutonium in the bomb. Fallout is particularly bad if the bomb is exploded near the ground. (The Hiroshima and Nagasaki bombs were exploded high in the air, to maximize the blast to as much of the city as possible.) If the bomb is exploded near the ground, then a lot of dirt and other material is caught up in the fireball of the explosion. This rises in the air. Ordinarily, much of the radioactivity would occur high in the air, where it hurts nobody. But if there is a lot of dirt mixed in, then the fission fragments tend to fall with the heavy dirt, and bring the radioactivity to the ground. This is a major problem for the larger bombs.

A large fraction (over 5%) of the fission fragments are the isotope strontium-90, a highly radioactive material with a half-life of 29 years, that gets into the food supply. Back in the 1950s, when many people were worried about the long term effects of nuclear testing, the term strontium-90 was well known by the general public.

Nuclear Reactors

A nuclear reactor is a device in which a "sustained" chain reaction takes place. It doesn't involve doubling; instead, from each fission only one of the emitted neutrons (on average) hits another nucleus to cause another fission. It is as if every man and woman had, on the average, two children. Then the population would not grow. The power output from a sustained nuclear reaction doesn’t grow, but is constant.

The power comes out in the form of heat, just as it does when burning coal or gasoline. Frequently the heat is used to boil water into steam. This steam is then used to run a turbine. (A turbine is really just a fan; as the steam expands through it, it makes the fan turn.) Imagine this: a super high-tech nuclear submarine really just uses uranium to boil water!

For their fuel, commercial nuclear reactors use primarily U-235, just as in a nuclear bomb. But the uranium is not enriched to bomb quality. Recall that natural uranium has only 0.7% U-235; the rest is U-238. For use in a bomb, the U-235 has to be enriched to about 80%. But for a nuclear reactor, it only has to be enriched to about 3%. (An exception is the Canadian reactor, called "Candu". We'll discuss this in a moment.)

Why can a reactor use less enriched fuel? There are two reasons: the first is that they don't require that both neutrons hit U-235; only one. So if one of the two neutrons is absorbed, that's ok -- for the reactor, not the bomb. Having a lot of U-238 around isn't so bad.

But there is a more important reason: a nuclear reactor uses a *"moderator."* A moderator is a chemical mixed in with the fuel that tends to slow down the neutrons without absorbing them. The most popular moderators are ordinary water, H_2O, heavy water: D_2O (the D stands for deuterium, which you recall is a hydrogen atom that has both a neutron and a proton in the nucleus), and graphite (which is nearly pure carbon). The moderators consist of nuclei which are light and don't absorb neutrons. The neutrons hit the moderator, and bounce off, but in the process they lose a little energy. After enough such bounces, the neutrons are no faster than expected from their temperature. They are called *"thermal neutrons"* to reflect the fact that they have slowed down to thermal velocities.

In the commercial nuclear reactor, the fast neutrons emitted in fission bounce off the moderator and become thermal (slow) neutrons. These neutrons are more readily absorbed on other U-235 nuclei, so the enrichment (concentration) of the U-235 need not be 80%, but only 3%.

Can a reactor turn into an atomic bomb?

In the atomic bomb, they had to use fast neutrons (not moderated) in order to have the entire 80 generations over with before the bomb blew itself apart. After 80 generations, the temperature was many millions of degrees. The only reason is hasn't yet blown apart is that there wasn't enough time! With moderated neutrons, the chain reaction is much slower, since the neutrons are slower.

This is an important fact: *commercial nuclear reactors depend on using slow neutrons*. The reason this is important, is that if the nuclear reactor begins to “run away”, i.e. if the operator makes a mistake[20] and the chain reaction begins to grow exponentially (doubling), then the slowness of the neutrons limits the size of the explosion. Once the temperature rises to a few thousand degrees K, the atoms are moving faster than the neutrons, and the relative speed of the collision is too fast for the reaction to occur; the chain reaction stops. The energy released will blow up the reactor, but that energy will be about the same that you would get from TNT. It's an explosion, but it is a million times smaller than a nuclear bomb.

A chain reaction that *depends* on slow neutrons cannot give rise to a nuclear explosion. For that reason, a commercial nuclear reactor cannot blow up like a nuclear bomb. It is

[20] Unfortunately, for public confidence, the best known commercial nuclear reactor operator is Homer Simpson.

important to know this, and to be able to explain the logic to the public, since this fact is not widely known.

There are real dangers from nuclear reactors (see the China Syndrome below). Blowing up like a nuclear bomb is not one of them.

> **Why are slow neutrons more likely to be absorbed on U-235?**
> The physical reason is simple: a slowly moving neutron feels the nuclear force for a longer time, and is more readily deflected towards the U-235 nucleus.Of course, the slow neutrons are also more strongly attracted to U-238. But the effect turns out to be much stronger for U-235. So having slow neutrons means you can use 3% enriched U-235 instead of 80%.

In Canada, their nuclear reactors use D_2O as a moderator. This is more expensive, but even more effective in slowing neutrons. As a result, they can use natural unenriched uranium, which is only 0.7% U-235. Their reactor is called a "Candu" reactor, after Canada and deuterium.

Plutonium Production

In a nuclear reactor, only one of the neutrons from uranium fission is used to produce another fission. The other is absorbed, possibly by control rods (to prevent a growing chain reaction) or possibly by U-238. When U-238 absorbs a neutron, it becomes U-239. This is radioactive, and decays (it emits an electron and a neutrino, and has a half-life of about 23 minutes) to an isotope of neptunium: Np-239. This isotope of neptunium is also radioactive. It emits an electron and a neutrino, with a half-life of 2.3 days, to turn into the very famous isotope of plutonium, the one that can be used for nuclear weapons, Pu-239.

That is how we manufacture plutonium. We make it from U-238 by hitting it with neutrons in a nuclear reactor. The plutonium is a different chemical element from uranium, so when the fuel is removed, the plutonium can be chemically separated. That is not hard to do. The extraction of plutonium is called "uranium reprocessing." When we give nuclear power plants to underdeveloped countries, we do not allow them to do their own reprocessing, for fear that they would get a supply of plutonium in this way. Of course, we do give them nuclear fuel to run the reactors – but that is a mixture of U-235 and U-238, with too large of a fraction of U-238 for it to work as a bomb.

Breeder Reactors

The Pu-239 is usually not considered nuclear waste, because it can be used itself to run a nuclear reactor. It is nuclear fuel. Moreover, if you put it in a nuclear reactor, you get three neutrons per fission instead of two. In a reactor, operating at constant (not exponentially growing) power, you want only one neutron per fission to produce another fission. What do you do with the extra two neutrons? Answer: put U-238 in the reactor, and make more plutonium.

Thus a reactor can make (out of U-238) more Pu-239 fuel than it consumes! Such a reactor is called a "breeder" reactor. It has the potential of turning all uranium, not just 0.7% of it, into nuclear fuel, and thereby increases the available fission fuel by a factor of 140. The time to double fuel, in a breeder reactor, is about ten years.

There has been public opposition to breeder reactors. The two most common objections are:

1. "The plutonium economy." Breeder reactors would allow much greater use of nuclear power, but it means that plutonium would be widespread. Besides the fact that plutonium is radioactive, and therefore dangerous, some might be diverted by terrorists to make nuclear bombs. Proponents respond that the dangers of plutonium has been greatly exaggerated, and that terrorists would not be able to make plutonium bombs because it is extremely difficult to get the required implosion to work adequately.
2. Reactor explosion. The most efficient kind of breeder reactor would use fast, not slow, neutrons. This is called a "fast breeder." But if fast neutrons are used, then the main safety aspect of the ordinary reactor is lost. In a fast breeder the chain reaction could spread uncontrolled, and instead of just a meltdown, the reactor really could explode like an atomic bomb. Proponents respond that they would put in lots of other safety systems that would prevent this from happening.

Dangers of Plutonium

Plutonium has been called "the most toxic material known to man." There is widespread fear of plutonium and of something referred to as a potential "plutonium economy". Because plutonium is so important in public discussion, it is worthwhile giving some of the physics facts.

Here are the key facts: Plutonium is toxic both because of its chemical effects and because of its radioactivity. The chemical toxicity is similar to that of other "heavy metals" and is not the cause for the widespread fear. The dangers are different for ingestion and for inhalation.

Ingestion of Plutonium

For acute radiation poisoning, the lethal dose is estimated to be 500 milligrams (mg), i.e. about 1/2 gram. A common poison, cyanide, requires a dose 5 times smaller to cause death: 100 mg. Thus for ingestion, plutonium is very toxic, but five times less toxic than cyanide. There is also a risk of cancer from ingestion, with a lethal doze (1 cancer) for 480 mg.

Inhalation of Plutonium Dust

For inhalation, the plutonium can cause death within a month (from pulmonary fibrosis or pulmonary edema); that requires 20 mg inhaled. To cause cancer with high probability, the amount that must be inhaled is 0.08 mg = 80 micrograms. The lethal dose for botulism toxin is estimated to be about 0.070 micrograms = 70 nanograms.[21] Thus botulism toxin is over a thousand times more toxic. The statement that plutonium is the most dangerous material known to man is false. But it is very dangerous, at least in dust form.

How easy is it to breathe in 0.08 mg = 80 micrograms? To get to the critical part of the lungs, the particle must be no larger than about 3 microns. A particle of that size has a mass of about 0.140 micrograms. To get to a dose of 80 micrograms requires 80/0.14 = 560 particles. In contrast, the lethal dose for anthrax is estimated to be 10,000 particles of a similar size. Thus plutonium dust, if spread in the air, is more dangerous than anthrax – although the effects are not as immediate.

Depleted Uranium

When U-235 is enriched, there is some U-238 left over. This is called “depleted uranium.” It is about half as radioactive as ordinary uranium since the U-235 and the radioactive isotope U-234 are gone. The remaining U-238 does decay by emitting an alpha particle, with a half life of 4.5 billon years, roughly the age of the Earth. That’s why there is so much left on the Earth – only half of the original U-238 has decayed.

In contrast, U-235 has a half-life of 0.7 billion years. In the 4.5 billion year age of the Earth, it has gone through 4.5/0.7 = 6.5 half lives. That has reduced its abundance by a factor of $2^{6.5} \approx 90$. That’s why there is so little left.

Depleted uranium is used by the military for certain kinds of weapons, particularly shells that are used to attack tanks and other armored vehicles. Depleted uranium is not used because of its radioactivity, but because of two other features it has:

1. It is very dense. With a density of 19 grams per cubic centimeter, it is almost twice as dense as lead. That is important for penetration.
2. When it hits a metal shield, it tends to form highly concentrated streams, instead of spreading out and splattering. This also helps it to penetrate armor.

[21] The toxicity of chemicals such as botulism toxin is not well known, since we don’t do experiments on humans, and many people feel that experiments on animals is also improper. Some people estimate that the LD50 for botulism may be as low as 3 ng (rather than 70).

People oppose the use of depleted uranium because it leaves radioactive material on the battlefield. Proponents say that the danger of radioactivity is small compared to the damage done by war, and that the alternative (lead) is also highly poisonous.

A nuclear reactor 1.7 billion years ago in Gabon, Africa

In 1972, the French discovered that the uranium they were mining in Gabon, Africa (at a location known as Oklo) did not have 0.7% U-235, but closer to 0.4%! At first they were worried that someone had been secretly stealing U-235, although no one had figured out how they could have extracted it from uranium ore.

French scientists finally discovered that the U-235 had been destroyed by fission, about 1.7 billion years ago. Back then, the fraction of U-235 had been much larger than it is now. (That's because it decays faster than U-238.) Instead of 0.7% of natural uranium (the current value) it would have been over 3%.

3% is large enough to use in a nuclear reactor, provided that there is water around to serve as a moderator. That is what we now believe happened in Gabon. Water seeping into the ground moderated the neutrons, and turned the uranium deposit into a natural nuclear reactor. When the reactor overheated, the water was vaporized, and the moderation stopped. So the reactor was self-regulating, and it didn't blow up. The power output has been estimated to be several kilowatts. Fifteen regions in three uranium ore deposits have been found in Gabon that were once nuclear reactors.

U-235 was burned, and it dropped below the (then) natural level of 3%. It produced plutonium and fission fragments. Eventually the uranium dropped to a lower level and the reactor turned off. Remarkably, despite abundant groundwater, the plutonium and fission fragments drifted through the rock less than 10 meters over the next 1.7 billion years.

Figure
A miner pointing to the uranium ore in Gabon that had once undergone a chain reaction.
(Image from Astronomical Image of the Day.)

Nuclear Reactor Fuel Requirements

To get a gigawatt of electric power from a nuclear reactor, for a year, you must consume some uranium. The amount is surprisingly small: about 1 ton of U-235, which (if pure) takes a volume of about a cubic foot. This has to be extracted from ordinary uranium that would fill up a cube 2 meters on a side. If you are interested in how I got this number, you can read the following calculation:

Uranium Fuel Calculation

We want to calculate the amount of U-235 needed to run a 1 gigawatt power plant for a year. We'll do a simplified calculation that will give us an approximate answer. As we said earlier, each fission of U-235 produces about 200 MeV of energy. Let's convert that to joules. $1\,\text{eV} = 1.6\times10^{-19}$ joules (J), so $200\,\text{MeV} = 200\times10^{6}\times\left(1.6\times10^{-19}\,\text{J}\right) \approx 3\times10^{-11}\,\text{J}$.

How many do we need for a gigawatt-year of energy? A year is 3×10^{7} seconds[22]. A gigawatt is 10^{9} joules per second. So the number of joules in one year is $E = 10^{9}\,\tfrac{\text{J}}{\text{s}}\times 3\times10^{7}\,\text{s} = 3\times10^{16}\,\text{J}$.

So the number of fissions needed N is the energy needed divided by the energy per fission: $N = \left(3\times10^{16}\text{ joules}\right)/\left(3\times10^{-11}\text{ joules per fisson}\right) = 10^{27}\text{ fissons}$. So we need 10^{27} atoms of U-235 to produce a gigawatt for a year.

We assumed that all of the energy goes into electric power. But that isn't true – only about a third does. So we really need 3×10^{27} U-235 atoms.

One mole contains 6×10^{23} atoms. So we need $\left(3\times10^{27}\text{ atoms}\right)/\left(6\times10^{23}\,\tfrac{\text{atoms}}{\text{mole}}\right) = 5000\,\text{moles}$. Each mole weighs 235 grams (since there are 235 protons and neutrons in each atom). So the weight of U-235 that we need is $5000\,\text{moles}\times 235\,\tfrac{\text{grams}}{\text{mole}} \approx 10^{6}\,\text{grams} = 1\,\text{ton}$ of U-235. Uranium has a density of 19 grams per cubic centimeter. So the amount of U-235 needed, 10^{6} grams, is $10^{6}/19 \approx 50{,}000$ cubic centimeters, which is a cube with sides of 37 cm, a little more than a foot. So remember it this way: the amount of U-235 required is about a cubic foot.

This U-235 is found in natural uranium, but it is only 0.7%, i.e. it is 0.007 of the natural uranium. So the amount of natural uranium it takes to run a nuclear reactor for a year is about 1 ton/0.007 = 140 tons = 140×10^{6} grams. With a density of 19 grams per cubic centimeter, this works out to $\left(140\times10^{6}\right)/19 \approx 7.4\times10^{6}\,\text{cm}^3$, which is a cube with sides of about 2 meters.

[22] That's the number you get if you take 60 seconds per minute, 60 minutes per hour, 24 hours per day, and 365 days per year:
$60\,\tfrac{\text{seconds}}{\text{minute}}\times 60\,\tfrac{\text{minutes}}{\text{hour}}\times 24\,\tfrac{\text{hours}}{\text{day}}\times 365\,\tfrac{\text{days}}{\text{year}} \approx 3\times10^{7}\,\text{seconds}$.

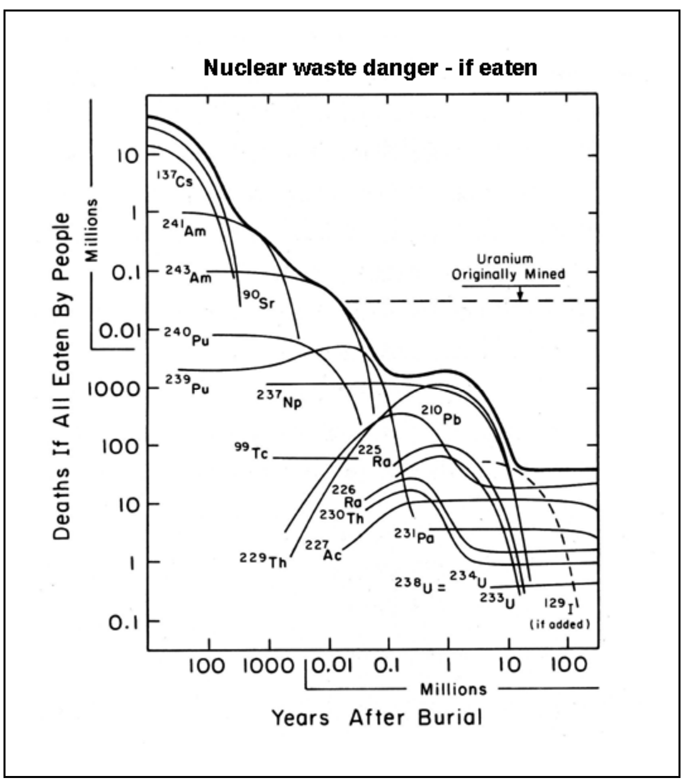
Nuclear Waste

The fission fragments from uranium all come from the uranium, so their weight is comparable. Thus, a year of operation of a nuclear power plant will produce about one ton of fission fragments. There may be a comparable amount of plutonium produced, but as discussed above, that is considered valuable fuel, and will be removed by chemical reprocessing. That's why the plutonium is not usually considered part of the waste. It is also much less radioactive than the fission fragments, since its half-life (24,000 years) is so long.

If they were concentrated, the fission fragments would take up a few cubic feet of volume. But it is expensive to do that, and so they are normally mixed in with larger amounts of unspent fuel, primarily U-238. This fuel with its fission fragments makes up the high level radioactive waste of nuclear energy.

Most of the fission fragments are radioactive. They are the same particles that cause radioactive fallout. Some of them have half-lives of a few seconds. Some have half-lives of years. We already discussed Strontium-90, which makes up 5% of the fission fragments, and has a half-life of 28 years.

The following interesting plot shows this decrease in radioactivity. It was made specifically to address the dangers of the lingering radioactivity. So the vertical axis shows the deaths that would occur this radiation were all put inside people. Obviously that can't happen, but the plot is interesting anyway.



Suppose we "turn off" the chain reaction. We can do this by removing the moderator, or by putting in special "control rods" that absorb neutrons. Then the reactor will still produce heat from the radioactive decay of the remaining fission fragments. So the reactor continues to produce power, although the power level continues to decrease.

Look at the dark line that lies on top of all the others. Notice the scale on both axes is not linear; each tick mark increases the amount by a factor of ten. In the units of this plot, right after the material is removed from the reactor there is enough radioactivity to kill between 10 and 100 million people. (Let's take the value to be 50 million.) Check: can you see that on the plot?

One hundred years later, there is still radioactivity. It has dropped to a level of about 1 million. In other words, it has decreased from 50 to 1, i.e. it is down by a factor of 50, to about 2% of its original level.

After 10,000 years, the radioactivity has dropped to about 0.1 million. At this point it is no more radioactive then the original uranium that was taken out of the ground to produce it. So the net effect, if this radioactivity is buried, is that the average radioactivity of the ground has been reduced by the "burning of uranium."

Yucca Mountain

The storage of nuclear waste has become a very contentious issue. A waste storage location is under construction in Nevada at a location called "Yucca mountain." Opponents argue that the nuclear waste stays radioactive for thousands of years, and nobody can certify that the site will remain leak-proof or secure against terrorists for that long a period. Proponents argue that the site has been geologically stable for a long time, and that the dangers of nuclear waste are exaggerated. They point to the fact that even in the ground-water rich area of the prehistoric Gabon nuclear reactor, the nuclear waste only drifted 10 meters. Opponents argue that the one example of Gabon does not prove that the risk at other locations would necessarily be low.

The China Syndrome

The term "China Syndrome" was originally invented by someone with a strange sense of humor to describe the worst possible nuclear reactor accident. (Most people seem to think there is something worse: a reactor becoming a nuclear bomb. But, as described above, that is not possible because the uranium is not sufficiently enriched.)

In the China Syndrome, the water that is usually being boiled by the chain reaction, suddenly leaks away. There is no water to boil. What would happen in this "loss of coolant" accident? Can you guess?

The first thing is surprising to most people: the chain reaction stops. The reason is that the cooling water is also a moderator; it slows neutrons. So when the water is gone, the neutrons are not moderated. That means that most neutrons are absorbed on U-238, which does not give a chain reaction. So the chain reaction stopped.

> **Interesting flub by Senator:** When the Chernobyl Nuclear Reactor underwent a similar accident, the Russians announced that the chain reaction had stopped. The chairman of the Senate Intelligence Committee announced on television that this was a "blatant lie." He was confusing the chain reaction with the decay of the remaining fission fragments. He knew the radioactivity hadn't stopped, but didn't realize that the Soviets were being completely honest. The fact that the chain reaction had stopped was important; it meant that the level of power being produced had dropped enormously. (Remember this, if you become a Senator!)

The chain reaction stops, but there is still the "waste heat" from the fission fragments. Without the cooling water, the reactor gets hotter and hotter. The fuel finally melts. It melts through its containers and forms a puddle at the bottom of the steel reactor vessel. The fuel puddle keeps on getting hotter and hotter. The steel reactor vessel melts. The fuel falls into the ground. It keeps on getting hotter. The soil and rock melts. The fuel just keeps on going -- all the way "to China".

No, obviously it won't reach China. (Besides, China isn't on the other side of the Earth.) It won't get too far, because it spreads out, and that allows it to cool. But in doing this, it has broken through the steel vessel that is supposed to keep it from the environment. Any gases that are in the fuel pellets will escape into the atmosphere. It is these gases (and some volatile elements, such as iodine) that caused the most damage at Chernobyl. There is a huge amount of radioactivity in the reactor -- enough to kill 50 million people (if they ate it). Even a small amount leaked into the atmosphere can do enormous damage. As we stated in Chapter 4, the number of expected deaths from Chernobyl (assuming the linear hypothesis) is about 24,000. It is difficult to imagine a worse accident than the Chernobyl one, so the 24,000 is a much more reasonable estimate than the 50 million.

24,000 deaths is a pretty frightening number. Is nuclear power worth it? Why not just use something else, such as solar power? Well -- people may not want to use solar energy until it is as cheap as oil. (That will happen sometime in your lifetime -- a Muller prediction.) So in the meantime, let's just use something safe: oil.

Is oil really safe? It pours lots of carbon dioxide into the atmosphere. The consequences of that are debated, but most people think the result will be serious global warming. How bad is that? How would you compare it to 24,000 deaths? Some people might argue that the Afghan war is one of the consequences of our use of oil. Why would we have bases in Saudi Arabia, if oil weren't so important to us?

Incidentally, the Chernobyl power plant had a terrible design. It didn't even have a containment building, like we have in the US. If it did, there may very well have been virtually no deaths. So is it fair to think of US Nuclear Power plants in terms of Chernobyl?

Other things are dangerous too. If you are unfamiliar with the tragedy of Bhopal, look it up on the web. In 1984, a gas leak from a chemical plant killed 5,000 people in the town of Bhopal in India. Some people have estimated that the total number of deaths from this accident will eventually reach 20,000.

Nuclear Waste Storage

What should we do with the waste, the fission fragments left over from burning U-235? Some people say "bury it." Put it back into the ground. But what if it gets into the ground water? Most people assume that that would be horribly bad. Therefore, they argue, it must be put in a very stable geologic mine, some place where it will undergo no disturbance for 10,000 years. Such a location has been prepared in Nevada, but opponents say that even this site can't be certified for 10,000 years. Who knows what kind of government we will have then!

Other people argue that the danger is greatly exaggerated. If all the radioactivity were released, in a fire, for example, then the number of deaths is about 24,000. (It happened -- Chernobyl. And these deaths aren't even observable, as we showed in the chapter on Radioactivity.) If instead, we bury the waste in the ground, it will easily be at least 24,000 times safer! So we don't even need a special location.

What do you think? Keep in mind that there is a real public fear of radioactivity that makes it very difficult to make a rational decision. Any governor who accepts radioactive waste storage in his state, is very likely to be challenged by people who feel that any level of radioactivity is too much. How would you handle this? How can you balance the real risks vs. the perceived risks, and still be reelected?
Some people say, avoid all this complication! Just put the waste into rockets, and send it into the Sun! But people who say this are ignoring the possibility of an accident. What is the probability that the rocket will fail, and fall back to earth, releasing all the radioactivity?

Is waste storage a technical problem? Many scientists think it is, and are trying to find a clever technical solution. But it seems that the issue is dominated by the public perception problem. The politician must find a solution that seems safe even to those who are unaware of the natural radioactivity in the environment. It is a very tough political problem.

Coal burning plants bury their waste in the ground. They are not radioactive, but the ashes are very high in carcinogens. What if these get into the ground water? How safe is coal, as an alternative to nuclear?

Present Stockpile

The United States currently has 12,500 nuclear weapons, although not all of these are in active use. In all these weapons, there are about 10 different "designs", but most involve a combination of fusion and fission. Russia has a similar stockpile. Both the US and Russia are in the process of dismantling many of the missiles. The historical reason for the huge stockpile is interesting. The US feared a surprise attack from Russia (replay of Pearl Harbor?). The US assumed that most of its weapons would be destroyed in such an attack. It wanted to make sure that even if only 1% of its weapons survived, they would be enough to destroy Russia. The assumption was that if Russia knew this, they would never attack. See the movie Dr. Strangelove for an ironic account of the possible consequences of this strategy.

The big issue for the stockpile now is reduction (through treaties) and "stockpile stewardship". This refers to the fact that as our weapons get old, some people argue that they may fail. In olden days, we would assure their functionality through periodic testing, but we have now entered an era when we have decided against more testing. (This is largely an attempt to keep other nations from developing nuclear weapons.) So there is a large program at Livermore and Los Alamos to try to develop methods of testing the reliability of the weapons without having to set off any of them. It is a big technical challenge.

Books

A fascinating historical book on this subject is "The Making of the Atomic Bomb" by Richard Rhodes. Another is "The Los Alamos Primer" by Robert Serber, published by the University of California Press. Robert Server was one of the principal designers of the nuclear weapons used in World War II, and his primer is based on the once highly-classified lectures he gave in Los Alamos to introduce physicists to nuclear weapons design.

Most of the material about nuclear weapons was once classified, but it is not now. The Department of Energy has a site that lists previously classified material: www.osti.gov/opennet/. In particular, see the document RDD-7. See also a paper by Richard Garwin titled "Maintaining Nuclear Weapons Safe and Reliable under a CTBT", dated May 31, 2000. CTBT stands for "Comprehensive Test Ban Treaty".

Quick Review

The doubling law takes you from small numbers, to extremely high numbers, in a relatively small number of generations (e.g. 64, as in the squares on a chess board). Chain reactions can involve doubling, tripling, or any other factor greater than one (such as 1.4). The classic chain reaction is the one that takes place in an atomic bomb. That is made possible because in a neutron-induced fission, 2 or 3 additional neutrons are released, and these can trigger further fissions. In 64 to 84 generations, an entire mole of material (6×10^{23} atoms) can be split.

Other chain reactions include the growth of the fetus, the growth of a cancer, and the spread of a virus (both biological and computer). The population bomb was thought by Malthus to be similar, but it is presently slowing down. (Population explosions do occur after mass extinctions, and when foreign animals are introduced into a land in which they have no natural enemies, such as rabbits in Australia.) The concept of a chain reaction was developed into a practical tool called the "polymer chain reaction" (PCR) which is very valuable in biology, and can be used to identify people from their DNA. Other examples of chain reactions include avalanches (rock, snow, and electrons – as in lightning and sparks). The math of the chain reaction is identical to that of compound interest, and to Moore's Law of computer technology growth. The number of layers in a Damascus sword doubles in the manufacture with every folding.

"Atomic bomb" is the common name for a nuclear weapons based on the chain reaction of U-235 or Pu-239. U-235 is a rare (0.7%) isotope of uranium that is difficult to separate. In World War II, Lawrence did this with a Calutron, and Saddam Hussein chose this same approach in his weapon program. Plutonium is manufactured in nuclear reactors, and it is easy to separate from other chemicals, but it requires a difficult design (using implosion) to use in a nuclear weapon. The bomb explodes when the uranium or plutonium is collected into a "critical mass", a blob of material big enough that most neutrons produced hit a nucleus and trigger fission instead of leaking out.

A thermonuclear bomb, also called a "hydrogen bomb", is a two-stage weapon, in which the fission primary ignites a secondary that contains two isotopes of hydrogen (deuterium and tritium). The fission fragments from the primary (and from a uranium shield) are the most dangerous part of the residual radioactivity. If the bomb was exploded at low altitude, then dirt mixed with the fission fragments causes these radioactive pieces to fall out rapidly, and that could cause greater deaths than the explosion itself. The worst fallout is strontium-90.

At the peak of the "cold war", the US and the Soviet Union had over 10,000 nuclear warheads which could be launched at the other country. Nuclear weapons are no longer tested (in part, to reduce proliferation), and "stockpile stewardship" refers to the problem of making sure that these weapons still work.

Nuclear reactors are based on the chain reaction, but they normally work with a neutron multiplication of 1, so the reaction doesn't grow. Nuclear reactors use moderators to slow the neutrons. This increases the probability that a neutron will be attracted to a nucleus. If the moderator is lost (e.g. the water leaks out) then the chain reaction stops. A nuclear reactor that depends on a moderator cannot explode like an atomic bomb. If the fuel collects at the bottom of the reactor, it will continue to heat from the remaining radioactivity of the fission fragments, and this can lead to the "China Syndrome."

Nuclear waste consists of the long-lived fission fragments. It takes about 10,000 years for the radioactivity to drop below the level of the original uranium that was removed from the ground. Proponents of nuclear power argue that the nuclear waste is far safer, since it is placed in special locations isolated from ground water, unlike the original uranium.