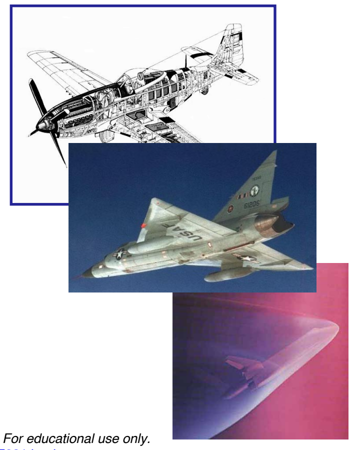
Induced Drag and High-Speed Aerodynamics

Robert Stengel, Aircraft Flight Dynamics, MAE 331, 2018

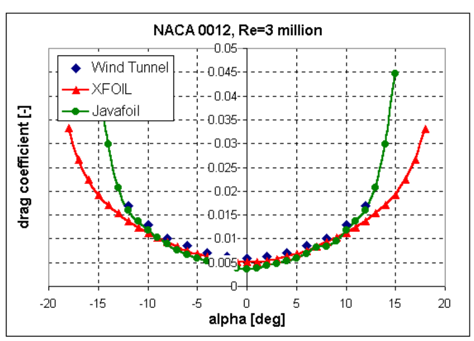
- Drag-due-to-lift and effects of wing planform
- Effect of angle of attack on lift and drag coefficients
- Mach number (i.e., air compressibility) effects on aerodynamics
- Newtonian approximation for lift and drag



Reading:
Flight Dynamics
Aerodynamic Coefficients, 85-96
Airplane Stability and Control
Chapter 1

http://www.princeton.edu/~stengel/MAE331.html
http://www.princeton.edu/~stengel/FlightDynamics.html

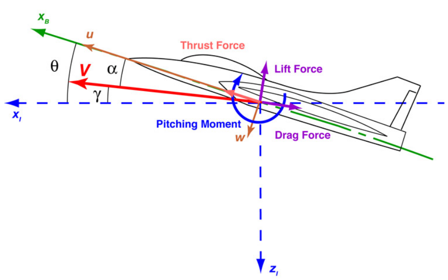
Induced Drag



Aerodynamic Drag

$$Drag = C_D \frac{1}{2}\rho V^2 S \approx \left(C_{D_0} + \varepsilon C_L^{\ 2}\right)\frac{1}{2}\rho V^2 S$$

$$\approx \left[C_{D_0} + \varepsilon\left(C_{L_o} + C_{L_\alpha}\alpha\right)^2\right]\frac{1}{2}\rho V^2 S$$



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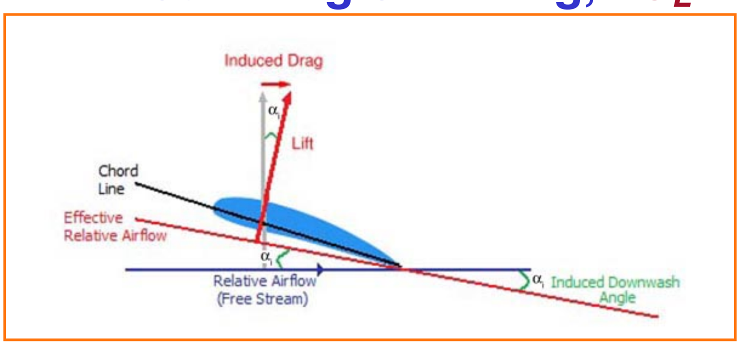
Induced Drag of a Wing, εC_L^2



- **Lift produces downwash (angle proportional to lift)**
  - **Downwash rotates local velocity vector clockwise in figure**
  - **Lift is perpendicular to velocity vector**
  - **Axial component of rotated lift induces drag**
- **But what is the proportionality factor, ε?**

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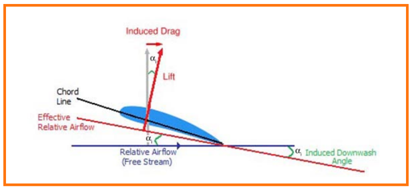
Induced Angle of Attack

$$C_{D_i} = C_L \sin \alpha_i, \quad \text{where}$$

$$\alpha_i = C_L / \pi e AR, \text{ Induced angle of attack}$$



$$C_{D_i} = C_L \sin\left(C_L / \pi e AR\right) \triangleq C_L^{\,2} / \pi e AR$$

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Three Expressions for Induced Drag of a Wing

$$C_{D_i} = \frac{C_L^{\,2}}{\pi e AR} \triangleq \frac{C_L^{\,2}(1+\delta)}{\pi AR} \triangleq \varepsilon C_L^{\,2}$$

$e =$ Oswald efficiency factor

$= 1$ for elliptical distribution

$\delta =$ departure from ideal elliptical lift distribution

$$\varepsilon = \frac{1}{\pi e AR} = \frac{(1+\delta)}{\pi AR}$$

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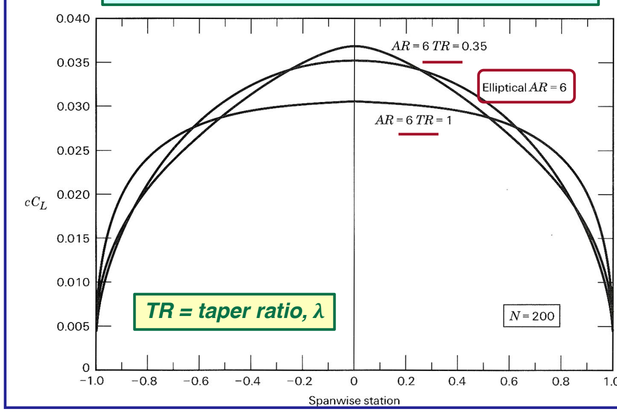
Spanwise Lift Distribution of Elliptical and Trapezoidal Wings

Straight Wings (@ 1/4 chord), McCormick



For some taper ratio between 0.35 and 1, trapezoidal lift distribution is nearly elliptical

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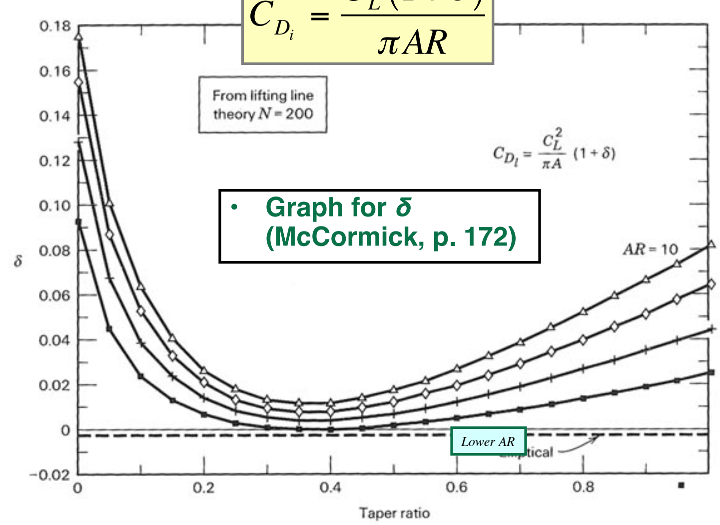
Induced Drag Factor, *δ*

$$C_{D_i} = \frac{C_L^2 (1+\delta)}{\pi AR}$$



- **Graph for *δ* (McCormick, p. 172)**

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Oswald Efficiency Factor, *e*

$$C_{D_i} = \frac{C_L^2}{\pi e AR}$$

Empirical approximations for *e*

Pamadi

$$\kappa = \frac{AR\,\lambda}{\cos\Lambda_{LE}}$$

$$R = 0.0004\kappa^3 - 0.008\kappa^2 + 0.05\kappa + 0.86$$

$$e \approx \frac{1.1 C_{L_\alpha}}{R C_{L_\alpha} + (1-R)\pi AR}$$

Raymer

$$e \approx 1.78\left(1 - 0.045 AR^{0.68}\right) - 0.64 \quad \text{[Straight wing]}$$

$$e \approx 4.61\left(1 - 0.045 AR^{0.68}\right)\left(\cos\Lambda_{LE}\right)^{0.15} - 3.1 \quad \text{[Swept wing]}$$

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Maximum Lift-to-Drag Ratio

Maximize *L/D* by proper choice of C_L

$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{C_L}{C_{D_o} + \varepsilon C_L^2}$$

$$\frac{\partial(L/D)}{\partial C_L} = 0$$

$$\frac{\partial(L/D)}{\partial C_L} = 0 = \frac{\left(C_{D_o} + \varepsilon C_L^2\right) - C_L\left(2\varepsilon C_L\right)}{\left(C_{D_o} + \varepsilon C_L^2\right)^2} = \frac{\left(C_{D_o} - \varepsilon C_L^2\right)}{\left(C_{D_o} + \varepsilon C_L^2\right)^2}$$

$$\left(C_L\right)_{(L/D)_{\max}} = \sqrt{\frac{C_{D_o}}{\varepsilon}}$$

$$\left(L/D\right)_{\max} = \frac{1}{2\sqrt{\varepsilon C_{D_o}}}$$

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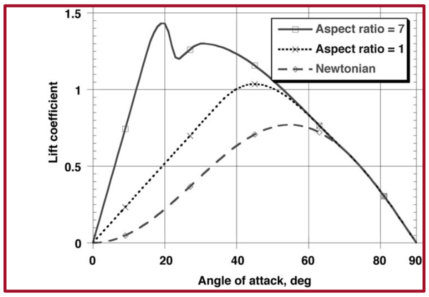
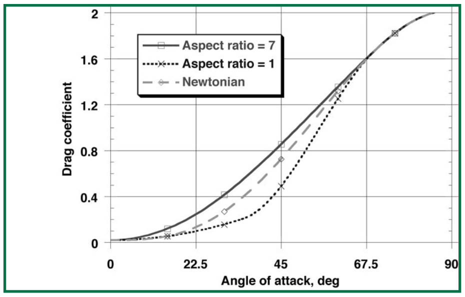
Lift and Drag Coefficients Over Large Angles of Attack ($0° < \alpha < 90°$)



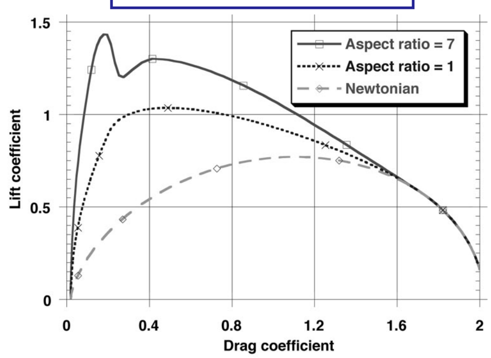


All coefficients converge to Newtonian-like values at very high angle of attack

Low-*AR* wing has less drag than high-*AR* wing at given *α*

Lift vs. Drag for Large Variation in Angle of Attack ($0° < \alpha < 90°$)

Subsonic Lift-Drag Polar



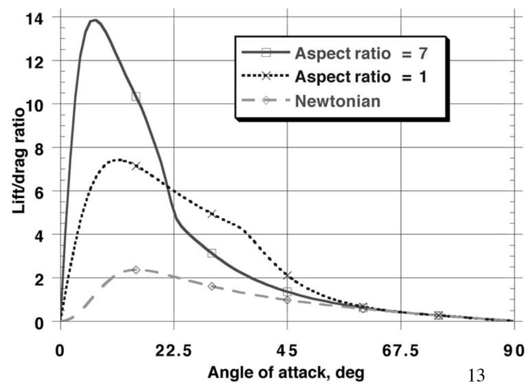
Low-*AR* wing has less drag than high-*AR* wing, but less lift as well

High-*AR* wing has the best overall subsonic *L/D*

Lift-to-Drag Ratio vs. Angle of Attack

- Performance metric for aircraft
- High-*AR* wing: Best overall subsonic L/D
- Low-*AR* wing: Best L/D at high angle of attack

$$\frac{L}{D} = \frac{C_L \bar{q} S}{C_D \bar{q} S} = \frac{C_L}{C_D}$$



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Newtonian Flow and Aerodynamic Forces

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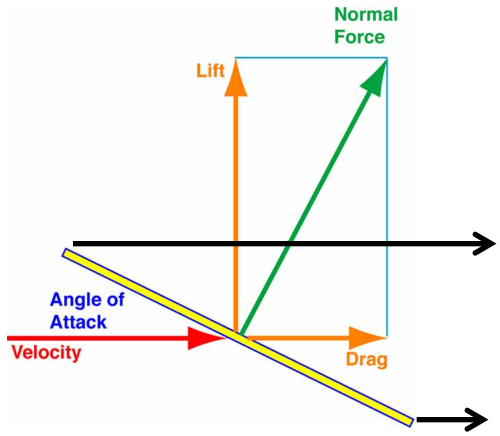
Newtonian Flow

- No circulation
- “Cookie-cutter” flow
- Equal pressure across bottom of a flat plate
- Flow brought to a halt at the surface



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Newtonian Flow

Normal Force

$$\text{Normal Force} = \left(\frac{\text{Mass flow rate}}{\text{Unit area}}\right) \times \left(\text{Change in velocity}\right) \times \left(\text{Projected Area}\right) \times \left(\text{Angle between plate and velocity}\right)$$

$$\begin{aligned} N &= (\rho V)(V-0)(S\sin\alpha)(\sin\alpha) \\ &= (\rho V^2)(S\sin^2\alpha) \\ &= (2\sin^2\alpha)\left(\frac{1}{2}\rho V^2\right)S \\ &\equiv C_N\left(\frac{1}{2}\rho V^2\right)S \end{aligned}$$

$$C_N = 2\sin^2\alpha$$

$\alpha =$ Incident flow angle

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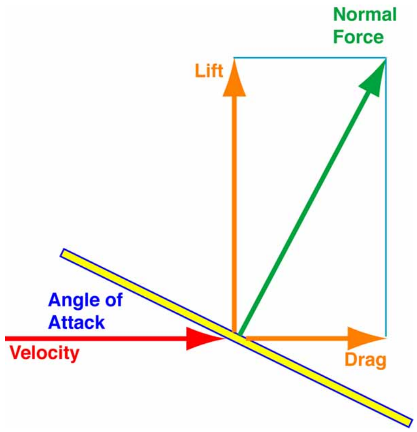
Newtonian Forces on a Flat Plate



Normal Force Coefficient

$$C_N = 2\sin^2\alpha$$

Lift and Drag

$$\textit{Lift} = N\cos\alpha$$

$$C_L = \left(2\sin^2\alpha\right)\cos\alpha$$

$$\textit{Drag} = N\sin\alpha$$

$$C_D = 2\sin^3\alpha$$

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Aerodynamic Force Estimation for a Hypersonic Aircraft

Integrate differential normal force over the aircraft surface, accounting for varying surface incidence (i.e., angle) to the flow

$$\mathbf{f}_B = \int_{Surface} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} dx\,dy\,dz = \begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix}$$

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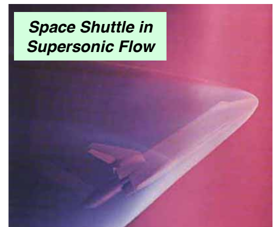
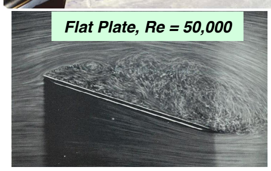
Application of Newtonian Flow

- But where does the airflow go?
- Hypersonic flow (M ~> 5)
  - Shock wave close to surface (thin shock layer), merging with the boundary layer
  - Flow is ~ parallel to the surface
  - Separated upper surface flow

- All Mach numbers at <u>high</u> angle of attack
  - Separated flow on upper (leeward) surfaces


Space Shuttle in Supersonic Flow


High-Angle-of-Attack Research Vehicle (F-18)


Flat Plate, Re = 50,000

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Historical Factoid

Conversions from Propellers to Jets


Douglas XB-42 Mixmaster


Douglas XB-43


Northrop YB-35


Northrop XB-49


Convair B-36


Convair YB-60

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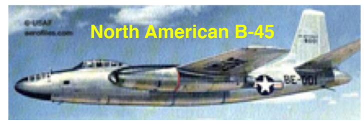
Historical Factoid

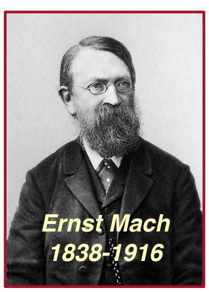
Jets at an Awkward Age

- **Performance of first jet aircraft outstripped stability and control technology**
  - Lacked satisfactory actuators, sensors, and control electronics
  - Transistor: 1947, integrated circuit: 1958
- **Dramatic dynamic variations over larger flight envelope**
  - Control mechanisms designed to lighten pilot loads were subject to instability
- **Reluctance of designers to embrace change, fearing decreased reliability, increased cost, and higher weight**

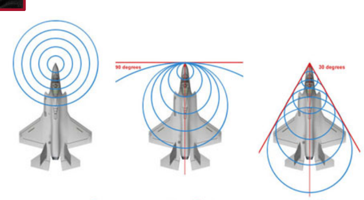


Mach Number Effects

$$\text{Mach Number} = \frac{\text{True Airspeed}}{\text{Speed of Sound}}$$

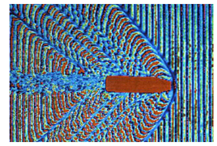


Schlieren photograph



No Speed *Mach 1* *Mach 2*

Drag Due to Pressure Differential



$$C_{D_{base}} = C_{pressure_{base}} \frac{S_{base}}{S} \approx \frac{0.029}{\sqrt{C_{friction} \frac{S_{wet}}{S_{base}}}} \frac{S_{base}}{S} \quad (M < 1) \quad [\text{Hoerner}]$$

Blunt base pressure drag

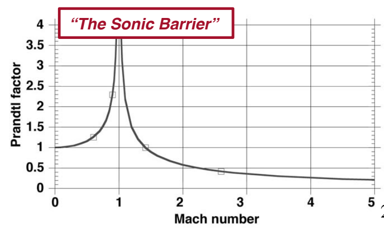
$$< \frac{2}{\gamma M^2} \left(\frac{S_{base}}{S} \right) \quad (M > 2, \quad \gamma = specific\ heat\ ratio)$$

Prandtl factor

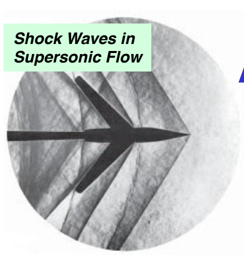
$$C_{D_{wave}} \approx \frac{C_{D_{incompressible}}}{\sqrt{1 - M^2}} \quad (M < 1)$$

$$\approx \frac{C_{D_{compressible}}}{\sqrt{M^2 - 1}} \quad (M > 1)$$

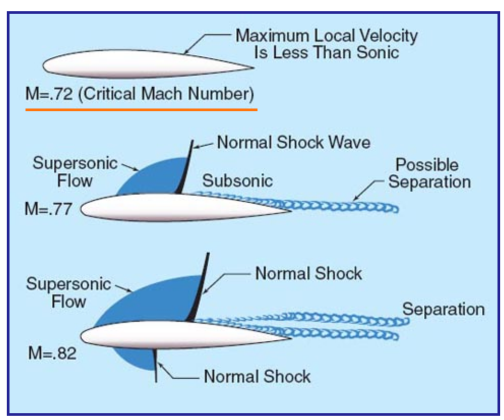
$$\approx \frac{C_{D_{M=\sqrt{2}}}}{\sqrt{M^2 - 1}} \quad (M > 1)$$



Air Compressibility Effect



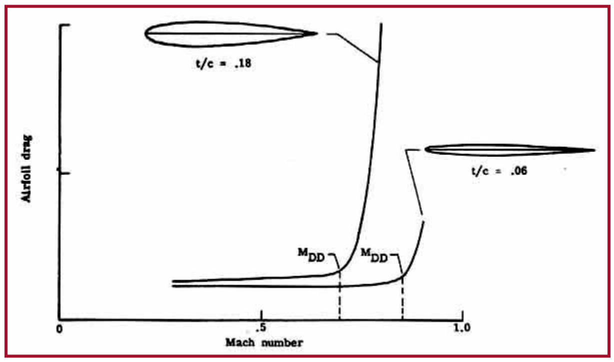
- **Drag rises due to pressure increase across a shock wave**
- **Subsonic flow**
  - **Local airspeed less than sonic (i.e., speed of sound) everywhere**
- **Transonic flow**
  - **Airspeed less than sonic at some points, greater than sonic elsewhere**
- **Supersonic flow**
  - **Local airspeed greater than sonic virtually everywhere**



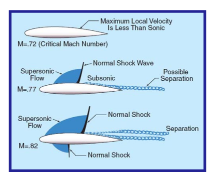
- **Critical Mach number**
  - **Mach number at which local flow first becomes sonic**
  - **Onset of drag-divergence**
  - **M_{crit} ~ 0.7 to 0.85**

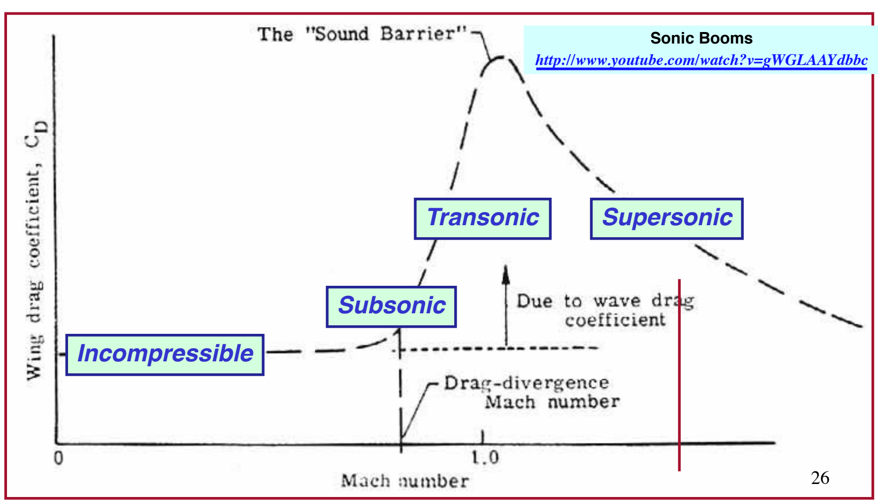
Effect of Chord Thickness on Wing Pressure Drag

- Thinner chord sections lead to higher M_{crit}, or drag-divergence Mach number

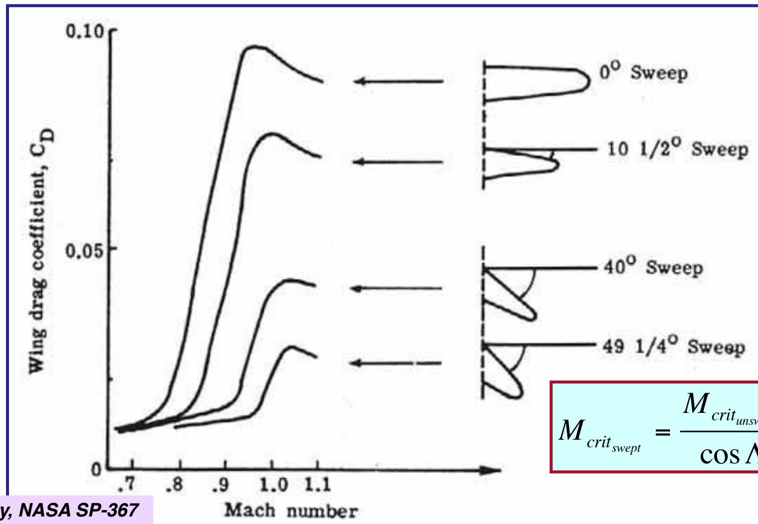


Air Compressibility Effect on Wing Drag





Pressure Drag on Wing Depends on Sweep Angle



$$M_{crit_{swept}} = \frac{M_{crit_{unswept}}}{\cos\Lambda}$$

Talay, NASA SP-367

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Historical Factoid

From Straight to Swept Wings

- **Straight-wing models were redesigned with swept wings to reduce compressibility effects on drag and increase speed**
- **Dramatic change in stability, control, and flying qualities**

North American FJ-1 and FJ-4 Fury

Republic F-84B Thunderbird and F-84F Thunderstreak

Grumman F9F-2 Panther and F9F-6 Cougar

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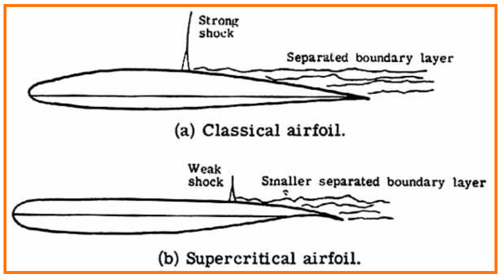
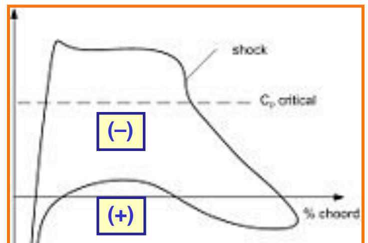
Supercritical Wing

- Richard Whitcomb's supercritical airfoil
  - Wing upper surface flattened to increase M_{crit}
  - Wing thickness can be restored
    - Important for structural efficiency, fuel storage, etc.





Pressure Distribution on Supercritical Airfoil ~ Section Lift

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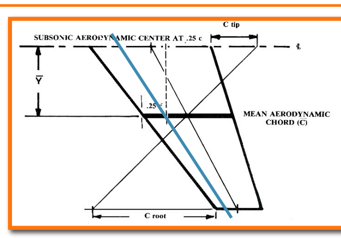
Subsonic Air Compressibility and Sweep Effects on 3-D Wing Lift Slope

- Subsonic 3-D wing, with sweep effect

$$C_{L_\alpha} = \frac{\pi AR}{\left[1 + \sqrt{1 + \left(\frac{AR}{2\cos\Lambda_{1/4}}\right)^2 \left(1 - M^2 \cos\Lambda_{1/4}\right)}\right]}$$



$\Lambda_{1/4}$ = *sweep angle of quarter chord*

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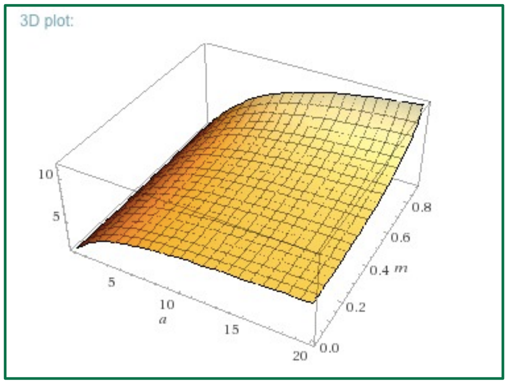
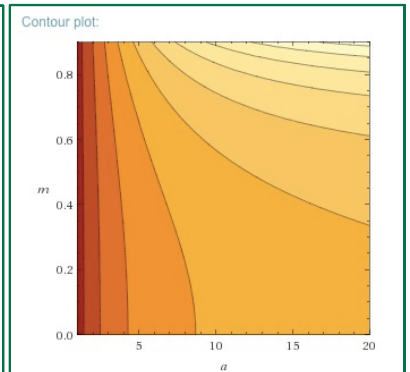
Subsonic Air Compressibility Effects on 3-D Wing Lift Slope

Subsonic 3-D wing, sweep = 0

plot(pi A / (1+sqrt(1 + ((A / 2)^2) (1 - M^2))), A=1 to 20, M = 0 to 0.9)





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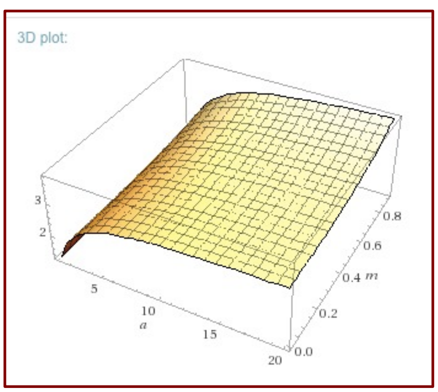
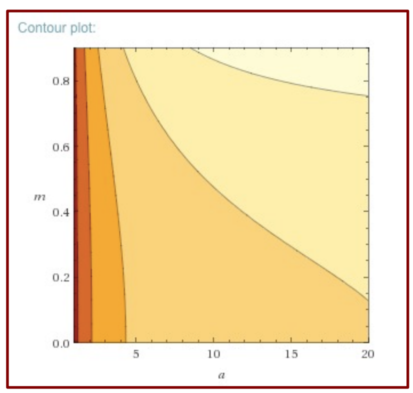
Subsonic Air Compressibility Effects on 3-D Wing Lift Slope

Subsonic 3-D wing, sweep = 60°

plot(pi A / (1+sqrt(1 + (A ^2) (1 – 0.5 M^2))), A=1 to 20, M = 0 to 0.9)





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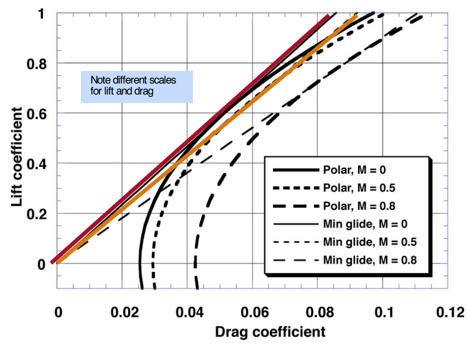
Lift-Drag Polar for a Typical Bizjet

- L/D = slope of line drawn from origin
  - Single maximum for a given polar
  - Two solutions for lower L/D (high and low airspeed)
  - Available L/D decreases with Mach number
- Intercept for L/D_{max} depends only on ε and zero-lift drag



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Wing Lift Slope at M = 1

Approximation for all wing planforms

$$C_{L_\alpha} = \frac{\pi AR}{2} = 2\pi\left(\frac{AR}{4}\right)$$

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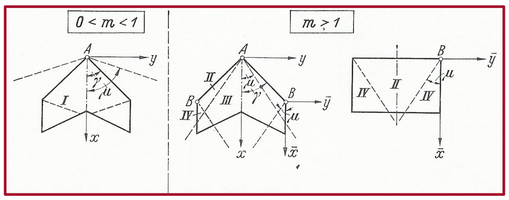
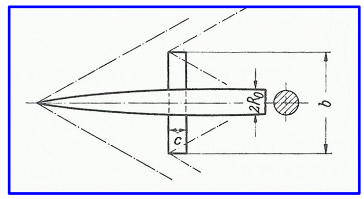
Supersonic Effects on Arbitrary Wing and Wing-Body Lift Slope

- **Impinging shock waves**
- **Discrete areas with differing *M* and local pressure coefficients, c_p**
- **Areas change with α**
- **No simple equations for lift slope**





$$m = \tan\gamma / \tan\mu$$

Schlicting & Truckenbrodt, 1979

γ, μ = sweep angles of shock and leading edge, from x axis

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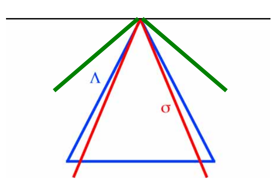
Supersonic Compressibility Effects on Triangular Wing Lift Slope

Supersonic delta (triangular) wing

Supersonic leading edge

$$C_{L_\alpha} = \frac{4}{\sqrt{M^2 - 1}}$$



Subsonic leading edge

$$C_{L_\alpha} = \frac{2\pi^2 \cot\Lambda}{(\pi + \lambda)}$$

where

$$\lambda = m\left(0.38 + 2.26m - 0.86m^2\right)$$

$$m = \cot\Lambda_{LE} / \cot\sigma$$

Λ_{LE} = sweep angle of leading edge, from y axis

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Historical Factoid

Fighter Jets of the 1950s: "Century Series"

- **Emphasis on supersonic speed**

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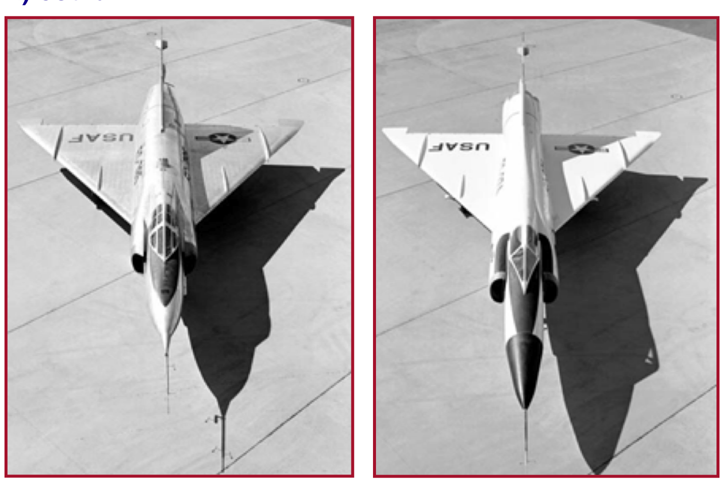
Transonic Drag Rise and the Area Rule

- **Richard Whitcomb (NASA Langley) and Wallace Hayes (Princeton)**
- ***YF-102A* (left) could not break speed of sound in level flight; *F-102A* (right) could**



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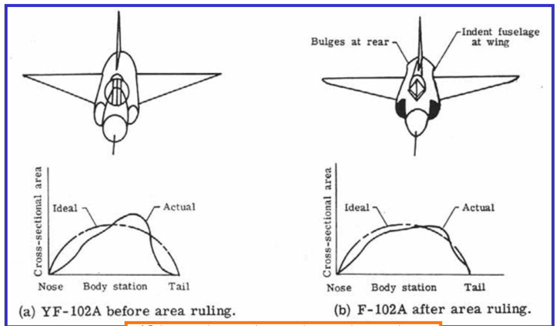
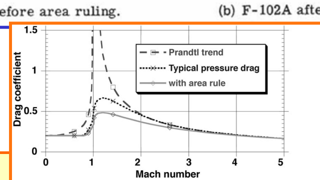
Transonic Drag Rise and the Area Rule

Cross-sectional area of total configuration should gradually increase and decrease to minimize transonic drag





Talay, NASA SP-367

Sears-Haack Body
http://en.wikipedia.org/wiki/Sears-Haack_body

Area Rule
https://en.wikipedia.org/wiki/Area_rule

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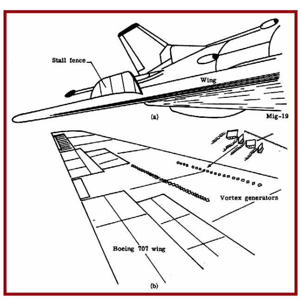
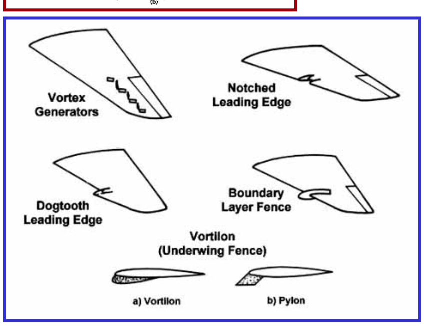
Secondary Wing Structures



- **Vortex generators, fences, vortilons, notched or dog-toothed wing leading edges**
  - **Boundary layer control**
  - **Maintain attached flow with increasing α**
  - **Avoid tip stall**



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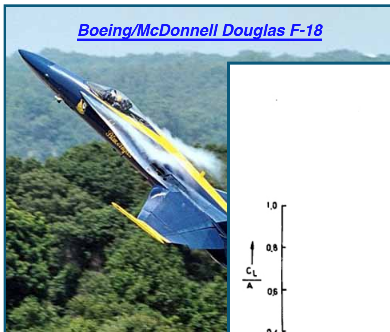
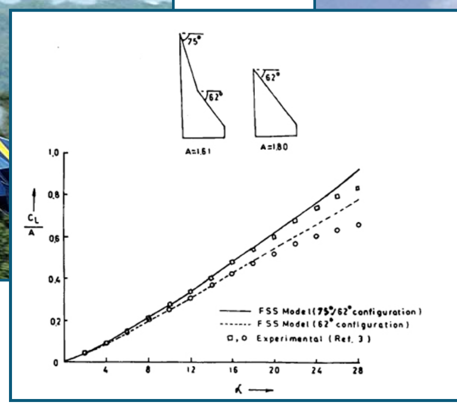
Leading-Edge Extensions

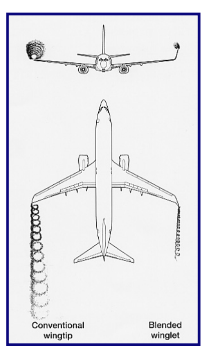
- Strakes or leading edge extensions
  - Maintain lift at high *α*
  - Reduce c.p. shift at high Mach number

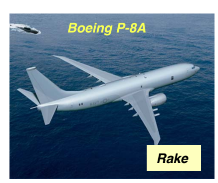




Wingtip Design

- **Winglets, rake, and Hoerner tip** reduce induced drag by controlling the tip vortices
- **End plate, wingtip fence** straightens flow, increasing apparent aspect ratio *(L/D)*
- **Chamfer** produces favorable roll w/ sideslip





Next Time: Aerodynamic Moments (i.e., Torques)

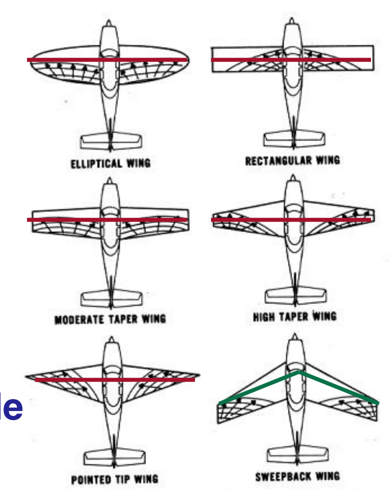
Reading:
Flight Dynamics
Aerodynamic Coefficients, 96-118
Airplane Dynamics and Control
Chapter 6

Learning Objectives

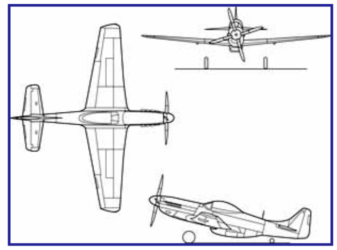
Expressions for aerodynamic balance and moment
Concepts of aerodynamic center, center of pressure, and static margin
Configuration and angle-of-attack effects on pitching moment and stability
Calculate configuration and sideslip-angle effects on lateral-directional (i.e., rolling and yawing) aerodynamic moments
Tail design effects on airplane aerodynamics

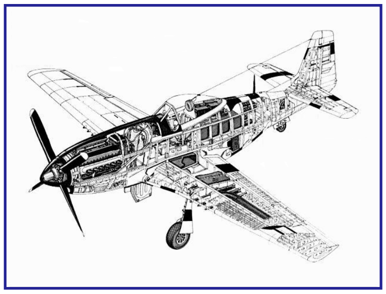
Straight, Swept, and Tapered Wings

- **Straight at the quarter chord**
- **Swept at the quarter chord**
- **Progression of separated flow from trailing edge with increasing angle of attack**



P-51 Mustang





$Wing\ Span = 37\ ft\ (9.83\ m)$

$Wing\ Area = 235\ ft\ (21.83\ m^2)$

$Loaded\ Weight = 9{,}200\ lb\ (3{,}465\ kg)$

$Maximum\ Power = 1{,}720\ hp\ (1{,}282\ kW)$

$C_{D_o} = 0.0163$

$AR = 5.83$

$\lambda = 0.5$

http://en.wikipedia.org/wiki/P-51_Mustang

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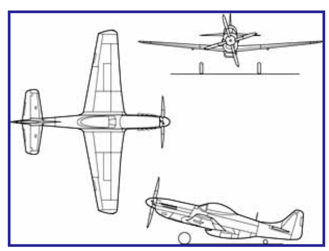
P-51 Mustang Example



$$C_{L_\alpha} = \frac{\pi AR}{\left[1 + \sqrt{1 + \left(\frac{AR}{2}\right)^2}\right]} = 4.49\ per\ rad\ (wing\ only)$$

$e = 0.947$

$\delta = 0.0557$

$\varepsilon = 0.0576$

$$C_{D_i} = \varepsilon C_L^2 = \frac{C_L^2}{\pi e AR} = \frac{C_L^2(1+\delta)}{\pi AR}$$

http://www.youtube.com/watch?v=WE0sr4vmZtU

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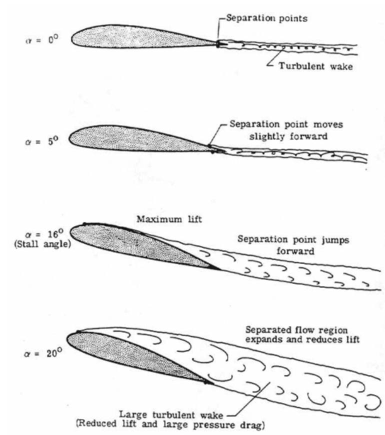
Design for Satisfactory Stalls

- Marked by noticeable, uncommanded changes in pitch, yaw, or roll and/or by a marked increase in buffet
- Stall must be detectable
- Aircraft must pitch down when it occurs
- Up to the stall break, ailerons and rudder should operate properly
- Inboard stall strips to prevent tip stall and loss of roll control before the stall
- Strakes for improved high-α flight



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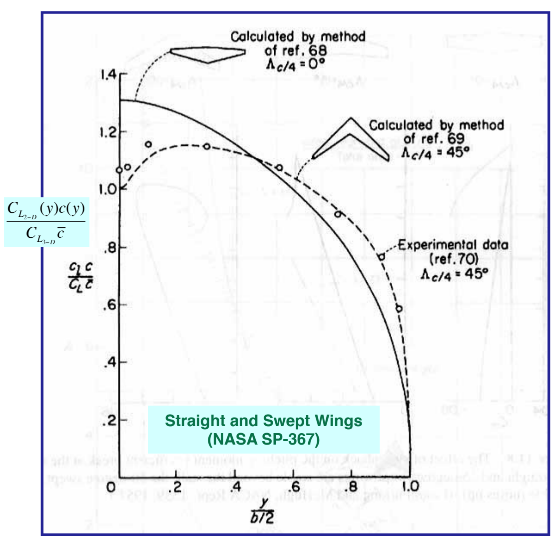
Spanwise Lift Distribution of 3-D Wings



- Wing does not have to have a geometrically elliptical planform to have a nearly elliptical lift distribution
- Sweep moves lift distribution toward tips

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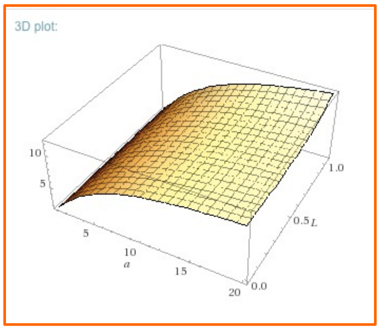
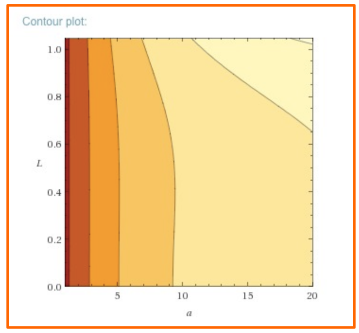
Transonic Sweep Effects on 3-D Wing Lift Slope

Subsonic 3-D wing, M = 0.85

```
plot(pi A / (1+sqrt(1 + ((A / 2 cos(L)) ^2) (1 – cos(L)  0.85^2))), A=1 to 20,
L = 0 to (pi / 3))
```





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Sweep Reduces Subsonic Lift Slope

Swept Wing

$$C_{L_\alpha} = \frac{\pi AR}{\left[1+\sqrt{1+\left(\frac{AR}{2\cos\Lambda_{1/4}}\right)^2\left(1-M^2\cos\Lambda_{1/4}\right)}\right]}$$

$$= \frac{\pi AR}{\left[1+\sqrt{1+\left(\frac{AR}{2\cos\Lambda_{1/4}}\right)^2}\right]}$$ [Incompressible flow]

Triangular Wing

$$C_{L_\alpha} = \frac{2\pi^2\cot\Lambda_{LE}}{(\pi+\lambda)}$$

$$where \quad \lambda = m\left(0.38+2.26m-0.86m^2\right)$$

$$m = \cot\Lambda_{LE}/\cot\sigma$$

Λ_{LE}, σ : measured from y axis

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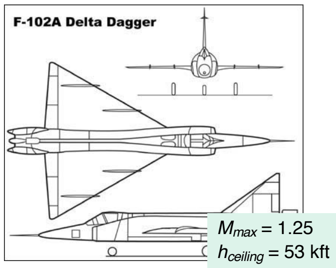
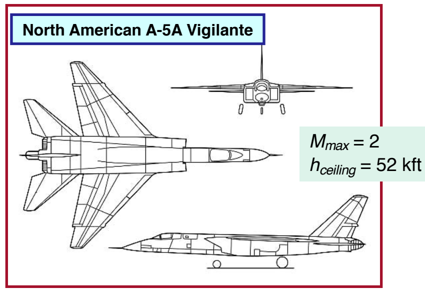
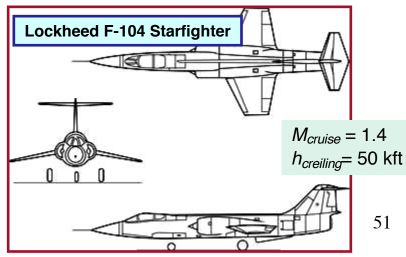
Low Aspect Ratio Configurations





- Typical for supersonic aircraft



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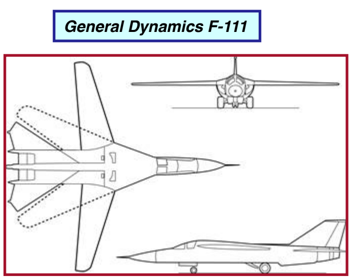
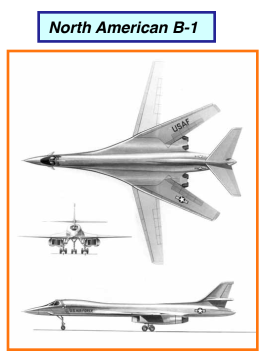
Variable Aspect Ratio Configurations





Aerodynamic efficiency at sub- and supersonic speeds

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Sweep Effect on Thickness Ratio

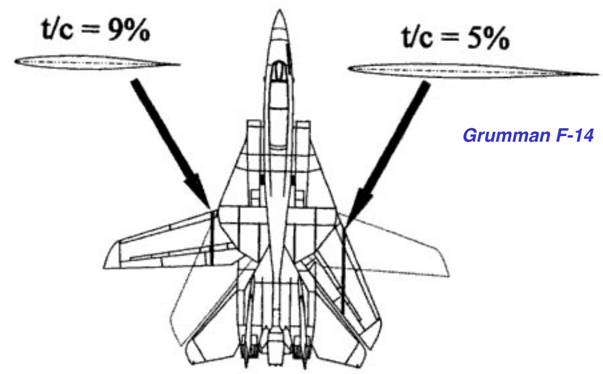
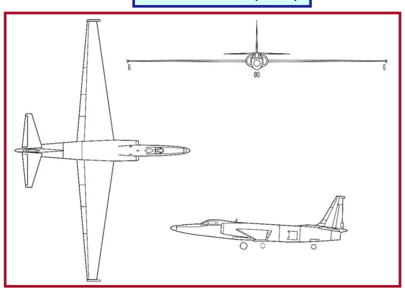


Fig. 1.18 Effect of sweep on relative thickness of wing sections.

from Asselin

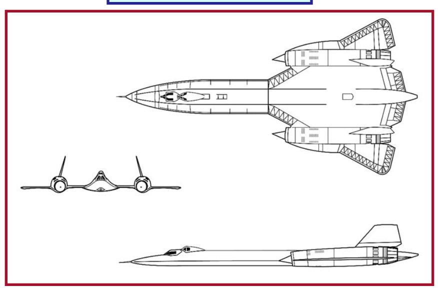
Reconnaissance Aircraft

Lockheed U-2 (ER-2)



- **Subsonic, high-altitude flight**

Lockheed SR-71 Trainer

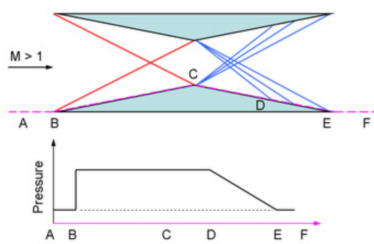


- **Supersonic, high-altitude flight**

Historical Factoid

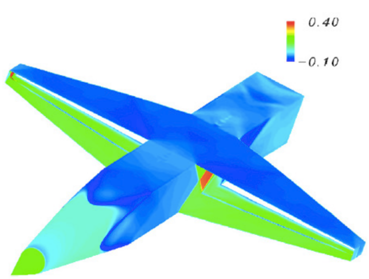
What Happened to the F-103?

Supersonic Biplane



- **Concept of Adolf Busemann (1935)**
  - **Shock wave cancellation at one specific Mach number**
  - **2-D wing**

http://en.wikipedia.org/wiki/Adolf_Busemann



- **Kazuhiro Kusunose *et al* , Tohoku U (PAS, 47, 2011, 53-87)**
  - **Adjustable flaps**
  - **Tapered, variably spaced 3-D wings**
  - **Fuselage added**

Supersonic Transport Concept

- **Rui Hu, Qiqi Wang (MIT), Antony Jameson (Texas A&M), AIAA-2011-1248**
  - **Optimization of biplane aerodynamics**
  - **Sketch of possible configuration**

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