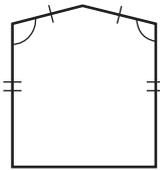# Polygons

A **polygon** is a closed plane figure formed by three or more line segments that meet at points called vertices. You can classify a polygon by the number of sides and the number of angles that it has.

**Congruent** figures have the same size and shape. In a **regular polygon,** all sides are congruent and all angles are congruent.

**Classify the polygon below.**

| Polygon | Sides | Angles | Vertices |
|---|---|---|---|
| Triangle | 3 | 3 | 3 |
| Quadrilateral | 4 | 4 | 4 |
| Pentagon | 5 | 5 | 5 |
| Hexagon | 6 | 6 | 6 |
| Heptagon | 7 | 7 | 7 |
| Octagon | 8 | 8 | 8 |
| Nonagon | 9 | 9 | 9 |
| Decagon | 10 | 10 | 10 |

How many sides does this polygon have? **5 sides**

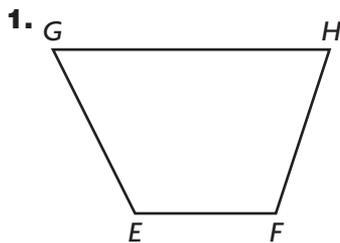How many angles does this polygon have? **5 angles**

Name the polygon. **pentagon**

Are all the sides congruent? **no**

Are all the angles congruent? **no**

So, the polygon above is a pentagon. It is *not* a regular polygon.

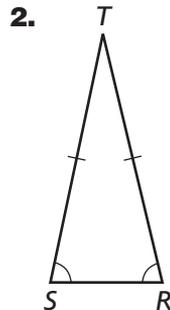**Name each polygon. Then tell whether it is a *regular polygon* or *not a regular polygon*.**

1.  G   H
    E   F

2.  T
    S   R

3.  Y
    U   X
    V   W

4.  U   N
    T       O
    S       P
    R   Q

1. _____   2. _____   3. _____   4. _____

   _____      _____      _____      _____

   _____      _____      _____      _____
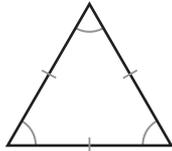
   _____      _____      _____      _____
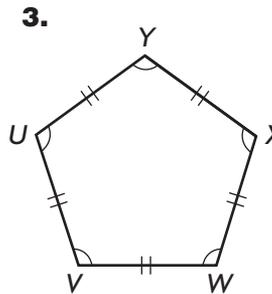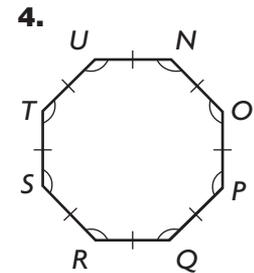
# Triangles

You can classify triangles by the length of their sides and by the measure of their angles. **Classify each triangle.**

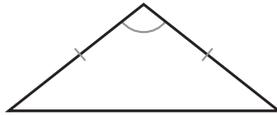Use a ruler to measure the side lengths.

- **equilateral triangle**
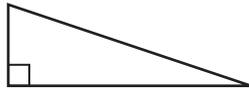  All sides are the same length.
- **isosceles triangle**
  Two sides are the same length.
- **scalene triangle**
  All sides are different lengths.

Use the corner of a sheet of paper to classify the angles.

- **acute triangle**
  All three angles are acute.
- **obtuse triangle**
  One angle is obtuse. The other two angles are acute.
- **right triangle**
  One angle is right. The other two angles are acute.

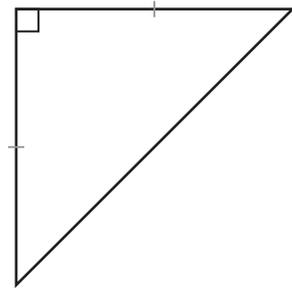**Classify the triangle according to its side lengths.**
It has two congruent sides.
## The triangle is an isosceles triangle.
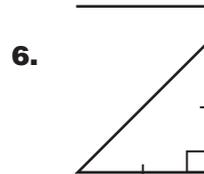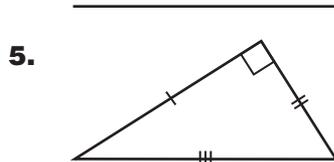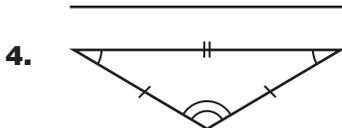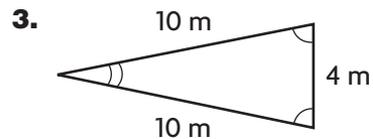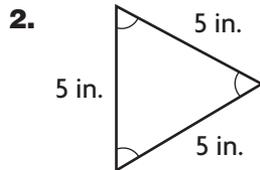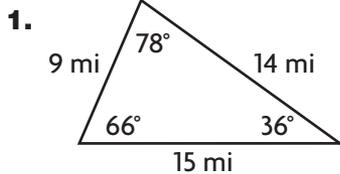
**Classify the triangle according to its angle measures.**
It has one right angle.
## The triangle is a right triangle.

**Classify each triangle. Write *isosceles*, *scalene*, or *equilateral*. Then write *acute*, *obtuse,* or *right*.**
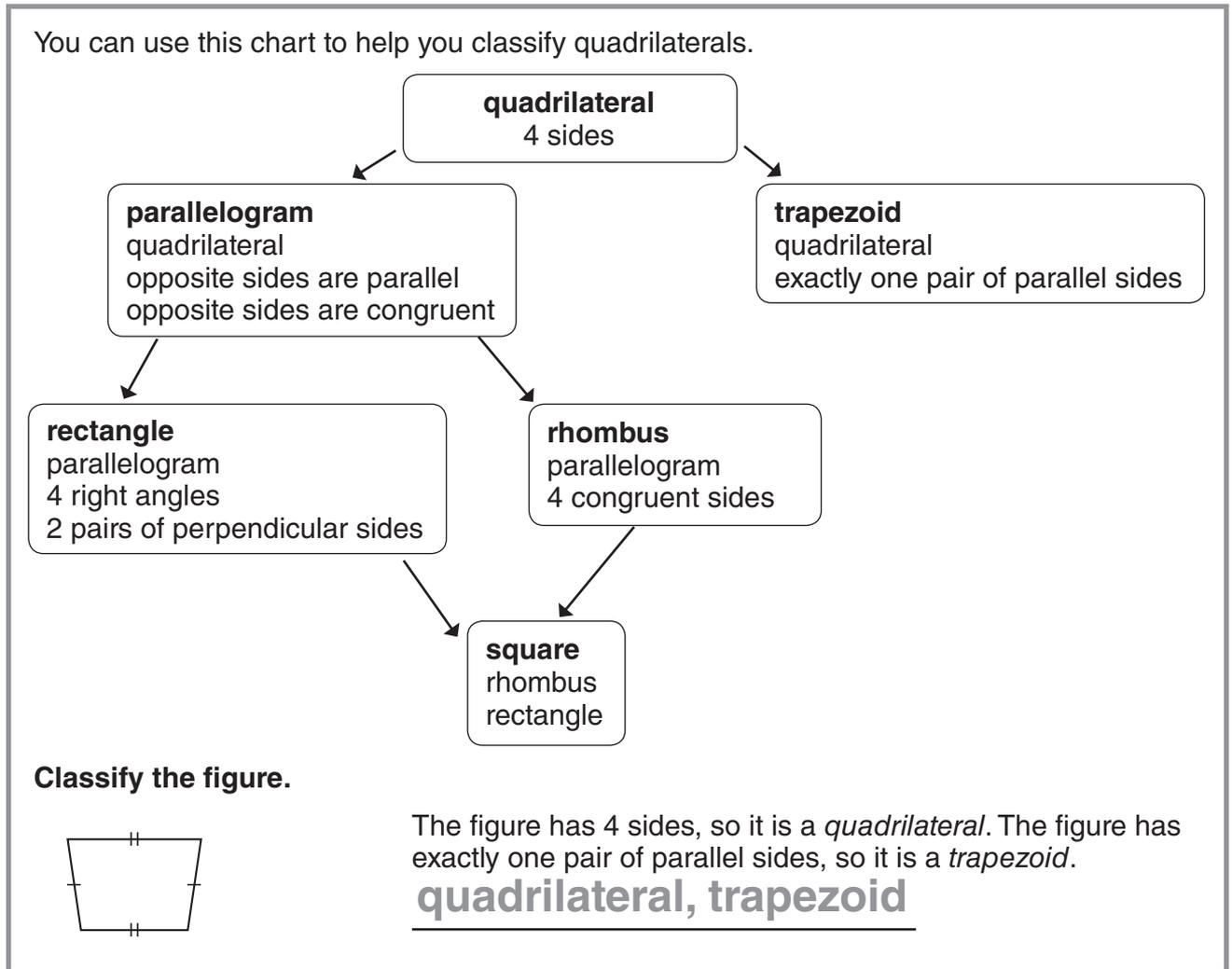
1. 9 mi, 78°, 14 mi, 66°, 36°, 15 mi

2. 5 in., 5 in., 5 in.

3. 10 m, 4 m, 10 m

4.

5.

6.

# Quadrilaterals

You can use this chart to help you classify quadrilaterals.

**quadrilateral**
4 sides

**parallelogram**
quadrilateral
opposite sides are parallel
opposite sides are congruent

**trapezoid**
quadrilateral
exactly one pair of parallel sides

**rectangle**
parallelogram
4 right angles
2 pairs of perpendicular sides

**rhombus**
parallelogram
4 congruent sides

**square**
rhombus
rectangle

**Classify the figure.**

The figure has 4 sides, so it is a *quadrilateral*. The figure has exactly one pair of parallel sides, so it is a *trapezoid*.
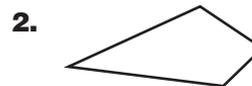
quadrilateral, trapezoid

**Classify the quadrilateral in as many ways as possible. Write *quadrilateral*, *parallelogram*, *rectangle*, *rhombus*, *square*, or *trapezoid*.**
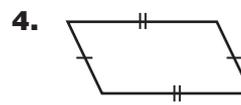
1. _____
   _____

2. _____
   _____

3. _____
   _____

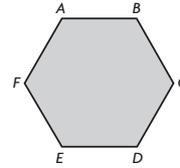4. _____
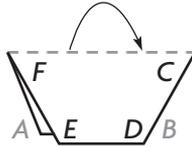   _____
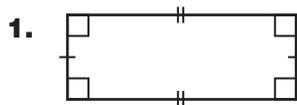
Name _____

# Problem Solving • Properties of Two-Dimensional Figures

Haley thinks hexagon *ABCDEF* has 6 congruent sides, but she does not have a ruler to measure the sides. Are the 6 sides congruent?

| Read the Problem | Solve the Problem |
|---|---|
| **What do I need to find?**<br><br>I need to determine if sides<br><br>*AB, BC, CD, DE, EF,* and *FA*<br><br>have the **same length**. | Trace the hexagon and cut out the shape.<br><br>**Step 1** Fold the hexagon to match the sides *AB* and *ED*, sides *FE* and *FA*, and sides *CD* and *CB*.<br><br><br><br>The sides match, so they are congruent. |
| **What information do I need to use?**<br><br>The figure is a **hexagon** with **6** sides and **6 congruent** angles. | **Step 2** Fold along the diagonal between *B* and *E* to match sides *BA* and *BC*, sides *AF* and *CD*, and sides *EF* and *ED*. Fold along the diagonal between *A* and *D* to match sides *AF* and *AB*, sides *FE* and *BC*, and sides *DE* and *DC*. |
| **How will I use the information?**<br><br>I will **act it out by tracing the figure and then folding the figure** to match all the sides to see if they are **congruent**. | **Step 3** Use logic to match sides *AB* and *CD*, sides *AB* and *EF*, sides *BC* and *DE*, and sides *DE* and *FA*.<br><br>The sides match, so they are congruent. |

1. Justin thinks square *STUV* has 4 congruent sides, but he does not have a ruler to measure the sides. Are the sides congruent? **Explain.**

   _____

   _____

   _____

   _____

   _____

   _____

2. Esther knows octagon *OPQRSTUV* has 8 congruent angles. How can she determine whether the octagon has 8 congruent sides without using a ruler?

   _____

   _____

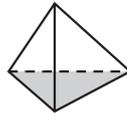   _____

   _____

   _____

   _____

Name _____

# Three-Dimensional Figures

A **polyhedron** is a solid figure with faces that are polygons. You can identify a polyhedron by the shape of its faces.
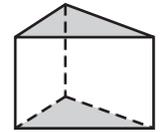
A **pyramid** is a polyhedron with one polygon base. The lateral faces of a pyramid are triangles that meet at a common vertex.

| | | |
|---|---|---|
| **triangular pyramid** | The base and faces are triangles. | |
| **rectangular pyramid** | The base is a rectangle. | |
| **square pyramid** | The base is a square. | |
| **pentagonal pyramid** | The base is a pentagon. | |
| **hexagonal pyramid** | The base is a hexagon. | |

A **prism** is a polyhedron with two congruent polygons as bases. The lateral faces of a prism are rectangles.

| | | |
|---|---|---|
| **triangular prism** | The two bases are triangles. | |
| **rectangular prism** | All faces are rectangles. | |
| **square prism or cube** | All faces are squares. | |
| **pentagonal prism** | The two bases are pentagons. | |
| **hexagonal prism** | The two bases are hexagons. | |
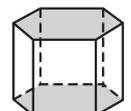
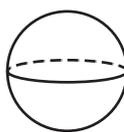A solid figure with curved surfaces is **not a polyhedron**.

| | | |
|---|---|---|
| **cone** | The one base is a circle. | |
| **sphere** | There is no base. | |

| | | |
|---|---|---|
| **cylinder** | The two bases are circles. | |
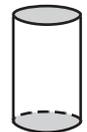
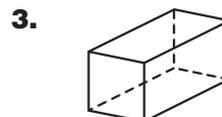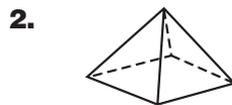**Classify the solid figure. Write *prism, pyramid, cone, cylinder,* or *sphere*.**

The solid figure has one base.

The rest of its faces are triangles.
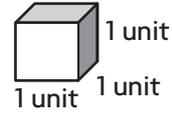
So, the solid figure is a ___pyramid___.

**Classify each solid figure. Write *prism, pyramid, cone, cylinder,* or *sphere.***

**1.** _____

**2.** _____

**3.** _____

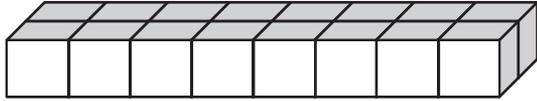**4.** _____

# Unit Cubes and Solid Figures

A **unit cube** is a cube that has a length, width, and height of 1 unit. You can use unit cubes to build a rectangular prism.

1 unit
1 unit
1 unit

**Count the number of cubes used to build the rectangular prism.**

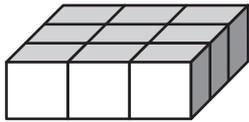The length of the prism is made up of __8__ unit cubes.

The width of the prism is made up of __2__ unit cubes.

The height of the prism is made up of __1__ unit cube.

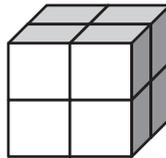The number of unit cubes used to build the rectangular prism is __16__.

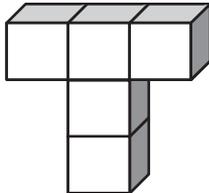**Count the number of unit cubes used to build each solid figure.**
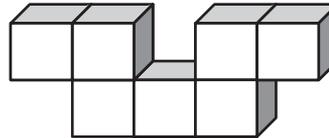
**1.**
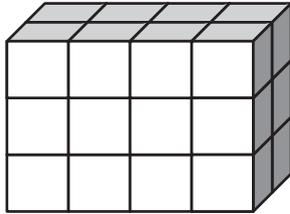
_____ unit cubes

**2.**

_____ unit cubes

**3.**

_____ unit cubes

**4.**

_____ unit cubes

# Understand Volume

The **volume** of a rectangular prism is equal to the number of unit cubes that make up the prism. Each unit cube has a volume of 1 cubic unit.

**Find the volume of the prism. 1 unit cube = 1 cubic inch**



**Step 1** Count the number of unit cubes in the bottom layer of the prism.

There are __4__ unit cubes that make up the length of the first layer.

There are __2__ unit cubes that make up the width of the first layer.

There is __1__ unit cube that makes up the height of the first layer.

So, altogether, there are __8__ unit cubes that make up the bottom layer of the prism.

**Step 2** Count the number of layers of cubes that make up the prism.

The prism is made up of __3__ layers of unit cubes.

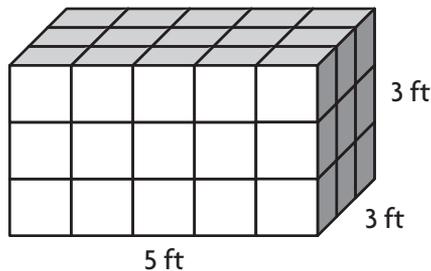**Step 3** Find the total number of cubes that fill the prism.

Multiply the number of layers by the number of cubes in each layer.

$3 \times 8 =$ __24__ unit cubes

Each unit cube has a volume of 1 cubic inch. So, the volume of the prism is $24 \times 1$, or __24__ cubic inches.
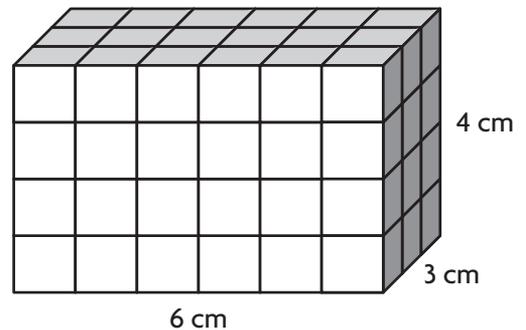
**Use the unit given. Find the volume.**

1.



3 ft
3 ft
5 ft

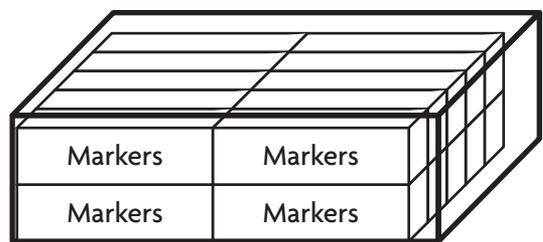Each cube = 1 cu ft

Volume = _____ cu _____

2.



4 cm
3 cm
6 cm

Each cube = 1 cu cm

Volume = _____ cu _____

# Estimate Volume

You can estimate the volume of a larger box by filling it with smaller boxes.

Mario packs boxes of markers into a large box. The volume of each box of markers is 15 cubic inches. Estimate the volume of the large box.

The volume of one box of markers is __15__ cubic inches.

Use the box of markers to estimate the volume of the large box.

- The large box holds __2__ layers of boxes of markers, a top layer and a bottom layer. Each layer contains __10__ boxes of markers. So, the large box holds about 2 × 10, or __20__ boxes of markers.

- Multiply the volume of 1 box of markers by the estimated number of boxes of markers that fit in the large box.

  __20__ × __15__ = __300__

So, the volume of the large box is about __300__ cubic inches.
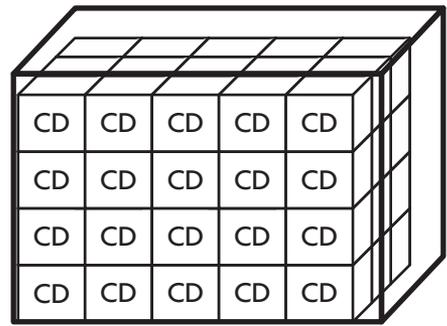
**Estimate the volume.**

1. Each box of toothpaste has a volume of 25 cubic inches.
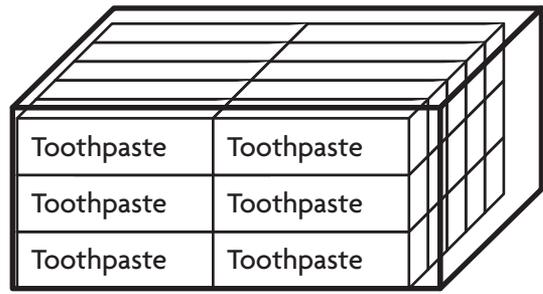
   There are _____ boxes of toothpaste in the large box.

   The estimated volume of the large box

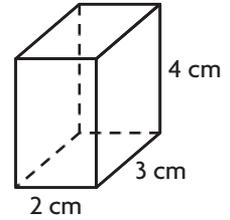   is _____ × 25 = _____ cubic inches.

2. Volume of CD case: 80 cu cm

   Volume of large box: _____

# Volume of Rectangular Prisms

Jorge wants to find the volume of this rectangular prism.
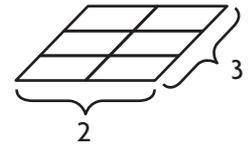He can use cubes that measure 1 centimeter on each side
to find the volume.

**Step 1** The base has a length of 2 centimeters
and a width of 3 centimeters. Multiply to find the area
of the base.
Base = __2__ × __3__
Base = __6__ cm²

**Step 2** The height of the prism is 4 centimeters. Add the
number of cubes in each layer to find the volume.
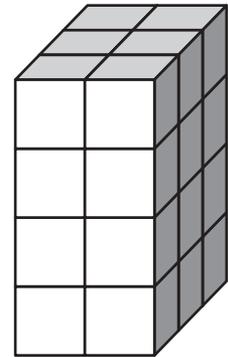
**Remember:** Each layer has 6 cubes.

**Step 3** Count the cubes. __24__ cubes
Multiply the base and the height to check your answer.
Volume = __6__ × __4__
Volume = __24__ cubic centimeters

So, the volume of Jorge's rectangular prism is __24__ cubic centimeters.

**Find the volume.**

**1.**

3 cm
2 cm
3 cm

Volume: _____

**2.**

2 ft
5 ft
2 ft

Volume: _____

**3.**

4 in.
2 in.
4 in.

Volume: _____

**4.**

3 cm
6 cm
3 cm
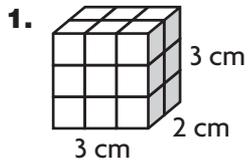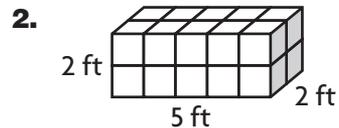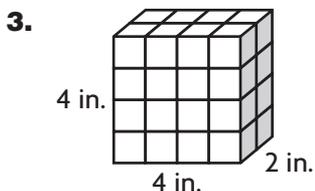
Volume: _____

# Algebra • Apply Volume Formulas

You can use a formula to find the volume of a rectangular prism.
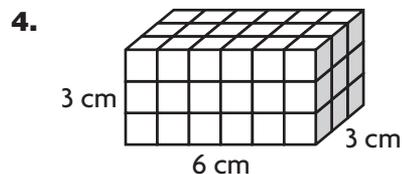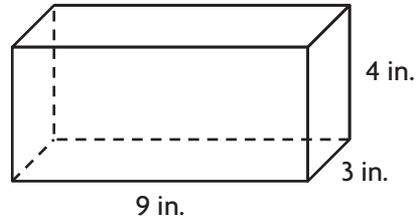
$$Volume = length \times width \times height$$
$$V = (l \times w) \times h$$

**Find the volume of the rectangular prism.**

**Step 1** Identify the length, width, and height
of the rectangular prism.

length = __9__ in.   width = __3__ in.   height = __4__ in.

**Step 2** Substitute the values of the length, width, and height
into the formula.

$$V = (l \times w) \times h$$
$$V = (\underline{\ 9\ } \times \underline{\ 3\ }) \times \underline{\ 4\ }$$

**Step 3** Multiply the length by the width.

$$V = (9 \times 3) \times 4$$
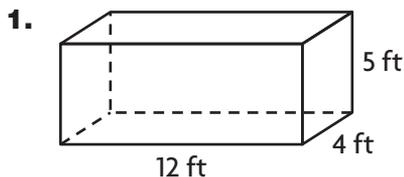$$V = \underline{\ 27\ } \times 4$$

**Step 4** Multiply the product of the length and width by the height.
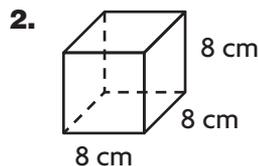
$$V = 27 \times \underline{\ 4\ }$$
$$= \underline{\ 108\ }$$

So, the volume of the rectangular prism is __108__ cubic inches.

**Find the volume.**

**1.**

5 ft
4 ft
12 ft

$V =$ _____

**2.**

8 cm
8 cm
8 cm

$V =$ _____

# Problem Solving • Compare Volumes

A company makes aquariums that come in three sizes of rectangular prisms. The length of each aquarium is three times its width and depth. The depths of the aquariums are 1 foot, 2 feet, and 3 feet. What is the volume of each aquarium?

| Read the Problem | Solve the Problem |
|---|---|
| **What do I need to find?**<br><br>I need to find the __volume__ of each aquarium. | **Think:** The depth of an aquarium is the same as the height of the prism formed by the aquarium |
| **What information do I need to use?**<br><br>I can use the formula for volume,<br>___$V = l \times w \times h$, or $V = B \times h$___. I can<br>use ___1 ft, 2 ft, and 3 ft___ as the depths.<br>I can use the clues __the length is three times__<br>__the width and depth__ _____. | |

<table>
<tr><td>Length (ft)</td><td>Width (ft)</td><td>Depth, or Height (ft)</td><td>Volume (cu ft)</td></tr>
<tr><td>3</td><td>1</td><td>1</td><td>3</td></tr>
<tr><td>6</td><td>2</td><td>2</td><td>24</td></tr>
<tr><td>9</td><td>3</td><td>3</td><td>81</td></tr>
</table>

**How will I use the information?**

I will use the __volume formula__ and a __table__ to list all of the possible combinations of lengths, widths, and depths.

So, the volumes of the aquariums are 3 cubic feet, 24 cubic feet, and 81 cubic feet.

1. Jamie needs a bin for her school supplies. A blue bin has a length of 12 inches, a width of 5 inches, and a height of 4 inches. A green bin has a length of 10 inches, a width of 6 inches, and a height of 5 inches. What is the volume of the bin with the greatest volume?

2. Suppose the blue bin that Jamie found had a length of 5 inches, a width of 5 inches, and a height of 12 inches. Would one bin have a greater volume than the other? **Explain.**

_____

_____

# Find Volume of Composed Figures

A composite figure is a solid made up of two or more solids. To find the volume of a composite figure, first find the volume of each solid that makes up the figure. Then find the sum of the volumes of the figures.
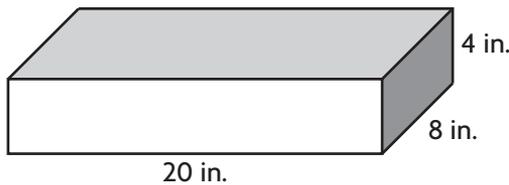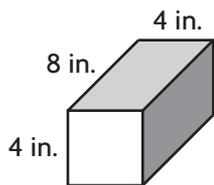
**Find the volume of the composite figure at right.**

**Step 1** Break apart the composite figure into two rectangular prisms. Label the dimensions of each prism.

4 in.

4 in.

8 in.

4 in.

8 in.

20 in.

**Prism 1**          **Prism 2**

4 in.

8 in.

4 in.

4 in.

8 in.

20 in.
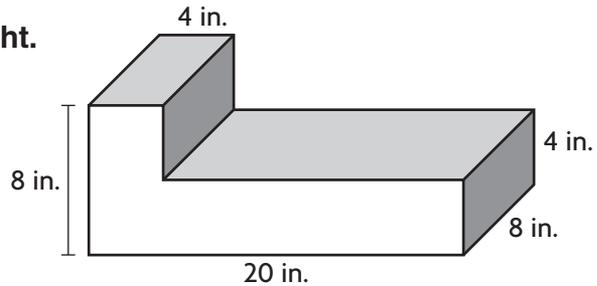
**Step 2** Find the volume of each prism.

**Prism 1**
$V = (l \times w) \times h$
$V = \underline{\ 4\ } \times \underline{\ 8\ } \times \underline{\ 4\ }$
$V = 128$ in.$^3$

**Prism 2**
$V = (l \times w) \times h$
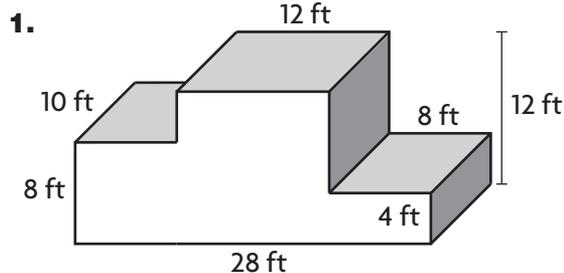$V = \underline{\ 20\ } \times \underline{\ 8\ } \times \underline{\ 4\ }$
$V = 640$ in.$^3$

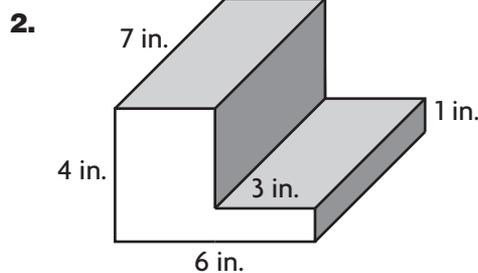**Step 3** Find the sum of the volumes of the two prisms.

Volume of Prism 1 + Volume of Prism 2 = Volume of Composite Figure
$\underline{128 \text{ in.}^3}$ + $\underline{640 \text{ in.}^3}$ = Volume of Composite Figure
$\underline{768 \text{ in.}^3}$ = Volume of Composite Figure

So, the volume of the composite figure is 768 in.$^3$

**Find the volume of the composite figure.**

**1.**

12 ft

10 ft

8 ft

8 ft

28 ft

8 ft

4 ft

12 ft

12 ft

$V = $ _____

**2.**

7 in.

4 in.

3 in.

1 in.

6 in.

$V = $ _____