Synchronous Machines

Revised November 3, 2008

Synchronous Machines:

Synchronous machines are AC machines that a field circuit supplied by an external DC source

Synchronous generators or alternators are synchronous machines used to convert mechanical energy into electrical energy.

Synchronous motors convert electrical energy into mechanical energy.

Synchronous Generators:

A DC current is applied to the rotor winding which produces the rotor magnetic field.

This rotor is rotated by a prime mover (a steam turbine, for example), which produces as rotating magnetic field in the machine. This rotating field induces a three-phase set of voltages within the stator windings of the generator.

Synchronous Motors:

A three-phase set of stator currents produces a rotating magnetic field.

This causes the rotor magnetic field to align with it.

Since the stator magnetic field is rotating the rotor rotates as it tries to keep up with the moving stator magnetic fields.

This supplies mechanical power to a load.

Synchronous Machines:

Terminology: *Field windings* are the windings that produce the main magnetic field in a machine.

Armature windings are the windings where the main voltage is induced.

For synchronous machines, the field windings are on the rotor, which can be either salient or non-salient in construction. The terms rotor windings and field windings are equivalent. The rotor is essentially a large electromagnet.

Similarly, the terms stator windings and armature windings are equivalent.

Synchronous Generators

Synchronous Machines:

A DC current must be supplied to the field circuit on the rotor.

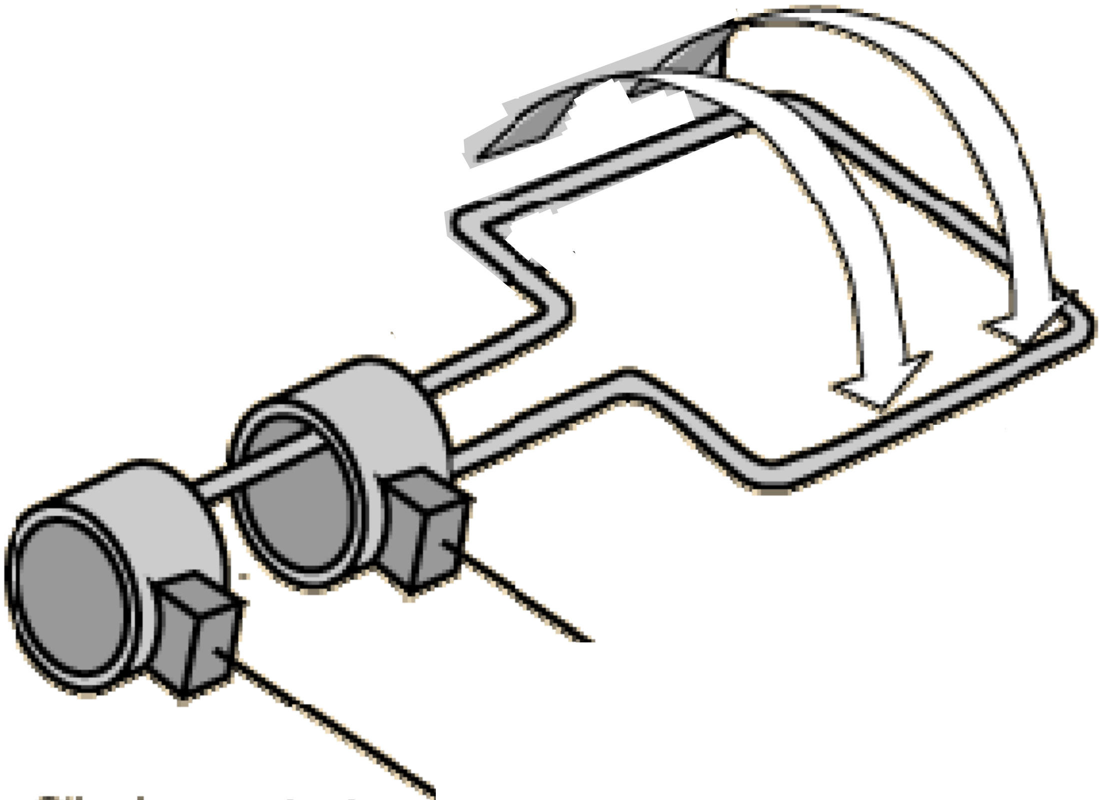
Since the rotor is rotating a special connection is required:

1. Supply DC power from an external source by means of *slip rings* and *brushes*

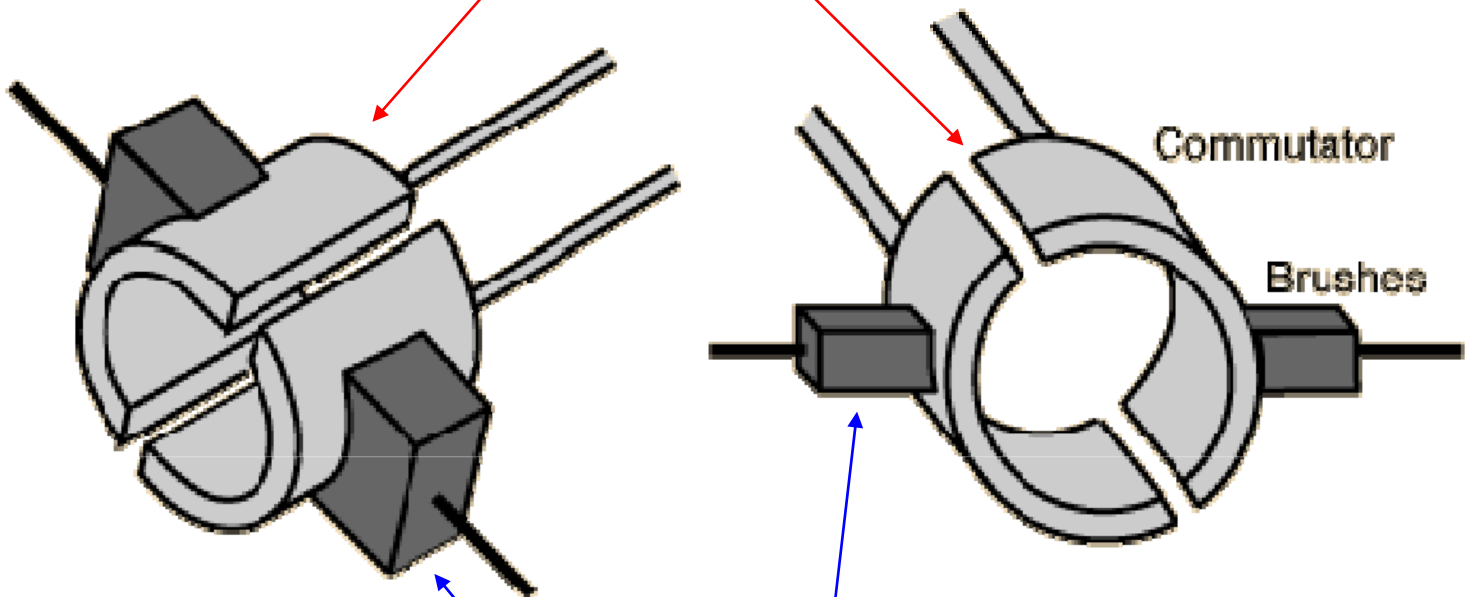
2. Supply DC power from a DC power source mounted on the rotating shaft of the synchronous machine.

Synchronous Machines:

This is for a generator, but the same picture applies.



Slip rings or Commutator



More on this later

Brushes – made with a low friction carbon compound.

Speed of Rotation of a Synchronous Generator

By definition, synchronous generators are termed as such because the electrical frequency produced is synchronized with the mechanical rate of rotation of the generator.

Since the rotor magnetic field is produced by a DC current, it forms an electromagnet that is directed in whatever position the rotor happens to be aligned.

Recall that the rate of rotation of the rotating magnetic field of the stator is given by (see Note Set 4, Slide 34):

$$f_{electrical} = \frac{n_m P}{120}, \quad P \;\; poles, n_m \text{ rev/min}$$

Since the rotor turns at the same speed as the magnetic field, this result also relates the speed of rotation to the electrical frequency produced.

To produce a frequency of 60 Hz, the rotor must turn at 3600 rpm for a two-pole machine (well suited for steam turbines) or at 1800 rpm for a four-pole machine, better suited for water turbines.

Internally Generated Voltage of a Synchronous Generator

Recall from Note Set 4, Slide 80 that the magnitude of the voltage induced in a given stator phase is:

$$E_A = \sqrt{2} N_C \pi f \phi \quad (rms)$$

The induced voltage depends on

- the flux ϕ
- the frequency (or speed of rotation)
- the construction of the machine

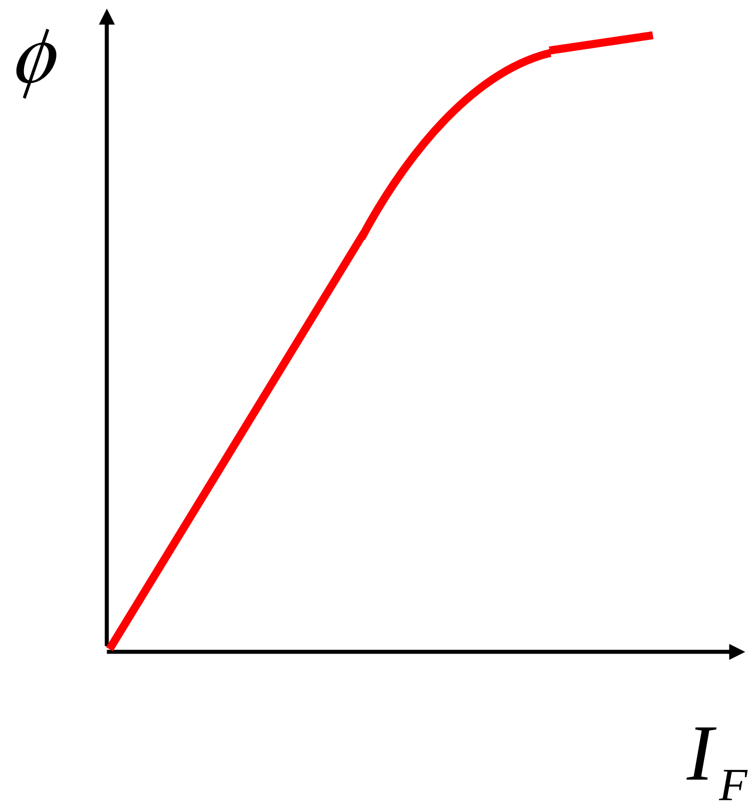
and the induce voltage is usually expressed as

$$E_A = K \omega \phi$$

where K is a constant that depends on machine construction.

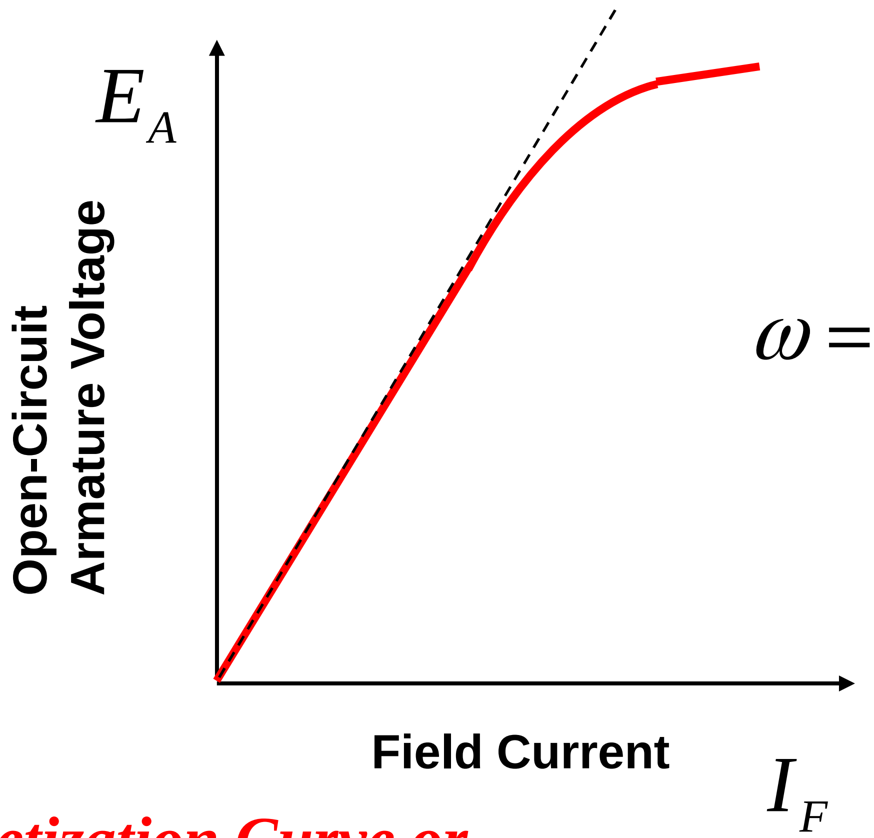
Internally Generated Voltage of a Synchronous Generator

The internally generated voltage is directly proportional to the flux, but the flux itself depends on the current flowing in the rotor field circuit.



Internally Generated Voltage of a Synchronous Generator

Hence the internally generated voltage is related to the field current.



$$\omega = \omega_{sync} \quad (\text{constant})$$

Magnetization Curve or
Open-Circuit Characteristics of the Machine

Equivalent Circuit of a Synchronous Generator

The voltage E_A (produced in one phase) is called "internally generated" because it is not generally the voltage that appears at the terminals of the generator.

The only time that this internal voltage appears at the output is when there is no armature current flowing in the machine.

The relationship between the internally generated voltage E_A and the output voltage V_ϕ is generally represented by a circuit model.

Equivalent Circuit of a Synchronous Generator

The factors which influence the relationship between E_A and V_ϕ include:

1. The distortion in the air-gap magnetic field caused by the current flowing in the stator – *called armature reaction – this tends to be the major influence*
2. The self-inductance of the armature coils
3. The resistance of the armature coils
4. The shape of the rotor in a salient pole machine – this effect tends to be small

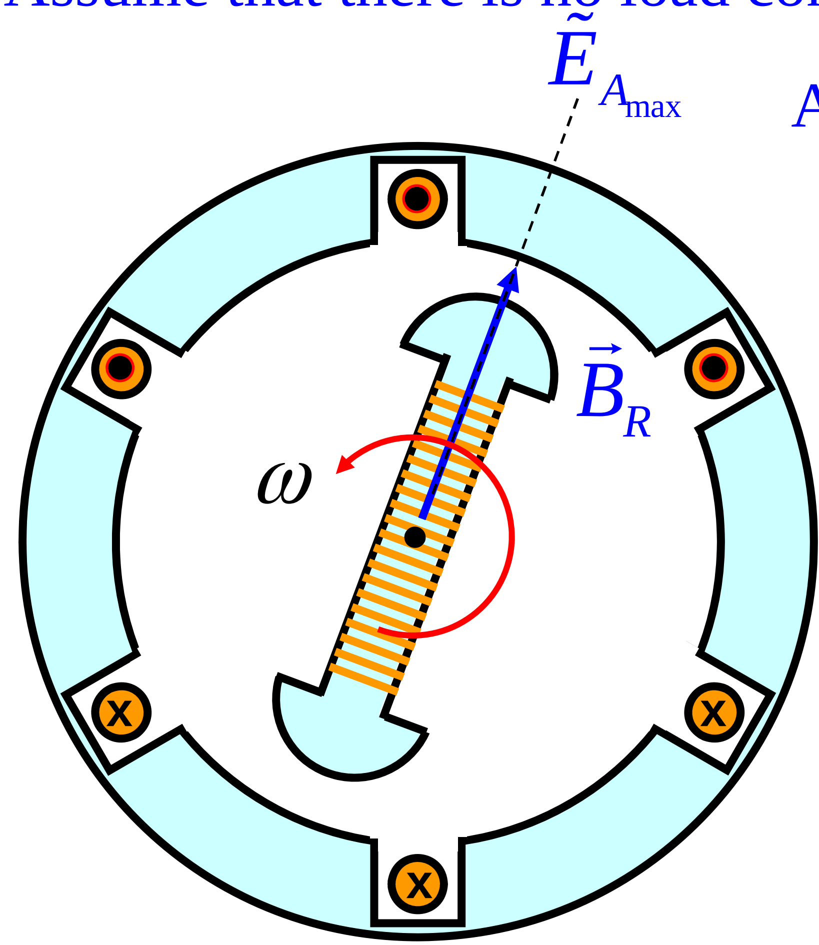
Equivalent Circuit of a Synchronous Generator

Armature Reaction:

Rotate the rotor $\longrightarrow$ induce voltage $E_A \longrightarrow$ a current flows in the load (the stator windings) $\longrightarrow$ this stator current produces its own magnetic field which adds to the original rotor field which changes the resulting phase voltage.

Armature Reaction:

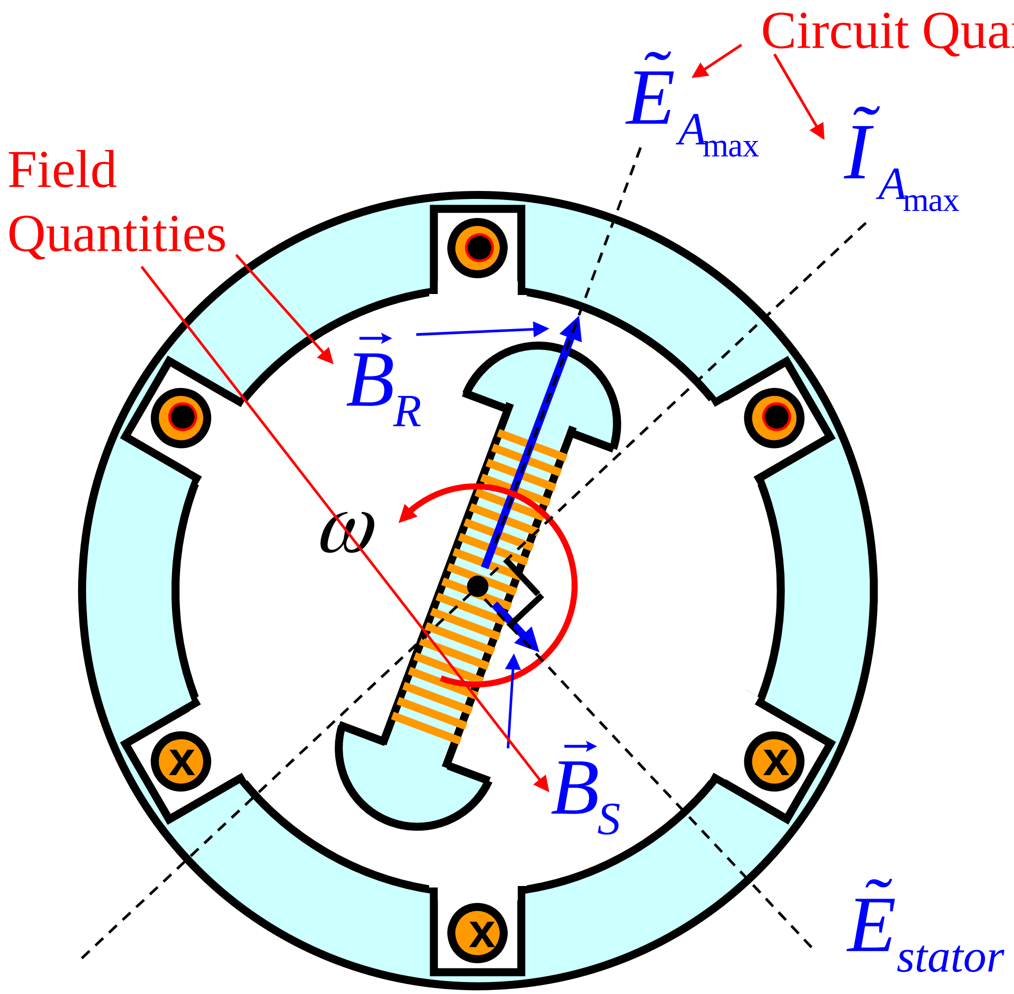
Consider the two-pole rotor spinning inside of a three-phase stator. Assume that there is no load connected to the stator.



As shown in Set 4, Slide 78, the rotor magnetic field B_R induces a voltage E_A whose peak coincides with B_R (positive on top and negative at bottom). With no load, and hence no armature current flow, $E_A = V_\phi$

Armature Reaction:

Now suppose that the generator is connected to a lagging load.



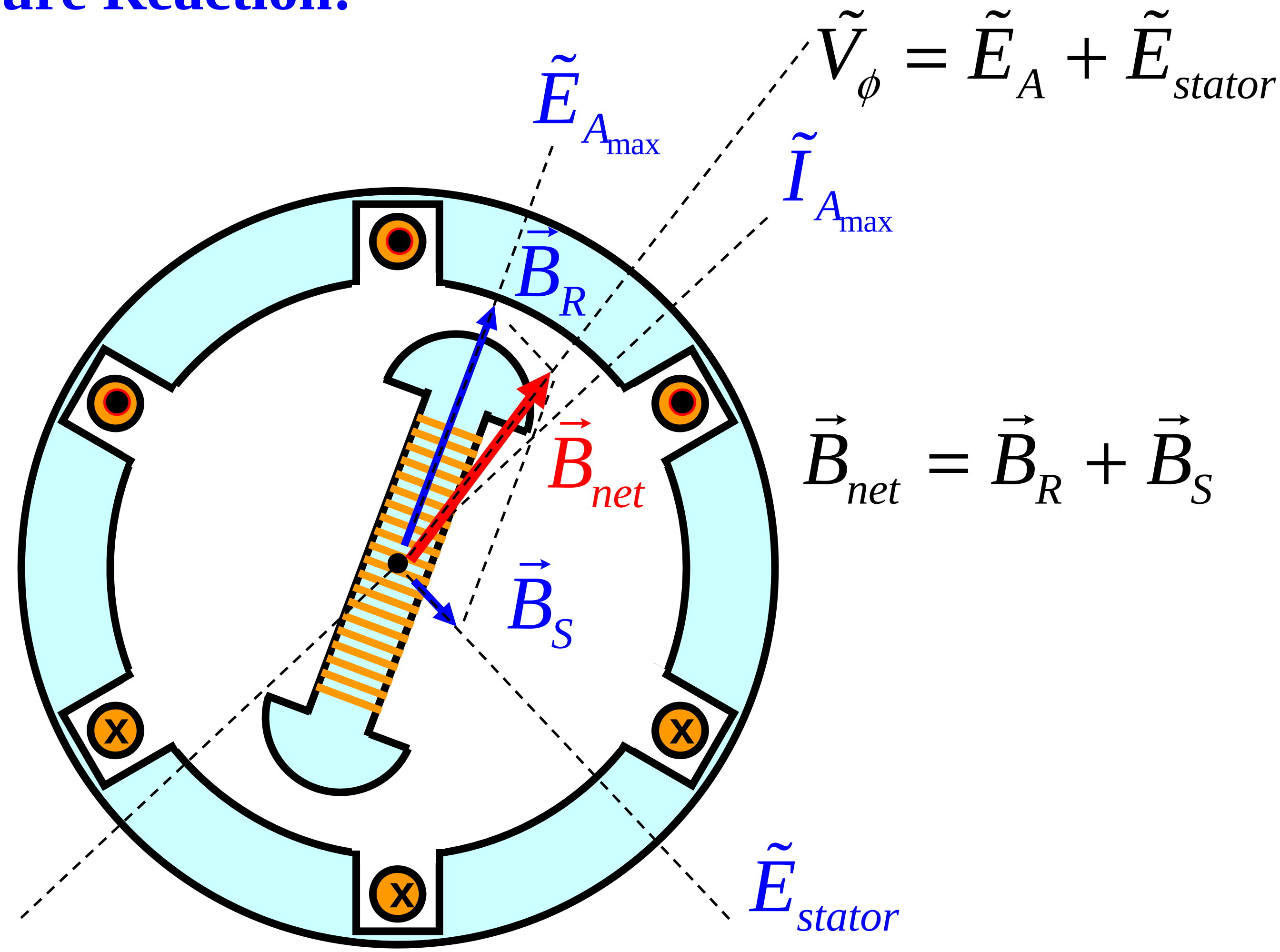
This lagging current in the stator windings will produce a magnetic field B_S, which will produce a voltage of its own, E_{stator}.

The total voltage and flux density is:

$$\tilde{V}_\phi = \tilde{E}_A + \tilde{E}_{stator}$$

$$\vec{B}_{net} = \vec{B}_R + \vec{B}_S$$

Armature Reaction:



Armature Reaction: How can this be modeled?

Note that E_{stator} lies 90° *behind* the plane of maximum current I_A.

Also note that the voltage E_{stator} is directly proportional to current I_A. Indeed, I_A is what produced E_{stator}.

This can be written

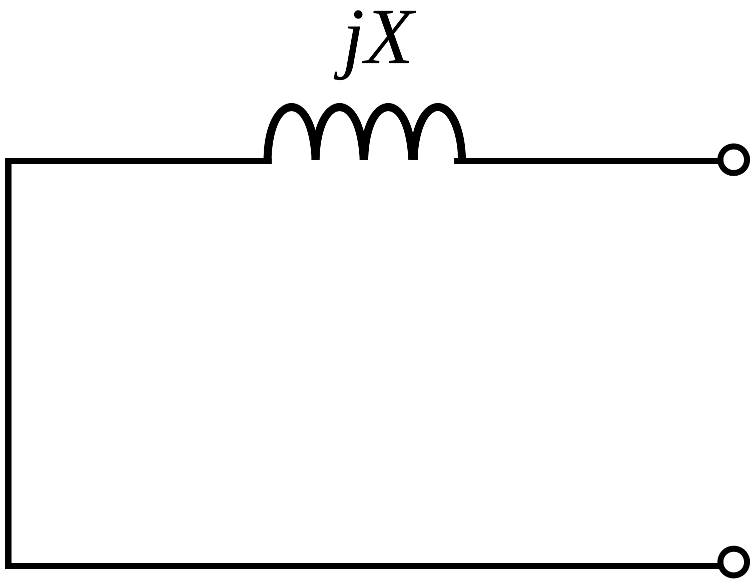
$$\tilde{E}_{stator} \propto \underset{\substack{\uparrow \\ lags}}{-} j\tilde{I}_A$$

Then

$$\tilde{V}_\phi = \tilde{E}_A + \tilde{E}_{stator}$$
$$= \tilde{E}_A - jX\tilde{I}_A$$

Armature Reaction: How can this be modeled?

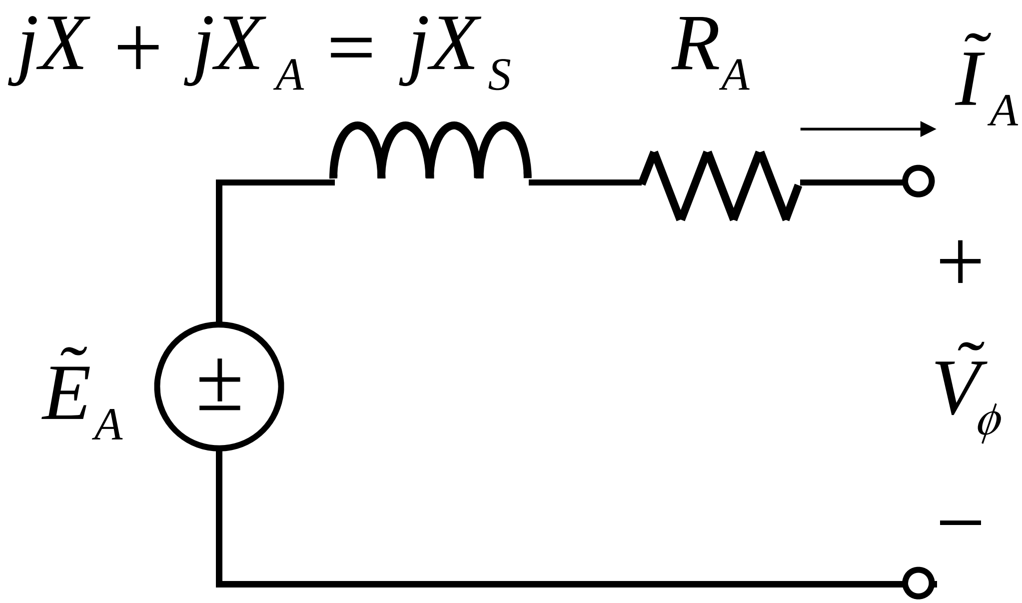
$$\tilde{V}_{\phi} = \tilde{E}_A - jX\tilde{I}_A$$



Model for Armature Reaction Only

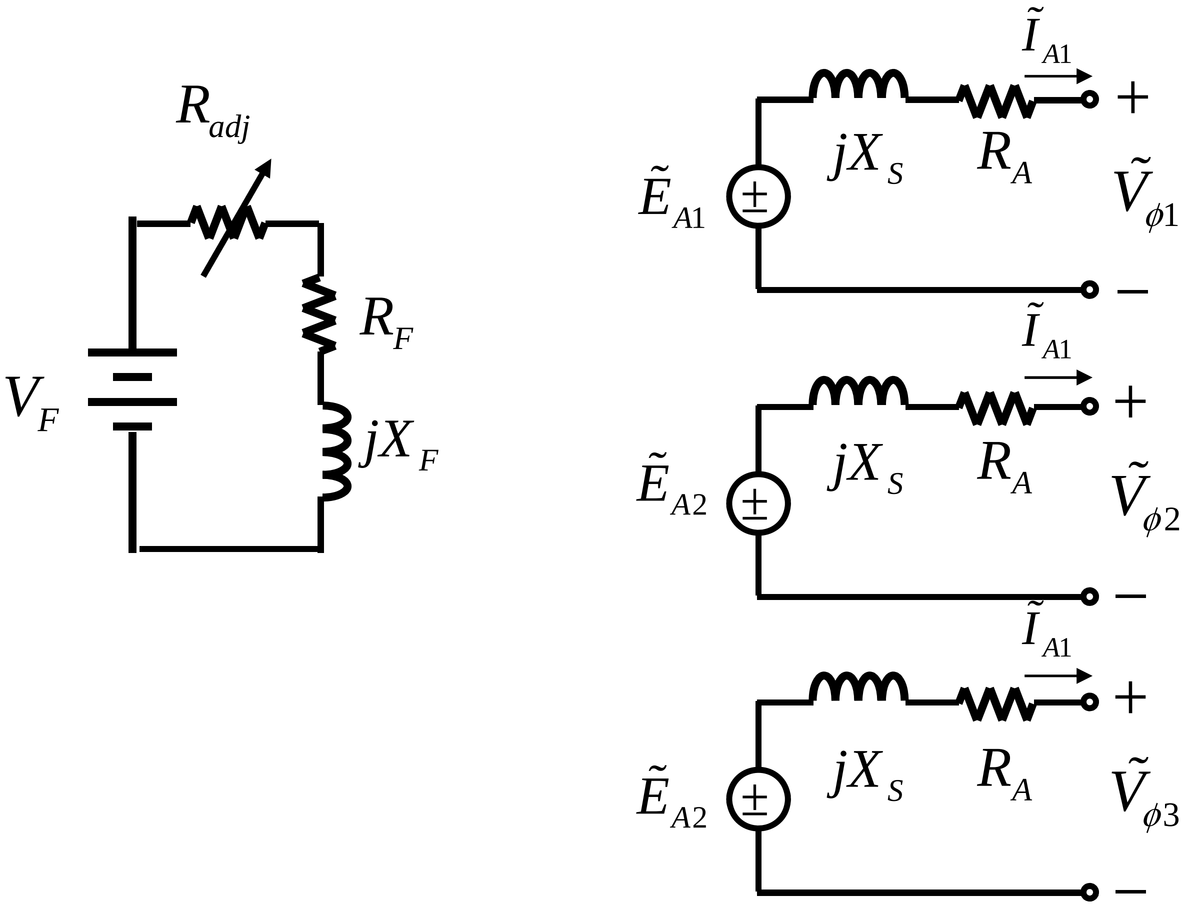
The stator winding also have a self inductance L_A (or reactance X_A) and a resistance R_A. Including these in the model gives:

$$\tilde{V}_\phi = \tilde{E}_A - jX\tilde{I}_A - jX_A\tilde{I}_A - R_A\tilde{I}_A$$

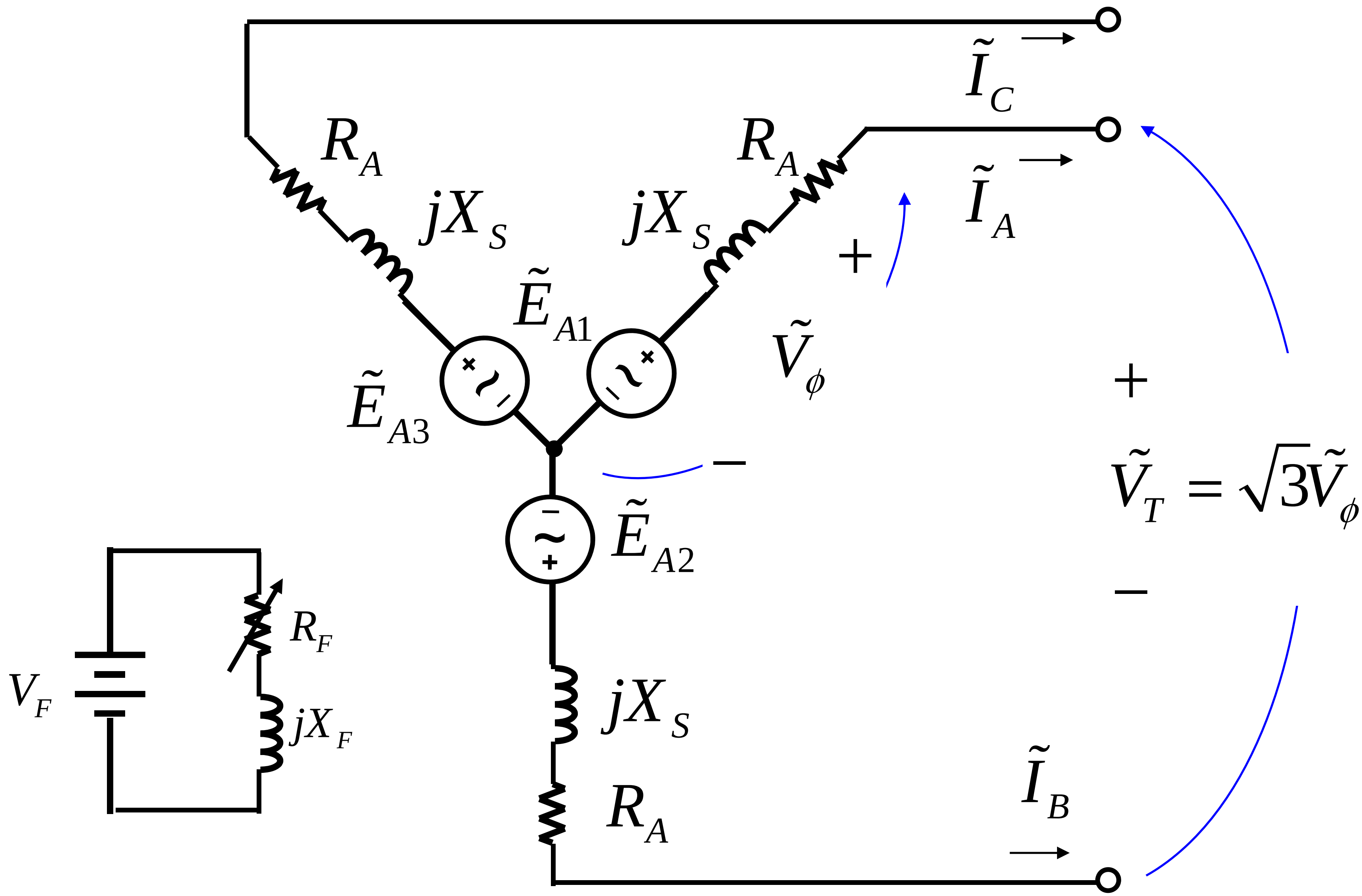


Model for Stator Phase Winding

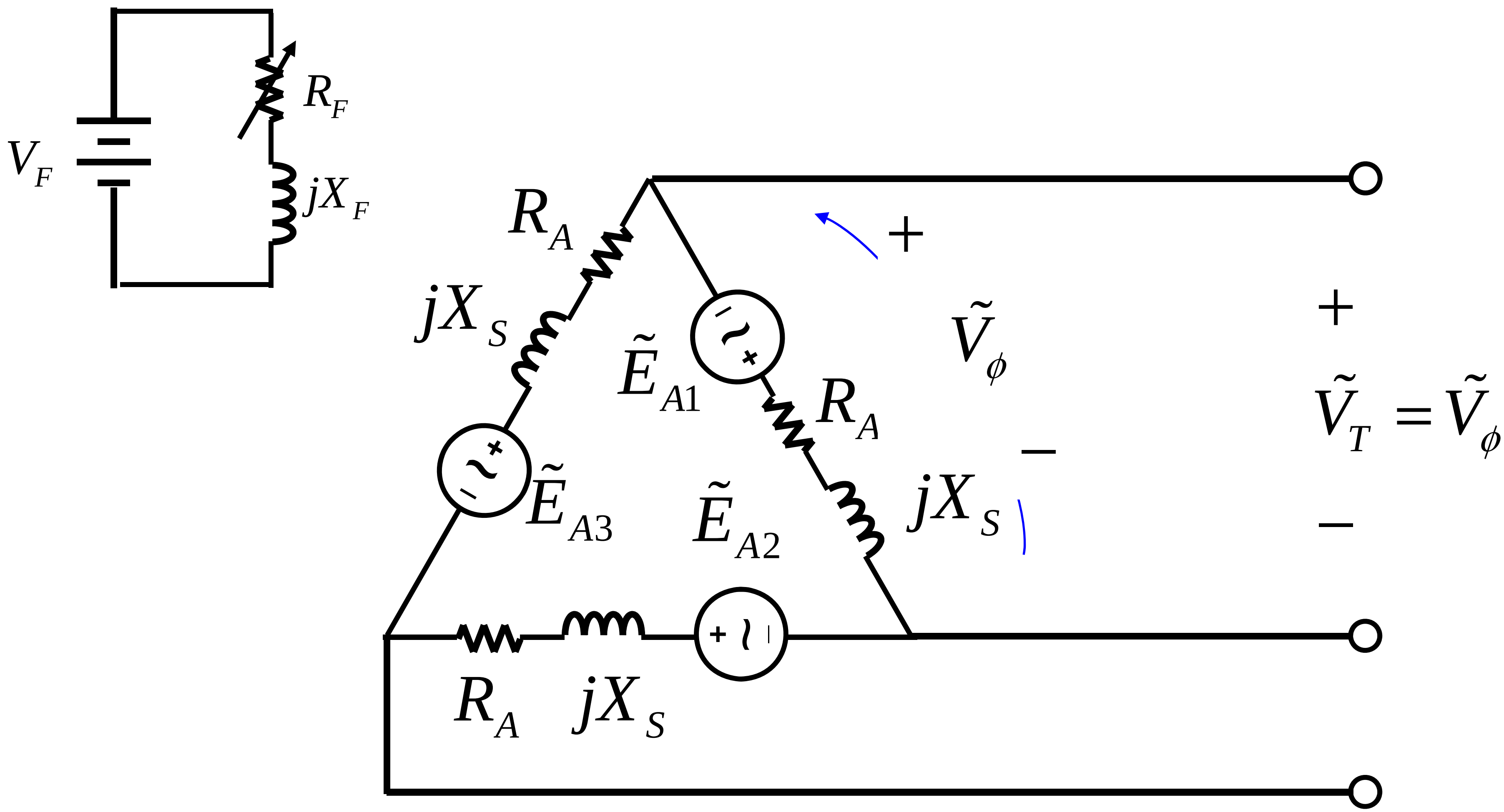
Full equivalent circuit of a three-phase synchronous generator.



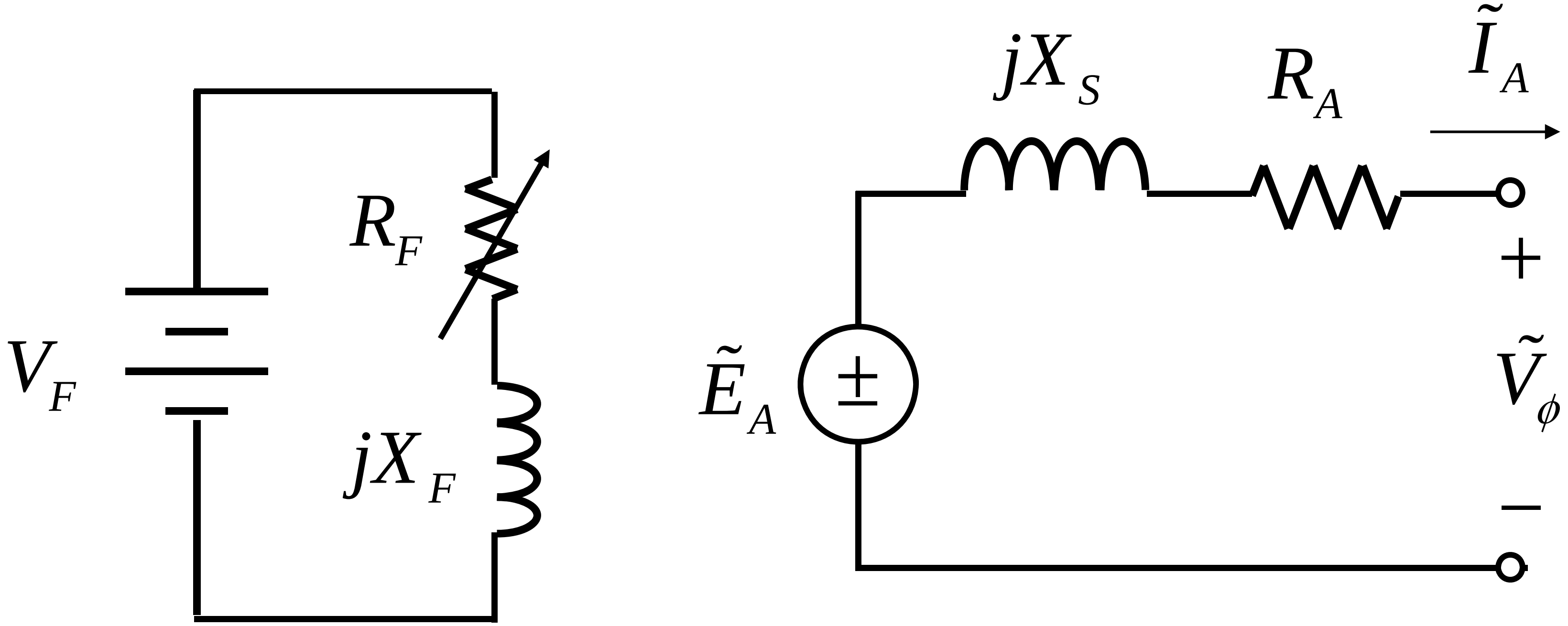
Full equivalent circuit of a three-phase synchronous generator.



Full equivalent circuit of a three-phase synchronous generator.

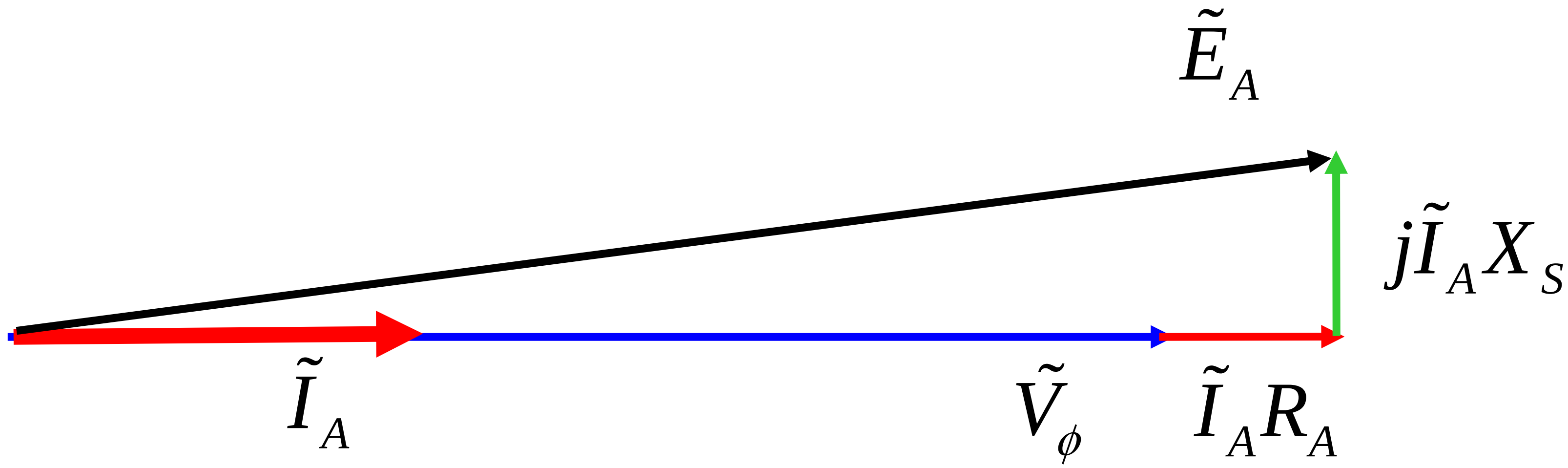


Per-phase equivalent circuit – Balanced Loads Only!.



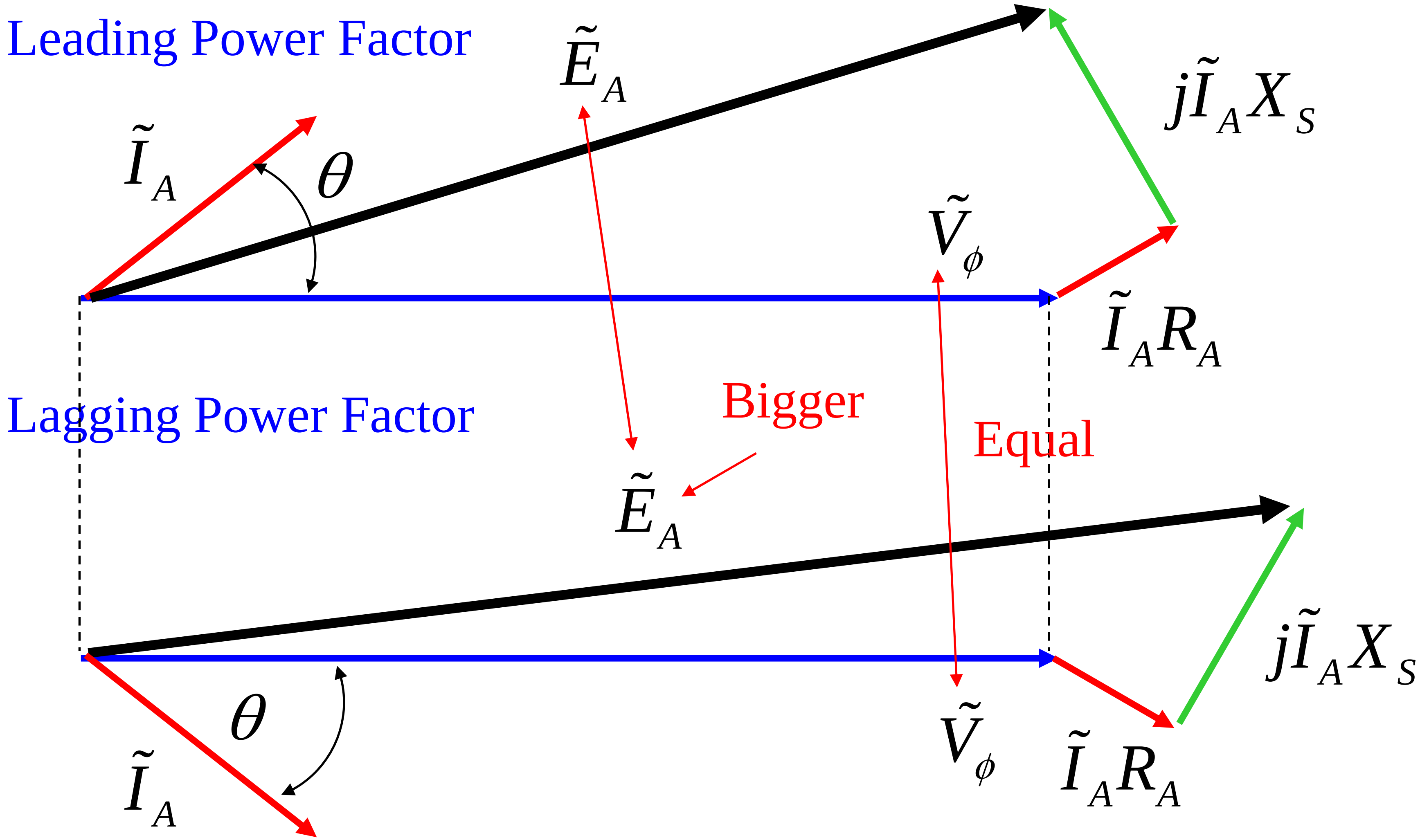
Phasor Diagrams for a Synchronous Generator.

Unity Power Factor



$$\tilde{V}_\phi = \tilde{E}_A - jX_S\tilde{I}_A - R_A\tilde{I}_A$$

Phasor Diagrams for a Synchronous Generator.



Phasor Diagrams for a Synchronous Generator.

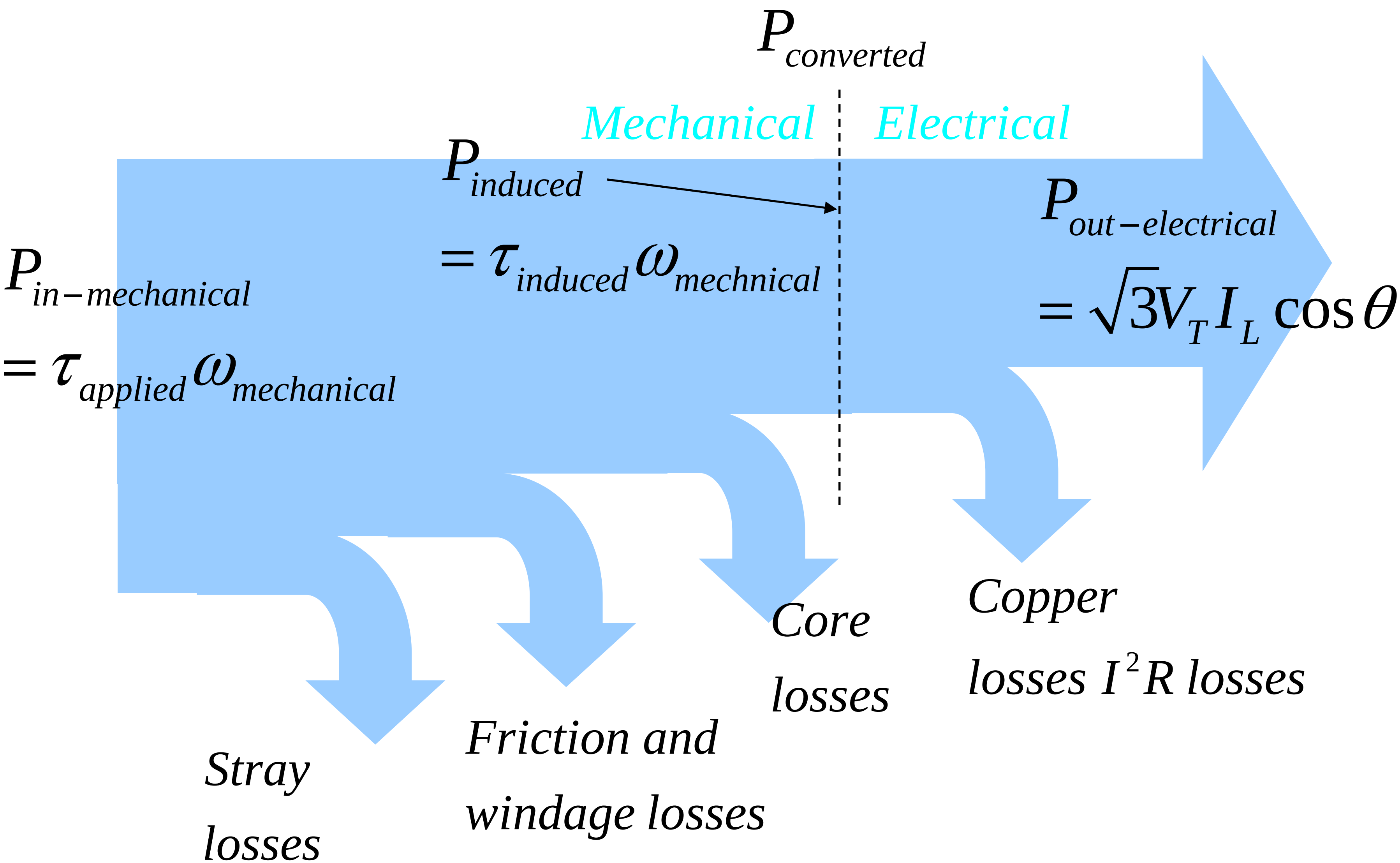
For a constant phase voltage V_ϕ and armature current I_A, *a larger internal voltage* E_A is *required for a lagging power factor*.

This in turn requires a larger flux, and hence a larger field current since

$$E_A = K\omega\phi$$

since K and ω are constant.

Power and Torque in a Synchronous Generator.



Power and Torque in a Synchronous Generator.

Recall the results from Note Set 1, Slide 100. The input mechanical power for a generator is the power applied to the shaft, which is given by

$$P_{in} = \tau_{app}\omega_m$$

while the power converted from mechanical to electrical energy is given by

$$P_{conv} = \tau_{ind}\omega_m$$

or

$$P_{conv} = 3E_A I_A \cos\gamma$$

Here γ is the angle between phasors $\tilde{E}_A$ and $\tilde{I}_A$

Power and Torque in a Synchronous Generator.

Recall the results from Note Set 2, Slides 70-71 that the real electrical output power of a (three phase) synchronous generator, for either a delta or wye connection is

$$P_{out} = \sqrt{3} V_T I_L \cos\theta$$
$$= 3 V_\phi I_A \cos\theta$$

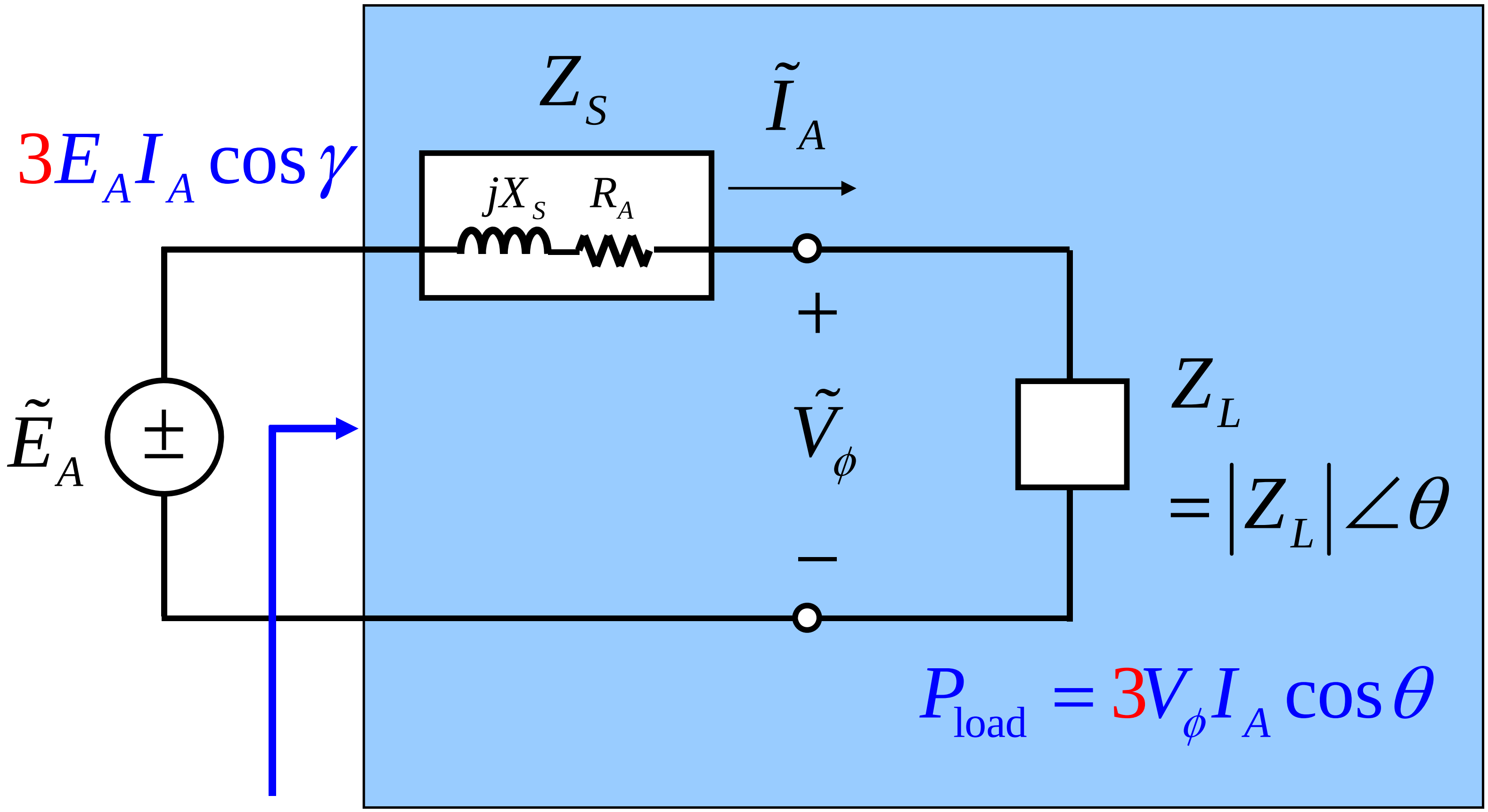
while the reactive power is

$$Q_{out} = \sqrt{3} V_T I_L \sin\theta$$
$$= 3 V_\phi I_A \sin\theta$$

Recall that the angle θ is the angle between V_ϕ and I_A, ***not*** V_T and I_L. (See Set 2, Slides 68-69)

Power and Torque in a Synchronous Generator.

$$P_{\text{supplied}} = 3E_A I_A \cos\gamma$$



$$P_{\text{load}} = 3V_\phi I_A \cos\theta$$

$$Z = |Z| \angle \gamma$$

Power and Torque in a Synchronous Generator.

In real synchronous machines of any size,

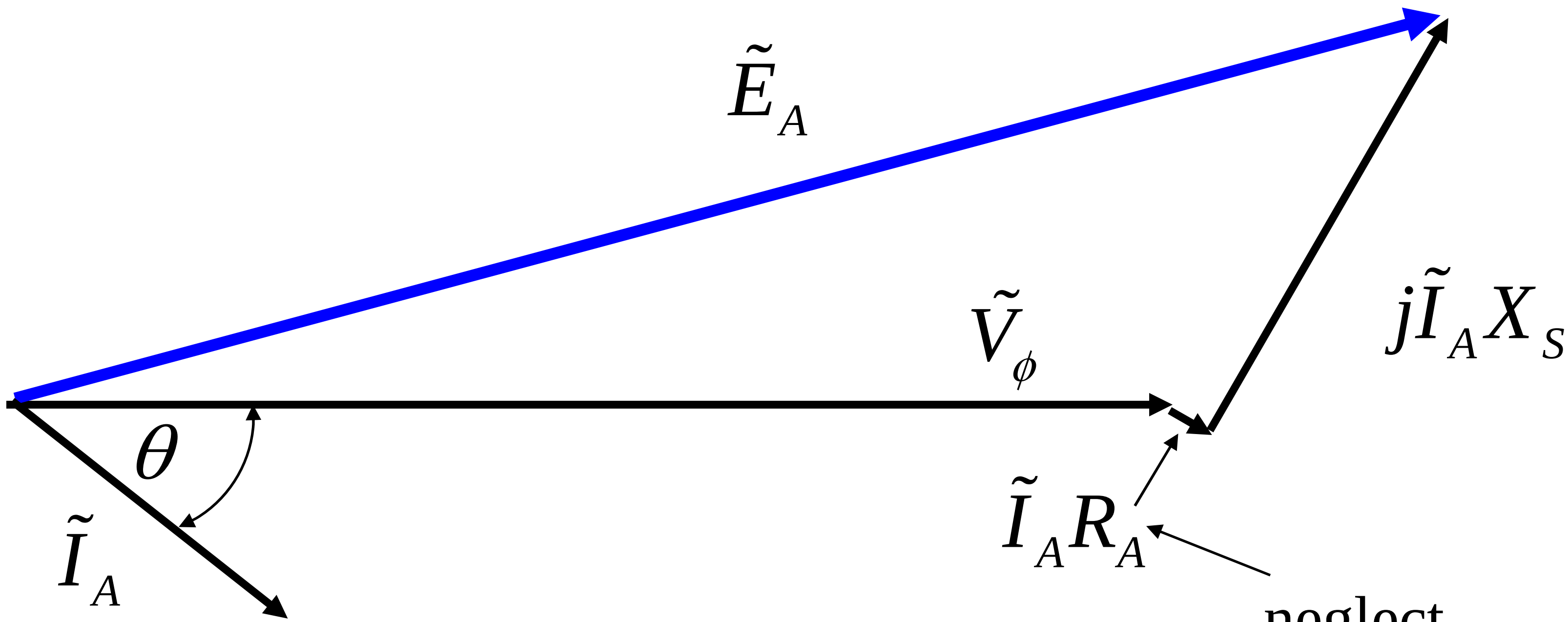
$$R_A \ll X_S$$

and can be ignored.

This approximation yields a simple expression for the output power of the generator.

Power and Torque in a Synchronous Generator.

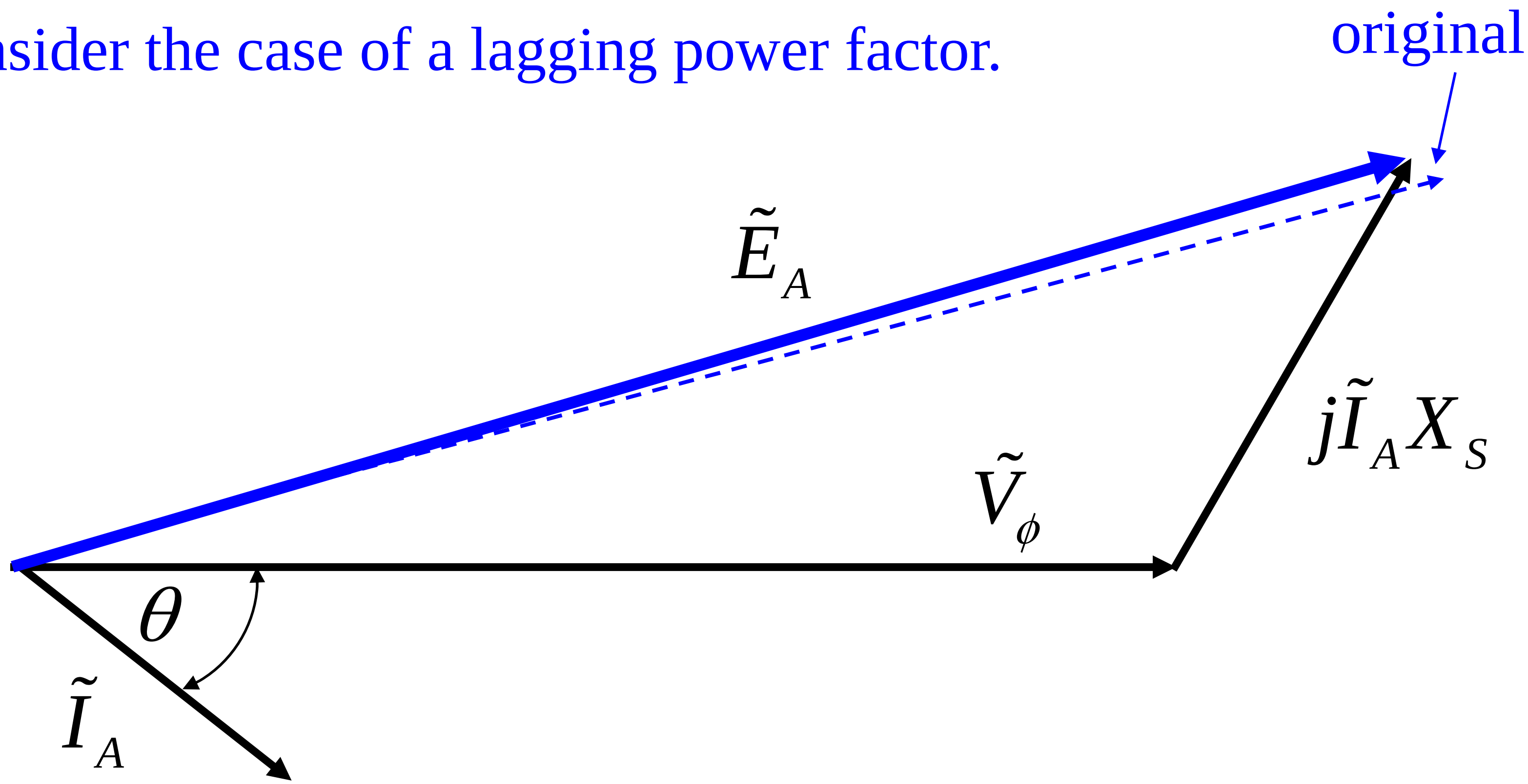
Consider the case of a lagging power factor.



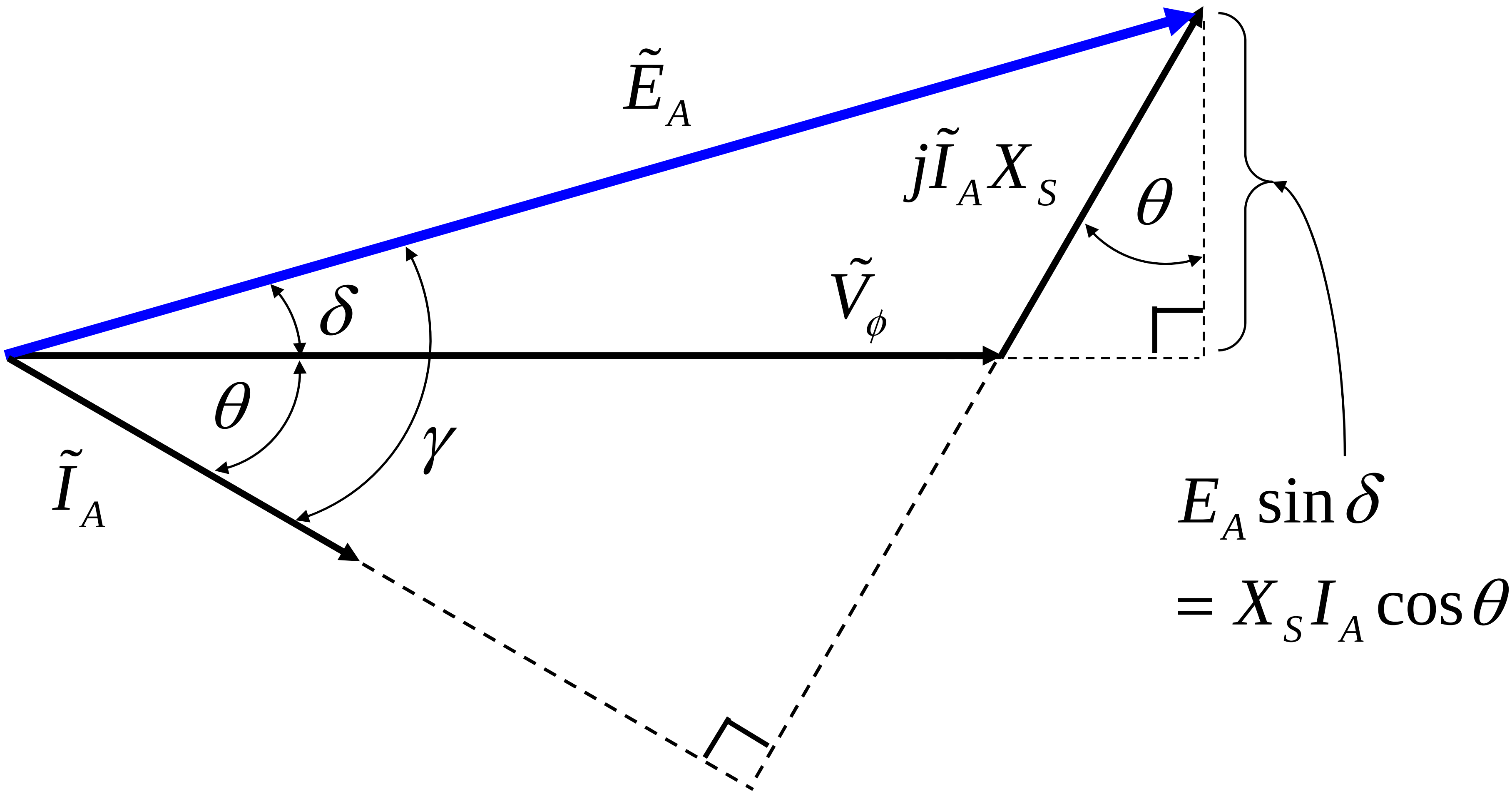
$$R_A \ll X_S$$

Power and Torque in a Synchronous Generator.

Consider the case of a lagging power factor.



Power and Torque in a Synchronous Generator. Consider the case of a lagging power factor.



Power and Torque in a Synchronous Generator.

From Slide 33,

$$P_{out} = \sqrt{3} V_T I_L \cos\theta$$
$$= 3 V_\phi I_A \cos\theta$$

$$E_A \sin\delta = X_S I_A \cos\theta \Rightarrow I_A \cos\theta = \frac{E_A \sin\delta}{X_S}$$

$$\Rightarrow P_{out} = \frac{3 V_\phi E_A \sin\delta}{X_S}$$

Power and Torque in a Synchronous Generator.

But since the resistance is zero (it was ignored) there are no electrical losses, hence,

$$P_{conv} = P_{out} = 3V_{\phi}\frac{E_A \sin\delta}{X_S}$$

Note that the maximum power that the generator can supply occurs when sin δ = 1, or,

$$P_{\max} = \frac{3V_{\phi}E_A}{X_S}$$

Power and Torque in a Synchronous Generator.

Static Stability Limit of the Generator:

$$P_{\max} = \frac{3V_{\phi}E_{A}}{X_{S}}$$

Typically,

$$P_{out} \ll P_{\max}$$

Power and Torque in a Synchronous Generator.

Recall Set 4, Slide 89, $\vec{\tau}_{ind} \propto \vec{B}_R \times \vec{B}_S$

or, Set 4, Slide 91, $\vec{\tau}_{ind} = k\vec{B}_R \times \vec{B}_{net}$

$$\left|\vec{\tau}_{ind}\right| = \tau_{ind} = kB_R B_S \sin\delta$$

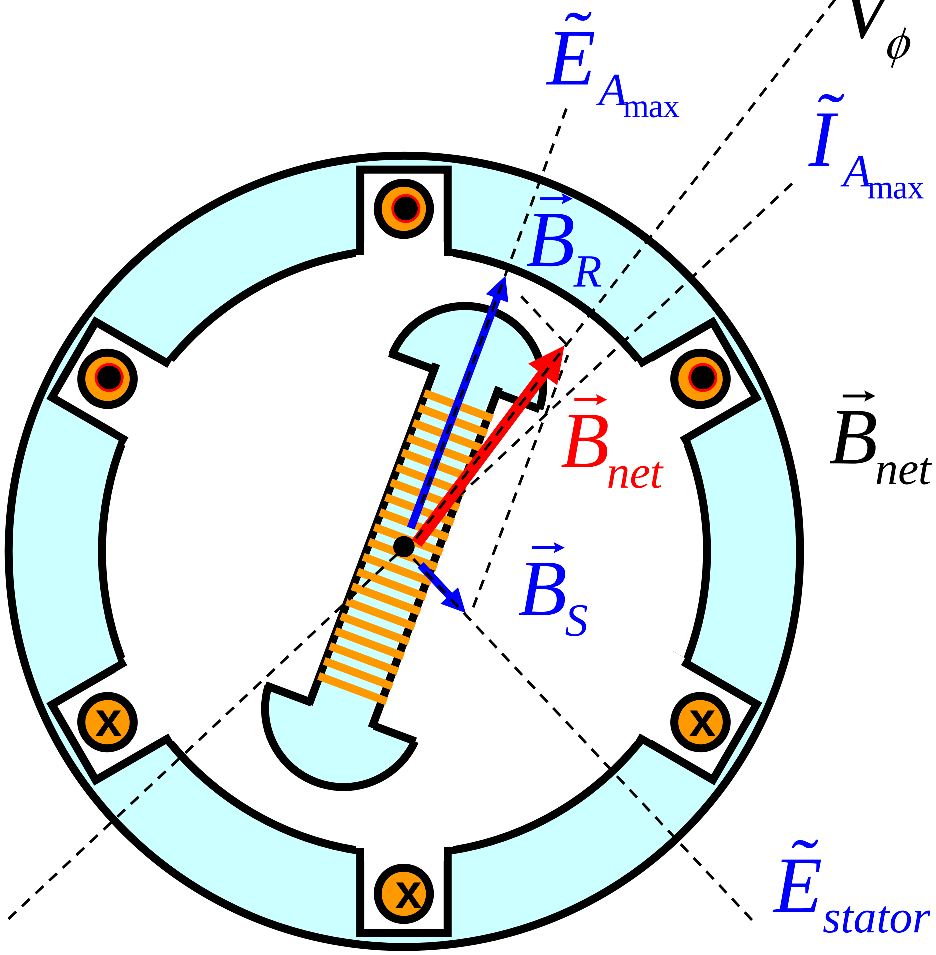
but, from Slide 20, (see next slide)

$$\vec{B}_R \Rightarrow \tilde{E}_A, \quad \vec{B}_{net} \Rightarrow \tilde{V}_\phi$$

$$\angle\left(\vec{B}_R, \vec{B}_{net}\right) = \angle\left(\tilde{E}_A, \tilde{V}_\phi\right) = \delta$$

Power and Torque in a Synchronous Generator.

$$\tilde{V}_{\phi} = \tilde{E}_A + \tilde{E}_{stator}$$



$$\vec{B}_{net} = \vec{B}_R + \vec{B}_S$$

Power and Torque in a Synchronous Generator.

From Slides 32 and 40,

$$P_{conv} = \tau_{ind}\omega_m,\ P_{conv} = 3V_\phi \frac{E_A \sin\delta}{X_S}$$

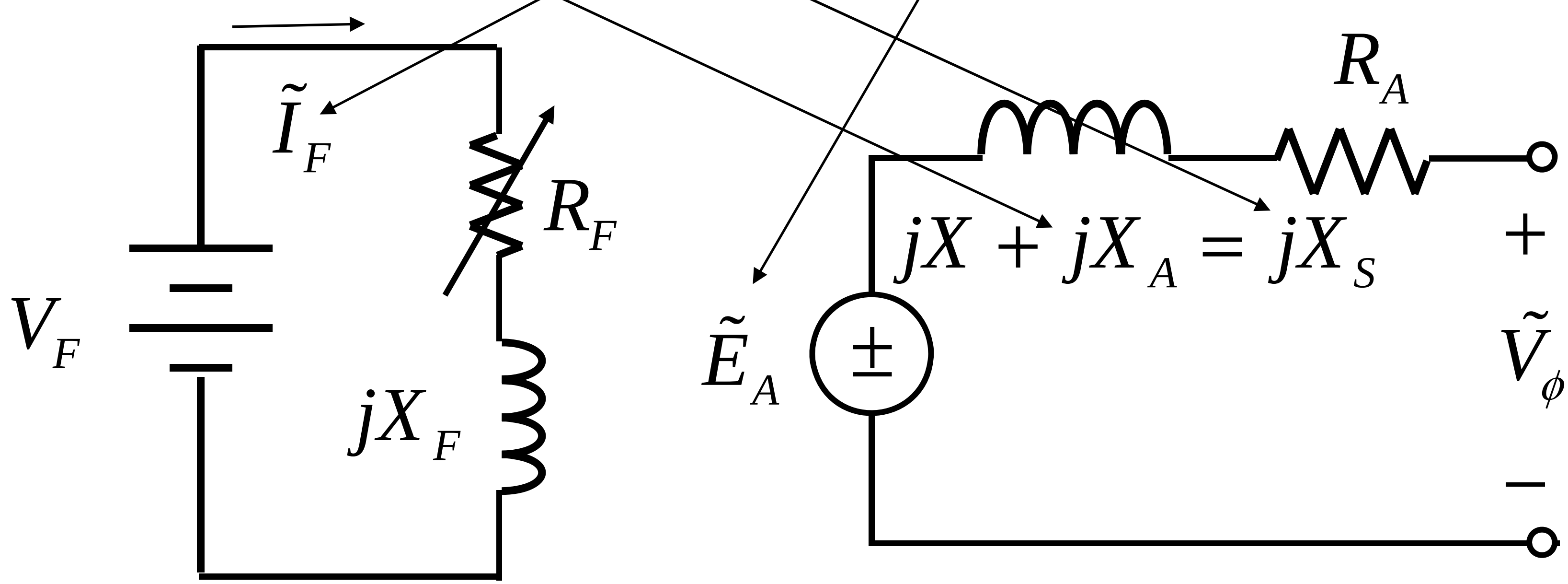
$$\Rightarrow \tau_{ind} = \frac{P_{conv}}{\omega_m} = 3V_\phi \frac{E_A \sin\delta}{\omega_m X_S}$$

$$\tau_{ind} = 3V_\phi \frac{E_A \sin\delta}{\omega_m X_S}$$

Measuring Synchronous Generator Model Parameters

We need to determine numerical values for:

1. The relationship between the field current I_F and the flux, or, equivalently between the field current and E_A
2. The synchronous reactance
3. The armature reactance



Measuring Synchronous Generator Model Parameters

1. Perform an open-circuit test.

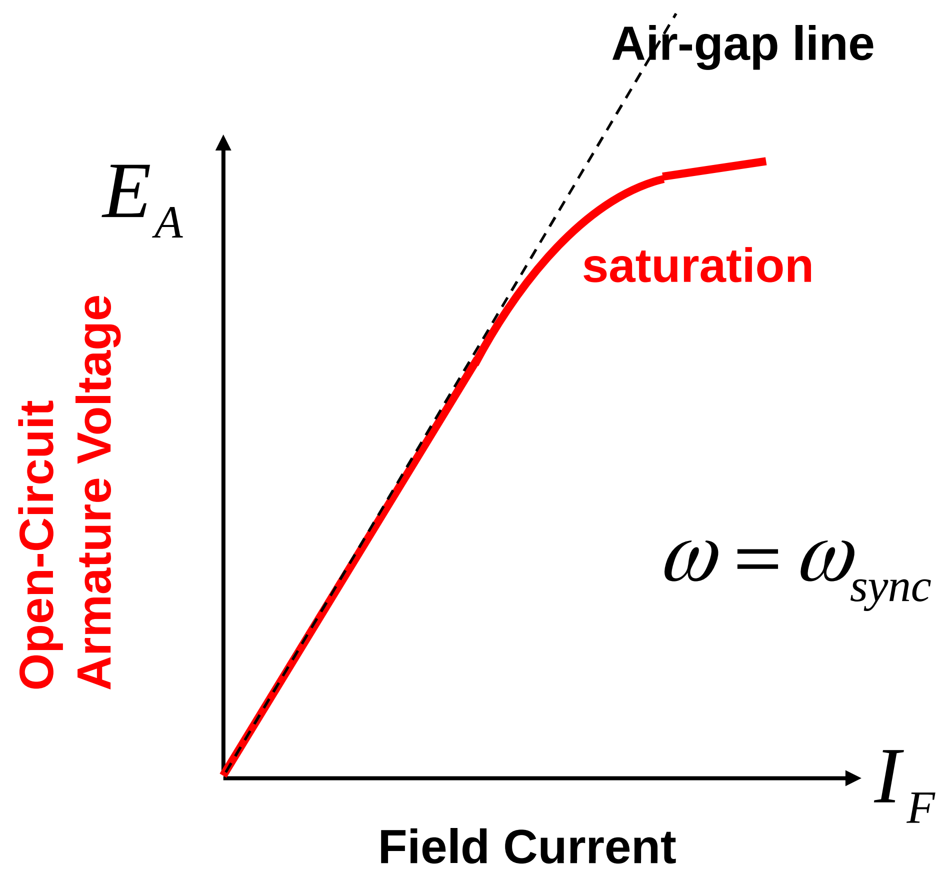
a. Turn the generator at rated speed.
b. Disconnect the generator from all loads
c. Set the field current to zero
d. Gradually increase the field current and measure the resulting terminal voltage

For the open-circuit test, $I_A = 0 \Rightarrow \tilde{E}_A = \tilde{V}_\phi$

We can thus make a plot of E_A vs. I_F.

Measuring Synchronous Generator Model Parameters

1. Perform an open-circuit test. (recall Slide 14)



Open-Circuit Characteristics (OCC)

Measuring Synchronous Generator Model Parameters

Note that curve is highly linear until saturation starts to occur at high field currents.

The unsaturated iron in the frame of the machine has a reluctance that is thousands of time smaller than the reluctance of the air gap, so almost all the magnetomotive force appears across the gap and the flux increase is linear.

When the iron begins to saturate its reluctance increases dramatically, and the flux increases at a much slower rate with an increase in mmf.

The linear portion of the Open Circuit Characteristics is called the *air-gap line* of the characteristic.

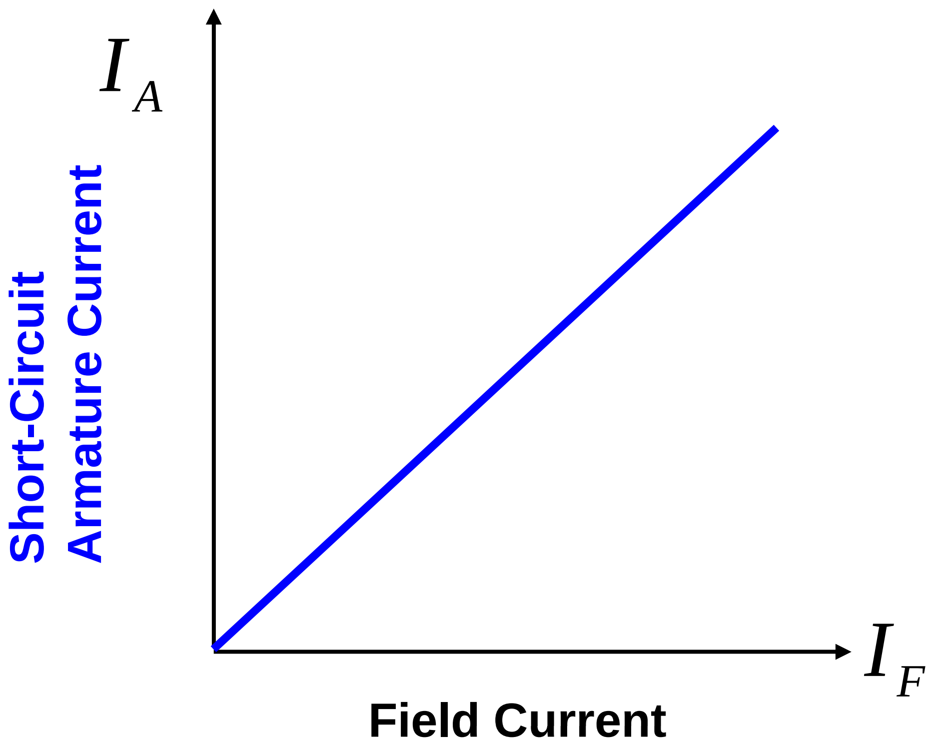
Measuring Synchronous Generator Model Parameters

2. Perform an <u>short-circuit</u> test.

a. **Set the field current to zero!**
b. Short-circuit the output of the generator (through ammeters)
c. Measure the armature current I_A or the line current I_L as the field current is increased.

We can thus make a plot of I_A vs. I_F. This plot is called the *Short-Circuit Characteristics.*

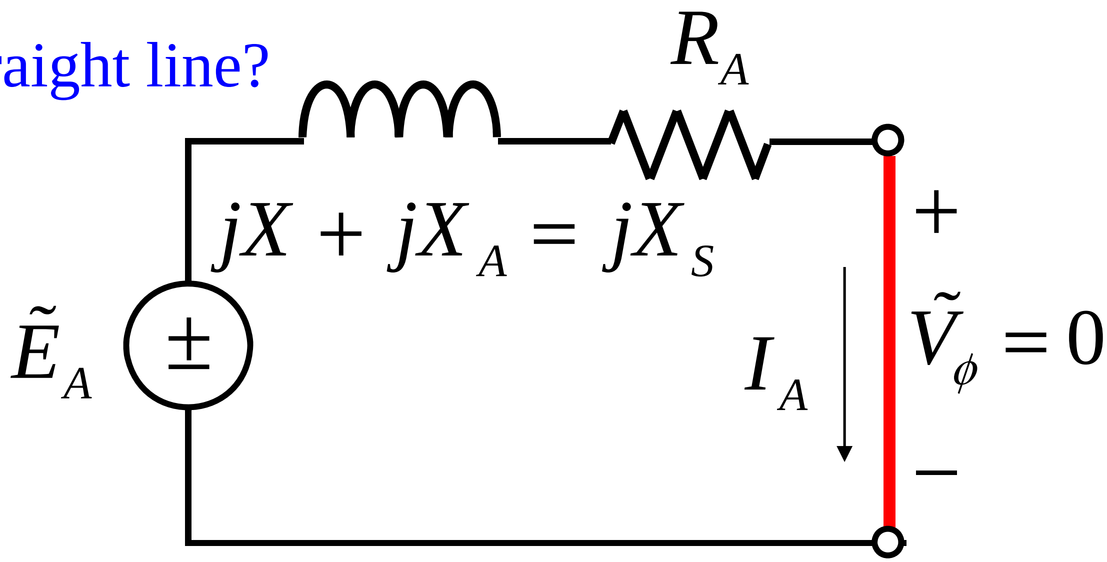
Measuring Synchronous Generator Model Parameters



Short-Circuit Characteristics (SCC)

Measuring Synchronous Generator Model Parameters

Why a straight line?



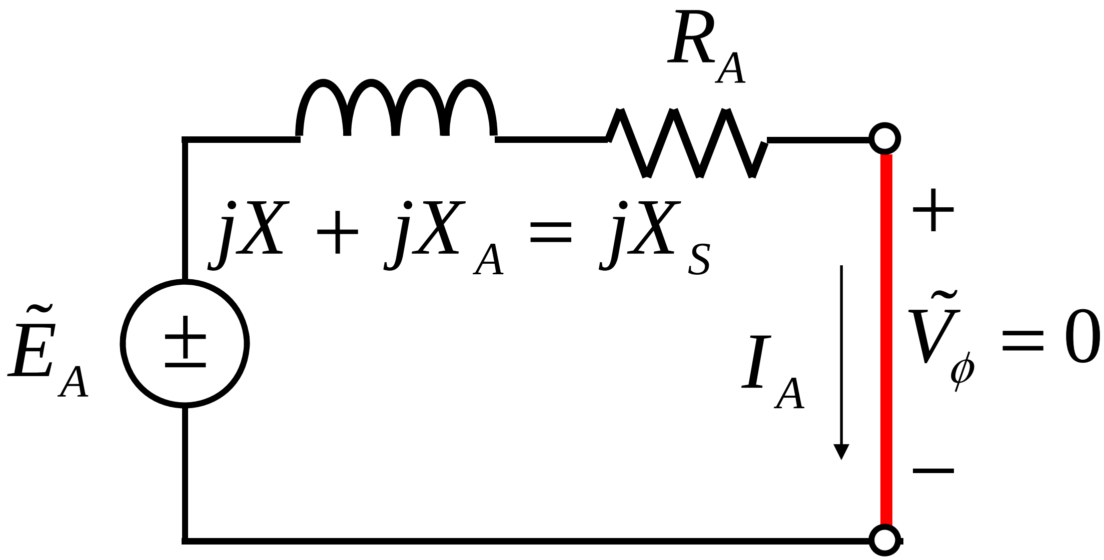
Recall from Slide 42 that $\vec{B}_R \Rightarrow \tilde{E}_A, \quad \vec{B}_{net} \Rightarrow \tilde{V}_\phi = 0$

$$\vec{B}_{net} = \vec{B}_R + \vec{B}_S$$

Hence B_{net} is very small, the iron is unsaturated, and the SCC curve is linear.

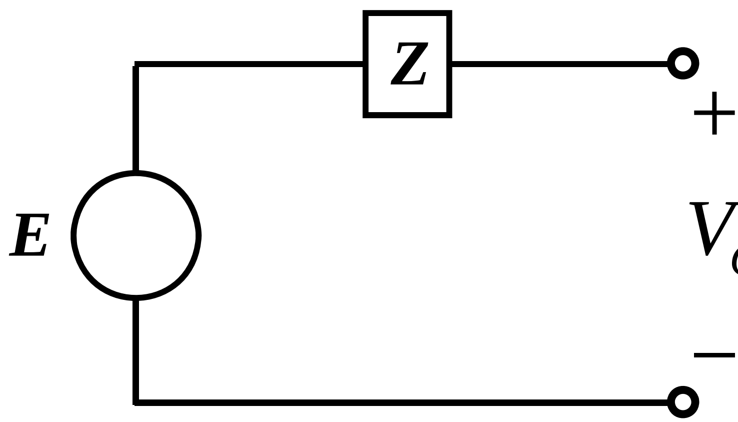
Measuring Synchronous Generator Model Parameters

Now we can get the necessary values.

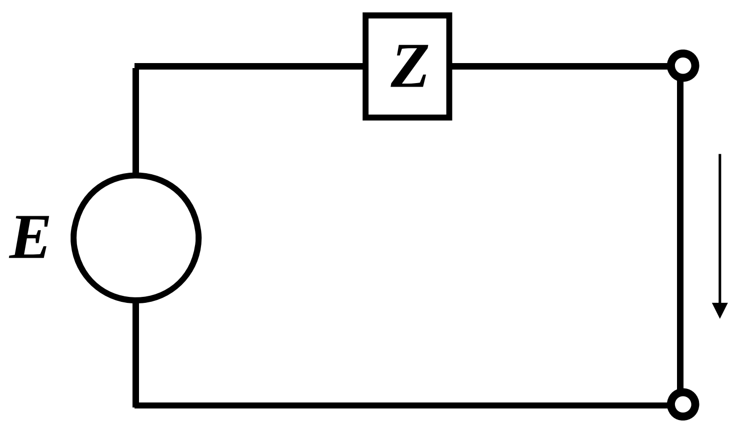


$$Z_S = \sqrt{R_A{}^2 + X_S{}^2} = \frac{E_A}{I_A} = \frac{V_\phi\big|_{\substack{open\\circuit}}}{I_A}$$

Note: Basic Circuits 101:



$$V_{OC} = E$$



$$I_{SC} = \frac{E}{Z}$$

$$Z = \frac{V\big|_{\substack{open\\circuit}}}{I\big|_{\substack{short\\circuit}}}$$

Measuring Synchronous Generator Model Parameters

$$X_S \gg R_A$$

$$Z_S = \sqrt{R_A^{\,2} + X_S^{\,2}} \approx X_S = \frac{E_A}{I_A} = \frac{\left. V_\phi \right|_{\substack{open \\ circuit}}}{\left. I_A \right|_{\substack{short \\ circuit}}}$$

$$\boxed{X_S = \frac{\left. V_\phi \right|_{\substack{open \\ circuit}}}{\left. I_A \right|_{\substack{short \\ circuit}}}}$$

Measuring Synchronous Generator Model Parameters

To get an approximate value of X_S:

For a given field current,

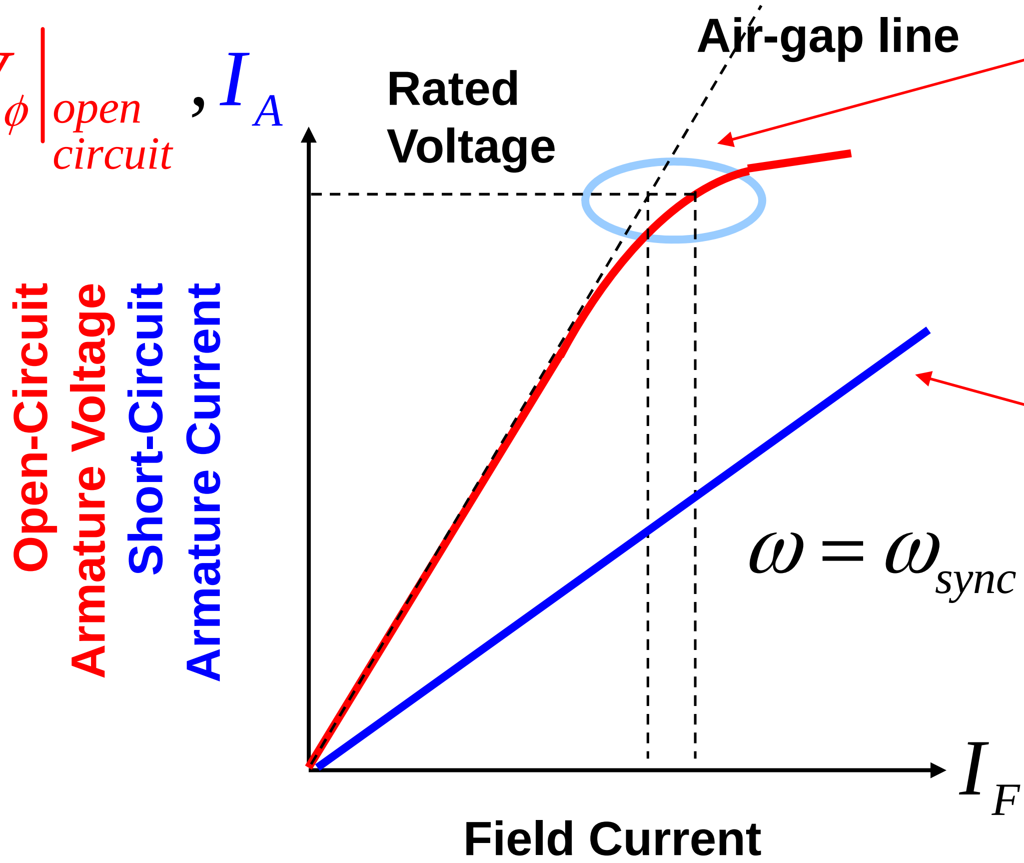
1. Determine E_A from the OCC
2. Determine I_A from the SCC
3. Determine X_S via:

$$X_S = \frac{V_\phi \big|_{\substack{open\\circuit}}}{I_A \big|_{\substack{short\\circuit}}}$$

1. R_S can be determined (approximately) from simple DC measurements

Measuring Synchronous Generator Model Parameters

$$E_A = V_\phi \Big|_{\substack{open \\ circuit}} , I_A$$



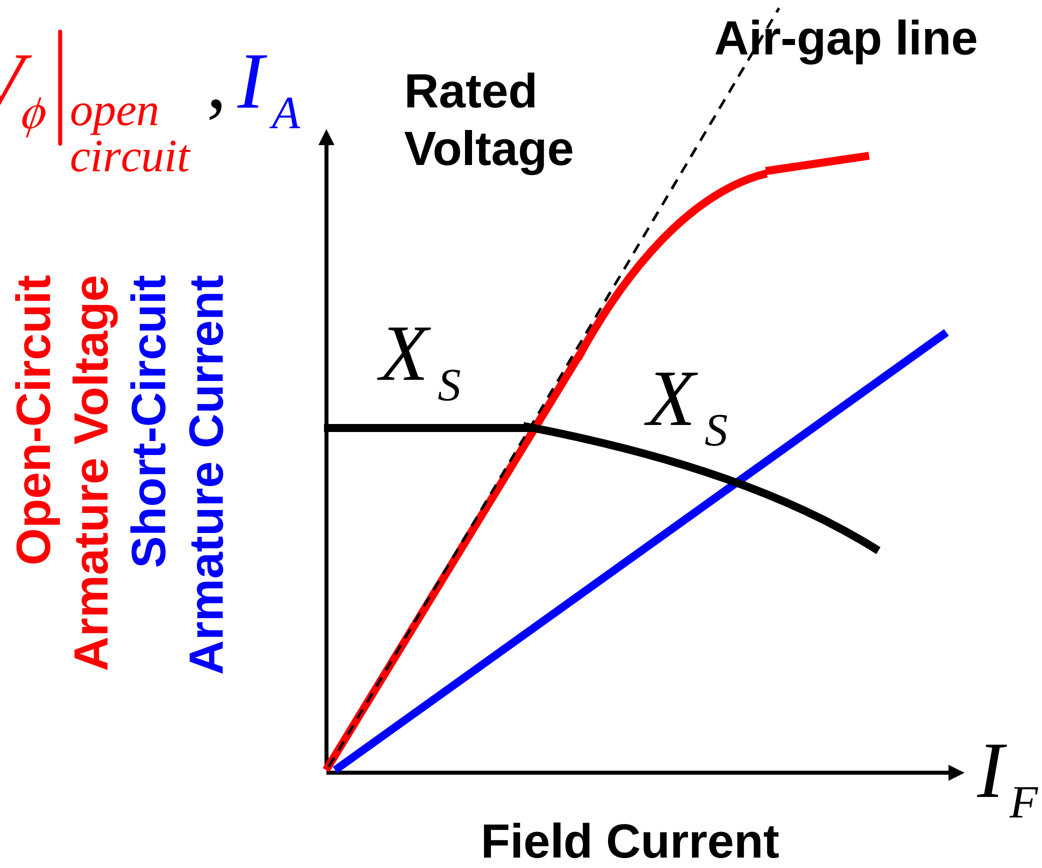
Slightly saturated

$$X_S = \frac{V_\phi \Big|_{\substack{open \\ circuit}}}{I_A \Big|_{\substack{short \\ circuit}}}$$

Very much unsaturated

Measuring Synchronous Generator Model Parameters

$$E_A = V_\phi \Big|_{\substack{open\\circuit}}, I_A$$



Effects of a Changing Load on a Synchronous Generator

Under increased load, the real/reactive power increases as does the load current. For a fixed field circuit (resistance) the field current and hence the flux is constant. Since we generally assume the machine is being turned at the constant, synchronous frequency ω, the magnitude of the internally generated voltage

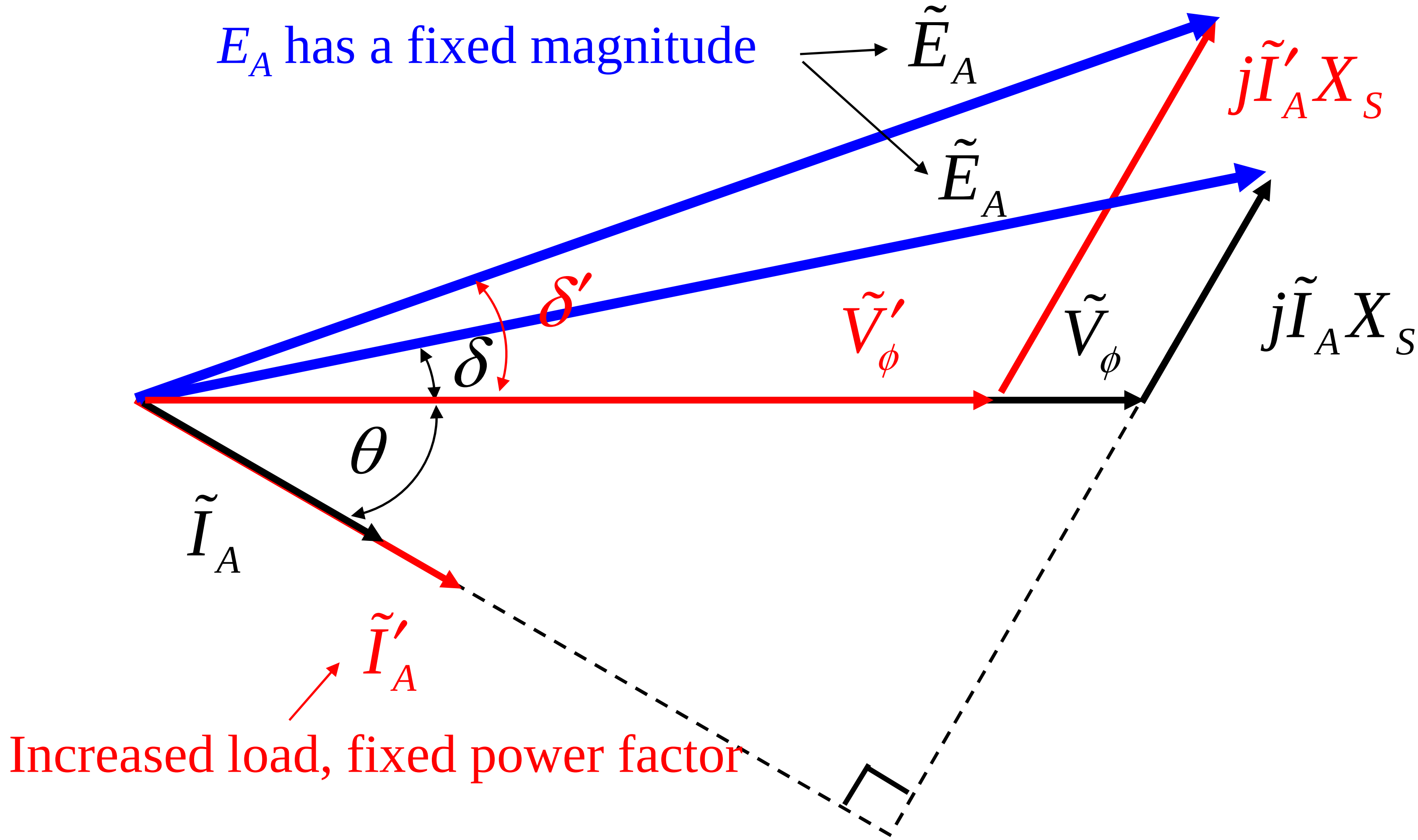
$$E_A = K\omega\phi$$

remains constant.

But if E_A is constant, what quantity varies with a changing load?

Effects of a Changing Load on a Synchronous Generator

First examine a load with a lagging power factor.



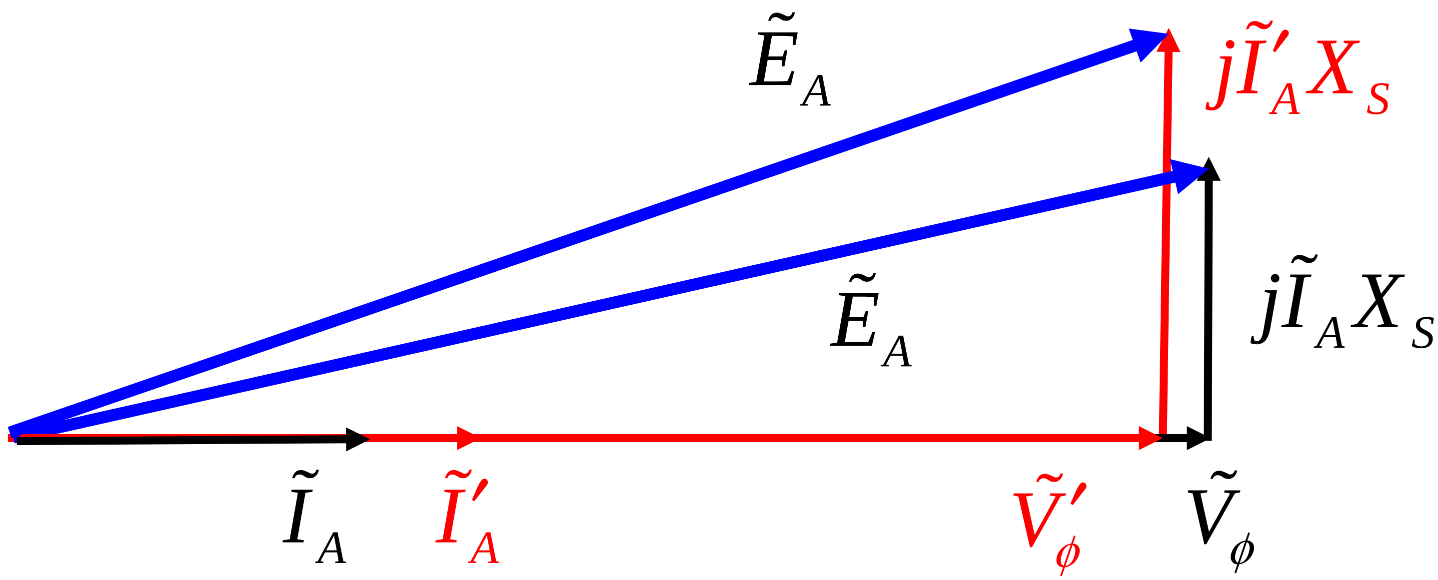
Effects of a Changing Load on a Synchronous Generator

First examine a load with a lagging power factor.

As the load increases, V_ϕ drops (sharply).

Effects of a Changing Load on a Synchronous Generator

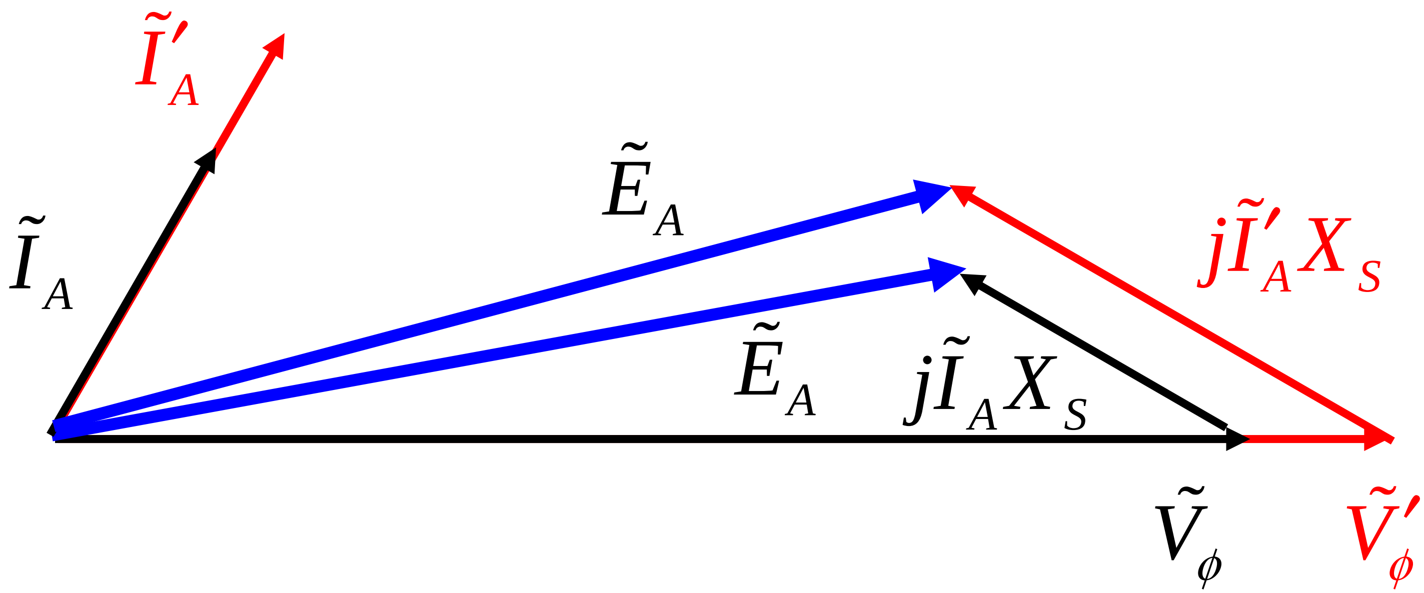
Next examine a load with a unity power factor.



As the load increases, V_ϕ still drops, but much less than before.

Effects of a Changing Load on a Synchronous Generator

Now examine a load with a leading power factor.



As the load increases, V_ϕ <u>*increases*</u>.

Effects of a Changing Load on a Synchronous Generator

Voltage Regulation:

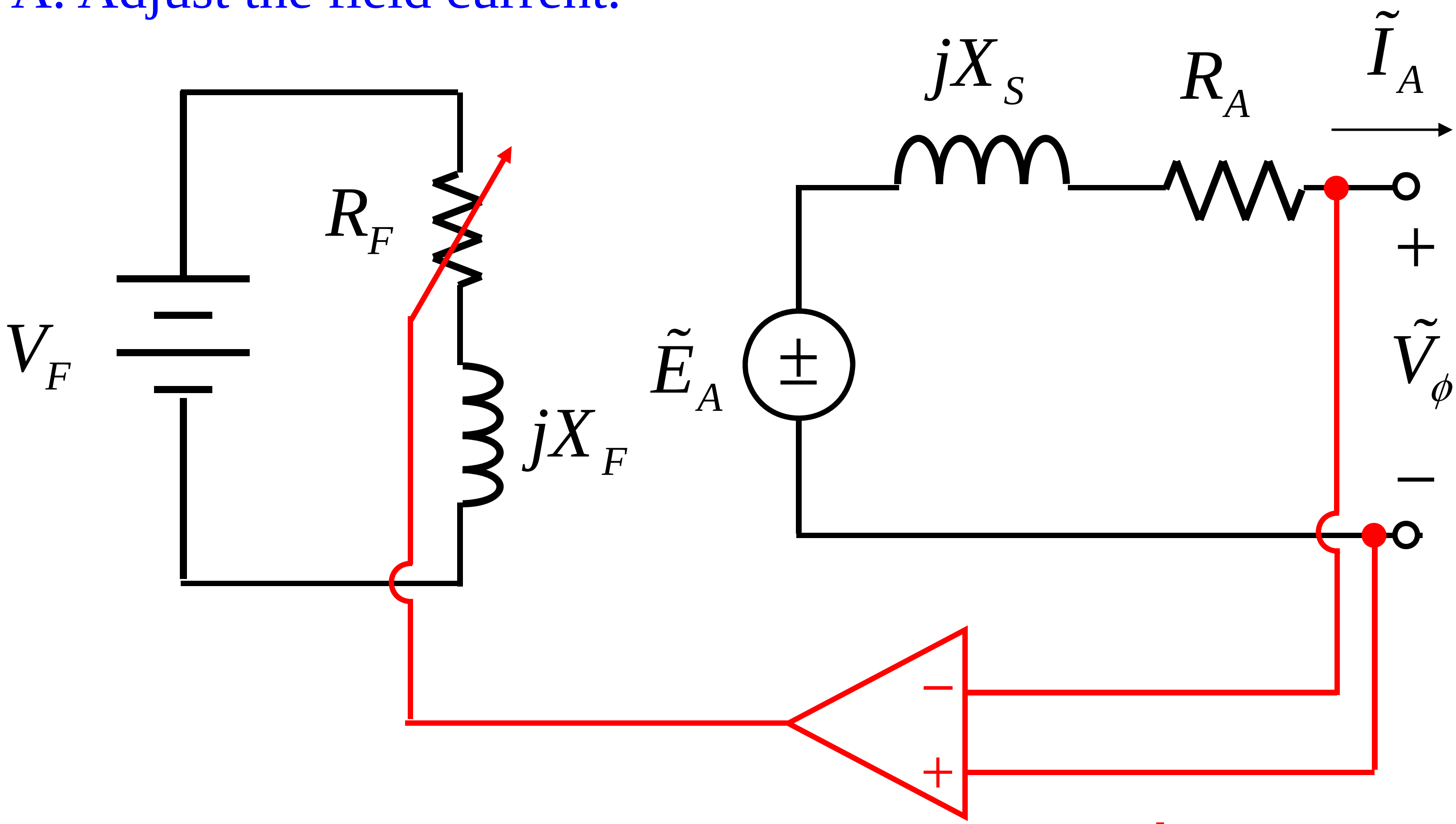
$$VR = \frac{V_{no\ load} - V_{full\ load}}{V_{full\ load}} \times 100\%$$

$$= \begin{cases} \text{postive (largish),} & \text{lagging PF} \\ \text{smallish (good),} & \text{unity PF} \\ \text{negative,} & \text{leading PF} \end{cases}$$

Effects of a Changing Load on a Synchronous Generator

Q. What can we do to compensate for changing loads?

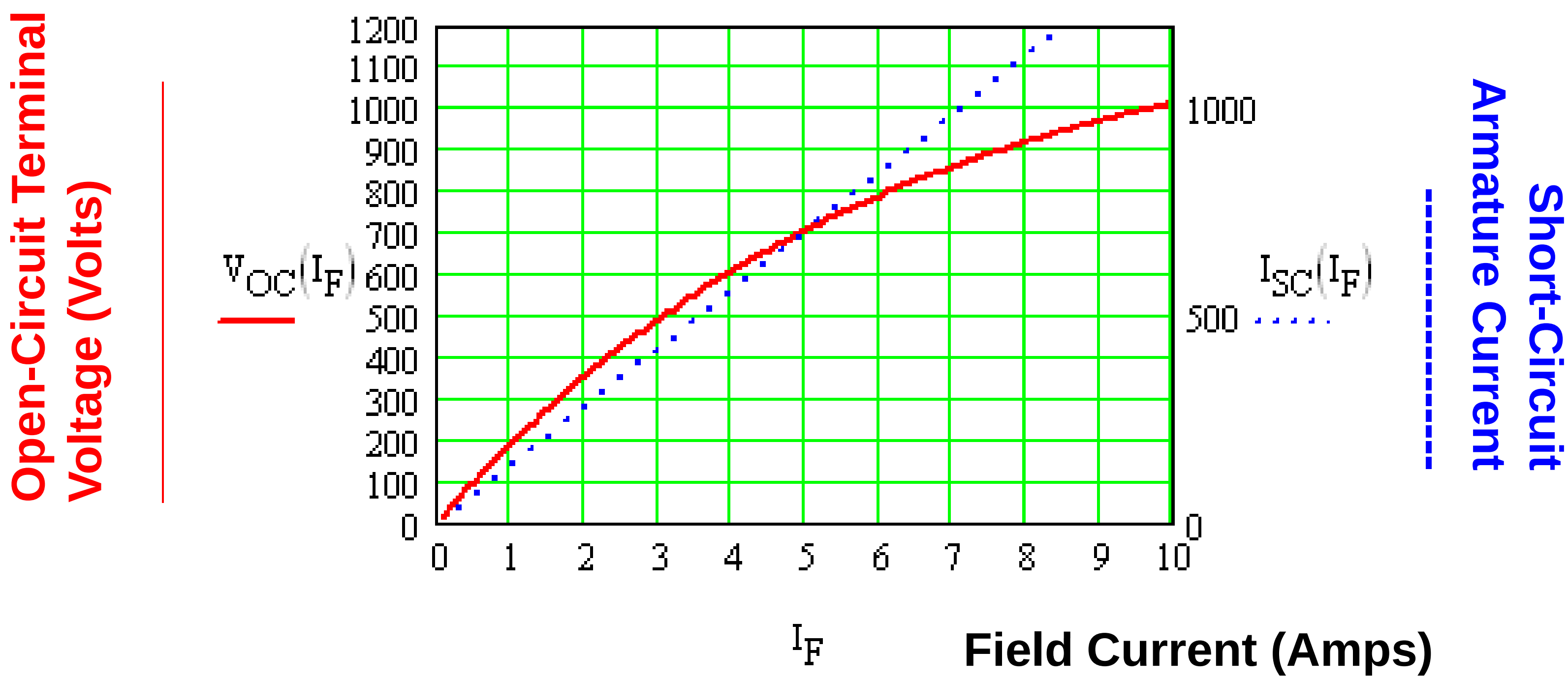
A. Adjust the field current.



another time...

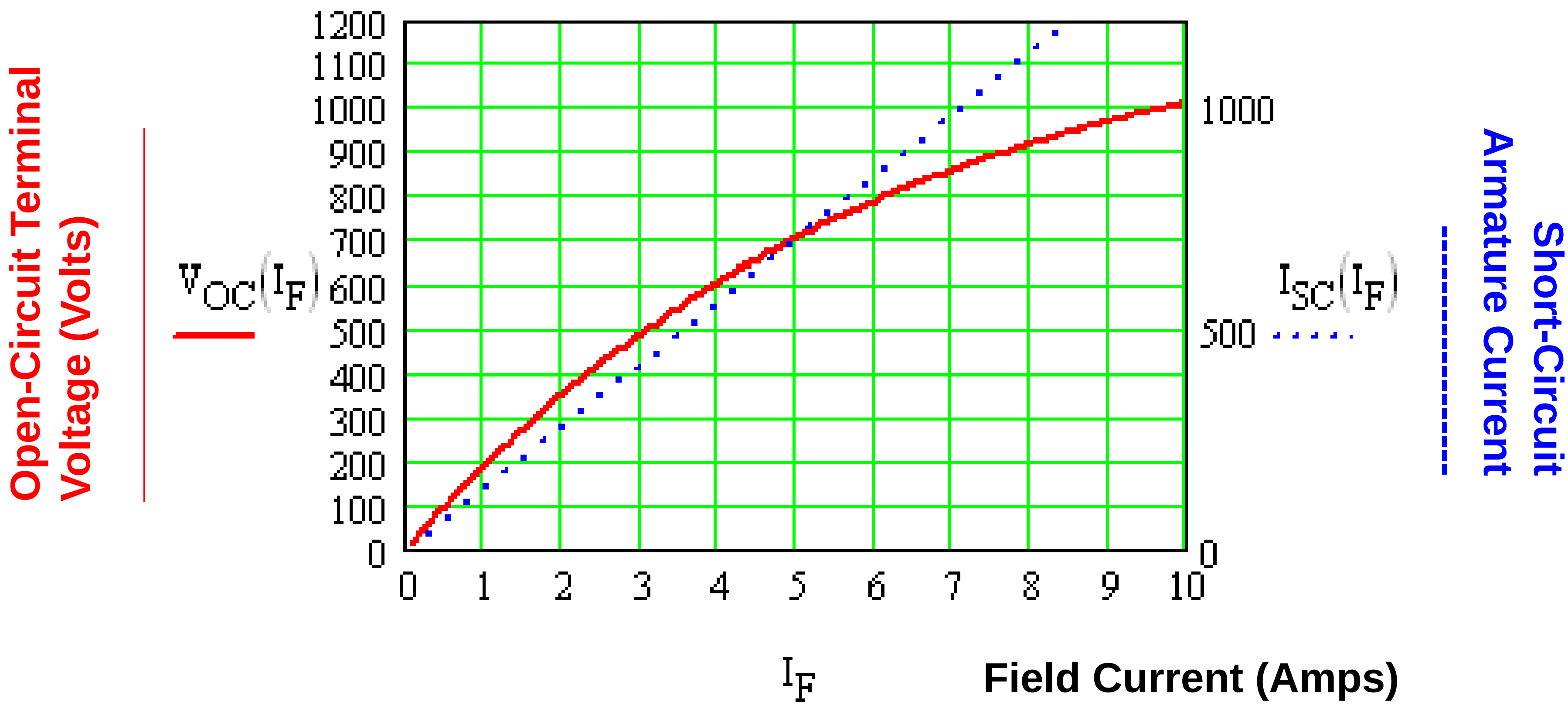
Synchronous Generator Examples

Text Problems 5.5 – 5.14: Consider a two-pole Y-connected synchronous generator rated at 300 kVA, 480 V, 60 Hz, and 0.85 PF lagging. The armature resistance is $R_A = 0.04\ \Omega$. The core losses of this generator at rated conditions 10 kW, and the friction and windage losses are 13 kW. The open-circuit and short-circuit characteristics are shown below.



$$V_{OC}(I_F) := 1250\cdot\left(1 - e^{-\frac{I_F}{6}}\right) \qquad I_{SC}(I_F) := \frac{1200}{8.5}\cdot I_F$$

$$I_F := 0, 0.1 .. 10$$

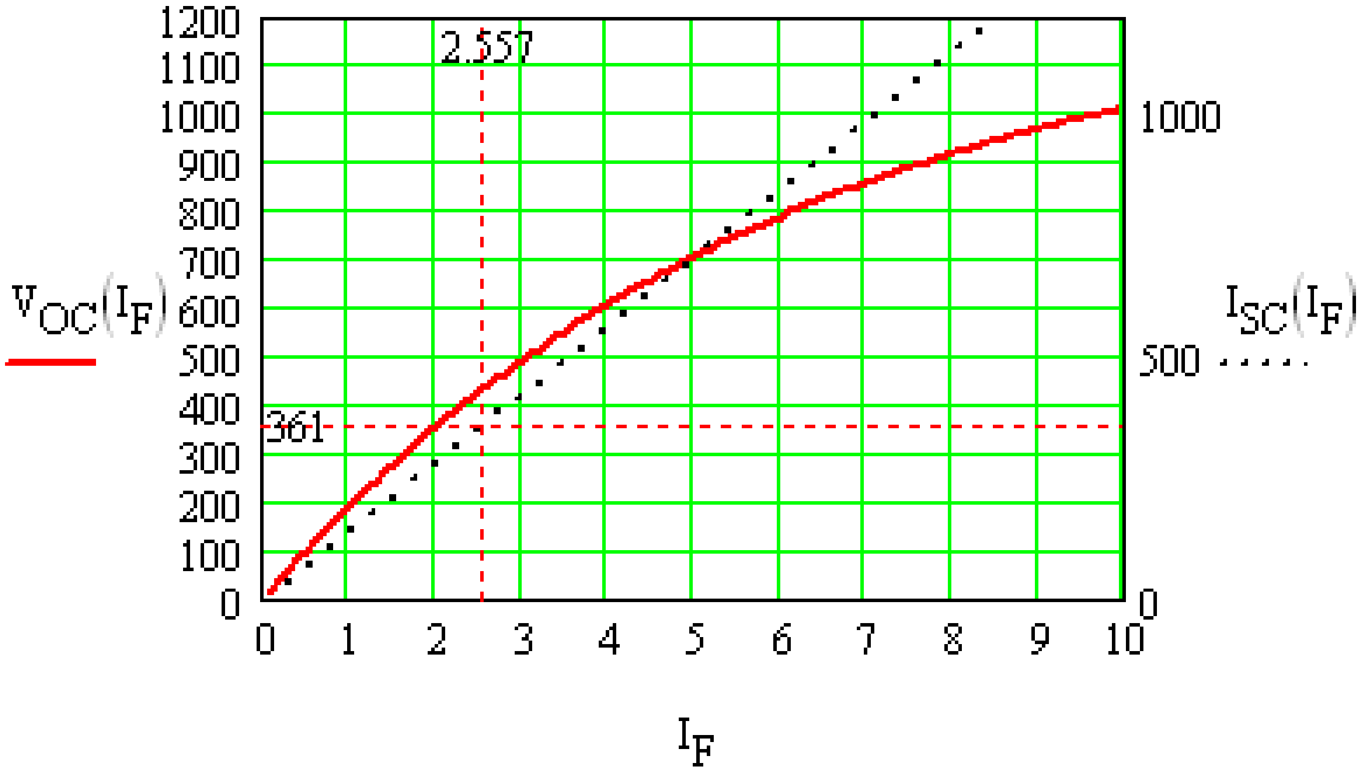


What is the saturated synchronous reactance of this generator at the rated conditions?

At rated conditions:

$$I_A = I_L = \frac{S}{\sqrt{3}V_T} = \frac{300kVA}{\sqrt{3}\left(480V\right)} = 361A$$

The field current required to produce this much short-circuit current is read off the graph from the short-circuit characteristics.



$I_F(I_{SC}) := \frac{8.5}{1200} \cdot I_{SC}$ $\qquad$ $I_F(361) = 2.557$

$V_{OC}(2.557) = 433.741$

The open-circuit voltage for a field current of 2.58 amps is about 434 volts. The open-circuit phase voltage (also = E_A) is

$$E_A = \frac{434}{\sqrt{3}} = 251\,volts$$

The (approximate) synchronous reactance is

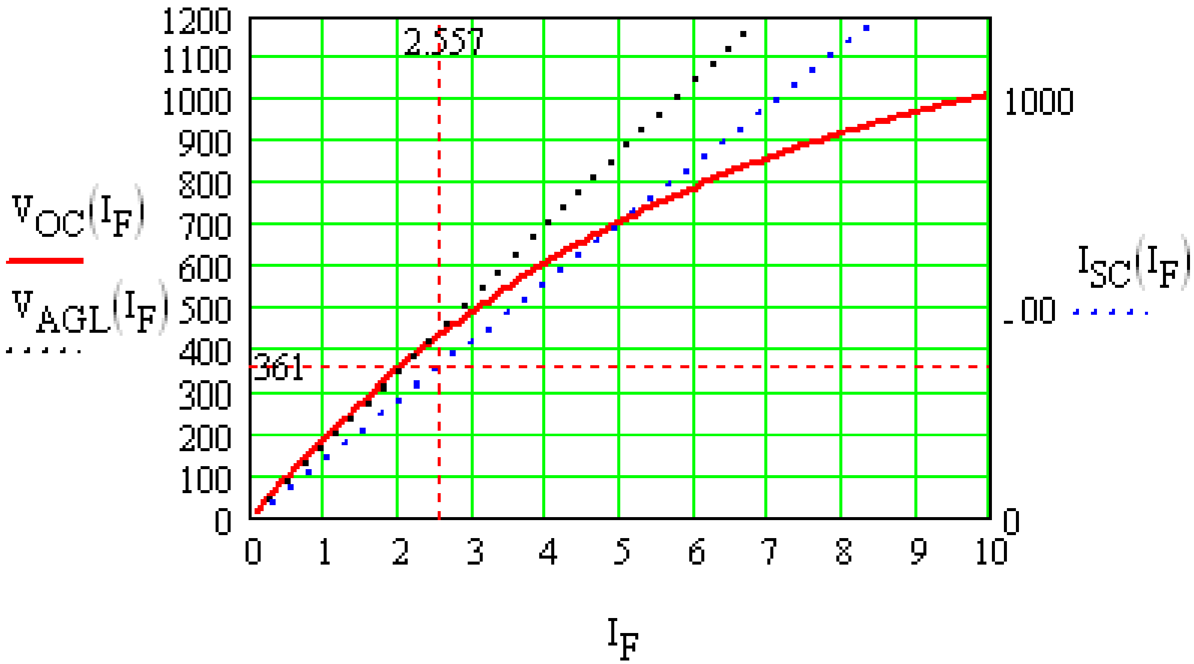
$$X_S = \frac{E_A}{I_A} = \frac{251}{361} = 0.695\,\Omega$$

What is the unsaturated synchronous reactance of this generator?

The unsaturated reactance X_{Su} is the ratio of the air-gap line voltage to the short-circuit current.

Air Gap Line: $V_{AGL}(I_F) := 175 \cdot I_F$ $V_{AGL}(I_F(361)) = 447.49$

$I_F := 0, 0.1 .. 10$



What is the unsaturated synchronous reactance of this generator?

The unsaturated reactance X_{Su} is the ratio of the air-gap line to the short-circuit current.

$$X_{Su} = \frac{E_A}{I_A} = \frac{\frac{447}{\sqrt{3}}}{361} = 0.71\,\Omega$$

What are the rated current and internal generated voltage of this generator?

Rated line and armature current:

$$I_A = I_L = \frac{S}{\sqrt{3}V_T} = \frac{300kVA}{\sqrt{3}\left(480V\right)} = 361A$$

$$PF = 0.85\left(lagging\right) \Rightarrow \theta = -31.8^\circ$$

$$\tilde{I}_A = 361e^{-j31.8^\circ}\ amps$$

Rated voltage:

$$V_\phi = \frac{480}{\sqrt{3}} = 277.13\,volts$$

What are the rated current and internal generated voltage of this generator?

The saturated synchronous reactance at rated condition we just found as:

$$X_S = \frac{E_A}{I_A} = \frac{251}{361} = 0.695\ \Omega$$

Therefore the internally generated voltage is (from slide 23):

$$\tilde{V}_\phi = \tilde{E}_A - jX\tilde{I}_A - jX_A\tilde{I}_A - R_A\tilde{I}_A$$

What are the rated current and internal generated voltage of this generator?

$$\tilde{V}_{\phi} = \tilde{E}_A - jX\tilde{I}_A - jX_S\tilde{I}_A - R_A\tilde{I}_A$$

$$\Rightarrow \tilde{E}_A = \tilde{V}_{\phi} + jX\tilde{I}_A + jX_S\tilde{I}_A + R_A\tilde{I}_A$$

$$E_A := 277 + 0.04 \cdot 361 \cdot e^{-j \cdot acos(0.85)} + j \cdot 0.695 \cdot \left(361 \cdot e^{-j \cdot acos(0.85)}\right)$$

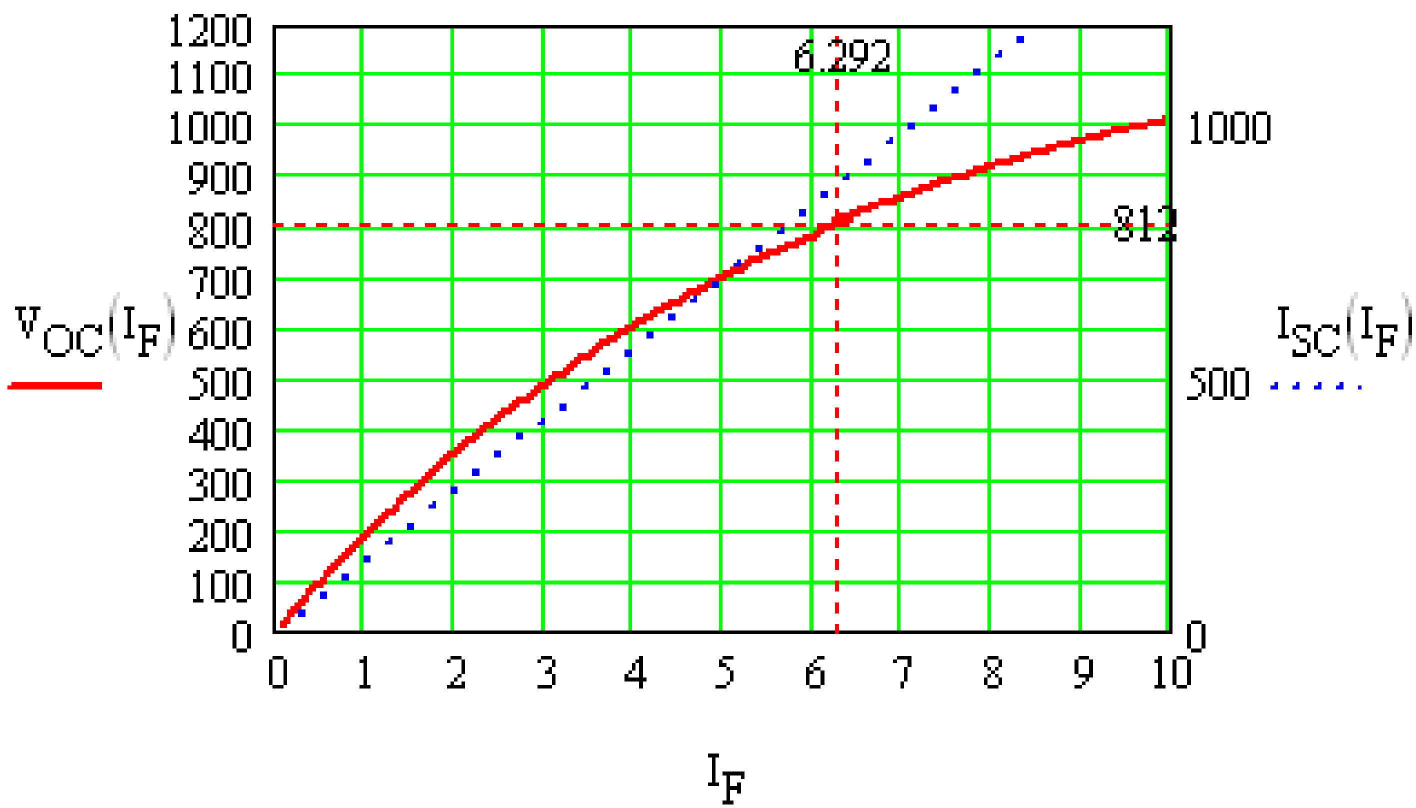
$$\left|E_A\right| = 468.942 \qquad \arg\left(E_A\right) \cdot \frac{180}{\pi} = 26.011$$

What field current does this generator require to operate at the rated current, voltage, and power factor?

The internally generated voltage corresponds to a no-load terminal voltage of

$$V_{OC} = 469\sqrt{3} = 812\,V$$

The corresponding field current is obtained from the graph:



$$I_F(V_{OC}) := -6 \cdot \ln\left(1 - \frac{V_{OC}}{1250}\right) \qquad I_F(812) = 6.292$$

What is the voltage regulation of this generator at the rated current and power factor?

$$VR = \frac{V_{T,no\ load} - V_{T,\ full\ load}}{V_{T,\ full\ load}} \times 100\%$$

$$= \frac{812 - 480}{480} \times 100\% = 69.2\%$$

If the generator is operating at the rated conditions and the load is suddenly removed, what will the terminal voltage be?

$$V_{OC} = 469\sqrt{3} = 812\,V$$

What are the electrical losses in this generator at rated conditions?

$$P_{Cu} = 3\left|I\right|^2 R_A = 3\left(361\right)^2\left(0.04\right) = 15.639\,kW$$

If the machine is operating at rated conditions, what torque must be applied to the shaft of the generator?

$$P_{out} = 300kVA \cdot PF = 300kVA \cdot 0.85 = 255\ kW$$

$$P_{Cu} = 15.639\ kW$$

$$P_{\substack{Friction \\ \& Windage}} = 13\ kW$$

$$P_{core} = 10\ kW$$

$$P_{stray} = 0$$

$$P_{in} = \left(255 + 15.6 + 13 + 10\right) kW = 293.6\ kW$$

$$\tau_{applied} = \frac{P_{in}}{\omega_m} = \frac{293.6\ kW}{\left(3600\,\frac{\cancel{rev}}{\cancel{min}}\right)\left(2\pi\,\frac{rad}{\cancel{rev}}\right)\left(\frac{1\,\cancel{min}}{60\ sec}\right)}$$

$$= 779\ watt - \sec = 779\ N - m$$

or

$$\tau_{applied} = 7.04\frac{P_{in}}{n_m} = 7.04\frac{293.6\ kW}{3600\,\frac{rev}{min}} = 574\ \ ft - lbs$$

What is the torque angle of this generator at rated conditions?

From:

$$\tilde{E}_A = \tilde{V}_\phi + jX\tilde{I}_A + jX_S\tilde{I}_A + R_A\tilde{I}_A$$

$$\tilde{E}_A = 469\angle 26^\circ$$

$$\Rightarrow \delta = 26^\circ$$

Assume that the generator field current is adjusted to supply 480 V under rated conditions. What is the static stability limit of this generator? Ignore R_a in the calculation. How close is the full-load condition of this generator to the static stability limit?

At rated conditions, $\tilde{E}_A = 469\angle 26°$

$$P_{\max} = \frac{3V_\phi E_A}{X_S} = \frac{3 \cdot 277 \cdot 469}{0.695} = 561\, kW$$

The full-load rated power, $P_{out} = 255\, kW$

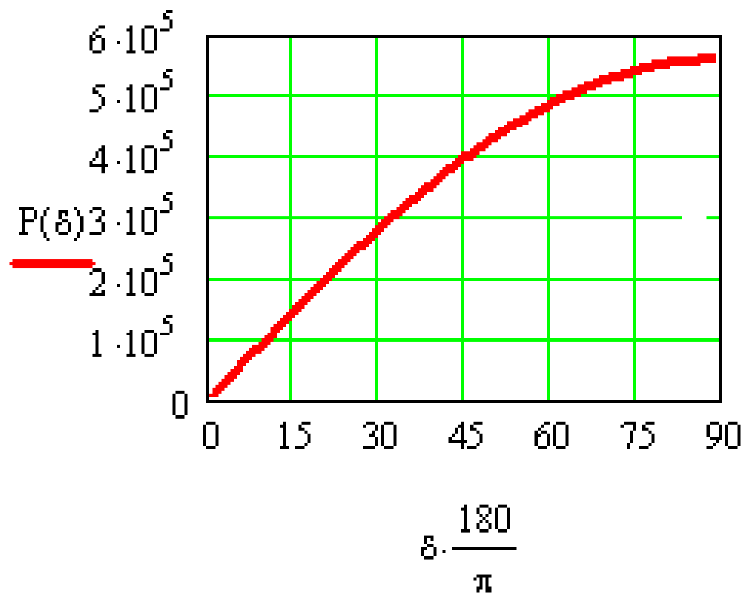
is less than half the static stability limit.

Assume that the generator field current is adjusted to supply 480 V under rated conditions. Plot the power supplied by the generator as a function of the torque angle δ. (Again neglect R_A.)

From Slide 44:

$$P_{conv} = 3V_{\phi}\frac{E_A \sin\delta}{X_S} = \frac{3(277)(469)}{0.695}\sin(\delta)\,Watts$$

$$= 561\,kW\,\sin(\delta)$$

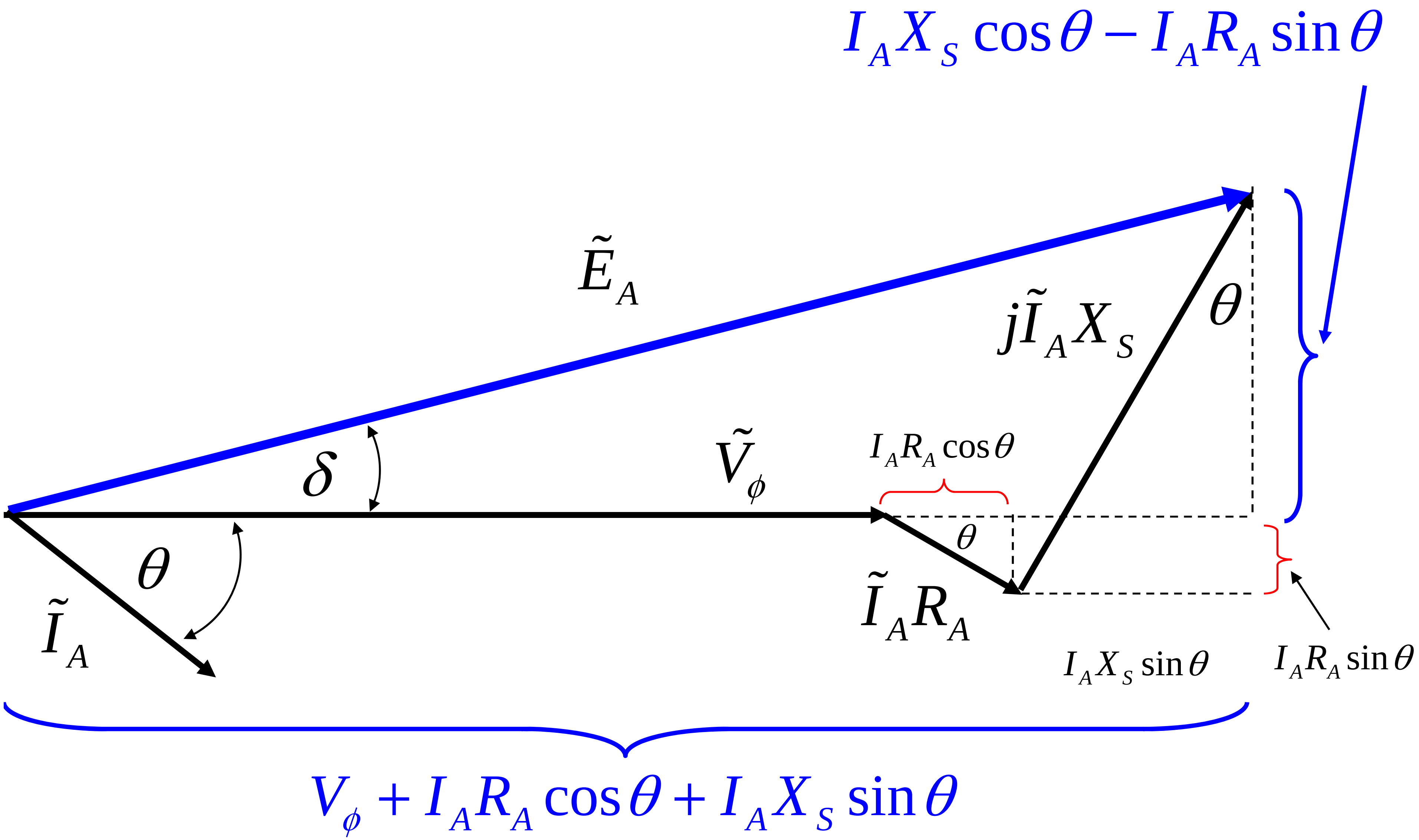


Assume that the generator field current is adjusted so that the generator supplies rated voltage at the rated load current and power factor. If the field current and the magnitude of the load current are held constant, how will the load terminal voltage change as the load power factor varies from 0.85 lagging to 0.85 leading? Plot the terminal voltage versus the impedance angle of the load being supplied by this generator.

If the field current is held constant, then the magnitude of E_A will be constant, although its angle δ will vary. Also, the magnitude of the armature current is constant. Since we know R_A, X_S, and the angle θ, we can solve for V_ϕ from:

$$\tilde{E}_A = \tilde{V}_\phi + R_A \tilde{I}_A + jX_S \tilde{I}_A$$

Lagging power factor:



$$\tilde{V}_\phi = V_\phi \angle 0^\circ$$

$$\left|\tilde{E}_A\right|^2 = E_A{}^2 = \left[V_\phi + I_A R_A \cos\theta + I_A X_S \sin\theta\right]^2$$

$$+\left[I_A X_S \cos\theta - I_A R_A \sin\theta\right]^2$$

$$\Rightarrow \left[V_\phi + I_A R_A \cos\theta + I_A X_S \sin\theta\right]^2$$

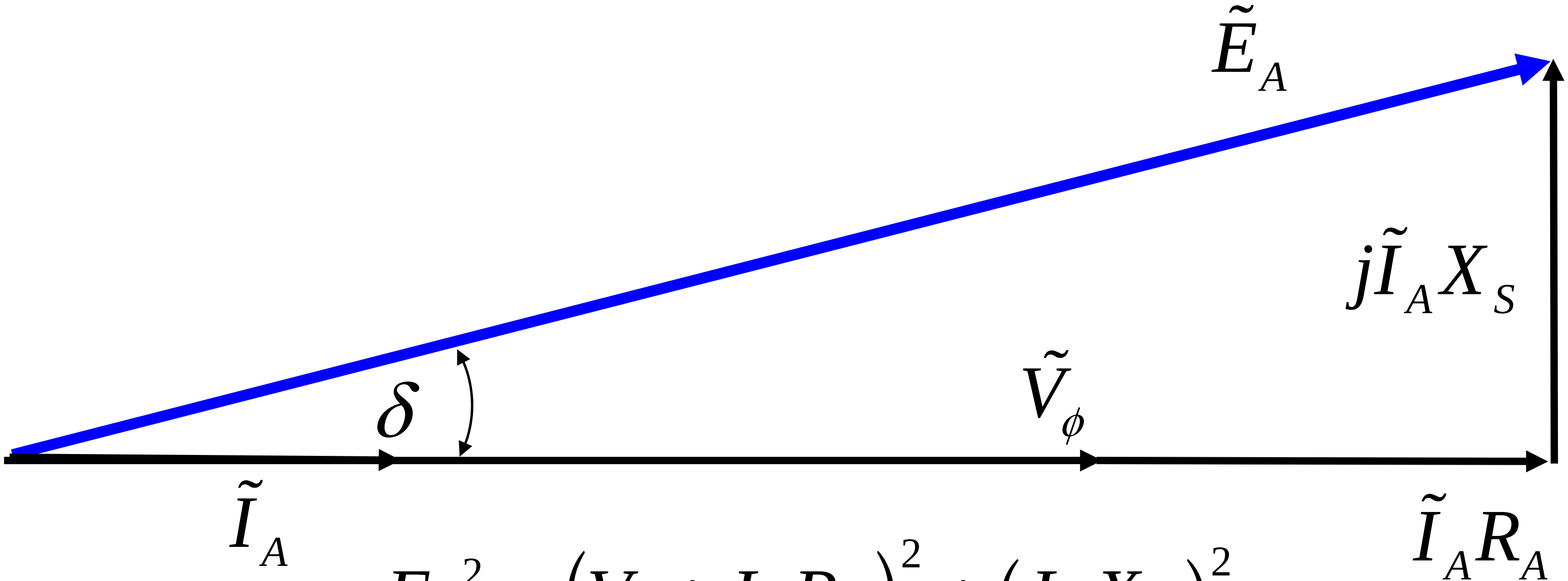
$$= E_A{}^2 - \left[\left(I_A X_S \cos\theta - I_A R_A \sin\theta\right)\right]^2$$

$$V_\phi = \sqrt{E_A{}^2 - \left[\left(I_A X_S \cos\theta - I_A R_A \sin\theta\right)\right]^2}$$

$$-\left(I_A R_A \cos\theta + I_A X_S \sin\theta\right)$$

Unity power factor:

In the previous result set $\theta = 0$ or use the picture below.



$$E_A^{\,2} = \left(V_\phi + I_A R_A\right)^2 + \left(I_A X_S\right)^2$$

$$V_\phi = \sqrt{E_A^{\,2} - \left(I_A X_S\right)^2} - I_A R_A$$

Leading power factor:

In the previous result make the replacement: $\sin\theta \rightarrow -\sin\theta$

or draw a phasor diagram again.

$$V_\phi = \sqrt{E_A{}^2 - \left[\left(I_A X_S \cos\theta + I_A R_A \sin\theta\right)\right]^2} - \left(I_A R_A \cos\theta - I_A X_S \sin\theta\right)$$

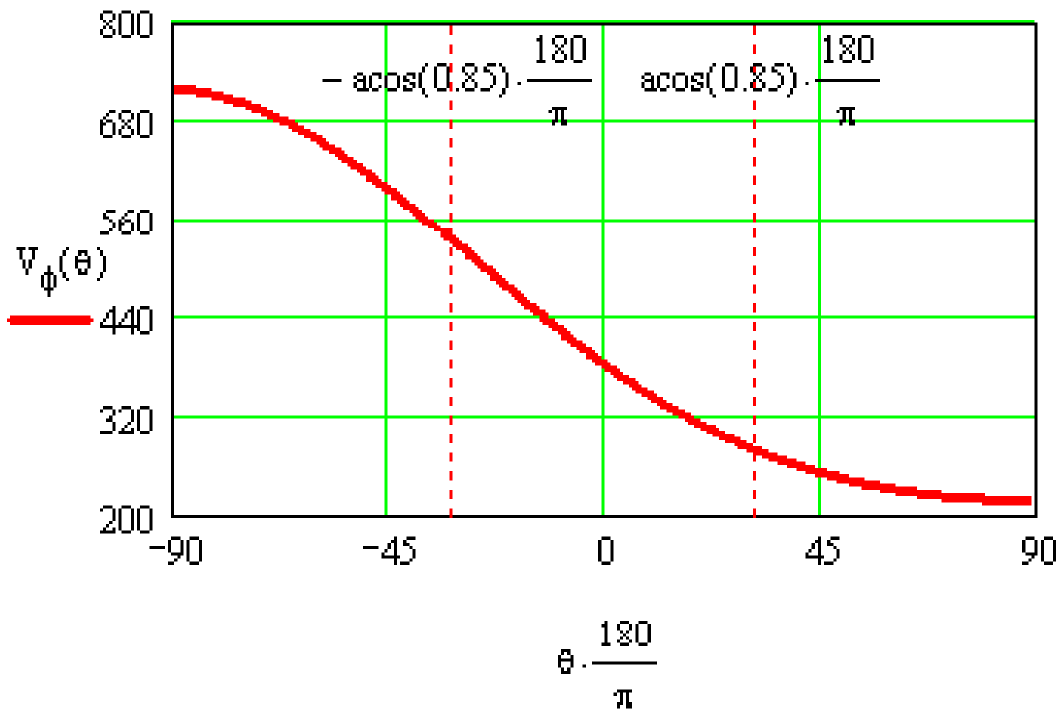
All three cases are captured in the following plot...

$E_A := 469$ $\qquad X_S := 0.695$

$I_A := 361$ $\qquad R_A := 0.04$

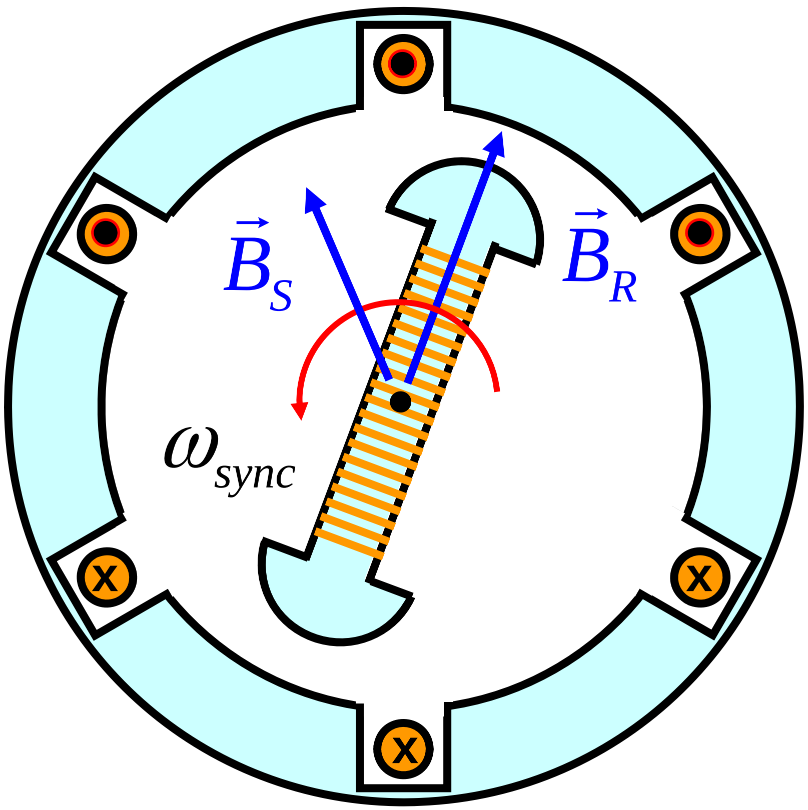
$$V_\phi(\theta) := \sqrt{E_A{}^2 - \left(I_A \cdot X_S \cdot \cos(\theta) - I_A \cdot R_A \cdot \sin(\theta)\right)^2} - \left(I_A \cdot R_A \cdot \cos(\theta) + I_A \cdot X_S \cdot \sin(\theta)\right)$$

$$\theta := -\frac{\pi}{2}, -\frac{\pi}{2} + 0.01 .. \frac{\pi}{2}$$



Synchronous Motors

Synchronous Motors



$$\vec{\tau}_{ind} = k\vec{B}_R \times \vec{B}_{net}$$
$$(counterclockwise)$$

The field current produces the magnetic field B_R.

A three-phase set of voltages applied to the stator (armature) windings produce a rotating magnetic field B_S.

The rotor field will tend to line up with the stator field.

Since the stator magnetic field is rotating the rotor field and hence the rotor itself will try to catch up.

The larger the angle between B_R and B_S, the greater the torque.

Synchronous Motors

The rotor "chases" the stator's rotating magnetic field, never quite catching up with it.

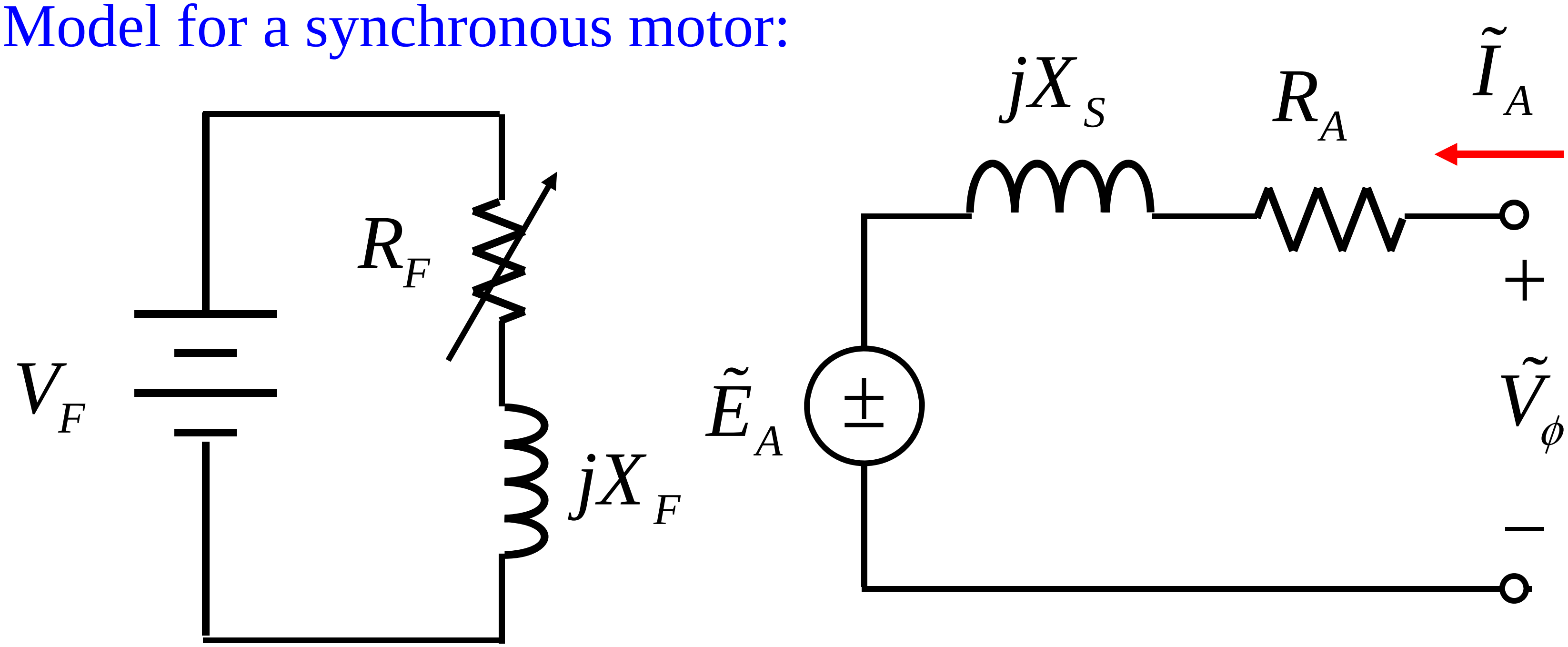
Since the synchronous motor is the same machine as a synchronous generator, all the results develop previously for power, torque, and speed apply here as well.

A synchronous motor is the same a synchronous generator in all respects except that the direction of power flow has been reversed. Consequently the direction of the stator current is also expected to reverse.

The model is...

Synchronous Motors

Model for a synchronous motor:

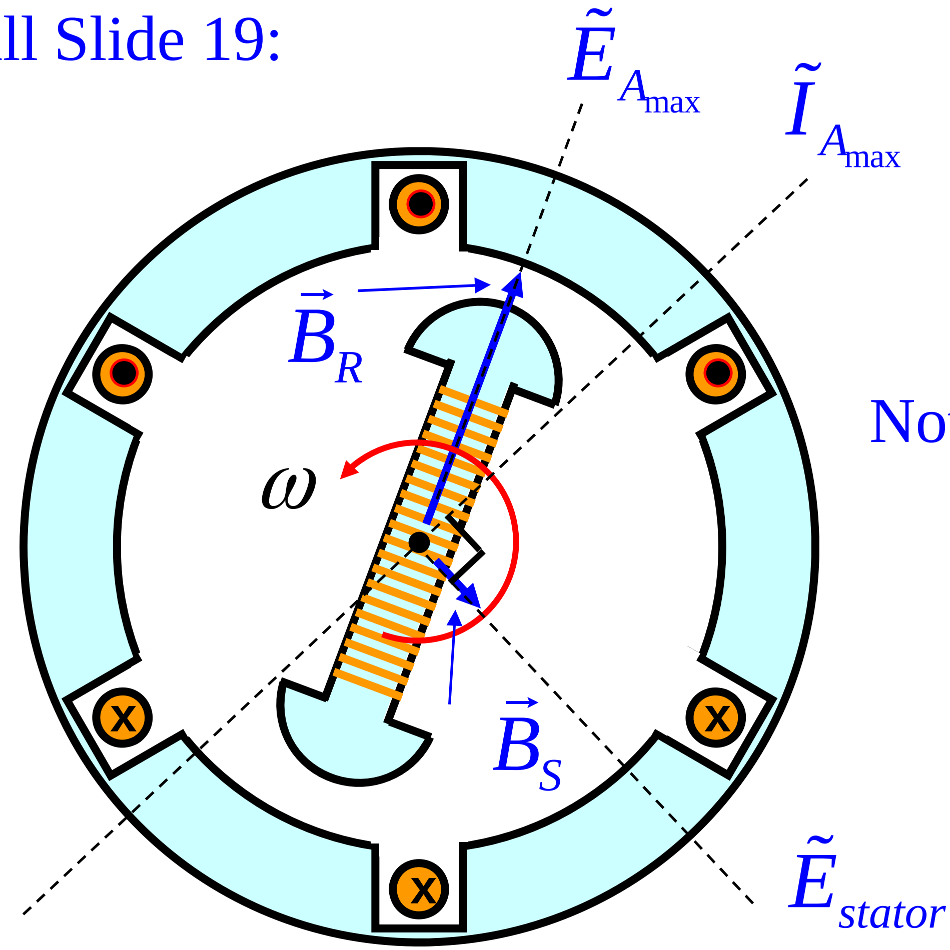


$$\tilde{V}_\phi = \tilde{E}_A - jX\tilde{I}_A - jX_A\tilde{I}_A - R_A\tilde{I}_A$$ Generator

$$\tilde{V}_\phi = \tilde{E}_A + jX\tilde{I}_A + jX_A\tilde{I}_A + R_A\tilde{I}_A$$ Motor

Synchronous Motors – Magnetic Field Perspective

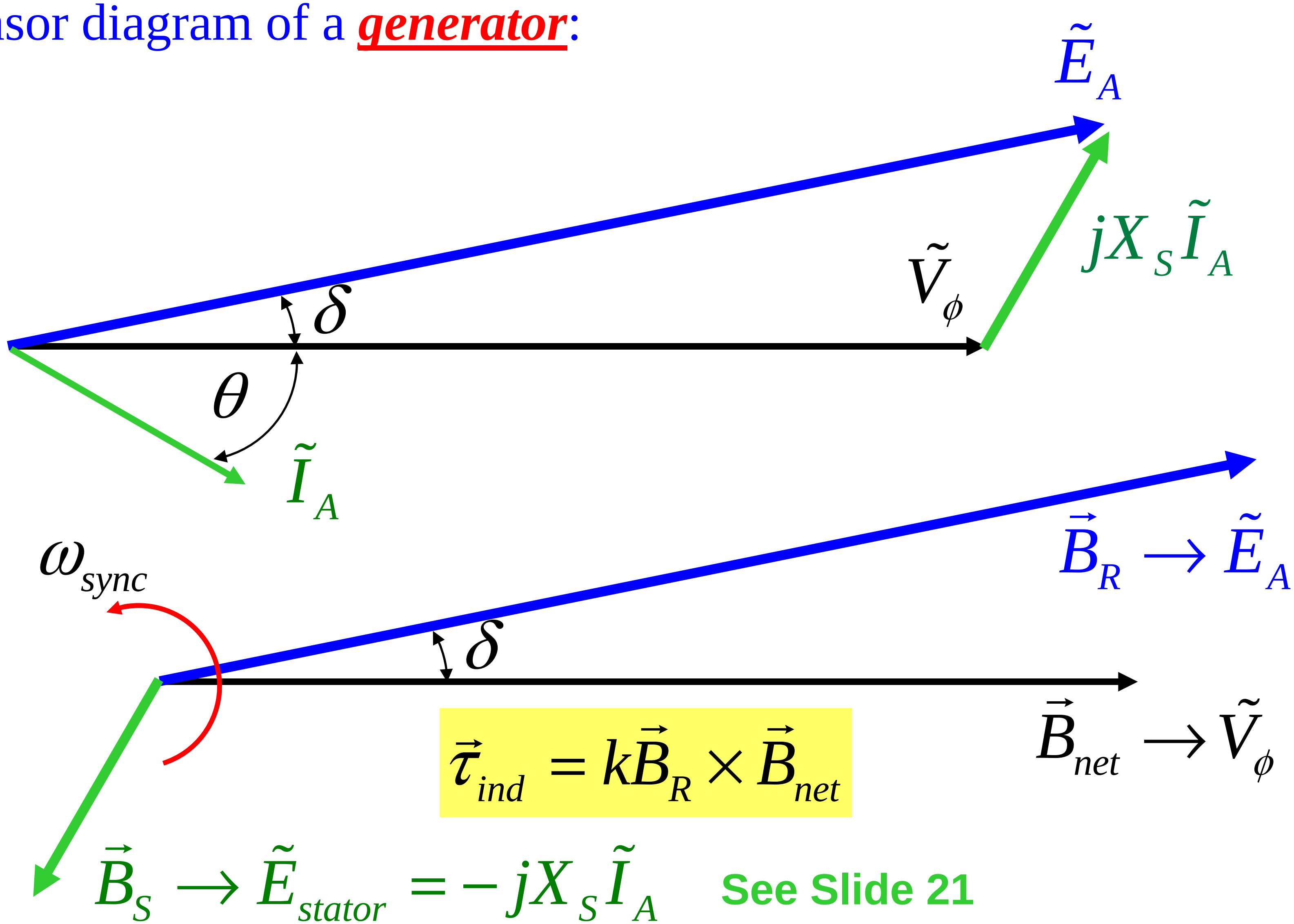
Recall Slide 19:



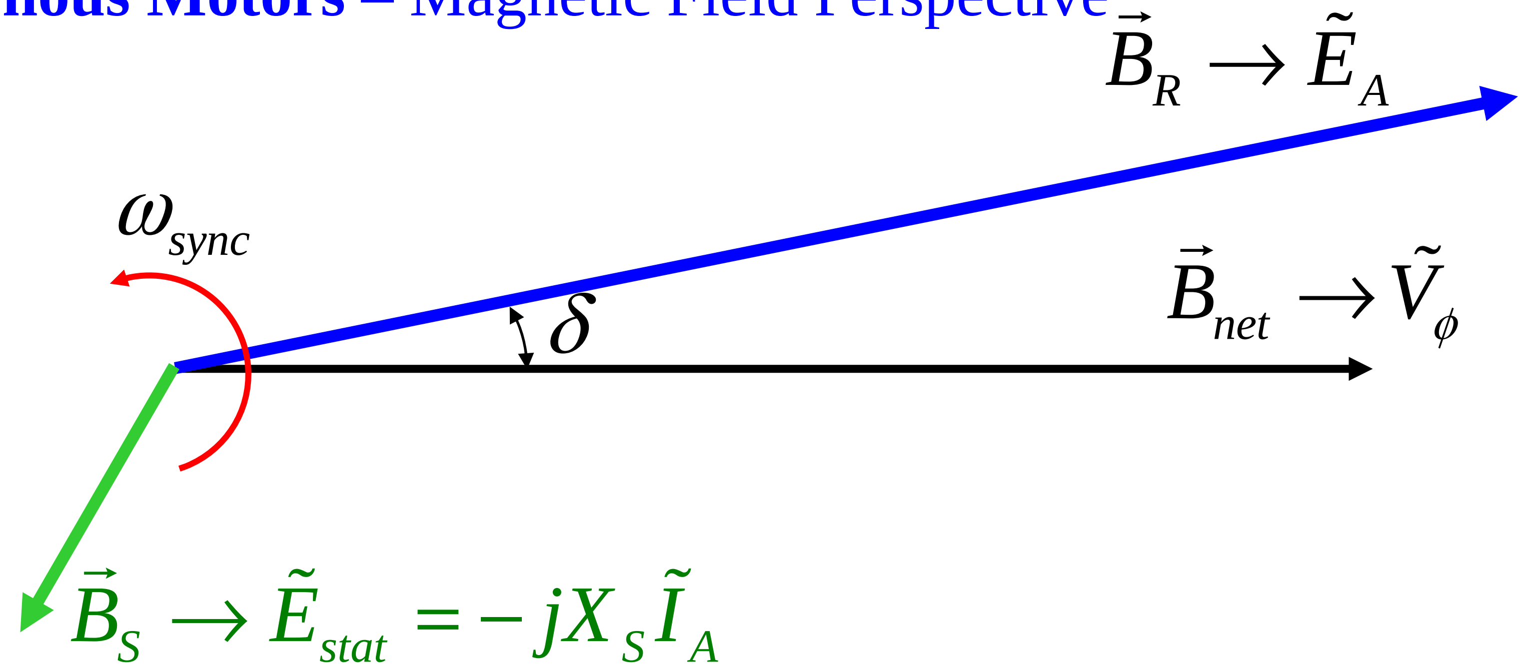
Note how B_R always *leads* B_S

GENERATOR

Synchronous Motors – Magnetic Field Perspective – Consider the phasor diagram of a ***generator***:



Synchronous Motors – Magnetic Field Perspective



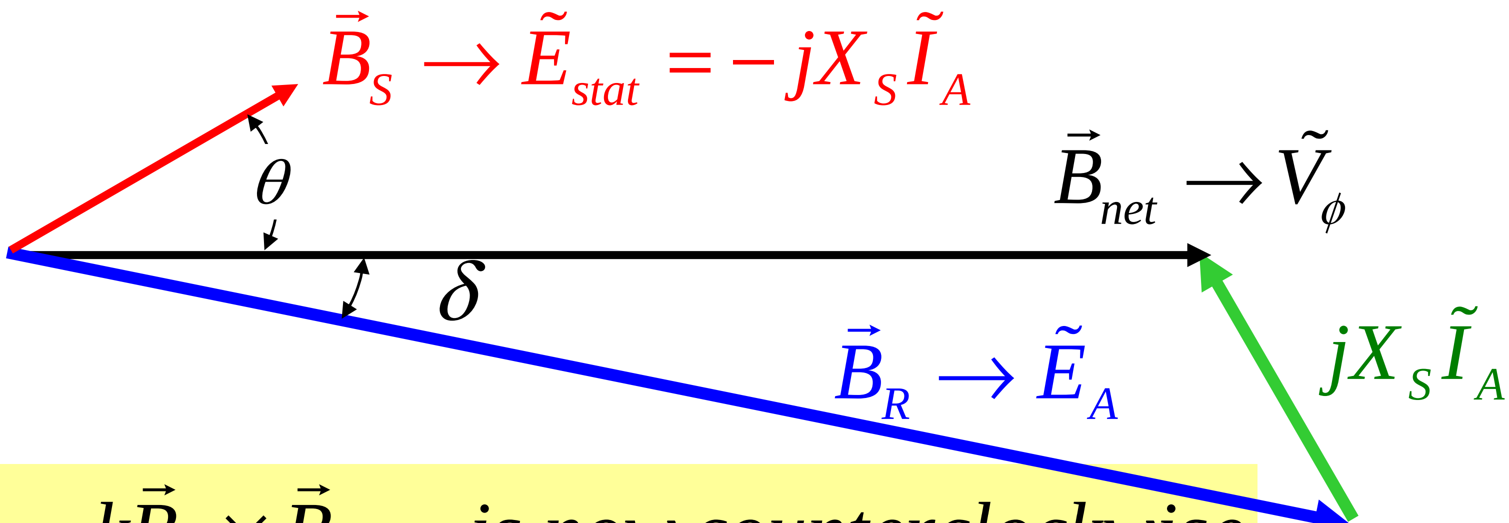
$$\vec{\tau}_{ind} = k\vec{B}_R \times \vec{B}_{net} \quad clockwise$$

The torque is *clockwise*, opposite the direction of rotation. The induced torque is *counter-torque*, opposing the rotation caused by the external applied torque.

Synchronous Motors – Magnetic Field Perspective

Suppose that instead of turning the shaft in the direction of motion the prime mover suddenly loses power and starts to drag on the machine's shaft. What happens?

The rotor slows down because of the drag and falls behind the net magnetic field in the machine. Then B_R no longer leads B_S (or B_{net})



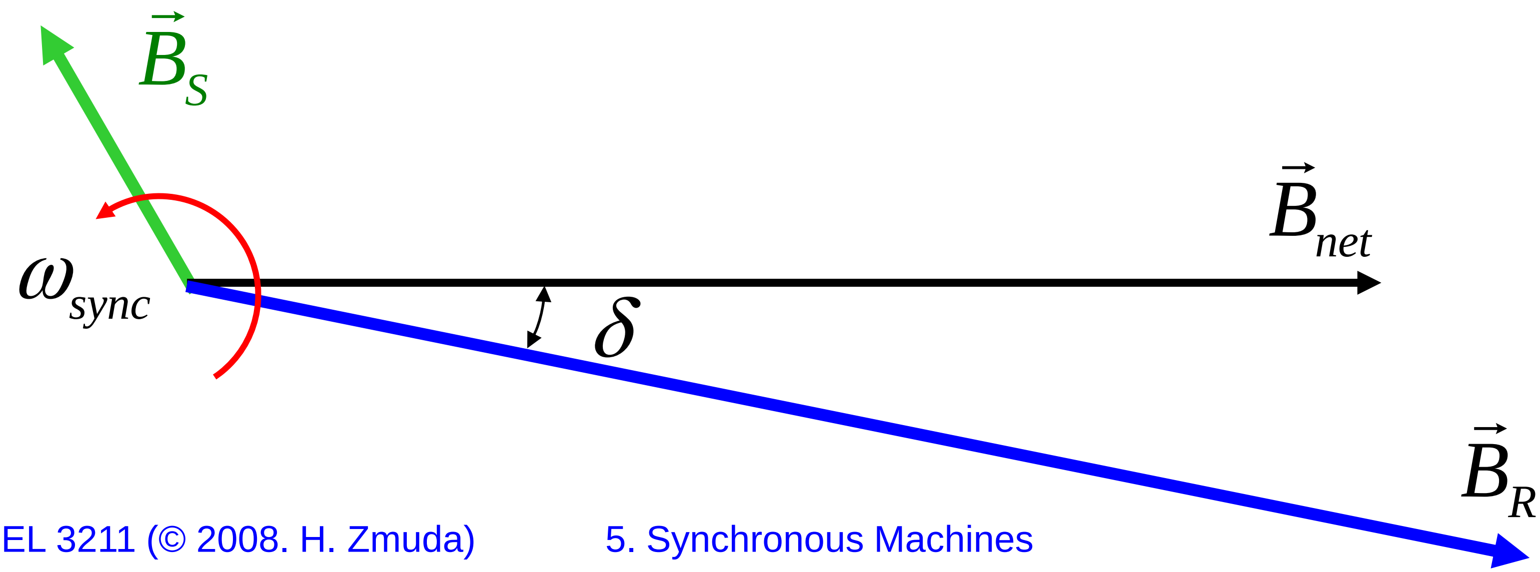
$\vec{\tau}_{ind} = k\vec{B}_R \times \vec{B}_{net}$ *is now counterclockwise*

Synchronous Motors – Magnetic Field Perspective

As the rotor (and hence B_R) slows down and falls behind the net B_{net}, the operation of the machine suddenly changes. Since

$$\vec{\tau}_{ind} = k\vec{B}_R \times \vec{B}_{net}$$

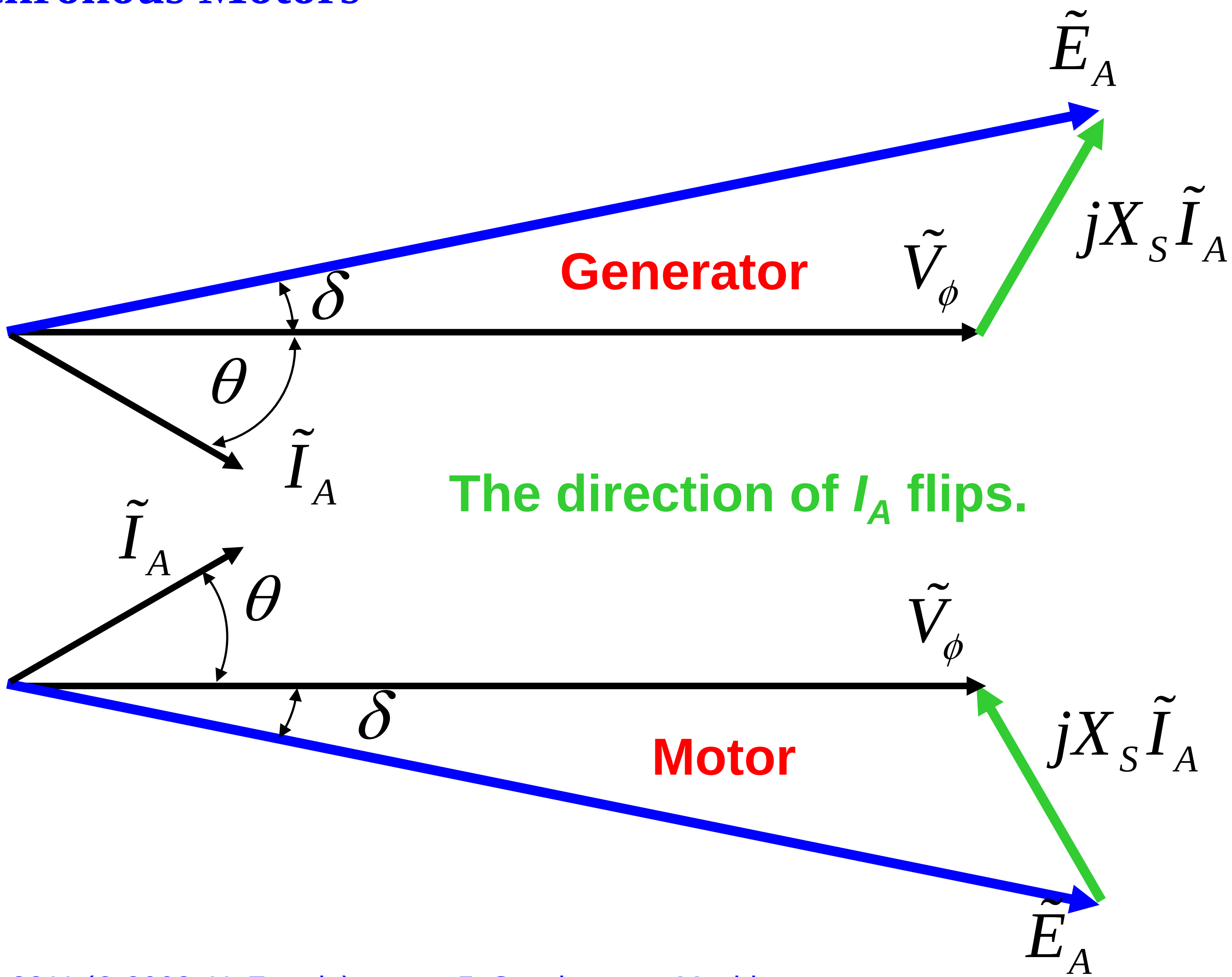
when B_R is behind B_{net} the direction of the induced torque reveres and becomes counterclockwise. Now the machine's torque is in the direction of motion and the machine is acting as a motor.



Synchronous Motors – Magnetic Field Perspective

The increasing torque angle δ results in a larger and large torque in the direction of rotation until eventually the motor's induced torque equals the load torque on the shaft. At this point the machine is operating at steady state and at synchronous speed again, but now as a motor.

Synchronous Motors



Synchronous Motors

The basic difference between motor and generator operation in synchronous machines can easily be seen from the either the magnetic field or phasor diagram.

In a *generator*, E_A lies ahead of V_ϕ, and B_R lies ahead of B_{net}.

In a *motor*, E_A lies behind V_ϕ, and B_R lies behind B_{net}.

In a *motor* the induced torque is in the direction of motion.

In a *generator*, the induced torque is a counter-torque opposing the direction of motion.

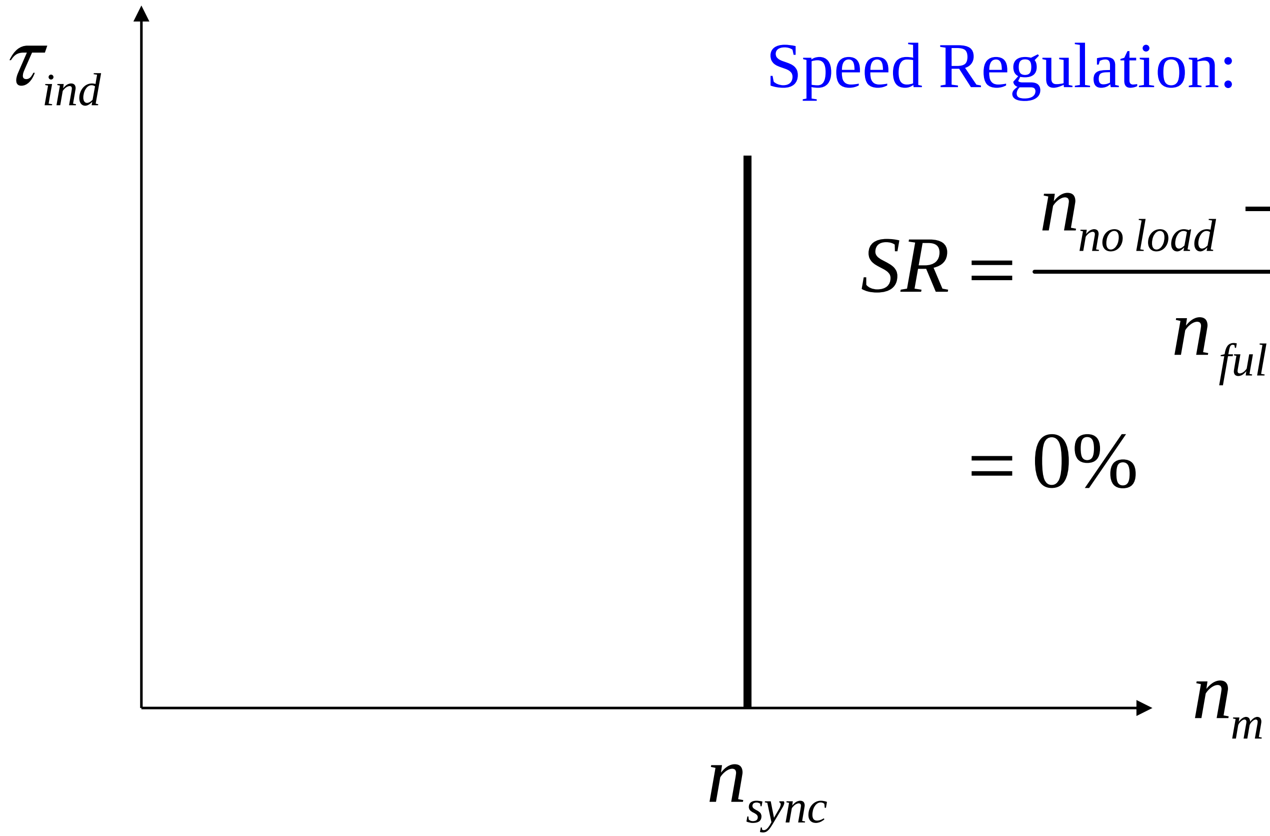
Synchronous Motors – Torque-Speed Characteristics

Synchronous motors supply power to loads that are basically constant speed devices. Motors are generally energized from power systems that are capable of supplying much more than the amount of energy needed by the motor. In other words, we assume that the motor power supply cannot be loaded down regardless of the amount of power being drawn by the motor.

The speed of the motor is locked to the applied electrical frequency, so the speed of the motor is a constant regardless of the load.

The resulting torque-speed characteristics are thus...

Synchronous Motors – Torque-Speed Characteristics



Speed Regulation:

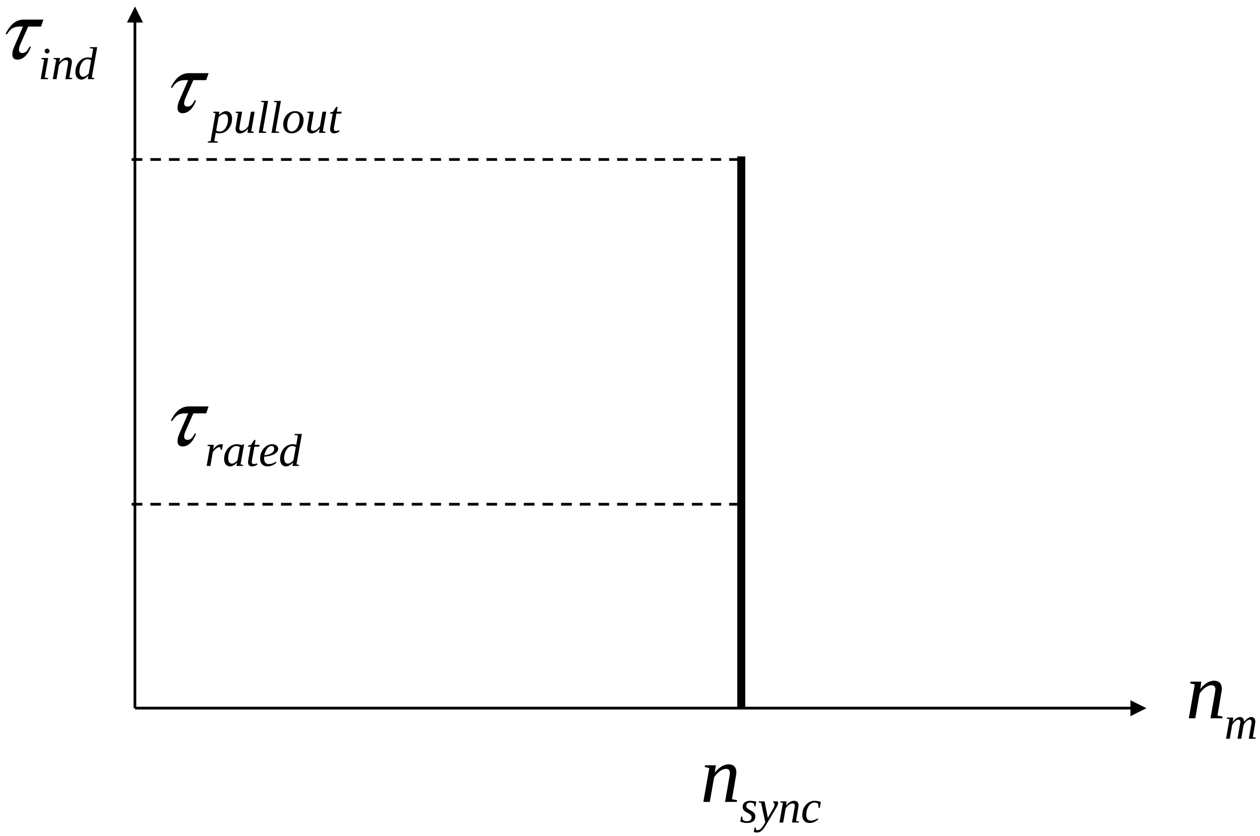
$$SR = \frac{n_{no\ load} - n_{full\ load}}{n_{full\ load}} \times 100\%$$

$$= 0\%$$

Synchronous Motors – Torque-Speed Characteristics

The maximum torque a motor can supply is called the *pullout torque.*

Typically $\tau_{pullout} \approx 3\tau_{rated}$



Synchronous Motors – Torque-Speed Characteristics

Recall from Slide 44,

$$\tau_{ind} = kB_R B_{net} \sin\delta = \frac{3V_\phi E_A \sin\delta}{\omega_m X_S}$$

Therefore

$$\tau_{pullout} = kB_R B_{net} = \frac{3V_\phi E_A}{\omega_m X_S}$$

Synchronous Motors – Torque-Speed Characteristics

Since $I_F \to E_A$

and $$\tau_{pullout} = kB_R B_{net} = \frac{3V_\phi E_A}{\omega_m X_S}$$

the larger the field current, the greater the maximum torque of the motor.

There is clearly a stability advantage in operating the motor with a large field current.

Synchronous Motors – Torque-Speed Characteristics

When the torque on the shaft of a synchronous motor exceeds the pullout torque the rotor can no longer remain locked to the stator and to the net magnetic fields. At this point the rotor begins to slip behind them.

As the rotor slow down, the stator magnetic field *laps* it repeatedly, and the direction of the induced torque in the rotor reverses with each pass.

This results in huge torque surges, first one way, then the other, and the motor vibrates severely.

The loss of synchronization after the pullout torque is exceeded is known as *slipping poles*.

Synchronous Motors – Effects of Load Changes

When a load is connected to the shaft of a synchronous motor, the motor will develop enough torque to keep the motor and its load turning at synchronous speed..

What happens when the load is changed?

Synchronous Motors – Effects of Load Changes

If the load on the shaft is increased (in a step fashion) the rotor will slow down. As it does the torque angle δ increases as does the induced torque, since

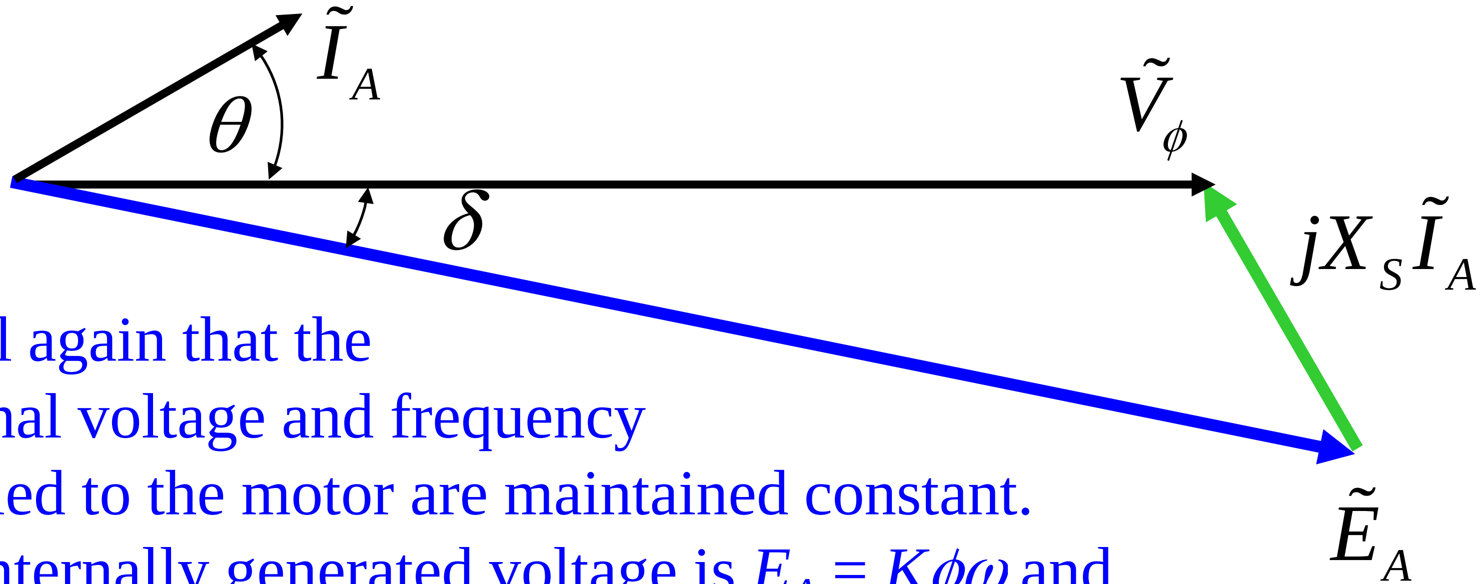
$$\tau_{ind} = \frac{3V_\phi E_A \sin\delta}{\omega_m X_S}$$

This increased torque speeds the motor back up to synchronous speed but now with a larger torque angle.

To see the effect of a changing load, again examine the phasor diagram.

Synchronous Motors – Effects of Load Changes

Consider first the phasor diagram before the load is increased.



Recall again that the terminal voltage and frequency supplied to the motor are maintained constant.

The internally generated voltage is $E_A = K\phi\omega$ and depends only on the field current and the frequency (speed of machine). Since the speed is constrained by the electrical frequency and the field current we assume is untouched, then E_A *must remain constant as the load changes.*

So what does change?

Synchronous Motors – Effects of Load Changes

If we neglect the armature resistance (we will) then the power converted from electrical to mechanical form by the motor will be the same as the input power. Again from Slides 33 and 44 recall,

$$P = 3V_\phi I_A \cos\theta \quad \text{and} \quad P = \frac{3V_\phi E_A \sin\delta}{X_S}$$

Since the phase voltage V_ϕ is held constant by the motor's power supply, the quantities

$$I_A \cos\theta \quad \text{and} \quad E_A \sin\delta$$

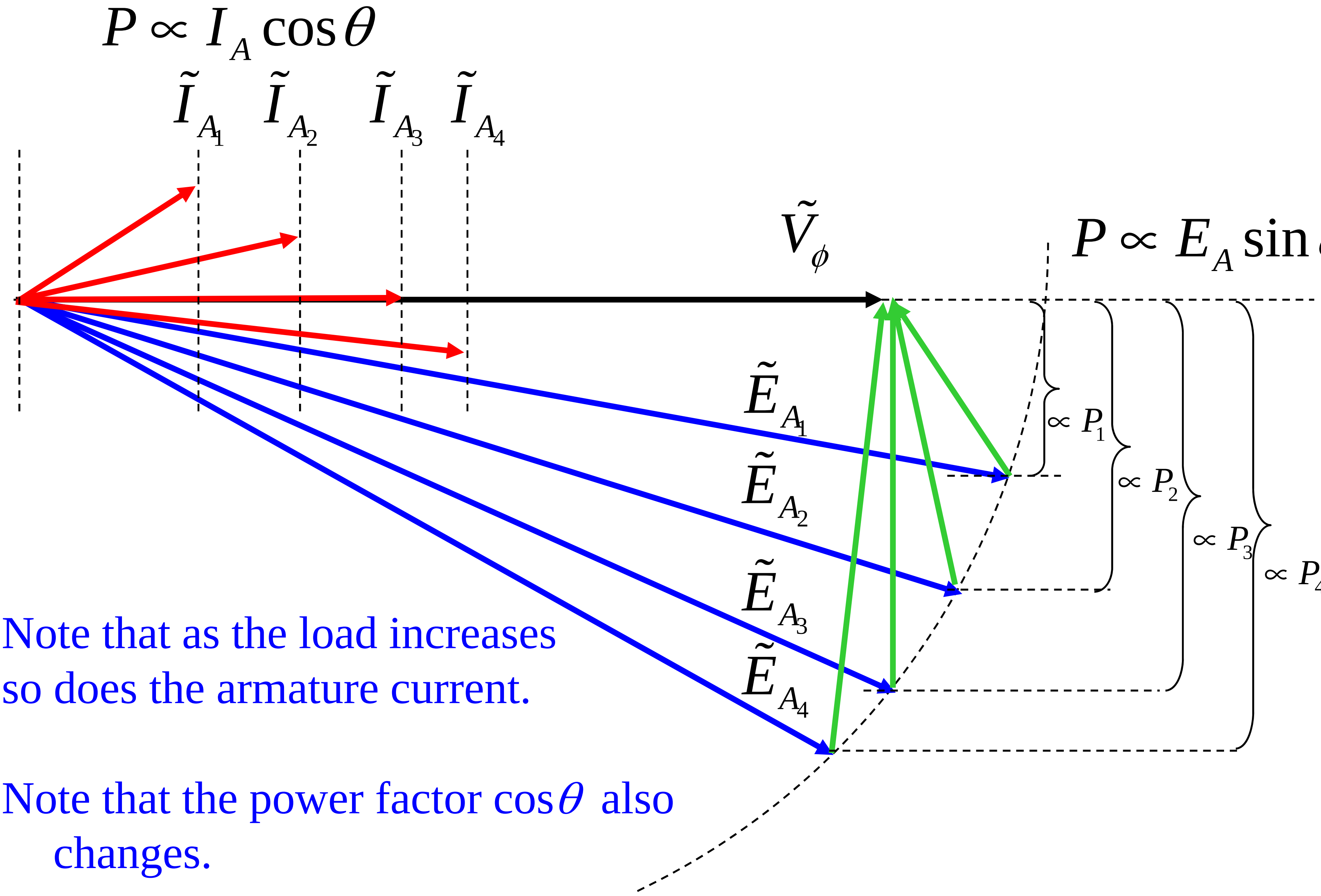
must be directly proportional to the power supplied by the motor.

Synchronous Motors – Effects of Load Changes

When the power supplied by the motor increases, then

$$I_A \cos\theta \text{ and } E_A \sin\delta$$

will increase, but in a way that keeps E_A constant, as shown on the next slide...



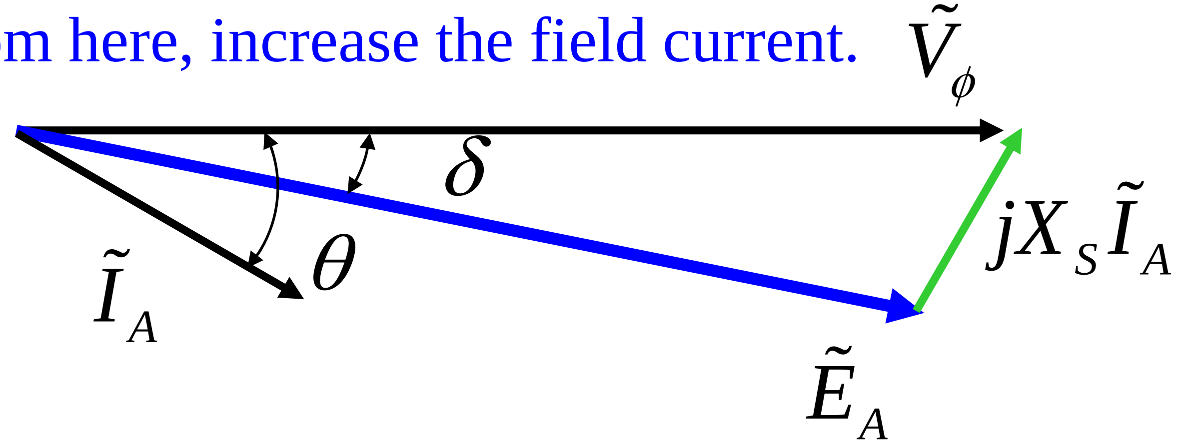
Note that as the load increases so does the armature current.

Note that the power factor $\cos\theta$ also changes.

Effects of Changes in Field Current

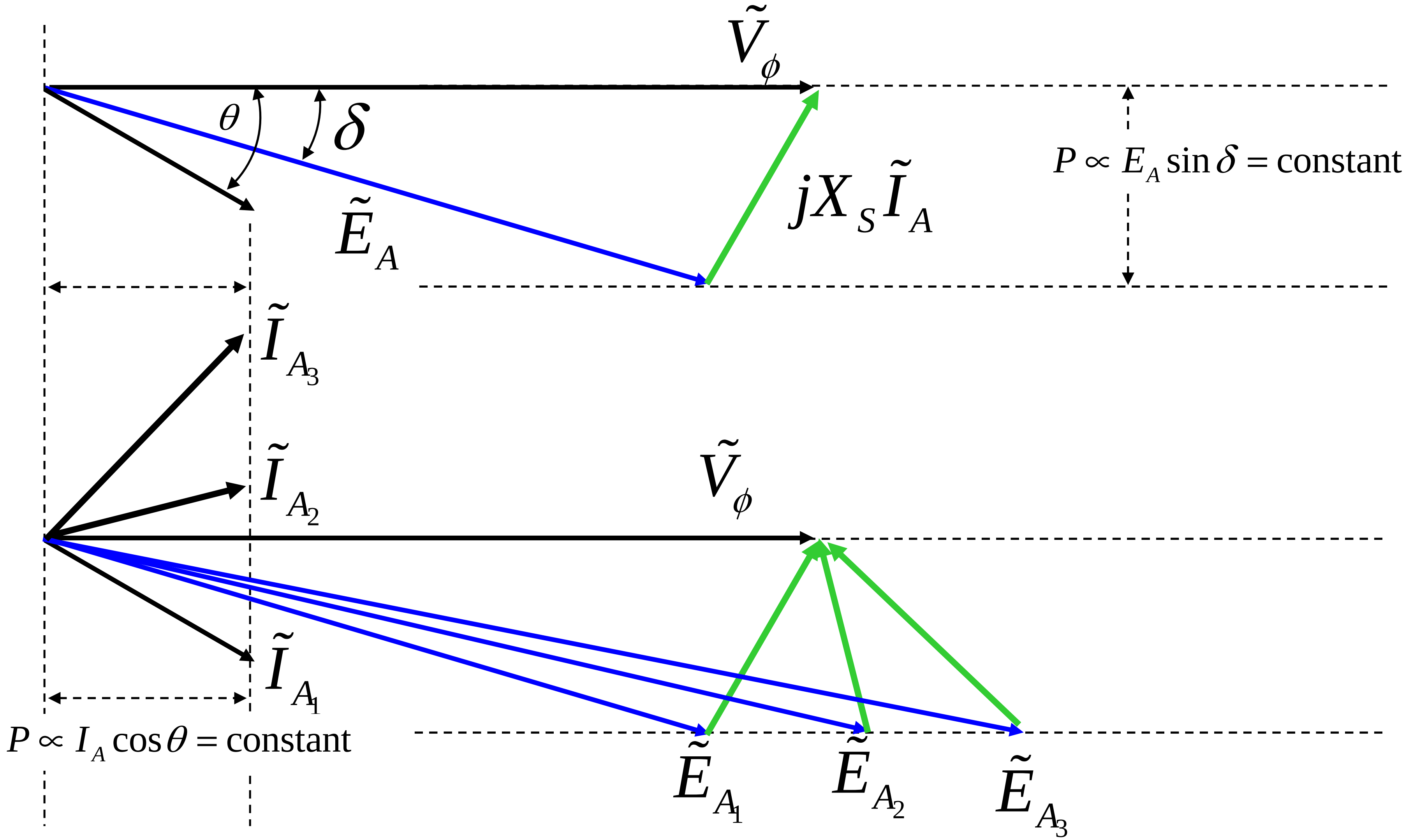
Effects of Changes in Field Current

The field current is really the only quantity that can be adjusted. Starting from here, increase the field current.



Note that increasing I_F means increasing E_A, but the power supplied by the motor, which is determined load torque, does not change. Further, I_A does not change the speed n_m, and since the load to the shaft is unchanged, the (real) power supplied is unchanged. Constant power means that $E_A \sin\delta$ and $I_A \cos\theta$ must remain constant. The terminal voltage V_ϕ is also maintained constant by the power source. Increasing E_A leaves one possibility...

Effects of Changes in Field Current



Effects of Changes in Field Current

Notice how, as E_A is increased, the armature current I_A first decreases then increases again.

For smaller E_A, I_A is lagging, and the motor is an inductive load. It consumes reactive power Q.

As the field current increases, the armature current I_A and terminal voltage V_ϕ line up and the motor looks resistive.

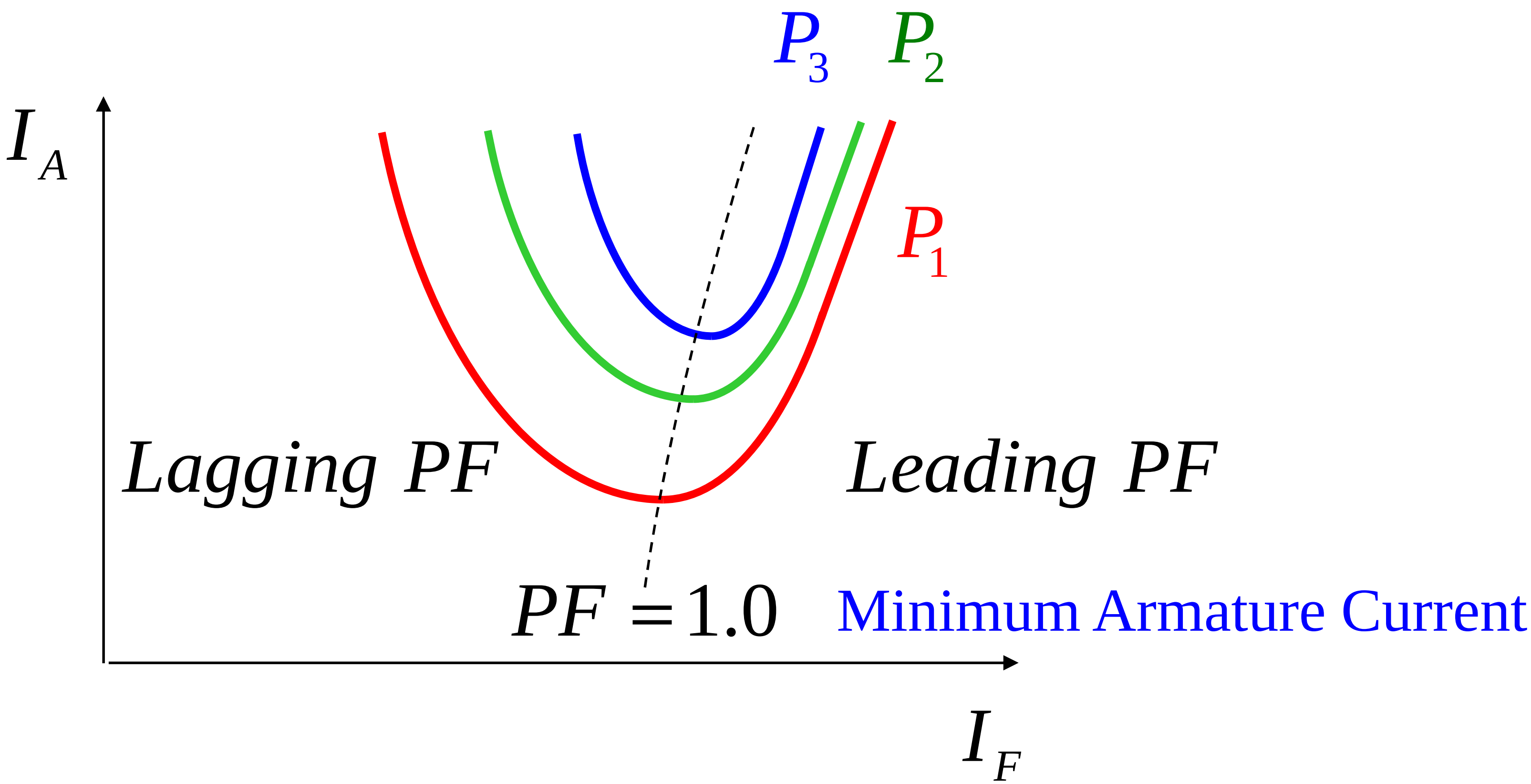
As the fields current increases further, the armature current becomes leading, and the motor look capacitive. It is consuming reactive power – Q, i.e., it is supplying reactive power Q to the system.

Effects of Changes in Field Current

Therefore, by controlling the field current of a synchronous motor, the reactive power (supplied or consumed) by the power system can be controlled.

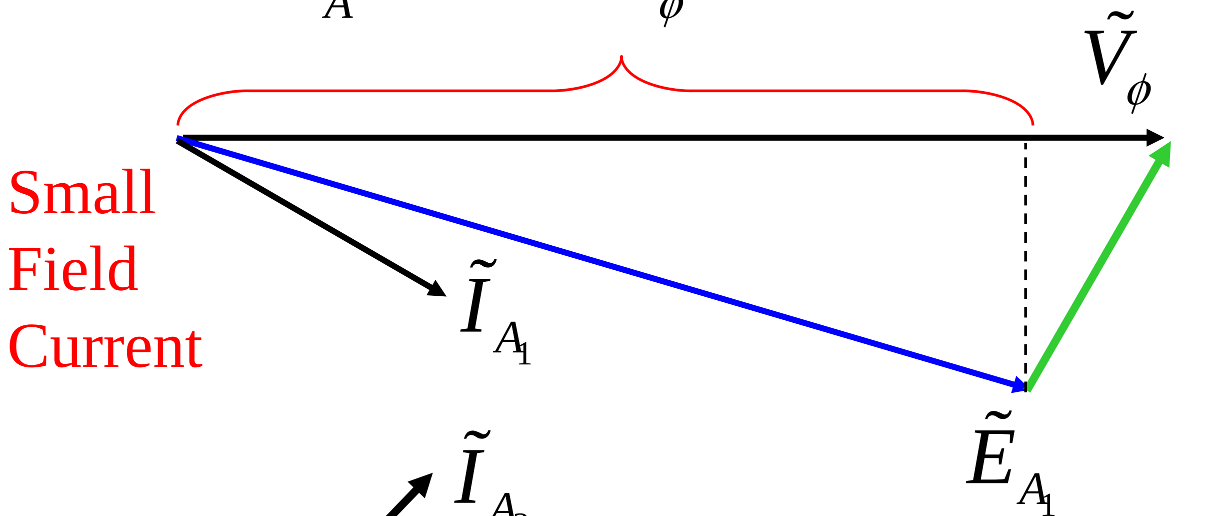
Synchronous Motor *V* Curve

The information just discussed is often expressed with the *V-curves* supplied with the motor.

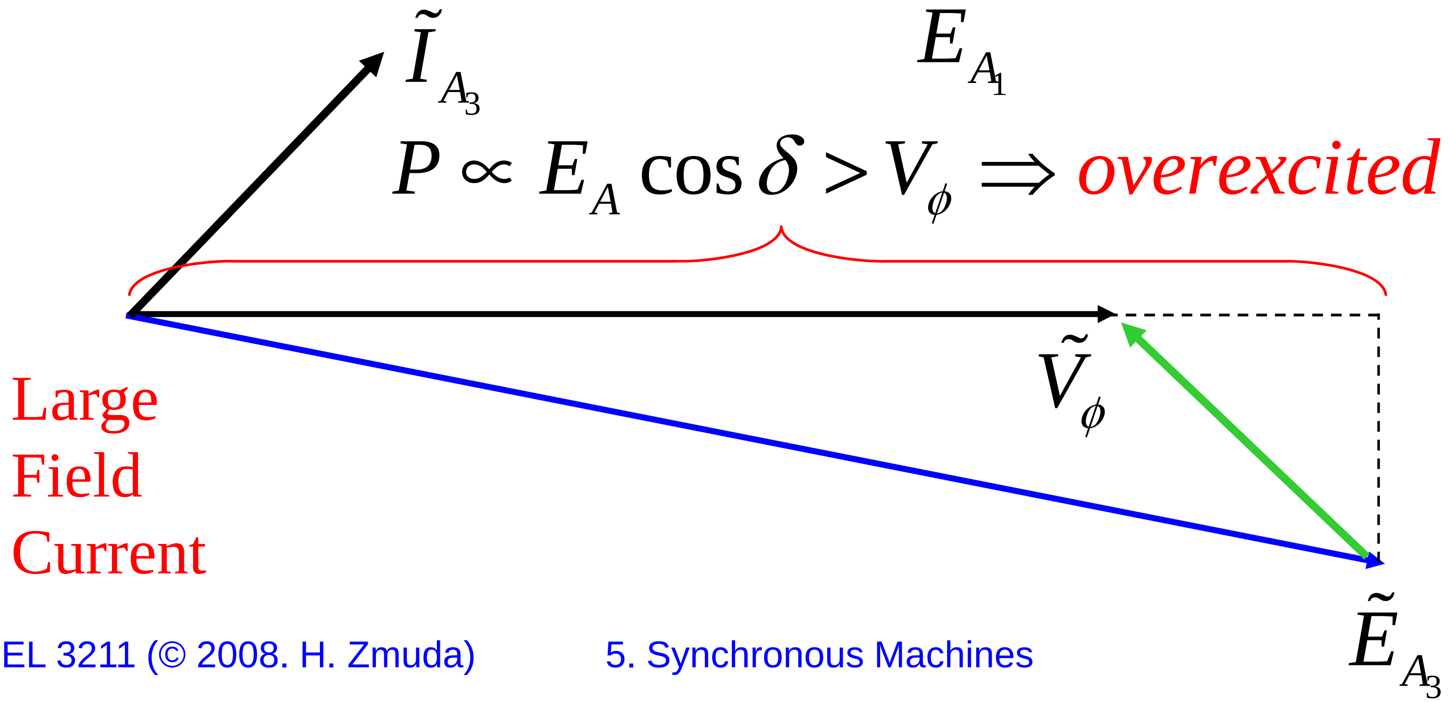


Effects of Changes in Field Current

$$P \propto E_A \cos\delta < V_\phi \Rightarrow \textit{underexcited}$$



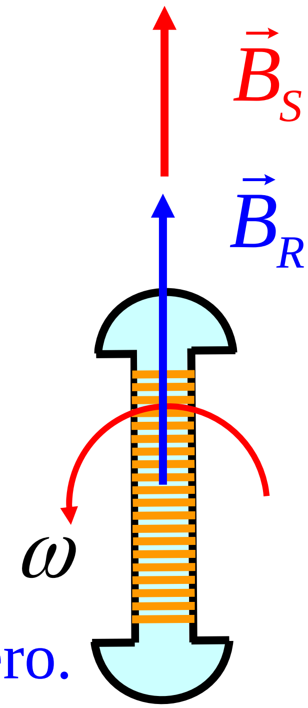
$$P \propto E_A \cos\delta > V_\phi \Rightarrow \textit{overexcited}$$



Starting Synchronous Motors

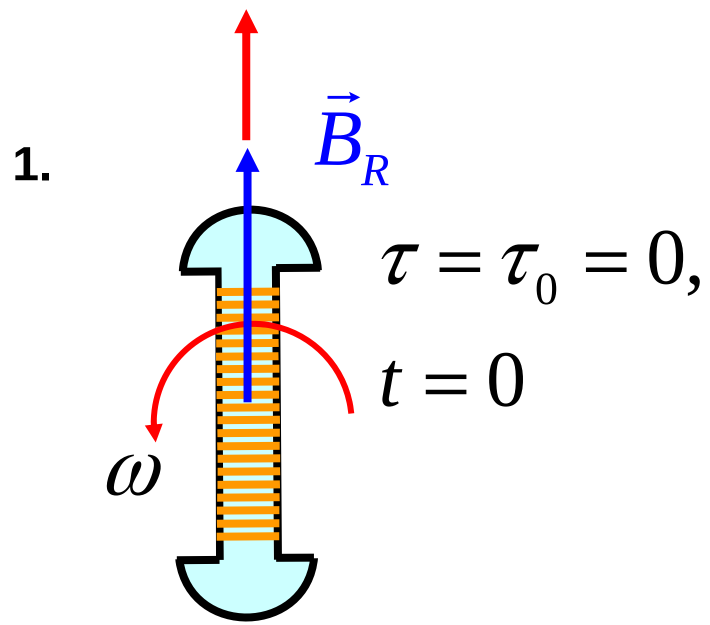
So far we've assumed that the motor was always turning at synchronous speed. How it got there is not so simple.

1. Rotor is initially stationary
2. Apply power to stator windings
3. The rotor is still stationary at first
4. Thus B_R is stationary
5. The stator field B_S immediately begins to sweep at synchronous speed



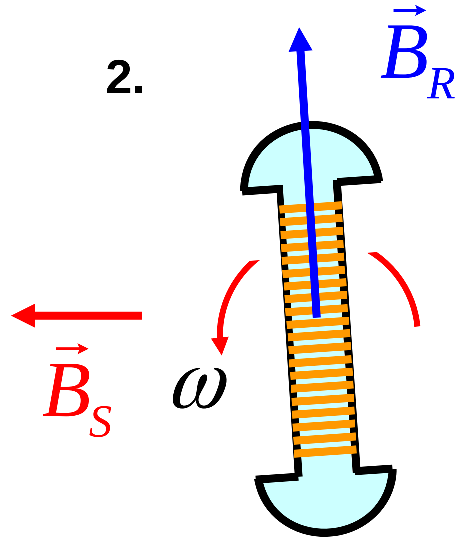
At $t = 0$, the torque is zero.

Starting Synchronous Motors



$$\vec{\tau}_{ind} \propto \vec{B}_R \times \vec{B}_S$$

Starting Synchronous Motors



For t = 1/240 s*, the rotor has barely moved (inertia) but the stator field has progressed by 90°

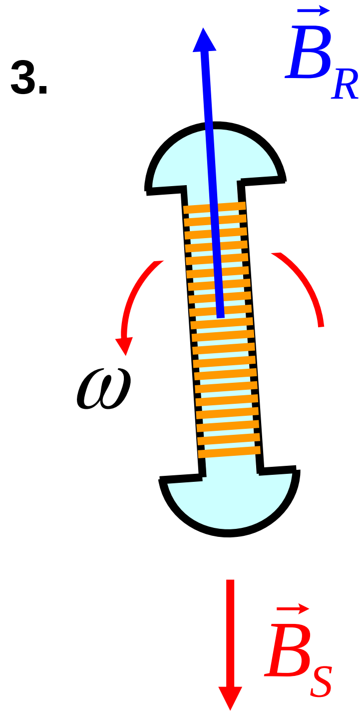
$$\tau = \tau_1, counterclockwise,$$

$$t = \frac{1}{240} s$$

$$\vec{\tau}_{ind} \propto \vec{B}_R \times \vec{B}_S$$

$$* \quad \omega t_1 = \frac{\pi}{2} \Rightarrow t_1 = \frac{\pi}{2} \frac{1}{2\pi f} = \frac{1}{4 \times 60} = \frac{1}{240} s$$

Starting Synchronous Motors



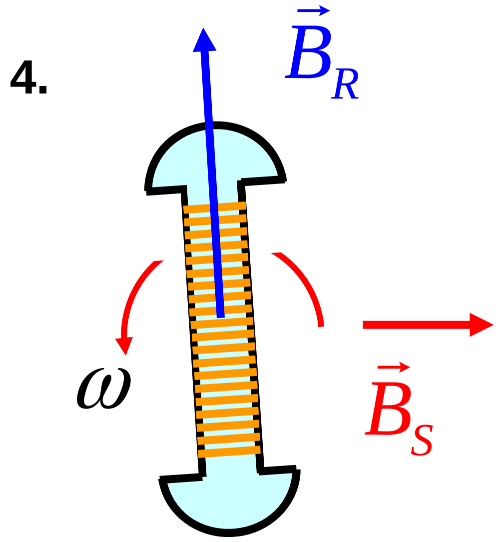
For t = 2/240 s, the rotor has still barely moved (inertia) but the stator field has progressed by 180°

$$\tau = \tau_2 = 0$$

$$t = \frac{2}{240} s$$

$$\vec{\tau}_{ind} \propto \vec{B}_R \times \vec{B}_S$$

Starting Synchronous Motors



For t = 3/240 s, the rotor has still barely moved (inertia) but the stator field has progressed by 270°

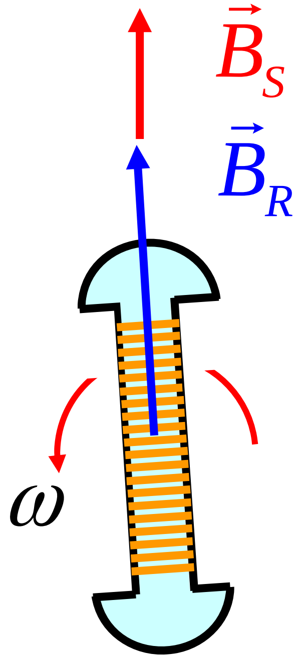
$$\tau = \tau_4 = \text{clockwise}$$

$$t = \frac{3}{240} s$$

$$\vec{\tau}_{ind} \propto \vec{B}_R \times \vec{B}_S$$

Starting Synchronous Motors

5.



For t = 4/240 s, the rotor has still barely moved (inertia) but the stator field is back to 0º

$$\tau = \tau_4 = 0$$

$$t = \frac{4}{240} = \frac{1}{60}s$$

Nothing has happened!

$$\vec{\tau}_{ind} \propto \vec{B}_R \times \vec{B}_S$$

Starting Synchronous Motors

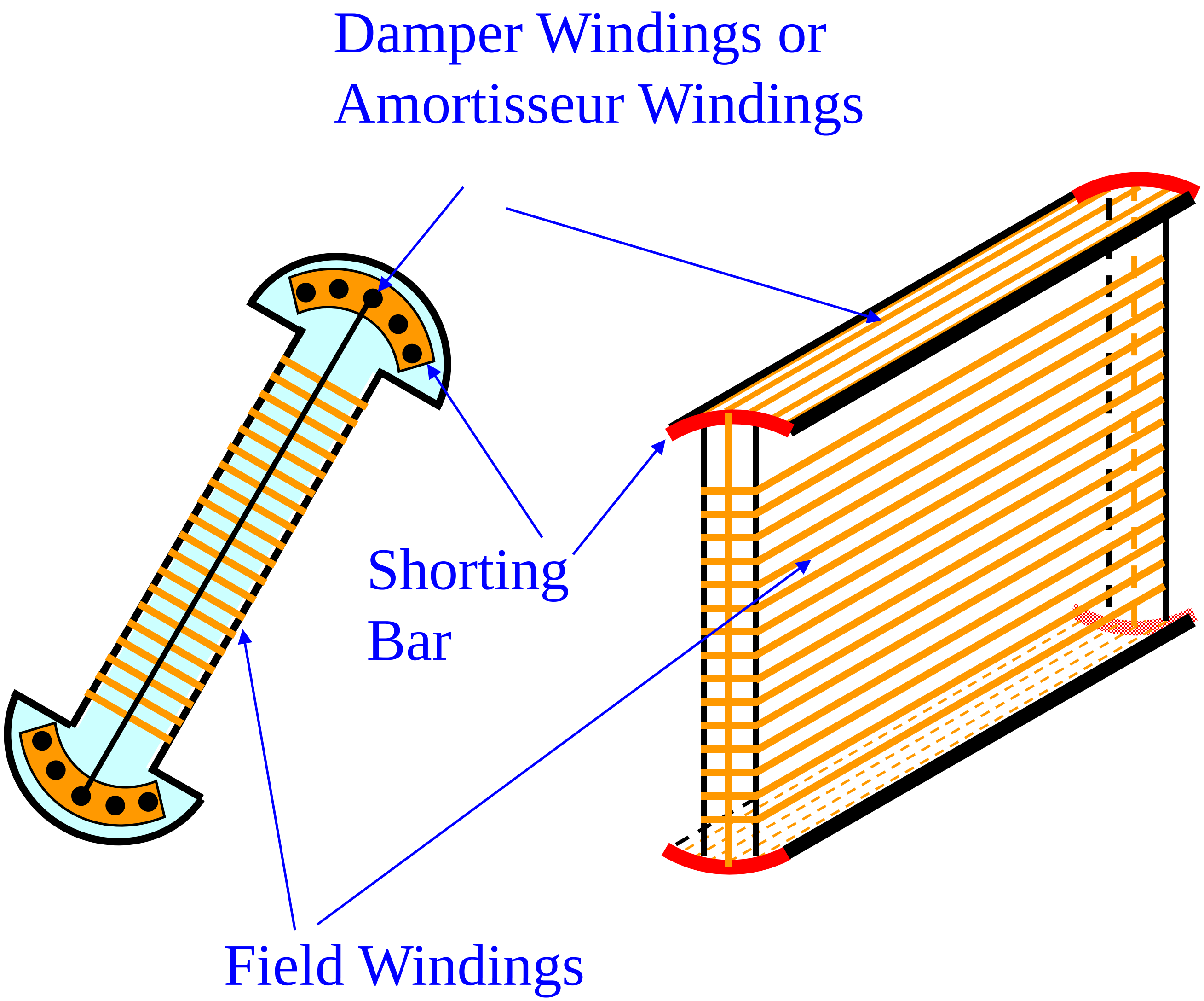
What can we do?

1. Reduce the electrical frequency to a low enough value so that the rotor can initially keep up. This requires additional electronics to control the frequency.

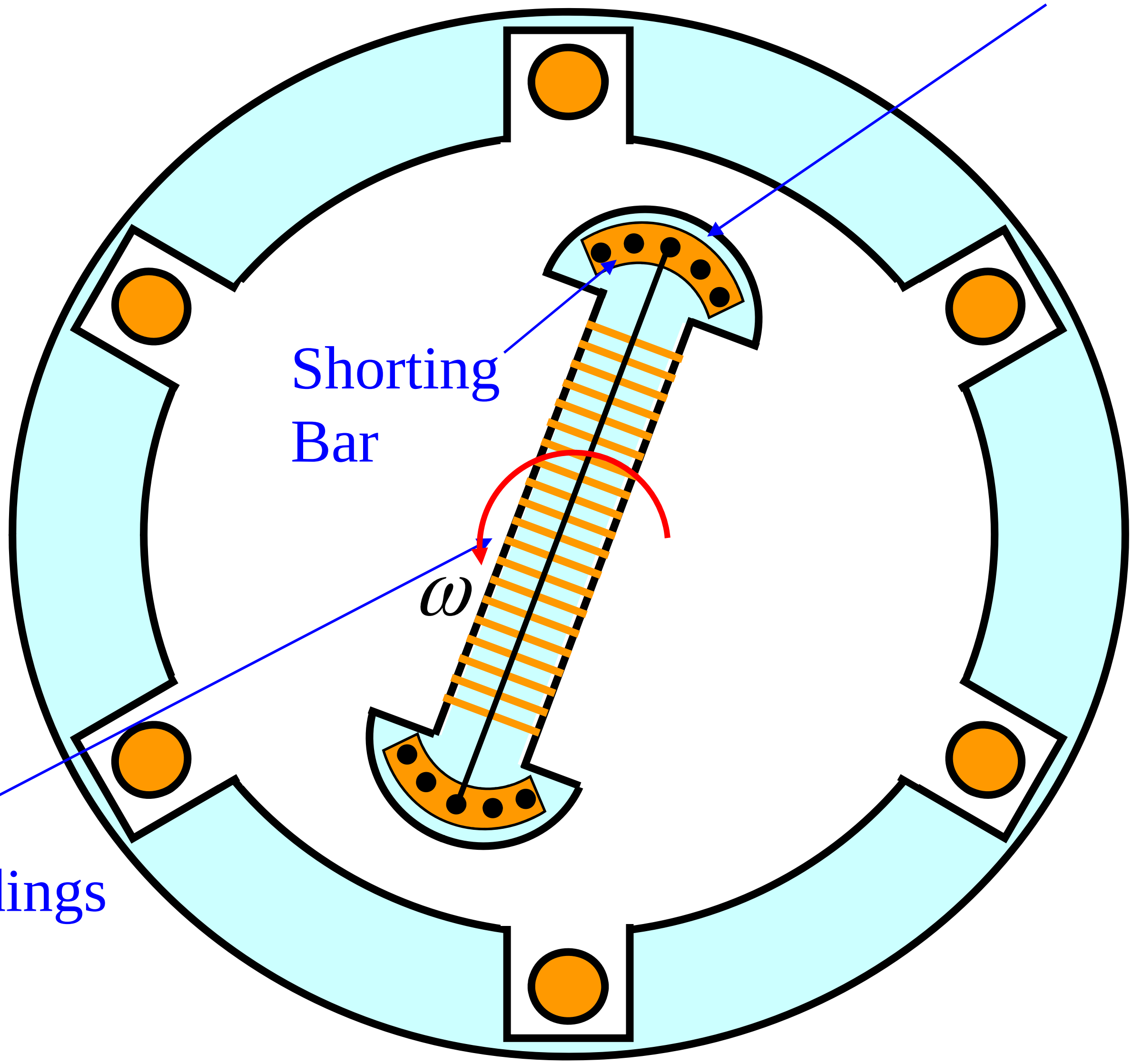
2. Start the rotor with a prime mover then let it go. We'd like a self-starting motor!

3. Use *damper windings* or *Amortisseur windings*. This is what is actually done.

Starting Synchronous Motors



Starting Synchronous Motors

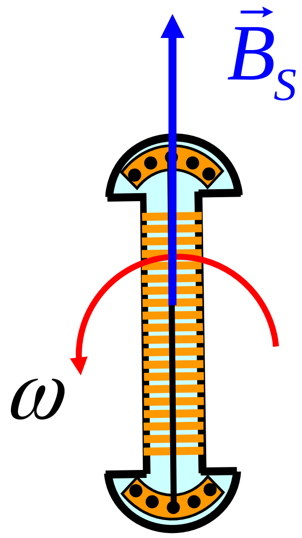


Starting Synchronous Motors

How damper winds get the motor started:

1. Assume that the field winding is disconnected and that a three-phase set of voltages is applied to the stator winding.

2. When power is first applied at $t = 0$ s, assume that the stator magnetic field is vertical as shown.



$$\vec{B}_F = 0$$

Starting Synchronous Motors

How damper winds get the motor started:

1. Assume that the field winding is disconnected and that a three-phase set of voltages is applied to the stator winding.

2. When power is first applied at $t = 0$ s, assume that the stator magnetic field is vertical as shown.

3. As the stator magnetic field sweeps across the field windings it induces a voltage in the usual way and expressed as,

$$e_{induced} = \left(\vec{v} \times \vec{B}\right) \bullet \vec{\ell}$$

where v is the velocity of the bar *relative to the magnetic field*.

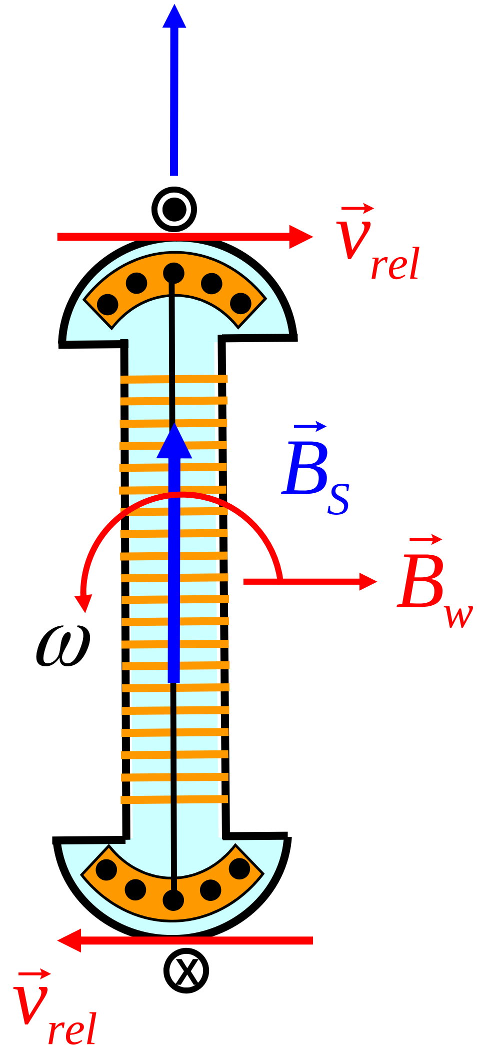
Starting Synchronous Motors

$$e_{induced} = \left(\vec{v}_{rel} \times \vec{B}_S\right) \bullet \vec{\ell}$$

$t = 0$:

The induced voltage will produce a current "out" if the top damper winding and "in" at the bottom.

This current will produce the winding magnetic field B_w shown.



The winding and stator magnetic fields will produce a torque $\tau_{induced}$

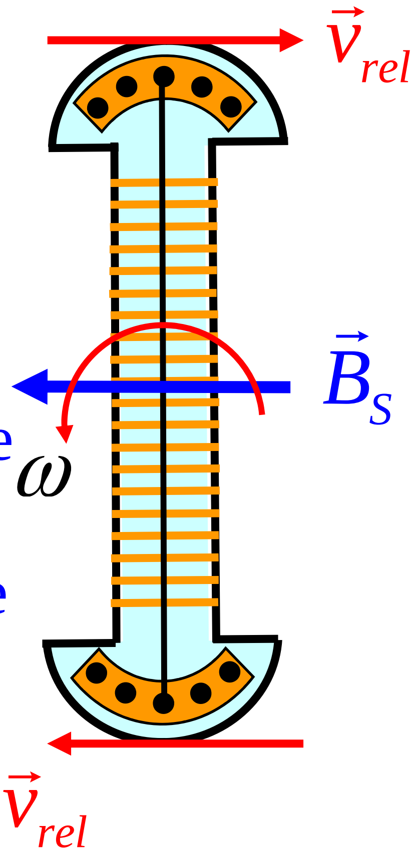
$$\vec{\tau}_{induced} = k\vec{B}_w \times \vec{B}_S$$

This torque on the bars (and hence on the rotor) is *counterclockwise*.

Starting Synchronous Motors

$$e_{induced} = \left(\vec{v}_{rel} \times \vec{B}_S\right) \bullet \vec{\ell}$$

At $t = 1/240$ s, the stator magnetic field has rotated 90° while the rotor has barely moved (inertia). Since the magnetic field and the velocity are parallel, no voltage is induced, hence no damper winding fields is produce, and the induced torque is zero,



$$\vec{B}_w = 0$$

$$\vec{\tau}_{induced} = k\vec{B}_w \times \vec{B}_S = 0$$

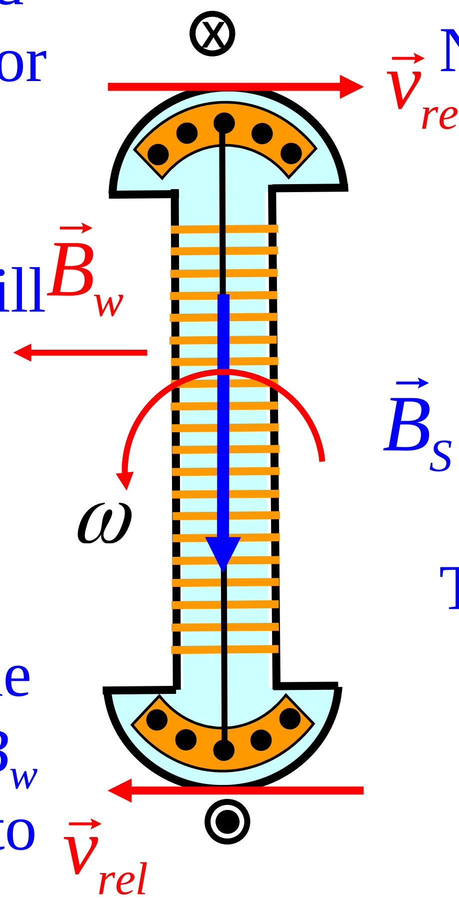
Starting Synchronous Motors

$$e_{induced} = \left(\vec{v}_{rel} \times \vec{B}_S\right) \bullet \vec{\ell}$$

At t = 2/240 s, the stator magnetic field has rotated another 90º while the rotor has hardly moved.

Now the induced voltage will produce a current "in" if the top damper winding and "out" at the bottom.

This current will produce the winding magnetic field B_w shown which not points to the left.



Now he winding and stator magnetic fields will produce a torque $\tau_{induced}$

$$\vec{\tau}_{induced} = k\vec{B}_w \times \vec{B}_S$$

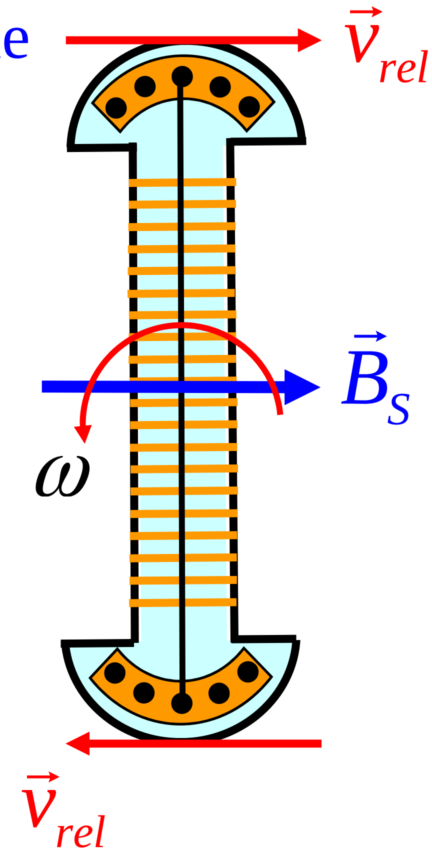
This torque on the bars (and hence on the rotor) is ***still*** *counterclockwise*.

Starting Synchronous Motors

$$e_{induced} = \left(\vec{v}_{rel} \times \vec{B}_S\right) \bullet \vec{\ell}$$

Finally at t = 3/240 s, the stator magnetic field has rotated another 90^o while the rotor has still hardly moved.

Similar to before, the induced torque is zero.



$$\vec{B}_w = 0$$

$$\vec{\tau}_{induced} = k\vec{B}_w \times \vec{B}_S = 0$$

Starting Synchronous Motors

Note that now the torque is *unidirectional*. As a consequence the rotor will speed up.

Note also that the rotor will never reach synchronous speed, though it will come close. If it did reach synchronous speed, there would no no relative velocity between the rotor and stator fields.

With no relative motion the induced voltage and hence the current would be zero. This would in turn eliminate the magnetic field of the damper winding and hence the induced torque would go to zero.

The speed will however get close to n_{sync}. When it does, the field current is then applied and the motor locks in step with the stator magnetic field.

Starting Synchronous Motors

In practice, the field winding would not be left open circuited during startup, since this would induce very high voltages across the field windings.

In practice they are short circuited, and the field induced in them will actually aid the start-up process.

Starting Synchronous Motors

The start-up process is as follows:

1. Disconnect the field winding from the DC source and short them.

2. Apply the three-phase voltages to the stator windings and let the rotor accelerate to near synchronous speed. The load should be removed from the shaft so that the synchronous speed can be reached as closely as possible.

3. Connect the DC field circuit power source, then add the loads to the shaft.

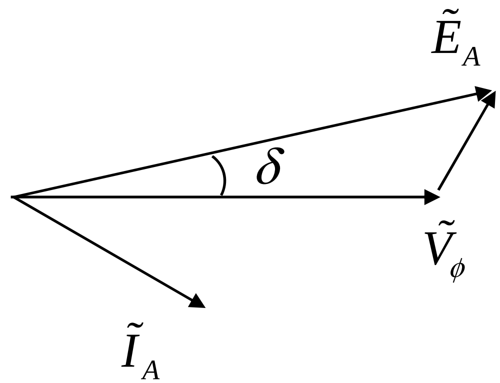
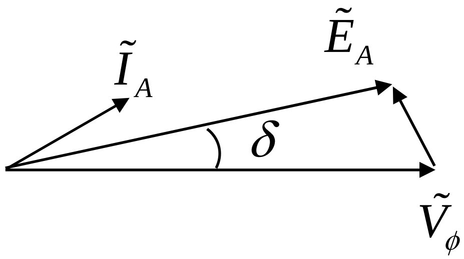
Summarizing Synchronous Machines

A synchronous machine can

1. supply real power to or
2. consume real power from a power system
3. or supply reactive power to or
4. consume reactive power a power system.

The generator converts mechanical power to electrical power while a motor converts electrical power to mechanical power, but in fact they are really the same machine.

Summarizing Synchronous Machines

<table>
<tr><th></th><th>Supply Reactive Power Q $E_A \sin\delta > V_\phi$</th><th>Consume Reactive Power Q $E_A \sin\delta < V_\phi$</th></tr>
<tr><td>Supply Power P
Generator
$\tilde{E}_A$ leads $\tilde{V}_\phi$</td><td>
</td><td>
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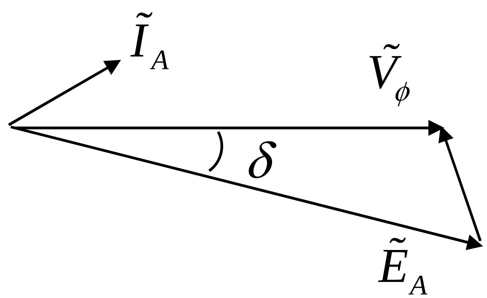
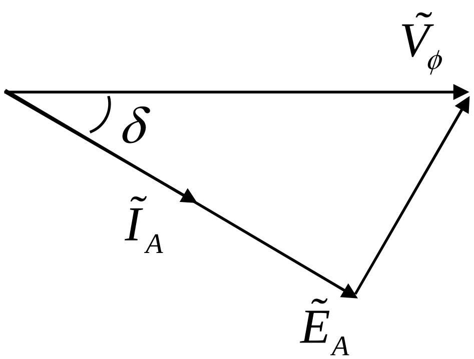
<tr><td>Consume Power P
Motor
$\tilde{E}_A$ lags $\tilde{V}_\phi$</td><td>
</td><td>
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Read Note Set 5a:
Examples of Synchronous Motors
Not covered in class.

Read Section 5.13 (Text) on Synchronous Machine Ratings