

Preface

Although much attention has been paid since 1990 to reforming calculus courses, precalculus textbooks have remained surprisingly traditional. Now that the College Board's AP* Calculus curriculum has been accepted as a model for a twenty-first-century calculus course, the path has been cleared for a new precalculus course to match the AP* goals and objectives. With this edition of *Precalculus: Graphical, Numerical, Algebraic* the authors of *Calculus: Graphical, Numerical, Algebraic*, the best-selling textbook in the AP* Calculus market, have designed such a precalculus course. For those students continuing on in a calculus course, this precalculus textbook concludes with a chapter that prepares students for the two central themes of calculus: instantaneous rate of change and continuous accumulation. This intuitively appealing preview of calculus is both more useful and more reasonable than the traditional, unmotivated foray into the computation of limits, and it is more in keeping with the stated goals and objectives of the AP* courses.

Recognizing that precalculus could be a capstone course for many students, the authors also include *quantitative literacy* topics such as probability, statistics, and the mathematics of finance. Their goal is to provide students with the good critical-thinking skills needed to succeed in any endeavor.

Continuing in the spirit of two earlier editions, the authors have integrated graphing technology throughout the course, not as an additional topic but as an essential tool for both mathematical discovery and effective problem solving. Graphing technology enables students to study a full catalog of basic functions at the beginning of the course, thereby giving them insights into function properties that are not seen in many books until later chapters. By connecting the algebra of functions to the visualization of their graphs, the authors are even able to introduce students to parametric equations, piecewise-defined functions, limit notation, and an intuitive understanding of continuity as early as Chapter 1.

Once students are comfortable with the language of functions, the authors guide them through a more traditional exploration of twelve basic functions and their algebraic properties, always reinforcing the connections among their algebraic, graphical, and numerical representations. The book uses a consistent approach to modeling, emphasizing in every chapter the use of particular types of functions to model behavior in the real world.

Finally, this textbook has faithfully incorporated not only the teaching strategies that have made *Calculus: Graphical, Numerical, Algebraic* so popular, but also some of the strategies from the popular Prentice Hall high-school algebra series, and thus has produced a seamless pedagogical transition from prealgebra through calculus for students. Although this book can certainly be appreciated on its own merits, teachers who seek continuity in their mathematics sequence might consider this deliberate alignment of pedagogy to be an additional asset of *Precalculus: Graphical, Numerical, Algebraic*.

Our Approach

The Rule of Four—A Balanced Approach

A principal feature of this edition is the balance among the algebraic, numerical, graphical, and verbal methods of representing problems: the rule of four. For instance, we obtain solutions algebraically when that is the most appropriate technique to use, and we obtain solutions graphically or numerically when algebra is difficult to use. We urge students to solve problems by one method and then support or confirm their solutions by using another method. We believe that students must learn the value of each of these methods or represen-

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tations and must learn to choose the one most appropriate for solving the particular problem under consideration. This approach reinforces the idea that to understand a problem fully, students need to understand it algebraically as well as graphically and numerically.

Problem-Solving Approach

Systematic problem solving is emphasized in the examples throughout the text, using the following variation of Polya's problem-solving process:

- *understand* the problem,
- *develop* a mathematical model,
- *solve* the mathematical model and support or confirm the solutions, and
- *interpret* the solution.

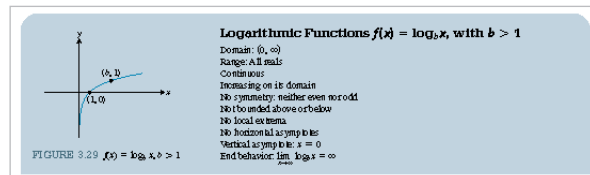
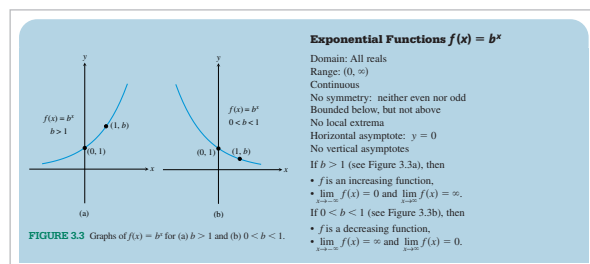
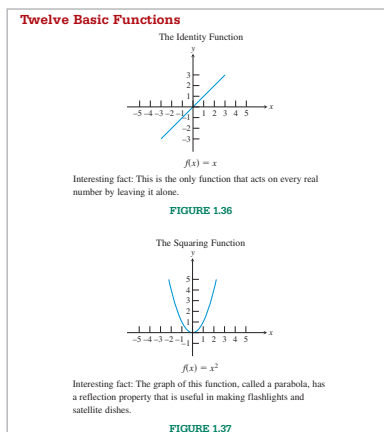
Students are encouraged to use this process throughout the text.

Twelve Basic Functions

Twelve Basic Functions are emphasized throughout the book as a major theme and focus. These functions are:

- The Identity Function
- The Squaring Function
- The Cubing Function
- The Reciprocal Function
- The Square Root Function
- The Exponential Function
- The Natural Logarithm Function
- The Sine Function
- The Cosine Function
- The Absolute Value Function
- The Greatest Integer Function
- The Logistic Function

One of the most distinctive features of this textbook is that it introduces students to the full vocabulary of functions early in the course. Students meet the twelve basic functions graphically in Chapter 1 and are able to compare and contrast them as they learn about concepts like domain, range, symmetry, continuity, end behavior, asymptotes, extrema, and even periodicity—concepts that are difficult to appreciate when the only examples a teacher can refer to are polynomials. With this book, students are able to characterize functions by their behavior within the first month of classes. (For example, thanks to graphing technology, it is no longer necessary to understand radians before one can learn that the sine function is bounded, periodic, odd, and continuous, with domain $(-\infty, \infty)$ and range $[-1, 1]$.) Once students have a comfortable understanding of functions in general, the rest of the course consists of studying the various types of functions in greater depth, particularly with respect to their algebraic properties and modeling applications.



These functions are used to develop the fundamental analysis skills that are needed in calculus and advanced mathematics courses. Section 1.2 provides an overview of these functions by examining their graphs. A complete gallery of basic functions is included in Appendix B for easy reference.

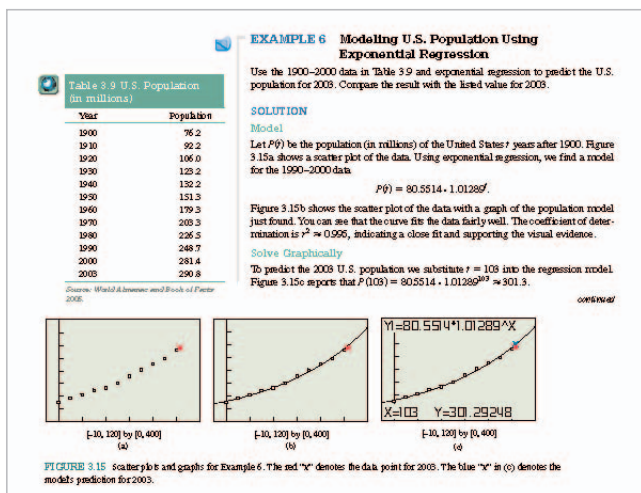
Each basic function is revisited later in the book with a deeper analysis that includes investigation of the algebraic properties.

General characteristics of families of functions are also summarized.

Applications and Real Data

The majority of the applications in the text are based on real data from cited sources, and their presentations are self-contained; students will not need any experience in the fields from which the applications are drawn.

As they work through the applications, students are exposed to functions as mechanisms for modeling data and are motivated to learn about how various functions can help model real-life problems. They learn to analyze and model data, represent data graphically, interpret from graphs, and fit curves. Additionally, the tabular representation of data presented in this text highlights the concept that a function is a correspondence between numerical variables. This helps students build the connection between the numbers and graphs and recognize the importance of a full graphical, numerical, and algebraic understanding of a problem. For a complete listing of applications, please see the Applications Index on page 1051.



Interpret

The model predicts the U.S. population will be 301.3 million in 2003. The actual population was 290.8 million. We overestimated by 10.5 million, less than a 4% error.

Now try Exercise 43.

Chapter P

Complex numbers are now introduced in Section P.6, which is earlier than their previous placement in Chapter 2.

Chapter 1

Section 1.4 from the previous edition has been split into two sections to give more practice at function composition and to give inverse functions their own section. Graphical representations of absolute value compositions have been added.

Chapter 2

The section on complex numbers has been moved to Chapter P to make the length of this chapter more teachable. Subsections titled “Applications of Quadratic Functions” and “Monomial Functions and Their Graphs” have been included to highlight these topics.

Chapter 4

Exploration exercises have been added to introduce the arcsecant and arccosecant functions and the domain options associated with them.

Chapter 6

The material of this chapter is now unified under the title “Applications of Trigonometry.” The section on vectors has been simplified, and there is a new subsection connecting the topics of polar curves and parametric curves. Geometric representation of complex numbers has been moved from Chapter 2 to Section 6.6.

Chapter 8

The updated Chapter Project titled “Ellipses as Models of Pendulum Motion” addresses the application of ellipses.

Chapter 9

There are now separate sections for sequences and series, with more examples and exercises involving each, as well as expanded treatment of sequence convergence.

Chapter 10

This preview of calculus first provides an historical perspective to calculus by presenting the classical studies of motion through the tangent line and area problems. Limits are then investigated further, and the chapter concludes with graphical and numerical examinations of derivatives and integrals.

New or Enhanced Features

Several features have been enhanced in this revision to assist students in achieving mastery of the skills and concepts of the course. We are pleased to offer the following new or enhanced features.


Chapter Openers include a motivating photograph and a general description of an application that can be solved with the topics in the chapter. The application is revisited later in the chapter with a specific problem that is solved. These problems enable students to explore realistic situations using graphical, numerical, and algebraic methods. Students are also asked to model problem situations using the functions studied in the chapter. In addition, the chapter sections are listed here.

A **Chapter Overview** begins each chapter to give you a sense of what you are going to learn. This overview provides a roadmap of the chapter, as well as tells how the different topics in the chapter are connected under one big idea. It is always helpful to remember that mathematics isn’t modular, but interconnected, and that the skills and concepts you are learning throughout the course build on one another to help you understand more complicated processes and relationships.

Similarly, the **What you’ll learn about ... and why** feature gives you the big ideas in each section and explains their purpose. You should read this as you begin the

CHAPTER 1

Functions and Graphs



- 1.1 Modeling and Equation Solving
- 1.2 Functions and Their Properties
- 1.3 Twelve Basic Functions
- 1.4 Building Functions from Functions
- 1.5 Parametric Relations and Inverses
- 1.6 Graphical Transformations
- 1.7 Modeling with Functions

One of the central principles of economics is that the value of money is not constant. It is a function of time. Since fortunes are made and lost by people attempting to predict the future value of money, much attention is paid to quantitative measures like the consumer price index, a basic measure of inflation in various sectors of the economy. See page 159 for a look at how the consumer price index for housing has behaved over time.

276 CHAPTER 3 Exponential, Logistic, and Logarithmic Functions

Chapter 3 Overview

In this chapter, we study three interrelated families of functions: exponential, logistic, and logarithmic functions. Polynomial functions, rational functions, and power functions with rational exponents are algebraic functions—functions obtained by adding, subtracting, multiplying, and dividing constants and an independent variable, and raising exponents to integer powers and extracting roots. In this chapter and the next one, we explore (and use) rational functions, which go beyond, or transcend, these algebraic operations.

Just like their algebraic cousins, exponential, logistic, and logarithmic functions have wide application. Exponential functions model growth and decay over time, such as unrestricted population growth and the decay of radioactive substances. Logistic functions model restricted population growth, certain chemical reactions, and the spread of tumors and diseases. Logarithmic functions are the basis of the Richter scale of earthquake intensity, the pH acidity scale, and the decibel measurement of sound.

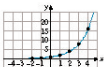
The chapter closes with a study of the mathematics of finance, an application of exponential and logarithmic functions often used when making investments.

3.1 Exponential and Logistic Functions

What you’ll learn about

- Exponential Functions and Their Graphs
- The Natural Base e
- Logistic Functions and Their Graphs
- Population Models
- ... and why

Exponential and logistic functions model many growth patterns, including the growth of human and animal populations.



Exponential Functions and Their Graphs

The functions $f(x) = a^x$ and $g(x) = 2^x$ each involve a base raised to a power, but the roles are reversed:

- For $f(x) = a^x$, the base is the variable x , and the exponent is the constant a ; f is a familiar exponential and power function.
- For $g(x) = 2^x$, the base is the constant 2 , and the exponent is the variable x ; g is an exponential function. See Figure 3.1.

DEFINITION Exponential Functions

Let a and b be real number constants. An exponential function in x is a function that can be written in the form

$$f(x) = a \cdot b^x,$$

when a is nonzero, b is positive, and $b \neq 1$. The constant a is the *initial value* of the value at $x = 0$, and b is the *base*.

Exponential functions are defined and continuous for all real numbers. It is important to recognize whether a function is an exponential function.

CHAPTER OPENER PROBLEM (from page 69)

PROBLEM: The table below shows the growth in the computer price index (CPI) for housing for selected years between 1980 and 2003 (based on 1983 dollars). How can we construct a function to predict the housing CPI for the years 2004–2010?

Year	Housing CPI
1980	81.1
1985	107.7
1990	128.5
1995	148.5
1998	160.4
1999	163.9
2000	169.6
2001	176.4
2002	180.3
2003	184.8

Source: Bureau of Labor Statistics, guide to the World Almanac, and Brook of Data 2005.

SOLUTION: A scatter plot of the data is shown in Figure 1.87, where x is the number of years since 1980. Since the data points fall near an upward-sloping line, we can use a calculator to compute a regression line to model the data. The equation of the regression line is found to be $y = 4.37x + 83.20$.

As Figure 1.88 shows, the line fits the data very well.

To predict the housing CPI for 2004, use $x = 24$ in the equation of the regression line. Similarly, we can predict the housing CPI for each of the years 2004–2010, as shown below:

Year	Predicted CPI (Housing)
2004	$y = 4.37(24) + 83.20 = 188.1$
2005	$y = 4.37(25) + 83.20 = 192.5$
2006	$y = 4.37(26) + 83.20 = 196.8$
2007	$y = 4.37(27) + 83.20 = 201.2$
2008	$y = 4.37(28) + 83.20 = 205.6$
2009	$y = 4.37(29) + 83.20 = 209.9$
2010	$y = 4.37(30) + 83.20 = 214.3$

Even with a regression fit as impressive as in Figure 1.88, it is risky to predict even this far beyond the data set. Statistics like the CPI are dependent on many volatile factors that can quickly render any mathematical model obsolete. In fact, many economists convinced that this growth could not be sustained, began warning in 2003 that the “housing bubble” would pop before 2010.

Common Logarithms—Base 10

Logarithms with base 10 are called **common logarithms**. Because of their connection to our base-ten number system, the metric system, and scientific notation, common logarithms are especially useful. We often drop the subscript of 10 for the base when using common logarithms. The common logarithmic function $\log_{10} x = \log x$ is the inverse of the exponential function $f(x) = 10^x$. So

$$y = \log x \text{ if and only if } 10^y = x.$$

Applying this relationship, we can obtain other relationships for logarithms with base 10.

Basic Properties of Common Logarithms

Let x and y be real numbers with $x > 0$.

- $\log 1 = 0$ because $10^0 = 1$.
- $\log 10 = 1$ because $10^1 = 10$.
- $\log 10^y = y$ because $10^y = 10^y$.
- $10^{\log x} = x$ because $\log x = \log x$.

EXPLORATION 3 Graphs of Exponential Functions

1. Graph each function in the viewing window $[-2, 2]$ by $[-1, 6]$.

(a) $y_1 = 2^x$ (b) $y_2 = 3^x$ (c) $y_3 = 4^x$ (d) $y_4 = 5^x$

- Which point is common to all four graphs?
- Analyze the functions for domain, range, continuity, increasing or decreasing behavior, symmetry, boundedness, extrema, asymptotes, and end behavior.

2. Graph each function in the viewing window $[-2, 2]$ by $[-1, 6]$.

(a) $y_1 = \left(\frac{1}{2}\right)^x$ (b) $y_2 = \left(\frac{1}{3}\right)^x$
 (c) $y_3 = \left(\frac{1}{4}\right)^x$ (d) $y_4 = \left(\frac{1}{5}\right)^x$

- Which point is common to all four graphs?
- Analyze the functions for domain, range, continuity, increasing or decreasing behavior, symmetry, boundedness, extrema, asymptotes, and end behavior.

In Exercises 71 and 72, use the data in Table 3.28.

Table 3.28 Populations of Two U.S. States (in millions)


Year	Georgia	Illinois
1900	2.2	4.8
1910	2.6	5.6
1920	2.9	6.5
1930	2.9	7.6
1940	3.1	7.9
1950	3.4	8.7
1960	3.9	10.1
1970	4.6	11.1
1980	5.5	11.4
1990	6.5	11.4
2000	8.2	12.4

Source: U.S. Census Bureau as reported in the World Almanac and Book of Facts 2005.

- 71. Modeling Population** Find an exponential regression model for Georgia's population, and use it to predict the population in 2005.
- 72. Modeling Population** Find a logistic regression model for Illinois's population, and use it to predict the population in 2010.

section and always review it after you have completed the section to make sure you understand all of the key topics that you have just studied.

Vocabulary is highlighted in yellow for easy reference. **Properties** are boxed in green so that you can easily find them.


Each example ends with a suggestion to **Now Try** a related exercise. Working the suggested exercise is an easy way for you to check your comprehension of the material while reading each section, instead of waiting to the end of each section or chapter to see if you “got it.” In the *Annotated Teacher's Edition*, various examples are marked for the instructor with the  icon. Alternates are provided for these examples in the *Acetates and Transparencies* package.

Explorations appear throughout the text and provide you with the perfect opportunity to become an active learner and discover mathematics on your own. This will help hone your critical thinking and problem-solving skills. Some are technology-based and others involve exploring mathematical ideas and connections.

Margin Notes on various topics appear throughout the text. *Tips* offer practical advice to you on using your grapher to obtain the best, most accurate results. *Margin notes* include historical information, hints about examples, and provide additional insight to help you avoid common pitfalls and errors.

A BIT OF HISTORY

Logarithmic functions were developed around 1594 as computational tools by Scottish mathematician John Napier (1550–1617). He originally called them “artificial numbers,” but changed the name to logarithms, which means “reckoning numbers.”

The **Looking Ahead to Calculus**  icon is found throughout the text next to many examples and topics to point out concepts that students will encounter again in calculus. Ideas that foreshadow calculus are highlighted, such as limits, maximum and minimum, asymptotes, and continuity. Early in the text, the idea of the limit using an intuitive and conceptual approach is introduced. Some calculus notation and language is introduced in the early chapters and used throughout the text to establish familiarity.

Graphs of Logarithmic Functions with Base b

Using the change-of-base formula we can rewrite any logarithmic function $g(x) = \log_b x$ as

$$g(x) = \frac{\ln x}{\ln b} = \frac{1}{\ln b} \ln x.$$

So every logarithmic function is a constant multiple of the natural logarithmic function $f(x) = \ln x$. If the base is $b > 1$, the graph of $g(x) = \log_b x$ is a vertical stretch or shrink of the graph of $f(x) = \ln x$ by the factor $1/\ln b$. If $0 < b < 1$, a reflection across the x -axis is required as well.

The **Web/Real Data**  icon is used to mark the examples and exercises that use real cited data.

The **Chapter Review** material at the end of each chapter are sections dedicated to helping students review the chapter concepts. **Key Ideas** has three parts: Properties, Theorems, and Formulas; Procedures; and Gallery of Functions. The **Review Exercises** represent the full range of exercises covered in the chapter and give additional practice with the ideas developed in the chapter. The exercises with red numbers indicate problems that would make up a good practice test. **Chapter Projects** conclude each chapter and require students to analyze data. They can be assigned as either individual or group work. Each project expands upon concepts and ideas taught in the chapter, and many projects refer to the Web for further investigation of real data.

CHAPTER 3 Key Ideas

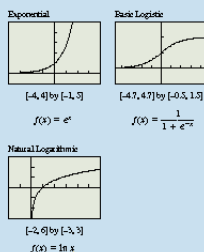
PROPERTIES, THEOREMS, AND FORMULAS

- Exponential Growth and Decay 279
- Exponential Functions $f(x) = A^x$ 280
- Exponential Functions and the Base e 282
- Exponential Population Model 280
- Changing Between Logarithmic and Exponential Form 300
- Basic Properties of Logarithms 301
- Basic Properties of Common Logarithms 302
- Basic Properties of Natural Logarithms 304
- Properties of Logarithms 310
- Change-of-Base Formula for Logarithms 313
- Logarithmic Functions $f(x) = \log_b x$, with $b > 1$ 314
- One-to-One Properties 320
- Inverse's Law of Cooling 325
- Interest Compounded Annually 334
- Interest Compounded k Times per Year 337
- Interest Compounded Continuously 337
- Future Value of an Annuity 339
- Present Value of an Annuity 340

PROCEDURES

- Re-regression of Data 314–316
- Logarithmic Re-expansion of Data 320–329

GALLERY OF FUNCTIONS



CHAPTER 3 Review Exercises

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1 and 2, compute the exact value of the function for the given x value without using a calculator.

- $f(x) = -3 \cdot 4^x$ for $x = \frac{1}{\log_4 3}$
- $f(x) = 6 \cdot 3^x$ for $x = -\frac{1}{\log_3 2}$

In Exercises 3 and 4, determine a formula for the exponential function whose graph is shown in the figure.

In Exercises 5–10, describe how to transform the graph of f into the graph of $g(x) = 2^x$ or $h(x) = e^x$. Sketch the graph by hand and support your answer with a grapher.

- $f(x) = 4^{x+3}$
- $f(x) = -8^{x-3}$
- $f(x) = 2^{x+9}$
- $f(x) = -4^{-x}$
- $f(x) = 8^{-x} + 3$
- $f(x) = 2^{x-4}$

In Exercises 11 and 12, find the y -intercept and the horizontal asymptote.

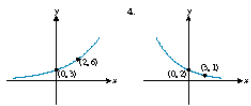
- $f(x) = \frac{100}{3 + 2e^{-0.05x}}$
- $f(x) = \frac{50}{3 + 2e^{-0.05x}}$
- $f(x) = e^{x+2}$
- $f(x) = 2(5^{x+2}) + 1$

In Exercises 13 and 14, state whether the function is an exponential growth function or an exponential decay function, and describe its end behavior using limits.

- $f(x) = e^{x+2}$
- $f(x) = 2(5^{x+2}) + 1$
- $f(x) = 15^{-x}$
- $f(x) = 3(4^{x+3}) - 2$
- $f(x) = \frac{6}{1 + 3 \cdot 0.4^x}$
- $f(x) = \frac{100}{4 + 2e^{-0.05x}}$

In Exercises 19–22, find the exponential function that satisfies the given conditions.

- Initial value = 24, increasing at a rate of 5.3% per day
- Initial population = 67,000, increasing at a rate of 1.67% per year
- Initial height = 18 cm, doubling every 3 weeks



In Exercises 31–34, rewrite the equation in exponential form.

- $\log_2 x = 5$
- $\log_3 x = y$
- $\ln \frac{1}{2} = -2$
- $\log_5 \frac{1}{5} = -3$

In Exercises 35–38, describe how to transform the graph of $y = \log_2 x$ into the graph of the given function. Sketch the graph by hand and support with a grapher.

- $f(x) = \log_2(x+4)$
- $g(x) = \log_2(4-x)$
- $h(x) = -\log_2(x-1) + 2$
- $k(x) = -\log_2(x+1) + 4$

In Exercises 39–42, graph the function, and analyze it for domain, range, continuity, increasing or decreasing behavior, symmetry, boundedness, extrema, asymptotes, and end behavior.

- $f(x) = x \ln x$
- $f(x) = \frac{1}{x} \ln x$
- $f(x) = x^2 \ln |x|$
- $f(x) = \ln |x|$

In Exercises 43–54, solve the equation.

- $10^x = 4$
- $10^{2x} = 3$
- $\log_2 x = -7$
- $3 \log_2 x + 1 = 7$
- $\frac{2}{x} - 3 = 5$
- $\log_2(x+2) + \log_2(x-1) = 4$
- $\ln(3x+4) - \ln(2x+1) = 5$
- An atomic nucleus has a diameter of about 0.00000000000001 m.
- An atom's mass is about 1.66×10^{-27} kg.
- $10^x = 4$
- $10^{2x} = 3$
- $\log_2 x = -7$
- $3 \log_2 x + 1 = 7$
- $\frac{2}{x} - 3 = 5$
- $\log_2(x+2) + \log_2(x-1) = 4$
- $\ln(3x+4) - \ln(2x+1) = 5$

QUICK REVIEW 3.5 (For help, go to Sections P.1 and 1.4.)

In Exercises 1–4, prove that each function in the given pair is the inverse of the other.

- $f(x) = e^{2x}$ and $g(x) = \ln(x^{1/2})$
- $f(x) = 10^{0.2x}$ and $g(x) = \log_2 x^2, x > 0$
- $f(x) = (1/3) \ln x$ and $g(x) = e^{3x}$
- $f(x) = 3 \log_2 x^2, x > 0$ and $g(x) = 10^{0.4x}$

In Exercises 5 and 6, write the number in scientific notation.

- The mean distance from Jupiter to the Sun is about 778,300,000 km.

In Exercises 7 and 8, write the number in decimal form.

- Avogadro's number is about 6.02×10^{23} .
- The atomic mass unit is about 1.66×10^{-27} kg.

In Exercises 9 and 10, use scientific notation to simplify the expression; leave your answer in scientific notation.

- $(86,000)(31,000,000)$
- $\frac{0.0000008}{0.000005}$

Standardized Test Questions

59. **True or False:** The order of magnitude of a positive number is its natural logarithm. Justify your answer.

60. **True or False:** According to Newton's Law of Cooling, an object will approach the temperature of the medium that surrounds it. Justify your answer.

In Exercises 61–64, solve the problem without using a calculator.

- Multiple Choice** Solve $2^{x-1} = 32$.
(A) $x = 1$ (B) $x = 2$ (C) $x = 4$
(D) $x = 11$ (E) $x = 13$
- Multiple Choice** Solve $\ln x = -1$.
(A) $x = -1$ (B) $x = 1/e$ (C) $x = 1$
(D) $x = e$ (E) No solution is possible.

61. **Multiple Choice** How many times more serious was the 2001 earthquake in Annapolis, Peru ($M_s = 8.1$) than the 1998 double earthquake in Taidir province, Algeria ($M_s = 6.1$)?

- (A) 2 (B) 6.1 (C) 8.1
(D) 14.2 (E) 100

62. **Multiple Choice** Newton's Law of Cooling is an exponential model. (E) a linear model. (C) a logarithmic model. (D) a logistic model. (B) a power model.

63. **Multiple Choice** Newton's Law of Cooling is an exponential model. (E) a linear model. (C) a logarithmic model. (D) a logistic model. (B) a power model.

Explorations

In Exercises 65 and 66, use the data in Table 3.26. Determine whether a linear, logarithmic, exponential, power, or logistic regression equation is the best model for the data. Explain your choice. Support your writing with tables and graphs as needed.

65. **Writing to Learn** Modeling Population Which regression equation is the best model for Alaska's population?

66. **Writing to Learn** Modeling Population Which regression equation is the best model for Hawaii's population?

67. **Group Activity** Normal Distribution The function $f(x) = ke^{-x^2/2}$, where k and e are positive constants, is a bell-shaped curve that is useful in probability and statistics.

(a) Graph f for $c = 1$ and $k = 0.1, 0.5, 1, 2, 10$. Explain the effect of changing k .

(b) Graph f for $c = 1$ and $k = 0.1, 0.5, 1, 2, 10$. Explain the effect of changing c .

68. **Writing to Learn** Prove if $a > 0$ and $b > 0$, then $\log a - \log b = \log \frac{a}{b}$. Explain how this result relates to powers of ten and orders of magnitude.

69. **Formal Discovery** The potential energy E (in joules) stored in two point charges q_1 and q_2 separated by a distance r in a certain molecular structure is modeled by the function

$$E = -\frac{5.6}{r} + 10e^{-r/9}$$

where r is the distance separating the nuclei.

(a) **Writing to Learn** Graph this function in the window $[-10, 10]$ by $[-10, 30]$, and explain which portion of the graph does not represent this potential energy situation.

(b) Identify a viewing window that shows that portion of the graph (with $r \leq 10$) which represents this situation, and find the maximum value for E .

70. In Example 8, the Newton's Law of Cooling model was $T(t) - T_a = (T_0 - T_a)e^{-kt} = 61.656 \times 0.92779^t$. Determine the value of k .

71. Verify the conclusion made about natural logarithmic regression on page 329.

72. Verify the conclusion made about power regression on page 329.

In Exercises 73–78, solve the equation or inequality.

- $e^x + x = 5$
- $e^x - 8x + 1 = 0$
- $e^x \leq 5 + \ln x$
- $\ln |x| - e^{2x} \leq 3$
- $2 \log x - 4 \log 3 > 0$
- $2 \log(x+1) - 2 \log 6 < 0$

73. **Table 3.26** Populations of Two U.S. States (in thousands)

Year	Alaska	Hawaii
1900	63.6	134
1910	64.4	132
1920	55.0	256
1930	59.2	368
1940	72.5	425
1950	128.6	500
1960	226.2	633
1970	302.6	770
1980	401.9	945
1990	550.0	1108
2000	626.9	1212

Source: U.S. Census Bureau.

CHAPTER 3 Project

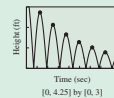
Analyzing a Bouncing Ball

When a ball bounces up and down on a flat surface, the maximum height of the ball decreases with each bounce. Each rebound is a percentage of the previous height. For most balls, the percentage is a constant. In this project, you will use a motion detector device to collect height data for a ball bouncing underneath a motion detector, then find a mathematical model that describes the maximum bounce height as a function of bounce number.

Collecting the Data

Set up the Calculator Based Laboratory (CBL™) system with a motion detector or a Calculator Based Ranger (CBR™) system to collect ball bounce data using a ball bounce program for the CBL or the Ball Bounce Application for the CBR. See the CBL/CBR guidebook for specific setup instructions.

Hold the ball at least 2 feet below the detector and release it so that it bounces straight up and down beneath the detector. These programs convert distance versus time data to height from the ground versus time. The graph shows a plot of sample data collected with a racketball and CBR. The data table below shows each maximum height collected.



Bounce Number	Maximum Height (feet)
0	2.7188
1	2.1426
2	1.6565
3	1.2640
4	0.98309
5	0.77783

EXPLORATIONS

- If you collected motion data using a CBL or CBR, a plot of height versus time should be shown on your graphing calculator or computer screen. Trace to the maximum height for each bounce and record your data in a table and use other lists in your calculator to enter this data. If you don't have access to a CBL/CBR, enter the data given in the table into your graphing calculator/computer.
- Bounce height 1. Is what percentage of bounce height 0? Calculate the percentage return for each bounce. The numbers should be fairly constant.
- Create a scatter plot for maximum height versus bounce number.
- For bounce 1, the height is predicted by multiplying bounce height 0, or H_0 , by the percentage P . The second height is predicted by multiplying this height HP by P which gives the H^2P^2 . Explain why $y = HP^n$ is the appropriate model for this data, where n is the bounce number.
- Enter this equation into your calculator using your values for H and P . How does the model fit your data?
- Use the statistical features of the calculator to find the exponential regression for this data. Compare it to the equation that you used as a model.
- How would your data and equation change if you used a different type of ball?
- What factors would change the H value and what factors affect the P value?
- Rewrite your equation using base e instead of using P as the base for the exponential equation.
- What do you predict the graph of \ln (bounce height) versus bounce number to look like?
- Plot \ln (bounce height) versus bounce number. Calculate the linear regression and use the concept of logarithmic regression to explain how the slope and y -intercept are related to P and H .

Exercise Sets

Each exercise set begins with a **Quick Review** to help you review skills needed in the exercise set, thus reminding you again that mathematics is not modular. There are also directions that give a *section to go to for help* so that students are prepared to do the Section Exercises.

There are over 6,000 exercises, including 680 Quick Review Exercises. Following the Quick Review are exercises that allow you to practice the algebraic skills learned in that section. These exercises have been carefully graded from routine to challenging. The following types of skills are tested in each exercise set:

- Algebraic and analytic manipulation
- Connecting algebra to geometry
- Interpretation of graphs
- Graphical and numerical representations of functions
- Data analysis

Also included in the exercise sets are thought-provoking exercises:

- Standardized Test Questions** include two true-false problems with justifications and four multiple-choice questions.
- Explorations** are opportunities for students to discover mathematics on their own or in groups. These exercises often require the use of critical thinking to explore the ideas.

- Writing to Learn** exercises give you practice at communicating about mathematics and opportunities to demonstrate your understanding of important ideas.
- Group Activity** exercises ask you to work on the problems in groups or solve them as individual or group projects.

- **Extending the Ideas** exercises go beyond what is presented in the textbook. These exercises are challenging extensions of the book’s material.

This variety of exercises provides sufficient flexibility to emphasize the skills most needed for each student or class.

Supplements and Resources

For the Student

Student Edition, ISBN 0-13-227650-X

Student’s Solutions Manual, ISBN 0-321-36994-7

- Contains detailed, worked-out solutions to odd-numbered exercises

Student Practice Workbook, ISBN 0-13-198580-9


- New examples that parallel key examples from each section in the book are provided, along with a detailed solution
- Related practice problems follow each example

Graphing Calculator Manual, ISBN 0-321-37000-7

- Grapher Workshop provides detailed instruction on important grapher features
- Features TI-83 Plus, Silver, TI-84, and TI-89 Titanium

For the Teacher

Annotated Teacher’s Edition, ISBN 0-321-37423-1

- Answers included on the same page for most exercises
- Various examples are marked with the  icon. Alternates are provided for these examples in the *Acetates and Transparencies* package.
- The *Annotated Teacher’s Edition* also includes notes written specifically for the teacher. These notes include chapter and section objectives, suggested assignments, lesson guides, and teaching tips.

Resource Manual, ISBN 0-321-36995-5

- Major Concepts Review, Group Activity Worksheets, Sample Chapter Tests, Standardized Test Preparation Questions, Contest Problems

Solutions Manual, ISBN 0-321-36993-9

- Complete solutions to all exercises, including Quick Reviews, Exercises, Explorations, and Chapter Reviews

Tests and Quizzes, ISBN 0-321-36992-0

- Two parallel tests per chapter, two quizzes for every 3–4 sections, two parallel mid-term tests covering Chapters P–5, two parallel end-of-year tests, covering Chapters 6–10

Acetates and Transparencies, ISBN 0-321-36997-1

- Full color transparencies of 10–15 useful general transparencies along with black and white transparency masters of all Quick Review Exercises
- One Alternate Example per lesson

FOLLOW-UP

Ask . . .

If $a > 0$, how can you tell whether $y = a \cdot b^x$ represents an increasing or decreasing function? (The function is increasing if $b > 1$ and decreasing if $0 < b < 1$.)

ASSIGNMENT GUIDE

Day 1: Ex. 1–13 odd, 15–39, multiples of 3, 45, 48

Day 2: Ex. 41, 44, 49, 52, 53, 55, 65, 67, 68, 70, 71

COOPERATIVE LEARNING

Group Activity: Ex. 39, 40

NOTES ON EXERCISES

Ex. 15–30 encourage students to think about the appearance of functions without using a grapher.

Ex. 59–64 provide practice for standardized tests.

Ex. 69–72 require students to think about the meaning of different kinds of functions.

ONGOING ASSESSMENT

Self-Assessment: Ex. 1, 7, 11, 15, 21, 41, 51, 55

Embedded Assessment: Ex. 53, 68

OBJECTIVE

Students will be able to use exponential growth, decay, and regression to model real-life problems.

MOTIVATE

Ask . . .

If a culture of 100 bacteria is put into a petri dish and the culture doubles every hour, how long will it take to reach 400,000? Is there a limit to growth?

LESSON GUIDE

Day 1: Constant Percentage Rate and Exponential Functions; Exponential Growth and Decay Models

Day 2: Using Regression to Model Population; Other Logistic Models

Technology Resources

***MathXL®*, www.mathxl.com**

MathXL® is a powerful online homework, tutorial, and assessment system that accompanies our textbooks in mathematics or statistics. With MathXL, instructors can create, edit, and assign online homework and tests using algorithmically generated exercises correlated at the objective level to the textbook. They can also create and assign their own online exercises and import TestGen tests for added flexibility. All student work is tracked in MathXL's online gradebook. Students can take chapter tests in MathXL and receive personalized study plans based on their test results. The study plan diagnoses weaknesses and links students directly to tutorial exercises for the objectives they need to study and retest. Students can also access video clips from selected exercises. For more information, visit our Web site at www.mathxl.com, or contact your local sales representative.

MathXL® Tutorials on CD

This interactive tutorial CD-ROM provides algorithmically generated practice exercises that are correlated at the objective level to the exercises in the textbook. Every practice exercise is accompanied by an example and a guided solution designed to involve students in the solution process. Selected exercises may also include a video clip to help students visualize concepts. The software provides helpful feedback for incorrect answers and can generate printed summaries of students' progress.

InterAct Math Tutorial Web site, www.interactmath.com

Get practice and tutorial help online! This interactive tutorial Web site provides algorithmically generated practice exercises that correlate directly to the exercises in the textbook. Students can retry an exercise as many times as they like with new values each time for unlimited practice and mastery. Every exercise is accompanied by an interactive guided solution that provides helpful feedback for incorrect answers, and students can also view a worked-out sample problem that steps them through an exercise similar to the one they're working on.

Video Lectures on CD

The video lectures for this text are also available on CD-ROM, making it easy and convenient for students to watch the videos from a computer at home or on campus. The complete digitized video set, affordable and portable for students, is ideal for distance learning or supplemental instruction. These videotaped lectures feature an engaging team of mathematics instructors who present comprehensive coverage of topics in the text.

TestGen®

TestGen® enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. Tests can be printed or administered online. The software is available on a dual-platform Windows/Macintosh CD-ROM.

Presentation Express CD-ROM (PowerPoint® slides)

This time saving component includes classroom presentation slides that correlate to the topic sequence of the textbook. In addition, all transparencies are included in *PowerPoint®* format making it easier for you to teach and to customize based on your teaching preferences. All slides can be customized and edited.

Teacher Express CD-ROM (with LessonView)

- This is a new suite of instructional tools on CD-ROM to help teachers plan, teach, and assess at the click of a mouse. Powerful lesson planning, resource management, testing, and an interactive *Annotated Teacher's Edition* all in one place make class preparation quick and easy!
- Contents include: Planning Express, *Annotated Teacher's Edition*, Program Teaching Resources, Correlations, Links to Other Resources.
- Online resources require an Internet Connection.

Student Express CD-ROM (with PDF Text)

The perfect tool for test review or studying, this CD provides the complete student textbook in an electronic format.

Web Site

Our Web site, www.awl.com/demana, provides dynamic resources for instructors and students. Some of the resources include TI graphing calculator downloads, online quizzing, teaching tips, study tips, Explorations, and end-of-chapter projects.