The Dublin School of Grinds

# 5th Year
# Maths
# Higher Level

# Functions & Graphs
# Exponential Functions



Reference: 5-mat-h-Functions and Graphs, Exponential Functions

# EASTER REVISION COURSES

## Looking to maximise your CAO points?

Easter is well known as a time for students to vastly improve on the points that they received in their mock exams. To help students take advantage of this valuable time, The Dublin School of Grinds is running intensive exam-focused Easter Revision Courses. Each course runs for five days (90 minutes per day).

The focus of these courses is to maximise students' CAO points. **Special offer:** Buy 1st course and get 2nd course free. To avail of this offer, early booking is required as courses were fully booked last year.

## What do students get at these courses?

- ✓ 90 minutes of intensive tuition per day for five days, with Ireland's leading teachers.
- ✓ Comprehensive study notes.
- ✓ A focus on simple shortcuts to raise students' grades and exploit the critically important marking scheme.
- ✓ Access to a free supervised study room.
- ✓ Access to food and beverage facilities.

**NOTE:** These courses are built on the fact that there are certain predicable trends that appear and reoccur over and over again in the State Examinations.

To book, call us on **01-442 4442** or book online at **www.dublinschoolofgrinds.ie**

**EASTER REVISION COURSE FEES:**

| | PRICE | TOTAL | SAVINGS |
|---|---|---|---|
| 1st Course | €295 | €295 | - |
| 2nd Course | FREE | €295 | €295 |
| 3rd Course | €100 | €395 | €490 |
| 4th Course | €100 | €495 | €685 |
| 5th Course | €100 | €595 | €880 |
| 6th Course | €100 | €695 | €1,075 |
| 7th Course | €100 | €795 | €1,270 |
| 8th Course | €100 | €895 | €1,465 |
| 9th Course | €100 | €995 | €1,660 |

**NOTE:** Any bookings for Junior Cert courses will also receive a weekly grind in one subject for the rest of the academic year, free of charge. This offer applies to 3rd and 2nd year students ONLY.

**FREE DAILY BUS SERVICE**

For full information on our Easter bus service, see 3 pages ahead.

## Oral Preparation Courses

Separate to the Easter Revision Courses, The Dublin School of Grinds is also running Oral Preparation Courses. With the Oral marking component of the Leaving Certificate worth up to 40%, it is of paramount importance that students are fully prepared for these examinations. These courses will show students how to lead the Examiner towards topics that the student is prepared in. This will provide students with the confidence they need to perform at their peak.

**ORAL PREPARATION COURSE FEES:**

| | PRICE | TOTAL | SAVINGS |
|---|---|---|---|
| 1st Oral Course | €140 | €140 | - |
| 2nd Oral Course | €100 | €240 | €40 |

# Timetable

An extensive range of course options are available over a two-week period to cater for students' timetable needs. Courses are held over the following weeks:

» Monday 21st March – Friday 25th March 2016
» Monday 28th March – Friday 1st April 2016

All Easter Revision Courses take place in The Talbot Hotel, Stillorgan (formerly known as The Stillorgan Park Hotel).

## 6th Year Easter Revision Courses

| SUBJECT | LEVEL | DATES | TIME |
|---|---|---|---|
| Accounting | H | Monday 21st March – Friday 25th March | 8:00am - 9:30am |
| Agricultural Science | H | Monday 28th March – Friday 1st April | 2:00pm - 3:30pm |
| Applied Maths | H | Monday 28th March – Friday 1st April | 8:00am - 9:30am |
| Art History | H | Monday 28th March – Friday 1 April | 8:00am - 9:30am |
| Biology Course A* | H | Monday 21st March – Friday 25th March | 8:00am - 9:30am |
| Biology Course A* | H | Monday 21st March – Friday 25th March | 12:00pm - 1:30pm |
| Biology Course A* | H | Monday 28th March – Friday 1st April | 10:00am - 11:30am |
| Biology Course B* | H | Monday 21st March – Friday 25th March | 10:00am - 11:30am |
| Biology Course B* | H | Monday 21st March – Friday 25th March | 2:00pm - 3:30pm |
| Biology Course B* | H | Monday 28th March – Friday 1st April | 8:00am - 9:30am |
| Business | H | Monday 21st March – Friday 25th March | 12:00pm - 1:30pm |
| Business | H | Monday 28th March – Friday 1st April | 8:00am - 9:30am |
| Chemistry Course A* | H | Monday 28th March – Friday 1st April | 12:00pm - 1:30pm |
| Chemistry Course B* | H | Monday 28th March – Friday 1st April | 2:00pm - 3:30pm |
| Classical Studies | H | Monday 21st March – Friday 25th March | 8:00am - 9:30am |
| Economics | H | Monday 21st March – Friday 25th March | 8:00am - 9:30am |
| Economics | H | Monday 28th March – Friday 1st April | 10:00am - 11:30am |
| English Paper 1* | H | Monday 21st March – Friday 25th March | 12:00pm - 1:30pm |
| English Paper 2* | H | Monday 21st March – Friday 25th March | 10:00am - 11:30am |
| English Paper 2* | H | Monday 21st March – Friday 25th March | 2:00pm - 3:30pm |
| English Paper 2* | H | Monday 28th March – Friday 1st April | 10:00am - 11:30am |
| English Paper 2* | H | Monday 28th March – Friday 1st April | 12:00pm - 1:30pm |
| French | H | Monday 21st March – Friday 25th March | 10:00am - 11:30am |
| French | H | Monday 28th March – Friday 1st April | 8:00am - 9:30am |
| Geography | H | Monday 28th March – Friday 1st April | 8:00am - 9:30am |
| Geography | H | Monday 28th March – Friday 1st April | 10:00am - 11:30am |
| German | H | Monday 21st March – Friday 25th March | 10:00am - 11:30am |
| History (Europe)* | H | Monday 21st March – Friday 25th March | 2:00pm - 3:30pm |
| History (Ireland)* | H | Monday 21st March – Friday 25th March | 12:00pm - 1:30pm |
| Home Economics | H | Monday 21st March – Friday 25th March | 10:00am - 11:30am |
| Irish | H | Monday 21st March – Friday 25th March | 10:00am - 11:30am |
| Irish | H | Monday 28th March – Friday 1st April | 12:00pm - 1:30pm |
| Maths Paper 1* | H | Monday 21st March – Friday 25th March | 8:00am - 9:30am |
| Maths Paper 1* | H | Monday 21st March – Friday 25th March | 12:00pm - 1:30pm |
| Maths Paper 1* | H | Monday 28th March – Friday 1st April | 10:00am - 11:30am |
| Maths Paper 1* | H | Monday 28th March – Friday 1st April | 2:00pm - 3:30pm |
| Maths Paper 2* | H | Monday 21st March – Friday 25th March | 10:00am - 11:30am |
| Maths Paper 2* | H | Monday 21st March – Friday 25th March | 2:00pm - 3:30pm |
| Maths Paper 2* | H | Monday 28th March – Friday 1st April | 12:00pm - 1:30pm |
| Maths Paper 2* | H | Monday 28th March – Friday 1st April | 4:00pm - 5:30pm |
| Maths | O | Monday 21st March – Friday 25th March | 8:00am - 9:30am |
| Maths | O | Monday 28th March – Friday 1st April | 12:00pm - 1:30pm |
| Physics | H | Monday 28th March – Friday 1st April | 10:00am - 11:30am |
| Spanish | H | Monday 21st March – Friday 25th March | 2:00pm - 3:30pm |
| Spanish | H | Monday 28th March – Friday 1st April | 10:00am - 11:30am |

* Due to large course content, these subjects have been divided into two courses. For a full list of topics covered in these courses, please see 3 pages ahead.

## 6th Year Oral Preparation Courses

| SUBJECT | LEVEL | DATES | TIME |
|---|---|---|---|
| French | H | Sunday 20th March | 10:00am - 2:00pm |
| German | H | Saturday 26th March | 10:00am - 2:00pm |
| Irish | H | Saturday 26th March | 10:00am - 2:00pm |
| Spanish | H | Saturday 19th March | 1:00pm - 5:00pm |

## 5th Year Easter Revision Courses

| SUBJECT | LEVEL | DATES | TIME |
|---|---|---|---|
| Maths | H | Monday 28th March – Friday 1st April | 8:00am - 9:30am |
| English | H | Monday 28th March – Friday 1st April | 4:00pm - 5:30pm |

Note: 5th year students are welcome to attend any 6th year course as part of our buy 1 get 1 free offer.

## 3rd Year Easter Revision Courses

| SUBJECT | LEVEL | DATES | TIME |
|---|---|---|---|
| Business Studies | H | Monday 28th March – Friday 1st April | 8:00am - 9:30am |
| English | H | Monday 21st March – Friday 25th March | 8:00am - 9:30am |
| English | H | Monday 28th March – Friday 1st April | 2:00pm - 3:30pm |
| French | H | Monday 28th March – Friday 1st April | 12:00pm - 1:30pm |
| Geography | H | Monday 28th March – Friday 1st April | 12:00pm - 1:30pm |
| German | H | Monday 21st March – Friday 25th March | 8:00am - 9:30am |
| History | H | Monday 21st March – Friday 25th March | 4:00pm - 5:30pm |
| Irish | H | Monday 28th March – Friday 1st April | 2:00pm - 3:30pm |
| Maths | H | Monday 21st March – Friday 25th March | 10:00am - 11:30am |
| Maths | H | Monday 21st March – Friday 25th March | 12:00pm - 1:30pm |
| Maths | H | Monday 28th March – Friday 1st April | 10:00am - 11:30am |
| Maths | O | Monday 28th March – Friday 1st April | 12:00pm - 1:30pm |
| Science | H | Monday 28th March – Friday 1st April | 2:00pm - 3:30pm |
| Science | H | Monday 21st March – Friday 25th March | 2:00pm - 3:30pm |
| Spanish | H | Monday 21st March – Friday 25th March | 12:00pm - 1:30pm |

## 2nd Year Easter Revision Courses

| SUBJECT | LEVEL | DATES | TIME |
|---|---|---|---|
| Maths | H | Monday 21st March – Friday 25th March | 2:00pm - 3:30pm |

**BUY 1ST COURSE GET 2ND COURSE FREE!**

**NOTE:** Any bookings for Junior Cert courses will also receive a weekly grind in one subject for the rest of the academic year, free of charge. This offer applies to 3rd and 2nd year students ONLY.

**BOOK EARLY TO AVAIL OF THE SPECIAL OFFER**

**Contents;**

# 1 Exponential functions

For Junior Certificate, you met functions that look like this…$f(x) = x^2 + 2x - 3$.

There were three basic types of question the Examiner could ask with functions like this.

- Draw a graph of the function.
- Find an x-value or x-values when given a y-value.
- Find a y-value when given an x-value.

But what about functions that look like this…$f(x) = 5^x$, where the letter is a power?
Another word for a power is an exponent, so this is called an **exponential** function.
Note: You are dealing with something to the power of x, not x to the power of something.

The Examiner can ask the same basic types of question with exponential functions.

Finding y-values is pretty straightforward. You just use your calculator.

## Question 1

A function is defined as $f(x) = 7^x$.

Find the value of
(i) $f(2)$
(ii) $f(0)$
(iii) $f(-1)$
(iv) $f(0{\cdot}5)$
(v) $f(0{\cdot}28)$

There is a number that crops up all the time in exponential functions.
It is called **Euler's number**, represented by the letter **e**, and is approximately 2·71828.
You will meet this constant on a few different occasions on the Leaving Certificate course.
It behaves like any other numbers in exponential equations.

## Question 2

A function is defined as $f(x) = e^x$.

Find the value of (i) $f(3)$

(ii) $f(0)$

(iii) $f(-1)$

(iv) $f(-2)$

(v) $f\left(\frac{1}{2}\right)$

(vi) $f(-0{\cdot}6)$

(vii) $f\left(-\frac{3}{2}\right)$

Finding x-values is trickier. It involves solving **exponential equations**!
Note: Any equation where the letter is part of a power is called an exponential equation.

Suppose $f(x) = 3^x$ and you are asked to find the value of x for which $f(x) = 81$.
You will end up with the equation $3^x = 81$.

You could do this by writing the 81 on the right hand side as a power of 3…$81 = 3^4$!
So $3^x = 3^4$, which means $x = 4$…by putting the powers equal to one another.

But what if the number on the right hand side cannot be easily written as a power of 3?
You need a method that will work for all exponential equations.
The way to do it is to use **logarithms**…usually abbreviated to **logs**.

We will just take a very quick look at logs here.

In the equation $5x = 25$, to get the x on its own you divide both sides by 5.
Why? You have a multiplication by 5 on the left hand side.
The inverse of multiplying by 5 is dividing by 5.
So dividing both sides by 5 gets rid of the 5 on the left hand side.

In the equation $\sqrt{x} = 6$, to get the x on its own you square both sides.
Why? You have a square root on the left hand side.
The inverse of finding a square root is squaring.
So squaring both sides gets rid of the square root on the left hand side.

In the equation $\sin x = 0{\cdot}5$, to get the x on its own you take the $\sin^{-1}$ of both sides.
Why? You have a sin on the left hand side.
The inverse of sin by 5 is $\sin^{-1}$.
So taking the $\sin^{-1}$ of both sides gets rid of the sin on the left hand side.

Logs are the inverse of exponents.
In the equation $3^x = 81$, the number that x is the power of is called the base…the base is 3 here.
So taking the log to the base 3 (written $\log_3$) will get rid of the 3 on the left hand side.
So $\log_3(3^x) = \log_3(81)$
$x = \log_3(81) = 4$…your calculator will tell you that $\log_3(81) = 4$!

## Question 3

Find the value of x if

(i) $4^x = 64$

(ii) $625^x = 5$

(iii) $5^x = 100$

(iv) $3^x = -6$

(v) $1{\cdot}5^x = 4$

Give your answers correct to two decimal places where relevant.

Logs to the base e ($\log_e$) are referred to as **natural logs**…written as $\ln x$.

## Question 4

A function is defined as $f(x) = e^{2x}$.

(i) Find the value of $f(4)$.

(ii) Find the value of $x$ for which $f(x) = 6$.
Give your answer correct to one decimal place.

(iii) Adam is asked to find the value of $x$ for which $f(x) = -3$.
Explain the difficulty Adam will have with this question.

## 2 Graphs of exponential functions

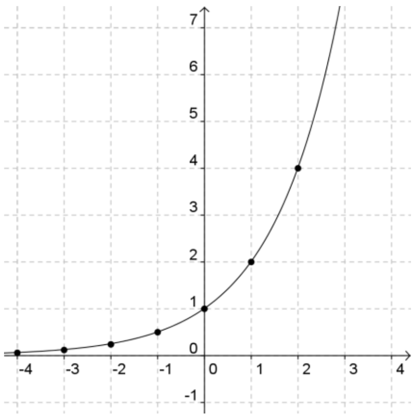
Questions on graphs of exponential functions are very common. The Examiner expects that you would be able to plot and recognise graphs of exponential functions.

All of the exponential functions you can be asked about are basically $f(x) = A \times b^x$.
The shape of the graph will depend mainly on the value of b…the base number.

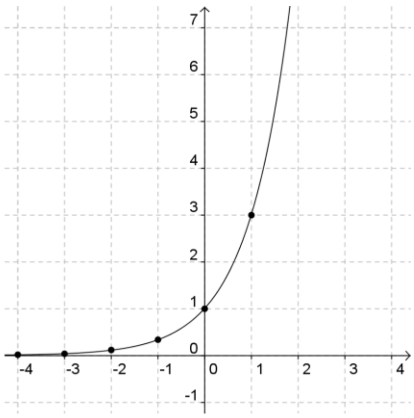
The value of b cannot ever be negative. If it was, the graph cannot be drawn!

<u>If b is bigger than 1…b > 1</u>

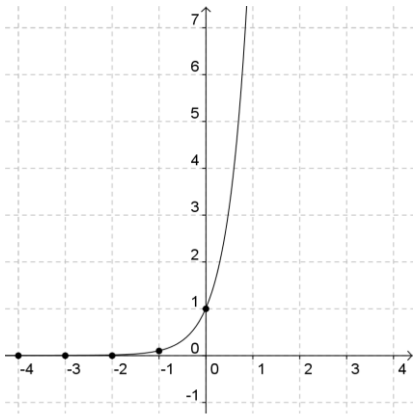
Look at the graphs below…




$f(x) = 2^x$




$f(x) = 3^x$



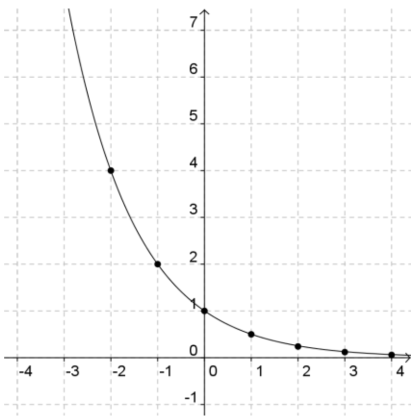

$f(x) = 10^x$

There are four important things to notice…

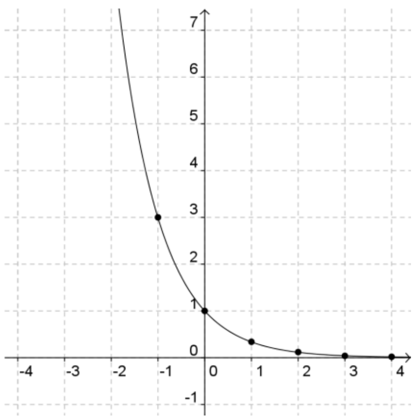
- If the base number b is bigger than 1, the graphs are increasing.
  This is called exponential increase or <u>exponential growth</u>.
- The graphs all pass through the point (0,1)…they all hit the y-axis at y = 1.
- The graphs pass through the point (1,b).
  So $y = 2^x$ passes through (1,2), $y = 3^x$ passes through (1,3), and so on…
- The bigger the value of b, the tighter the curve is to the y-axis.

<u>If b is between 0 and 1…0 < b < 1</u>

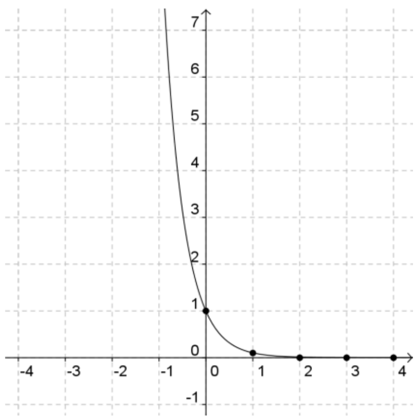
Look at the graphs below…




$f(x) = \left(\frac{1}{2}\right)^x$




$f(x) = \left(\frac{1}{3}\right)^x$



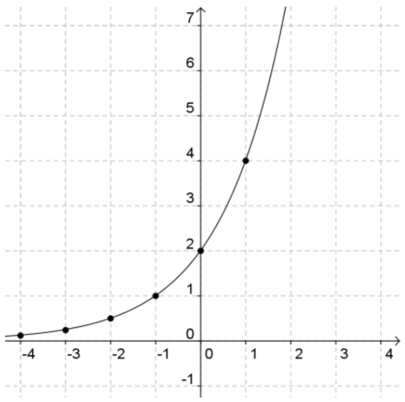

$f(x) = \left(\frac{1}{10}\right)^x$

Again, there are four important things to notice…

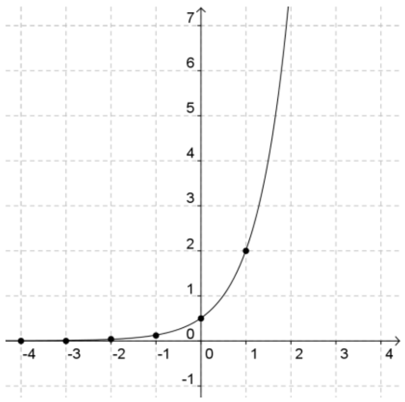
- If the base number b is between 0 and 1, the graphs are decreasing.
  This is called exponential decrease or exponential decay.
- The graphs all pass through the point (0,1) again…they all hit the y-axis at $y = 1$.
- The graphs pass through the point (1,b) again.
  So $y = \left(\frac{1}{2}\right)^x$ passes through $\left(1, \frac{1}{2}\right)$, $y = \left(\frac{1}{3}\right)^x$ passes through $\left(1, \frac{1}{3}\right)$, and so on…
- The smaller the value of b, the tighter the curve is to the y-axis.

If the function is $f(x) = A \times b^x$, what effect does the value of A have on the shape of the graph?

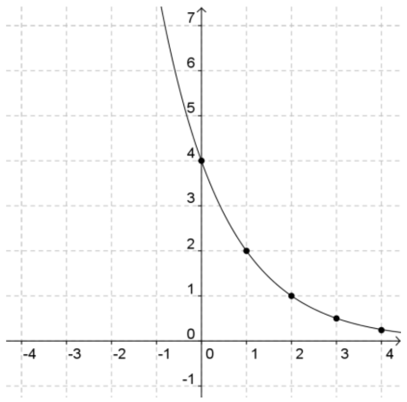
Look at the graphs below…



$f(x) = 2(2^x)$



$f(x) = \frac{1}{2}(4^x)$



$f(x) = 4\left(\frac{1}{2}\right)^x$

Two things to point out here…

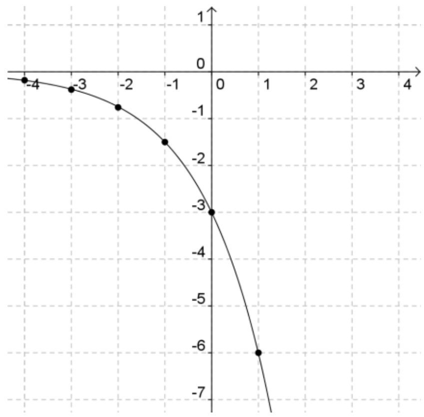
- Exponential graphs usually pass through the point (0,1).
  When there is a number in front of the function, the graph does not hit the y-axis at $y = 1$.
  The graph hits the y-axis at $y = A$ instead, so the graph passes through the point (0,A)!
  So $y = 2(2^x)$ hits the y-axis at $y = 2$, $y = \frac{1}{2}(4^x)$ hits the y-axis at $y = \frac{1}{2}$, and so on…
- Exponential graphs usually pass through the point (1,b) as well.
  When there is a number in front of the function, the graph does not pass through (1,b).
  The graph passes through the point (1,Ab) instead!
  So $y = 2(2^x)$ passes through (1,4), $y = \frac{1}{2}(4^x)$ passes through (1,2), and so on…

Look at the graph shown opposite…

If the value of A is negative, it turns the graph upside down.
It still passes through the points (0,A) and (1,Ab).

You can identify an exponential graph by looking at these two points. The y-value where the curve hits the y-axis is the value of A. Dividing the y-value of the point where $x = 1$ by this number will give the value of b.

The Examiner can give you a graph and ask you to identify it.



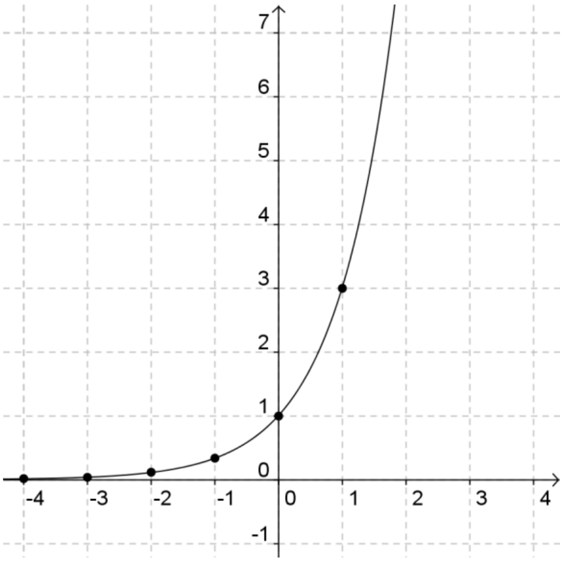
$f(x) = -3(2^x)$

## Question 5

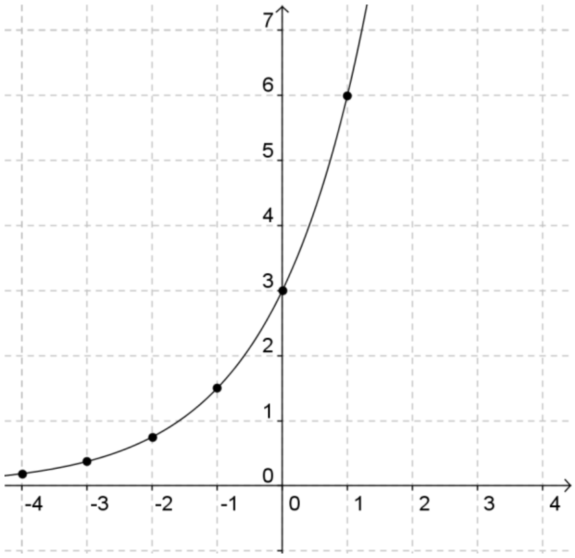
Look at the graphs numbered 1 to 4 below.

Each graph shows a function of the form $f(x) = A.b^x$ where $A, b \in \mathbb{Q}$.

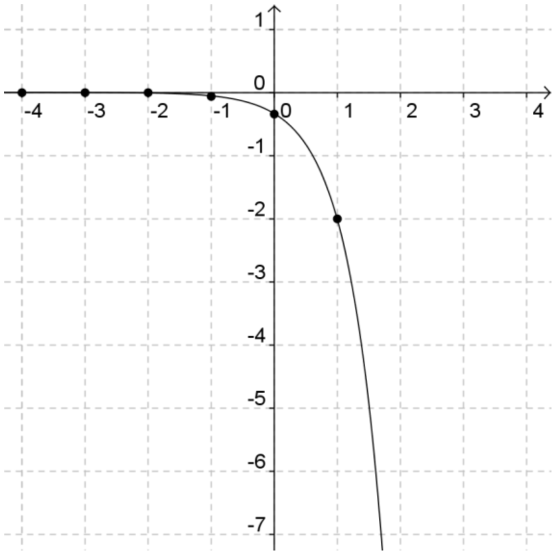
Identify the function shown in each graph.



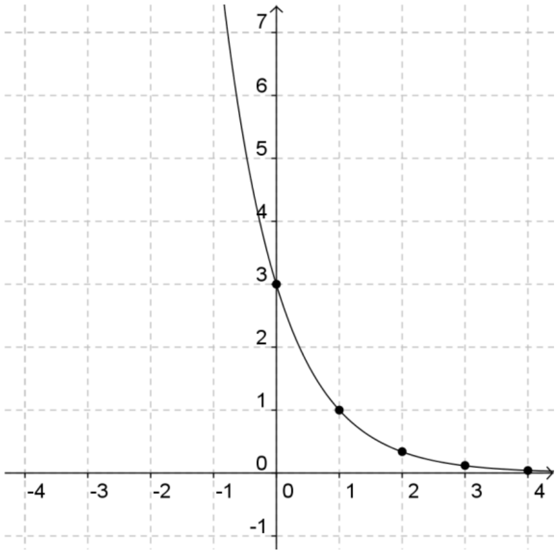

Graph 1




Graph 2




Graph 3




Graph 4

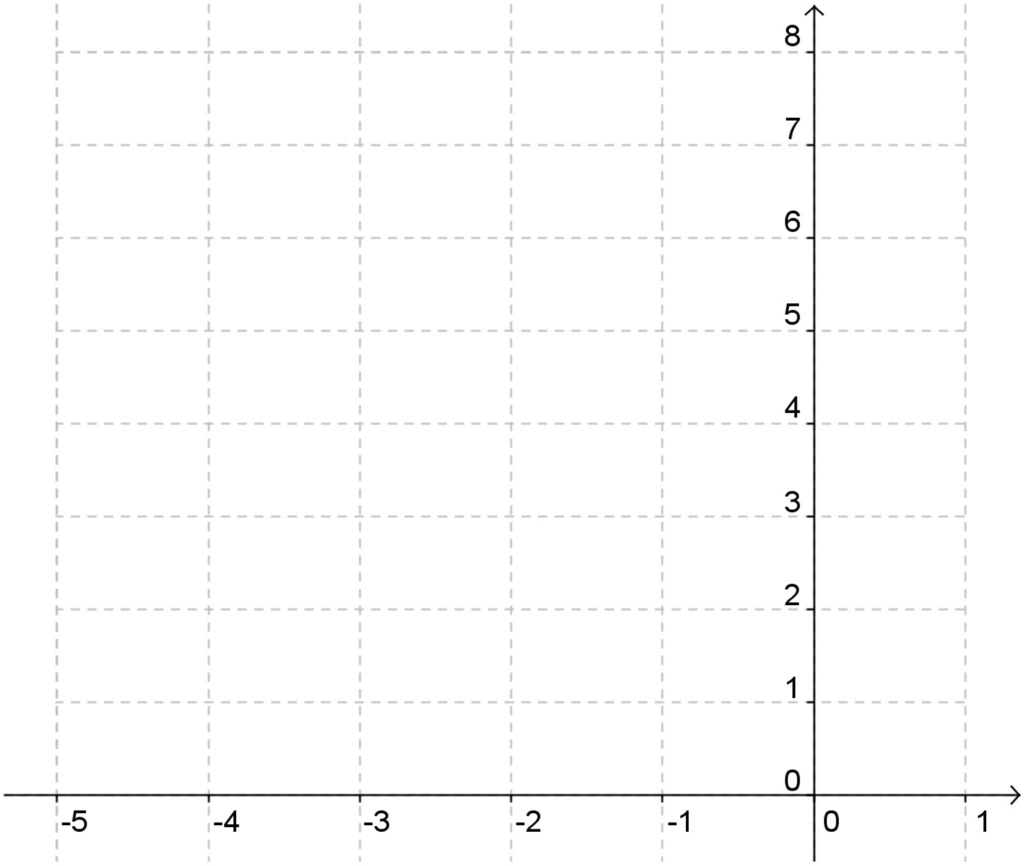
## 3 Plotting graphs of exponential functions

The Examiner can also ask you to plot exponential graphs for yourself.
To do this, you need to work out a set of points, plot them on a co-ordinate diagram and join them together with a smooth curve.

### Question 6

A function is defined as $f(x) = 4(2^x)$.

(i) On the graph below, plot the function f in the domain $-5 \leq x \leq 1$.




(ii) From your graph, estimate the value of $f(-1{\cdot}25)$.

(iii) From your graph, estimate the value of x for which $f(x) = 6$.

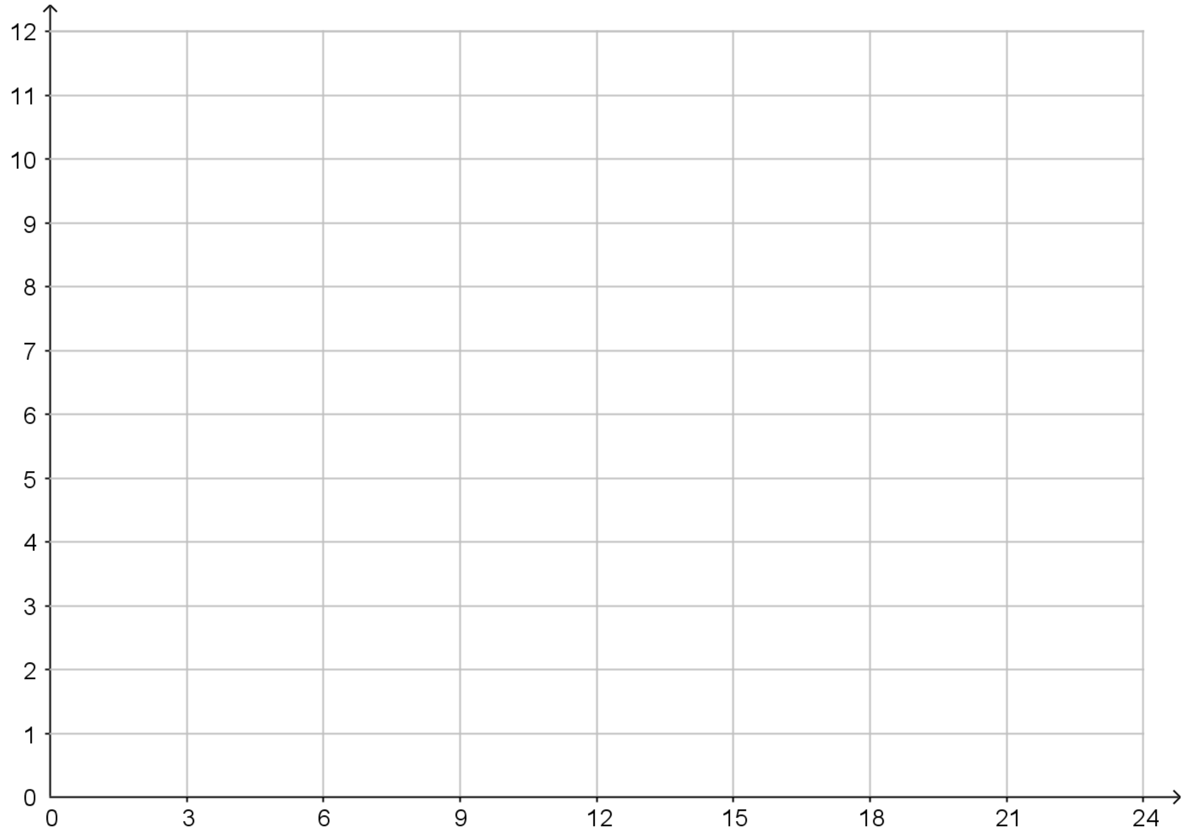
## Question 7

A function is defined as $f(x) = 12(0{\cdot}9^x)$.

(i) Complete the following table.

| x | 0 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 |
|---|---|---|---|---|---|---|---|---|---|
| f(x) | | | | | | | | | |

(ii) Hence, on the graph below plot the function f in the domain $0 \leq x \leq 24$.




(iii) Use your graph to estimate the value of f(17).

(iv) Confirm your answer by substituting into the function.

## 4 Practical questions involving exponential functions

### Question 8

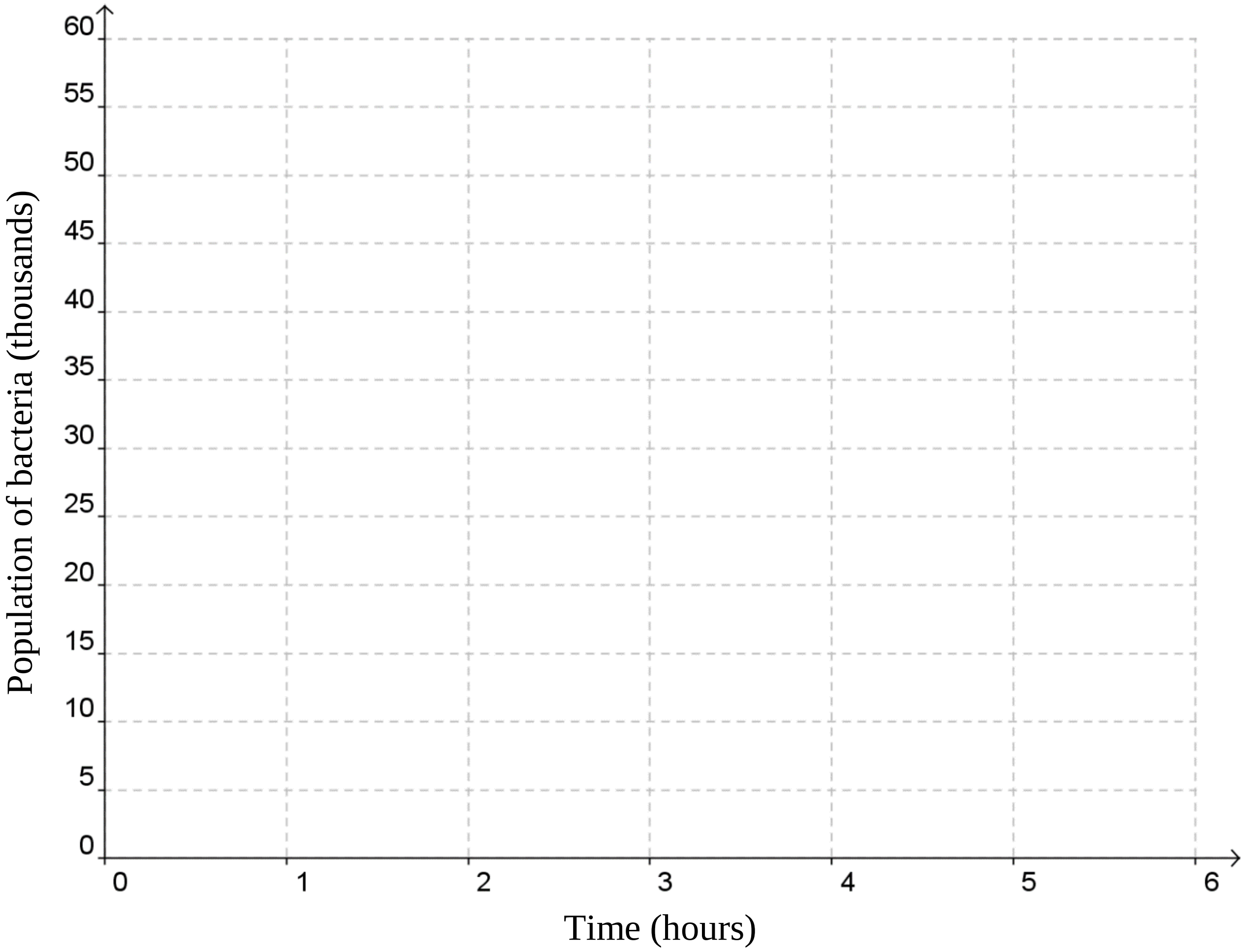
A scientist is growing bacteria in an incubator.
At the start of his experiment he places an initial population ($P_0$) of 15000 bacteria into the incubator.
As the bacteria are left to multiply in the incubator, the population after t hours ($P_t$) is given by the following formula.

$$P_t = P_0\left(2^{0 \cdot 3t}\right)$$

(i) Complete the following table.
Give your answers correct to one decimal place.

| t (hours) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $P_t$ (thousands of bacteria) | 15 | | | | | | |

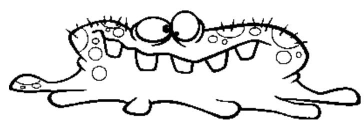
(ii) On the graph below, plot the change in population of bacteria over the first 6 hours.



(iii) What would the population of bacteria be after 9·2 hours?
Give your answer in thousands correct to one decimal place.

(iv) The "doubling time" is the time taken for the population of bacteria to double.
Calculate the doubling time for the bacteria in this scientist's experiment.
Give your answer in minutes.

(v) A second scientist is growing a different type of bacteria.
The population of these bacteria after t hours ($P_t$) is given by the following formula.



$$P_t = P_0\left(2^{0\cdot 6t}\right)$$

Assuming the starting populations were the same, how will the growth of the bacteria in the second scientist's experiment compare with the growth of the bacteria in the first scientist's experiment?

## Question 9

During a zombie outbreak, the number of people (Z) in thousands turned into zombies after t days can be represented by the following formula.

$$Z = A\left(3^{nt}\right) \text{ where A and n are constants}$$

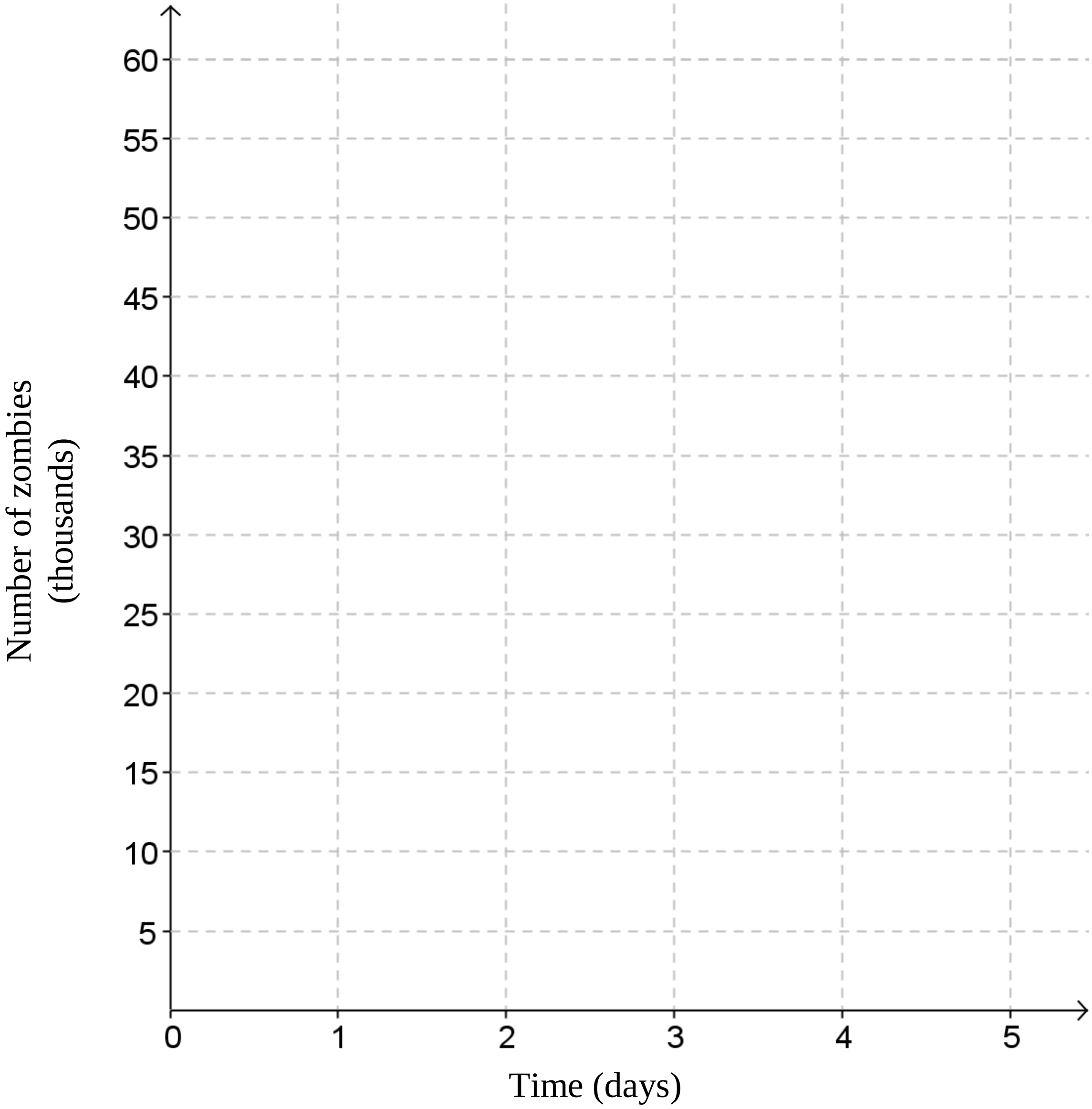
The number of people turned into zombies over the first three days of the outbreak is shown in the table below.

| Time (days) | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Number of zombies (thousands) | 14·000 | 18·425 | 24·249 | 31·913 |

(i) Use the information from the table to find the values of A and n. Give your value of n correct to two decimal places.

(ii) Hence, find the number of zombies after 4 and 5 days.

(iii) On the graph below, plot the increase in the number of zombies over the first 5 days.




(iv) The population of the island of Ireland is 6·398 million.
How long will it take for the entire population to be turned into zombies?
Give your answer correct to one decimal place.

## Question 10

Carbon has an unstable radioactive form (isotope), Carbon-14.
Carbon-14 decays exponentially slowly over time according to the following formula,

$$A = e^{-dt}$$

where A = the amount of Carbon-14 remaining in a sample (in decimal form),
t = the time elapsed in years,
d = the decay constant which is 0·000121.

(i) The half-life of Carbon-14 is the length of time it takes for the amount of the element in a sample to drop to half the original amount.
Calculate the half-life of Carbon-14.
Give your answer correct to the nearest year.

(ii) How long will it take the amount of Carbon-14 in a sample to drop to zero?

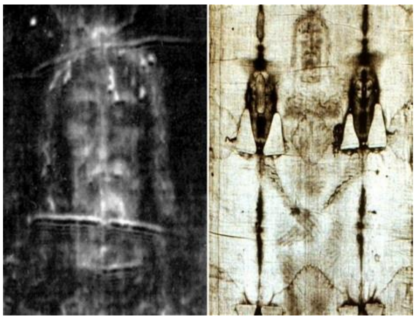
By finding the proportion of Carbon-14 remaining in a material or sample, the age of that sample can be estimated quite reliably. This is called radiocarbon dating.

(iii) The Dead Sea Scrolls were dated using this method in 1995.
The scrolls were found to contain 73·9% of their original Carbon-14 content.
Estimate the age of the scrolls.
Give your answer correct to the nearest year.

(iv) The Shroud of Turin is a linen cloth bearing the image of a man and is believed by some Christians to be the burial cloth of Jesus Christ.

Believers claim that the shroud is 2000 years old.

After years of discussion, the Catholic Church finally granted permission for the shroud to be dated using radiocarbon dating in 1988.

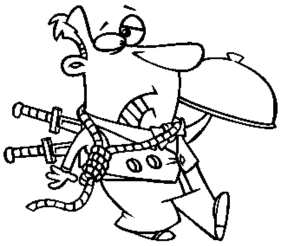


If the Shroud of Turin is genuinely 2000 years old, what percentage of Carbon-14 would have to remain in any sample of material taken from it?
Give your answer correct to one decimal place.

(v) Researchers found that material taken from the shroud actually contained 92·7% of its original amount of Carbon-14.
Give an estimate for the year in which the shroud was created.

## Question 11

In murder cases, crime scene analysis often involves estimating the time of death by determining the drop in temperature of the body.



A dead body cools according to Newton's Law of Cooling which is

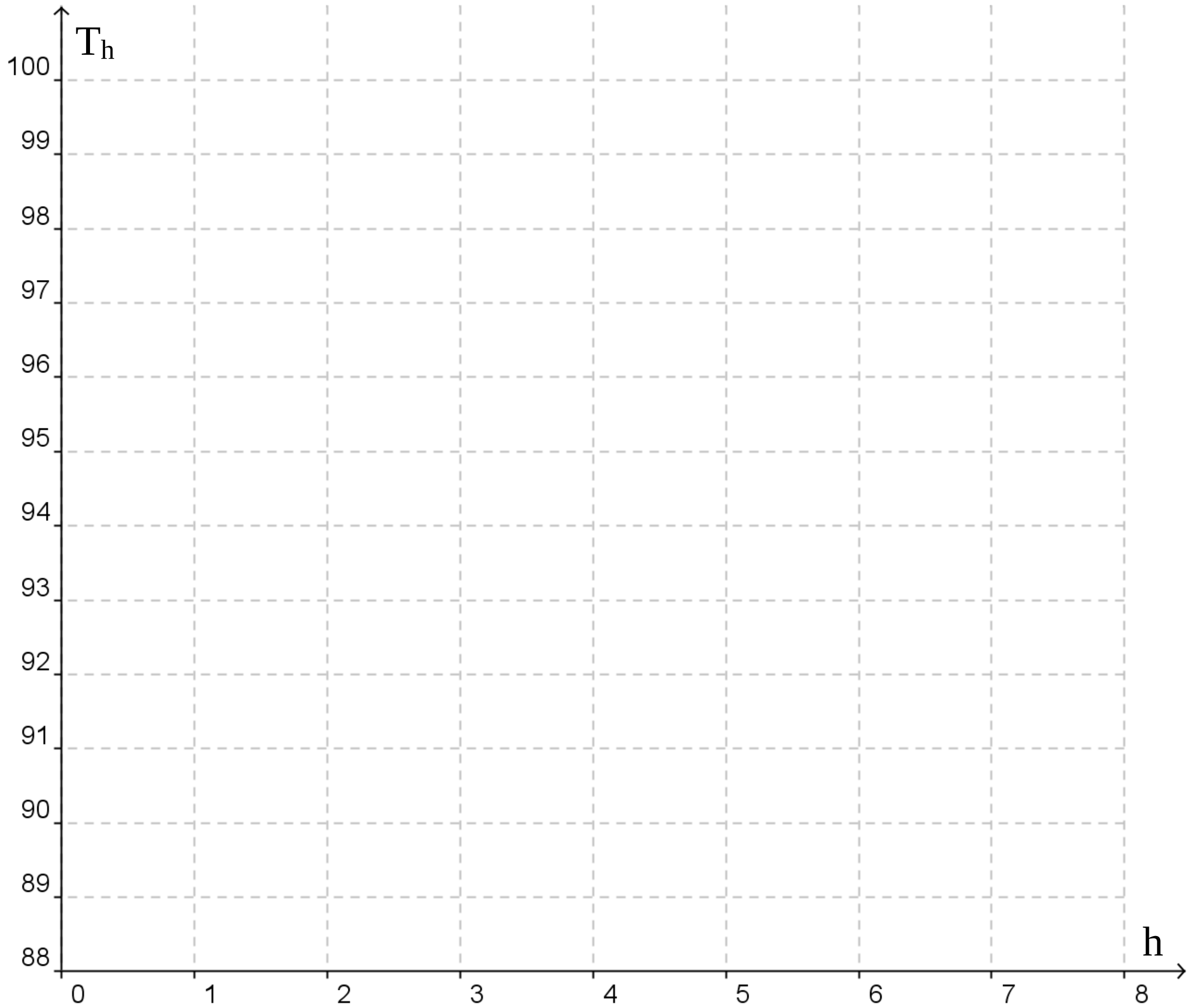
$$T_h = (T_0 - T_e)e^{-0{\cdot}05h} + T_e$$

where $T_h$ = the temperature of the body after h hours,
$T_0$ = the initial temperature of the body,
$T_e$ = the temperature of the surrounding environment,
h = the time in hours.

Part (a)

A body is left lying in a room in which the temperature is a constant 70°F.
Assume that the initial temperature of the body is normal body temperature, 98·6°F.

(i) On the graph below, plot the change in body temperature over the first 8 hours.



(ii) Use your graph to estimate the temperature of the body after 4·5 hours.

(iii) Use your graph to estimate how long the body has been dead if the body temperature is measured as 94°F.

Part (b)

(i) At 11:00, a coroner arrives at a crime scene where a murder has been committed. He measures the temperature of the body as 93·3°F and the temperature of the room as 72·5°F. Estimate the time at which the murder was committed.

(ii) The murderer had been very clever and had wrapped the body in an electric blanket. This blanket kept the temperature around the body at 85°F for the first 5 hours. Estimate the real time at which the murder was committed.

## Past Leaving Certificate Exam Question
## 2014 Higher Level Exam Paper 1 Question 9

Ciarán is preparing food for his baby and must use cooled boiled water.
The equation $y = Ae^{kt}$ describes how the boiled water cools.
In this equation:

- t is the time, in minutes, from when the water boiled,
- y is the difference between the water temperature and room temperature at time t, measured in degrees Celsius,
- A and k are constants.

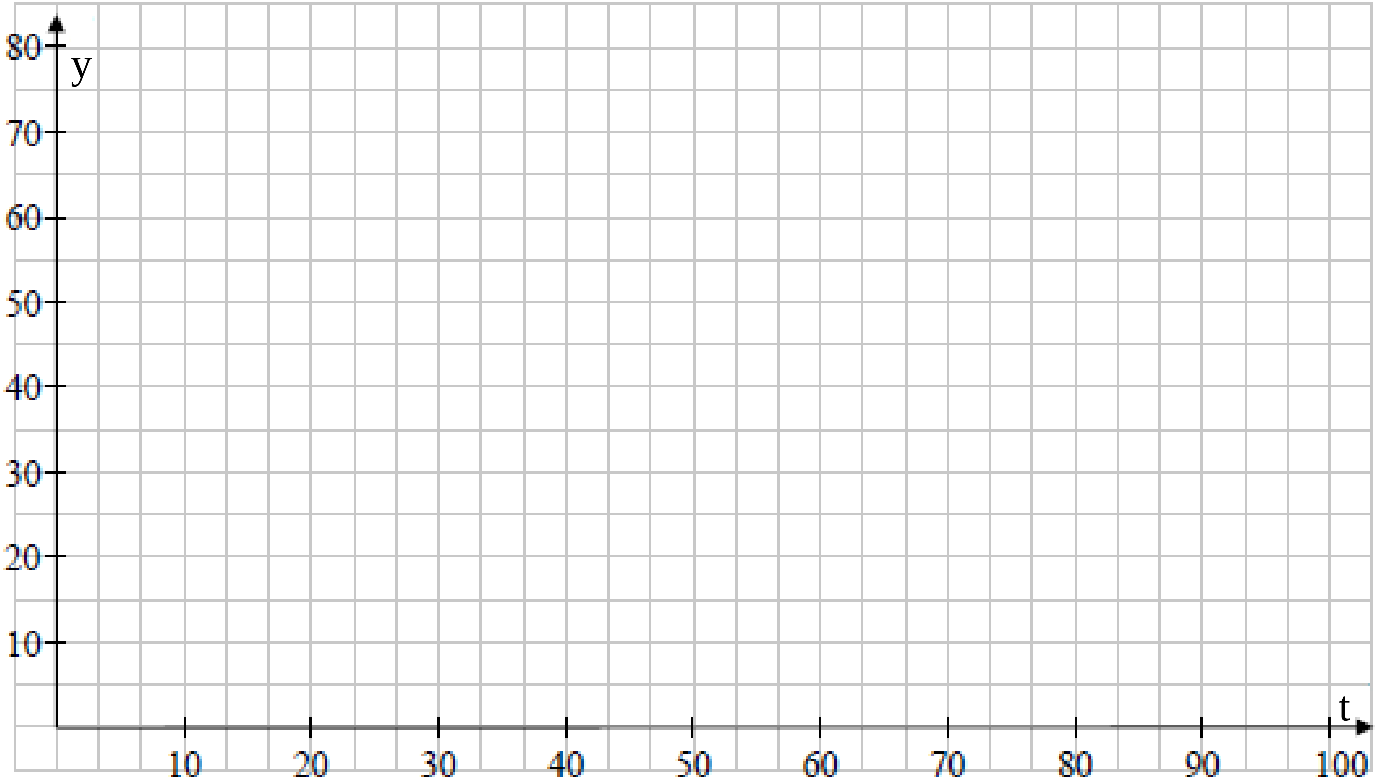
The temperature of the water when it boils is 100°C and the room temperature is a constant 23°C.

(a) Write down the value of the temperature difference, y, when the water boils, and find the value of A.

(b) After five minutes, the temperature of the water is 88°C.
Find the value of k, correct to three significant figures.

(c) Ciarán prepares the food for his baby when the water has cooled to 50°C.
How long does it take, correct to the nearest minute, for the water to cool to this temperature?

(d) Using your values for A and k, sketch the curve $f(t) = Ae^{kt}$ for $0 \leq t \leq 100$, $t \in \mathbb{R}$.



(e) (i) On the same diagram, sketch a curve $g(t) = Ae^{mt}$, showing the water cooling at a faster rate, where A is the value from part (a), and m is a constant.
Label each graph clearly.

(ii) Suggest one possible value for m for the sketch you have drawn and give a reason for your choice.