Quadratic Patterns
5th Year Ordinary Level

For the lesson on 8/12/2016
At Ardscoil Na Mara, 5th Year Ordinary Level class
Teacher: John Hartery
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1. **Title of the Lesson:** Finding the general term (T_n) of a quadratic pattern in many ways.

2. **Brief description of the lesson** A problem solving lesson involving triangular shapes, patterns and the general form of a quadratic ($an^2 + bn + c$)

3. **Aims of the Lesson:**

Long-Term Goals:
I'd like my students to become more creative when devising approaches and methods to solve problems.
I'd like to emphasise to students that a problem can have several equally valid methods of solution.
I'd like my students to connect and review number patterns in a different context.
I'd like my students to visualise patterns in a variety of ways.

Short-Term Goals:
I'd like my students to recognise a pattern as being either linear or quadratic.

I'd like my students to understand that finding patterns and developing algebra are both relevant techniques that can be applied to problems that initially may seem unconnected.

I'd like my students to be able to generate a formula for the triangular numbers, rectangular/square numbers using variables and to be able to check whether this formula is correct.

4. **Learning Outcomes:**

At the end of this lesson students should be able to find the General Term of a Quadratic Pattern using more than one method.

5. **Background and Rationale**

The group chose this topic and specifically parts of the 2015 ordinary level paper 1, question 7 to attempt to work through different ways to approach this question with the class.
This is a challenging question for this level and the group wanted to think carefully about the prior knowledge needed to tackle it and the different ways that students could approach it.

6. **Research**

We considered and used a lot of physical resources including shapes, stack cubes, tangrams, drawing squares and parabolas.We consulted textbooks for ideas on different approaches and used the website http://www.visualpatterns.org/as a source of pictorial patterns

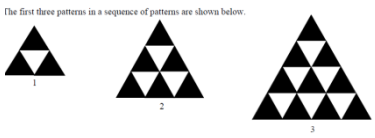
7. About the Unit and the Lesson

This research lesson is designed as a type two problem i.e. a problem that is clearly related to recent work. It is designed to come after approximately 2 weeks investigating patterns.

8. Flow of the Unit:

Lesson	To explore a pattern in a variety of ways leading to a general form of pattern and an expression in algebra	**# of lesson periods**
1	Linear patterns	5
2	Quadratic Patterns	5
3	Exponential	2
4	Revision of above	1

9. Flow of the Lesson

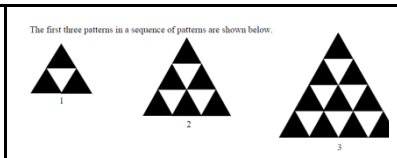
Teaching Activity	Points of Consideration
1. Introduction Very quick check to make sure all students know what a general term is.	Linear, Arithmetic Quadratic, Exponential Shapes of Patterns
2. Posing the Task The class will be given a problem worksheet and a placemat page to present their solutions. Allow 12 minutes for individual student work on the problem.  Draw the 4th pattern Complete table Come up with as many ways as possible of forming the General Term	Individual work followed by group discussion in groups of 3
3. Anticipated Student Responses R1: Geometric shape (next shape(s)) R2: Table R3: Sequence (4, 9, 16, ….) R4: Graph R5: Expression in words (add one, then square) R6: Expression in Algebra $((n+1)^2)$ R7: Formula $(n^2 + 2n + 1)$	Observe student work and offer individual suggestions Can you see a pattern? How can this pattern help you find an answer? How did you find that out? Why do you think that? Who has the same answer as this? Who has a different solution? Are everybody's results the same?
4. Comparing and Discussing We would be prepared to go through responses R1 to R7. Working towards presenting R7 as the "most sophisticated " and transferable approach.	Showcase responses and methods of problem solving in order of sophistication. Students called to board to demonstrate their work and take questions from class & teacher. Can anyone think of another way?
5. Summing up Restate problem and recap on solutions.	Numbers of ways and responses. Preferred method.

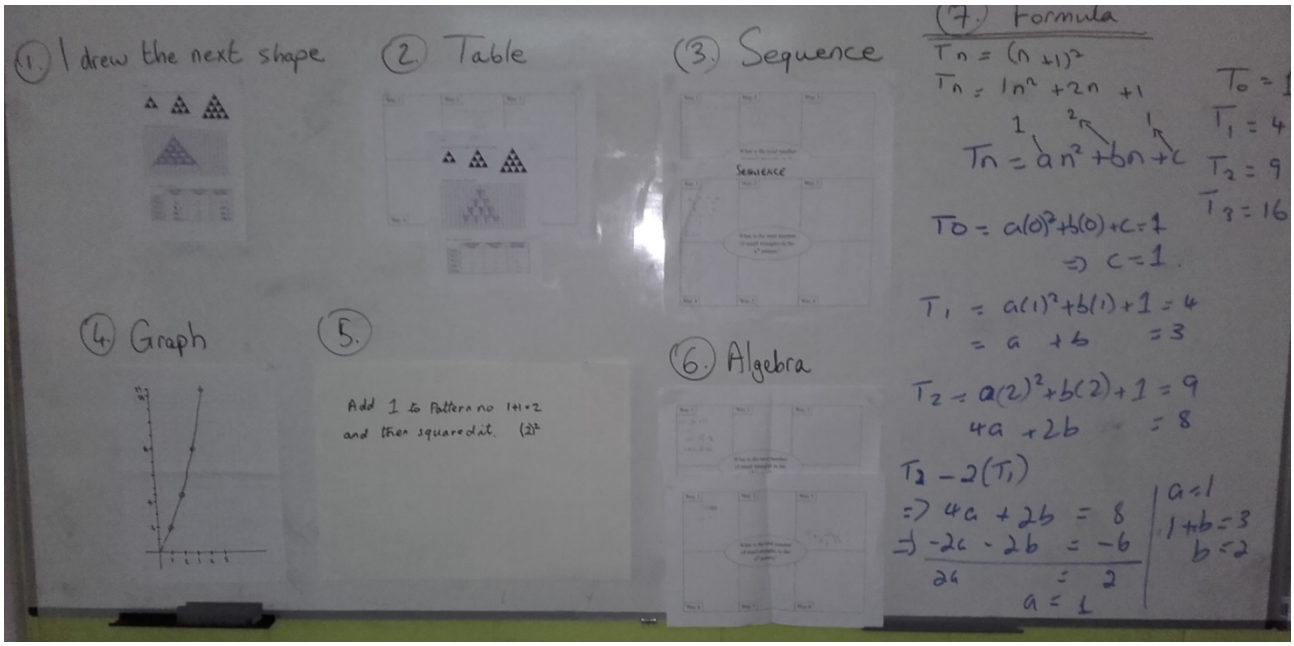
Homework	Homework of quadratic.

10. Evaluation

There will be 3 observers monitoring 6 students each. Each observer will use a form to record observations.
Student's work will be collected at the end of the lesson.

11. Board Plan

Word box Keywords		R1 | R2 | R3 | R7 R4 | R5 | R6



12. Post-lesson reflection

What are the major patterns and tendencies in the evidence? Discuss
The timing of the lesson worked out very well and students had ample time to complete the task.
The lesson took place during first period in the morning and students were slow to arrive to class and were slow initially to engage in the lesson. They were shy and maybe a little intimidated by the extra teachers in the room.

The first few questions on the work sheet were attempted very well which was expected but many students struggled to correctly identify the general term of the quadratic pattern.
Some students made very basic numeracy errors like miscounting the triangles therefore making it impossible to generalise the pattern.

What are the key observations or representative examples of student learning and thinking?

Students were very capable of drawing the patterns. Visualising the pattern did greatly help them. Some students tended to persist with the question with more determination than others and we saw this persistence as crucial in successfully solving the problem.

What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?

As expected the majority of students had difficulty with generalising a quadratic pattern. Most could not come up with a general formula by observation and trial and error. It can be a difficult concept for ordinary level students.

In what ways did students achieve or not achieve the learning goals?

It was felt that goals were only partly met by some students, and that this remains a challenging topic for students at this level. During the research lesson the teacher (in discussion with students) effectively demonstrated ways to find the general term for this pattern. The teacher reported further progress with this class group since the research lesson.

Based on your analysis, how would you change or revise the lesson?

- Put the table and the questions on the one side of the paper so students can see both simultaneously. (see Appendix B)
- Depending on the group: possibly ask students to work in pairs.
- Give students the frame of a table with just the two headings relevant in finding the general term. One heading would be **'Term Number'** and the other would be **'Total Number of Triangles'.**(see Appendix B)
- Ask students to find an earlier term than the 100^{th} (maybe the 10^{th} or 15^{th}) and then pose the question how would we find the 100th. They may have then been able to generalise then.
- The time of the lesson didn't lend itself to interaction with the students (first class in the morning!!!). The lesson may have been more successful if it was later in the day.

What are the implications for teaching in your field?

This remains a challenging topic for ordinary level students and there is only so much time that can be devoted to the topic. Working through a variety of ways of approaching such a problem in one lesson is a time efficient and effective way of assessing the class and teaching at the same time. One teacher in the group commented that she had started using this teaching style in her maths classes and that herself and her students are enjoying using the method.

Appendix A:2-sided problem worksheet given to students for the research lesson.

The first three patterns in a sequence of patterns are shown below.

1

2

3

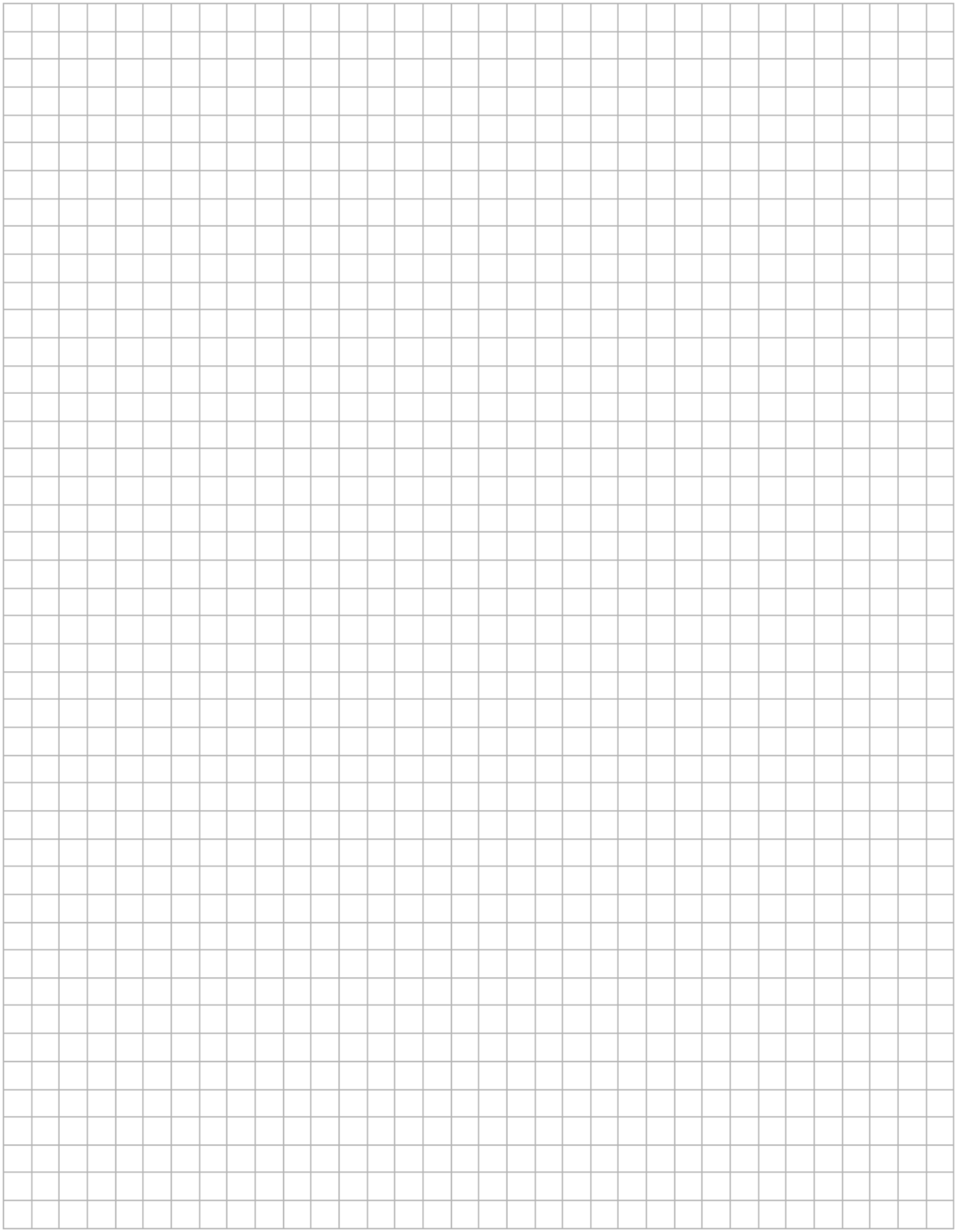
(a) Draw the fourth pattern in the sequence.

(b) Complete the table below.

	Number of Black Triangles	Number of White Triangles	Total Number of Small Triangles
Pattern 1	3	1	$T_1 = 4$
Pattern 2			$T_2 =$
Pattern 3			$T_3 =$
Pattern 4			$T_4 =$
Pattern 5			$T_5 =$

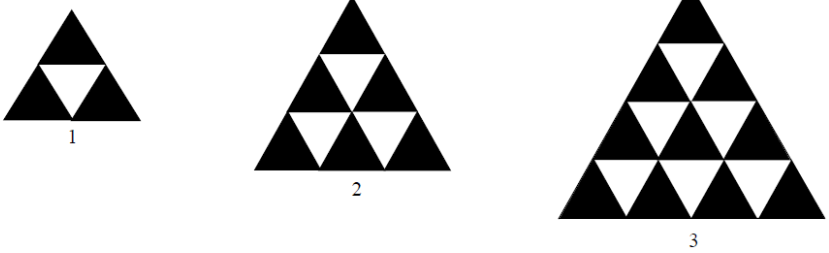
How many small triangles would there be in pattern 100?

Write an expression in n for the total number of triangles in the n^{th} pattern.



Appendix B: suggested problem worksheet to be given to students in future.

The first three patterns in a sequence of patterns are shown below.



1

2

3

1. Draw the fourth pattern in the sequence

2. Complete the table below

	Total number of small triangles
Pattern 1	4
Pattern 2	
Pattern 3	
Pattern 4	
Pattern 5	

3. How many small triangles in total are in the 9^{th} pattern?

3. How many small triangles in total are in the 100^{th} pattern?

4. Write an expression for the total number of triangles in the n^{th} pattern.