6.1 Solving Equations by Using Inverse Operations

Inverse Operations "undo" or reverse each others results.

Examples:

$+$ and $-$ are inverse operations.

$\times$ and $\div$ are inverse operations.

$\sqrt{x}$ and x^2 are inverse operations.

Example 1: Writing Then Solving One-Step Equations

For each statement below, write then solve an equation to determine each number. Verify the solution.

a) Three times a number is 12.

$$3x = 12$$

$$x = 4$$

b) A number divided by 4 is 8.

$$\frac{x}{4} = 8 \qquad x = 32$$

c) A number plus 7 is 15.

$$x + 7 = 15$$

$$x = 8$$

Example 2: Solving One-Step Equations.

a) $m+7=2$

$m = 2-7$

$m = -5$

b) $-4x=20$

$\frac{-4x}{-4} = \frac{20}{-4}$

$x = -5$

c) $6-y=-2$

$-y = -2-6$

$-y = -8$

$y = 8$

d) $7x=-21$

$\frac{7x}{7} = \frac{-21}{7}$

$x = -3$

e) $\frac{x}{2}=9$

$\cancel{2}\cdot\frac{x}{\cancel{2}} = 9\cdot 2$

$x = 18$

f) $\frac{x}{-3}=2$

$\cancel{-3}\cdot\frac{x}{\cancel{-3}} = 2\cdot -3$

$x = -6$

Example 3: Solving a Two-Step Equation

a) $2x+1=7$

$2x = 7-1$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$

b) $-3x-2=-11$

$-3x = -11+2$

$\frac{-3x}{-3} = \frac{-9}{-3}$

$x = 3$

c) $5m+2m=7+2$

$\frac{7m}{7} = \frac{9}{7}$

$m = \frac{9}{7}$

d) $\frac{w}{2}-1=11$

$\frac{w}{2} = 11+1$

$2\cdot\frac{w}{2} = 12\cdot 2$

$w = 24$

e) $-4x-6=12$

$-4x = 12+6$

$\frac{-4x}{-4} = \frac{18}{-4}$

$x = -\frac{18}{4}$ or $-\frac{9}{2}$

f) $1 = 4 + \frac{n}{3}$

$1-4 = \frac{n}{3}$

$3 \cdot -3 = \frac{n}{\cancel{3}} \cdot \cancel{3}$

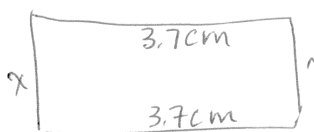
$-9 = n$ or $n = -9$

Example 4: Using an Equation to Model and Solve a Problem

A rectangle has length 3.7 cm and perimeter 13.2 cm.

a) Write an equation that can be used to determine the width of the rectangle.
b) Solve and verify the equation.

a)



$P = 13.2\text{cm}$

let x = width

$P = x + x + 3.7 + 3.7$

$P = 2x + 7.4$

$13.2 = 2x + 7.4$

b) $2x + 7.4 = 13.2$

$2x = 13.2 - 7.4$

$\frac{2x}{2} = \frac{5.8}{2}$

$x = 2.9$

Verify $2.9 + 2.9 + 3.7 + 3.7 =$
$= 13.2$

Example 5: Using an Equation to Solve a Percent Problem

a) 7% of a number is 56.7. Find the number.

$7\% \, x = 56.7$

$\frac{0.07x}{.07} = \frac{56.7}{.07}$

$x = 810$

b) 4 ½% of a number is 80. Find the number.

$4.5\% \, x = 80$

$\frac{0.045x}{.045} = \frac{80}{.045}$

$x = 1777.\overline{7}$

6.2 Solving Equations by Using Balance Strategies

Investigate: Try the Balance Puzzle.

When solving linear equations, think of the equation as a balance, with the = sign as the middle or fulcrum.

When an operation is done to one side of an equation, it must be done to the other in order to keep balance.

The idea is to isolate variables on one side and constants on the other side.

Example 1: Modelling Equations with Variables on Both Sides

a) $6x+2=10+4x$

$6x-4x=10-2$

$2x=8$

$x=4$

b) $7x+1=2x-6$

$7x-2x=-6-1$

$5x=--7$

$x=\frac{-7}{5}$

c) $-3x+1=-4x+4$

$-3x+4x=4-1$

$x=3$

d) $8c+2c=3c+9$

$10c=3c+9$

$10c-3c=9$

$7c=9$

$c=9/7$

e) $-8m+7=-6m+3$

$-8m+6m=3-7$

$-2m=-4$

$m=2$

f) $-8y+2=8y+3$

$-8y-8y=3-2$

$-16y=1$

$y=\frac{-1}{16}$

Use the distributive property to remove brackets and then solve.

g) $2(x+1)=7$

$2x+2=7$

$2x=7-2$

$2x=5$

$x=5/2$

h) $-3(x+1)=2(x-4)$

$-3x-3=2x-8$

$-3x-2x=-8+3$

$-5x=-5$

$x=1$

i) $4(x+2)+2(x+1)=7$

$4x+8+2x+2=7$

$6x+10=7$

$6x=7-10$

$6x=-3$

$x=\frac{-3}{6}=\frac{-1}{2}$

j) $5(c+3)-(c+4)=1$

$5c+15-c-4=1$

$4c+11=1$

$4c=-10$

$c=\frac{-10}{4}=\frac{-5}{2}$

k) $3(m-3)-2(m+4)=6$

$3m-9-2m-8=6$

$m-17=6$

$m=6+17$

$m=23$

Example 2: Solving Equations with Rational Coefficients (Fractions)

To get rid of a fraction, multiply each term by the common denominator.

a) $\frac{2x}{3}=\frac{4x}{5}+7$

LCD = 15

$(15)\frac{2x}{3}=(15)\frac{4x}{5}+(15)7$

$10x=12x+105$

$10x-12x=105$

$-2x=105$

$\boxed{x=\frac{-105}{2}}$

b) $\frac{x}{2}-\frac{3x}{4}=1$

LCD = 4

$(4)\frac{x}{2}-(4)\frac{3x}{4}=(4)1$

$2x-3x=4$

$-1x=4$

$\boxed{x=-4}$

c) $\frac{2x+1}{4}+\frac{3x-2}{3}=2$

LCD = 12

$\frac{12(2x+1)}{4}+\frac{12(3x-2)}{3}=12(2)$

$3(2x+1)+4(3x-2)=24$

$6x+3+12x-8=24$

$18x-5=24$

$18x=24+5$

$18x=29$

$\boxed{x=\frac{29}{18}}$

d) $\frac{(x-4)}{7}=\frac{(2x+1)}{3}$

LCD = 21

$\frac{21(x-4)}{7}=\frac{21(2x+1)}{3}$

$3(x-4)=7(2x+1)$

$3x-12=14x+7$

$3x-14x=7+12$

$-11x=19$

$\boxed{x=\frac{-19}{11}}$

Example 3: Using an Equation to Model and Solve a Problem

A cell phone company offers two plans.

Plan A: 120 free minutes, \$0.75 per additional minute
Plan B: 30 free minutes, \$0.25 per additional minute

Which time for calls will result in the same cost for both plans?

Model the problem with an equation and solve the problem. let x = time.

A : $0.75(x - 120)$

B : $0.25(x - 30)$

$$0.75(x-120) = 0.25(x-30)$$

$$0.75x - 90 = 0.25x - 7.5$$

$$0.75x - 0.25x = -7.5 + 90$$

$$0.5x = 82.5$$

$$x = 165 \text{ min.}$$

6.3 Introduction to Linear Inequalities

Recall the following symbols:

= equal to

< less than

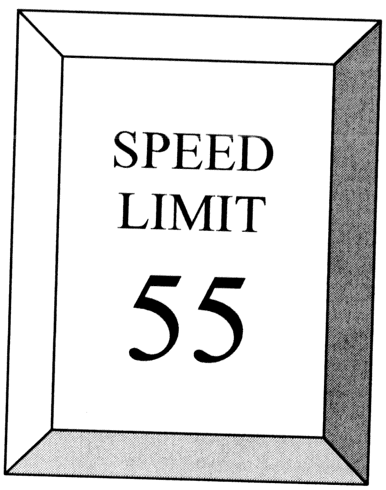
> greater than

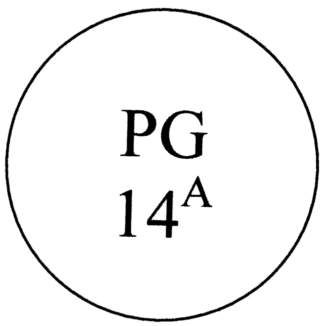
≤ less than OR equal to

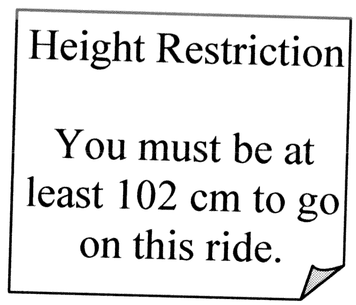
≥ greater than OR equal to.

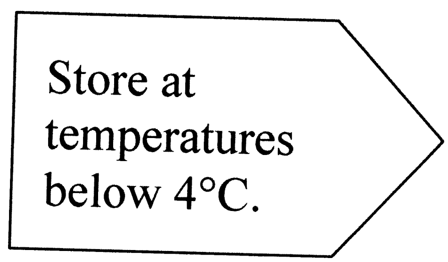
Investigate:

Many real-world situations can be modeled by inequalities.









Write an inequality for each situation.

Speed limit: $s \leq 55$

PG movie: $a \geq 14$

Height restriction: $h \geq 102$

Temperature: $t < 4$

Example 1: Writing an Inequality to Describe a Situation

Define a variable and write an inequality to describe each situation.

a) Contest entrants must be at least 18 years old. $a \geq 18$

b) The temperature has been below -5°C for the last week. $t < -5$

c) You must have 7 items or less to use the express checkout line. $i \leq 7$

d) Scientists have identified over 400 species of dinosaurs. $s > 400$

We use an inequality to describe a range of numbers instead of a single number.

A linear equation is true for only one value of the variable.

A linear inequality may be true for many values of the variable. There are usually too many values to list, so we show them on a number line.

Example 2: Determining Whether a Number Is a Solution of an Inequality

a) Consider $x \geq -4$. Which number is part of the solution?

-8 ✗
-3.5 ✓
-4 ✓
-4.5 ✗
0 ✓

b) Consider $x < 9$. Which number is part of the solution?

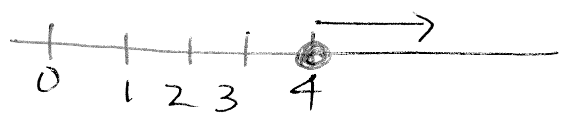
17 ✗
0 ✓
-2 ✓
9 ✗
12 ✗

When graphing inequalities on a number line, a closed circle (shaded) or dot will indicate the inclusion of that point and an open circle (non-shaded) or hole will indicate the non-inclusion of that point.

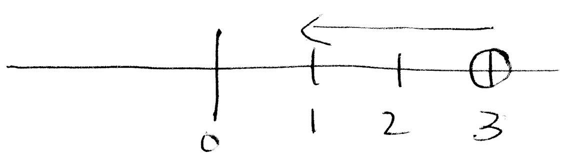
$\geq, \leq$ means	●
$>, <$ means	○

Example 3: Graphing Inequalities on a Number Line

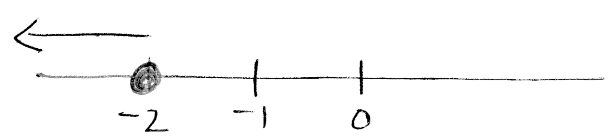
a) $x \geq 4$



b) $n < 3$

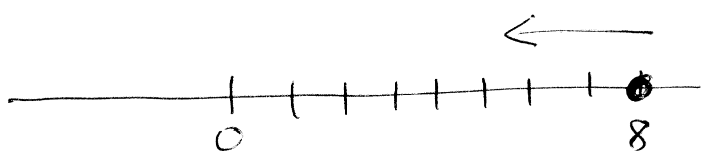


c) $x \leq -2$



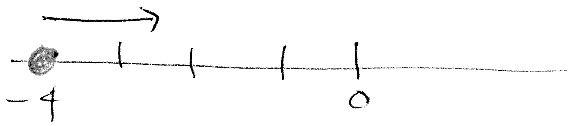
d) $8 \geq x$

$x \leq 8$

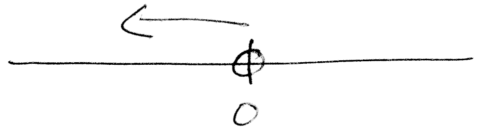


e) $-4 \le g$

$g \ge -4$

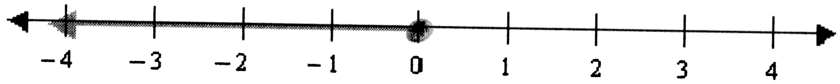


f) $x < 0$



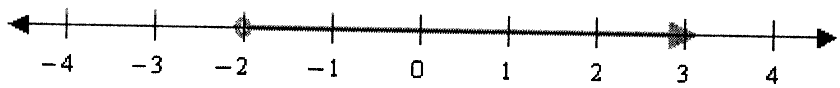
Now give the inequality for each number line below.

a)



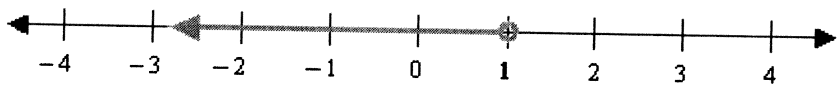
$x \le 0$

b)



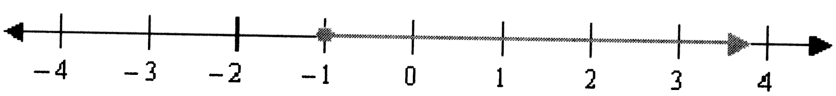
$x > -2$

c)



$x < 1$

d)



$x \ge -1$

6.4 Solving Linear Inequalities by Using Addition and Subtraction

Solving an Inequality

When the same number is added to or subtracted from each side of an inequality, the resulting inequality is still true.

For Example:

$$3 < 7$$

$$3+1 < 7+1 \qquad 4 < 8 \quad \checkmark \text{ still true!}$$

or

$$3-6 < 7-6 \qquad -3 < 1 \quad \checkmark \text{ still true!}$$

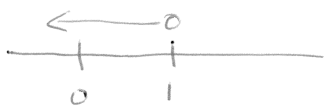
Example 1: Solving and Inequality

a) $2x+1<3$

$$2x < 3-1$$

$$\frac{2x}{2} < \frac{2}{2}$$

$$x < 1$$



b) $6x-7 \geq 8$

$$6x \geq 8+7$$

$$\frac{6x}{6} \geq \frac{15}{6}$$

$$x \geq \frac{15}{6} \quad \text{or} \quad x \geq \frac{5}{2} \quad \text{or } x \geq 2.5$$

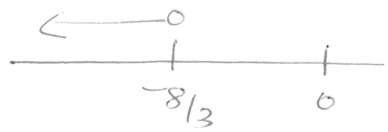
c) $3x+1>9$

$$3x > 9-1$$

$$\frac{3x}{3} > \frac{8}{3}$$

$$x > 8/3$$

d) $5x+1<2x-7$

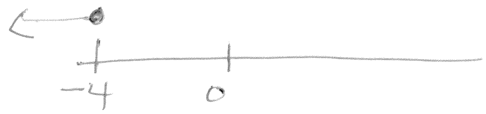


$5x-2x < -7-1$

$\frac{3x}{3} < \frac{-8}{3}$

$x < -8/3$.

e) $4x+2 \le x-10$



$4x - x \le -10 - 2$

$\frac{3x}{3} \le \frac{-12}{3}$

$x \le -4$

Example 2: Using an Inequality to Model and Solve a Problem

Jake plans to board his dog while he is away on vacation. Boarding House A charges $90 plus $5 per day. Boarding House B charges $100 plus $4 per day. For how many days must Jake board his dog for Boarding House A to be **less** expensive than Boarding House B?

a) Choose a variable and write an inequality. let x = no. of days.
b) Solve the problem.
c) Graph on a number line.

a) A: $90+5x$ A less than B
B: $100+4x$

$90+5x < 100+4x$

$90-100 < 4x-5x$

$-10 < -1x$

$\frac{}{-1}$ flip $\frac{}{-1}$

$10 > x$

not till next lesson

OR

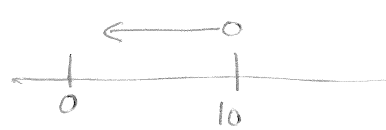
$90+5x < 100+4x$

$5x-4x < 100-90$

$1x < 10$

$x < 10$.

Jake must board his dog for less than 10 days.

c) 

6.5 Solving Linear Inequalities by Using Multiplication and Division

Investigate:

$8 > 3$ Now multiply both sides by 2.

$\underline{\ 16\ } > \underline{\ 6\ }$ This is still true.

$8 > 3$ Now multiply both sides by -2.

$\underline{\ -16\ } > \underline{\ -6\ }$ This is not true.

To overcome this, follow the following properties of inequalities:

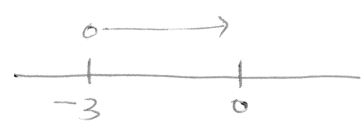
- When each side of an inequality is multiplied or divided by the same positive number, the resulting inequality is still true.
- When each side of the inequality is multiplied or divided by the same negative number, the inequality sign must be **reversed** for the inequality to remain true.

So from above, $\underline{\ -16\ } < \underline{\ -6\ }$ This is now true.

Example 1: Solving a One-Step Inequality

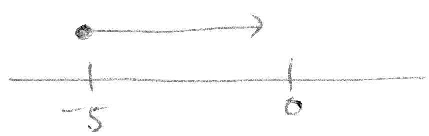
a) $\frac{7a}{7} > \frac{-21}{7}$

$a > -3$



b) $\frac{-5s}{-5} \leq \frac{25}{-5}$ flip when ÷ by "–"

$s \geq -5$



c) $\dfrac{y}{-4} > -3$

$\cancel{-4} \cdot \dfrac{y}{\cancel{-4}} > -3 \cdot -4$

$y < 12$

flip when x by "−"

d) $\dfrac{k}{3} \geq -2$

$\cancel{3} \cdot \dfrac{k}{\cancel{3}} \geq -2 \cdot 3$

$k \geq -6$

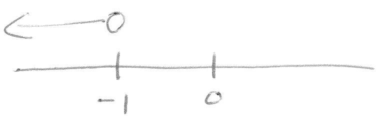
Example 2: Solving a Multi-Step Inequality

a) $3x + 1 > 6x + 4$

$3x - 6x > 4 - 1$

$\dfrac{-3x}{-3} > \dfrac{3}{-3}$ flip!

$x < -1$

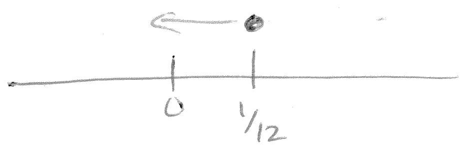


b) $-5x + 2 \geq 7x + 1$

$-5x - 7x \geq 1 - 2$

$\dfrac{-12x}{-12} \geq \dfrac{-1}{-12}$ flip!

$x \leq 1/12$



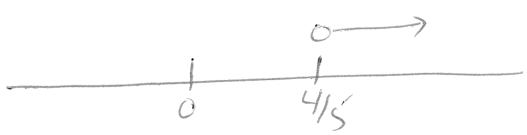
c) $-2(x+1) < 3(x-2)$ distribute first!

$-2x - 2 < 3x - 6$

$-2x - 3x < -6 + 2$

$\dfrac{-5x}{-5} < \dfrac{-4}{-5}$ flip!

$x > 4/5$



Example 3: Using an Inequality to Model and Solve a Problem

A super-slide charges \$1.25 to rent a mat and \$0.75 per ride. Haru has \$10.25. How many rides can she go on?

a) Choose a variable, then write an inequality to solve this problem.
b) Solve the problem.
c) Graph the solution.

a) let n = number of rides.

$$1.25 + 0.75n \le 10.25$$

b)
$$0.75n \le 10.25 - 1.25$$

$$\frac{0.75n}{.75} \le \frac{9}{.75}$$

$$n \le 12$$

She can go on 12 rides or less.

c)