6.1 Evaluate *n*th Roots and Use Rational Exponents

Goal • Evaluate *n*th roots and study rational exponents.

Your Notes

VOCABULARY

*n*th root of *a*

Index of a radical

REAL *n*th ROOTS OF *a*

Let n be an integer ($n > 1$) and let a be a real number.

If *n* is an even integer:

- $a < 0$ No real *n*th roots.
- $a = 0$ One real *n*th root: $\sqrt[n]{0} =$ ___
- $a > 0$ Two real *n*th roots: $\pm\sqrt[n]{a} =$ ______

If *n* is an odd integer:

- $a < 0$ One real *n*th root: $\sqrt[n]{a} =$ _____
- $a = 0$ One real *n*th root: $\sqrt[n]{0} =$ ___
- $a > 0$ One real *n*th root: $\sqrt[n]{a} =$ _____

Example 1 ***Find nth roots***

Find the indicated real *n*th root(s) of *a*.

a. $n = 3, a = -64$

b. $n = 6, a = 729$

Solution

a. Because $n = 3$ is odd and $a = -64$ ___ 0, -64 has ____________________. Because $(\_\_\_\_)^3 = -64$, you can write $\sqrt[3]{-64} =$ _____ or $(-64)^{1/3} =$ _____.

b. Because $n = 6$ is even and $a = 729$ ___ 0, 729 has ____________________. Because $\_\_\_^6 = 729$ and $(\_\_\_\_)^6 = 729$, you can write $\pm\sqrt[6]{729} =$ _____ or $\pm 729^{1/6} =$ _____.

6.1 Evaluate *n*th Roots and Use Rational Exponents

Goal • Evaluate *n*th roots and study rational exponents.

Your Notes

VOCABULARY

*n*th root of *a* For an integer *n* greater than 1, if $b^n = a$, then *b* is an *n*th root of *a*.

Index of a radical An *n*th root of *a* is written as $\sqrt[n]{a}$, where *n* is the index of the radical.

REAL *n*th ROOTS OF *a*

Let *n* be an integer ($n > 1$) and let *a* be a real number.

If *n* is an even integer:

- $a < 0$ No real *n*th roots.
- $a = 0$ One real *n*th root: $\sqrt[n]{0} = \underline{0}$
- $a > 0$ Two real *n*th roots: $\pm\sqrt[n]{a} = \underline{\pm a^{1/n}}$

If *n* is an odd integer:

- $a < 0$ One real *n*th root: $\sqrt[n]{a} = \underline{a^{1/n}}$
- $a = 0$ One real *n*th root: $\sqrt[n]{0} = \underline{0}$
- $a > 0$ One real *n*th root: $\sqrt[n]{a} = \underline{a^{1/n}}$

Example 1 ***Find nth roots***

Find the indicated real *n*th root(s) of *a*.

a. $n = 3, a = -64$ **b.** $n = 6, a = 729$

Solution

a. Because $n = 3$ is odd and $a = -64$ $\underline{<}$ 0, −64 has _one real cube root_. Because $(\underline{-4})^3 = -64$, you can write $\sqrt[3]{-64} = \underline{-4}$ or $(-64)^{1/3} = \underline{-4}$.

b. Because $n = 6$ is even and $a = 729$ $\underline{>}$ 0, 729 has _two real sixth roots_. Because $\underline{3}^6 = 729$ and $(\underline{-3})^6 = 729$, you can write $\pm\sqrt[6]{729} = \underline{\pm 3}$ or $\pm 729^{1/6} = \underline{\pm 3}$.

Your Notes

Checkpoint Find the indicated real nth roots of a.

1. $n = 4, a = 256$	**2.** $n = 3, a = 512$

RATIONAL EXPONENTS

Let a be a real number, and let m and n be positive integers with $n > 1$.

$a^{m/n} = (a^{1/n})^m = (\_\_\_\_)^m$

and $a^{m/n} = (a^m)^{1/n} = (\sqrt[n]{\_\_\_\_})$

$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(a^{1/n})^m} = \frac{1}{(\_\_\_\_)^m}, a \neq 0$

Example 2 *Evaluate an expression with rational exponents*

Evaluate $8^{-4/3}$.

Solution

Rational Exponent Form	Radical Form
$8^{-4/3} =$ ____	$8^{-4/3} =$ ____
$=$ ____	$=$ ____
$=$ ____	$=$ ____
$=$ ____	$=$ ____

Example 3 *Solve equations using nth roots*

a. $2x^6 = 1458$

$x^6 =$ ____

$x =$ ____

$x =$ ____

b. $(x + 4)^3 = 12$

$x + 4 =$ ____

$x =$ ____

$x \approx$ ____

Your Notes

Checkpoint Find the indicated real *n*th roots of *a*.

1. $n = 4$, $a = 256$ ± 4	**2.** $n = 3$, $a = 512$ 8

RATIONAL EXPONENTS

Let a be a real number, and let m and n be positive integers with $n > 1$.

$a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$

and $a^{m/n} = (a^m)^{1/n} = (\sqrt[n]{a^m})$

$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(a^{1/n})^m} = \frac{1}{(\sqrt[n]{a})^m}$, $a \neq 0$

Example 2 *Evaluate an expression with rational exponents*

Evaluate $8^{-4/3}$.

Solution

Rational Exponent Form

$$8^{-4/3} = \frac{1}{8^{4/3}} = \frac{1}{(8^{1/3})^4} = \frac{1}{2^4} = \frac{1}{16}$$

Radical Form

$$8^{-4/3} = \frac{1}{8^{4/3}} = \frac{1}{(\sqrt[3]{8})^4} = \frac{1}{2^4} = \frac{1}{16}$$

Example 3 *Solve equations using nth roots*

a. $2x^6 = 1458$

$x^6 = 729$

$x = \pm\sqrt[6]{729}$

$x = \pm 3$

b. $(x + 4)^3 = 12$

$x + 4 = \sqrt[3]{12}$

$x = \sqrt[3]{12} - 4$

$x \approx -1.71$

Your Notes

Example 4 ***Use nth roots in problem solving***

Animal Population The population P of a certain animal species after t months can be modeled by $P = C(1.21)^{t/3}$ where C is the initial population. Find the population after 19 months if the initial population was 75.

Solution

$P = C(1.21)^{t/3}$	**Write model for population.**
$= \underline{\qquad\qquad}$	**Substitute for C and t.**
$\approx \underline{\qquad}$	**Use a calculator.**

The population of the species is about ______ after 19 months.

✔ ***Checkpoint*** **Complete the following exercises.**

3. Evaluate $(-125)^{-2/3}$.	**4.** Solve $(y - 3)^4 = 200$.

5. The volume of a cone is given by $V = \frac{\pi r^2 h}{3}$, where h is the height of the cone and r is the radius. Find the radius of a cone whose volume is 25 cubic inches and whose height is 6 inches.

Homework

Your Notes

Example 4 *Use nth roots in problem solving*

Animal Population The population P of a certain animal species after t months can be modeled by $P = C(1.21)^{t/3}$ where C is the initial population. Find the population after 19 months if the initial population was 75.

Solution

$P = C(1.21)^{t/3}$	**Write model for population.**
$= \underline{75(1.21)^{19/3}}$	**Substitute for C and t.**
$\approx \underline{250.8}$	**Use a calculator.**

The population of the species is about $\underline{251}$ after 19 months.

✔ ***Checkpoint*** **Complete the following exercises.**

3. Evaluate $(-125)^{-2/3}$.

$\frac{1}{25}$

4. Solve $(y - 3)^4 = 200$.

$\sqrt[4]{200} + 3 \approx 6.76$ or

$-\sqrt[4]{200} + 3 \approx -0.76$

5. The volume of a cone is given by $V = \frac{\pi r^2 h}{3}$, where h is the height of the cone and r is the radius. Find the radius of a cone whose volume is 25 cubic inches and whose height is 6 inches.

1.99 in.

Homework

6.2 Apply Properties of Rational Exponents

Goal • Simplify expressions involving rational exponents.

Your Notes

VOCABULARY

Simplest form of a radical

Like radicals

PROPERTIES OF RATIONAL EXPONENTS

Let a and b be real numbers and let m and n be rational numbers. The following properties have the same names as those in Lesson 5.1, but now apply to rational exponents.

Property

1. $a^m \cdot a^n = a^{m+n}$ $\qquad 4^{1/2} \cdot 4^{3/2} = 4^{(1/2 + 3/2)}$ ______
2. $(a^m)^n = a^{mn}$ $\qquad (2^{5/2})^2 = 2^{(5/2 \cdot 2)}$ ______
3. $(ab)^m = a^m b^m$ $\qquad (16 \cdot 4)^{1/2} = 16^{1/2} \cdot 4^{1/2}$ ______
4. $a^{-m} = \frac{1}{a^m}, a \neq 0$ $\qquad 25^{-1/2} = \frac{1}{25^{1/2}} =$ ______
5. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$ $\qquad \frac{3^{5/2}}{3^{1/2}} = 3^{(5/2 - 1/2)} =$ ______
6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$ $\qquad \left(\frac{27}{8}\right)^{1/3} = \frac{27^{1/3}}{8^{1/3}} =$ ______

6.2 Apply Properties of Rational Exponents

Goal • Simplify expressions involving rational exponents.

Your Notes

VOCABULARY

Simplest form of a radical A radical with index n is in simplest form if the radicand has no perfect nth powers as factors and any denominator has been rationalized.

Like radicals Two radical expressions with the same index and radicand.

PROPERTIES OF RATIONAL EXPONENTS

Let a and b be real numbers and let m and n be rational numbers. The following properties have the same names as those in Lesson 5.1, but now apply to rational exponents.

Property

1. $a^m \cdot a^n = a^{m+n}$ $\quad 4^{1/2} \cdot 4^{3/2} = 4^{(1/2 + 3/2)} = 4^2 = 16$
2. $(a^m)^n = a^{mn}$ $\quad (2^{5/2})^2 = 2^{(5/2 \cdot 2)} = 2^5 = 32$
3. $(ab)^m = a^m b^m$ $\quad (16 \cdot 4)^{1/2} = 16^{1/2} \cdot 4^{1/2} = 4 \cdot 2 = 8$
4. $a^{-m} = \frac{1}{a^m}, a \neq 0$ $\quad 25^{-1/2} = \frac{1}{25^{1/2}} = \frac{1}{5}$
5. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$ $\quad \frac{3^{5/2}}{3^{1/2}} = 3^{(5/2 - 1/2)} = 3^2 = 9$
6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$ $\quad \left(\frac{27}{8}\right)^{1/3} = \frac{27^{1/3}}{8^{1/3}} = \frac{3}{2}$

Your Notes

Example 1 *Use properties of exponents*

Use the properties of rational exponents to simplify the expression.

a. $9^{1/2} \cdot 9^{3/4} =$ ______________

b. $(7^{2/3} \cdot 5^{1/6})^3 =$ ______________

$=$ ______________

$=$ ______________

c. $\dfrac{3^{5/6}}{3^{1/3}} =$ ______________

d. $\left(\dfrac{16^{2/3}}{4^{2/3}}\right)^4 =$ ______________

PROPERTIES OF RADICALS

Product Property of Radicals

$\sqrt[n]{a \cdot b} =$ ______________

Quotient Property of Radicals

$\sqrt[n]{\dfrac{a}{b}} = \dfrac{}{\_\_\_\_}$, $b \neq 0$

Example 2 *Use properties of radicals*

Use the properties of radicals to simplify the expression.

a. $\sqrt[5]{27} \cdot \sqrt[5]{9} =$ __________ $=$ ________ $=$ ____ **Product property**

b. $\dfrac{\sqrt[3]{192}}{\sqrt[3]{3}} = \dfrac{}{\_\_\_\_}$ $=$ ________ $=$ ____ **Quotient property**

✔ ***Checkpoint*** Simplify the expression.

1. $(6^6 \cdot 5^6)^{-1/6}$	**2.** $\dfrac{\sqrt{245}}{\sqrt{5}}$

Your Notes

Example 1 *Use properties of exponents*

Use the properties of rational exponents to simplify the expression.

a. $9^{1/2} \cdot 9^{3/4} = 9^{(1/2 + 3/4)} = 9^{5/4}$

b. $(7^{2/3} \cdot 5^{1/6})^3 = (7^{2/3})^3 \cdot (5^{1/6})^3$

$= 7^{(2/3 \cdot 3)} \cdot 5^{(1/6 \cdot 3)}$

$= 7^2 \cdot 5^{1/2} = 49 \cdot 5^{1/2}$

c. $\dfrac{3^{5/6}}{3^{1/3}} = 3^{(5/6 - 1/3)} = 3^{3/6} = 3^{1/2}$

d. $\left(\dfrac{16^{2/3}}{4^{2/3}}\right)^4 = \left[\left(\dfrac{16}{4}\right)^{2/3}\right]^4 = (4^{2/3})^4 = 4^{(2/3 \cdot 4)} = 4^{8/3}$

PROPERTIES OF RADICALS

Product Property of Radicals

$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Quotient Property of Radicals

$\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$, $b \neq 0$

Example 2 *Use properties of radicals*

Use the properties of radicals to simplify the expression.

a. $\sqrt[5]{27} \cdot \sqrt[5]{9} = \sqrt[5]{27 \cdot 9} = \sqrt[5]{243} = 3$ **Product property**

b. $\dfrac{\sqrt[3]{192}}{\sqrt[3]{3}} = \sqrt[3]{\dfrac{192}{3}} = \sqrt[3]{64} = 4$ **Quotient property**

Checkpoint **Simplify the expression.**

1. $(6^6 \cdot 5^6)^{-1/6}$ $\dfrac{1}{30}$	**2.** $\dfrac{\sqrt{245}}{\sqrt{5}}$ 7

Your Notes

Example 3 *Write radicals in simplest form*

Write the expression in simplest form.

$\sqrt[5]{128}$ = ________ **Factor out perfect fifth power.**

= _____ • _____ **Product property**

= _____ **Simplify.**

Example 4 *Add and subtract like radicals and roots*

Simplify the expression.

a. $2(12^{2/3}) + 7(12^{2/3})$ = ______________________

b. $\sqrt[4]{48} - \sqrt[4]{3}$ = _____ • _____ − _____

= ____________________________

✔ ***Checkpoint*** **Write the expression in simplest form.**

3. $\sqrt[3]{\frac{5}{9}}$	**4.** $6\sqrt[4]{6} + 2\sqrt[4]{6}$

Example 5 *Simplify expressions involving variables*

Simplify the expression. Assume all variables are positive.

a. $\sqrt[5]{32x^{15}}$ = ______________________________

b. $(36m^4n^{10})^{1/2}$ = _________________

= _________________________

c. $\sqrt[3]{\frac{a^9}{b^6}}$ =

____

d. $\frac{42x^4z^7}{6x^{3/2}y^{-3}z^5}$ = ______________________________

Your Notes

Example 3 *Write radicals in simplest form*

Write the expression in simplest form.

$\sqrt[5]{128}$	$= \sqrt[5]{32 \cdot 4}$	Factor out perfect fifth power.
	$= \sqrt[5]{32} \cdot \sqrt[5]{4}$	Product property
	$= 2\sqrt[5]{4}$	Simplify.

Example 4 *Add and subtract like radicals and roots*

Simplify the expression.

a. $2(12^{2/3}) + 7(12^{2/3}) = (2 + 7)(12^{2/3}) = 9(12^{2/3})$

b. $\sqrt[4]{48} - \sqrt[4]{3} = \sqrt[4]{16} \cdot \sqrt[4]{3} - \sqrt[4]{3}$

$= 2\sqrt[4]{3} - \sqrt[4]{3} = (2 - 1)\sqrt[4]{3} = \sqrt[4]{3}$

✔ ***Checkpoint*** **Write the expression in simplest form.**

3. $\sqrt[3]{\frac{5}{9}}$	**4.** $6\sqrt[4]{6} + 2\sqrt[4]{6}$
$\frac{\sqrt[3]{15}}{3}$	$8\sqrt[4]{6}$

Example 5 *Simplify expressions involving variables*

Simplify the expression. Assume all variables are positive.

a. $\sqrt[5]{32x^{15}} = \sqrt[5]{2^5 \cdot (x^3)^5} = \sqrt[5]{2^5} \cdot \sqrt[5]{(x^3)^5} = 2x^3$

b. $(36m^4n^{10})^{1/2} = 36^{1/2}(m^4)^{1/2}(n^{10})^{1/2}$

$= 6m^{(4 \cdot 1/2)}n^{(10 \cdot 1/2)} = 6m^2n^5$

c. $\sqrt[3]{\frac{a^9}{b^6}} = \frac{\sqrt[3]{a^9}}{\sqrt[3]{b^6}} = \frac{\sqrt[3]{(a^3)^3}}{\sqrt[3]{(b^2)^3}} = \frac{a^3}{b^2}$

d. $\frac{42x^4z^7}{6x^{3/2}y^{-3}z^5} = 7x^{(4 - 3/2)}y^{-(-3)}z^{(7 - 5)} = 7x^{5/2}y^3z^2$

Your Notes

You must multiply the original expression by a form of 1, in this case ______, when simplifying so that the new expression is equivalent.

Example 6 *Write variable expressions in simplest form*

Write the expression in simplest form. Assume all variables are positive.

$\sqrt[4]{\dfrac{a^2}{b^6}} = \dfrac{}{\underline{}}$ **Multiply to make denominator a perfect fourth power.**

$= \dfrac{}{\underline{}}$ **Simplify.**

$= \dfrac{}{\underline{}}$ **Quotient property.**

$= \dfrac{}{\underline{}}$ **Simplify.**

Example 7 *Add and subtract expressions involving variables*

Perform the indicated operation. Assume all variables are positive.

a. $10\sqrt[5]{y} - 6\sqrt[5]{y} =$ ______________________

b. $3a^2b^{1/4} + 4a^2b^{1/4} =$ ______________________

✔ ***Checkpoint*** **Simplify the expression. Assume all variables are positive.**

5. $\sqrt[3]{8x^7y^3z^{11}}$	**6.** $7\sqrt[3]{2a^5} - a\sqrt[3]{128a^2}$

Homework

Your Notes

You must multiply the original expression by a form of 1, in this case $\sqrt[4]{\frac{b^2}{b^2}}$, when simplifying so that the new expression is equivalent.

Example 6 *Write variable expressions in simplest form*

Write the expression in simplest form. Assume all variables are positive.

$$\sqrt[4]{\frac{a^2}{b^6}} = \sqrt[4]{\frac{a^2}{b^6}} \cdot \sqrt[4]{\frac{b^2}{b^2}}$$ **Multiply to make denominator a perfect fourth power.**

$$= \sqrt[4]{\frac{a^2b^2}{b^8}}$$ **Simplify.**

$$= \frac{\sqrt[4]{a^2b^2}}{\sqrt[4]{b^8}}$$ **Quotient property.**

$$= \frac{\sqrt[4]{a^2b^2}}{b^2}$$ **Simplify.**

Example 7 *Add and subtract expressions involving variables*

Perform the indicated operation. Assume all variables are positive.

a. $10\sqrt[5]{y} - 6\sqrt[5]{y} = (10 - 6)\sqrt[5]{y} = 4\sqrt[5]{y}$

b. $3a^2b^{1/4} + 4a^2b^{1/4} = (3 + 4)a^2b^{1/4} = 7a^2b^{1/4}$

✔ ***Checkpoint*** **Simplify the expression. Assume all variables are positive.**

5. $\sqrt[3]{8x^7y^3z^{11}}$ $2x^2yz^3\sqrt[3]{xz^2}$	**6.** $7\sqrt[3]{2a^5} - a\sqrt[3]{128a^2}$ $3a\sqrt[3]{2a^2}$

Homework

6.3 Perform Function Operations and Composition

Goal • Perform operations with functions.

Your Notes

VOCABULARY

Power function

Composition

OPERATIONS ON FUNCTIONS

Let f and g be any two functions. A new function h can be defined by performing any of the four basic operations on f and g.

Operation and Definition	**Example:** $f(x) = 3x,\ g(x) = x + 3$
Addition $h(x) = f(x) + g(x)$	$h(x) = 3x + (x + 3)$ $= \underline{\hspace{3cm}}$
Subtraction $h(x) = f(x) - g(x)$	$h(x) = 3x - (x + 3)$ $= \underline{\hspace{3cm}}$
Multiplication $h(x) = f(x) \cdot g(x)$	$h(x) = 3x(x + 3)$ $= \underline{\hspace{3cm}}$
Division $h(x) = \dfrac{f(x)}{g(x)}$	$h(x) = \dfrac{}{\underline{\hspace{2cm}}}$

The domain of h consists of the x-values that are in the domains of ______________. Additionally, the domain of a quotient does not include x-values for which $g(x) =$ ____.

6.3 Perform Function Operations and Composition

Goal • Perform operations with functions.

Your Notes

VOCABULARY

Power function A function of the form $y = ax^b$ where a is a real number and b is a rational number

Composition The composition of a function g with a function f is $h(x) = g(f(x))$. The domain of h is the set of all x-values such that x is in the domain of f and $f(x)$ is in the domain of g.

OPERATIONS ON FUNCTIONS

Let f and g be any two functions. A new function h can be defined by performing any of the four basic operations on f and g.

Operation and Definition	Example: $f(x) = 3x$, $g(x) = x + 3$
Addition $h(x) = f(x) + g(x)$	$h(x) = 3x + (x + 3)$ $= 4x + 3$
Subtraction $h(x) = f(x) - g(x)$	$h(x) = 3x - (x + 3)$ $= 2x - 3$
Multiplication $h(x) = f(x) \cdot g(x)$	$h(x) = 3x(x + 3)$ $= 3x^2 + 9x$
Division $h(x) = \frac{f(x)}{g(x)}$	$h(x) = \frac{3x}{x + 3}$

The domain of h consists of the x-values that are in the domains of both f and g. Additionally, the domain of a quotient does not include x-values for which $g(x) = 0$.

Your Notes

Example 1 ***Add and subtract functions***

Let $f(x) = 3x^{1/2}$ and $g(x) = -5x^{1/2}$. Find the following.

a. $f(x) + g(x)$

b. $f(x) - g(x)$

c. the domains of $f + g$ and $f - g$

Solution

a. $f(x) + g(x) = 3x^{1/2} + (-5x^{1/2})$

$= \underline{\hspace{6cm}}$

b. $f(x) - g(x) = 3x^{1/2} - (-5x^{1/2})$

$= \underline{\hspace{6cm}}$

c. The functions f and g each have the same domain: __________________________. So, the domains of $f + g$ and $f - g$ also consist of __________________________________.

Example 2 ***Multiply and divide functions***

Let $f(x) = 7x$ and $g(x) = x^{1/6}$. Find the following.

a. $f(x) \cdot g(x)$

b. $\frac{f(x)}{g(x)}$

c. the domains of $f \cdot g$ and $\frac{f}{g}$

Solution

a. $f(x) \cdot g(x) = (7x)(x^{1/6}) =$ ____________________

b. $\frac{f(x)}{g(x)} =$ __________________________

c. The domain of f consists of ____________________, and the domain of g consists of ________________________________. So, the domain of $f \cdot g$ consists of ______________________________. Because $g(0) =$ ____, the domain of $\frac{f}{g}$ is restricted to ___________________________.

Your Notes

Example 1 ***Add and subtract functions***

Let $f(x) = 3x^{1/2}$ and $g(x) = -5x^{1/2}$. Find the following.

a. $f(x) + g(x)$

b. $f(x) - g(x)$

c. the domains of $f + g$ and $f - g$

Solution

a. $f(x) + g(x) = 3x^{1/2} + (-5x^{1/2})$

$= [3 + (-5)]x^{1/2} = -2x^{1/2}$

b. $f(x) - g(x) = 3x^{1/2} - (-5x^{1/2})$

$= [3 - (-5)]x^{1/2} = 8x^{1/2}$

c. The functions f and g each have the same domain: all nonnegative real numbers. So, the domains of $f + g$ and $f - g$ also consist of all nonnegative real numbers.

Example 2 ***Multiply and divide functions***

Let $f(x) = 7x$ and $g(x) = x^{1/6}$. Find the following.

a. $f(x) \cdot g(x)$

b. $\frac{f(x)}{g(x)}$

c. the domains of $f \cdot g$ and $\frac{f}{g}$

Solution

a. $f(x) \cdot g(x) = (7x)(x^{1/6}) = 7x^{(1 + 1/6)} = 7x^{7/6}$

b. $\frac{f(x)}{g(x)} = \frac{7x}{x^{1/6}} = 7x^{(1 - 1/6)} = 7x^{5/6}$

c. The domain of f consists of all real numbers, and the domain of g consists of all nonnegative real numbers. So, the domain of $f \cdot g$ consists of all nonnegative real numbers. Because $g(0) = 0$, the domain of $\frac{f}{g}$ is restricted to all positive real numbers.

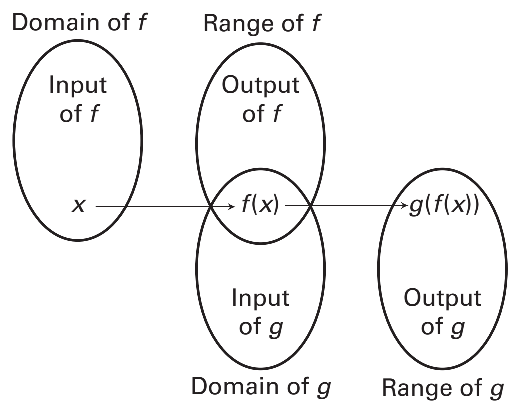
Your Notes

✔ *Checkpoint* Complete the following exercise.

1. Let $f(x) = 5x^{3/2}$ and $g(x) = -2x^{3/2}$. Find (a) $f + g$, (b) $f - g$, (c) $f \cdot g$, (d) $\frac{f}{g}$, and (e) the domains.

COMPOSITION OF FUNCTIONS

The composition of a function g with a function f is $h(x) =$ ________. The domain of h is the set of all x-values such that x is in the domain of ___ and $f(x)$ is in the domain of ___.



Your Notes

Checkpoint Complete the following exercise.

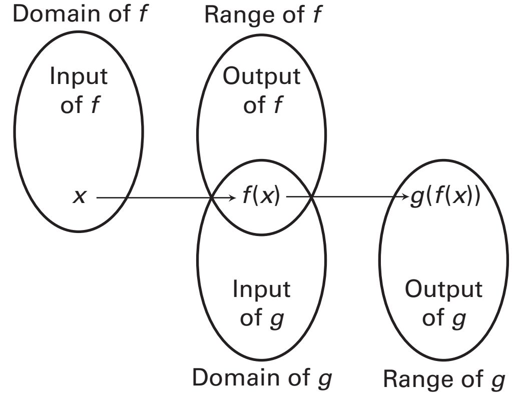
1. Let $f(x) = 5x^{3/2}$ and $g(x) = -2x^{3/2}$. Find (a) $f + g$, (b) $f - g$, (c) $f \cdot g$, (d) $\frac{f}{g}$, and (e) the domains.

a. $3x^{3/2}$ b. $7x^{3/2}$ c. $-10x^3$ d. $-\frac{5}{2}$

e. The domain of $f + g$, $f - g$, and $f \cdot g$ is all nonnegative real numbers. The domain of $\frac{f}{g}$ is all positive real numbers.

COMPOSITION OF FUNCTIONS

The composition of a function g with a function f is $h(x) =$ $g(f(x))$. The domain of h is the set of all x-values such that x is in the domain of f and $f(x)$ is in the domain of g.



Your Notes

Example 3 *Find compositions of functions*

Let $f(x) = 6x^{-1}$ and $g(x) = 3x + 5$. Find the following.

a. $f(g(x))$ **b.** $g(f(x))$ **c.** $f(f(x))$

d. the domain of each composition

Solution

a. $f(g(x)) = f(3x + 5) =$ ______________________

b. $g(f(x)) = g(6x^{-1})$

$=$ ____________________________________

c. $f(f(x)) = f(6x^{-1}) =$ ________________________________

d. The domain of $f(g(x))$ consists of __________________

except $x =$ _____ because $g($ _____ $) = 0$ is not in

the ______________. (Note that $f(0) =$ ___ , which

is ____________.) The domains of $g(f(x))$ and $f(f(x))$ consist of __________________ except $x =$ ___, again because ____________________________.

Checkpoint **Complete the following exercise.**

2. Let $f(x) = 5x - 4$ and $g(x) = 3x^{-1}$. Find (a) $f(g(x))$, (b) $g(f(x))$, (c) $f(f(x))$, and (d) the domain of each composition.

Homework

Your Notes

Example 3 *Find compositions of functions*

Let $f(x) = 6x^{-1}$ and $g(x) = 3x + 5$. Find the following.

a. $f(g(x))$ **b.** $g(f(x))$ **c.** $f(f(x))$

d. the domain of each composition

Solution

a. $f(g(x)) = f(3x + 5) =$ $6(3x + 5)^{-1} = \frac{6}{3x + 5}$

b. $g(f(x)) = g(6x^{-1})$

$= 3(6x^{-1}) + 5 = 18x^{-1} + 5 = \frac{18}{x} + 5$

c. $f(f(x)) = f(6x^{-1}) =$ $6(6x^{-1})^{-1} = 6(6^{-1}x) = 6^0 x = x$

d. The domain of $f(g(x))$ consists of all real numbers except $x = -\frac{5}{3}$ because $g\left(-\frac{5}{3}\right) = 0$ is not in the domain of f. (Note that $f(0) = \frac{6}{0}$, which is undefined.) The domains of $g(f(x))$ and $f(f(x))$ consist of all real numbers except $x = 0$, again because 0 is not in the domain of f.

Checkpoint **Complete the following exercise.**

2. Let $f(x) = 5x - 4$ and $g(x) = 3x^{-1}$. Find (a) $f(g(x))$, (b) $g(f(x))$, (c) $f(f(x))$, and (d) the domain of each composition.

a. $\frac{15}{x} - 4$ b. $\frac{3}{5x - 4}$ c. $25x - 24$

d. The domain of $f(g(x))$ and $f(f(x))$ is all real numbers except $x = 0$. The domain of $g(f(x))$ is all real numbers except $x = \frac{4}{5}$.

Homework

6.4 Use Inverse Functions

Goal • Find inverse functions.

Your Notes

VOCABULARY

Inverse relation

Inverse function

Example 1 *Find an inverse relation*

Find an equation for the inverse of the relation $y = 7x - 4$.

$y = 7x - 4$	Write original equation.
______	Switch x and y.
______	Add ___ to each side.
______	Solve for y. This is the inverse relation.

INVERSE FUNCTIONS

Functions f and g are inverses of each other provided:

$f(g(x)) =$ ___ and $g(f(x)) =$ ___

The function g is denoted by f^{-1}, read as "f inverse."

Example 2 *Verify that functions are inverses*

Verify that $f(x) = 7x - 4$ and $f^{-1}(x) = \frac{1}{7}x + \frac{4}{7}$ are inverses.

Show that $f(f^{-1}(x)) = x$.

$f(f^{-1}(x)) = f\left(\frac{1}{7}x + \frac{4}{7}\right)$

= ______

= ______

= ___

Show that $f^{-1}(f(x)) = x$.

$f^{-1}(f(x)) = f^{-1}(7x - 4)$

= ______

= ______

= ___

6.4 Use Inverse Functions

Goal • Find inverse functions.

Your Notes

VOCABULARY

Inverse relation A relation that interchanges the input and output values of the original relation

Inverse function The original relation and its inverse relation whenever both relations are functions

Example 1 ***Find an inverse relation***

Find an equation for the inverse of the relation $y = 7x - 4$.

$y = 7x - 4$	**Write original equation.**
$x = 7y - 4$	**Switch x and y.**
$x + 4 = 7y$	**Add 4 to each side.**
$\frac{1}{7}x + \frac{4}{7} = y$	**Solve for y. This is the inverse relation.**

INVERSE FUNCTIONS

Functions f and g are inverses of each other provided:

$f(g(x)) = x$ and $g(f(x)) = x$

The function g is denoted by f^{-1}, read as "f inverse."

Example 2 ***Verify that functions are inverses***

Verify that $f(x) = 7x - 4$ and $f^{-1}(x) = \frac{1}{7}x + \frac{4}{7}$ are inverses.

Show that $f(f^{-1}(x)) = x$.

$$f(f^{-1}(x)) = f\left(\frac{1}{7}x + \frac{4}{7}\right)$$
$$= 7\left(\frac{1}{7}x + \frac{4}{7}\right) - 4$$
$$= x + 4 - 4$$
$$= x$$

Show that $f^{-1}(f(x)) = x$.

$$f^{-1}(f(x)) = f^{-1}(7x - 4)$$
$$= \frac{1}{7}(7x - 4) + \frac{4}{7}$$
$$= x - \frac{4}{7} + \frac{4}{7}$$
$$= x$$

Your Notes

✔ ***Checkpoint*** **Find the inverse of the function. Then verify that your result and the original function are inverses.**

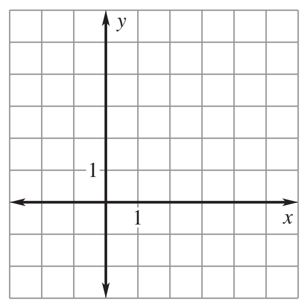
1. $f(x) = -3x + 5$

Example 3 *Find the inverse of a power function*

Find the inverse of $f(x) = 4x^2$, $x \leq 0$. Then graph f and f^{-1}.

$f(x) = 4x^2$	**Write original function.**
$y = 4x^2$	**Replace $f(x)$ with y.**
__________	**Switch x and y.**
__________	**Divide each side by 4.**
__________	**Take square roots of each side.**

The domain of f is restricted to negative values of x. So, the range of f^{-1} must also be restricted to negative values, and therefore the inverse is $f^{-1}(x) =$ ________. (If the domain were restricted to $x \geq 0$, you would choose $f^{-1}(x) =$ ________.)

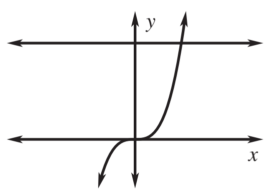


You can check the solution by noting that the graph of $f^{-1}(x) = -\frac{1}{2}\sqrt{x}$ is the reflection of the graph of $f(x) = 4x^2$, $x \leq 0$, in the line $y = x$.

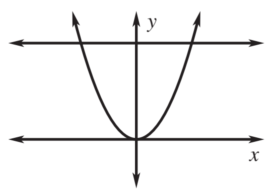
HORIZONTAL LINE TEST

The inverse of a function f is also a function if and only if no horizontal line intersects the graph of f ______ __________.

Function



Not a function



Your Notes

✔ ***Checkpoint*** **Find the inverse of the function. Then verify that your result and the original function are inverses.**

1. $f(x) = -3x + 5$

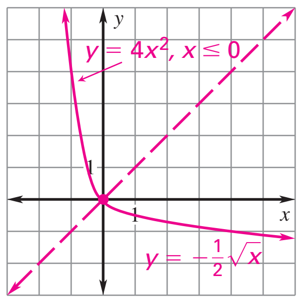
$f^{-1}(x) = -\frac{1}{3}x + \frac{5}{3}$

Example 3 ***Find the inverse of a power function***

Find the inverse of $f(x) = 4x^2$, $x \le 0$. Then graph f and f^{-1}.

$f(x) = 4x^2$	**Write original function.**
$y = 4x^2$	**Replace $f(x)$ with y.**
$x = 4y^2$	**Switch x and y.**
$\frac{1}{4}x = y^2$	**Divide each side by 4.**
$\pm\frac{1}{2}\sqrt{x} = y$	**Take square roots of each side.**

The domain of f is restricted to negative values of x. So, the range of f^{-1} must also be restricted to negative values, and therefore the inverse is $f^{-1}(x) = -\frac{1}{2}\sqrt{x}$. (If the domain were restricted to $x \ge 0$, you would choose $f^{-1}(x) = \frac{1}{2}\sqrt{x}$.)

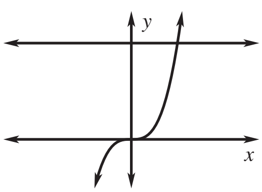


You can check the solution by noting that the graph of $f^{-1}(x) = -\frac{1}{2}\sqrt{x}$ is the reflection of the graph of $f(x) = 4x^2$, $x \le 0$, in the line $y = x$.

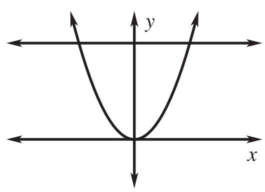
HORIZONTAL LINE TEST

The inverse of a function f is also a function if and only if no horizontal line intersects the graph of f more than once.

Function



Not a function

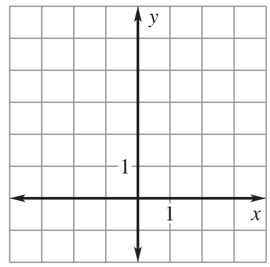


Your Notes

Example 4 ***Find the inverse of a cubic function***

Consider the function $f(x) = \frac{1}{4}x^3 + 3$. Determine whether the inverse of f is a function. Then find the inverse.

Solution



Graph the function *f*. Notice that no ________________ intersects the graph more than once. So, the inverse of *f* is itself a __________. To find an equation for f^{-1}, complete the following steps.

$f(x) = \frac{1}{4}x^3 + 3$	**Write original function.**
$y = \frac{1}{4}x^3 + 3$	**Replace $f(x)$ with y.**
______________	**Switch x and y.**
______________	**Subtract ___ from each side.**
______________	**Multiply each side by ___.**
______________	**Take cube root of each side.**

The inverse of *f* is $f^{-1}(x) =$ ___________.

✔ ***Checkpoint*** **Find the inverse of the function.**

2. $f(x) = 2x^4 + 1$	**3.** $g(x) = \frac{1}{32}x^5$

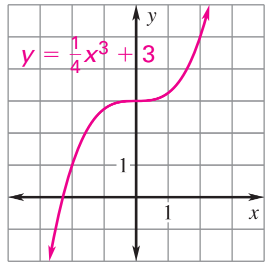
Homework

Your Notes

Example 4 ***Find the inverse of a cubic function***

Consider the function $f(x) = \frac{1}{4}x^3 + 3$. Determine whether the inverse of f is a function. Then find the inverse.

Solution



Graph the function *f*. Notice that no horizontal line intersects the graph more than once. So, the inverse of *f* is itself a function. To find an equation for f^{-1}, complete the following steps.

$f(x) = \frac{1}{4}x^3 + 3$	**Write original function.**
$y = \frac{1}{4}x^3 + 3$	**Replace $f(x)$ with y.**
$x = \frac{1}{4}y^3 + 3$	**Switch x and y.**
$x - 3 = \frac{1}{4}y^3$	**Subtract 3 from each side.**
$4x - 12 = y^3$	**Multiply each side by 4.**
$\sqrt[3]{4x - 12} = y$	**Take cube root of each side.**

The inverse of f is $f^{-1}(x) = \sqrt[3]{4x - 12}$.

Checkpoint **Find the inverse of the function.**

2. $f(x) = 2x^4 + 1$ $f^{-1}(x) = \sqrt[4]{\frac{1}{2}x - \frac{1}{2}}$	**3.** $g(x) = \frac{1}{32}x^5$ $g^{-1}(x) = 2\sqrt[5]{x}$

Homework

6.5 Graph Square Root and Cube Root Functions

Goal • Graph square root and cube root functions.

Your Notes

VOCABULARY

Radical function

PARENT FUNCTIONS FOR SQUARE ROOT AND CUBE ROOT FUNCTIONS

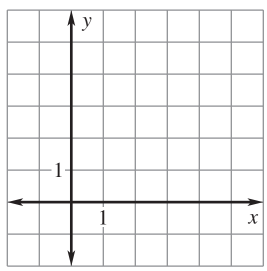
- The parent function for the family of square root functions is $f(x) = \sqrt{x}$. The domain is x _____, and the range is y _____.
- The parent function for the family of cube root functions is $g(x) = \sqrt[3]{x}$. The domain and range are _________________.

Example 1 ***Graph a square root function***

Graph $y = 2\sqrt{x}$, and state the domain and range. Compare the graph with the graph of $y = \sqrt{x}$.

Solution

Make a table of values and sketch the graph.



x	0	1	2	3	4
y	____	____	_______	_______	____

The radicand of a square root is always nonnegative. So, the domain is x ___ 0. The range is y ___ 0.

The graph of $y = 2\sqrt{x}$ is a vertical ________ of the parent graph of $y = \sqrt{x}$.

6.5 Graph Square Root and Cube Root Functions

Goal • Graph square root and cube root functions.

Your Notes

VOCABULARY

Radical function A function containing a radical such as $y = \sqrt{x}$

PARENT FUNCTIONS FOR SQUARE ROOT AND CUBE ROOT FUNCTIONS

- The parent function for the family of square root functions is $f(x) = \sqrt{x}$. The domain is x ≥ 0, and the range is y ≥ 0.
- The parent function for the family of cube root functions is $g(x) = \sqrt[3]{x}$. The domain and range are all real numbers.

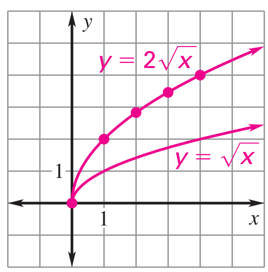
Example 1 ***Graph a square root function***

Graph $y = 2\sqrt{x}$, and state the domain and range. Compare the graph with the graph of $y = \sqrt{x}$.

Solution

Make a table of values and sketch the graph.

x	0	1	2	3	4
y	0	2	2.83	3.46	4



The radicand of a square root is always nonnegative. So, the domain is $x \geq 0$. The range is $y \geq 0$.

The graph of $y = 2\sqrt{x}$ is a vertical stretch of the parent graph of $y = \sqrt{x}$.

Your Notes

Example 2 ***Graph a cube root function***

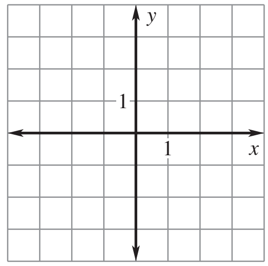
Graph $y = -\frac{1}{2}\sqrt[3]{x}$, and state the domain and range. Compare the graph with the graph of $y = \sqrt[3]{x}$.

Solution

Make a table of values and sketch the graph.

x	−2	−1	0
y	______	______	___

x	1	2
y	______	______

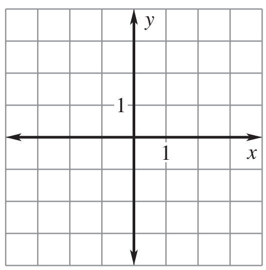


The domain and range are ________________.

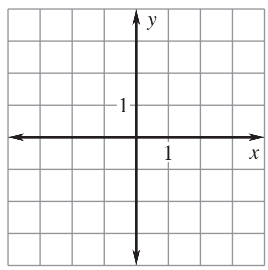
The graph of $y = -\frac{1}{2}\sqrt[3]{x}$ is a vertical ________ of the parent graph of $y = \sqrt[3]{x}$ by a factor of ___ followed by a reflection in the *x*-axis.

✔ ***Checkpoint*** **Graph the function. Then state the domain and range.**

1. $y = 2\sqrt[3]{x}$



2. $y = -2\sqrt{x}$



Your Notes

Example 2 *Graph a cube root function*

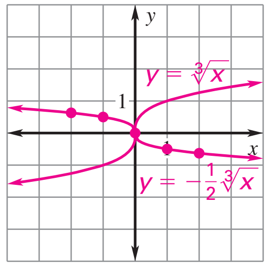
Graph $y = -\frac{1}{2}\sqrt[3]{x}$, and state the domain and range. Compare the graph with the graph of $y = \sqrt[3]{x}$.

Solution

Make a table of values and sketch the graph.

x	−2	−1	0
y	0.63	0.5	0

x	1	2
y	−0.5	−0.63

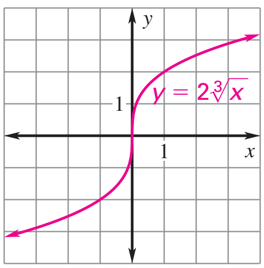


The domain and range are all real numbers.

The graph of $y = -\frac{1}{2}\sqrt[3]{x}$ is a vertical shrink of the parent graph of $y = \sqrt[3]{x}$ by a factor of $\frac{1}{2}$ followed by a reflection in the x-axis.

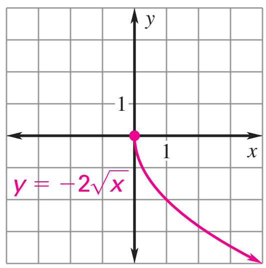
Checkpoint **Graph the function. Then state the domain and range.**

1. $y = 2\sqrt[3]{x}$



The domain and range are all real numbers.

2. $y = -2\sqrt{x}$



domain $x \geq 0$, range $y \leq 0$

Your Notes

GRAPHS OF RADICAL FUNCTIONS

To graph $y = a\sqrt{x - h} + k$ or $y = a\sqrt[3]{x - h} + k$, follow these steps:

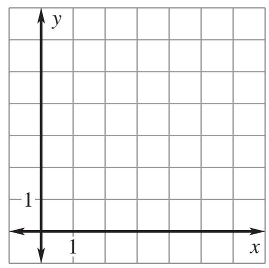
Step 1 ________ the graph of $y = a\sqrt{x}$ or $y = a\sqrt[3]{x}$.

Step 2 **Translate** the graph ___ units horizontally and ___ units vertically.

Example 3 ***Graph a translated square root function***

Graph $y = 3\sqrt{x - 1} + 2$. Then state the domain and range.

Solution



1. **Sketch** the graph of $y = 3\sqrt{x}$. Notice that it begins at the origin and passes through the point (1, ___).
2. **Translate** the graph. For $y = 3\sqrt{x - 1} + 2$, h = ___ and k = ___. So, shift the graph ____________ and ______ ______. The resulting graph starts at (___, ___) and passes through (___, ___).

From the graph, you can see that the domain of the function is ______ and the range of the function is ______.

Your Notes

GRAPHS OF RADICAL FUNCTIONS

To graph $y = a\sqrt{x - h} + k$ or $y = a\sqrt[3]{x - h} + k$, follow these steps:

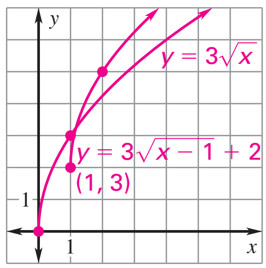
Step 1 Sketch the graph of $y = a\sqrt{x}$ or $y = a\sqrt[3]{x}$.

Step 2 **Translate** the graph h units horizontally and k units vertically.

Example 3 *Graph a translated square root function*

Graph $y = 3\sqrt{x - 1} + 2$. Then state the domain and range.

Solution



1. **Sketch** the graph of $y = 3\sqrt{x}$. Notice that it begins at the origin and passes through the point (1, 3).

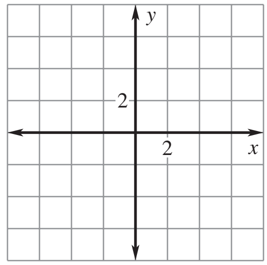
2. **Translate** the graph. For $y = 3\sqrt{x - 1} + 2$, $h = 1$ and $k = 2$. So, shift the graph right 1 unit and up 2 units. The resulting graph starts at (1, 2) and passes through (2, 5).

From the graph, you can see that the domain of the function is $x \geq 1$ and the range of the function is $y \geq 2$.

Your Notes

Example 4 ***Graph a translated cube root function***

Graph $y = -2\sqrt[3]{x + 3} - 2$. Then state the domain and range.



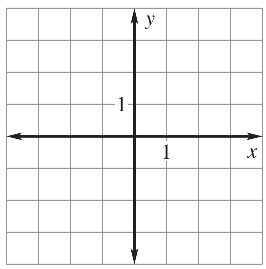
Solution

1. **Sketch** the graph of $y = -2\sqrt[3]{x}$. Notice that it passes through the origin and the points (_____, ___) and (___, _____).
2. **Note** that for $y = -2\sqrt[3]{x + 3} - 2$, h = _____ and k = _____. So, shift the graph ____________ and _______________. The resulting graph passes through the points (_____, ___), (_____, _____), and (_____, _____).

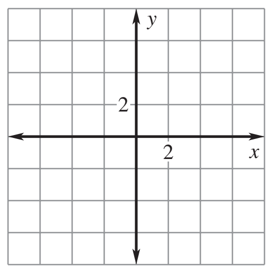
From the graph, you can see that the domain and range of the function are both __________________.

Checkpoint **Graph the function. Then state the domain and range.**

3. $y = -\frac{1}{2}\sqrt{x + 3} + 2$



4. $y = 3\sqrt[3]{x} + 2$

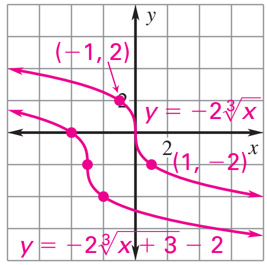


Homework

Your Notes

Example 4 *Graph a translated cube root function*

Graph $y = -2\sqrt[3]{x + 3} - 2$. Then state the domain and range.



Solution

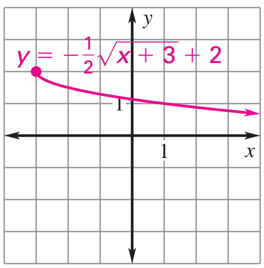
1. **Sketch** the graph of $y = -2\sqrt[3]{x}$. Notice that it passes through the origin and the points (−1 , 2) and (1 , −2).

2. **Note** that for $y = -2\sqrt[3]{x + 3} - 2$, $h =$ −3 and $k =$ −2 . So, shift the graph left 3 units and down 2 units . The resulting graph passes through the points (−4 , 0), (−3 , −2), and (−2 , −4).

From the graph, you can see that the domain and range of the function are both all real numbers .

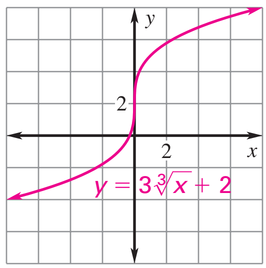
Checkpoint **Graph the function. Then state the domain and range.**

3. $y = -\frac{1}{2}\sqrt{x + 3} + 2$



domain $x \geq -3$, range $y \leq 2$

4. $y = 3\sqrt[3]{x} + 2$



The domain and range are all real numbers.

Homework

6.6 Solve Radical Equations

Goal • Solve radical equations.

Your Notes

VOCABULARY

Radical equation

SOLVING RADICAL EQUATIONS

To solve a radical equation, follow these steps:

Step 1 ________ the radical on one side of the equation, if necessary.

Step 2 **Raise** each side of the equation to the same ________ to eliminate the radical and obtain a linear, quadratic, or other polynomial equation.

Step 3 ________ the polynomial equation using techniques you learned in previous chapters. Check your solution.

Example 1 ***Solve a radical equation***

Solve $\sqrt{x + 6} = 3$.

$\sqrt{x + 6} = 3$ — **Write original equation.**

___________ = ____ — **Square each side to eliminate the radical.**

________ = ___ — **Simplify.**

___ = ___ — **Subtract ___ from each side.**

The solution is ___. Check this in the original equation.

Checkpoint **Solve the equation. Check your solution.**

1. $\sqrt[3]{x - 5} + 1 = -1$

6.6 Solve Radical Equations

Goal • Solve radical equations.

Your Notes

VOCABULARY

Radical equation An equation with a radical that has variables in the radicand

SOLVING RADICAL EQUATIONS

To solve a radical equation, follow these steps:

Step 1 Isolate the radical on one side of the equation, if necessary.

Step 2 **Raise** each side of the equation to the same power to eliminate the radical and obtain a linear, quadratic, or other polynomial equation.

Step 3 Solve the polynomial equation using techniques you learned in previous chapters. Check your solution.

Example 1 ***Solve a radical equation***

Solve $\sqrt{x + 6} = 3$.

$\sqrt{x+6} = 3$		**Write original equation.**
$(\sqrt{x+6})^2 = 3^2$		**Square each side to eliminate the radical.**
$x + 6 = 9$		**Simplify.**
$x = 3$		**Subtract 6 from each side.**

The solution is 3. Check this in the original equation.

Checkpoint Solve the equation. Check your solution.

1. $\sqrt[3]{x - 5} + 1 = -1$

-3

Your Notes

Example 2 ***Solve an equation with a rational exponent***

$(3x + 4)^{2/3} = 16$	**Original equation**
________ = ______	**Raise each side to the power $\frac{3}{2}$.**
______ = ______	**Apply properties of exponents.**
______ = ____	**Simplify.**
____ = ____	**Subtract ___ from each side.**
___ = ____	**Divide each side by ___.**

The solution is ____. Check this in the original equation.

Example 3 ***Solve an equation with an extraneous solution***

$x - 2 = \sqrt{x + 10}$	**Original equation**
______ = ________	**Square each side.**
________ = ______	**Expand left side and simplify right side.**
________ = 0	**Write in standard form.**
________ = 0	**Factor.**
_____ = 0 or _____ = 0	**Zero product property**
x = ___ or x = ____	**Solve for x.**

CHECK

Check x = ___.

$x - 2 = \sqrt{x + 10}$

_____ $\stackrel{?}{=}$ _______

___ $\stackrel{?}{=}$ ____

______

Check x = −___.

$x - 2 = \sqrt{x + 10}$

______ $\stackrel{?}{=}$ ________

____ $\stackrel{?}{=}$ ____

______

The only solution is ___. (The apparent solution _____ is extraneous.)

Your Notes

Example 2 ***Solve an equation with a rational exponent***

$(3x + 4)^{2/3} = 16$		**Original equation**
$[(3x + 4)^{2/3}]^{3/2} = 16^{3/2}$		**Raise each side to the power $\frac{3}{2}$.**
$3x + 4 = (16^{1/2})^3$		**Apply properties of exponents.**
$3x + 4 = 64$		**Simplify.**
$3x = 60$		**Subtract 4 from each side.**
$x = 20$		**Divide each side by 3.**

The solution is 20. Check this in the original equation.

Example 3 ***Solve an equation with an extraneous solution***

$x - 2 = \sqrt{x + 10}$	**Original equation**
$(x - 2)^2 = (\sqrt{x + 10})^2$	**Square each side.**
$x^2 - 4x + 4 = x + 10$	**Expand left side and simplify right side.**
$x^2 - 5x - 6 = 0$	**Write in standard form.**
$(x - 6)(x + 1) = 0$	**Factor.**
$x - 6 = 0$ or $x + 1 = 0$	**Zero product property**
$x = 6$ or $x = -1$	**Solve for *x*.**

CHECK

Check $x = 6$.

$x - 2 = \sqrt{x + 10}$

$6 - 2 \stackrel{?}{=} \sqrt{6 + 10}$

$4 \stackrel{?}{=} \sqrt{16}$

$4 = 4$

Check $x = -1$.

$x - 2 = \sqrt{x + 10}$

$-1 - 2 \stackrel{?}{=} \sqrt{-1 + 10}$

$-3 \stackrel{?}{=} \sqrt{9}$

$-3 \neq 3$

The only solution is 4. (The apparent solution -1 is extraneous.)

Your Notes

Example 4 *Solve an equation with two radicals*

Solve $\sqrt{x + 6} + 2 = \sqrt{10 - 3x}$.

$\sqrt{x + 6} + 2 = \sqrt{10 - 3x}$	**Write original equation.**
________ = ________	**Square each side.**
________ = ______	**Expand left side and simplify right side.**
______ = ____	**Isolate radical expression.**
______ = ____	**Divide each side by 4.**
______ = ____	**Square each side again.**
______ = ____	**Simplify.**
0 = ________	**Write in standard form.**
0 = ________	**Factor.**
_____ = 0 or _____ = 0	**Zero product property**
x = ___ or x = ___	**Solve for x.**

CHECK Check x = ___.

________ $\stackrel{?}{=}$ ________

______ $\stackrel{?}{=}$ ____

___ = ___

Check x = −____.

________ $\stackrel{?}{=}$ ________

______ $\stackrel{?}{=}$ ____

___ = ___

The only solution is ____. (The apparent solution ___ is extraneous.)

Your Notes

Example 4 ***Solve an equation with two radicals***

Solve $\sqrt{x+6}+2=\sqrt{10-3x}$.

$\sqrt{x+6}+2=\sqrt{10-3x}$	**Write original equation.**
$(\sqrt{x+6}+2)^2=(\sqrt{10-3x})^2$	**Square each side.**
$x+6+4\sqrt{x+6}+4=10-3x$	**Expand left side and simplify right side.**
$4\sqrt{x+6}=-4x$	**Isolate radical expression.**
$\sqrt{x+6}=-x$	**Divide each side by 4.**
$(\sqrt{x+6})^2=(-x)^2$	**Square each side again.**
$x+6=x^2$	**Simplify.**
$0=x^2-x-6$	**Write in standard form.**
$0=(x-3)(x+2)$	**Factor.**
$x-3=0$ or $x+2=0$	**Zero product property**
$x=3$ or $x=-2$	**Solve for x.**

CHECK Check $x=$ 3.

$\sqrt{3+6}+2\overset{?}{=}\sqrt{10-3(3)}$

$\sqrt{9}+2\overset{?}{=}\sqrt{1}$

$5=1$

Check $x=-$ -2.

$\sqrt{(-2)+6}+2\overset{?}{=}\sqrt{10-3(-2)}$

$\sqrt{4}+2\overset{?}{=}\sqrt{16}$

$4=4$

The only solution is -2. (The apparent solution 3 is extraneous.)

Your Notes

✔ ***Checkpoint*** **Solve the equation. Check for extraneous solutions.**

2. $-2x^{4/3} - 21 = -53$

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3. $x + 2 = \sqrt{2x + 7}$

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4. $\sqrt{3x + 4} - 1 = \sqrt{x + 5}$

Homework

Your Notes

Checkpoint **Solve the equation. Check for extraneous solutions.**

2. $-2x^{4/3} - 21 = -53$

8

3. $x + 2 = \sqrt{2x + 7}$

1

4. $\sqrt{3x + 4} - 1 = \sqrt{x + 5}$

4

Homework

Words to Review

Give an example of the vocabulary word.

*n*th root of *a*	Index of a radical
Simplest form of a radical	Like radicals
Power function	Composition
Inverse relation	Inverse function
Radical function	Radical equation

Review your notes and Chapter 6 by using the Chapter Review on pages 459–461 of your textbook.

Words to Review

Give an example of the vocabulary word.

*n*th root of *a* 3 is the cube root of 27.	Index of a radical 3 is the index of $\sqrt[3]{27}$.
Simplest form of a radical $3\sqrt{2x}$ is the simplest form of $\sqrt{18x}$.	Like radicals $7(11^{1/3})$ and $18(11^{1/3})$ are like radicals.
Power function $f(x) = 6x^4$	Composition If $f(x) = 3x^2$ and $g(x) = x - 1$, then $g(f(x)) = 3x^2 - 1$.
Inverse relation $y = \frac{1}{6}x + \frac{1}{3}$ is the inverse relation for $y = 6x - 2$.	Inverse function $f(x) = 3x + 6$ and $f^{-1}(x) = \frac{1}{3}x - 2$ are inverse functions.
Radical function $y = \sqrt[3]{x+5} - 6$	Radical equation $\sqrt{x+7} = -3$

Review your notes and Chapter 6 by using the Chapter Review on pages 459–461 of your textbook.