6-2

1. Plan

Objectives

1 To use relationships among sides and among angles of parallelograms
2 To use relationships involving diagonals of parallelograms or transversals

Examples

1 Using Consecutive Angles
2 Using Algebra
3 Using Algebra
4 Real-World Connection

Professional Development

Math Background

The parallelogram has the most subsets in this text's hierarchy of quadrilaterals, so it is natural to develop its properties first. In this way, any property of a parallelogram can be applied to any rectangle, rhombus, or square.

More Math Background: p. 304C

Lesson Planning and Resources

See p. 304E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need
For intervention, direct students to:

Congruent Figures
Lesson 4-1: Example 1
Extra Skills, Word Problems, Proof Practice, Ch. 4

Using the ASA Postulate
Lesson 4-3: Example 1
Extra Skills, Word Problems, Proof Practice, Ch. 4

6-2 Properties of Parallelograms

What You'll Learn

- To use relationships among sides and among angles of parallelograms
- To use relationships involving diagonals of parallelograms or transversals.

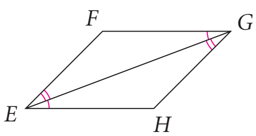
. . . And Why

To divide a card into three parts of equal heights without a ruler, as in Example 4

Check Skills You'll Need

GO for Help Lessons 4-1 and 4-3

Use the figure at the right.



1. Name the postulate or theorem that justifies the congruence $\triangle EFG \cong \triangle GHE$. **ASA**

2. Complete each statement.

a. $\angle FEG \cong$ __?__ **b.** $\angle EFG \cong$ __?__
c. $\angle FGE \cong$ __?__ **d.** $\overline{EF} \cong$ __?__
e. $\overline{FG} \cong$ __?__ **f.** $\overline{GE} \cong$ __?__

a. $\angle HGE$ **b.** $\angle GHE$ **c.** $\angle HEG$ **d.** $\overline{GH}$ **e.** $\overline{HE}$ **f.** $\overline{EG}$

3. What other relationship exists between $\overline{FG}$ and $\overline{EH}$? **They are ∥.**

New Vocabulary • consecutive angles

1 Properties: Sides and Angles

You can use what you know about parallel lines and transversals to prove some theorems about parallelograms.

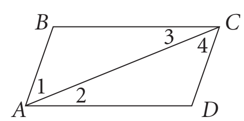
Key Concepts

Theorem 6-1

Opposite sides of a parallelogram are congruent.

Proof

Proof of Theorem 6-1



Given: $\square ABCD$

Prove: $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{DA}$

$ABCD$ is a ▱. (Given) → $\overline{AB} \parallel \overline{DC}$ (Def. of ▱) → $\angle 1 \cong \angle 4$ (If lines ∥, then alt. int. ∠s are ≅.)

$ABCD$ is a ▱. (Given) → $\overline{BC} \parallel \overline{AD}$ (Def. of ▱) → $\angle 3 \cong \angle 2$ (If lines ∥, then alt. int. ∠s are ≅.)

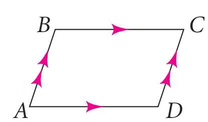
$\overline{AC} \cong \overline{AC}$ (Reflexive Prop. of ≅)

$\angle 1 \cong \angle 4$, $\angle 3 \cong \angle 2$, $\overline{AC} \cong \overline{AC}$ → $\triangle ABC \cong \triangle CDA$ (ASA) → $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{DA}$ (CPCTC)

Vocabulary Tip

Read "▱" as "parallelogram." The plural is "▱s."

Angles of a polygon that share a side are **consecutive angles.** A parallelogram has opposite sides parallel. Its consecutive angles are same-side interior angles so they are supplementary. In $\square ABCD$, consecutive angles B and C are supplementary, as are consecutive angles C and D.



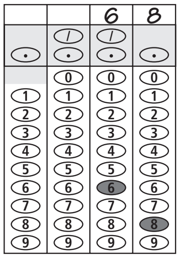
Differentiated Instruction Solutions for All Learners

Special Needs L1
Have students draw a parallelogram and, using two different colors, draw its diagonals. Students then cut out the four triangles formed. Students observe that the two pieces of each diagonal are congruent

learning style: tactile

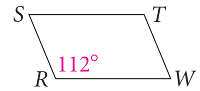
Below Level L2
Students can draw a parallelogram, cut along a diagonal, and manipulate the triangles thus formed to see the congruence relationships described in Theorems 6-1 and 6-2.

learning style: tactile



1 EXAMPLE Using Consecutive Angles

Gridded Response Find $m\angle S$ in $\square RSTW$.



$\angle R$ and $\angle S$ are consecutive angles of a parallelogram. They are supplementary.

$m\angle R + m\angle S = 180$	**Definition of supplementary angles**
$112 + m\angle S = 180$	**Substitute.**
$m\angle S = 68$	**Subtract 112 from each side.**

Quick Check

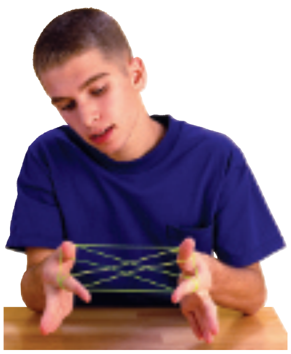
1 **Critical Thinking** If consecutive angles of a quadrilateral are supplementary, must the quadrilateral be a parallelogram? Explain. **Yes; by the Converse of the Same-Side Int. ∠s Thm., both pairs of opp. sides are ∥.**

A proof of Theorem 6-2 uses the consecutive angles of a parallelogram, and the fact that supplements of the same angle are congruent.

Key Concepts

Theorem 6-2

Opposite angles of a parallelogram are congruent.



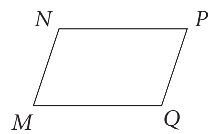
Real-World Connection

Opposite angles in the "cat's cradle" parallelogram (center) are congruent.

Plan for Proof of Theorem 6-2

Given: $\square MNPQ$

Prove: $\angle M \cong \angle P$ and $\angle N \cong \angle Q$



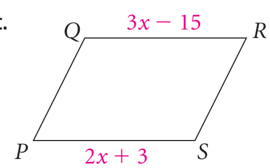
Plan: $\angle M \cong \angle P$ if they are supplements of the same angle, $\angle N$. Each is a supplement of $\angle N$ because same side interior angles are supplementary.
$\angle N \cong \angle Q$ using similar reasoning with $\angle M$.

You can use this plan to write a proof of Theorem 6-2 in Exercise 36. If you choose to write a flow proof, you'll find a guide on page 319.

You can use Theorems 6-1 and 6-2 along with algebra to find unknown values in parallelograms.

2 EXAMPLE Using Algebra

Algebra Find the value of x in $\square PQRS$. Then find QR and PS.

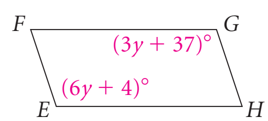


$3x - 15 = 2x + 3$	**Opposite sides of a ▱ are congruent.**
$x - 15 = 3$	**Subtract 2x from each side.**
$x = 18$	**Add 15 to each side.**
$QR = 3x - 15 = 39$	**Substitute.**

$\overline{PS} \cong \overline{QR}$, so $PS = 39$.

Quick Check

2 Find the value of y in $\square EFGH$. Then find $m\angle E$, $m\angle G$, $m\angle F$, and $m\angle H$.
11; $m\angle E = 70$, $m\angle G = 70$, $m\angle F = 110$, $m\angle H = 110$



2. Teach

Guided Instruction

1 EXAMPLE

Have students find all four angle measures in *RSTW* to prepare them for Theorem 6-2.

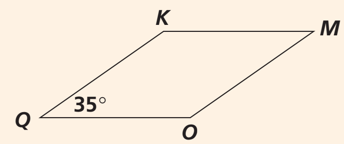
Alternative Method

Another way to prove Theorem 6-2 is to draw a diagonal and compare corresponding parts of congruent triangles twice, as was done once in the proof of Theorem 6-1.

PowerPoint

Additional Examples

1 Use $\square KMOQ$ to find $m\angle O$.



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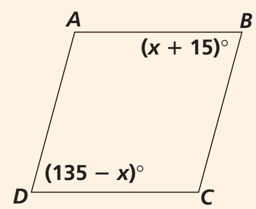
2 Find the value of x in $\square ABCD$. Then find $m\angle A$.



60; 105

Advanced Learners L4

Before reading the proof of Theorem 6-1, copy the diagram, and ask students which segments and angles can be marked congruent.

learning style: verbal

English Language Learners ELL

Make sure students understand the difference between *consecutive angles* and *adjacent angles*. Adjacent angles share a common vertex and a common side but no common interior points.

learning style: verbal

Guided Instruction

Technology Tip
Have students use geometry software to demonstrate Theorem 6-4.

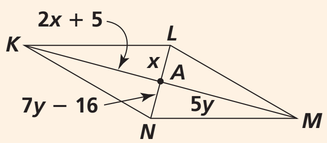
4 EXAMPLE Tactile Learners

Instruct students to repeat the example for themselves, tracing the edge of the blank card on the lined paper to help them see that the segments are congruent.

PowerPoint

Additional Examples

3 Find the values of x and y in ▱$KLMN$.



$x = 5, y = 3$

4 Explain how to divide a blank card into five equal rows using Theorem 6-4 and a piece of lined paper. **Place the card on the lined paper so that one corner of the card touches the first line of the paper and a consecutive corner of the card touches the sixth line. Mark the points where the lines intersect the card. Repeat the procedure on the opposite edge of the card. Connect the marks on opposite edges of the card using a straightedge.**

Resources

- Daily Notetaking Guide 6-2 L3
- Daily Notetaking Guide 6-2—Adapted Instruction L1

Closure

Lesson 6-1 defined a rectangle as a parallelogram with four right angles. Explain why you can now define a rectangle as a parallelogram with one right angle. **Because opposite ∠s of a parallelogram are ≅, there are two rt. ∠s. Because consecutive ∠s are supplementary, there are two more rt. ∠s.**

2 Properties: Diagonals and Transversals

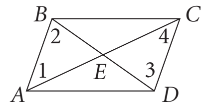
The diagonals of parallelograms have a special property.

Key Concepts

Theorem 6-3
The diagonals of a parallelogram bisect each other.

Proof

Proof of Theorem 6-3



Given: ▱$ABCD$
Prove: $\overline{AC}$ and $\overline{BD}$ bisect each other at E.

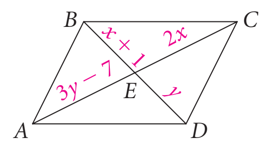
Statements	Reasons
1. $ABCD$ is a parallelogram.	**1.** Given
2. $\overline{AB} \parallel \overline{DC}$	**2.** Definition of parallelogram
3. $\angle 1 \cong \angle 4$; $\angle 2 \cong \angle 3$	**3.** Parallel lines form ≅ alt. int. ∠s.
4. $\overline{AB} \cong \overline{DC}$	**4.** Opposite sides of a ▱ are ≅.
5. $\triangle ABE \cong \triangle CDE$	**5.** ASA
6. $\overline{AE} \cong \overline{CE}$; $\overline{BE} \cong \overline{DE}$	**6.** CPCTC
7. $\overline{AC}$ and $\overline{BD}$ bisect each other at E.	**7.** Definition of bisector

Real-World Connection
The railing braces are diagonals of parallelograms so they bisect each other.

You can use Theorem 6-3 to find unknown lengths in parallelograms.

3 EXAMPLE Using Algebra

Solve a system of linear equations to find the values of x and y in ▱$ABCD$. Then find AE, EC, BE, and ED.



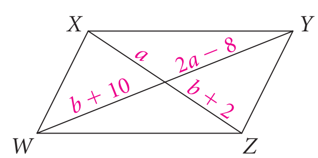
① $3y - 7 = 2x$ — **The diagonals of a parallelogram bisect each other.**
② $y = x + 1$

$3(x + 1) - 7 = 2x$ — **Substitute $x + 1$ for y in equation ①.**
$3x + 3 - 7 = 2x$ — **Distribute.**
$3x - 4 = 2x$ — **Simplify.**
$3x = 2x + 4$ — **Add 4 to each side.**
$x = 4$ — **Subtract $2x$ from each side.**
$3y - 7 = 2(4) = 8$ — **Substitute 4 for x in equations ① and ②.**
$y = 4 + 1 = 5$

$AE = EC = 8$ and $BE = ED = 5$.

Quick Check

3 Find the values of a and b.
$a = 16, b = 14$



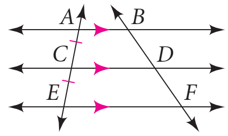
In Exercise 52, you will use Theorem 6-1, opposite sides of a parallelogram are congruent, to prove the following theorem.

Key Concepts

Theorem 6-4

If three (or more) parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

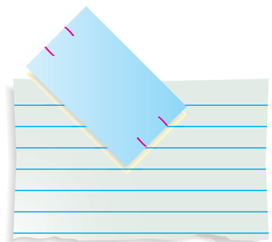
$\overline{BD} \cong \overline{DF}$



4 EXAMPLE **Real-World Connection**

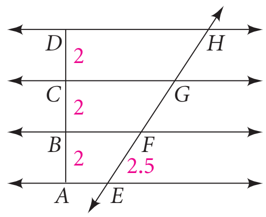
Measurement Show how to separate a blank card into three strips that are the same height by using lined paper, a straightedge, and Theorem 6-4.

The lines of the paper are parallel and equally spaced. Place a corner of the top edge of the card on the first line of the paper. Place the corner of the bottom edge on the fourth line. Mark the points where the second and third lines intersect the card. The marks will be equally spaced because the edge of the card is a transversal for the equally spaced parallel lines of the paper. Repeat for the other side of the card. Connect the marks using a straightedge.



Quick Check

4. In the figure at the right, $\overleftrightarrow{DH} \parallel \overleftrightarrow{CG} \parallel \overleftrightarrow{BF} \parallel \overleftrightarrow{AE}$, $AB = BC = CD = 2$, and $EF = 2.5$. Find EH. **7.5**



EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

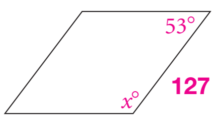
Practice and Problem Solving

A Practice by Example

Algebra **Find the value of x in each parallelogram.**

Example 1 (page 313)

GO for Help

1. 

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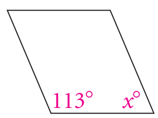
2. 

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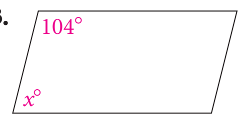
3. 

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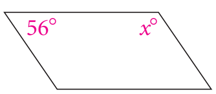
4. 

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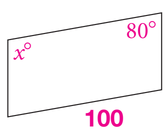
5. 

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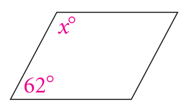
6. 

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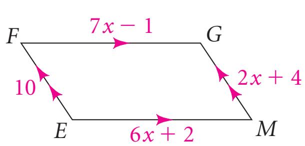
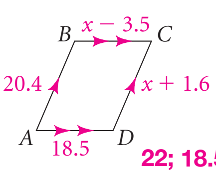
Example 2 (page 313)

Algebra **Find the value of x and the length of each side.**

7. 

3; 10, 20, 20

8. 

22; 18.5, 23.6, 23.6

3. Practice

Assignment Guide

1 A B 1-13, 31-39, 41-44, 48-50

2 A B 14-30, 40, 45-47

C Challenge 51-53

Test Prep 54-60
Mixed Review 61-67

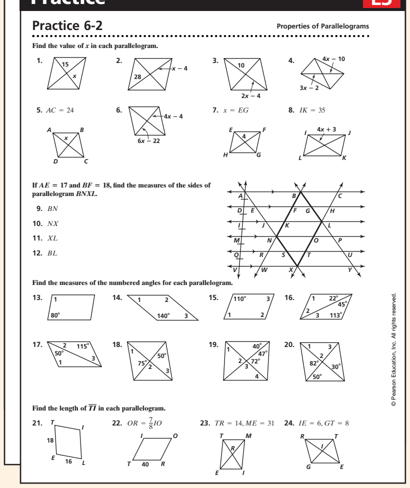
Homework Quick Check
To check students' understanding of key skills and concepts, go over Exercises 9, 16, 34, 41, 44.

Exercises 1–6 These exercises can be done orally using mental math. As students answer, have them explain and justify their methods of solving for x.

Exercises 7–13 Ask students to name the property described in the theorem they use.

Differentiated Instruction Resources

GPS Guided Problem Solving L3
Enrichment L4
Reteaching L2
Adapted Practice L1
Practice L3



Error Prevention!

Exercises 14–18 Some students may misinterpret Theorem 6-3, *The diagonals of a parallelogram bisect each other*, and think that the diagonals of a parallelogram are congruent.

Connection to History

Exercise 20 Isaac Merrit Singer made the first practical sewing machine with a foot treadle replacing the hand crank in 1851. His first machines sold for $75 each, a huge amount in those days, inspiring him to devise an installment buying plan for his customers.

Exercise 33 Ask: *What relationships can you write for the parallelogram?* $3x = y$, $y + 3 = 180$, and $3y + 3x = 180$

Tactile Learners

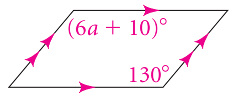
Exercise 51 Part a can be demonstrated quickly by threading string through four pieces of straws and manipulating the shape to form different parallelograms.

12. 12; $m\angle Q = m\angle S = 36$, $m\angle P = m\angle R = 144$
13. 6; $m\angle H = m\angle J = 30$, $m\angle I = m\angle K = 150$
14. $x = 6, y = 8$
15. $x = 5, y = 7$
16. $x = 7, y = 10$
17. $x = 6, y = 9$
18. $x = 3, y = 4$
31. $BC = AD = 14.5$ in.; $AB = CD = 9.5$ in.
32. $BC = AD = 33$ cm; $AB = CD = 13$ cm
34. The opp. ∠s are ≅, so they have = measures. Consecutive ∠s are suppl., so their sum is 180.

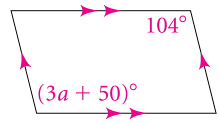
20. Pick 4 equally spaced lines on the paper. Place the paper so that the first button is on the first line and the last button is on the fourth line. Draw a line between the first and last buttons. The remaining buttons should be placed where the drawn line crosses the 2 ∥ lines on the paper.

Algebra Find the value of a.

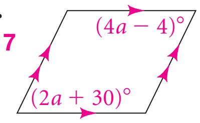
9. 20



10. 18

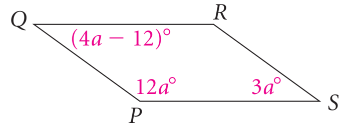


11. 17

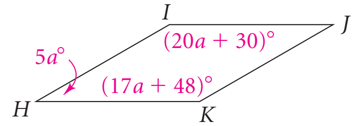


Find the value of a and the measure of each angle in each parallelogram.

12.



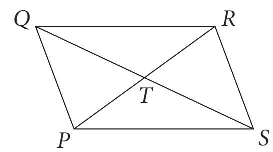
13.



12–18. See margin.

Example 3 (page 314)

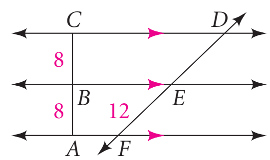
Algebra Find the values of x and y in ▱$PQRS$.



14. $PT = 2x, TR = y + 4, QT = x + 2, TS = y$
15. $PT = x + 2, TR = y, QT = 2x, TS = y + 3$
16. $PT = y, TR = x + 3, QT = 2y, TS = 3x - 1$
17. $PT = 2x, TR = y + 3, QT = 3x, TS = 2y$
18. $PT = 8x, TR = 6y, QT = 2x + 2, TS = 2y$

Example 4 (page 315)

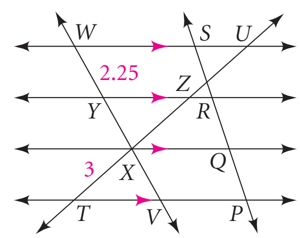
19. Find ED and FD in the figure at the right. 12; 24



Exercise 20

20. **Sewing** Suppose you don't have a ruler. Explain how to space four buttons equally on a shirt if you know where the first and last buttons must be placed and you have a large piece of lined paper. See above left.

In the figure, the horizontal lines are parallel and $PQ = QR = RS$. Find each length.



21. ZU 3
22. XZ 3
23. XU 6
24. TZ 6
25. TU 9
26. XV 2.25
27. YX 2.25
28. YV 4.5
29. WX 4.5
30. WV 6.75

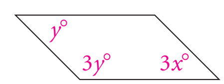
B Apply Your Skills

Algebra Use the given information to find the lengths of all four sides of ▱$ABCD$.

31–32. See margin.

31. The perimeter is 48 in. AB is 5 in. less than BC.
32. The perimeter is 92 cm. AD is 7 cm more than twice AB.

33. **Multiple Choice** What is the value of x in the parallelogram? A



Ⓐ 15 Ⓑ 45
Ⓒ 60 Ⓓ 135

34. **Writing** Explain how to find the measures of the remaining three angles of a parallelogram if you already know the measure of one of the angles. See margin.

40. The lines going accross may not be ∥ since they are not marked as ∥.
42. Answers may vary. Sample:

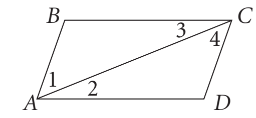
1. *LENS* and *NGTH* are ▱s. (Given)
2. ∠*ELS* ≅ ∠*ENS* and ∠*GTH* ≅ ∠*GNH* (Opp. ∠s of a ▱ are ≅.)
3. ∠*ENS* ≅ ∠*GNH* (Vertical ∠s are ≅.)
4. ∠*ELS* ≅ ∠*GTH* (Trans. Prop. of ≅)

35 a. $\overline{DC}$
b. $\overline{AD}$
c. $\cong$
d. Reflexive
e. ASA
f. CPCTC

35. Developing Proof Complete this paragraph proof of Theorem 6-1 by filling in the blanks.

Given: $\square ABCD$

Prove: $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$



Proof: $ABCD$ is a parallelogram, therefore $\overline{AB} \parallel$ **a.** _?_ and $\overline{BC} \parallel$ **b.** _?_. $\angle 1 \cong \angle 4$ and $\angle 3 \cong \angle 2$, because alternate interior angles are **c.** _?_. $\overline{AC} \cong \overline{AC}$ by the **d.** _?_ Property of Congruence. Therefore $\triangle ABC \cong \triangle CDA$ by **e.** _?_. So, $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$ because **f.** _?_.

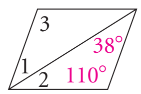
GO for Help

For a guide to solving Exercise 36, see p. 319.

Proof **36.** Write a proof for Theorem 6-2. You may wish to follow the plan on page 313. **See back of book.**

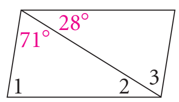
Find the measures of the numbered angles for each parallelogram.

37.



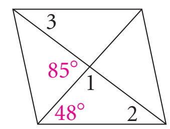
38, 32, 110

38.



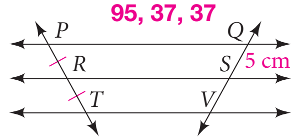
81, 28, 71

39.



95, 37, 37

40. Error Analysis Brian states that $QV = 10$ cm in the figure at the right. Explain why Brian's statement may not be correct. **See margin.**



41. Algebra In a parallelogram one angle is 9 times the size of another. Find the measures of the angles. **18, 162**

Proof **Write a paragraph proof, flow proof, or two-column proof.** **42–44. See margin, pp. 317–318.**

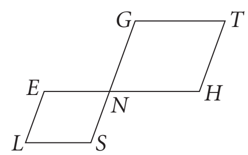
Problem Solving Hint

For each of Exercises 42–44, sketch the diagram and mark it as you think through a proof.

42. Given: $\square LENS$ and $\square NGTH$
Prove: $\angle L \cong \angle T$

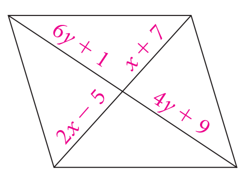
43. Given: $\square LENS$ and $\square NGTH$
Prove: $\overline{LS} \parallel \overline{GT}$

44. Given: $\square LENS$ and $\square NGTH$
Prove: $\angle E$ is supplementary to $\angle T$.



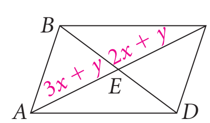
Algebra **Find the value(s) of the variable(s) in each parallelogram.**

45.



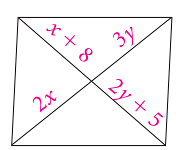
$x = 12, y = 4$

46. $AC = 4x + 10$



$x = 0, y = 5$

47.

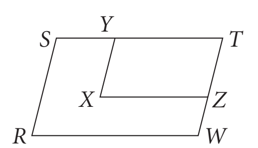


$x = 9, y = 6$

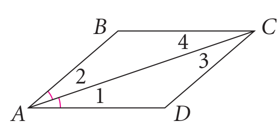
Proof **Write a paragraph proof, flow proof, or two-column proof.**

48. Given: $\square RSTW$ and $\square XYTZ$ **48–50. See margin.**
Prove: $\angle R \cong \angle X$

49. Given: $\square RSTW$ and $\square XYTZ$
Prove: $\overline{XY} \parallel \overline{RS}$



50. Given: $\square ABCD$ and $\overline{AC}$ bisects $\angle DAB$.
Prove: $\overline{AC}$ bisects $\angle DCB$.



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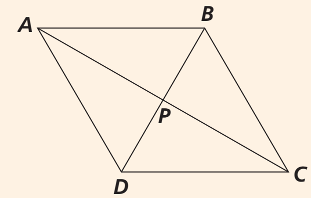
4. Assess & Reteach

PowerPoint

Lesson Quiz

Use parallelogram *ABCD* for Exercises 1–5.



1. If $AB = 3x + 11$, $BC = 2x + 19$, and $CD = 7x - 17$, find x. **7**
2. If $m\angle BAD = y$ and $m\angle ADC = 4y - 70$, find y. **50**
3. If $m\angle ABC = 2x + 100$ and $m\angle ADC = 6x + 84$, find $m\angle BCD$. **72**
4. If $m\angle BCD = 80$ and $m\angle CAD = 34$, find $m\angle ACD$. **46**
5. If $AP = 3x$, $BP = y$, $CP = x + y$, and $DP = 6x - 40$, find x and y. **$x = 10, y = 20$**

Alternative Assessment

Have students draw and label a parallelogram and then name all the congruent sides, angles, and diagonals.

48. Answers may vary. Sample: In $\square RSTW$ and $\square XYTZ$, $\angle R \cong \angle T$ and $\angle X \cong \angle T$ because opp. ∠s of a ▱ are ≅. Then $\angle R \cong \angle X$ by the Trans. Prop. of ≅.

49. In $\square RSTW$ and $\square XYTZ$, $\overline{XY} \parallel \overline{TW}$ and $\overline{RS} \parallel \overline{TW}$ by the def. of a ▱. Then $\overline{XY} \parallel \overline{RS}$ because if 2 lines are ∥ to the same line, then they are ∥ to each other.

50. $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$ by def. of ▱. $\angle 2 \cong \angle 3$ and $\angle 1 \cong \angle 4$ because if 2 lines are ∥, then alt. int. ∠s are ≅. $\angle 3 \cong \angle 4$ because if 2 ∠s are each ≅ to 2 ≅ ∠s, then they are ≅. By def. of bisect, $\overline{AC}$ bisects $\angle DCB$.

43. Answers may vary. Sample: In ▱s $LENS$ and $NGTH$, $\overline{GT} \parallel \overline{EH}$ and $\overline{EH} \parallel \overline{LS}$ by the Def. of a ▱. Therefore $\overline{LS} \parallel \overline{GT}$ because if 2 lines are ∥ to the same line then they are ∥ to each other.

44. Answers may vary. Sample:
1. $LENS$ and $NGTH$ are ▱s (Given)
2. $\angle GTH \cong \angle GNH$ (Opp. ∠s of a ▱ are ≅.)
3. $\angle ENS \cong \angle GNH$ (Vertical ∠s are ≅.)
4. $\angle LEN$ is supp. to $\angle ENS$ (Consec. ∠s in a ▱ are suppl.)
5. $\angle ENS \cong \angle GTH$ (Trans. Prop. of ≅)
6. $\angle E$ is suppl. to $\angle T$ (Suppl. of ≅ ∠s are suppl.)

Test Prep

A sheet of blank grids is available in the Test-Taking Strategies with Transparencies booklet. Give this sheet to students for practice with filling in the grids.

Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 361
- Test-Taking Strategies, p. 356
- Test-Taking Strategies with Transparencies

Challenge

51. a. Open-Ended Sketch two parallelograms whose corresponding sides are congruent but whose corresponding angles are not congruent.

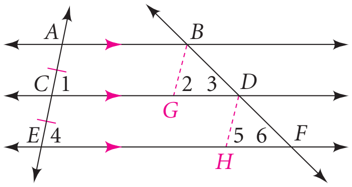
b. Critical Thinking Is there an SSSS congruence theorem for parallelograms? Explain. **a–b. See margin.**

Proof **52.** Prove Theorem 6-4.

Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$ and $\overline{AC} \cong \overline{CE}$

Prove: $\overline{BD} \cong \overline{DF}$ **See margin.**

(*Hint*: Draw lines through B and D parallel to $\overleftrightarrow{AE}$ and intersecting $\overleftrightarrow{CD}$ at G and $\overleftrightarrow{EF}$ at H.)



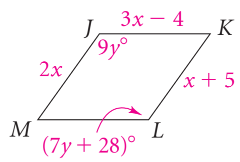
Proof **53. a.** Prove that if two sides and the included angle of one parallelogram are congruent to corresponding parts of another parallelogram, then the parallelograms are congruent. (*Hint:* Prove that all the corresponding parts of the parallelograms are congruent.) **a–b. See margin.**

b. Is there a theorem similar to SAS for trapezoids? Explain.

Test Prep

Gridded Response

Use the parallelogram at the right for Exercises 54–57. Find the indicated segment length or angle measure.



54. JM **10** **55.** ML **11** **56.** $m\angle L$ **126** **57.** $m\angle J$ **126**

58. The measures of three angles in a parallelogram are 20, 160, and 20. Find the measure of the fourth angle. **160**

59. The measures of two angles in a parallelogram are 32 and 32. Find the measure of one of the other two angles. **148**

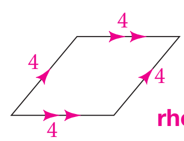
60. Two consecutive angles in a parallelogram have measures $x + 5$ and $4x - 10$. Find the measure of the smaller angle. **42**

Mixed Review

GO for Help

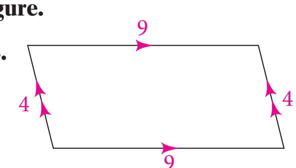
Lesson 6-1 **Determine the most precise name for each figure.**

61.



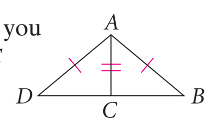
rhombus

62.

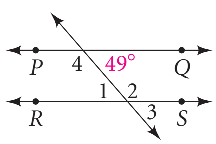


parallelogram

Lesson 4-6 **63.** What additional information do you need to prove $\triangle ADC \cong \triangle ABC$ by the HL Theorem? $\overline{AC} \perp \overline{DB}$



Lesson 3-1 **In the figure at the right, $\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$. Find each measure.**



64. $m\angle 1$ **49** **65.** $m\angle 2$ **131** **66.** $m\angle 3$ **49** **67.** $m\angle 4$ **131**

51. a. Answers may vary. Check students' work.

b. No; the corr. sides can be ≅ but the ∡ may not be.

52. a. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$ and $\overline{AC} \cong \overline{CE}$ (Given).

b. $ABGC$ and $CDHE$ are parallelograms. (Def. of a ▱)

c. $\overline{BG} \cong \overline{AC}$ and $\overline{DH} \cong \overline{CE}$ (Opp. sides of a ▱ are ≅.)

d. $\overline{BG} \cong \overline{DH}$ (Trans. Prop. of ≅.)

e. $\overline{BG} \parallel \overline{DH}$ (If two lines are ∥ to the same line, then they are ∥ to each other.)

f. $\angle 2 \cong \angle 1$, $\angle 1 \cong \angle 4$, $\angle 4 \cong \angle 5$, and $\angle 3 \cong \angle 6$ (If 2 lines are ∥, then the corr. ∡ are ≅.)

g. $\angle 2 \cong \angle 5$ (Trans. Prop. of ≅)

h. $\triangle BGD \cong \triangle DHF$ (AAS)

i. $\overline{BD} \cong \overline{DF}$ (CPCTC)

53. a. Given: 2 sides and the included ∠ of ▱$ABCD$ are ≅ to the corr. parts of ▱$WXYZ$. Let $\angle A \cong \angle W$, $\overline{AB} \cong \overline{WX}$ and $\overline{AD} \cong \overline{WZ}$. Since opp. ∡ of a ▱ are ≅, $\angle A \cong \angle C$ and $\angle W \cong \angle Y$. Thus $\angle C \cong \angle Y$ by the Trans. Prop. of ≅. Similarly, opp. sides of a ▱ are ≅, thus $\overline{AB} \cong \overline{CD}$ and $\overline{WX} \cong \overline{ZY}$. Using the Trans. Prop. of ≅, $\overline{CD} \cong \overline{ZY}$. The same can be done to prove $\overline{BC} \cong \overline{XY}$. Since consec. ∡ of a ▱ are suppl., $\angle A$ is suppl. to $\angle D$, and $\angle W$ is suppl. to $\angle Z$. Suppls. of ≅ ∡ are ≅, thus $\angle D \cong \angle Z$. The same can be done to prove $\angle B \cong \angle X$. Therefore, since all corr. ∡ and sides are ≅, ▱$ABCD$ ≅ ▱$WXYZ$.

b. No; opp. ∡ and sides are not necessarily ≅ in a trapezoid.