Use substitution to solve each system of equations.

1. $y = x + 5$
   $3x + y = 25$

SOLUTION:

$y = x + 5$
$3x + y = 25$

Substitute $x + 5$ for y in the second equation.

$$
\begin{aligned}
3x + y &= 25 && \text{Second equation} \\
3x + (x + 5) &= 25 && \text{Substitution.} \\
4x + 5 &= 25 && \text{Simplify.} \\
4x + 5 - 5 &= 25 - 5 && \text{Subtract 5 from each side.} \\
4x &= 20 && \text{Simplify.} \\
\frac{4x}{4} &= \frac{20}{4} && \text{Divide each side by 20.} \\
x &= 5 && \text{Simplify.}
\end{aligned}
$$

Substitute the solution for x into either equation to find y.

$$
\begin{aligned}
y &= x + 5 && \text{First equation} \\
&= 5 + 5 && \text{Substitution} \\
&= 10 && \text{Simplify.}
\end{aligned}
$$

The solution is (5, 10).

2. $x = y - 2$
$4x + y = 2$

SOLUTION:

$x = y - 2$
$4x + y = 2$
Substitute $y - 2$ for x in the second equation.

$$4x + y = 2 \qquad \text{Second equation}$$
$$4(y - 2) + y = 2 \qquad \text{Substitution}$$
$$4y - 8 + y = 2 \qquad \text{Distributive Property}$$
$$5y - 8 = 2 \qquad \text{Simplify.}$$
$$5y - 8 + 8 = 2 + 8 \qquad \text{Add 8 to each side.}$$
$$5y = 10 \qquad \text{Simplify.}$$
$$\frac{5y}{5} = \frac{10}{5} \qquad \text{Divide each side by 5.}$$
$$y = 2 \qquad \text{Simplify.}$$

Use the solution for y and either equation to find x.

$$x = y - 2 \qquad \text{First equation}$$
$$= 2 - 2 \qquad \text{Substitution}$$
$$= 0 \qquad \text{Simplify.}$$

The solution is (0, 2).

6-2 Substitution

3. $3x + y = 6$
   $4x + 2y = 8$

SOLUTION:

$3x + y = 6$
$4x + 2y = 8$

First, solve the first equation for y to get $y = -3x + 6$. Then substitute $-3x + 6$ for y in the second equation.

$4x + 2y = 8$	Second equation
$4x + 2(-3x + 6) = 8$	Substitution
$4x - 6x + 12 = 8$	Distributive Property
$-2x + 12 = 8$	Simplify.
$-2x + 12 - 12 = 8 - 12$	Add 12 to each side.
$-2x = -4$	Simplify.
$\dfrac{-2x}{-2} = \dfrac{-4}{-2}$	Divide each side by -2.
$x = 2$	Simplify.

Use the solution for x and either equation to find y.

$3x + y = 6$	First equation
$3(2) + y = 6$	Substitution
$6 + y = 6$	Simplify.
$6 - 6 + y = 6 - 6$	Subtract 6 from each side.
$y = 0$	Simplify.

The solution is (2, 0).

4. $2x + 3y = 4$
   $4x + 6y = 9$

SOLUTION:

$2x + 3y = 4$
$4x + 6y = 9$
First, solve the first equation for x.

$$2x + 3y = 4 \qquad \text{First equation}$$
$$2x + 3y - 3y = 4 - 3y \qquad \text{Subtract } 3y \text{ from each side.}$$
$$2x = 4 - 3y \qquad \text{Simplify.}$$
$$\frac{2x}{2} = \frac{4 - 3y}{2} \qquad \text{Divide each side by 2.}$$
$$x = 2 - \frac{3}{2}y \qquad \text{Simplify.}$$

Then substitute $2 - \frac{3}{2}y$ for x in the second equation.

$$4x + 6y = 9 \qquad \text{Second equation}$$
$$4\left(2 - \frac{3}{2}y\right) + 6y = 9 \qquad \text{Substitution}$$
$$8 - 6y + 6y = 9 \qquad \text{Simplify.}$$
$$8 \neq 9 \qquad \text{Simplify.}$$

Since the left side does not equal the right, there is no solution to this system of equations.

5. $x - y = 1$
   $3x = 3y + 3$

SOLUTION:

$x - y = 1$
$3x = 3y + 3$
First, solve the first equation for x to get $x = y + 1$. Then substitute $y + 1$ for x in the second equation.

$$3x = 3y + 3 \qquad \text{Second equation}$$
$$3(y + 1) = 3y + 3 \qquad \text{Substitution}$$
$$3y + 3 = 3y + 3 \qquad \text{Distributive Property}$$
$$3y - 3y + 3 = 3y - 3y + 3 \qquad \text{Subtract } 3y \text{ from each side.}$$
$$3 = 3 \qquad \text{Simplify.}$$

This equation is an identity. So, there are infinitely many solutions.

6. $2x - y = 6$
$-3y = -6x + 18$

SOLUTION:

$2x - y = 6$
$-3y = -6x + 18$
First, solve the first equation for y to get $2x - 6 = y$. Then substitute $2x - 6$ for y in the second equation.

$$
\begin{aligned}
-3y &= -6x + 18 && \text{Second equation} \\
-3(2x - 6) &= -6x + 18 && \text{Substitution} \\
-6x + 18 &= -6x + 18 && \text{Distributive Property} \\
-6x + 6x + 18 &= -6x + 6x + 18 && \text{Add } 6x \text{ to each side.} \\
18 &= 18 && \text{Simplify.}
\end{aligned}
$$

This equation is an identity. So, there are infinitely many solutions.

7. **GEOMETRY** The sum of the measures of angles X and Y is 180°. The measure of angle X is 24° greater than the measure of angle Y.

a. Define the variables, and write equations for this situation.

b. Find the measure of each angle.

SOLUTION:

a. Let $x = m\angle X$, and $y = m\angle Y$; The sum means to add so the first equation is $x + y = 180$. Greater than means addition as well, so the second equation is $x = 24 + y$.

b. $x + y = 180$
$x = 24 + y$
Substitute the second equation into the first equation.

$$\begin{aligned} x + y &= 180 && \text{Second equation} \\ (24 + y) + y &= 180 && \text{Substitution} \\ 24 + 2y &= 180 && \text{Simplify.} \\ 24 - 24 + 2y &= 180 - 24 && \text{Subtract 24 from each side.} \\ 2y &= 156 && \text{Simplify.} \\ \frac{2y}{2} &= \frac{156}{2} && \text{Divide each side by 2.} \\ y &= 78 && \text{Simplify.} \end{aligned}$$

Use the solution for y and either equation to find x.

$$\begin{aligned} x &= 24 + y && \text{First equation} \\ &= 24 + (78) && \text{Substitution} \\ &= 102 && \text{Simplify.} \end{aligned}$$

So, $m\angle X = 102°$ and $m\angle Y = 78°$.

Use substitution to solve each system of equations.

8. $y = 5x + 1$
   $4x + y = 10$

SOLUTION:

$y = 5x + 1$
$4x + y = 10$
Substitute $5x + 1$ for y in the second equation.

$4x + y = 10$	Second equation
$4x + (5x + 1) = 10$	Substitute $5x + 1$ for y.
$9x + 1 = 10$	Combine like terms.
$9x + 1 - 1 = 10 - 1$	Subtract 1 from each side.
$9x = 9$	Simplify.
$\frac{9x}{9} = \frac{9}{9}$	Divide each side by 9.
$x = 1$	Simplify.

Use the solution for x and either equation to find y.

$y = 5x + 1$	First equation
$y = 5(1) + 1$	Substitute 1 for x.
$y = 6$	Simplify.

The solution is (1, 6).

9. $y = 4x + 5$
$2x + y = 17$

SOLUTION:

$y = 4x + 5$
$2x + y = 17$
Substitute $4x + 5$ for y in the second equation.

$$
\begin{array}{lll}
2x + y &= 17 & \text{Second equation} \\
2x + (4x + 5) &= 17 & \text{Substitute } 4x + 5 \text{ for } y. \\
6x + 5 &= 17 & \text{Combine like terms.} \\
6x + 5 - 5 &= 17 - 5 & \text{Subtract 5 from each side.} \\
6x &= 12 & \text{Simplify.} \\
\dfrac{6x}{6} &= \dfrac{12}{6} & \text{Divide each side by 6.} \\
x &= 2 & \text{Simplify.}
\end{array}
$$

Use the solution for x and either equation to find y.

$$
\begin{array}{lll}
y &= 4x + 5 & \text{First equation} \\
y &= 4(2) + 5 & \text{Substitute 2 for } x. \\
y &= 13 & \text{Simplify.}
\end{array}
$$

The solution is (2, 13).

6-2 Substitution

10. $y = 3x - 34$
    $y = 2x - 5$

 SOLUTION:

$y = 3x - 34$
$y = 2x - 5$
Substitute $2x - 5$ for y in the first equation.

$$
\begin{aligned}
y &= 3x - 34 & &\text{First equation} \\
2x - 5 &= 3x - 34 & &\text{Substitute } 2x - 5 \text{ for } y. \\
2x - 5 + 5 &= 3x - 34 + 5 & &\text{Add 5 to each side.} \\
2x &= 3x - 29 & &\text{Simplify.} \\
2x - 3x &= 3x - 29 - 3x & &\text{Subtract } 3x \text{ from each side.} \\
-1x &= -29 & &\text{Simplify.} \\
x &= 29 & &\text{Divide each side by } -1.
\end{aligned}
$$

Use the solution for x and either equation to find y.

$$
\begin{aligned}
y &= 2x - 5 & &\text{Second equation} \\
y &= 2(29) - 5 & &\text{Substitute 29 for } x. \\
y &= 58 - 5 & &\text{Multiply.} \\
y &= 53 & &\text{Simplify.}
\end{aligned}
$$

The solution is (29, 53).

11. $y = 3x - 2$
   $y = 2x - 5$

   SOLUTION:

   $y = 3x - 2$
   $y = 2x - 5$

   Substitute $2x - 5$ for y in the first equation.

   $$
   \begin{aligned}
   y &= 3x - 2 && \text{First equation} \\
   2x - 5 &= 3x - 2 && \text{Substitute } 2x - 5 \text{ for } y. \\
   2x - 5 + 5 &= 3x - 2 + 5 && \text{Add 5 to each side.} \\
   2x &= 3x + 3 && \text{Simplify.} \\
   2x - 3x &= 3x + 3 - 3x && \text{Subtract } 3x \text{ from each side.} \\
   -1x &= 3 && \text{Simplify.} \\
   x &= -3 && \text{Divide each side by } -1.
   \end{aligned}
   $$

   Use the solution for x and either equation to find y.

   $$
   \begin{aligned}
   y &= 3x - 2 && \text{Second equation} \\
   y &= 3(-3) - 2 && \text{Substitute } -3 \text{ for } x. \\
   y &= -9 - 2 && \text{Multiply.} \\
   y &= -11 && \text{Simplify.}
   \end{aligned}
   $$

   The solution is $(-3, -11)$.

12. $2x + y = 3$
    $4x + 4y = 8$

SOLUTION:

$2x + y = 3$
$4x + 4y = 8$

First, solve the first equation for y to get $y = -2x + 3$. Then substitute $-2x + 3$ for y in the second equation.

$$
\begin{aligned}
4x + 4y &= 8 && \text{Second equation} \\
4x + 4(-2x + 3) &= 8 && \text{Substitute } -2x + 3 \text{ for } y. \\
4x - 8x + 12 &= 8 && \text{Distributive Property} \\
-4x + 12 &= 8 && \text{Combine like terms.} \\
-4x + 12 - 12 &= 8 - 12 && \text{Subtract 12 from each side.} \\
-4x &= -4 && \text{Simplify.} \\
\frac{-4x}{-4} &= \frac{-4}{-4} && \text{Divide each side by } -4. \\
x &= 1 && \text{Simplify.}
\end{aligned}
$$

Use the solution for x and either equation to find y.

$$
\begin{aligned}
2x + y &= 3 && \text{First equation} \\
2(1) + y &= 3 && \text{Substitute 1 for } x. \\
2 + y &= 3 && \text{Multiply.} \\
y &= 1 && \text{Subtract 2 from each side.}
\end{aligned}
$$

The solution is (1, 1).

13. $3x + 4y = -3$
    $x + 2y = -1$

    SOLUTION:

    $3x + 4y = -3$
    $x + 2y = -1$

    First, solve the second equation for x to get $x = -2y - 1$. Then, substitute $-2y - 1$ for x in the first equation.

$$
\begin{aligned}
3x + 4y &= -3 && \text{First equation} \\
3(-2y - 1) + 4y &= -3 && \text{Substitute } -2y - 1 \text{ for } x. \\
-6y - 3 + 4y &= -3 && \text{Distributive Property} \\
-2y - 3 &= -3 && \text{Combine like terms.} \\
-2y - 3 + 3 &= -3 + 3 && \text{Add 3 to each side.} \\
-2y &= 0 && \text{Simplify.} \\
\frac{-2y}{-2} &= \frac{0}{-2} && \text{Divide each side by } -2. \\
y &= 0 && \text{Simplify.}
\end{aligned}
$$

Use the solution for y and either equation to find x.

$$
\begin{aligned}
x + 2y &= -1 && \text{Second equation} \\
x + 2(0) &= -1 && \text{Substitute 0 for } y. \\
x + 0 &= -1 && \text{Multiply.} \\
x &= -1 && \text{Simplify.}
\end{aligned}
$$

The solution is $(-1, 0)$.

14. $y = -3x + 4$
    $-6x - 2y = -8$

    SOLUTION:

    $y = -3x + 4$
    $-6x - 2y = -8$

    Substitute $-3x + 4$ for y in the second equation.

$$
\begin{aligned}
-6x - 2y &= -8 && \text{Second equation} \\
-6x - 2(-3x + 4) &= -8 && \text{Substitute } -3x + 4 \text{ for } y. \\
-6x + 6x - 8 &= -8 && \text{Distributive Property} \\
-8 &= -8 && \text{Combine like terms.}
\end{aligned}
$$

This equation is an identity. So, there are infinitely many solutions.

15. $-1 = 2x - y$
$8x - 4y = -4$

SOLUTION:

$-1 = 2x - y$
$8x - 4y = -4$
First, solve the first equation for y to get $1 + 2x = y$. Then substitute $1 + 2x$ for y in the second equation.

$$
\begin{aligned}
8x - 4y &= -4 && \text{Second equation} \\
8x - 4(1 + 2x) &= -4 && \text{Substitute } 1 + 2x \text{ for } y. \\
8x - 4 - 8x &= -4 && \text{Distributive Property} \\
-4 &= -4 && \text{Combine like terms.}
\end{aligned}
$$

This equation is an identity. So, there are infinitely many solutions.

16. $x = y - 1$
$-x + y = -1$

SOLUTION:

$x = y - 1$
$-x + y = -1$
Substitute $y - 1$ for x in the second equation.

$$
\begin{aligned}
-x + y &= -1 && \text{Second equation} \\
-(y - 1) + y &= -1 && \text{Substitute } y - 1 \text{ for } x. \\
-y + 1 + y &= -1 && \text{Distributive Property} \\
1 &\neq -1 && \text{Combine like terms.}
\end{aligned}
$$

Since the left side does not equal the right, there is no solution to this system of equations.

6-2 Substitution

17. $y = -4x + 11$
    $3x + y = 9$

SOLUTION:

$y = -4x + 11$
$3x + y = 9$
Substitute $-4x + 11$ for y in the second equation.

$$
\begin{aligned}
3x + y &= 9 && \text{Second equation} \\
3x + (-4x + 11) &= 9 && \text{Substitute } -4x + 11 \text{ for } y. \\
-1x + 11 &= 9 && \text{Combine like terms.} \\
-1x + 11 - 11 &= 9 - 11 && \text{Subtract 11 from each side.} \\
-1x &= -2 && \text{Simplify.} \\
\frac{-1x}{-1} &= \frac{-2}{-1} && \text{Divide each side by } -1. \\
x &= 2 && \text{Simplify.}
\end{aligned}
$$

Use the solution for x and either equation to find y.

$$
\begin{aligned}
y &= -4x + 11 && \text{First equation} \\
y &= -4(2) + 11 && \text{Substitute 2 for } x. \\
y &= -8 + 11 && \text{Multiply.} \\
y &= 3 && \text{Simplify.}
\end{aligned}
$$

The solution is (2, 3).

18. $y = -3x + 1$
    $2x + y = 1$

    SOLUTION:

    $y = -3x + 1$
    $2x + y = 1$
    Substitute $-3x + 1$ for y in the second equation.

$$
\begin{array}{rll}
2x + y &= 1 & \text{Second equation} \\
2x + (-3x + 1) &= 1 & \text{Substitute } -3x + 1 \text{ for } y. \\
-1x + 1 &= 1 & \text{Combine like terms.} \\
-1x + 1 - 1 &= 1 - 1 & \text{Subtract 1 from each side.} \\
-1x &= 0 & \text{Simplify.} \\
\frac{-1x}{-1} &= \frac{0}{-1} & \text{Divide each side by } -1. \\
x &= 0 & \text{Simplify.}
\end{array}
$$

Use the solution for x and either equation to find y.

$$
\begin{array}{rll}
y &= -3x + 1 & \text{First equation} \\
y &= -3(0) + 1 & \text{Substitute 0 for } x. \\
y &= 0 + 1 & \text{Multiply.} \\
y &= 1 & \text{Simplify.}
\end{array}
$$

The solution is (0, 1).

19. $3x + y = -5$
    $6x + 2y = 10$

    SOLUTION:

    $3x + y = -5$
    $6x + 2y = 10$
    First, solve the first equation for y to get $y = -3x - 5$. Then substitute $-3x - 5$ for y in the second equation.

$$
\begin{array}{rll}
6x + 2y &= 10 & \text{Second equation} \\
6x + 2(-3x - 5) &= 10 & \text{Substitute } -3x - 5 \text{ for } y. \\
6x - 6x - 10 &= 10 & \text{Distributive Property} \\
-10 &\neq 10 & \text{Combine like terms.}
\end{array}
$$

Since the left side does not equal the right, there is no solution to this system of equations.

20. $5x - y = 5$
$-x + 3y = 13$

SOLUTION:

$5x - y = 5$
$-x + 3y = 13$
First, solve the second equation for x to get $x = 3y - 13$. Then substitute $3y - 13$ for x in first equation.

$$
\begin{aligned}
5x - y &= 5 & \text{First equation} \\
5(3y - 13) - y &= 5 & \text{Substitute } 3y - 13 \text{ for } x. \\
15y - 65 - y &= 5 & \text{Distributive Property} \\
14y - 65 &= 5 & \text{Combine like terms.} \\
14y - 65 + 65 &= 5 + 65 & \text{Add 65 to each side.} \\
14y &= 70 & \text{Simplify.} \\
\frac{14y}{14} &= \frac{70}{14} & \text{Divide each side by 14.} \\
y &= 5 & \text{Simplify.}
\end{aligned}
$$

Use the solution for y and either equation to find x.

$$
\begin{aligned}
-x + 3y &= 13 & \text{Second equation} \\
-x + 3(5) &= 13 & \text{Substitute 5 for } y. \\
-x + 15 &= 13 & \text{Multiply.} \\
-x &= -2 & \text{Subtract 15 from each side.} \\
x &= 2 & \text{Divide each side by } -1.
\end{aligned}
$$

The solution is (2, 5).

21. $2x + y = 4$
$-2x + y = -4$

SOLUTION:
$2x + y = 4$
$-2x + y = -4$
First, solve the first equation for y to get $y = -2x + 4$. Then, substitute $-2x + 4$ for y in the second equation.

$$
\begin{aligned}
-2x + y &= -4 & \text{Second equation} \\
-2x + (-2x + 4) &= -4 & \text{Substitute } -2x + 4 \text{ for } y. \\
-4x + 4 &= -4 & \text{Combine like terms.} \\
-4x + 4 - 4 &= -4 - 4 & \text{Subtract 4 from each side.} \\
-4x &= -8 & \text{Simplify.} \\
\frac{-4x}{-4} &= \frac{-8}{-4} & \text{Divide each side by } -2. \\
x &= 2 & \text{Simplify.}
\end{aligned}
$$

Use the solution for x and either equation to find y.

$$
\begin{aligned}
2x + y &= 4 & \text{First equation} \\
2(2) + y &= 4 & \text{Substitute 2 for } x. \\
4 + y &= 4 & \text{Multiply.} \\
y &= 0 & \text{Subtract 4 from each side.}
\end{aligned}
$$

The solution is (2, 0).

22. $-5x + 4y = 20$
$10x - 8y = -40$

SOLUTION:
$-5x + 4y = 20$
$10x - 8y = -40$
First, solve the first equation for y.

$$-5x + 4y = 20 \qquad \text{First equation}$$
$$-5x + 5x + 4y = 20 + 5x \qquad \text{Add } 5x \text{ to each side.}$$
$$\frac{4y}{4} = \frac{20 + 5x}{4} \qquad \text{Divide each side by 4.}$$
$$y = 5 + \frac{5}{4}x \qquad \text{Simplify.}$$

Then, substitute $5 + \frac{5}{4}x$ for y in the second equation.

$$10x - 8y = -40 \qquad \text{Second equation}$$
$$10x - 8\left(5 + \frac{5}{4}x\right) = -40 \qquad \text{Substitute } 5 + \frac{5}{4}x \text{ for } y.$$
$$10x - 40 - 10x = -40 \qquad \text{Distributive Property}$$
$$-40 = -40 \qquad \text{Combine like terms.}$$

This equation is an identity. So, there are infinitely many solutions.

23. **ECONOMICS** In 2000, the demand for nurses was 2,000,000, while the supply was only 1,890,000. The projected demand for nurses in 2020 is 2,810,414, while the supply is only projected to be 2,001,998.

    a. Define the variables, and write equations to represent these situations.

    b. Use substitution to determine during which year the supply of nurses was equal to the demand.

    SOLUTION:

    a. Let x = number of years since 2000, and let y = the number of nurses. Find the slope for the supply and the demand.

    $$m_{demand} = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope formula}$$

    $$m_{demand} = \frac{2{,}001{,}998 - 1{,}890{,}000}{20 - 0} \qquad \text{Substitute.}$$

    $$m_{demand} = \frac{111{,}998}{20} \qquad \text{Simplify.}$$

    $$m_{demand} = 5599.9 \qquad \text{Use a calculator.}$$

    $$m_{supply} = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope formula}$$

    $$m_{supply} = \frac{2{,}810{,}414 - 2{,}000{,}000}{20 - 0} \qquad \text{Substitute.}$$

    $$m_{supply} = \frac{810{,}414}{20} \qquad \text{Simplify.}$$

    $$m_{supply} = 40{,}520.7 \qquad \text{Use a calculator.}$$

    Next, write an equation for each relationship.
    An equation for the supply is $y = 5599.9x + 1{,}890{,}000$ and an equation for the demand is $y = 40{,}520.7x + 2{,}000{,}000$.

    b. $y = 5599.9x + 1{,}890{,}000$
    $y = 40{,}520.7x + 2{,}000{,}000$
    Substitute $5599.9x + 1{,}890{,}000$ in for y into the second equation.

    $$y = 40{,}520.7x + 2{,}000{,}000 \qquad \text{Second equation}$$

    $$5599.9x + 1{,}890{,}000 = 40{,}520.7x + 2{,}000{,}000 \qquad \text{Substitute.}$$

    $$5599.9x = 40{,}520.7x + 110{,}000 \qquad \text{Subtract.}$$

    $$-34{,}920.8x = 110{,}000 \qquad \text{Subtract.}$$

    $$\frac{-34{,}920.8x}{-34{,}920.8} = \frac{110{,}000}{-34{,}920.8} \qquad \text{Divide.}$$

    $$x \approx 3.1 \qquad \text{Simplify.}$$

    Since x is the number of years since 2000, subtract 3.1 from 2000. So, the year when the supply of nurses was equal to the demand for nurses was during 1996.

6-2 Substitution

24. **CCSS REASONING** The table shows the approximate number of tourists in two areas of the world during a recent year and the average rates of change in tourism.

Destination	Number of Tourists	Average Rates of Change in Tourists (millions per year)
South America and the Caribbean	40.3 million	increase of 0.8
Middle East	17.0 million	increase of 1.8

a. Define the variables, and write an equation for each region's tourism rate.

b. If the trends continue, in how many years would you expect the number of tourists in the regions to be equal?

SOLUTION:

a. Let x = number of years, and y = number of tourists in millions. So, the South America and Caribbean tourism rate is $y = 0.8x + 40.3$ and the Middle East is $y = 1.8x + 17.0$.

b. $y = 0.8x + 40.3$
$y = 1.8x + 17.0$
Substitute $0.8x + 40.3$ for y in the second equation.

$$
\begin{aligned}
y &= 1.8x + 17.0 & &\text{Second equation} \\
0.8x + 40.3 &= 1.8x + 17.0 & &\text{Substitute.} \\
0.8x - 0.8x + 40.3 &= 1.8x - 0.8x + 17.0 & &\text{Subtract.} \\
40.3 - 17 &= x + 17 - 17 & &\text{Subtract.} \\
23.3 &= x & &\text{Simplify.}
\end{aligned}
$$

So, in 23.3 years, or about 23 years 4 months, the number of tourists in South America and the Caribbean will be equal to the number of tourists in the Middle East.

25. **SPORTS** The table shows the winning times for the Triathlon World Championship.

Year	Men's	Women's
2000	1:51:39	1:54:43
2009	1:44:51	1:59:14

a. The times are in hours, minutes, and seconds. Rewrite the times rounded to the nearest minute.

b. Let the year 2000 be 0. Assume that the rate of change remains the same for years after 2000. Write an equation to represent each of the men's and women's winning times y in any year x.

c. If the trend continues, when would you expect the men's and women's winning times to be the same? Explain your reasoning.

SOLUTION:

a. 1:51:39 represents 1 hr 51 min 39 sec = $(60 + 51 + \frac{39}{60})$ min or about 111 min.

1:44:51 represents 1 hr 44 min 51 sec = $(60 + 44 + \frac{51}{60})$ min or about 105 min.

1:54:43 represents 1 hr 54 min 43 sec = $(60 + 54 + \frac{43}{60})$ min or about 115 min.

1:59:14 represents 1 hr 59 min 14 sec = $(60 + 59 + \frac{14}{60})$ min or about 119 min.

So, the men's times would be 112 minutes in 2000 and 105 minutes in 2009, and the women's times would be 115 minutes in 2000 and 119 minutes in 2009.

b. If the times change in a constant manner, find the rate of change for each set of times and write a linear equation in slope-intercept form. Let $(x_1, y_1) = (0, 112)$ and $(x_2, y_2) = (9, 105)$.

$$m_m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope formula}$$

$$m_m = \frac{105 - 112}{9 - 0} \qquad \text{Substitute.}$$

$$m_m = \frac{-7}{9} \qquad \text{Simplify.}$$

$$m_m \approx -0.8 \qquad \text{Simplify.}$$

The y-intercept is 112, so the equation for the men's times is $y = -0.8x + 112$. Let $(x_1, y_1) = (0, 115)$ and $(x_2, y_2) = (9, 119)$.

$$m_w = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope formula}$$

$$m_w = \frac{119 - 115}{9 - 0} \qquad \text{Substitute.}$$

$$m_w = \frac{4}{9} \qquad \text{Simplify.}$$

$$m_w \approx 0.4 \qquad \text{Simplify.}$$

The y-intercept is 115, so the equation for the women's times is $y = 0.4x + 115$.

c. To find when the winning times would be the same, find the solution for the system of equations.
$y = -0.8x + 112$
$y = 0.4x + 115$
Substitute $0.4x + 115$ in for y into the first equation.

$$
\begin{aligned}
y &= -0.8x + 112 & \text{First equation} \\
0.4x + 115 &= -0.8x + 112 & \text{Substitute.} \\
0.4x + 115 + 0.8x &= -0.8x + 112 + 0.8x & \text{Add.} \\
1.2x + 115 &= 112 & \text{Simplify.} \\
1.2x &= -3 & \text{Subtract.} \\
\frac{1.2x}{1.2} &= \frac{-3}{1.2} & \text{Divide.} \\
x &= -2.5 & \text{Simplify.}
\end{aligned}
$$

Since the solution for x is less than 0, there is never a year after 2000 that the winning times for the men and women will be the same if the current trend continues.

26. **CONCERT TICKETS** Booker is buying tickets online for a concert. He finds tickets for himself and his friends for $65 each plus a one-time fee of $10. Paula is looking for tickets to the same concert. She finds them at another Web site for $69 and a one-time fee of $13.60.

   a. Define the variables, and write equations to represent this situation.

   b. Create a table of values for 1 to 5 tickets for each person's purchase.

   c. Graph each of these equations.

   d. Use the graph to determine who received the better deal. Explain why.

   SOLUTION:

   a. Let x = number of tickets purchased and let y = the cost. Booker's equation is $y = 65x + 10$ and Paula's equation is $y = 69x + 13.60$.

   b. Booker's table is the following.

   Booker's Concert Tickets

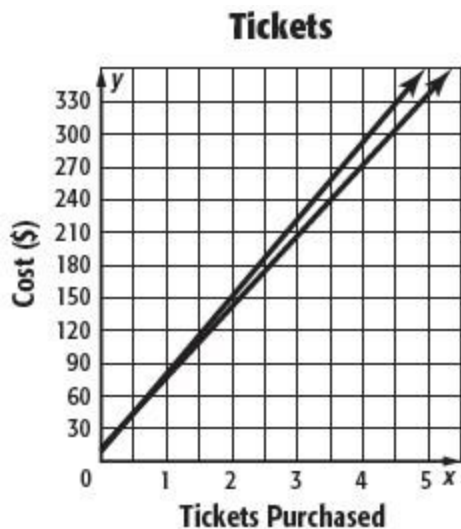 | Number of Tickets | Cost($) |
   |---|---|
   | 1 | 75 |
   | 2 | 140 |
   | 3 | 205 |
   | 4 | 270 |
   | 5 | 335 |

   Paula's Concert Tickets

Number of Tickets	Cost($)
1	82.60
2	151.60
3	220.60
4	289.60
5	358.60

   c. Both equations are in slope-intercept form.
   $y = 65x + 10$
   $y = 69x + 13.60$

Tickets

d. Since the graphs do not intersect in the first quadrant, Paula is always paying more for the tickets than Booker. Booker received the better deal.

27. **ERROR ANALYSIS** In the system $a + b = 7$ and $1.29a + 0.49b = 6.63$, a represents pounds of apples, and b represents pounds of bananas. Guillermo and Cara are finding and interpreting the solution. Is either of them correct? Explain.

Guillermo

$$1.29a + 0.49b = 6.63$$

$$1.29a + 0.49(a + 7) = 6.63$$

$$1.29 + 0.49a + 3.43 = 6.63$$

$$0.49a = 3.2$$

$$a = 1.9$$

$a + b = 7$, so $b = 5$. The solution $(2, 5)$ means that 2 pounds of apples and 5 pounds of bananas were bought.

6-2 Substitution

Cara

$1.29a + 0.49b = 6.63$

$1.29(7 - b) + 0.49b = 6.63$

$9.03 - 1.29b + 0.49b = 6.63$

$-0.8b = -2.4$

$b = 3$

The solution $b = 3$ means that 3 pounds of apples and 3 pounds of bananas were bought.

SOLUTION:

If $a + b = 7$, then by subtracting a from each side, $b = 7 - a$. In step 2, Guillermo substituted $a + 7$ for b and he should have substituted $7 - a$.

Cara solved the first equation for a by subtracting b from each side. So, $a = 7 - b$. She then substituted for a in the second equation.

$1.29a + 0.49b = 6.63$	Second equation
$1.29(7 - b) + 0.49b = 6.63$	Substitute $7 - b$ for a.
$9.03 - 1.29b + 0.49b = 6.63$	Distributive Property
$9.03 - 0.8b = 6.63$	Combine like terms.
$-0.8b = -2.4$	Subtract 9.03 from each side.
$\frac{-0.8b}{-0.8} = \frac{-2.4}{-0.8}$	Divide each side by -0.8.
$b = 3$	Simplify.

She should have then substituted the solution for b into the first equation to find the value of a which is the variable for the number of apples.

$a + b = 7$	First equation
$a + 3 = 7$	Substitute 3 for b.
$a = 4$	Subtract 3 from each side.

The number of apples should be 4.

Therefore, neither Guillermo nor Cara is correct. Guillermo substituted incorrectly for b. Cara solved correctly for a, but misinterpreted the pounds of apples bought.

28. **CCSS PERSEVERANCE** A local charity has 60 volunteers. The ratio of boys to girls is 7:5. Find the number of boy and the number of girl volunteers.

SOLUTION:

Let g represent the number of girls and b represent the number of boys. Since the total number of volunteers is 60, one equation is $g + b = 60$. A second equation can be written as $\dfrac{b}{g} = \dfrac{7}{5}$, since the ratio of boys to girls is 7:5. To find the number of boys and girls, solve the system of equations.

First, solve the first equation for g by subtracting b from each side to get $g = 60 - b$. Then, substitute $60 - b$ into the second equation for g.

$$\dfrac{b}{g} = \dfrac{7}{5} \qquad \text{Second equation}$$

$$\dfrac{b}{60-b} = \dfrac{7}{5} \qquad \text{Substitute } 60 - b \text{ for } g.$$

$$5b = 7(60 - b) \qquad \text{Cross products are equal.}$$

$$5b = 420 - 7b \qquad \text{Distributive Property}$$

$$5b + 7b = 420 - 7b + 7b \qquad \text{Add } 7b \text{ to each side.}$$

$$12b = 420 \qquad \text{Simplify.}$$

$$\dfrac{12b}{12} = \dfrac{420}{12} \qquad \text{Divide each side by 12.}$$

$$b = 35 \qquad \text{Simplify.}$$

Use the solution for b and either equation to find g.

$$g + b = 60 \qquad \text{Second equation}$$

$$g + 35 = 60 \qquad \text{Substitute 35 for } b.$$

$$g = 60 - 35 \qquad \text{Subtract 35 from each side.}$$

$$g = 25 \qquad \text{Simplify.}$$

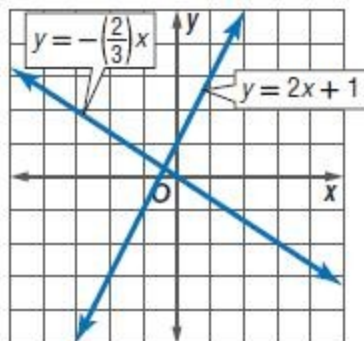So, there were 25 girl volunteers and 35 boy volunteers.

29. **REASONING** Compare and contrast the solution of a system found by graphing and the solution of the same system found by substitution.

SOLUTION:

Solve the following system of equations by graphing.

$y = 2x + 1$ and $y = -\frac{2}{3}x$.

From the graph, it appears the solution is close to $\left(-\frac{1}{2}, \frac{1}{2}\right)$.

Next, solve the same system by substitution.

$$
\begin{aligned}
y &= 2x + 1 & \text{First equation} \\
-\frac{2}{3}x &= 2x + 1 & \text{Substitute } -\frac{2}{3}x \text{ for } y. \\
-\frac{2}{3}x - 2x &= 2x + 1 - 2x & \text{Subtract } 2x \text{ from each side.} \\
-\frac{8}{3}x &= 1 & \text{Simplify.} \\
\left(-\frac{3}{8}\right) \cdot \left(-\frac{8}{3}x\right) &= 1 \cdot \left(-\frac{3}{8}\right) & \text{Multiply each side by } -\frac{3}{8}. \\
x &= -\frac{3}{8} & \text{Simplify.}
\end{aligned}
$$

Substitute $-\frac{3}{8}$ for x in either equation to find the value of y.

$$
\begin{aligned}
y &= -\frac{2}{3}x & \text{Second equation} \\
y &= -\frac{2}{3}\left(-\frac{3}{8}\right) & \text{Substitute } -\frac{3}{8} \text{ for } x. \\
y &= \frac{1}{4} & \text{Multiply.}
\end{aligned}
$$

The exact solution is $\left(-\frac{3}{8}, \frac{1}{4}\right)$ which is close, but not equal to, $\left(-\frac{1}{2}, \frac{1}{2}\right)$.

The solutions to a system will be the same, no matter the method used to solve. However, estimation may need to be used when graphing the system. To find a precise solution, the preferred method is substitution.

30. **OPEN ENDED** Create a system of equations that has one solution. Illustrate how the system could represent a real-world situation and describe the significance of the solution in the context of the situation.

    SOLUTION:

    Sample answer: Let a = the number of tops Allison bought, and let b = the number of tops Beth bought. Together, Allison and Beth bought 25 tops. Allison spent $24 per top, and Beth spent $16 per top. Together they spent $464. How many tops did each girl buy?

    $a + b = 25$

    $24a + 16b = 464$

    First, solve the first equation for a to get $a = 25 - b$. Then, substitute $25 - b$ into the second equation.

    $$
    \begin{aligned}
    24a + 16b &= 464 && \text{Second equation} \\
    24(25 - b) + 16b &= 464 && \text{Substitute.} \\
    600 - 24b + 16b &= 464 && \text{Distributive Property} \\
    600 - 600 - 8b &= 464 - 600 && \text{Subtract.} \\
    \frac{-8b}{-8} &= \frac{-136}{-8} && \text{Divide.} \\
    b &= 17 && \text{Simplify.}
    \end{aligned}
    $$

    Use the solution for b and either equation to find a.

    $$
    \begin{aligned}
    a + b &= 25 && \text{First equation} \\
    a + (17) &= 25 && \text{Substitute.} \\
    a + 17 - 17 &= 25 - 17 && \text{Subtract.} \\
    a &= 8 && \text{Simplify.}
    \end{aligned}
    $$

    So, Allison bought 8 tops, and Beth bought 17 tops.

31. **WRITING IN MATH** Explain how to determine what to substitute when using the substitution method of solving systems of equations.

    SOLUTION:

    First, look for a variable that can easily be solved for. An equation containing a variable with a coefficient of 1 can easily be solved for the variable. That expression can then be substituted into the second equation for the variable.

32. The debate team plans to make and sell trail mix. They can spend $34.

Item	Cost Per Pound
sunflower seeds	$4.00
raisins	$1.50

The pounds of raisins in the mix is to be 3 times the pounds of sunflower seeds. Which system can be used to find r, the pounds of raisins, and p, pounds of sunflower seeds, they should buy?

A $3p = r$
$4p + 1.5r = 34$

B $3p = r$
$4r + 1.5p = 34$

C $3r = p$
$4p + 1.5r = 34$

D $3r = p$
$4r + 1.5p = 34$

SOLUTION:
Use the total amount spent and cost per pound to find the first equation of $4p + 1.5r = 34$. Then, use the pounds of raisins in the mix is to be 3 times the pounds of sunflower seeds to find the second equation of $3p = r$.
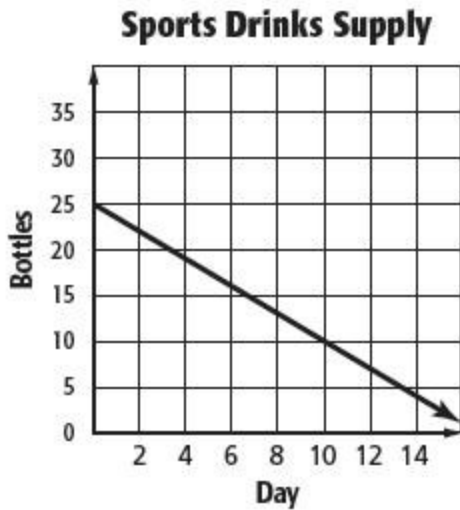
So, the correct choice is A.

33. **GRIDDED RESPONSE** The perimeters of two similar polygons are 250 centimeters and 300 centimeters, respectively. What is the scale factor of the first polygon to the second?

SOLUTION:

$$\frac{250}{300} = \frac{5}{6}$$

34. Based on the graph, which statement is true?

Sports Drinks Supply

F Mary started with 30 bottles.

G On day 10, Mary will have 10 bottles left.

H Mary will be out of sports drinks on day 14.

J Mary drank 5 bottles the first two days.

SOLUTION:

Examining the graph, the y-intercept is 25 and the x-intercept is greater than 16. This means that Mary started with 25 bottles and it will be more than 16 days before she is out of drinks. On day 2 it appears that she has 21 bottles left. This would mean she drank less than 5 bottles the first two days. The line appears to contain the point (10, 10) which would mean that on day 10, she will have 10 bottles left.

So, the correct choice is G.

35. If p is an integer, which of the following is the solution set for $2|p| = 16$?

   A $\{0, 8\}$

   B $\{-8, 0\}$

   C $\{-8, 8\}$

   D $\{-8, 0, 8\}$

   SOLUTION:

   $$2|p| = 16$$
   $$\frac{2|p|}{2} = \frac{16}{2}$$
   $$|p| = 8$$

   By definition of absolute value, p could be 8 or -8.

   So, the correct choice is C.

Graph each system and determine how many solutions it has. If it has one solution, name it.
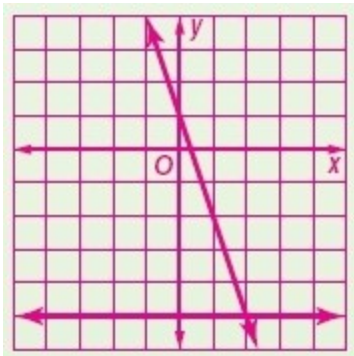
36. $y = -5$
$3x + y = 1$

SOLUTION:

To graph the system, write both equations in slope-intercept form.
$y = -5$
$y = -3x + 1$

The graph appears to intersect at the point $(2, -5)$. You can check this by substituting 2 for x and -5 for y.

$$y = -5 \qquad y = -3x + 1$$
$$-5 = -5 \qquad -5 \overset{?}{=} -3(2) + 1$$
$$-5 \overset{?}{=} -6 + 1$$
$$-5 = -5$$

The solution is $(2, -5)$.

37. $x = 1$
    $2x - y = 7$

SOLUTION:

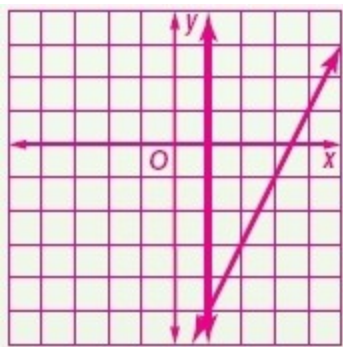To graph the system, write the second equations in slope-intercept form.

$x = 1$

$2x - y = 7$	Second equation
$2x - 2x - y = -2x + 7$	Subtract.
$-y = -2x + 7$	Simplify.
$-1(-y) = -1(-2x + 7)$	Multiply.
$y = 2x - 7$	Divide.

The graph appears to intersect at the point (1, −5). You can check this by substituting 1 for x and −5 for y.

$x = 1$ $2x - y = 7$
$1 = 1$ $2(1) - (-5) \overset{?}{=} 7$
                   $2 + \overset{?}{=} 7$
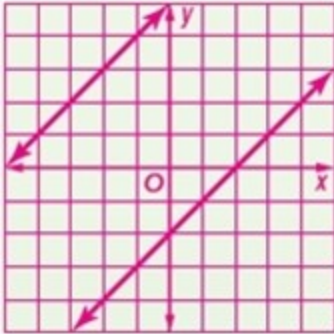 $7 = 7$

The solution is (1, −5).

38. $y = x + 5$
    $y = x - 2$

   SOLUTION:

   $y = x + 5$
   $y = x - 2$

The lines have the same slope but different y-intercepts, so the lines are parallel. Since they do not intersect, there is no solution of this system. The system is inconsistent.

39. $x + y = 1$
    $3y + 3x = 3$

SOLUTION:

To graph the system, write both equations in slope-intercept form.

$$x + y = 1 \qquad \text{First equation}$$
$$x - x + y = -x + 1 \quad \text{Subtract.}$$
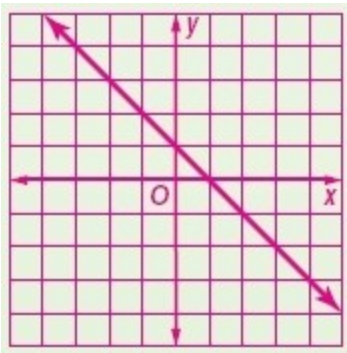$$y = -x + 1 \quad \text{Simplify.}$$

$$3x + 3y = 3 \qquad \text{Second equation}$$
$$3x - 3x + 3y = -3x + 3 \quad \text{Subtract.}$$
$$3y = -3x + 3 \quad \text{Simplify.}$$
$$\frac{3y}{3} = \frac{-3x + 3}{3} \quad \text{Divide.}$$
$$y = -x + 1 \quad \text{Simplify.}$$

The two lines are identical, so there are an infinite number of solutions to the system. The system is dependent.

40. **ENTERTAINMENT** Coach Ross wants to take the soccer team out for pizza after their game. Her budget is at most $70.

a. Using the sign shown, write an inequality that represents this situation.

b. Are there any restrictions on the variables? Explain.

SOLUTION:

a. Let p represent the number of pizzas, and let d represent the number of pitchers of soft drinks. The amount spent on pizzas is represented by $12p$, and the amount spent on pitchers of soft drink is represented by $2d$. The total spent is at most $70, so the inequality $12p + 2d \leq 70$ represents the situation.

b. The coach cannot purchase a fractional part of a pizza or pitcher of drinks. So, the number of pizzas purchased and the number of pitchers of soft drinks ordered must be an integer that is greater than or equal to 0. The cost of the pizzas and the cost of the pitchers must be at most $70. If $12p \leq 70$, then $p < 6$. If $2d \leq 70$, then $d < 35$. Therefore, the restrictions on the variables are that $0 \leq p < 6$, $0 \leq d \leq 35$, and p and d are integers;

Solve each inequality. Check your solution.

41. $6v + 1 \geq -11$

SOLUTION:

$$6v + 1 \geq -11$$
$$6v + 1 - 1 \geq -11 - 1$$
$$6v \geq -12$$
$$\frac{6v}{6} \geq \frac{-12}{6}$$
$$v \geq -2$$

To check this answer, substitute a number greater than or equal to -2 into the original inequality.

$$6v + 1 \geq -11$$
$$6(-1) + 1 \overset{?}{\geq} -11$$
$$-6 + 1 \overset{?}{\geq} -11$$
$$-5 \geq -11$$

So the solution checks.

42. $24 > 18 + 2n$

SOLUTION:

$$24 > 18 + 2n$$
$$24 - 18 > 18 - 18 + 2n$$
$$6 > 2n$$
$$\frac{6}{2} > \frac{2n}{2}$$
$$3 > n$$
$$n < 3$$

To check this answer, substitute a number less than 3 into the original inequality.

$$24 > 18 + 2n$$
$$24 \overset{?}{>} 18 + 2(0)$$
$$24 > 18$$

So the solution checks.

43. $-11 \geq \frac{2}{5}q + 5$

SOLUTION:

$$-11 \geq \frac{2}{5}q + 5$$
$$-11 - 5 \geq \frac{2}{5}q + 5 - 5$$
$$-16 \geq \frac{2}{5}q$$
$$-16 \cdot \frac{5}{2} \geq \frac{5}{2} \cdot \frac{2}{5}q$$
$$-40 \geq q$$
$$q \leq -40$$

To check this answer, substitute a number less than or equal to -40 into the original inequality.

$$-11 \geq \frac{2}{5}q + 5$$
$$-11 \overset{?}{\geq} \frac{2}{5}(-50) + 5$$
$$-11 \overset{?}{\geq} -20 + 5$$
$$-11 \geq -15$$

So the solution checks.

44. $\frac{a}{8} - 10 > -3$

SOLUTION:

$$\frac{a}{8} - 10 > -3$$

$$\frac{a}{8} - 10 + 10 > -3 + 10$$

$$\frac{a}{8} > 7$$

$$\frac{a}{8} \cdot 8 > 7 \cdot 8$$

$$a > 56$$

To check this answer, substitute a number greater than 56 into the original inequality.

$$\frac{a}{8} - 10 > -3$$

$$\frac{(64)}{8} - 10 \overset{?}{>} -3$$

$$8 - 10 \overset{?}{>} -3$$

$$-2 > -3$$

So the solution checks.

45. $-3t + 9 \leq 0$

SOLUTION:

$$-3t + 9 \leq 0$$

$$-3t + 9 - 9 \leq 0 - 9$$

$$-3t \geq -9$$

$$\frac{-3t}{-3} \geq \frac{-9}{-3}$$

$$t \geq 3$$

To check this answer, substitute a number greater than or equal to 3 into the original inequality.

$$-3t + 9 \leq 0$$

$$-3(4) + 9 \overset{?}{\leq} 0$$

$$-12 + 9 \overset{?}{\leq} 0$$

$$-3 \leq 0$$

So the solution checks.

46. $54 > -10 - 8n$

SOLUTION:

$$54 > -10 - 8n$$
$$54 + 10 > -10 + 10 - 8n$$
$$64 < -8n$$
$$\frac{64}{-8} < \frac{-8n}{-8}$$
$$-8 < n$$
$$n > -8$$

To check this answer, substitute a number greater than −8 into the original inequality.

$$54 > -10 - 8n$$
$$54 \overset{?}{>} -10 - 8(0)$$
$$54 \overset{?}{>} -10 - 0$$
$$54 > -10$$

So the solution checks.

Rewrite each product using the Distributive Property. Then simplify.

47. $10b + 5(3 + 9b)$

SOLUTION:

$$10b + 5(3 + 9b) = 10b + 15 + 45b$$
$$= 55b + 15$$

48. $5(3t^2 + 4) - 8t$

SOLUTION:

$$5(3t^2 + 4) - 8t = 15t^2 + 20 - 8t$$
$$= 15t^2 - 8t + 20$$

49. $7h^2 + 4(3h + h^2)$

SOLUTION:

$$7h^2 + 4(3h + h^2) = 7h^2 + 12h + 4h^2$$
$$= 11h^2 + 12h$$

50. $-2(7a + 5b) + 5(2a - 7b)$

SOLUTION:

$$-2(7a + 5b) + 5(2a - 7b) = -14a - 10b + 10a - 35b$$
$$= -4a - 45b$$