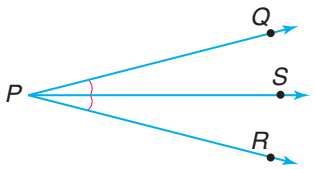
# 6–3 Angle Bisectors of Triangles

**What You'll Learn**
You'll learn to identify and use angle bisectors in triangles.

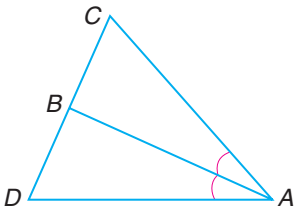
**Why It's Important**
**Engineering** Angle bisectors of triangles can be found in bridges.
*See Exercise 19.*

Recall that the bisector of an angle is a ray that separates the angle into two congruent angles.




$\overrightarrow{PS}$ bisects $\angle QPR$.
$\angle QPS \cong SPR$
$m\angle QPS = m\angle SPR$

An **angle bisector** of a triangle is a segment that separates an angle of the triangle into two congruent angles. One of the endpoints of an angle bisector is a vertex of the triangle, and the other endpoint is on the side opposite that vertex.



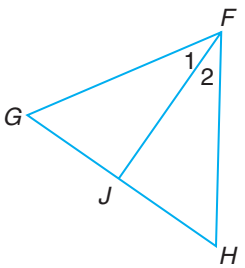

$\overline{AB}$ is an angle bisector of $\triangle DAC$.
$\angle DAB \cong \angle CAB$
$m\angle DAB = m\angle CAB$

Just as every triangle has three medians, three altitudes, and three perpendicular bisectors, every triangle has three angle bisectors.

| Special Segments in Triangles | | | |
|---|---|---|---|
| **Segment** | • altitude | • perpendicular bisector | • angle bisector |
| **Type** | • line segment | • line<br>• line segment | • ray<br>• line segment |
| **Property** | from the vertex, a line perpendicular to the opposite side | bisects the side of a triangle | bisects the angle of a triangle |

An angle bisector of a triangle has all of the characteristics of any angle bisector. In $\triangle FGH$, $\overline{FJ}$ bisects $\angle GFH$.

1. $\angle 1 \cong \angle 2$, so $m\angle 1 = m\angle 2$.
2. $m\angle 1 = \frac{1}{2}(m\angle GFH)$ or $2(m\angle 1) = m\angle GFH$
3. $m\angle 2 = \frac{1}{2}(m\angle GFH)$ or $2(m\angle 2) = m\angle GFH$



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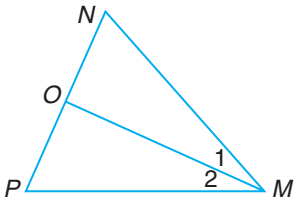
## Examples

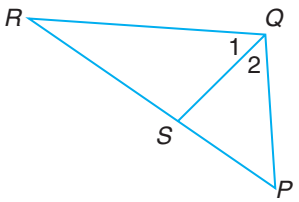
**1** **In $\triangle MNP$, $\overline{MO}$ bisects $\angle NMP$. If $m\angle 1 = 33$, find $m\angle 2$.**




Since $\overline{MO}$ bisects $\angle NMP$, $m\angle 1 = m\angle 2$.

Since $m\angle 1 = 33$, $m\angle 2 = 33$.

**2** **In $\triangle PQR$, $\overline{QS}$ bisects $\angle PQR$. If $m\angle PQR = 70$, what is $m\angle 2$?**

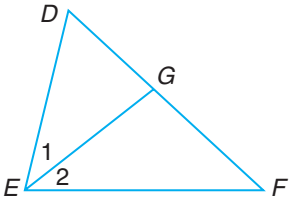



$m\angle 2 = \frac{1}{2}(m\angle PQR)$ *Definition of bisector*

$m\angle 2 = \frac{1}{2}(70)$ *Substitution*

$m\angle 2 = 35$ *Multiply.*

**3** **In $\triangle DEF$, $\overline{EG}$ bisects $\angle DEF$. If $m\angle 1 = 43$, find $m\angle DEF$.**



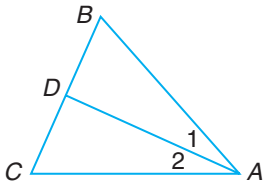

$m\angle DEF = 2(m\angle 1)$ *Definition of bisector*

$m\angle DEF = 2(43)$ *Substitution*

$m\angle DEF = 86$ *Multiply.*

## Your Turn

**In $\triangle ABC$, $\overline{AD}$ bisects $\angle BAC$.**



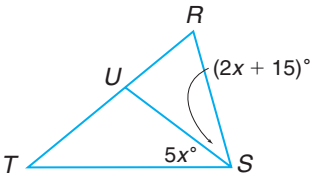

**a.** If $m\angle 1 = 32$, find $m\angle 2$.

**b.** Find $m\angle 1$ if $m\angle BAC = 52$.

**c.** What is $m\angle CAB$ if $m\angle 1 = 28$?

**Algebra Link**

**4** **In $\triangle RST$, $\overline{SU}$ is an angle bisector. Find $m\angle UST$.**




**Algebra Review**

Solving Multi-Step Equations, p. 723

$m\angle UST = m\angle RSU$ *Definition of bisector*

$5x = 2x + 15$ *Substitution*

$5x - 2x = 2x + 15 - 2x$ *Subtract 2x from each side.*

$3x = 15$ *Simplify.*

$\frac{3x}{3} = \frac{15}{3}$ *Divide each side by 3.*

$x = 5$ *Simplify.*

So, $m\angle UST = 5(5)$ or 25.

## Check for Understanding

**Communicating Mathematics**

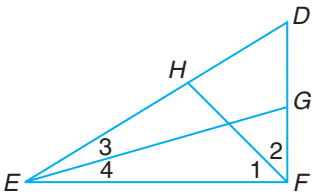
1. **Describe** an angle bisector of a triangle.
2. **Draw** an acute scalene triangle. Then use a compass and straightedge to construct the angle bisector of one of the angles.

**Vocabulary**
angle bisector

**Guided Practice**

**Examples 1–3**

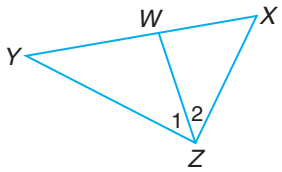
**In $\triangle DEF$, $\overline{EG}$ bisects $\angle DEF$, and $\overline{FH}$ bisects $\angle EFD$.**




3. If $m\angle 4 = 24$, what is $m\angle DEF$?
4. Find $m\angle 2$ if $m\angle 1 = 36$.
5. What is $m\angle EFD$ if $m\angle 1 = 42$?

**Example 4**

6. **Algebra** In $\triangle XYZ$, $\overline{ZW}$ bisects $\angle YZX$. If $m\angle 1 = 5x + 9$ and $m\angle 2 = 39$, find $x$.

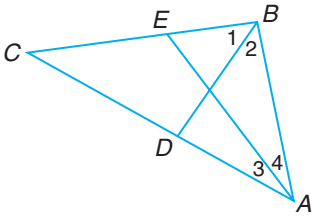



## Exercises

**Practice**

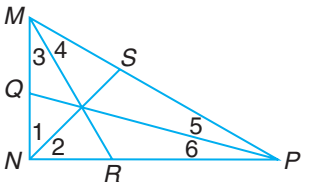
| Homework Help | |
|---|---|
| For Exercises | See Examples |
| 7, 11, 15, 17 | 3 |
| 8, 10, 13, 14, 16, 20 | 2 |
| 9, 12 | 1 |
| 18 | 4 |
| 19 | 2, 3 |
| Extra Practice | |
| See page 736. | |

**In $\triangle ABC$, $\overline{BD}$ bisects $\angle ABC$, and $\overline{AE}$ bisects $\angle BAC$.**

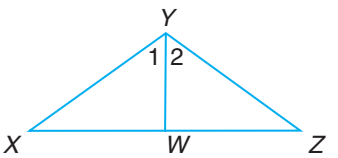



7. If $m\angle 1 = 55$, what is $m\angle ABC$?
8. Find $m\angle 3$ if $m\angle BAC = 38$.
9. What is $m\angle 4$ if $m\angle 3 = 22$?
10. Find $m\angle 2$ if $m\angle ABC = 118$.
11. What is $m\angle BAC$ if $m\angle 3 = 20$?

**In $\triangle MNP$, $\overline{NS}$ bisects $\angle MNP$, $\overline{MR}$ bisects $\angle NMP$, and $\overline{PQ}$ bisects $\angle MPN$.**

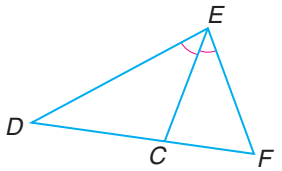



12. Find $m\angle 4$ if $m\angle 3 = 31$.
13. If $m\angle MPN = 34$, what is $m\angle 6$?
14. What is $m\angle 3$ if $m\angle NMP = 64$?
15. Find $m\angle MNP$ if $m\angle 1 = 44$.
16. What is $m\angle 2$ if $\angle MNP$ is a right angle?
17. In $\triangle XYZ$, $\overline{YW}$ bisects $\angle XYZ$. What is $m\angle XYZ$ if $m\angle 2 = 62$?




**Applications and Problem Solving**

18. **Algebra** In $\triangle DEF$, $\overline{EC}$ is an angle bisector. If $m\angle CEF = 2x + 10$ and $m\angle DEC = x + 25$, find $m\angle DEC$.



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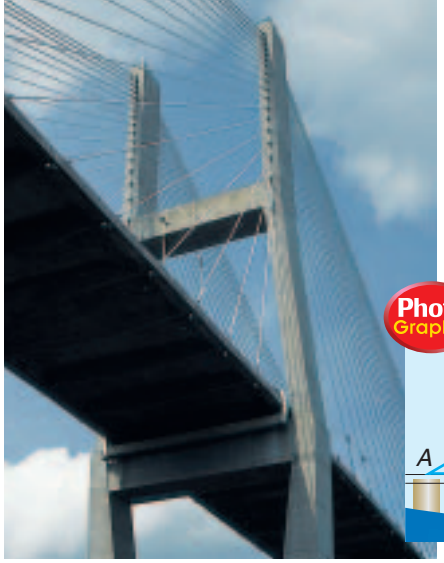


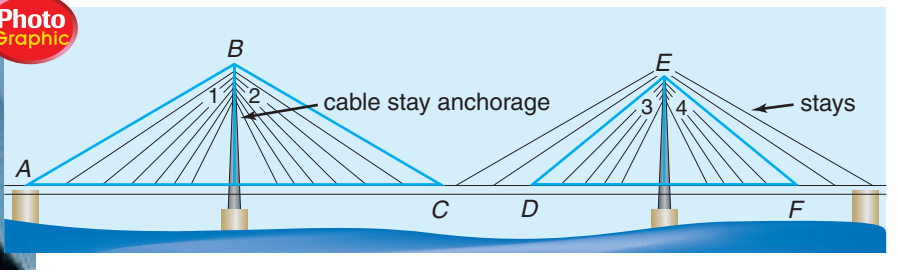
Tallmadge Bridge, Savannah, Georgia

**19. Engineering** One type of bridge, a *cable-stayed bridge* is shown. Notice that the *cable stay anchorage* is an angle bisector of each triangle formed by the cables called *stays* and the roadway.

**a.** Suppose $m\angle ABC = 120$, what is $m\angle 2$?

**b.** Suppose $m\angle 4 = 48$, what is $m\angle DEF$?

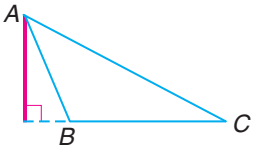
Photo Graphic




**20. Critical Thinking** What kind of angles are formed when you bisect an obtuse angle of a triangle? Explain.

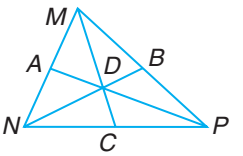
## Mixed Review

**21.** Tell whether the red segment in $\triangle ABC$ is an *altitude*, a *perpendicular bisector*, *both*, or *neither*. *(Lesson 6–2)*




**22.** In $\triangle MNP$, $\overline{MC}$, $\overline{NB}$, and $\overline{PA}$ are medians. Find $PD$ if $DA = 6$. *(Lesson 6–1)*

**23. Algebra** The measures of the angles of a triangle are $x + 2$, $4x + 3$, and $x + 7$. Find the measure of each angle. *(Lesson 5–2)*




Exercise 22

## Standardized Test Practice

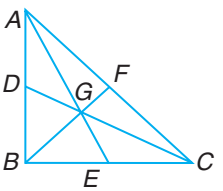
**24. Short Response** Triangle $DEF$ has sides that measure 6 feet, 6 feet, and 9 feet. Classify the triangle by its sides. *(Lesson 5–1)*

**25. Multiple Choice** Multiply $2r + s$ by $r - 3s$. *(Algebra Review)*

(A) $2r^2 - 3s^2$ (B) $2r^2 - 5rs - 3s^2$ (C) $2r^2 - 3rs$ (D) $-6r^2s^2$

## Quiz 1 — Lessons 6–1 through 6–3

**In $\triangle ABC$, $\overline{AE}$, $\overline{BF}$, and $\overline{CD}$ are medians.** *(Lesson 6–1)*

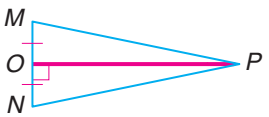



**1.** Find $GE$ if $AG = 9$.

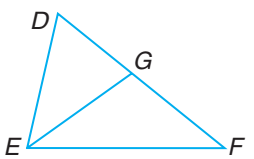
**2.** What is $BF$ if $BG = 5$?

**3.** If $DG = 12$, what is the measure of $\overline{DC}$?

**4.** Tell whether $\overline{PO}$ is an *altitude*, a *perpendicular bisector*, *both*, or *neither*. *(Lesson 6–2)*




**5. Algebra** In $\triangle DEF$, $\overline{EG}$ is an angle bisector. If $m\angle DEG = 2x + 7$ and $m\angle GEF = 4x - 1$, find $m\angle GEF$. *(Lesson 6–3)*



Chapter 6

# Investigation

# What a CIRCLE!

## Materials

- ruler
- protractor
- compass

## Circumcenter, Centroid, Orthocenter, and Incenter

Is there a relationship between the perpendicular bisectors of the sides of a triangle, the medians, the altitudes, and the angle bisectors of a triangle? Let's find out!

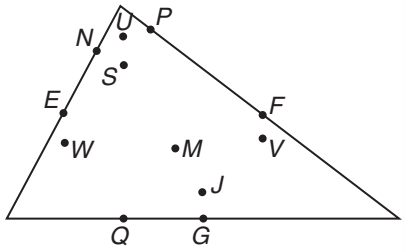
### Investigate

1. Use construction tools to locate some interesting points on a triangle.
   a. Draw a large acute scalene triangle.
   b. On a separate sheet of paper, copy the following table.

| Description of Points | Label of Points |
|---|---|
| midpoints of the three sides (3) | |
| circumcenter (1) | |
| centroid (1) | |
| intersection points of altitudes with the sides (3) | |
| orthocenter (1) | |
| midpoints of segments from orthocenter to each vertex (3) | |
| incenter (1) | |
| midpoint of segment joining circumcenter and orthocenter (1) | |

   c. Construct the perpendicular bisector of each side of your triangle. Label the midpoints *E*, *F*, and *G*. Record these letters in your table. The **circumcenter** is the point where the perpendicular bisectors meet. Label this point *J* and record it. To avoid confusion, erase the perpendicular bisectors, but not the circumcenter.
   d. Draw the medians of your triangle. The point where the medians meet is the *centroid*. Label this point *M* and record it in your table. Erase the medians, but not the centroid.

e. Construct the altitudes of the triangle. Label the points where the altitudes intersect the sides *N*, *P*, and *Q*. Record these points. The point where the altitudes meet is the **orthocenter**. Label this point *S* and record it. Erase the altitudes, but not the orthocenter.




f. Draw three segments, each having the orthocenter as one endpoint and a vertex of your triangle as the other endpoint. Find the midpoint of each segment. Label the midpoints *U*, *V*, and *W* and record these points in your table. Erase the segments.

g. Construct the bisector of each angle of the triangle. The point where the angle bisectors meet is the **incenter**. Label this point *X* and record it. Erase the angle bisectors, but not the incenter.

2. You should now have 13 points labeled. Follow these steps to construct a special circle, called a **nine-point circle**.

a. Locate the circumcenter and orthocenter. Draw a line segment connecting these two points. Bisect this line segment. Label the midpoint *Z* and record it in the table. Do not erase this segment.

b. Draw a circle whose center is point *Z* and whose radius extends to a midpoint of the side of your triangle. How many of your labeled points lie on or very close to this circle?

c. Extend the segment drawn in Step 2a. This line is called the **Euler** (OY-ler) **line**. How many points are on the Euler line?

## Extending the Investigation

In this extension, you will determine whether a special circle exists for other types of triangles.

Use paper and construction tools to investigate these cases.

1. an obtuse scalene triangle  2. a right triangle  3. an equilateral triangle

### Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Make a booklet of your constructions. For each triangle, include a table in which all of the points are recorded.
- Research Leonhard Euler. Write a brief report on his contributions to mathematics, including the nine-point circle.

**Investigation** For more information on the nine-point circle, visit: www.geomconcepts.com