Lesson 1: Introducing Circles

Common Core Georgia Performance Standards

MCC9–12.G.C.1

MCC9–12.G.C.2

Essential Questions

1. Why are all circles similar?
2. What are the relationships among inscribed angles, radii, and chords of a circle?
3. What are the relationships among circumscribed angles, central angles, and inscribed angles?
4. What is the relationship between a tangent line and the radius of a circle?

WORDS TO KNOW

arc — part of a circle's circumference

central angle — an angle with its vertex at the center of a circle

chord — a segment whose endpoints lie on the circumference of the circle

circle — the set of all points that are equidistant from a reference point, the center. The set of points forms a 2-dimensional curve that measures 360°.

circumference — the distance around a circle; $C = 2\pi r$ or $C = \pi d$, for which C represents circumference, r represents the circle's radius, and d represents the circle's diameter.

circumscribed angle — the angle formed by two tangent lines whose vertex is outside of the circle

concentric circles — coplanar circles that have the same center

congruent arcs — two arcs that have the same measure and are either of the same circle or of congruent circles

diameter	a straight line passing through the center of a circle connecting two points on the circle; equal to twice the radius
inscribed angle	an angle formed by two chords whose vertex is on the circle
intercepted arc	an arc whose endpoints intersect the sides of an inscribed angle and whose other points are in the interior of the angle
major arc	part of a circle's circumference that is larger than its semicircle
minor arc	part of a circle's circumference that is smaller than its semicircle
pi **(π)**	the ratio of circumference of a circle to the diameter; equal to approximately 3.14
radius	the distance from the center to a point on the circle; equal to one-half the diameter
secant line	a line that intersects a circle at two points
semicircle	an arc that is half of a circle
tangent line	a line that intersects a circle at exactly one point and is perpendicular to the radius of the circle

Recommended Resources

- Math Open Reference. "Central Angle Theorem."

  http://www.walch.com/rr/00048

  This site describes the Central Angle Theorem and allows users to explore the relationship between inscribed angles and central angles.

- Math Warehouse. "What Is the Tangent of a Circle?"

  http://www.walch.com/rr/00049

  This site reviews the properties of tangent lines, and allows users to interactively explore the idea that a tangent line is perpendicular to a radius at the point of tangency. This site also provides limited practice problems, as well as solutions.

- RicksMath.com. "Geometry and the Circle."

  http://www.walch.com/rr/00050

  This site defines and provides examples of relationships of circles and their properties.

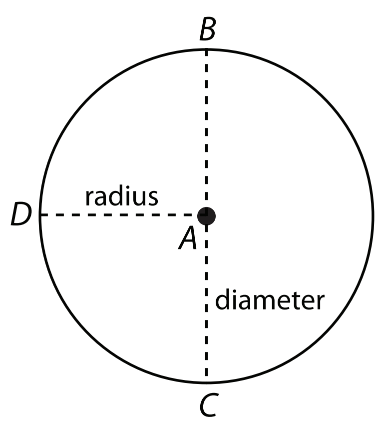
Lesson 3.1.1: Similar Circles and Central and Inscribed Angles

Introduction

In the third century B.C., Greek mathematician Euclid, often referred to as the "Father of Geometry," created what is known as Euclidean geometry. He took properties of shape, size, and space and postulated their unchanging relationships that cultures before understood but had not proved to always be true. Archimedes, a fellow Greek mathematician, followed that by creating the foundations for what is now known as calculus. In addition to being responsible for determining things like the area under a curve, Archimedes is credited for coming up with a method for determining the most accurate approximation of *pi*, π. In this lesson, you will explore and practice applying several properties of circles including proving that all circles are similar using a variation of Archimedes' method.

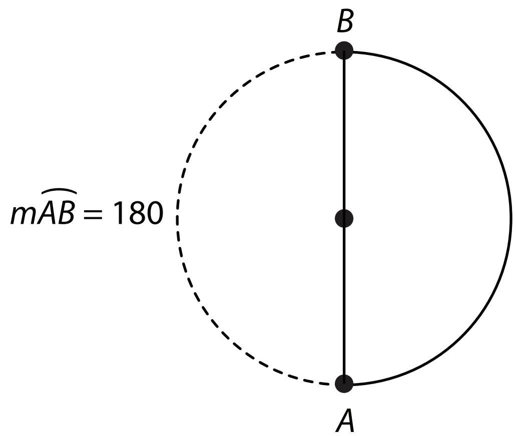
Key Concepts

- ***Pi***, (π), is the ratio of the circumference to the diameter of a circle, where the **circumference** is the distance around a circle, the **diameter** is a segment with endpoints on the circle that passes through the center of the circle, and a **circle** is the set of all points that are equidistant from a reference point (the center) and form a 2-dimensional curve.
- A circle measures 360°.
- **Concentric circles** share the same center.
- The diagram below shows circle A $(\odot A)$ with diameter $\overline{BC}$ and radius $\overline{AD}$. The **radius** of a circle is a segment with endpoints on the circle and at the circle's center; a radius is equal to half the diameter.

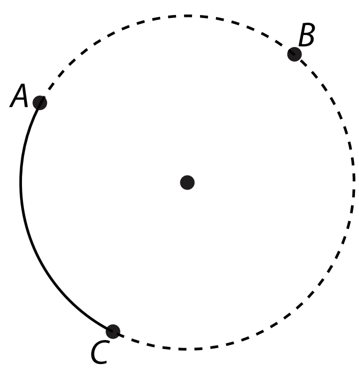


- All circles are similar and measure 360°.

- A portion of a circle's circumference is called an **arc**.
- The measure of a **semicircle**, or an arc that is equal to half of a circle, is 180°.
- Arcs are named by their endpoints.
- The semicircle in the following diagram can be named $\overset{\frown}{AB}$.

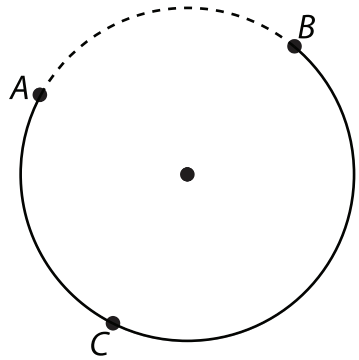


- A part of the circle that is larger than a semicircle is called a **major arc**.
- It is common to identify a third point on the circle when naming major arcs.
- The major arc in the following diagram can be named $\overset{\frown}{ABC}$.

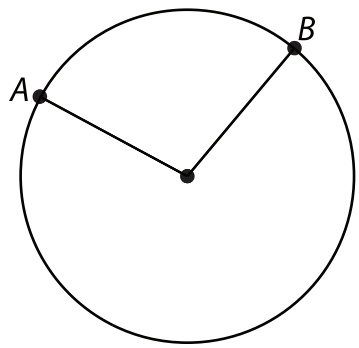


- A **minor arc** is a part of a circle that is smaller than a semicircle.

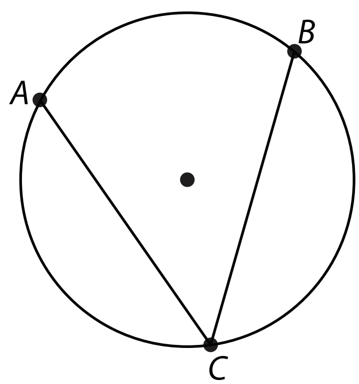
- The minor arc in the following diagram can be named $\overset{\frown}{AB}$.



- Two arcs of the same circle or of congruent circles are **congruent arcs** if they have the same measure.
- The measure of an arc is determined by the central angle.
- A **central angle** of a circle is an angle with its vertex at the center of the circle and sides that are created from two radii of the circle.



- A **chord** is a segment whose endpoints lie on the circumference of a circle.
- An **inscribed angle** of a circle is an angle formed by two chords whose vertex is on the circle.

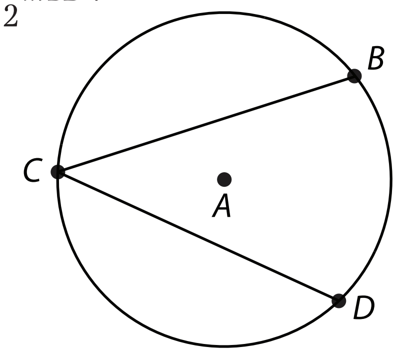


- An inscribed angle is half the measure of the central angle that intercepts the same arc. Conversely, the measure of the central angle is twice the measure of the inscribed angle that intercepts the same arc. This is called the Inscribed Angle Theorem.

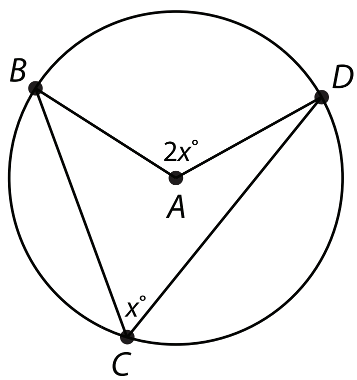
Inscribed Angle Theorem

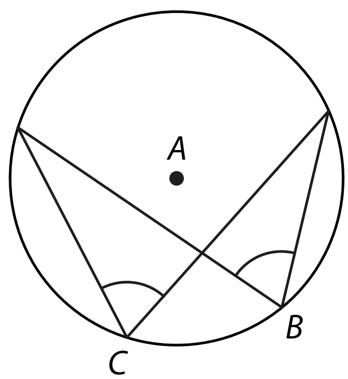
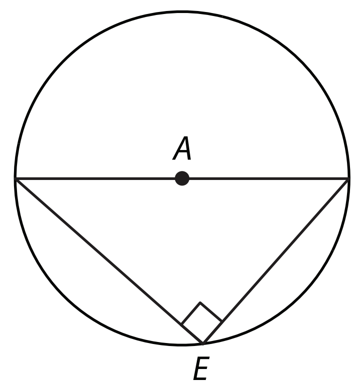
The measure of an inscribed angle is half the measure of its intercepted arc's angle.

Given $\odot A$, $m\angle C = \frac{1}{2} m\overset{\frown}{BD}$.



- In the following diagram, $\angle BCD$ is the inscribed angle and $\angle BAD$ is the central angle. They both intercept the minor arc $\overset{\frown}{BD}$.



Corollaries to the Inscribed Angle Theorem	
Corollary 1 Two inscribed angles that intercept the same arc are congruent. 	**Corollary 2** An angle inscribed in a semicircle is a right angle. 

Guided Practice 3.1.1

Example 1

Prove that the measure of a central angle is twice the measure of an inscribed angle that intercepts the same arc.

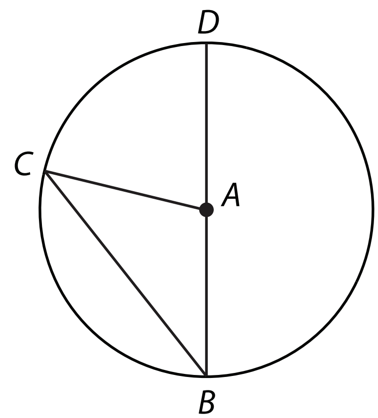
Given: $\odot A$ with inscribed $\angle B$ and central $\angle CAD$ intercepting $\overset{\frown}{CD}$.

Prove: $2m\angle B = m\angle CAD$

1. Identify the known information.

   Circle A with inscribed $\angle B$ and central $\angle CAD$ intercepts $\overset{\frown}{CD}$.

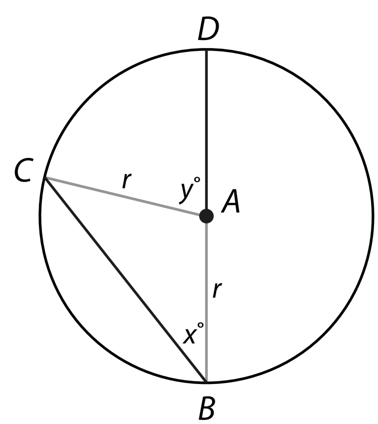
   Let $\overline{BD}$ be a diameter of the circle.

   By definition, $\overline{AB}$ and $\overline{AC}$ are radii of the circle.

   Mark them as r.

   Identify $\angle B$ as x and $\angle CAD$ as y.

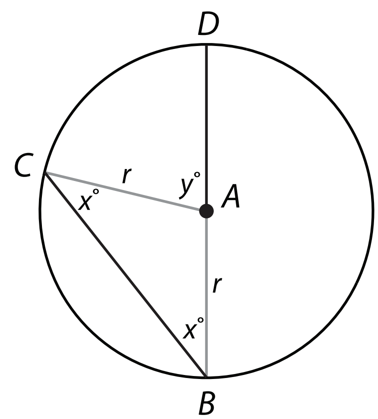
   

2. Identify what information is known about the angles of the triangle.

$\triangle ABC$ is isosceles since both legs are the same length, r.

By the Isosceles Triangle Theorem, both base angles of the triangle are congruent; therefore, $\angle ABC \cong \angle ACB = x$.

Also, the vertex angle of the isosceles triangle is a linear pair with y and thus $m\angle CAB = 180 - y$.



By the Triangle Sum Theorem:

$x + x + 180 - y = 180$

$2x - y = 0$

$2x = y$

Substituting the names for x and y yields $2m\angle B = m\angle CAD$.

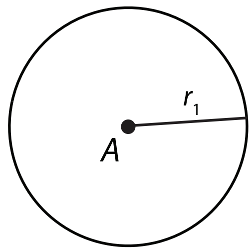
Therefore, the measure of a central angle is twice the measure of an inscribed angle that intercepts the same arc.

Example 2

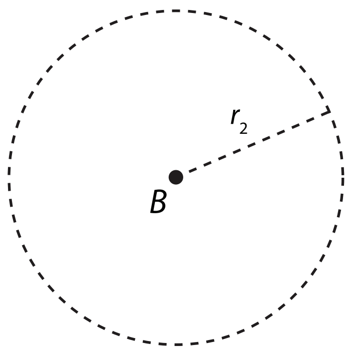
Prove that all circles are similar using the concept of similarity transformations.

1. Create a diagram to help with the proof.

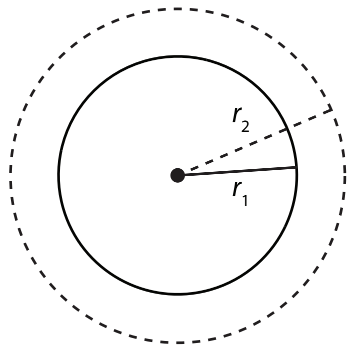
   Draw $\odot A$ with radius r_1.



   Draw $\odot B$ with radius r_2 such that $r_2 > r_1$.

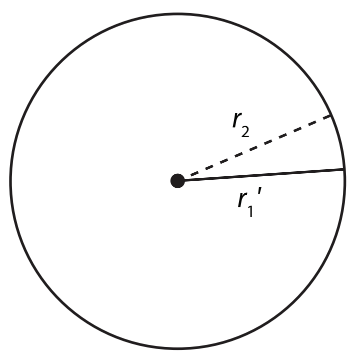


2. Using properties of rigid motion, translate $\odot B$ so that it is concentric with $\odot A$.



3. Determine the scale factor necessary to dilate $\odot A$ so that it maps to $\odot B$.

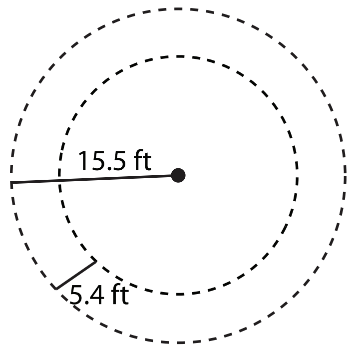
   To determine the ratio, note that each circle has a radius that by definition is the segment from the center to a point on the circle. To map $\odot A \rightarrow \odot B$, divide r_2 by r_1. The resulting ratio is the scale factor that produces the following image.



   This scale factor, $\frac{r_2}{r_1}$, is true for all circles and a similarity transformation of this sort can always be used because all circles are similar.

Example 3

A car has a circular turning radius of 15.5 feet. The distance between the two front tires is 5.4 feet. To the nearest foot, how much farther does a tire on the outer edge of the turning radius travel than a tire on the inner edge?



1. Calculate the circumference of the outer tire's turn.

$C = 2\pi r$	Formula for the circumference of a circle
$C = 2\pi(15.5)$	Substitute 15.5 for the radius (r).
$C = 31\pi$	Simplify.

2. Calculate the circumference of the inside tire's turn.

   First, calculate the radius of the inner tire's turn.

   Since all tires are similar, the radius of the inner tire's turn can be calculated by subtracting the distance between the two front wheels (the distance between each circle) from the radius of the outer tire's turn.

   $15.5 - 5.4 = 10.1$ ft

$C = 2\pi r$	Formula for the circumference of a circle
$C = 2\pi(10.1)$	Substitute 10.1 for the radius (r).
$C = 20.2\pi$	Simplify.

3. Calculate the difference in the circumference of each tire's turn.

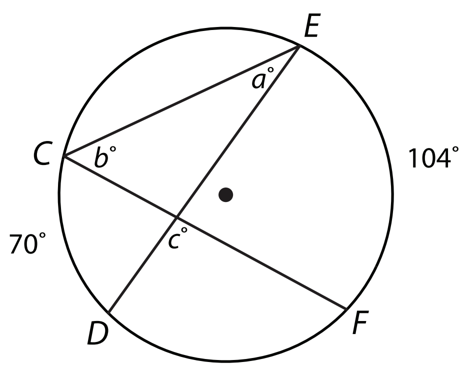
   Find the difference in the circumference of each tire's turn.

   $31\pi - 20.2\pi = 10.8\pi \approx 33.93$

   The outer tire travels approximately 34 feet farther than the inner tire.

Example 4

Find the value of each variable.



1. Identify the inscribed angle and the intercepted arc and use the proper theorem or corollary to find the value of a.

   a is the inscribed angle creating the intercepting arc $\overset{\frown}{CD}$.

   The measure of the intercepted arc is $70°$.

   Use the Inscribed Angle Theorem to find a.

$m\angle E = \frac{1}{2} m\overset{\frown}{CD}$	Inscribed Angle Theorem
$a = \frac{1}{2}(70)$	Substitute the given information.
$a = 35$	Solve for a.

   The value of a is 35°.

2. Identify the inscribed angle and the intercepted arc and use the proper theorem or corollary to find the value of *b*.

   b is the inscribed angle creating the intercepting arc $\overset{\frown}{EF}$.

   The measure of the intercepted arc is 104°.

   Use the Inscribed Angle Theorem to find *b*.

$m\angle C = \frac{1}{2} m\overset{\frown}{EF}$	Inscribed Angle Theorem
$b = \frac{1}{2}(104)$	Substitute the given information.
$b = 52$	Solve for *b*.

   The value of *b* is 52°.

3. Use the Triangle Sum Theorem and the Vertical Angles Theorem to find the value of *c*.

   Note that *a* and *b*, in addition to being the measures of inscribed angles, are interior angles of a triangle.

   The third interior angle is a vertical angle to *c* and is therefore congruent to *c*.

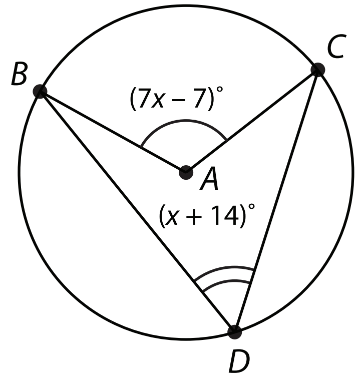
   Finding that third angle will yield the value of *c*.

$a + b + c = 180$	Triangle Sum Theorem and Vertical Angles Theorem
$35 + 52 + c = 180$	Substitute the measures of *a* and *b*.
$87 + c = 180$	Combine like terms.
$c = 93$	Solve for *c*.

   The value of *c* is 93°.

Example 5

Find the measures of $\angle BAC$ and $\angle BDC$.



1. Set up an equation to solve for x.

   $\angle BAC$ is a central angle and $\angle BDC$ is an inscribed angle in $\odot A$.

$m\angle BAC = 2m\angle BDC$	Central Angle/Inscribed Angle Theorem
$7x - 7 = 2(x + 14)$	Substitute values for $\angle BAC$ and $\angle BDC$.
$7x - 7 = 2x + 28$	Distributive Property
$5x = 35$	Solve for x.
$x = 7$	

2. Substitute the value of x into the expression for $\angle BDC$ to find the measure of the inscribed angle.

   $m\angle BDC = (x + 14)$

   $m\angle BDC = (7) + 14$

   $m\angle BDC = 21$

   The measure of $\angle BDC$ is 21°.

3. Find the value of the central angle, $\angle BAC$.

By the Inscribed Angle Theorem, $m\angle BAC = 2m\angle BDC$.

$m\angle BAC = 21(2)$

$m\angle BAC = 42$

The measure of $\angle BAC$ is 42°.

Practice 3.1.1: Similar Circles and Central and Inscribed Angles

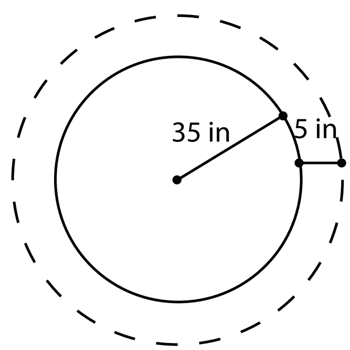
Given that all circles are similar, determine the scale factor necessary to map $\odot C \to \odot D$.

1. $\odot C$ has a radius of 144 units and $\odot D$ has a radius of 3 units.

2. $\odot C$ has a diameter of 50 units and $\odot D$ has a diameter of 12 units.

Use your knowledge of similar circles to complete problems 3 and 4.

3. A circular waterfall is surrounded by a brick wall. The radius of the inner wall is 35 inches. If the bricks are 5 inches wide, what scale factor was used to determine the radius of the outer wall?



4. A small car has a tire with a 17-inch diameter. A truck has a tire with a 32-inch diameter. How much farther than the car does the truck have to drive for its tire to complete one revolution?

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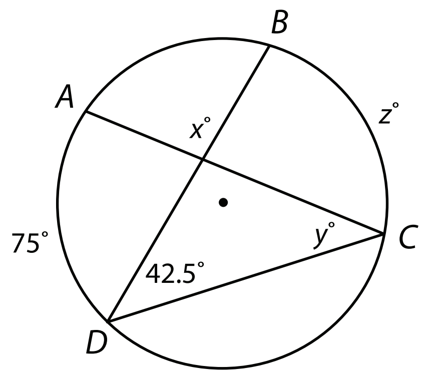
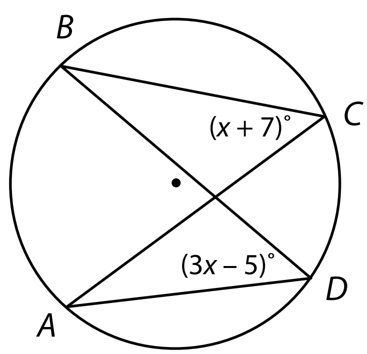
Use your knowledge of angles to complete the problems that follow.

5. Find the values of x, y, and z.



6. Find the value of x and the measure of $\overset{\frown}{AB}$.



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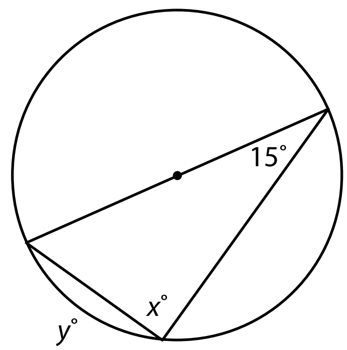
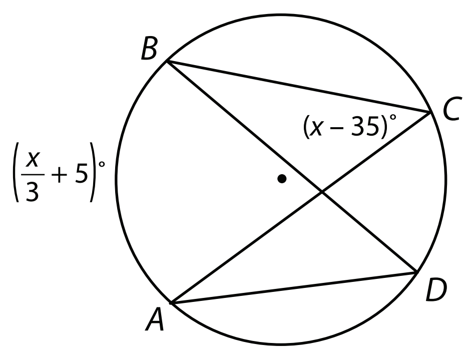
7. Find the values of x and y.



8. Find $m\angle C$ and $m\angle D$.



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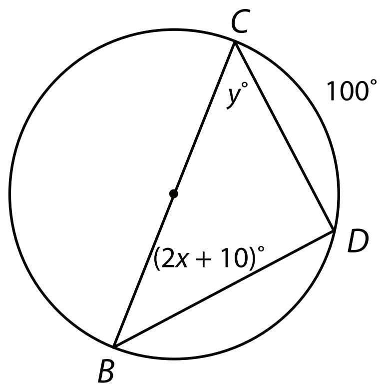
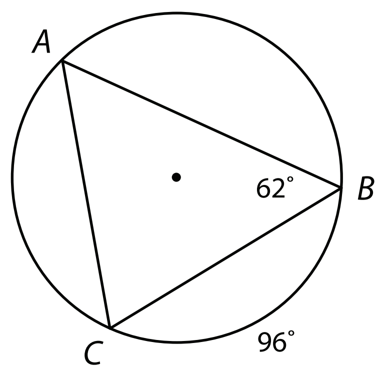
9. Find $m\angle B$ and $m\angle C$.



10. Find $m\overset{\frown}{AB}$ and $m\overset{\frown}{CA}$.



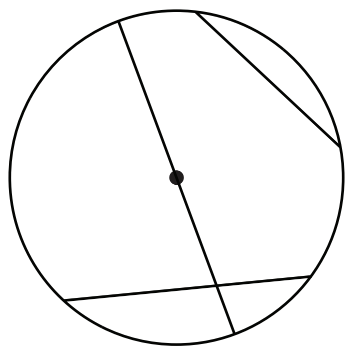
Lesson 3.1.2: Chord Central Angles Conjecture

Introduction

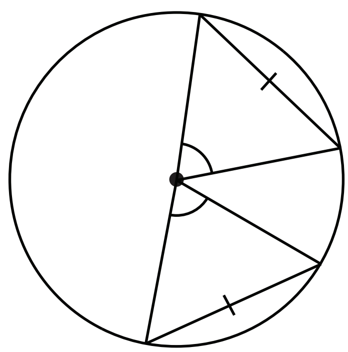
Circles have several special properties, conjectures, postulates, and theorems associated with them. This lesson focuses on the relationship between chords and the angles and arcs they create.

Key Concepts

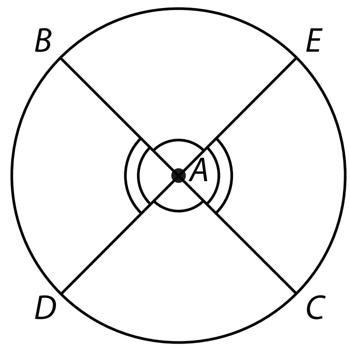
- Chords are segments whose endpoints lie on the circumference of a circle.
- Three chords are shown on the circle below.



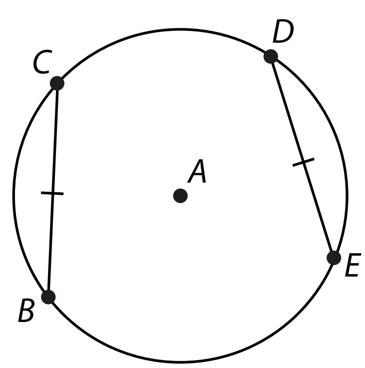
- Congruent chords of a circle create one pair of congruent central angles.



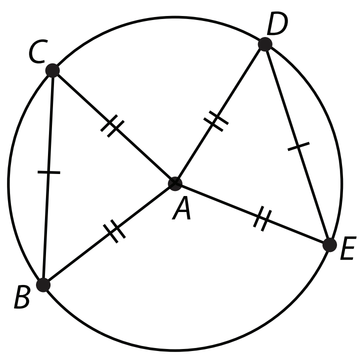
- When the sides of the central angles create diameters of the circle, vertical angles are formed. This creates two pairs of congruent central angles.



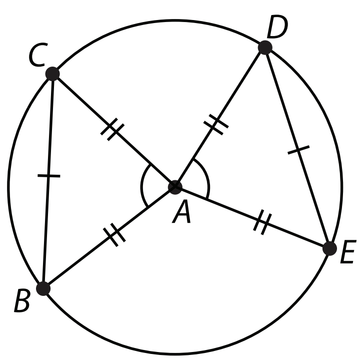
- Congruent chords also intercept congruent arcs.
- An **intercepted arc** is an arc whose endpoints intersect the sides of an inscribed angle and whose other points are in the interior of the angle.
- Remember that the measure of an arc is the same as the measure of its central angle.
- Also, recall that central angles are twice the measure of their inscribed angles.
- Central angles of two different triangles are congruent if their chords and circles are congruent.
- In the circle below, chords $\overline{BC}$ and $\overline{DE}$ are congruent chords of $\odot A$.



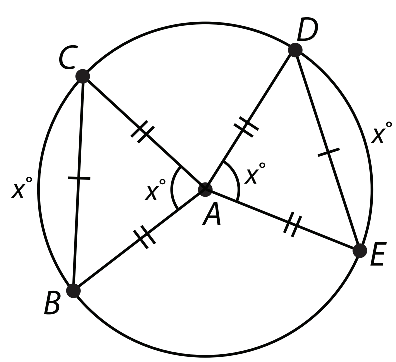
- When the radii are constructed such that each endpoint of the chord connects to the center of the circle, four central angles are created, as well as two congruent isosceles triangles by the SSS Congruence Postulate.



- Since the triangles are congruent and both triangles include two central angles that are the vertex angles of the isosceles triangles, those central angles are also congruent because Corresponding Parts of Congruent Triangles are Congruent (CPCTC).



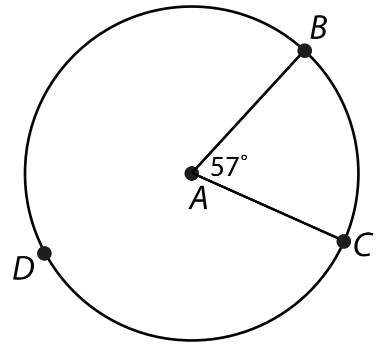
- The measure of the arcs intercepted by the chords is congruent to the measure of the central angle because arc measures are determined by their central angle.



Guided Practice 3.1.2

Example 1

In $\odot A$, $m\angle BAC = 57$. What is $m\overset{\frown}{BDC}$?



1. Find the measure of $\overset{\frown}{BC}$.

   $m\angle BAC = 57$

   The measure of $\angle BAC$ is equal to the measure of $\overset{\frown}{BC}$ because central angles are congruent to their intercepted arc; therefore, the measure of $\overset{\frown}{BC}$ is also 57°.

2. Find the measure of $\overset{\frown}{BDC}$.

   Subtract the measure of $\overset{\frown}{BC}$ from 360°.

   $360 - 57 = 303$

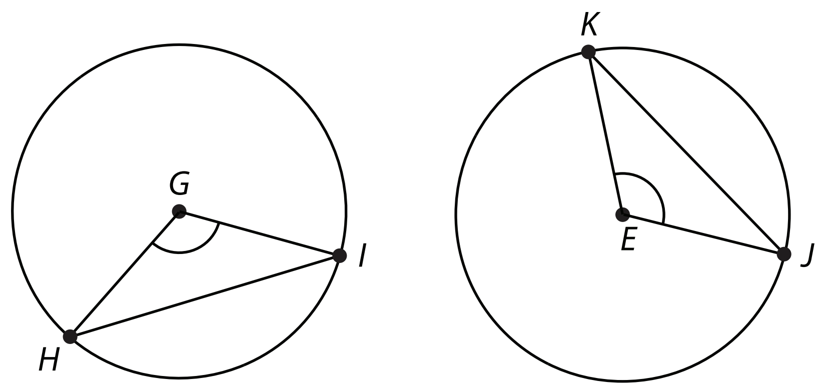
   $m\overset{\frown}{BDC} = 303$

3. State your conclusion.

   The measure of $\overset{\frown}{BDC}$ is 303°.

Example 2

$\odot G \cong \odot E$. What conclusions can you make?



1. What type of angles are $\angle G$ and $\angle E$, and what does that tell you about the chords?

   $\angle G$ and $\angle E$ are congruent central angles of congruent circles, so $\overline{HI}$ and $\overline{JK}$ are congruent chords.

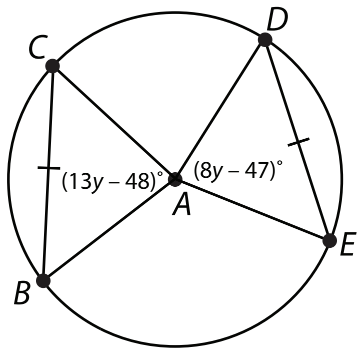
2. Since the central angles are congruent, what else do you know?

   The measures of the arcs intercepted by congruent chords are congruent, so minor arc *HI* is congruent to minor arc *JK*.

   Deductively, then, major arc *IH* is congruent to major arc *KJ*.

Example 3

Find the value of y.



1. Set up an equation to solve for y.

   The angles marked are central angles created by congruent chords, and are therefore congruent.

   Set the measure of each angle equal to each other.

   $13y - 48 = 8y + 47$

2. Solve for y.

   $13y - 48 = 8y + 47$ Central angles created by congruent chords are congruent.

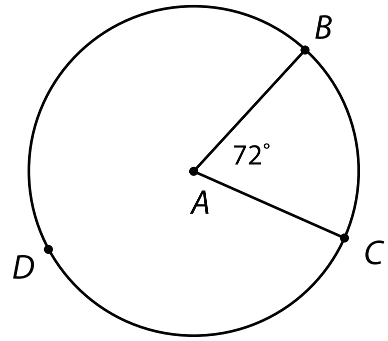
   $5y = 95$

   $y = 19$

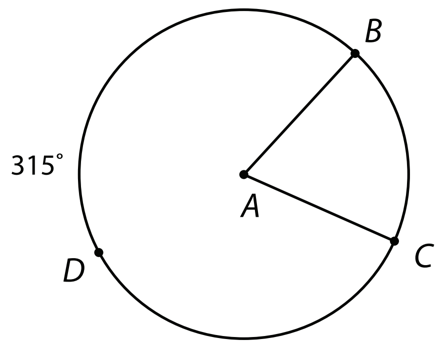
Practice 3.1.2: Chord Central Angles Conjecture

Use what you've learned about chords and central angles to solve.

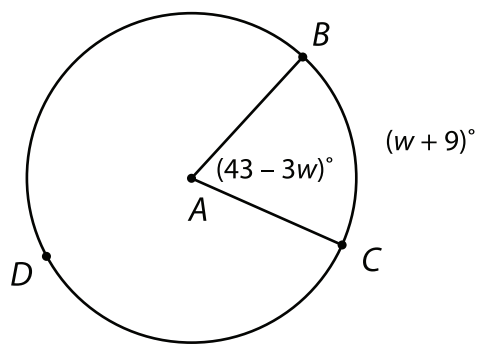
1. In $\odot A$, $m\angle BAC = 72$. What is $m\overset{\frown}{BDC}$?



2. In $\odot A$, $\overset{\frown}{BDC} = 315$. What is $m\angle BAC$?



3. What is the value of *w*?



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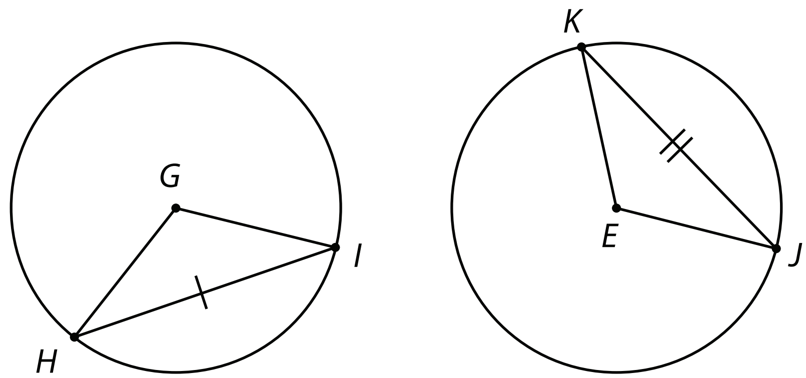
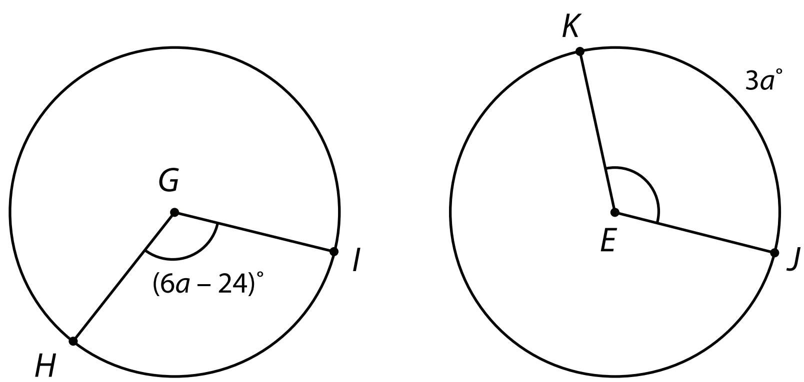
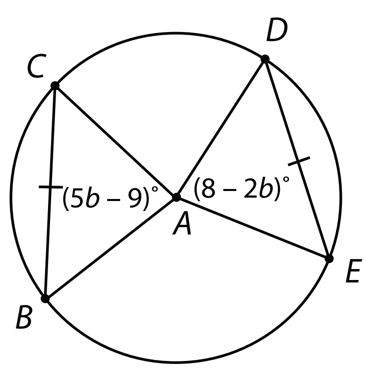
4. What can you conclude about $\odot G$ and $\odot E$?



5. $\odot G \cong \odot E$. What is the value of a?



6. Find the value of b.



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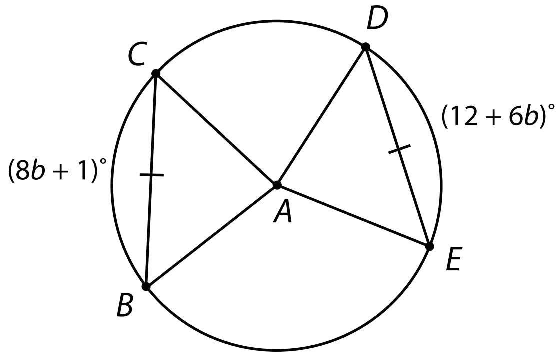
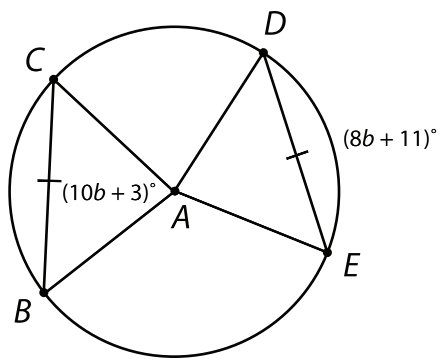
7. Find the value of b.



8. Find the value of b.



9. If the circumference of one circle is 54 millimeters and the major arc of a second circle measures 182°, what is the length of this major arc of the second circle?

10. You're sewing the seam on a circular lampshade for interior design class. The total circumference of the lampshade is 3 feet. The amount of the seam you've sewn so far measures approximately 23° around the circumference of the lampshade. What length of lampshade still needs to be sewn?

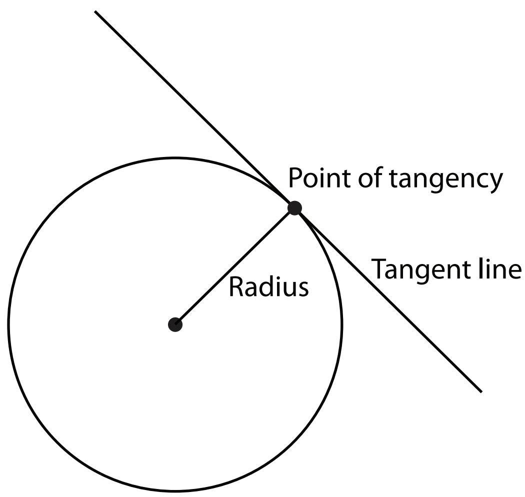
Lesson 3.1.3: Properties of Tangents of a Circle

Introduction

Circles and tangent lines can be useful in many real-world applications and fields of study, such as construction, landscaping, and engineering. There are many different types of lines that touch or intersect circles. All of these lines have unique properties and relationships to a circle. Specifically, in this lesson, we will identify what a tangent line is, explore the properties of tangent lines, prove that a line is tangent to a circle, and find the lengths of tangent lines. We will also identify and use secant lines, as well as discuss how they are different from tangent lines.

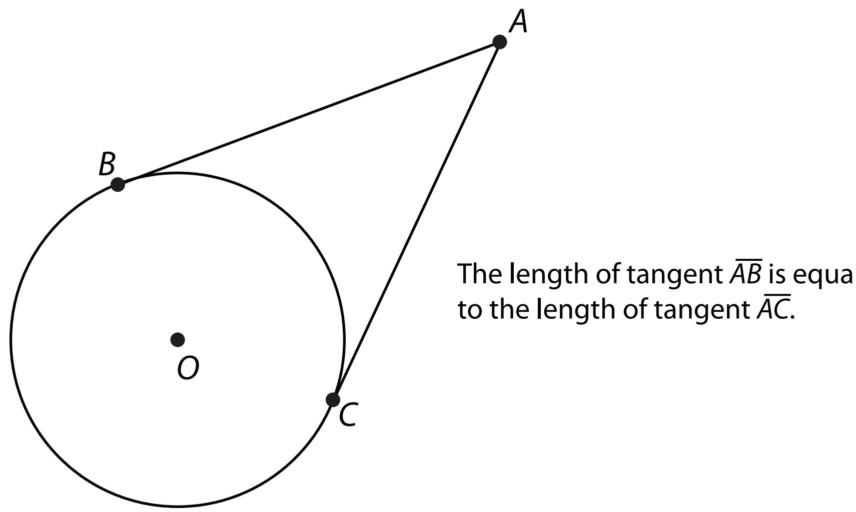
Key Concepts

- A **tangent line** is a line that intersects a circle at exactly one point.
- Tangent lines are perpendicular to the radius of the circle at the point of tangency.

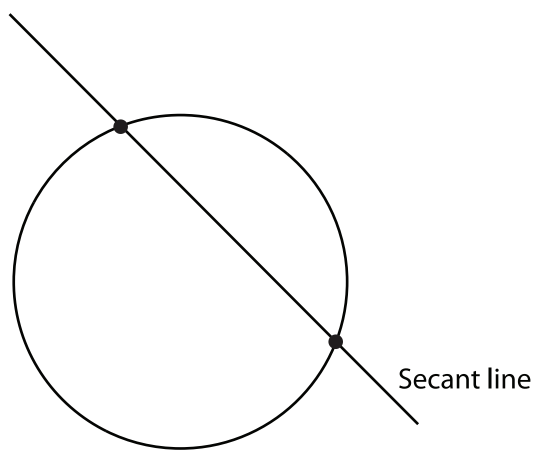


- You can verify that a line is tangent to a circle by constructing a right triangle using the radius, and verifying that it is a right triangle by using the Pythagorean Theorem.
- The slopes of a line and a radius drawn to the possible point of tangency must be negative reciprocals in order for the line to be a tangent.
- If two segments are tangent to the same circle, and originate from the same exterior point, then the segments are congruent.
- The angle formed by two tangent lines whose vertex is outside of the circle is called the **circumscribed angle**.

- $\angle BAC$ is a circumscribed angle.



- The angle formed by two tangents is equal to one half the positive difference of the angle's intercepted arcs.
- A **secant line** is a line that intersects a circle at two points.

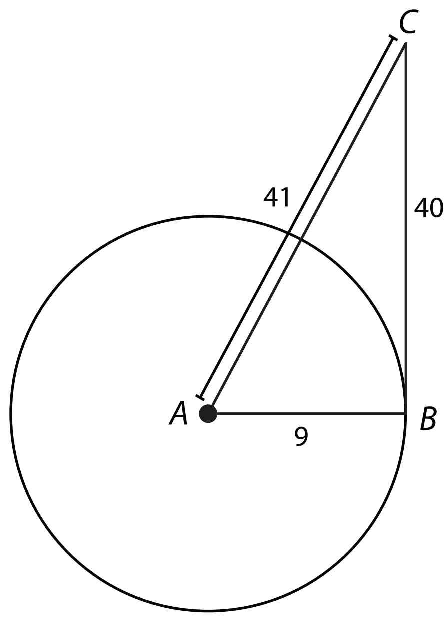


- An angle formed by a secant and a tangent is equal to the positive difference of its intercepted arcs.

Guided Practice 3.1.3

Example 1

Determine whether $\overline{BC}$ is tangent to $\odot A$ in the diagram below.



1. Identify the radius.

   The radius of the circle is the segment from the center of the circle to a point on the circle.

   $\overline{AB}$ is the radius of $\odot A$.

2. Determine the relationship between $\overline{AB}$ and $\overline{BC}$ at point B in order for $\overline{BC}$ to be tangent to $\odot A$.

   $\overline{AB}$ must be perpendicular to $\overline{BC}$ at point B in order for $\overline{BC}$ to be tangent to $\odot A$.

3. Show that $\angle ABC$ is a right angle by using the converse of the Pythagorean Theorem.

The converse of the Pythagorean Theorem states that when the sum of the squares of two sides of a triangle is equal to the square of the third side of the triangle, the triangle is a right triangle.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$(AB)^2 + (BC)^2 = (AC)^2$	Substitute segment names for a, b, and c.
$9^2 + 40^2 = 41^2$	Substitute values for $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$.
$81 + 1600 = 1681$	Simplify, then solve.
$1681 = 1681$	

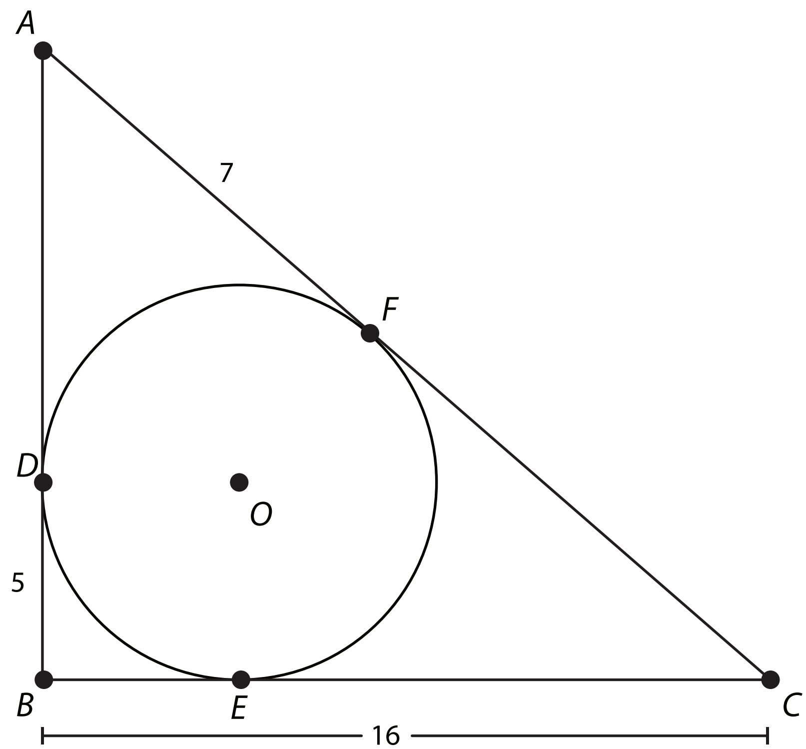
The result is a true statement; therefore, $\angle ABC$ is a right angle.

4. State your conclusion.

Since the converse of the Pythagorean Theorem is true, $\triangle ABC$ is a right triangle; therefore, $\angle ABC$ is a right angle. This makes $\overline{AB}$ perpendicular to $\overline{BC}$; therefore, $\overline{BC}$ is tangent to $\odot A$.

Example 2

Each side of $\triangle ABC$ is tangent to circle O at the points D, E, and F. Find the perimeter of $\triangle ABC$.



1. Identify the lengths of each side of the triangle.

   $\overline{AD}$ is tangent to the same circle as $\overline{AF}$ and extends from the same point; therefore, the lengths are equal.

   $AD = 7$ units

   $\overline{BE}$ is tangent to the same circle as $\overline{BD}$ and extends from the same point; therefore, the lengths are equal.

   $BE = 5$ units

   To determine the length of $\overline{CE}$, subtract the length of $\overline{BE}$ from the length of $\overline{BC}$.

   $16 - 5 = 11$

   $CE = 11$ units

   $\overline{CF}$ is tangent to the same circle as $\overline{CE}$ and extends from the same point; therefore, the lengths are equal.

   $CF = 11$ units

2. Calculate the perimeter of $\triangle ABC$.

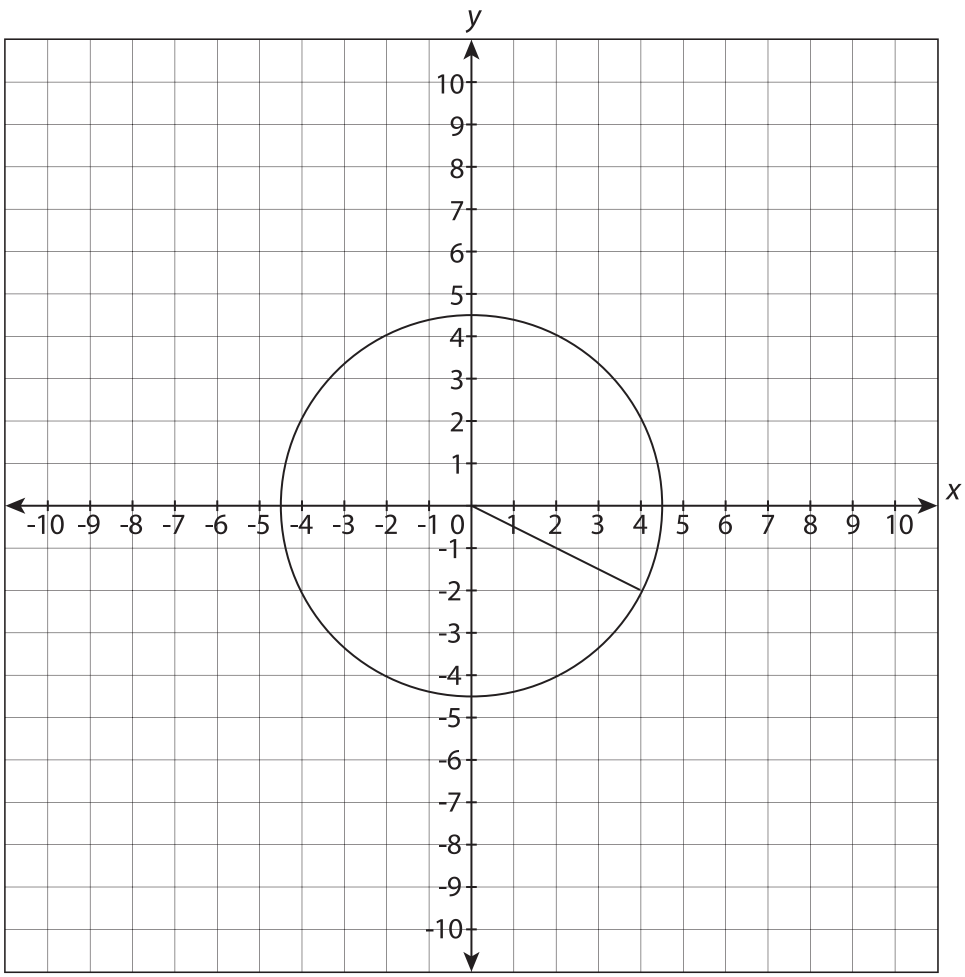
Add the lengths of $\overline{AD}$, $\overline{AF}$, $\overline{BD}$, $\overline{BE}$, $\overline{CE}$, and $\overline{CF}$ to find the perimeter of the polygon.

$$7 + 7 + 5 + 5 + 11 + 11 = 46 \text{ units}$$

The perimeter of $\triangle ABC$ is 46 units.

Example 3

A landscaper wants to build a walkway tangent to the circular park shown in the diagram below. The other walkway pictured is a radius of the circle and has a slope of $-\frac{1}{2}$ on the grid. If the walkways should intersect at (4, –2) on the grid, what equation can the landscaper use to graph the new walkway on the grid?



1. Determine the slope of the walkway.

   One of the properties of tangent lines states that if a line is tangent to a circle, then it is perpendicular to the radius at the point of tangency.

   Since the slope of the radius is $-\frac{1}{2}$, then the slope of the new walkway will be 2 because perpendicular lines have slopes that are negative reciprocals.

2. Determine the equation that represents the walkway.

   Recall that the point-slope form of a line is $y-y_1=m(x-x_1)$.

   Since the new walkway is tangent to the circle at (4, –2), then the point (4, –2) is on the tangent line, so it can be used to write the equation.

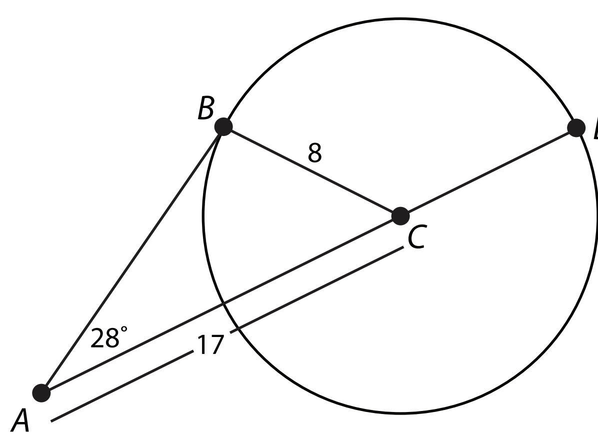
   $y-(-2)=2(x-4)$

   $y+2=2(x-4)$

   This equation can be rearranged into slope-intercept form, $y=2x-10$, for easier graphing.

Example 4

$\overline{AB}$ is tangent to $\odot C$ at point B as shown below. Find the length of $\overline{AB}$ as well as $m\widehat{BD}$.



1. Find the length of $\overline{AB}$.

Since $\overline{AB}$ is tangent to $\odot C$, then $\angle ABC$ is right angle because a tangent and a radius form a right angle at the point of tangency.

Since $\angle ABC$ is a right angle, $\triangle ABC$ is a right triangle.

Use the Pythagorean Theorem to find the length of $\overline{AB}$.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$8^2 + (AB)^2 = 17^2$	Substitute values for *a*, *b*, and *c*.
$64 + (AB)^2 = 289$	Simplify.
$(AB)^2 = 225$	
$AB = 15$	

The length of $\overline{AB}$ is 15 units.

2. Find $m\overset{\frown}{BD}$.

First, determine the unknown measure of $\angle ACB$.

Recall that the sum of all three angles of a triangle is 180°.

$\angle ABC$ is a right angle, so it is 90°.

$\angle BAC$ is 28°, as shown in the diagram.

Set up an equation to determine the measure of $\angle ACB$.

$28 + 90 + m\angle ACB = 180$

$118 + m\angle ACB = 180$

$m\angle ACB = 62$

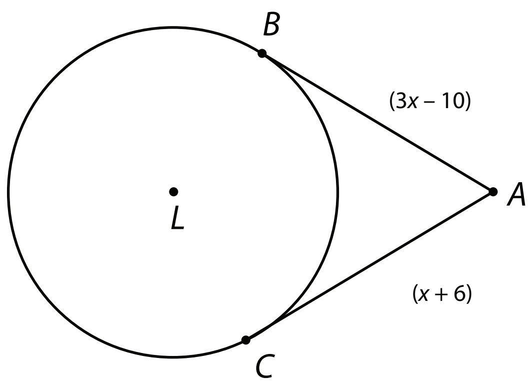
Since $m\angle ACB = 62$, then $m\angle BCD = 118$ because $\angle ACB$ and $\angle BCD$ are a linear pair.

$\angle BCD$ is a central angle, and recall that the measure of a central angle is the same as its intercepted arc, so $\overset{\frown}{BD}$ is 118°.

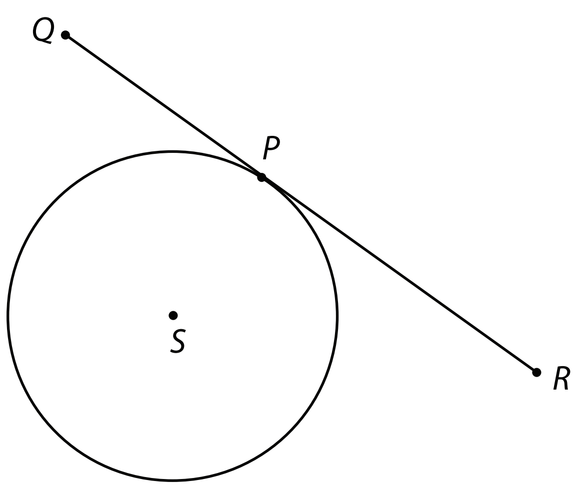
Practice 3.1.3: Properties of Tangents of a Circle

Use what you have learned about tangent lines and secant lines to answer the questions.

1. $\overline{AB}$ and $\overline{AC}$ are tangent to $\odot L$ in the diagram below. What is the value of x?



2. You know that $\overline{PR}$ is tangent to $\odot S$ in the diagram below. What must you prove to show that this is true?



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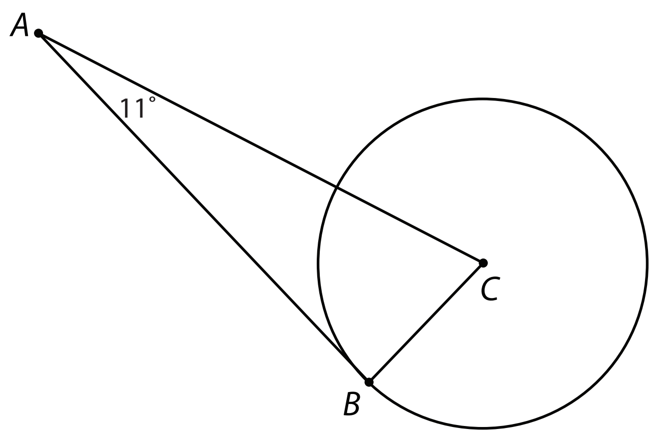
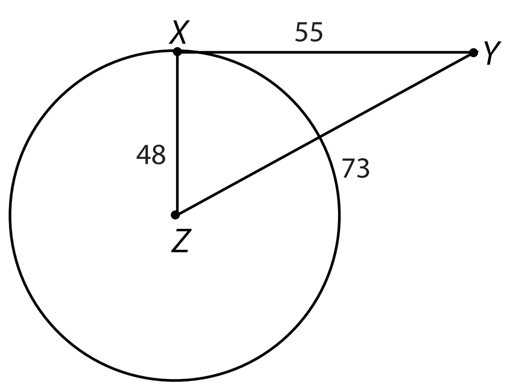
3. $\overline{AB}$ is tangent to $\odot C$ at point B in the diagram below. What is the measure of $\angle ACB$?



4. A space station orbiting the Earth is sending two signals that are tangent to the Earth. If the intercepted arc of the Earth's surface that is visible to the satellite is 168°, what is the measure of the angle formed by the two signal beams?

5. Is $\overline{XY}$ tangent to $\odot Z$ at point X in the diagram below? Explain.



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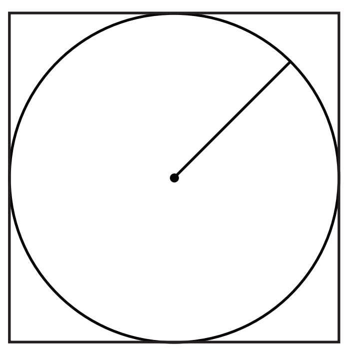
6. How many tangents can be drawn that contain a point on a circle? Explain your answer.

7. The slope of radius $\overline{PQ}$ in circle Q is $-\frac{2}{3}$. A student wants to draw a tangent to $\odot Q$ at point P. What will be the slope of this tangent line?

8. A homeowner is building a square fence around his circular patio so that each side of the fence is tangent to the circle, as shown in the diagram below. The radius of his patio is 18 yards. What is the perimeter of his fence?



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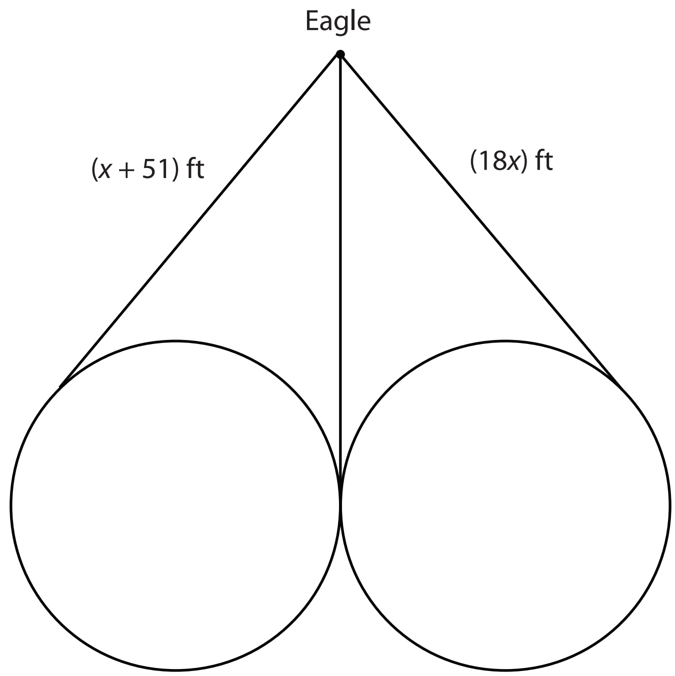
9. You are using binoculars to look at an eagle. The sides of the binoculars seem to extend from the eagle, and are tangent to the circular portion. How far away is the eagle?



10. Your friend Truman argues that chords and tangents are the same thing. You disagree. What do you tell Truman to convince him that he is incorrect?