6-5

1. Plan

Objectives

1 To verify and use properties of trapezoids and kites

Examples

1 Finding Angle Measures in Trapezoids
2 Real-World Connection
3 Finding Angle Measures in Kites

Professional Development

Math Background

A kite can be described as the union of two isosceles triangles without their common base or the figure formed by the radii from the centers of two intersecting circles to the points of intersection. Many construction methods depend on this relationship to circles and on the perpendicularity of the diagonals of a kite.

More Math Background: p. 304D

Lesson Planning and Resources

See p. 304E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need
For intervention, direct students to:

Using Properties of Special Quadrilaterals
Lesson 6-1: Example 3
Extra Skills, Word Problems, Proof Practice, Ch. 6

6-5

Trapezoids and Kites

What You'll Learn

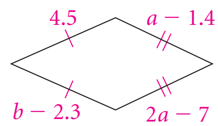
- To verify and use properties of trapezoids and kites

. . . And Why

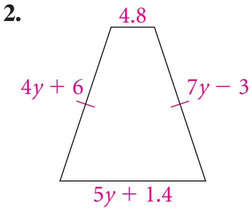
To find angle measures of trapezoidal windows, as in Example 2

Check Skills You'll Need GO for Help Lesson 6-1

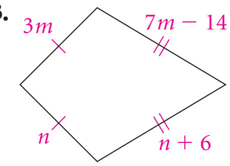
x^2 **Algebra** **Find the values of the variables. Then find the lengths of the sides.**

1.


a = 5.6, b = 6.8; 4.5, 4.2, 4.5, 4.2

2.


2. 3; 4.8, 16.4, 18, 18

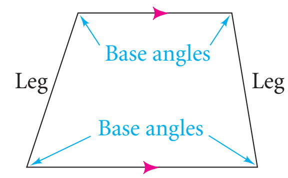
3.


m = 5, n = 15; 15, 15, 21, 21

New Vocabulary • base angles of a trapezoid

1 Properties of Trapezoids and Kites

The parallel sides of a trapezoid are its bases. The nonparallel sides are its legs. Two angles that share a base of a trapezoid are **base angles** of the trapezoid.



The following theorem is about each pair of base angles. You will be asked to prove it in Exercise 38.

Key Concepts

Theorem 6-15

The base angles of an isosceles trapezoid are congruent.

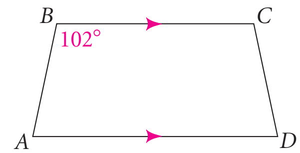
Real-World Connection

In the isosceles trapezoids at the top of this electric tea kettle, each pair of base angles are congruent.

The bases of a trapezoid are parallel. Therefore the two angles that share a leg are supplementary. This fact and Theorem 6-15 allow you to solve problems involving the angles of a trapezoid.

1 EXAMPLE Finding Angle Measures in Trapezoids

$ABCD$ is an isosceles trapezoid and $m\angle B = 102$. Find $m\angle A$, $m\angle C$, and $m\angle D$.



$m\angle A + m\angle B = 180$ Two angles that share a leg are supplementary.

$m\angle A + 102 = 180$ Substitute.

$m\angle A = 78$ Subtract 102 from each side.

By Theorem 6-15, $m\angle C = m\angle B = 102$ and $m\angle D = m\angle A = 78$.

Differentiated Instruction Solutions for All Learners

Special Needs L1
Point out that the diagonals of a kite and a rhombus are both perpendicular. Therefore, knowing a quadrilateral has perpendicular diagonals is not sufficient to conclude it is a kite, because it could be a rhombus.
learning style: tactile

Below Level L2
Help students associate Theorem 6-15 with the Isosceles Triangle Theorem by extending the nonparallel sides of an isosceles trapezoid to form an isosceles triangle.
learning style: visual

Quick Check

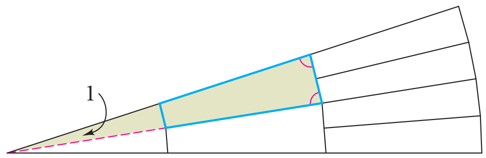
1. In the isosceles trapezoid, $m\angle S = 70$. Find $m\angle P$, $m\angle Q$, and $m\angle R$. **110, 110, 70**

2 EXAMPLE Real-World Connection

You are looking up at Harbour Centre Tower in Vancouver, Canada.

Architecture The second ring of the ceiling shown at the left is made from congruent isosceles trapezoids that create the illusion of circles. What are the measures of the base angles of these trapezoids?

Each trapezoid is part of an isosceles triangle whose base angles are the acute base angles of the trapezoid. The isosceles triangle has a vertex angle that is half as large as one of the 20 angles at the center of the ceiling.



The measure of each angle at the center of the ceiling is $\frac{360}{20}$ or 18.

The measure of $\angle 1$ is $\frac{18}{2}$, or 9.

The measure of each acute base angle is $\frac{180 - 9}{2}$, or 85.5.

The measure of each obtuse base angle is $180 - 85.5$, or 94.5.

Quick Check

2. A glass ceiling like the one above has 18 angles meeting at the center instead of 20. What are the measures of the base angles of the trapezoids in its second ring? **85, 95**

Like the diagonals of parallelograms, the diagonals of an isosceles trapezoid have a special property.

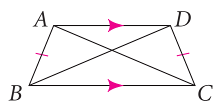
Key Concepts

Theorem 6-16

The diagonals of an isosceles trapezoid are congruent.

Proof

Proof of Theorem 6-16



Given: Isosceles trapezoid $ABCD$ with $\overline{AB} \cong \overline{DC}$

Prove: $\overline{AC} \cong \overline{DB}$

It is given that $\overline{AB} \cong \overline{DC}$. Because the base angles of an isosceles trapezoid are congruent, $\angle ABC \cong \angle DCB$. By the Reflexive Property of Congruence, $\overline{BC} \cong \overline{BC}$. Then, by the SAS Postulate, $\triangle ABC \cong \triangle DCB$. Therefore, $\overline{AC} \cong \overline{DB}$ by CPCTC.

Another special quadrilateral that is not a parallelogram is a kite. The diagonals of a kite, like the diagonals of a rhombus, are perpendicular. A proof of this for a kite (next page) is quite like its proof for a rhombus (at the top of page 330).

Advanced Learners L4

After students read the proof of Theorem 6-16, have them write a paragraph explaining whether the diagonals of an isosceles trapezoid bisect each other.

learning style: verbal

English Language Learners ELL

Draw several trapezoids in different orientations and have students identify the *bases, legs* and *base angles*. Emphasize that the parallel sides are called bases and are independent of their orientation.

learning style: verbal

2. Teach

Guided Instruction

1 EXAMPLE Error Prevention

Some students may think the base angles of an isosceles trapezoid have vertices only on the "bottom" side. This misconception stems from the common use of the word *base* to mean "the side of a figure on which it rests." Point out that each isosceles trapezoid has two bases, which may lie in any orientation, and two pairs of base angles.

2 EXAMPLE Careers

Architects design modern office buildings using not only trapezoids and squares but also triangles, circles, and ellipses.

Connection to Engineering

Have students find how a keystone is used and how its shape relates to this lesson.

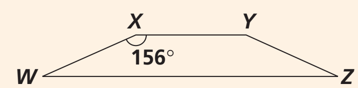
Visual Learners

When proving that the diagonals of an isosceles trapezoid are congruent, have students separately draw and label the overlapping triangles *ABC* and *DCB* to help them see how the parts correspond and why the triangles are congruent.

PowerPoint

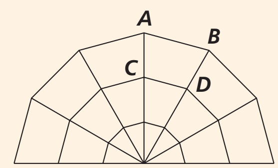
Additional Examples

1. *XYZW* is an isosceles trapezoid, and $m\angle X = 156$. Find $m\angle Y$, $m\angle Z$, and $m\angle W$.



$m\angle Y = 156$, $m\angle Z = m\angle W = 24$

2. Half of a spider's web is shown below, formed by layers of congruent isosceles trapezoids. Find the measures of the angles in *ABDC*.



$m\angle A = m\angle B = 75$, $m\angle C = m\angle D = 105$

Math Tip

After students read the proof of Theorem 6-17, ask: *Are both diagonals bisected?* **No; the figure then would be a parallelogram.**

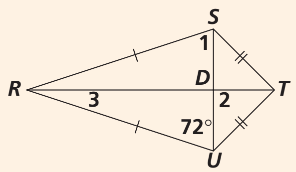
3 EXAMPLE Teaching Tip

Discuss as a class how to prove $\triangle DBA \cong \triangle DBC$ using SSS.

PowerPoint

Additional Examples

3 Find $m\angle 1$, $m\angle 2$, and $m\angle 3$ in the kite.



$m\angle 1 = 72$, $m\angle 2 = 90$, $m\angle 3 = 18$

Resources

- Daily Notetaking Guide 6-5 L3
- Daily Notetaking Guide 6-5—Adapted Instruction L1

Closure

Draw and label an isosceles trapezoid, a (convex) kite, and their diagonals. Then write congruence statements for all pairs of triangles that you can prove congruent. **Students should find three pairs of congruent triangles for each figure.**

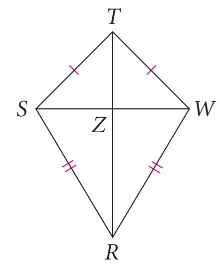
Key Concepts

Theorem 6-17

The diagonals of a kite are perpendicular.

Proof

Proof of Theorem 6-17



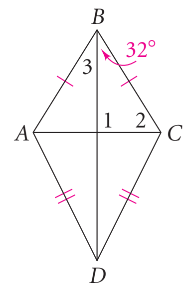
Given: Kite $RSTW$ with $\overline{TS} \cong \overline{TW}$ and $\overline{RS} \cong \overline{RW}$

Prove: $\overline{TR} \perp \overline{SW}$

Both T and R are equidistant from S and W. By the Converse of the Perpendicular Bisector Theorem, T and R lie on the perpendicular bisector of $\overline{SW}$. Since there is exactly one line through any two points (Postulate 1-1), $\overleftrightarrow{TR}$ must be the perpendicular bisector of $\overline{SW}$. Therefore, $\overline{TR} \perp \overline{SW}$.

You can use Theorem 6-17 to find angle measures in kites.

3 EXAMPLE Finding Angle Measures in Kites



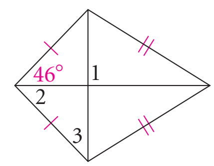
Find $m\angle 1$, $m\angle 2$, and $m\angle 3$ in the kite.

$m\angle 1 = 90$	**Diagonals of a kite are perpendicular.**
$90 + m\angle 2 + 32 = 180$	**Triangle Angle-Sum Theorem**
$122 + m\angle 2 = 180$	**Simplify.**
$m\angle 2 = 58$	**Subtract 122 from each side.**

$\triangle ABD \cong \triangle CBD$ by SSS.

By CPCTC, $m\angle 3 = m\angle DBC = 32$.

Quick Check **3** Find $m\angle 1$, $m\angle 2$, and $m\angle 3$ in the kite. **90, 46, 44**



EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

Practice and Problem Solving

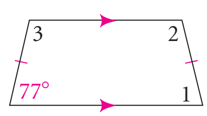
A Practice by Example

Example 1 (page 336)

GO for Help

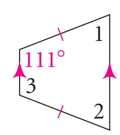
Each trapezoid is isosceles. Find the measure of each angle.

1.



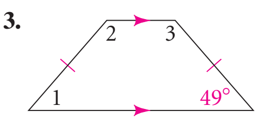
77, 103, 103

2.



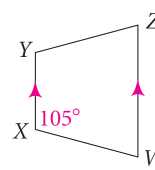
69, 69, 111

3.



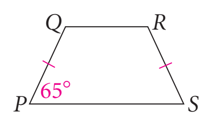
49, 131, 131

4.



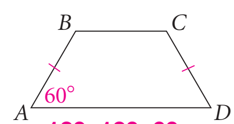
105, 75, 75

5.



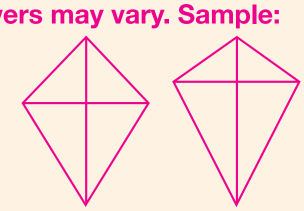
115, 115, 65

6.



120, 120, 60

17. Answers may vary. Sample:



Example 2 (page 337)

7. **Design** Each patio umbrella is made of eight panels that are congruent isosceles triangles with parallel stripes. A sample panel is shown at the right. The vertex angle of the panel measures 42.

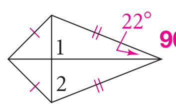
a. Classify the quadrilaterals shown as blue stripes on the panel.

7a. isosc. trapezoids

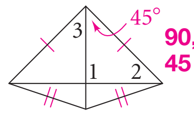
b. Find the measures of the quadrilaterals' interior angles.
69, 69, 111, 111

Example 3 (page 338)

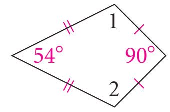
Find the measures of the numbered angles in each kite.

8. 

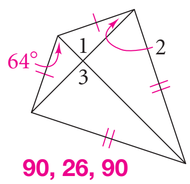
90, 68

9. 

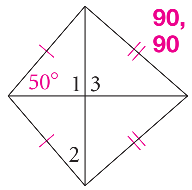
90, 45, 45

10. 

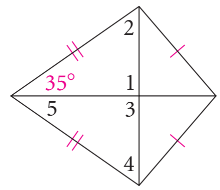
108, 108

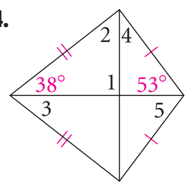
11. 

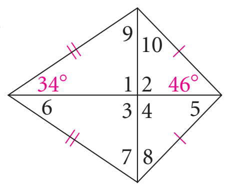
90, 26, 90

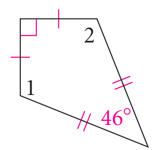
12. 

90, 40, 90

13. 

14. 

15. 

16. 

13. 90, 55, 90, 55, 35
14. 90, 52, 38, 37, 53
15. 90, 90, 90, 90, 46, 34, 56, 44, 56, 44
16. 112, 112

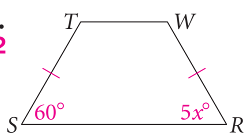
B Apply Your Skills

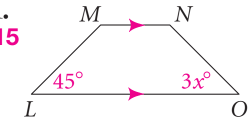
17. **Open-Ended** Sketch two kites that are not congruent, but with the diagonals of one congruent to the diagonals of the other. See margin.

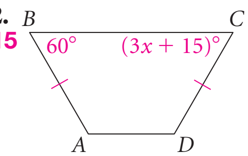
18. The perimeter of a kite is 66 cm. The length of one of its sides is 3 cm less than twice the length of another. Find the length of each side of the kite.
12, 12, 21, 21

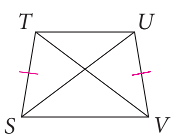
19. **Critical Thinking** If $KLMN$ is an isosceles trapezoid, is it possible for $\overline{KM}$ to bisect $\angle LMN$ and $\angle LKN$? Explain. See margin.

Algebra Find the value of the variable in each isosceles trapezoid.

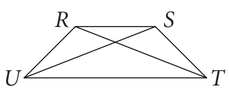
20. 12


21. 15


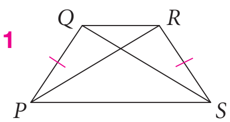
22. 15


23. 3


$TV = 2x - 1$
$US = x + 2$

24. 4


$SU = x + 1$
$TR = 2x - 3$

25. 1


$QS = x + 5$
$RP = 3x + 3$

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19. Explanations may vary. Sample: If both ∠s are bisected, then this combined with $\overline{KM} \cong \overline{KM}$ by the Reflexive Prop. means $\triangle KLM \cong \triangle KNM$ by SAS. By CPCTC, opp. $\angle L \cong \angle N$. $\angle L$ and $\angle N$ are opp., but $KLMN$ is isos., both pairs of base ∠'s are also ≅. By the Trans. Prop., all 4 angles are ≅, so $KLMN$ must be a rect. or a square. This contradicts what is gien, so $\overline{KM}$ cannot bisect $\angle LMN$ and $\angle LKN$.

3. Practice

Assignment Guide

1 A B	1-39
C Challenge	40-44
Test Prep	45-50
Mixed Review	51-56

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 6, 14, 29, 37, 38.

Diversity

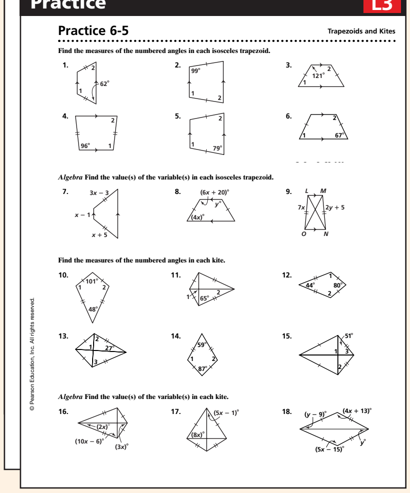
Exercise 7 The word *umbrella* comes from a Latin word meaning "shaded area or shadow," suggesting protection against the rain or sun. Ask whether students know the word for umbrella in other languages. For example, the Spanish word *paraguas* literally means "for water," and a *sombrilla* is a parasol.

Exercise 10 Discuss ways to prove $m\angle 1 = m\angle 2$.

Exercise 38 Have students work together to write this proof. Even with the Plan, the proof is complex and worthy of class discussion.

Differentiated Instruction Resources

- GPS Guided Problem Solving L3
- Enrichment L4
- Reteaching L2
- Adapted Practice L1
- Practice L3

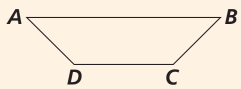


4. Assess & Reteach

PowerPoint

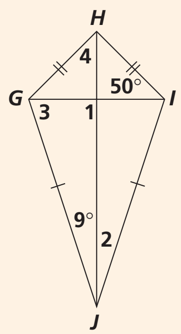
Lesson Quiz

Use isosceles trapezoid *ABCD* for Exercises 1 and 2.



1. If $m\angle A = 45$, find $m\angle B$, $m\angle C$, and $m\angle D$. **$m\angle B = 45$, $m\angle C = m\angle D = 135$**
2. If $AC = 3x - 16$ and $BD = 10x - 86$, find x. **10**

Use kite *GHIJ* for Exercises 3–6.



3. Find $m\angle 1$. **90**
4. Find $m\angle 2$. **9**
5. Find $m\angle 3$. **81**
6. Find $m\angle 4$. **40**

Alternative Assessment

Have students work in pairs to write answers to the following questions:

- How are a kite and a rhombus similar? How are they different?
- How are an isosceles trapezoid and a rectangle similar? How are they different?

34. No; if two consecutive ∠s are suppl., then another pair must be also because one pair of opp. ∠s is ≅. Therefore, the opp. ∠s would be ≅, which means the figure would be a ▱ and not a kite.

35. Yes; the ≅ ∠s must be 45° or 135° each.

36. No; if two consecutive ∠s were compl., then the kite would be concave.

37. Rhombuses and squares would be kites since opp. sides can be ≅ also.

39. Answers may vary. Sample: Draw $\overline{TA}$ and $\overline{RP}$.

1. isosc. trapezoid *TRAP* (Given)

2. $\overline{TA} \cong \overline{PR}$ (Diagonals of an isosc. trap. are ≅.)

3. $\overline{TR} \cong \overline{PA}$ (Given)

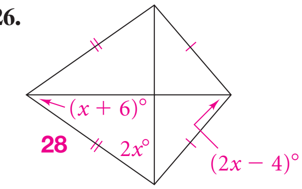
4. $\overline{RA} \cong \overline{RA}$ (Refl. Prop. of ≅)

5. $\triangle TRA \cong \triangle PAR$ (SSS)

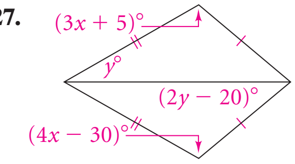
6. $\angle RTA \cong \angle APR$ (CPCTC)

Algebra Find the value(s) of the variable(s) in each kite.

26.

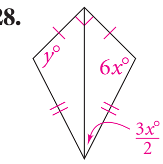


27.

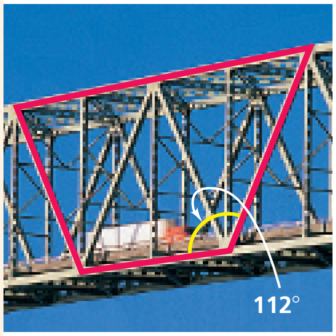


x = 35, y = 30

28.



x = 18, y = 108



Exercises 29–30

Bridge Design A quadrilateral is formed by the beams of the bridge at the left.

29. Classify the quadrilateral. Explain your reasoning. **Isosc. trapezoid; all the large rt. ∠s appear to be ≅.**
30. Find the measures of the other interior angles of the quadrilateral. **112, 68, 68**

Critical Thinking Can two angles of a kite be as follows? Explain. **31–33. See below left.**

31. opposite and acute
32. consecutive and obtuse
33. opposite and supplementary
34. consecutive and supplementary
35. opposite and complementary
36. consecutive and complementary **34–36. See margin.**

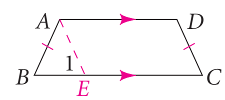
37. **Writing** A kite is sometimes defined as a quadrilateral with two pairs of consecutive sides congruent. Compare this to the definition you learned in Lesson 6-1. Are parallelograms, trapezoids, rhombuses, rectangles, or squares special kinds of kites according to the changed definition? Explain. **See margin.**

38. **Developing Proof** The plan suggests a proof of Theorem 6-15. Write a proof that follows the plan. **See back of book.**

GPS

Given: Isosceles trapezoid *ABCD* with $\overline{AB} \cong \overline{DC}$

Prove: $\angle B \cong \angle C$ and $\angle BAD \cong \angle D$



Plan: Begin by drawing $\overline{AE} \parallel \overline{DC}$ to form parallelogram *AECD* so that $\overline{AE} \cong \overline{DC} \cong \overline{AB}$. $\angle B \cong \angle C$ because $\angle B \cong \angle 1$ and $\angle 1 \cong \angle C$. Also, $\angle BAD \cong \angle D$ because they are supplements of the congruent angles, $\angle B$ and $\angle C$.

31. Yes, the ≅ ∠s can be obtuse.

32. Yes, the ≅ ∠s can be obtuse, as well as one other ∠.

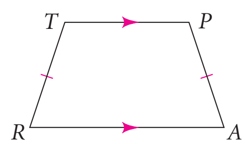
33. Yes; if 2 ≅ ∠s are rt. ∠s, they are suppl. The other 2 ∠s are also suppl.

43. *D* is any point on $\overleftrightarrow{BN}$ such that $ND \neq BN$ and *D* is below *N*.

Proof **Write a proof. Use the given figure with additional lines as needed.**

39. **Given:** Isosceles trapezoid *TRAP* with $\overline{TR} \cong \overline{PA}$

Prove: $\angle RTA \cong \angle APR$ **See margin.**



40. **Given:** Isosceles trapezoid *TRAP* with $\overline{TR} \cong \overline{PA}$; $\overline{BI}$ is the perpendicular bisector of $\overline{RA}$ intersecting $\overline{RA}$ at *B* and $\overline{TP}$ at *I*. **See margin.**

Prove: $\overline{BI}$ is the perpendicular bisector of $\overline{TP}$.

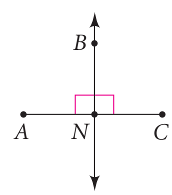
Challenge

For a trapezoid, consider the segment joining the midpoints of the two given segments. How are its length and the lengths of the two parallel sides of the trapezoid related? Justify your answer.

41. the two nonparallel sides **See left.**
42. the diagonals **See margin.**

41. It is one half the sum of the lengths of the bases; draw a diag. of the trap. to form 2 △s. The bases *B* and *b* of the trap. are each a base of a △. Then the segment joining the midpts. of the non-∥ sides is the sum of the midsegments of the △s. This sum is $\frac{1}{2}B + \frac{1}{2}b = \frac{1}{2}(B + b)$.

43. $\overleftrightarrow{BN}$ is the perpendicular bisector of $\overline{AC}$ at *N*. Describe the set of points, *D*, for which *ABCD* is a kite. **See above left.**



Proof 44. Prove that the angles formed by the noncongruent sides of a kite are congruent. (*Hint:* Draw a diagonal of the kite.) **See back of book.**

Test Prep

Multiple Choice

45. Which statement is true for every trapezoid? **B**

A. Exactly two sides are congruent. **B.** Exactly two sides are parallel.
C. Opposite angles are supplementary. **D.** The diagonals bisect each other.

46. Which statement is true for every kite? **J**

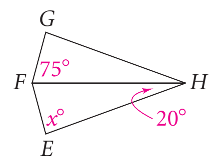
F. Opposite sides are congruent. **G.** At least two sides are parallel.
H. Opposite angles are supplementary. **J.** The diagonals are perpendicular.

47. Two consecutive angles of a trapezoid are right angles. Three of the following statements about the trapezoid could be true. Which statement CANNOT be true? **A**

A. The two right angles are base angles.
B. The diagonals are not congruent.
C. Two of the sides are congruent.
D. No two sides are congruent.

48. Quadrilateral *EFGH* is a kite. What is the value of *x*? **H**

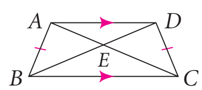
F. 15
G. 70
H. 85
J. 160



Short Response

49. In the trapezoid at the right, $BE = 2x - 8$, $DE = x - 4$, and $AC = x + 2$.

a. Write and solve an equation for *x*.
b. Find the length of each diagonal.



49. [2] a. $(2x - 8) + (x - 4) = x + 2$ **OR equivalent equation;** $x = 7$
b. 9, 9
[1] one computational error

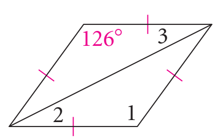
50. Diagonal $\overline{RB}$ of kite *RHBW* forms an equilateral triangle with two of the sides. $m\angle BWR = 40$. Draw and label a diagram showing the diagonal and the measures of all the angles. Which angles of the kite are largest? **See margin.**

Mixed Review

GO for Help

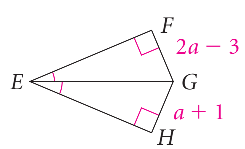
Lesson 6-4 **Find the indicated angle measures for the rhombus.**

51. $m\angle 1$ **126**   **52.** $m\angle 2$ **27**   **53.** $m\angle 3$ **27**

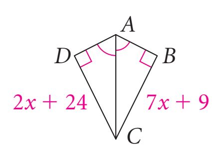


Lesson 5-2 **Algebra** **Find the values indicated.**

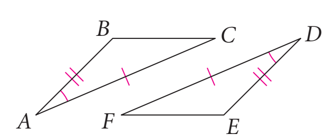
54. a. *a*
b. *FG*
c. *GH*
a. 4
b. 5
c. 5



55. a. *x*
b. *CD*
c. *BC*
a. 3
b. 30
c. 30



Lesson 4-2 **56.** State the postulate that justifies the statement $\triangle ABC \cong \triangle DEF$. **SAS**



Test Prep

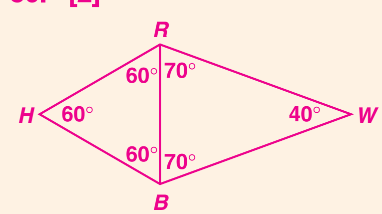
Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 361
- Test-Taking Strategies, p. 356
- Test-Taking Strategies with Transparencies

42. It is one half the difference of the lengths of the bases; from Ex. 41, the length of the segment joining the midpts. of the non-‖ sides is $\frac{1}{2}(B + b)$. By the Side-Splitter Thm., the middle part of this segment joins the midpts. of the diags. Each outer segment measures $\frac{1}{2}b$. So the length of the segment connecting the midpts. of the diags. is $\frac{1}{2}(B - b)$.

50. [2]



$\angle HRW$ and $\angle HBW$

[1] incorrect diagram OR no work shown

40. Draw $\overline{BI}$ as described, then draw $\overline{BT}$ and $\overline{BP}$.

1. $\overline{TR} \cong \overline{PA}$ (Given)
2. $\angle R \cong \angle A$ (Base ∠s of isosc. trap. are ≅.)
3. $\overline{RB} \cong \overline{AB}$ (Def. of bisector)
4. $\triangle TRB \cong \triangle PAB$ (SAS)
5. $\overline{BT} \cong \overline{BP}$ (CPCTC)
6. $\angle RBT \cong \angle ABP$ (CPCTC)
7. $\angle TBI \cong \triangle PBI$ (Compl. of ≅ ∠s are ≅.)
8. $\overline{BI} \cong \overline{BI}$ (Refl. Prop. of ≅)
9. $\triangle TBI \cong \triangle PBI$ (SAS)
10. $\angle BIT \cong \angle BIP$ (CPCTC)
11. $\angle BIT$ and $\angle BIP$ are rt. ∠s. (≅ supp. ∠s are rt. ∠s.)
12. $\overline{TI} \cong \overline{PI}$ (CPCTC)
13. $\overline{BI}$ is ⊥ bis. of $\overline{TP}$. (Def. of ⊥ bis.)