

6. Flow of Fluid and Bernoulli's Equation

Chapter Objectives

- Define various types of flow, *volume flow rate*, *weight flow rate*, and *mass flow rate* and their units.
- Define *steady flow* and the *principle of continuity*.
- Write the continuity equation, and use it to relate the volume flow rate, area, and velocity of flow between two points in a fluid flow system.
- Describe five types of commercially available pipe and tubing: steel pipe, ductile iron pipe, steel tubing, copper tubing, and plastic pipe and tubing.
- Specify the desired size of pipe or tubing for carrying a given flow rate of fluid at a specified velocity.

6. Flow of Fluid and Bernoulli's Equation

Laminar and Turbulent flow

- As the water flows from a faucet at a very low velocity, the flow appears to be smooth and steady. The stream has a fairly uniform diameter and there is little or no evidence of mixing of the various parts of the stream. This is called *laminar flow*.
- When the faucet is nearly fully open, the water has a rather high velocity. The elements of fluid appear to be mixing chaotically within the stream. This is a general description of *turbulent flow*.

6. Flow of Fluid and Bernoulli's Equation

Chapter Outline

1. Fluid Flow Rate and the Continuity Equation
2. Commercially Available Pipe and Tubing
3. Recommended Velocity of Flow in Pipe and Tubing
4. Conservation of Energy – Bernoulli's Equation
5. Interpretation of Bernoulli's Equation
6. Restrictions on Bernoulli's Equation
7. Applications of Bernoulli's Equation
8. Torricelli's Theorem
9. Flow Due to a Falling Head

6. Flow of Fluid and Bernoulli's Equation

6.1 Fluid Flow Rate and the Continuity Equation

- The quantity of fluid flowing in a system per unit time can be expressed by the following three different terms:
- **Q** The *volume flow rate* is the volume of fluid flowing past a section per unit time.
- **W** The *weight flow rate* is the weight of fluid flowing past a section per unit time.
- **M** The *mass flow rate* is the mass of fluid flowing past a section per unit time.

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6.1 Fluid Flow Rate and the Continuity Equation

- The most fundamental of these three terms is the volume flow rate Q , which is calculated from

$$Q = Av \quad (6-1)$$

- where A is the area of the section and v is the average velocity of flow. The units of Q can be derived as follows, using SI units for illustration:

$$Q = Av = \text{m}^2 \times \text{m/s} = \text{m}^3/\text{s}$$

6. Flow of Fluid and Bernoulli's Equation

6.1 Fluid Flow Rate and the Continuity Equation

- The weight flow rate W is related to Q by

$$W = \gamma Q \quad (6-2)$$

- where γ is the specific weight of the fluid. The units of W are then

$$W = \gamma Q = \text{N/m}^3 \times \text{m}^3/\text{s} = \text{N/s}$$

- The mass flow rate M is related to Q by

$$M = \rho Q \quad (6-3)$$

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6.1 Fluid Flow Rate and the Continuity Equation

- The units of M are then

$$M = \rho Q = \text{kg/m}^3 \times \text{m}^3/\text{s} = \text{kg/s}$$

- Table 6.1 shows the flow rates.
- Useful conversions are

$$1.0 \text{ L/min} = 0.06 \text{ m}^3/\text{h}$$

$$1.0 \text{ m}^3/\text{s} = 60\,000 \text{ L/min}$$

$$1.0 \text{ gal/min} = 3.785 \text{ L/min}$$

$$1.0 \text{ gal/min} = 0.2271 \text{ m}^3/\text{h}$$

$$1.0 \text{ ft}^3/\text{s} = 449 \text{ gal/min}$$

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6.1 Fluid Flow Rate and the Continuity Equation

Symbol	Name	Definition	SI Units	U.S. Customary System Units
Q	Volume flow rate	$Q = Av$	m^3/s	ft^3/s
W	Weight flow rate	$W = \gamma Q$ $W = \gamma Av$	N/s	lb/s
M	Mass flow rate	$M = \rho Q$ $M = \rho Av$	kg/s	slugs/s

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6.1 Fluid Flow Rate and the Continuity Equation

- Table 6.2 shows the typical volume flow rates.

Flow rate			Flow rate
m ³ /h	(L/min)		(gal/min)
0.9–7.5	15–125	Reciprocating pumps handling heavy fluids and slurries	4–33
0.60–6.0	10–100	Industrial oil hydraulic systems	3–30
6.0–36	100–600	Hydraulic systems for mobile equipment	30–150
2.4–270	40–4500	Centrifugal pumps in chemical processes	10–1200
12–240	200–4000	Flood control and drainage pumps	50–1000
2.4–900	40–15 000	Centrifugal pumps handling mine wastes	10–4000
108–570	1800–9500	Centrifugal fire-fighting pumps	500–2500

6. Flow of Fluid and Bernoulli's Equation

Example 6.1

Convert a flow rate of 30 gal/min to ft³/s.

The flow rate is

$$Q = 30 \text{ gal/min} \left(\frac{1.0 \text{ ft}^3/\text{s}}{449 \text{ gal/min}} \right) = 6.68 \times 10^{-2} \text{ ft}^3/\text{s}$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.2

Convert a flow rate of 600 L/min to m³/s.

$$Q = 600 \text{ L/min} \left(\frac{1.0 \text{ m}^3/\text{s}}{60\,000 \text{ L/min}} \right) = 0.010 \text{ m}^3/\text{s}$$

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Example 6.3

Convert a flow rate of 600 L/min to m³/s.

$$Q = 30 \text{ gal/min} \left(\frac{3.785 \text{ L/min}}{1.0 \text{ gal/min}} \right) = 113.6 \text{ L/min}$$

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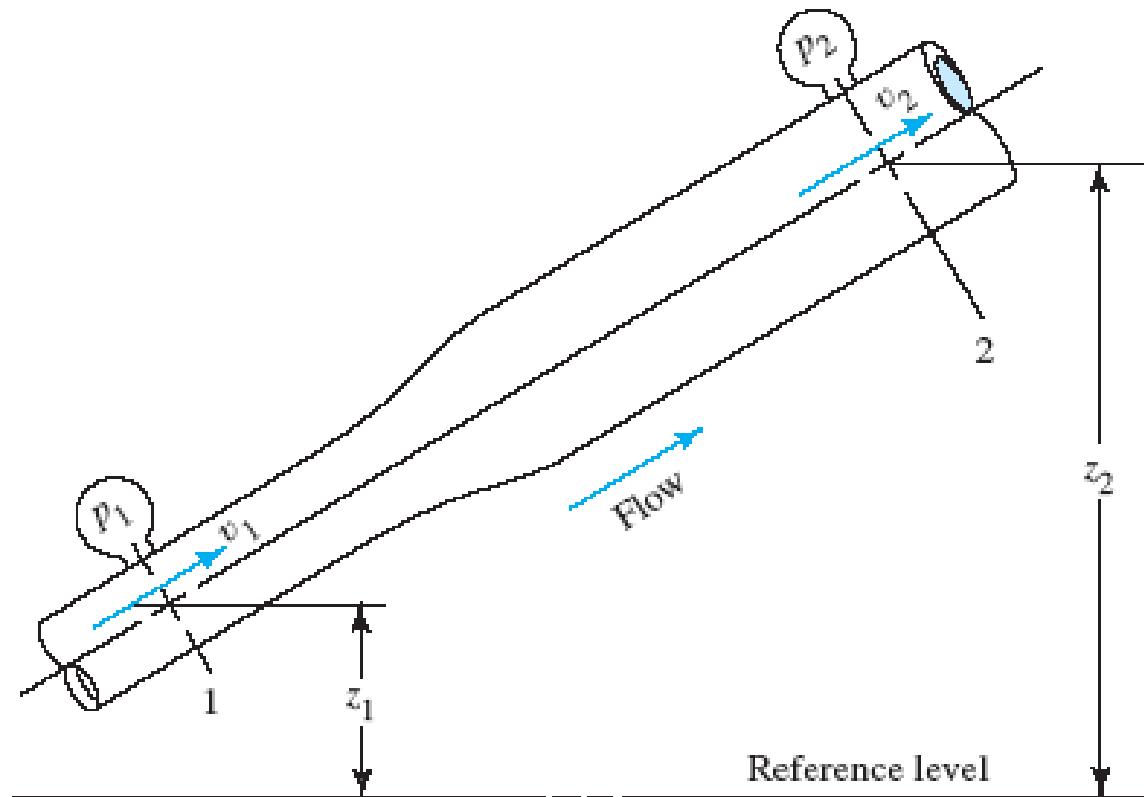
6.1 Fluid Flow Rate and the Continuity Equation

- The method of calculating the velocity of flow of a fluid in a closed pipe system depends on the *principle of continuity*.
- Fig 6.1 shows the portion of a fluid distribution system showing variations in velocity, pressure, and elevation.
- This can be expressed in terms of the mass flow rate as

$$M_1 = M_2$$

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6.1 Fluid Flow Rate and the Continuity Equation



6. Flow of Fluid and Bernoulli's Equation

6.1 Fluid Flow Rate and the Continuity Equation

- As $M = \rho Av$, we have

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \quad (6-4)$$

- Equation (6–4) is a mathematical statement of the principle of continuity and is called the *continuity equation*.
- It is used to relate the fluid density, flow area, and velocity of flow at two sections of the system in which there is steady flow.
- It is valid for all fluids, whether gas or liquid.

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6.1 Fluid Flow Rate and the Continuity Equation

- If the fluid in the pipe in Fig. 6.1 is a liquid that can be considered incompressible, then the terms Q_1 and Q_2 is the same.

$$A_1v_1 = A_2v_2$$

- Since $Q = Av$,

$$Q_1 = Q_2$$

- Equation (6–5) is the continuity equation as applied to liquids; it states that for steady flow the volume flow rate is the same at any section.

6. Flow of Fluid and Bernoulli's Equation

Example 6.4

In Fig. 6.1 the inside diameters of the pipe at sections 1 and 2 are 50 mm and 100 mm, respectively. Water is flowing with an average velocity of 8 m/s at section 1.

Calculate the following:

- (a)** Velocity at section 2
- (b)** Volume flow rate
- (c)** Weight flow rate
- (d)** Mass flow rate

6. Flow of Fluid and Bernoulli's Equation

Example 6.4

(a) Velocity at section 2.

From Eq. (6–5) we have

$$A_1 v_1 = A_2 v_2$$

$$v_2 = v_1 \left(\frac{A_1}{A_2} \right)$$

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi (50 \text{ mm})^2}{4} = 1963 \text{ mm}^2$$

$$A_2 = \frac{\pi D_2^2}{4} = \frac{\pi (100 \text{ mm})^2}{4} = 7854 \text{ mm}^2$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.4

Then the velocity at section 2 is

$$v_2 = v_1 \left(\frac{A_1}{A_2} \right) = \frac{8.0 \text{ m}}{\text{s}} \times \frac{1963 \text{ mm}^2}{7854 \text{ mm}^2} = 2.0 \text{ m/s}$$

Notice that for steady flow of a liquid, as the flow area increases, the velocity decreases.

This is independent of pressure and elevation.

6. Flow of Fluid and Bernoulli's Equation

Example 6.4

(b) Volume flow rate Q .

From Table 6.1, $Q = vA$. Because of the principle of continuity we could use the conditions either at section 1 or at section 2 to calculate Q . At section 1 we have

$$Q = A_1 v_1 = 1963 \text{ mm}^2 \times \frac{8.0 \text{ m}}{\text{s}} \times \frac{1 \text{ m}^2}{(10^3 \text{ mm})^2} = 0.0157 \text{ m}^3/\text{s}$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.4

(c) Weight flow rate W .

From Table 6.1, $W = \gamma Q$. At 70°C, the specific weight of water is 9.59 kN/m³. Then the weight flow rate is

$$W = \gamma Q = \frac{9.59 \text{ kN}}{\text{m}^3} \times \frac{0.0157 \text{ m}^3}{\text{s}} = 0.151 \text{ kN/s}$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.4

(d) Mass flow rate M .

From Table 6.1, $M = \rho Q$. At the density of water is 978 kg/m^3 . Then the mass flow rate is

$$M = \rho Q = \frac{978 \text{ kg}}{\text{m}^3} \times \frac{0.0157 \text{ m}^3}{\text{s}} = 15.36 \text{ kg/s}$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.5

According to the continuity equation for gases, Eq. (6–4), we have

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

Then, we can calculate the area of the two sections and solve for v_2

$$\rho_2 = \rho_1 \left(\frac{A_1}{A_2} \right) \left(\frac{v_1}{v_2} \right)$$

$$A_1 = (30.5 \text{ cm})(30.5 \text{ cm}) = 930.25 \text{ cm}^2$$

$$A_2 = \frac{\pi D_2^2}{4} = \frac{\pi(45.7 \text{ cm})^2}{4} = 1640 \text{ cm}^2$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.5

(a) Then, the density of the air in the round section is

$$\rho_2 = (1.134 \text{ kg/m}^3) \left(\frac{930.25 \text{ cm}^2}{1640 \text{ cm}^2} \right) \left(\frac{6.1 \text{ m/s}}{4.57 \text{ m/s}} \right)$$
$$\rho_2 = 0.859 \text{ kg/m}^3$$

(b) The weight flow rate can be found at section 1 from .
Then, the weight flow rate is

$$W = \gamma_1 A_1 v_1$$
$$W = (11.14 \text{ N/m}^3)(930.25 \text{ cm}^2) \left(\frac{1 \text{ m}^2}{10\,000 \text{ cm}^2} \right) \left(\frac{6.1 \text{ m}}{\text{s}} \right)$$
$$W = 6.32 \text{ N/s}$$

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6.2 Commercially Available Pipe and Tubing

- The nominal sizes for commercially available pipe still refer to an “inch” size even though the transition to the SI system is an international trend.
- For many applications, codes and standards must be followed as established by governmental agencies or organizations such as the following:

American Water Works Association (AWWA)

American Fire Sprinkler Association (AFSA)

National Fire Protection Association (NFPA)

ASTM International (ASTM) [Formerly American Society for Testing and Materials]

NSF International (NSF) [Formerly National Sanitation Foundation]

International Association of Plumbing and Mechanical Officials (IAPMO)

International Organization for Standardization (ISO)

6. Flow of Fluid and Bernoulli's Equation

6.2.1 Steel Pipe

- General-purpose pipe lines are often constructed of steel pipe.
- Standard pipe sizes are designated by the nominal size and schedule number.
- Schedule numbers are related to the permissible operating pressure of the pipe and to the allowable stress of the steel in the pipe.

6. Flow of Fluid and Bernoulli's Equation

6.2.1 Steel Pipe

Nominal Pipe Sizes in Metric Units

- Because of the long experience with manufacturing standard pipe according to the standard schedule numbers, they continue to be used often even when the piping system is specified in metric units.
- The following set of equivalents has been established by the International Standards Organization (ISO).

6. Flow of Fluid and Bernoulli's Equation

6.2.1 Steel Pipe

Customary Standard Size (in)	Metric DN Size (mm)	Customary Standard Size (in)	Metric DN Size (mm)	Customary Standard Size (in)	Metric DN Size (mm)
1/8	6	6	150	30	750
1/4	8	8	200	32	800
3/8	10	10	250	36	900
1/2	15	12	300	40	1000
3/4	20	14	350	42	1100
1	25	16	400	48	1200
1 1/4	32	18	450	54	1400
1 1/2	40	20	500	60	1500
2	50	22	550	64	1600
2 1/2	65	24	600	72	1800
3	80	26	650	80	2000
4	100	28	700	88	2200

6. Flow of Fluid and Bernoulli's Equation

6.2.2 Steel Tubing

- Standard steel tubing is used in fluid power systems, condensers, heat exchangers, engine fuel systems, and industrial fluid processing systems.
- Sizes are designated by outside diameter and wall thickness.

6. Flow of Fluid and Bernoulli's Equation

6.2.3 Copper Tubing

- There are six types of copper tubing offered, and the choice of which to use depends on the application, considering the environment, fluid pressure, and fluid properties.
- Copper tubing is available in either a soft, annealed condition or hard drawn.
- Drawn tubing is stiffer and stronger, maintains a straight form, and can carry higher pressures.
- Annealed tubing is easier to form into coils and other special shapes.

6. Flow of Fluid and Bernoulli's Equation

6.2.3 Ductile Iron Pipe

- Water, gas, and sewage lines are often made of ductile iron pipe because of its strength, ductility, and relative ease of handling.
- It has replaced cast iron in many applications.

6. Flow of Fluid and Bernoulli's Equation

6.2.3 Plastic Pipe and Tubing

- Plastic pipe and tubing are being used in a wide variety of applications where their light weight, ease of installation, corrosion and chemical resistance, and very good flow characteristics present advantages.

6. Flow of Fluid and Bernoulli's Equation

6.2.4 Hydraulic Hose

- Hose materials include butyl rubber, synthetic rubber, silicone rubber, thermoplastic elastomers, and nylon.
- Braided reinforcement may be made from steel wire, Kevlar, polyester, and fabric.
- Industrial applications include steam, compressed air, chemical transfer, coolants, heaters, fuel transfer, lubricants, refrigerants, paper stock, power steering fluids, propane, water, foods, and beverages.

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6.3 Recommended Velocity of Flow in Pipe and Tubing

- Many factors affect the selection of a satisfactory velocity of flow in fluid systems.
- Some of the important ones are the type of fluid, the length of the flow system, the type of pipe or tube, the pressure drop that can be tolerated, the devices (such as pumps, valves, etc.) that may be connected to the pipe or tube, the temperature, the pressure, and the noise.

6. Flow of Fluid and Bernoulli's Equation

6.3 Recommended Velocity of Flow in Pipe and Tubing

- The resulting flow velocities from the recommended pipe sizes in Fig. 6.2 are generally lower for the smaller pipes and higher for the larger pipes, as shown for the following data.

Volume Flow Rate		Suction Line			Discharge Line		
		Pipe	Velocity		Pipe	Velocity	
gal/min	m ³ /h	Size (in)	ft/s	m/s	Size (in)	ft/s	m/s
10	2.3	1	3.7	1.1	¾	6.0	1.8
100	22.7	2½	6.7	2.0	2	9.6	2.9
500	114	5	8.0	2.4	3½	16.2	4.9
2000	454	8	12.8	3.9	6	22.2	6.8

6. Flow of Fluid and Bernoulli's Equation

6.3 Recommended Velocity of Flow in Pipe and Tubing

Recommended Flow Velocities for Specialized Systems

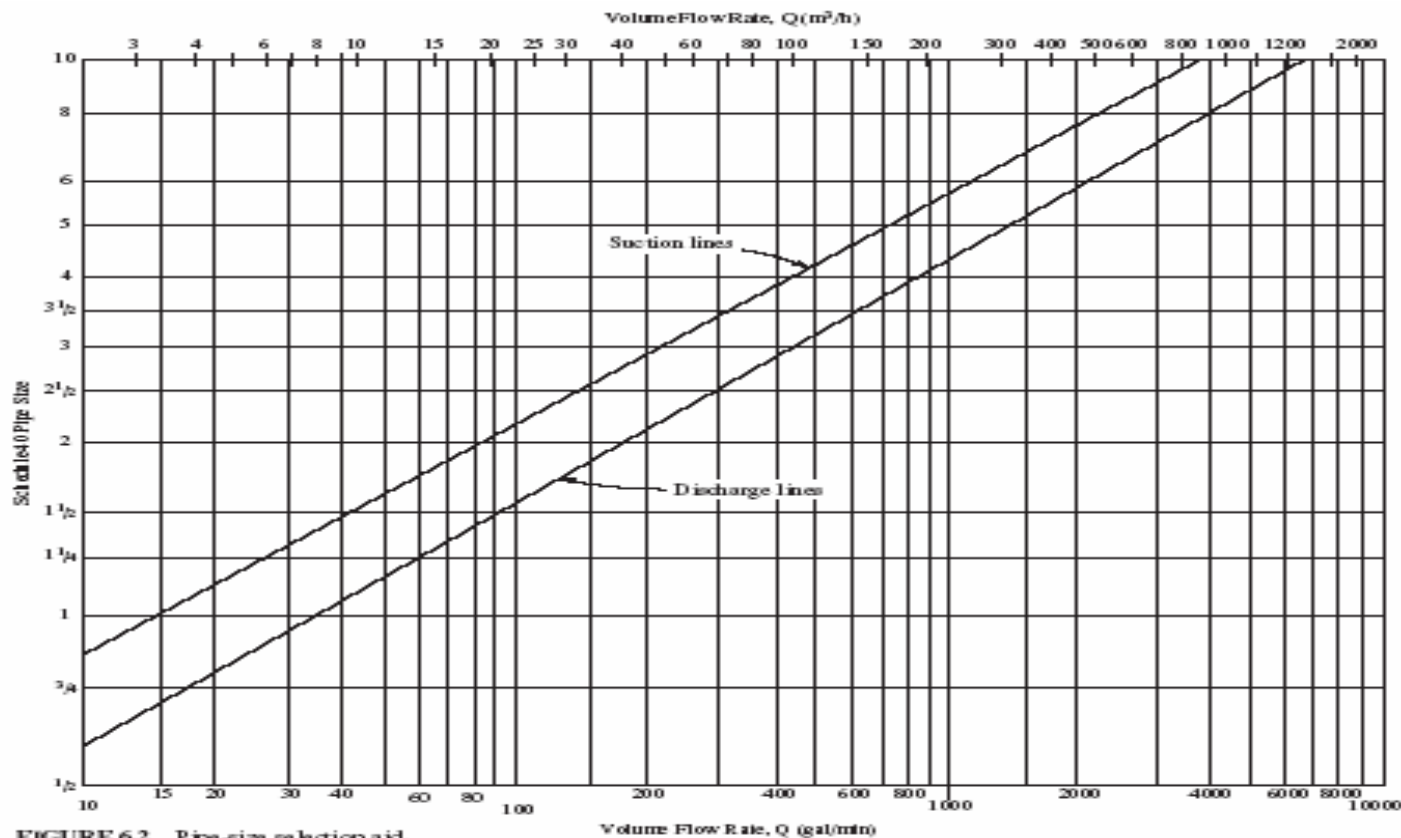


FIGURE 6.2 Pipe-size selection aid.

6. Flow of Fluid and Bernoulli's Equation

6.3 Recommended Velocity of Flow in Pipe and Tubing

- For example, recommended flow velocities for fluid power systems are as follows:

Type of Service	Recommended Range of Velocity	
	ft/s	m/s
Suction lines	2–4	0.6–1.2
Return lines	4–13	1.5–4
Discharge lines	7–18	2–5.5

6. Flow of Fluid and Bernoulli's Equation

6.3 Recommended Velocity of Flow in Pipe and Tubing

- The suction line delivers the hydraulic fluid from the reservoir to the intake port of the pump.
- A discharge line carries the high-pressure fluid from the pump outlet to working components such as actuators or fluid motors.
- A return line carries fluid from actuators, pressure relief valves, or fluid motors back to the reservoir.

6. Flow of Fluid and Bernoulli's Equation

Example 6.6

Determine the maximum allowable volume flow rate in L/min that can be carried through a standard steel tube with an outside diameter of 1.25 in and a 0.065 in wall thickness if the maximum velocity is to be 3.0 m/s.

Using the definition of volume flow rate, we have

$$Q = Av$$

$$A = 6.356 \times 10^{-4} \text{ m}^2 \quad (\text{from Appendix G})$$

$$Q = (6.356 \times 10^{-4} \text{ m}^2)(3.0 \text{ m/s}) = 1.907 \times 10^{-3} \text{ m}^3/\text{s}$$

$$Q = 1.907 \times 10^{-3} \text{ m}^3/\text{s} \left(\frac{60\,000 \text{ L/min}}{1.0 \text{ m}^3/\text{s}} \right) = 114 \text{ L/min}$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.7

Because Q and v are known, the required area can be found from

$$Q = Av$$

$$A = Q/v$$

First, we must convert the volume flow rate to the units of m^3/s :

$$Q = 192 \text{ m}^3/\text{h}(1 \text{ h}/3600 \text{ s}) = 0.0533 \text{ m}^3/\text{s}$$

$$A = \frac{Q}{v} = \frac{0.0533 \text{ m}^3/\text{s}}{6.0 \text{ m/s}} = 0.00888 \text{ m}^2 = 8.88 \times 10^{-3} \text{ m}^2$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.7

This must be interpreted as the *minimum* allowable area because any smaller area would produce a velocity higher than 6.0 m/s. Therefore, we must look in Appendix F for a standard pipe with a flow area just larger than $8.88 \times 10^{-3} \text{ m}^2$. A standard 5-in Schedule 40 steel pipe, with a flow area of $1.291 \times 10^{-2} \text{ m}^2$ is required. The actual velocity of flow when this pipe carries $0.0533 \text{ m}^3/\text{s}$ of water is

$$v = \frac{Q}{A} = \frac{0.0533 \text{ m}^3/\text{s}}{1.291 \times 10^{-2} \text{ m}^2} = 4.13 \text{ m/s}$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.7

If the next-smaller pipe (a 4-in Schedule 40 pipe) is used, the velocity is

$$v = \frac{Q}{A} = \frac{0.0533 \text{ m}^3/\text{s}}{8.213 \times 10^{-3} \text{ m}^2} = 6.49 \text{ m/s} \quad (\text{too high})$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.8

Entering Fig. 6.2 at $Q = 400$ gal/min, we select the following:

Suction pipe, 4-in Schedule 40: $A_s = 8.21 \times 10^{-3} \text{ m}^2$ (from Appendix F)

Discharge pipe, 3-in Schedule 40: $A_d = 4.77 \times 10^{-3} \text{ m}^2$ (from Appendix F)

The actual average velocity of flow in each pipe is

$$v_s = \frac{Q}{A_s} = \frac{0.025 \text{ m}^3/\text{s}}{8.21 \times 10^{-3} \text{ m}^2} = 3.05 \text{ m/s}$$

$$v_d = \frac{Q}{A_d} = \frac{0.025 \text{ m}^3/\text{s}}{4.77 \times 10^{-3} \text{ m}^2} = 5.24 \text{ m/s}$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.8 - Comments

Although these pipe sizes and velocities should be acceptable in normal service, there are situations where lower velocities are desirable to limit energy losses in the system. Compute the velocities resulting from selecting the next-larger standard Schedule 40 pipe size for both the suction and discharge lines:

Suction pipe, 5-in Schedule 40: $A_s = 1.291 \times 10^{-2} \text{ m}^2$ (from Appendix F)

Discharge pipe, 3 1/2-in Schedule 40: $A_d = 6.38 \times 10^{-3} \text{ m}^2$ (from Appendix F)

6. Flow of Fluid and Bernoulli's Equation

Example 6.8 - Comments

The actual average velocity of flow in each pipe is

$$v_s = \frac{Q}{A_s} = \frac{0.025 \text{ m}^3/\text{s}}{1.291 \times 10^{-2} \text{ m}^2} = 1.936 \text{ m/s}$$

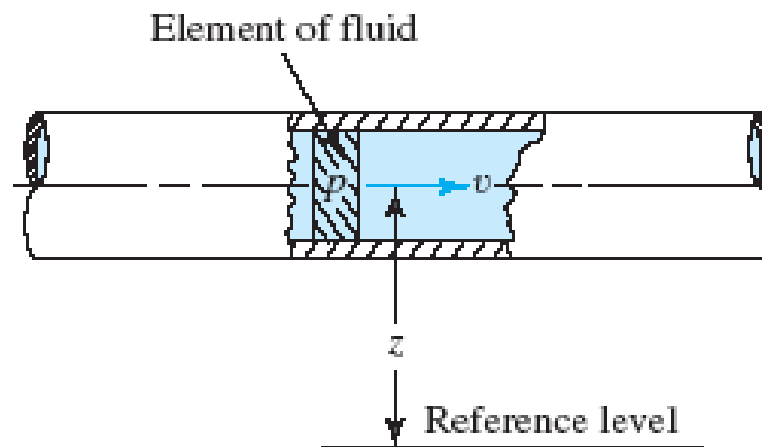
$$v_d = \frac{Q}{A_d} = \frac{0.025 \text{ m}^3/\text{s}}{6.38 \times 10^{-3} \text{ m}^2} = 3.92 \text{ m/s}$$

If the pump connections were the 4-in and 3-in sizes from the initial selection, a gradual reducer and gradual enlargement could be designed to connect these pipes to the pump.

6. Flow of Fluid and Bernoulli's Equation

6.4 Conservation of Energy – Bernoulli's Equation

- In physics you learned that energy can be neither created nor destroyed, but it can be transformed from one form into another.
- This is a statement of the law of *conservation of energy*.
- Fig 6.3 shows the element of a fluid in a pipe.



6. Flow of Fluid and Bernoulli's Equation

6.4 Conservation of Energy – Bernoulli's Equation

- There are three forms of energy that are always considered when analyzing a pipe flow problem.
 1. *Potential Energy*. Due to its elevation, the potential energy of the element relative to some reference level is

$$PE = wz \quad (6-6)$$

where w is the weight of the element.

6. Flow of Fluid and Bernoulli's Equation

6.4 Conservation of Energy – Bernoulli's Equation

2. *Kinetic Energy*. Due to its velocity, the kinetic energy of the element is

$$KE = wv^2/2g \quad (6-7)$$

3. *Flow Energy*. Sometimes called *pressure energy* or *flow work*, this represents the amount of work necessary to move the element of fluid across a certain section against the pressure p . Flow energy is abbreviated FE and is calculated from

$$FE = wp/\gamma \quad (6-8)$$

6. Flow of Fluid and Bernoulli's Equation

6.4 Conservation of Energy – Bernoulli's Equation

- Equation (6–8) can be derived as follows.
- The work done is

$$\text{Work} = pAL = pV$$

where V is the volume of the element. The weight of the element w is

$$w = \gamma V$$

where γ is the specific weight of the fluid. Then, the volume of the element is

$$V = w/\gamma$$

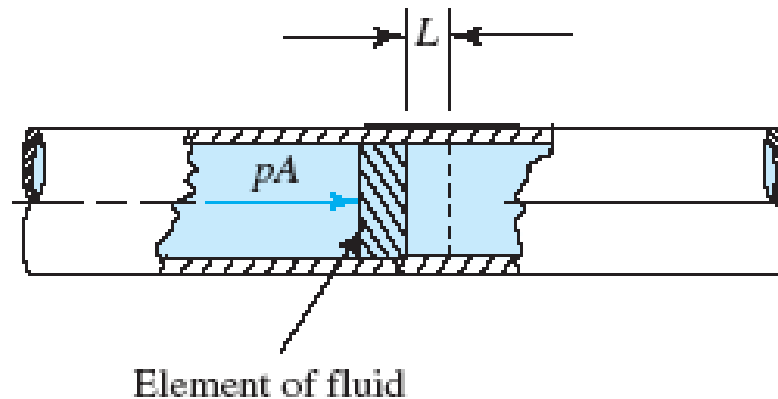
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6.4 Conservation of Energy – Bernoulli's Equation

- And we have Eq. (6-8)

$$\text{Work} = pV = pw/\gamma$$

- Fig 6.4 shows the flow energy.



6. Flow of Fluid and Bernoulli's Equation

6.4 Conservation of Energy – Bernoulli's Equation

- The total amount of energy of these three forms possessed by the element of fluid is the sum E ,

$$E = FE + PE + KE$$

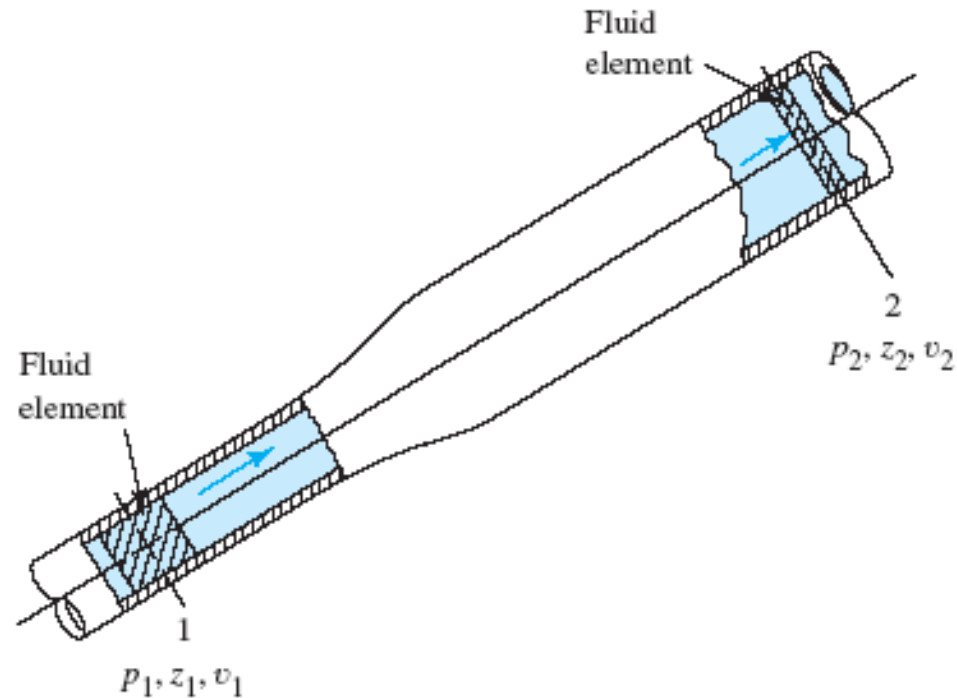
$$E = wp/\gamma + wz + wv^2/2g$$

- Each of these terms is expressed in units of energy, which are Newton-meters (Nm) in the SI unit system and foot-pounds (ft-lb) in the U.S. Customary System.

6. Flow of Fluid and Bernoulli's Equation

6.4 Conservation of Energy – Bernoulli's Equation

- Fig 6.5 shows the fluid elements used in Bernoulli's equation.



6. Flow of Fluid and Bernoulli's Equation

6.4 Conservation of Energy – Bernoulli's Equation

- At section 1 and 2, the total energy is

$$E_1 = \frac{wp_1}{\gamma} + wz_1 + \frac{wv_1^2}{2g}$$

$$E_2 = \frac{wp_2}{\gamma} + wz_2 + \frac{wv_2^2}{2g}$$

- If no energy is added to the fluid or lost between sections 1 and 2, then the principle of conservation of energy requires that

$$\begin{aligned} E_1 &= E_2 \\ \frac{wp_1}{\gamma} + wz_1 + \frac{wv_1^2}{2g} &= \frac{wp_2}{\gamma} + wz_2 + \frac{wv_2^2}{2g} \end{aligned}$$

6. Flow of Fluid and Bernoulli's Equation

6.4 Conservation of Energy – Bernoulli's Equation

- The weight of the element w is common to all terms and can be divided out.
- The equation then becomes

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g} \quad (6-9)$$

- This is referred to as *Bernoulli's equation*.

6. Flow of Fluid and Bernoulli's Equation

6.5 Interpretation of Bernoulli's Equation

- Each term in Bernoulli's equation, Eq. (6–9), resulted from dividing an expression for energy by the weight of an element of the fluid.
- ***Each term in Bernoulli's equation is one form of the energy possessed by the fluid per unit weight of fluid flowing in the system.***
- The units for each term are “energy per unit weight.” In the SI system the units are Nm/N and in the U.S. Customary System the units are $lb.ft/lb$.

6. Flow of Fluid and Bernoulli's Equation

6.5 Interpretation of Bernoulli's Equation

- Specifically,

p/γ is called the pressure head.

z is called the elevation head.

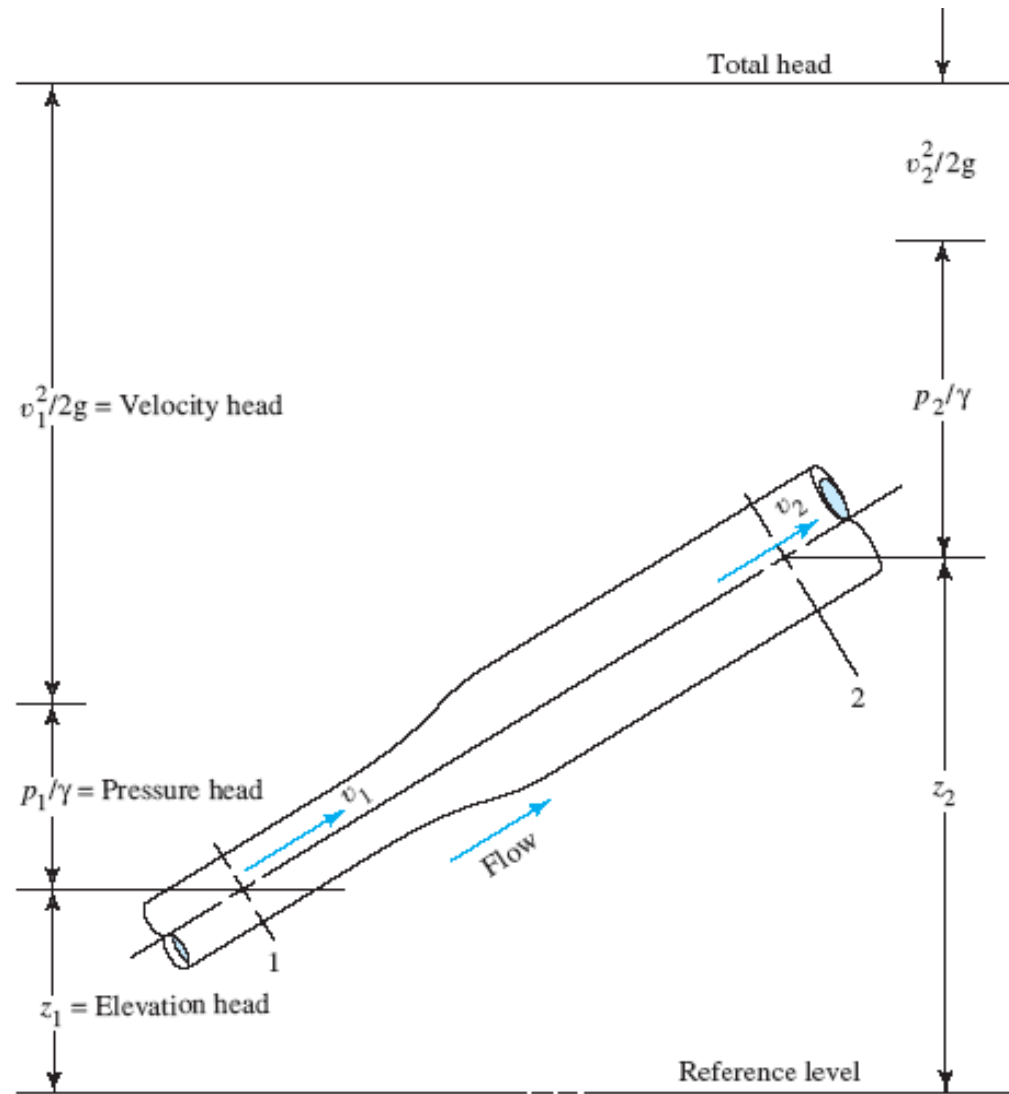
$v^2/2g$ is called the velocity head.

The sum of these three terms is called the total head.

- Fig 6.6 shows the pressure head, elevation head, velocity head, and total head.

6. Flow of Fluid and Bernoulli's Equation

6.5 Interpretation of Bernoulli's Equation



6. Flow of Fluid and Bernoulli's Equation

6.5 Interpretation of Bernoulli's Equation

- In Fig. 6.6 you can see that the velocity head at section 2 will be less than that at section 1. This can be shown by the continuity equation,

$$A_1v_1 = A_2v_2$$

$$v_2 = v_1(A_1/A_2)$$

- In summary,
- ***Bernoulli's equation accounts for the changes in elevation head, pressure head, and velocity head between two points in a fluid flow system. It is assumed that there are no energy losses or additions between the two points, so the total head remains constant.***

6. Flow of Fluid and Bernoulli's Equation

6.6 Restriction on Bernoulli's Equation

- Although Bernoulli's equation is applicable to a large number of practical problems, there are several limitations that must be understood to apply it properly.
 1. It is valid only for incompressible fluids because the specific weight of the fluid is assumed to be the same at the two sections of interest.
 2. There can be no mechanical devices between the two sections of interest that would add energy to or remove energy from the system, because the equation states that the total energy in the fluid is constant.

6. Flow of Fluid and Bernoulli's Equation

6.6 Restriction on Bernoulli's Equation

3. There can be no heat transferred into or out of the fluid.
4. There can be no energy lost due to friction.
 - In reality no system satisfies all these restrictions.
 - However, there are many systems for which only a negligible error will result when Bernoulli's equation is used.
 - Also, the use of this equation may allow a fast estimate of a result when that is all that is required.

6. Flow of Fluid and Bernoulli's Equation

6.7 Applications of Bernoulli's Equation

- Below is the procedure for applying Bernoulli's equation:
 1. Decide which items are known and what is to be found.
 2. Decide which two sections in the system will be used when writing Bernoulli's equation. One section is chosen for which much data is known. The second is usually the section at which something is to be calculated.

6. Flow of Fluid and Bernoulli's Equation

6.7 Applications of Bernoulli's Equation

3. Write Bernoulli's equation for the two selected sections in the system. It is important that the equation is written *in the direction of flow*. That is, the flow must proceed *from* the section on the left side of the equation *to* that on the right side.
4. Be explicit when labeling the subscripts for the pressure head, elevation head, and velocity head terms in Bernoulli's equation. You should note where the reference points are on a sketch of the system.
5. Simplify the equation, if possible, by canceling terms that are zero or those that are equal on both sides of the equation.

6. Flow of Fluid and Bernoulli's Equation

6.7 Applications of Bernoulli's Equation

6. Solve the equation algebraically for the desired term.
7. Substitute known quantities and calculate the result, being careful to use consistent units throughout the calculation.

6. Flow of Fluid and Bernoulli's Equation

Example 6.9

In Fig. 6.6, water at 10°C is flowing from section 1 to section 2. At section 1, which is 25 mm in diameter, the gage pressure is 345 kPa and the velocity of flow is 3.0 m/s. Section 2, which is 50 mm in diameter, is 2.0 m above section 1. Assuming there are no energy losses in the system, calculate the pressure p_2 .

List the items that are known from the problem statement before looking at the next panel.

$$\begin{array}{lll} D_1 = 25 \text{ mm} & v_1 = 3.0 \text{ m/s} & z_2 - z_1 = 2.0 \text{ m} \\ D_2 = 50 \text{ mm} & p_1 = 345 \text{ kPa(gage)} & \end{array}$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.9

The pressure p_2 is to be found. In other words, we are asked to calculate the pressure at section 2, which is different from the pressure at section 1 because there is a change in elevation and flow area between the two sections.

We are going to use Bernoulli's equation to solve the problem. Which two sections should be used when writing the equation?

6. Flow of Fluid and Bernoulli's Equation

Example 6.9

Now write Bernoulli's equation.

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

The three terms on the left refer to section 1 and the three on the right refer to section 2.

$$\begin{aligned}\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} &= \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g} \\ \frac{p_2}{\gamma} &= \frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} - z_2 - \frac{v_2^2}{2g} \\ p_2 &= \gamma \left(\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} - z_2 - \frac{v_2^2}{2g} \right)\end{aligned}$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.9

The final solution for p_2 should be

$$p_2 = p_1 + \gamma \left(z_1 - z_2 + \frac{v_1^2 - v_2^2}{2g} \right) \quad (6-10)$$

The continuity equation is used:

$$\begin{aligned} A_1 v_1 &= A_2 v_2 \\ v_2 &= v_1 (A_1 / A_2) \end{aligned}$$

This is found from

$$\begin{aligned} A_1 &= \pi D_1^2 / 4 = \pi (25 \text{ mm})^2 / 4 = 491 \text{ mm}^2 \\ A_2 &= \pi D_2^2 / 4 = \pi (50 \text{ mm})^2 / 4 = 1963 \text{ mm}^2 \\ v_2 &= v_1 (A_1 / A_2) = 3.0 \text{ m/s} (491 \text{ mm}^2 / 1963 \text{ mm}^2) = 0.75 \text{ m/s} \end{aligned}$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.9

Now substitute the known values into Eq. (6–10).

$$p_2 = 345 \text{ kPa} + \frac{9.81 \text{ kN}}{\text{m}^3} \left(-2.0 \text{ m} + \frac{(3.0 \text{ m/s})^2 - (0.75 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} \right)$$

The details of the solution are

$$p_2 = 345 \text{ kPa} + \frac{9.81 \text{ kN}}{\text{m}^3} \left(-2.0 \text{ m} + \frac{(9.0 - 0.563) \text{ m}^2/\text{s}^2}{2(9.81) \text{ m/s}^2} \right)$$

$$p_2 = 345 \text{ kPa} + \frac{9.81 \text{ kN}}{\text{m}^3} (-2.0 \text{ m} + 0.43 \text{ m})$$

$$p_2 = 345 \text{ kPa} - 15.4 \text{ kN/m}^2 = 345 \text{ kPa} - 15.4 \text{ kPa}$$

$$p_2 = 329.6 \text{ kPa}$$

6. Flow of Fluid and Bernoulli's Equation

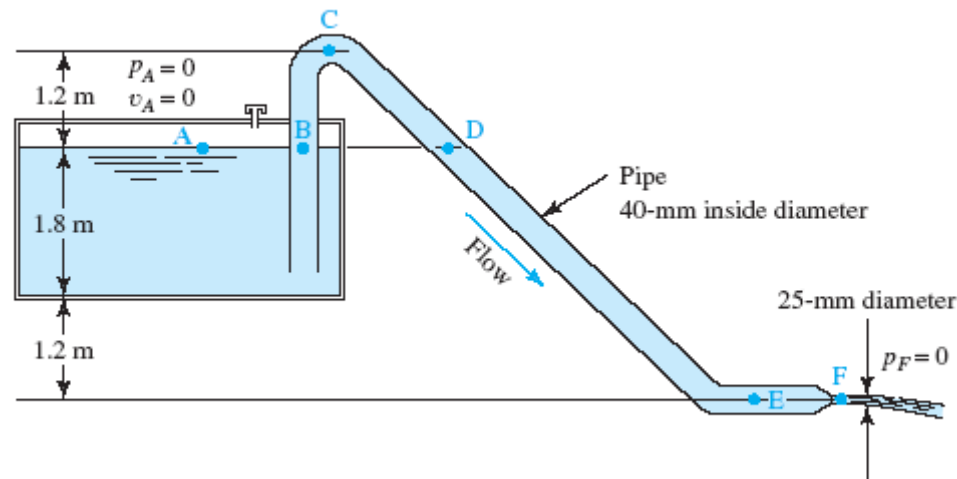
Example 6.9

The pressure p_2 is a gage pressure because it was computed relative to p_1 , which was also a gage pressure. In later problem solutions, we will assume the pressures to be gage unless otherwise stated.

6. Flow of Fluid and Bernoulli's Equation

6.7.1 Tanks, Reservoirs and Nozzles Exposed to the Atmosphere

- ***When the fluid at a reference point is exposed to the atmosphere, the pressure is zero and the pressure head term can be cancelled from Bernoulli's equation.***
- Fig 6.7 shows the siphon.



6. Flow of Fluid and Bernoulli's Equation

6.7.1 Tanks, Reservoirs and Nozzles Exposed to the Atmosphere

- The tank from which the fluid is being drawn can be assumed to be quite large compared to the size of the flow area inside the pipe.
- ***The velocity head at the surface of a tank or reservoir is considered to be zero and it can be cancelled from Bernoulli's equation.***

6. Flow of Fluid and Bernoulli's Equation

6.7.2 When Both Reference Points Are in the Same Pipe

- ***When the two points of reference for Bernoulli's equation are both inside a pipe of the same size, the velocity head terms on both sides of the equation are equal and can be cancelled.***

6. Flow of Fluid and Bernoulli's Equation

6.7.3 When Elevation Are Equal at Both Reference Point

- ***When the two points of reference for Bernoulli's equation are both at the same elevation, the elevation head terms z_1 and z_2 are equal and can be cancelled.***

6. Flow of Fluid and Bernoulli's Equation

Example 6.10

Figure 6.7 shows a siphon that is used to draw water from a swimming pool. The pipe that makes up the siphon has an inside diameter of 40 mm and terminates with a 25-mm diameter nozzle. Assuming that there are no energy losses in the system, calculate the volume flow rate through the siphon and the pressure at points B–E.

6. Flow of Fluid and Bernoulli's Equation

Example 6.10

The first step in this problem solution is to calculate the volume flow rate Q , using Bernoulli's equation. The two most convenient points to use for this calculation are A and F. What is known about point A?

Point A is the free surface of the water in the pool. Therefore, $p_A = 0Pa$. Also, because the surface area of the pool is very large, the velocity of the water at the surface is very nearly zero. Therefore, we will assume $v_A = 0$.

6. Flow of Fluid and Bernoulli's Equation

Example 6.10

Point F is in the free stream of water outside the nozzle. Because the stream is exposed to atmospheric pressure, the pressure $p_F = 0$ Pa. We also know that point F is 3.0 m below point A.

$$\frac{p_A}{\gamma} + z_A + \frac{v_A^2}{2g} = \frac{p_F}{\gamma} + z_F + \frac{v_F^2}{2g}$$

Because $p_A = 0$ Pa, $p_F = 0$ Pa, and v_A is approximately zero, we can cancel the from the equation. What remains is

$$\frac{p_A}{\gamma} + z_A + \frac{v_A^2}{2g} = \frac{p_F}{\gamma} + z_F + \frac{v_F^2}{2g}$$
$$z_A = z_F + \frac{v_F^2}{2g}$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.10

The objective is to calculate the volume flow rate, which depends on the velocity.

$$v_F = \sqrt{(z_A - z_F)2g}$$

The result is

$$v_F = \sqrt{(3.0 \text{ m})(2)(9.81 \text{ m/s}^2)} = \sqrt{58.9} \text{ m/s} = 7.67 \text{ m/s}$$

Using the continuity equation, compute the volume flow rate.

$$Q = A_F v_F$$

$$v_F = 7.67 \text{ m/s}$$

$$A_F = \pi(25 \text{ mm})^2/4 = 491 \text{ mm}^2$$

$$Q = 491 \text{ mm}^2 \left(\frac{7.67 \text{ m}}{\text{s}} \right) \left(\frac{1 \text{ m}^2}{10^6 \text{ mm}^2} \right) = 3.77 \times 10^{-3} \text{ m}^3/\text{s}$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.10

Points A and B are the best. As shown in the previous panels, using point A allows the equation to be simplified greatly, and because we are looking for p_B , we must choose point B. Write Bernoulli's equation for points A and B, simplify it as before, and solve for p_B .

$$\frac{p_A}{\gamma} + z_A + \frac{v_A^2}{2g} = \frac{p_B}{\gamma} + z_B + \frac{v_B^2}{2g}$$

Because $p_A = 0$ and $v_A = 0$, we have

$$\begin{aligned} z_A &= \frac{p_B}{\gamma} + z_B + \frac{v_B^2}{2g} \\ p_B &= \gamma[(z_A - z_B) - v_B^2/2g] \end{aligned} \quad (6-11)$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.10

We can calculate v_A by using the continuity equation:

$$Q = A_B v_B$$

$$v_B = Q/A_B$$

$$Q = 3.77 \times 10^{-3} \text{ m}^3/\text{s}$$

$$A_B = 1.257 \times 10^{-3} \text{ m}^2 \quad (\text{from Appendix J})$$

$$v_B = \frac{3.77 \times 10^{-3} \text{ m}^3}{\text{s}} \times \frac{1}{1.257 \times 10^{-3} \text{ m}^2} = 3.00 \text{ m/s}$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.10

The pressure at point B is

$$p_B = \gamma[(z_A - z_B) - v_B^2/2g]$$
$$\frac{v_B^2}{2g} = \frac{(3.00)^2 \text{ m}^2}{\text{s}^2} \times \frac{\text{s}^2}{(2)(9.81) \text{ m}} = 0.459 \text{ m}$$
$$p_B = (9.81 \text{ kN/m}^3)(0 - 0.459 \text{ m})$$
$$p_B = -4.50 \text{ kN/m}^2$$
$$p_B = -4.50 \text{ kPa}$$

The negative sign indicates that p_B is 4.50 kPa below atmospheric pressure. Notice that when we deal with fluids in motion, the concept that points on the same level have the same pressure does *not* apply as it does with fluids at rest.

6. Flow of Fluid and Bernoulli's Equation

Example 6.10

The next three panels present the solutions for the pressures p_C , p_D , and p_E , which can be found in a manner very similar to that used for p_B . Complete the solution for p_C before looking at the next panel.

The answer is $p_C = -16.27\text{kPa}$. We use Bernoulli's equation.

$$\frac{p_A}{\gamma} + z_A + \frac{v_A^2}{2g} = \frac{p_C}{\gamma} + z_C + \frac{v_C^2}{2g}$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.10

Because $p_A = 0$ and $v_A = 0$, the pressure at point C is

$$z_A = \frac{p_C}{\gamma} + z_C + \frac{v_C^2}{2g}$$

$$p_C = \gamma[(z_A - z_C) - v_C^2/2g]$$

$$z_A - z_C = -1.2 \text{ m} \quad (\text{negative, because } z_C \text{ is greater than } z_A)$$

$$v_C = v_B = 3.00 \text{ m/s} \quad (\text{because } A_C = A_B)$$

$$\frac{v_C^2}{2g} = \frac{v_B^2}{2g} = 0.459 \text{ m}$$

$$p_C = (9.81 \text{ kN/m}^3)(-1.2 \text{ m} - 0.459 \text{ m})$$

$$p_C = -16.27 \text{ kN/m}^2$$

$$p_C = -16.27 \text{ kPa}$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.10

The pressure at point E is 24.93 kPa. We use Bernoulli's equation:

$$\frac{p_A}{\gamma} + z_A + \frac{v_A^2}{2g} = \frac{p_E}{\gamma} + z_E + \frac{v_E^2}{2g}$$

Because $p_A = 0$ and $v_A = 0$, we have

$$z_A = \frac{p_E}{\gamma} + z_E + \frac{v_E^2}{2g}$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.10

$$p_E = \gamma[(z_A - z_E) - v_E^2/2g]$$

$$z_A - z_E = +3.0 \text{ m}$$

$$v_E = v_B = 3.00 \text{ m/s}$$

$$\frac{v_E^2}{2g} = \frac{v_B^2}{2g} = 0.459 \text{ m}$$

$$p_E = (9.81 \text{ kN/m}^3)(3.0 \text{ m} - 0.459 \text{ m})$$

$$p_E = 24.93 \text{ kN/m}^2$$

$$p_E = 24.93 \text{ kPa}$$

6. Flow of Fluid and Bernoulli's Equation

6.7.3 When Elevation Are Equal at Both Reference Point

SUMMARY OF THE RESULTS

1. The velocity of flow from the nozzle, and therefore the volume flow rate delivered by the siphon, depends on the elevation difference between the free surface of the fluid and the outlet of the nozzle.
2. The pressure at point B is below atmospheric pressure even though it is on the same level as point A, which is exposed to the atmosphere. In Eq. (6–11), Bernoulli's equation shows that the pressure head at B is decreased by the amount of the velocity head. That is, some of the energy is converted to kinetic energy, resulting in a lower pressure at B.

6. Flow of Fluid and Bernoulli's Equation

6.7.3 When Elevation Are Equal at Both Reference Point

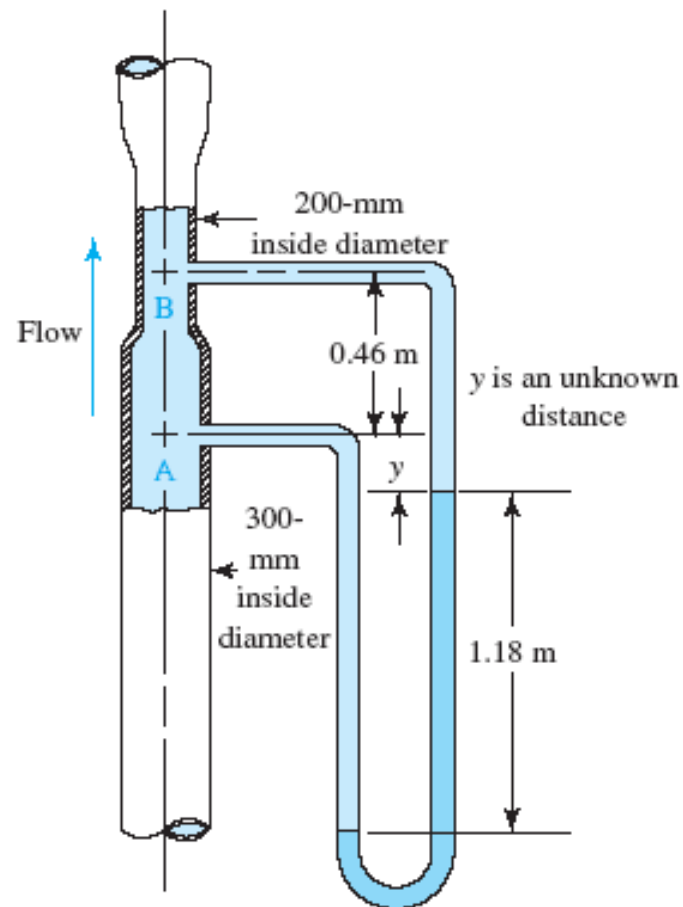
SUMMARY OF THE RESULTS

- When steady flow exists, the velocity of flow is the same at all points where the pipe size is the same.
- The pressure at point C is the lowest in the system because point C is at the highest elevation.
- The pressure at point D is the same as that at point B because both are on the same elevation and the velocity head at both points is the same.
- The pressure at point E is the highest in the system because point E is at the lowest elevation.

6. Flow of Fluid and Bernoulli's Equation

6.7.4 Venturi Meters and Other Closed Systems with Unknown Velocities

- Figure 6.8 shows a device called a *venturi meter* that can be used to measure the velocity of flow in a fluid flow system.



6. Flow of Fluid and Bernoulli's Equation

6.7.4 Venturi Meters and Other Closed Systems with Unknown Velocities

- The analysis of such a device is based on the application of Bernoulli's equation.
- The reduced-diameter section at B causes the velocity of flow to increase there with a corresponding decrease in the pressure.
- It will be shown that the velocity of flow is dependent on the *difference* in pressure between points A and B. Therefore, a differential manometer as shown is convenient to use.

6. Flow of Fluid and Bernoulli's Equation

Example 6.11

The venturi meter shown in Fig. 6.8 carries water at 60°C . The specific gravity of the gage fluid in the manometer is 1.25. Calculate the velocity of flow at section A and the volume flow rate of water.

The problem solution will be shown in the steps outlined at the beginning of this section but the programmed technique will not be used.

6. Flow of Fluid and Bernoulli's Equation

Example 6.11

1. *Decide what items are known and what is to be found.* The elevation difference between points A and B is known. The manometer allows the determination of the difference in pressure between points A and B. The sizes of the sections at A and B are known. The velocity is not known at any point in the system and the velocity at point A was specifically requested.
2. *Decide on sections of interest.* Points A and B are the obvious choices.

6. Flow of Fluid and Bernoulli's Equation

Example 6.11

3. *Write Bernoulli's equation between points A and B:*

$$\frac{p_A}{\gamma} + z_A + \frac{v_A^2}{2g} = \frac{p_B}{\gamma} + z_B + \frac{v_B^2}{2g}$$

4. *Simplify the equation, if possible, by eliminating terms that are zero or terms that are equal on both sides of the equation. No simplification can be done here.*

6. Flow of Fluid and Bernoulli's Equation

Example 6.11

5. *Solve the equation algebraically for the desired term.* This step will require significant effort. First, note that *both* of the velocities are unknown. However, we can find the difference in pressures between A and B and the elevation difference is known. Therefore, it is convenient to bring both pressure terms and both elevation terms onto the left side of the equation in the form of differences. Then the two velocity terms can be moved to the right side. The result is

$$\frac{p_A - p_B}{\gamma} + (z_A - z_B) = \frac{v_B^2 - v_A^2}{2g} \quad (6-12)$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.11

6. *Calculate the result.* Several steps are required. The elevation difference is

$$z_A - z_B = -0.46 \text{ m} \quad (6-13)$$

The value is negative because B is higher than A. This value will be used in Eq. (6-12) later. The pressure-head difference term can be evaluated by writing the equation for the manometer. We will use γ_g for the specific weight of the gage fluid, where

$$\gamma_g = 1.25(\gamma_w \text{ at } 4^\circ\text{C}) = 1.25(9.81 \text{ kN/m}^3) = 12.26 \text{ kN/m}^3$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.11

A new problem occurs here because the data in Fig. 6.8 do not include the vertical distance from point A to the level of the gage fluid in the right leg of the manometer. We will show that this problem will be eliminated by simply calling this unknown distance y or any other variable name. Now we can write the manometer equation starting at A:

$$p_A + \gamma(y) + \gamma(1.18 \text{ m}) - \gamma_g(1.18 \text{ m}) - \gamma(y) - \gamma(0.46 \text{ m}) = p_B$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.11

Note that the two terms containing the unknown y variable can be cancelled out. Solving for the pressure difference, we find

$$p_A - p_B = \gamma(0.46 \text{ m} - 1.18 \text{ m}) + \gamma_g(1.18 \text{ m})$$

$$p_A - p_B = \gamma(-0.72 \text{ m}) + \gamma_g(1.18 \text{ m})$$

$$\begin{aligned} \frac{p_A - p_B}{\gamma} &= -0.72 \text{ m} + \frac{\gamma_g(1.18 \text{ m})}{\gamma} \\ &= -0.72 \text{ m} + \frac{12.26 \text{ kN/m}^3 (1.18 \text{ m})}{9.65 \text{ kN/m}^3} \end{aligned}$$

$$(p_A - p_B)/\gamma = -0.72 \text{ m} + 1.50 \text{ m} = 0.78 \text{ m} \quad (6-14)$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.11

The entire left side of Eq. (6–12) has now been evaluated. Note, however, that there are still *two* unknowns on the right side, v_A and v_B . We can eliminate one unknown by finding another independent equation that relates these two variables. A convenient equation is the *continuity equation*,

$$A_A v_A = A_B v_B$$

$$v_B = v_A (A_A / A_B)$$

$$v_B = v_A (7.069 \times 10^{-2} / 3.142 \times 10^{-2}) = 2.25 v_A$$

$$v_B^2 = 5.06 v_A^2$$

$$v_B^2 - v_A^2 = 5.06 v_A^2 - v_A^2 = 4.06 v_A^2 \quad (6-15)$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.11

We can now take these results, the elevation head difference [Eq. (6–13)] and the pressure head difference [Eq. (6–14)], back into Eq. (6–12) and complete the solution. Equation (6–12) becomes

$$0.78 \text{ m} - 0.46 \text{ m} = 4.06 v_A^2 / 2g$$

$$v_A = \sqrt{\frac{2g(0.32 \text{ m})}{4.06}} = \sqrt{\frac{2(9.81 \text{ m/s}^2)(0.32 \text{ m})}{4.06}}$$

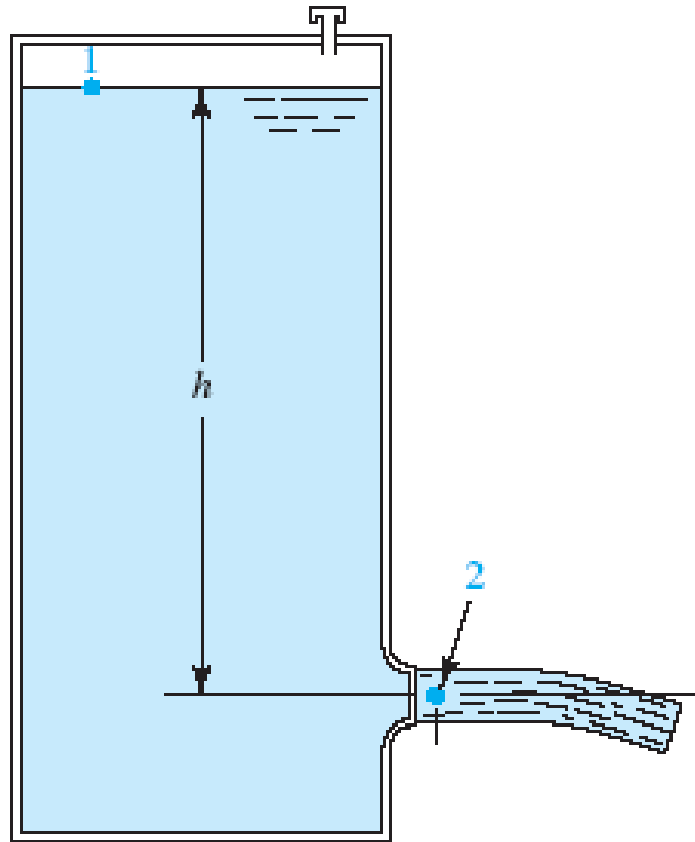
$$v_A = 1.24 \text{ m/s}$$

$$Q = A_A v_A = (7.069 \times 10^{-2} \text{ m}^2)(1.24 \text{ m/s}) = 8.77 \times 10^{-2} \text{ m}^3/\text{s}$$

6. Flow of Fluid and Bernoulli's Equation

6.8 Torricelli's Theorem

- Fig 6.9 shows the flow from a tank.



6. Flow of Fluid and Bernoulli's Equation

6.8 Torricelli's Theorem

- Fluid is flowing from the side of a tank through a smooth, rounded nozzle.
- To determine the velocity of flow from the nozzle, write Bernoulli's equation between a reference point on the fluid surface and a point in the jet issuing from the nozzle:

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

$$\frac{p_1^0}{\gamma} + z_1 + \frac{v_1^0}{2g} = \frac{p_2^0}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

$$v_2 = \sqrt{2g(z_1 - z_2)}$$

$$v_2 = \sqrt{2gh}$$

(6-16)

6. Flow of Fluid and Bernoulli's Equation

6.8 Torricelli's Theorem

- Equation (6–16) is called *Torricelli's theorem* in honor of Evangelista Torricelli, who discovered it in approximately 1645.

6. Flow of Fluid and Bernoulli's Equation

Example 6.12

For the tank shown in Fig. 6.9, compute the velocity of flow from the nozzle for a fluid depth h of 3.00 m.

This is a direct application of Torricelli's theorem:

$$v_2 = \sqrt{2gh} = \sqrt{(2)(9.81 \text{ m/s}^2)(3.0 \text{ m})} = 7.67 \text{ m/s}$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.13

For the tank shown in Fig. 6.9, compute the velocity of flow from the nozzle and the volume flow rate for a range of depth from 3.0 m to 0.50 m in steps of 0.50 m. The diameter of the jet at the nozzle is 50 mm.

The same procedure used in Example Problem 6.12 can be used to determine the velocity at any depth. So, at $h = 3.0\text{m}$, $v_2 = 7.67\text{ m/s}$. The volume flow rate is computed by multiplying this velocity by the area of the jet:

$$A_j = 1.963 \times 10^{-3} \text{ m}^2 \text{ (from Appendix J)}$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.13

Then,

$$Q = A_j v_2 = (1.963 \times 10^{-3} \text{ m}^2)(7.67 \text{ m/s}) = 1.51 \times 10^{-2} \text{ m}^3/\text{s}$$

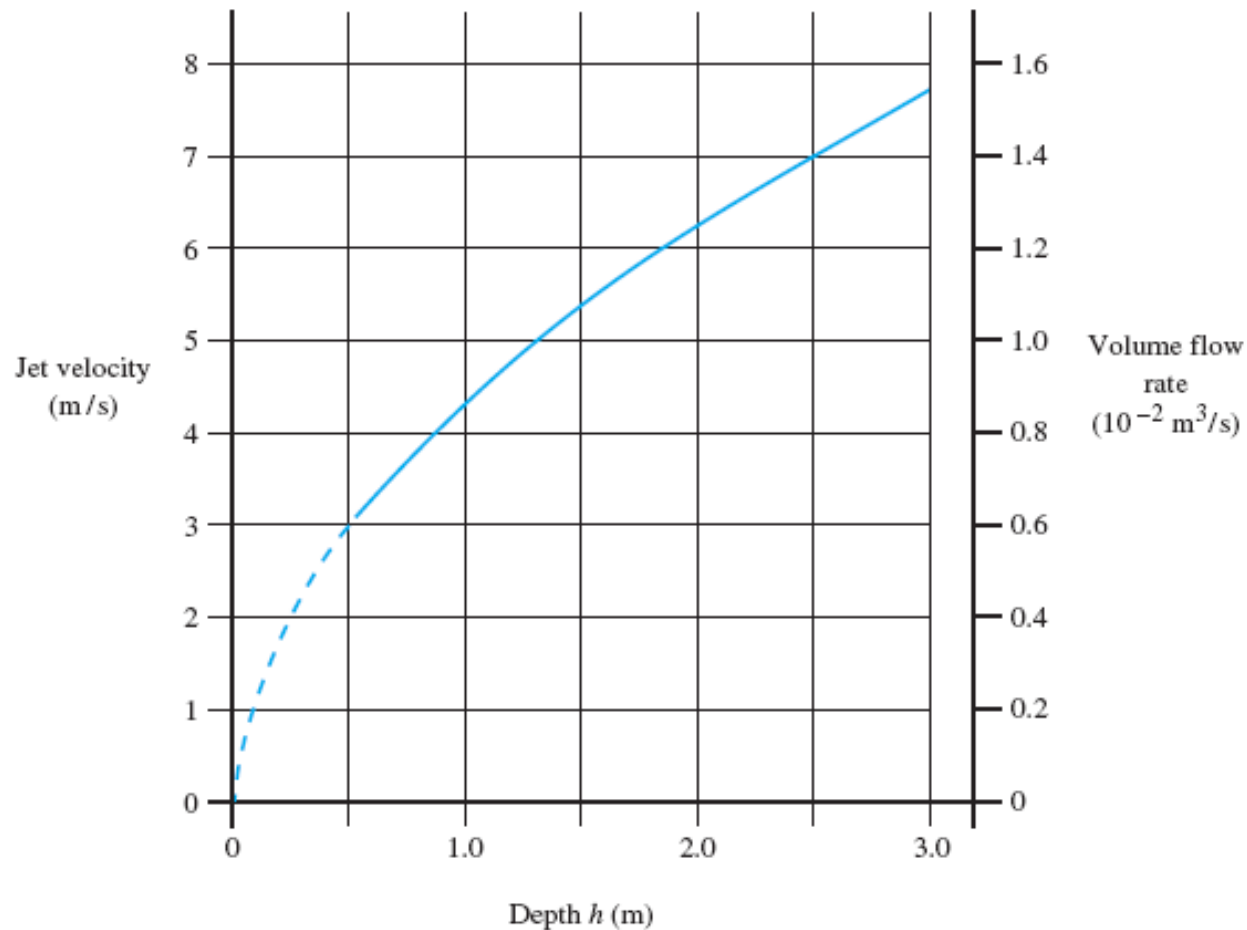
Using the same procedure, we compute the following data:

Depth h (m)	v_2 (m/s)	Q (m^3/s)
3.0	7.67	1.51×10^{-2}
2.5	7.00	1.38×10^{-2}
2.0	6.26	1.23×10^{-2}
1.5	5.42	1.07×10^{-2}
1.0	4.43	0.87×10^{-2}
0.5	3.13	0.61×10^{-2}

6. Flow of Fluid and Bernoulli's Equation

Example 6.13

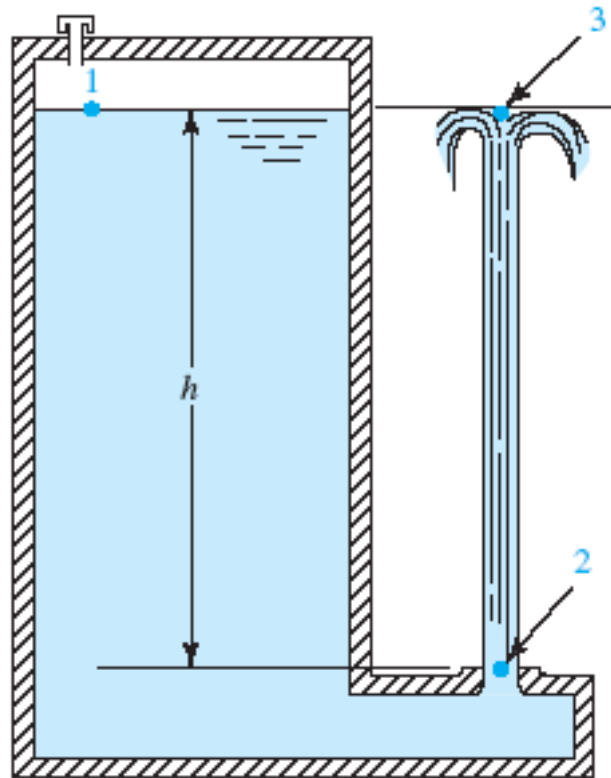
Figure 6.10 is a plot of velocity and volume flow rate versus depth.



6. Flow of Fluid and Bernoulli's Equation

6.8 Torricelli's Theorem

- Another interesting application of Torricelli's theorem is shown in Fig. 6.11, in which a jet of fluid is shooting upward.



6. Flow of Fluid and Bernoulli's Equation

6.8 Torricelli's Theorem

- If no energy losses occur, the jet will reach a height equal to the elevation of the free surface of the fluid in the tank.
- Of course, at this height the velocity in the stream is zero. This can be demonstrated using Bernoulli's equation.
- First obtain an expression for the velocity of the jet at point 2:

$$\frac{p_1}{\rho} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\rho} + z_2 + \frac{v_2^2}{2g}$$

$$v_2 = \sqrt{2gh}$$

6. Flow of Fluid and Bernoulli's Equation

6.8 Torricelli's Theorem

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6. Flow of Fluid and Bernoulli's Equation

6.8 Torricelli's Theorem

- Now, write Bernoulli's equation between point 2 and point 3 at the level of the free surface of the fluid, but in the fluid stream:

$$\frac{p_2^0}{\gamma} + z_2 + \frac{v_2^2}{2g} = \frac{p_3^0}{\gamma} + z_3 + \frac{v_3^2}{2g}$$

$$v_3 = \sqrt{v_2^2 + 2g(z_2 - z_3)}$$

- From Eq. (6-16), $v_2^2 = 2gh$. Also, $(z_2 - z_1)$. Then,

$$v_3 = \sqrt{2gh + 2g(-h)} = 0$$

6. Flow of Fluid and Bernoulli's Equation

6.8 Torricelli's Theorem

- This result verifies that the stream just reaches the height of the free surface of the fluid in the tank.
- To make a jet go higher (as with some decorative fountains, for example), a greater pressure can be developed above the fluid in the reservoir or a pump can be used to develop a higher pressure.

6. Flow of Fluid and Bernoulli's Equation

Example 6.14

First, use Bernoulli's equation to obtain an expression for the velocity of flow from the nozzle as a function of the air pressure.

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

$$v_2 = \sqrt{2g[(p_1/\gamma) + (z_1 - z_2)]}$$

$$v_2 = \sqrt{2g[(p_1/\gamma) + h]} \quad (6-17)$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.14

Then, in this problem, if we want a height of 12.2 m and $h=1.83$ m,

$$p_1/\gamma + h = 12.2 \text{ m}$$

$$p_1/\gamma = 12.2 \text{ m} - h = 12.2 \text{ m} - 1.83 \text{ m} = 10.37 \text{ m}$$

$$p_1 = \gamma(10.37 \text{ m})$$

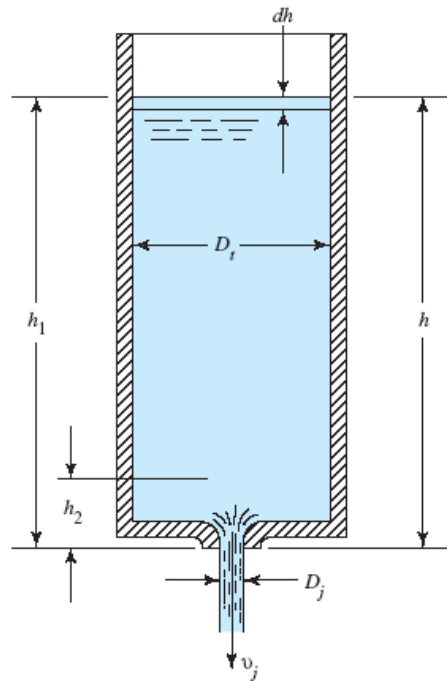
$$p_1 = (9.8 \text{ kN/m}^3)(10.37 \text{ m})$$

$$p_1 = 101.63 \text{ kN/m}^2 = 101.63 \text{ kPa}$$

6. Flow of Fluid and Bernoulli's Equation

6.9 Flow Due to Falling Head

- Figure 6.13 shows a tank with a smooth, well-rounded nozzle in the bottom through which fluid is discharging.
- For a given depth of fluid h , Torricelli's theorem tells us that the velocity of flow in the jet is



$$v_j = \sqrt{2gh}$$

6. Flow of Fluid and Bernoulli's Equation

6.9 Flow Due to Falling Head

- In a small amount of time dt , the volume of fluid flowing through the nozzle is

$$\text{Volume flowing} = Q(dt) = A_j v_j(dt) \quad (6-18)$$

- Meanwhile, because fluid is leaving the tank, the fluid level is decreasing.
- During the small time increment dt , the fluid level drops a small distance dh . Then, the volume of fluid removed from the tank is

$$\text{Volume removed} = -A_r dh \quad (6-19)$$

6. Flow of Fluid and Bernoulli's Equation

6.9 Flow Due to Falling Head

- These two volumes must be equal. Then,

$$A_j v_j (dt) = -A_t dh \quad (6-20)$$

- Solving for the time dt , we have

$$dt = \frac{-(A_t/A_j)}{v_j} dh \quad (6-21)$$

- From Torricelli's theorem, we can substitute

$$v_j = \sqrt{2gh}.$$

$$dt = \frac{-(A_t/A_j)}{\sqrt{2gh}} dh \quad (6-22)$$

6. Flow of Fluid and Bernoulli's Equation

6.9 Flow Due to Falling Head

- Rewriting to separate the terms involving h gives

$$dt = \frac{-(A_t/A_j)}{\sqrt{2g}} h^{-1/2} dh \quad (6-23)$$

- The time required for the fluid level to fall from a depth h_1 to a depth h_2 can be found by integrating Eq. (6-23):

$$\int_{t_1}^{t_2} dt = \frac{-(A_t/A_j)}{\sqrt{2g}} \int_{h_1}^{h_2} h^{-1/2} dh \quad (6-24)$$

$$t_2 - t_1 = \frac{-(A_t/A_j)}{\sqrt{2g}} \frac{\left(h_2^{1/2} - h_1^{1/2} \right)}{\frac{1}{2}} \quad (6-25)$$

6. Flow of Fluid and Bernoulli's Equation

6.9 Flow Due to Falling Head

- We can reverse the two terms involving h and remove the minus sign.

$$t_2 - t_1 = \frac{2(A_t/A_j)}{\sqrt{2g}} (h_1^{1/2} - h_2^{1/2}) \quad (6-26)$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.15

For the tank shown in Fig. 6.13, find the time required to drain the tank from a level of 3.0 m to 0.50 m. The tank has a diameter of 1.50 m and the nozzle has a diameter of 50 mm.

To use Eq. (6–26), the required areas are

$$A_t = \pi(1.50 \text{ m})^2/4 = 1.767 \text{ m}^2$$

$$A_j = \pi(0.05 \text{ m})^2/4 = 0.001963 \text{ m}^2$$

6. Flow of Fluid and Bernoulli's Equation

Example 6.15

The ratio of these two areas is required:

$$\frac{A_t}{A_j} = \frac{1.767 \text{ m}^2}{0.001963 \text{ m}^2} = 900$$

Now, in Eq. (6–26),

$$t_2 - t_1 = \frac{2(A_t/A_j)}{\sqrt{2g}} (h_1^{1/2} - h_2^{1/2})$$

$$t_2 - t_1 = \frac{2(900)}{\sqrt{2(9.81 \text{ m/s}^2)}} [(3.0 \text{ m})^{1/2} - (0.5 \text{ m})^{1/2}]$$

$$t_2 - t_1 = 417 \text{ s}$$

This is equivalent to 6 min and 57 s.

6. Flow of Fluid and Bernoulli's Equation

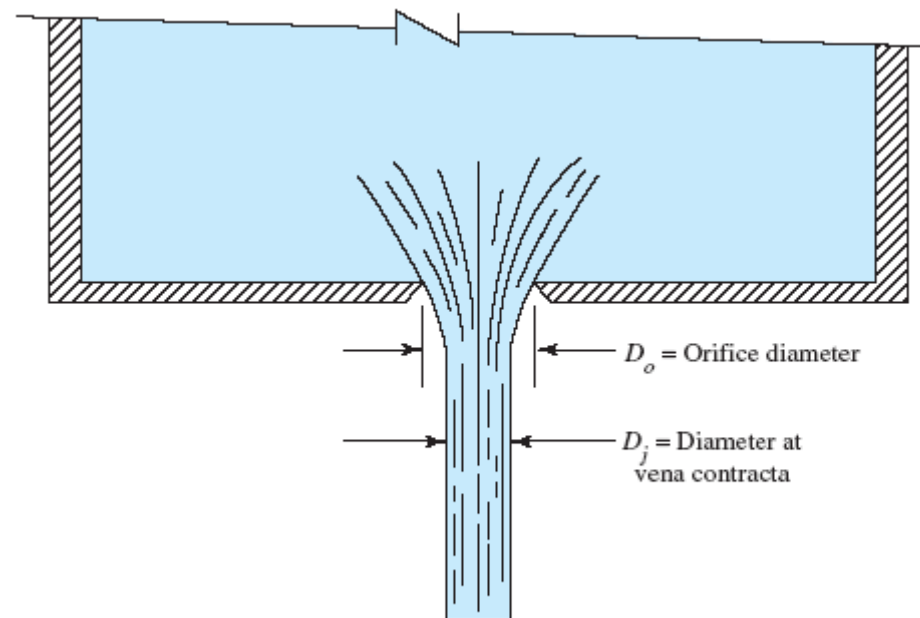
6.9.1 Draining a Pressurized Tank

- If the tank in Fig. 6.13 is sealed with a pressure above the fluid, the piezometric head $p/\rho g$ should be added to the actual liquid depth before completing the calculations called for in Eq. (6–25).

6. Flow of Fluid and Bernoulli's Equation

6.9.2 Effect of Type of Nozzle

- The development of Eq. (6–26) assumes that the diameter of the jet of fluid flowing from the nozzle is the same as the diameter of the nozzle itself.
- Fig 6.14 shows the flow through a sharp-edged orifice.



6. Flow of Fluid and Bernoulli's Equation

6.9.2 Effect of Type of Nozzle

- The proper area A_j to use for in Eq. (6–26) is that at the smallest diameter.
- This point, called the *vena contracta*, occurs slightly outside the orifice.
- For this sharp-edged orifice, $A_j = 0.62A_0$ is a good approximation.