6. Fluid mechanics: fluid statics; fluid dynamics

(internal flows, external flows)

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6.1 Fluid statics

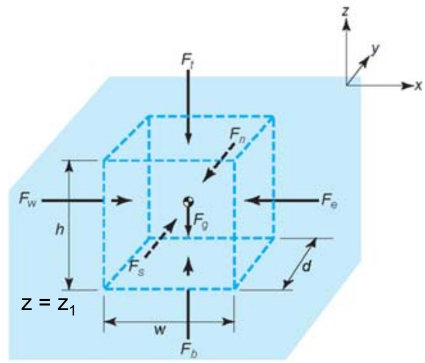
Fluid statics, static pressure /1

- In engineering applications, a **fluid** *(sv: fluid)* is **a liquid or a gas**
- The behaviour of **stationary fluids** is described by **fluid statics**
- A **liquid** in a container forms a layer with a distinct surface, and exerts forces on the walls supporting it, while a **gas** will fill the whole container.
- Two types of forces act on a fluid volume element:

  surface (pressure) forces and **body (gravitational) forces**: see Figure →

- **Pressure** (a scalar!) is defined as surface force / area, <u>for example</u>

  $p_b = F_b / (d \cdot w) = p$ @ $z = z_l$



Fluid volume h·d·w with density ρ and mass m = h·d·w·ρ

Picture: KJ05

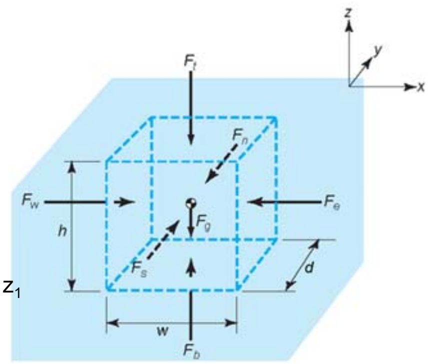
Fluid statics, static pressure /2

- For the horizontal forces $F_n + F_s = 0$ or $- p_y \cdot h \cdot w + p_y \cdot h \cdot w = 0 \rightarrow p_y = 0$
- Similarly $F_w + F_e = 0$ gives $p_x = 0$,
- There are three vertical forces: $-F_t \cdot h \cdot d - m \cdot g + F_b \cdot h \cdot d = 0$ (gravity g)
- The **pressure difference** between $z = z_l$ and $z = z_l + h$ follows from $- F_t - \rho \cdot h \cdot d \cdot w \cdot g = - F_b$, with $- F_b / (d \cdot w) = -p_z$ @ $z = z_l$; and $F_t / (d \cdot w) = -p_z$ @ $z = z_l + h$; gives

  $$\boxed{p_z(z_l) = p_z(z_l + h) + \rho \cdot h \cdot g}$$

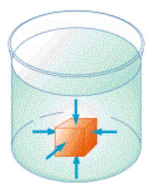
- If $z = z_l + h$ is at the fluid surface exposed to atmosperic pressure p^0 then

  $$\boxed{p_z(z_l) = p^0 + \rho \cdot h \cdot g}$$



Fluid volume h·d·w with density ρ and mass m = h·d·w·ρ

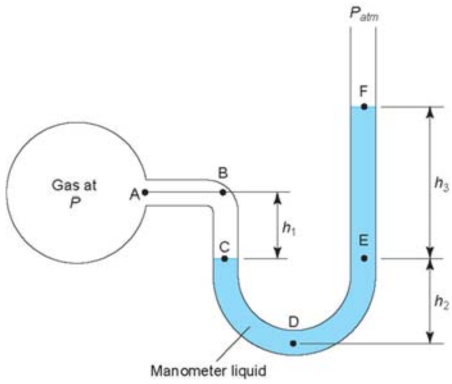
Picture: KJ05



Picture: http://www.physics.uc.edu/~sitko/CollegePhysicsIII/9-Solids&Fluids/Solids&Fluids.htm

U-tube manometer

- The **U-tube manometer** is based on the relation between depth and pressure in static fluids, with one end open to the atmosphere at p_{atm}
- For the Figure, with gravity g and densities ρ_g and ρ_l for gas and liquid:
  $p_C = \rho_g \cdot h_1 \cdot g + p_B$
  $p_D = \rho_l \cdot h_2 \cdot g + p_C = \rho_l \cdot h_2 \cdot g + \rho_g \cdot h_1 \cdot g + p_B$
  and also, from the other side
  $p_D = \rho_l \cdot (h_3+h_2) \cdot g + p_F = \rho_l \cdot (h_3+h_2) \cdot g + p_{atm}$
  which gives, with $p_B = p_A$
  $\rho_l \cdot h_2 \cdot g + \rho_g \cdot h_1 \cdot g + p_A = \rho_l \cdot (h_3+h_2) \cdot g + p_{atm}$
  $p_A - p_{atm} = \rho_l \cdot h_3 \cdot g - \rho_g \cdot h_1 \cdot g$
  and noting that $\rho_l \gg \rho_g$: $\boxed{\mathbf{p_A - p_{atm} = \rho_l \cdot h_3 \cdot g}}$



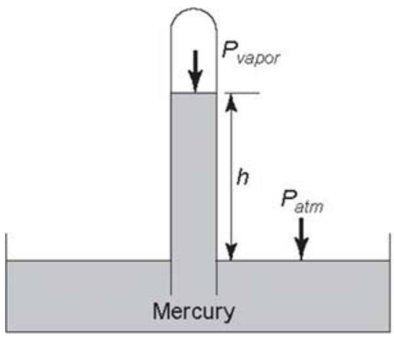
Note that the U-tube manometer measures pressure differences

Picture: KJ05

Barometer

- A device for measuring atmospheric pressure (which cannot be done using an U-tube manometer) is referred to as **barometer**
- A closed tube filled with mercury (Hg) is quickly put upside-down in an open container filled with Hg
- Gravity causes the Hg level in the tube to fall, but no air can enter the tube. The small gas volume trapped is Hg vapour at equilibrium with liquid Hg.
- For the tube $p_{vapor,Hg} + \rho_{Hg} \cdot h_{Hg} \cdot g = p_{atm}$
- At 20°C, $p_{vapor,Hg} = 0.158$ Pa « p_{atm}, thus

$\boxed{p_{atm} \approx \rho_{Hg} \cdot h_{Hg} \cdot g}$



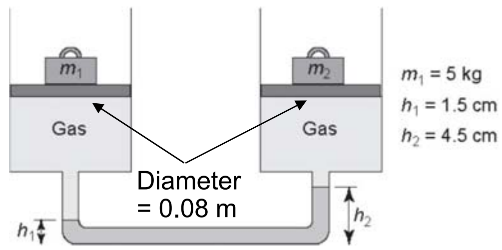
the density of liquid Hg is 13546.2 kg/m³ at 20°C

after Torricelli:
1 torr = 1 mm Hg pressure
1 atm = 760 torr at 0°C

Picture: KJ05

Example: a manometer

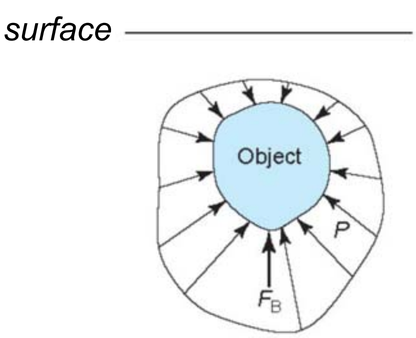
- Two piston-cylinder assemblies are connected by a tube filled with mercury (Hg) at 20°C (density 13546 kg/m^3)
- The diameter of each piston is 0.08 m, the mass of each piston is 0.40 kg. Mass m_1 = 5.00 kg
- Use the data to <u>calculate</u> mass m_2.



Picture: KJ05

Buoyancy /1

- **Buoyancy** *(sv: flytkraft, fi: nostovoima)* or **buoyant force** acts on all objects immersed or submerged *(sv: sänkad)* in a fluid
- It is an overall **upwards force** as the result of the fact that pressure p in a static fluid increases with depth



Picture: KJ05

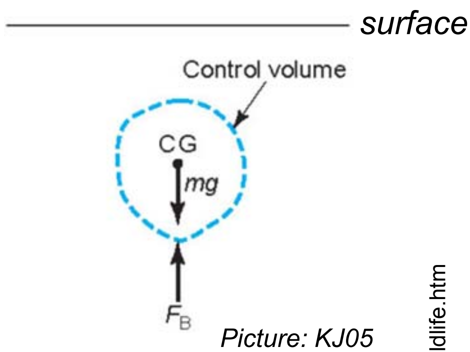
Picture: http://www.math.nyu.edu/~crorres/Archimedes/Crown/Vitruvius.html

Picture: http://www.physics.lsa.umich.edu/demolab/graphics/2b40_u2.jpg

Buoyancy /2

- For an immersed object, horizontal forces cancel each other, and the two vertical forces are gravity and buoyancy.
- The forces on the surface of the object are the same as when that surface would be filled with the fluid
- Thus, the **buoyant force** on a mass with volume V is equal (but opposite in sign) to the weight of the fluid in the volume V, and acts on the same centre of gravity (CG):

$$F_B = - m_{fluid} \cdot g = - \rho_{fluid} \cdot V \cdot g$$



Picture: KJ05

Picture: http://www.hi.is/~oi/svalbard_wildlife.htm

Buoyancy /3

- For any object the buoyancy force it experiences may be less than, equal to or larger than its weight
- If F_B > weight, the object will rise / float

  If F_B < weight, the object will sink

  If F_B = weight, the will float in suspension
- For example, for the two fluids geometry →

  $$F_B = (\rho_1 \cdot V_1 + \rho_2 \cdot V_2) \cdot g$$

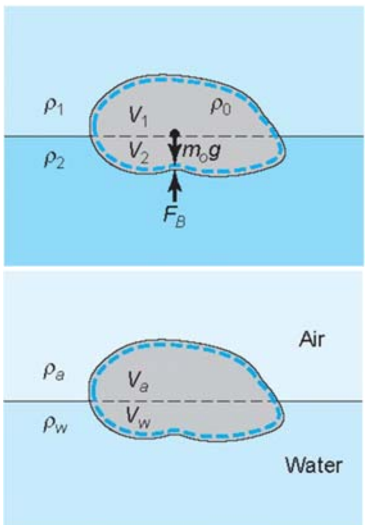
  in equilibrium with $F_{gravity} = m_0 \cdot g = \rho_0 \cdot V_{tot} \cdot g$

  for object mass m_0 (kg).

  → $\boldsymbol{\rho_0 \cdot V_{tot} = \rho_1 \cdot V_1 + \rho_2 \cdot V_2}$ **and** $\boldsymbol{V_{tot} = V_1 + V_2}$
- For example, for cases with water + air →

  $F_B = (\rho_a \cdot V_a + \rho_w \cdot V_w) \cdot g \approx \rho_w \cdot V_w \cdot g \qquad (\rho_a >> \rho_w)$

  → $\boldsymbol{\rho_0 \cdot V_{tot} = \rho_w \cdot V_w}$, **or :** $\boldsymbol{\rho_0 / \rho_w = V_w / V_{tot}}$



Pictures: KJ05

Example: buoyancy

- The tip of a certain iceberg (which is the volume of the iceberg above the water surface) is V_{tip} = 79 m^3, in seawater of with density ρ_{sea} = 1027 kg/m^3. <u>Calculate</u> the submerged (*i.e.* under water) volume of the iceberg. For ice the density is ρ_{ice} = 920 kg/m^3.

picture: http://www.sgisland.org/pages/zone/download.htm

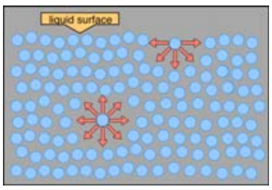
Source: KJ05

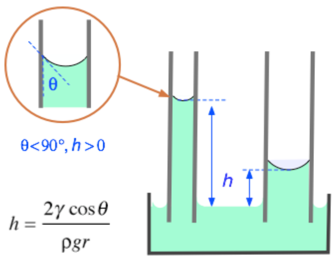
Surface tension

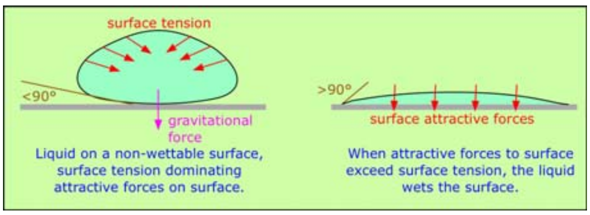
- A liquid at a material interface, usually liquid-gas, exerts **a force F_{int} per unit length L along the surface**.
- It is the result of molecular attraction at a liquid surface being different from that "in" the liquid → the surface acts like a stretched membrane
- **Surface tension** (σ or γ, unit: N/m) **quantifies this force**:

$$F_{int} = \gamma \cdot L$$

- Result phenomena:
  - Contact angle
  - Capillary action (rise or drop)
  - Bubbles, droplets







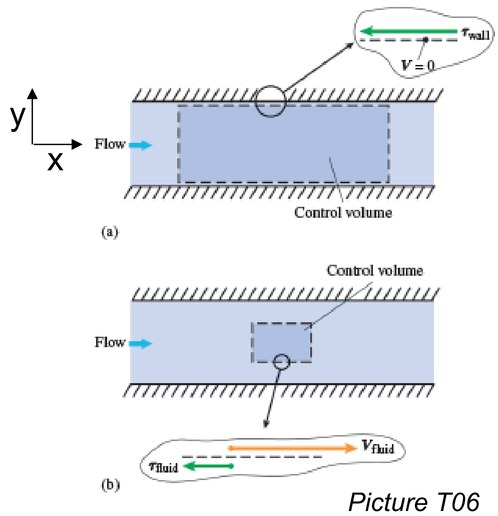
For ambient water-air: γ = 0.073 N/m

http://www.chem1.com/acad/sci/aboutwater.html
http://www.chem1.com/acad/webtext/states/state-images/caprise1.png
http://3.bp.blogspot.com/_ADtjVgTw6-Q/TJIE_2usjgI/AAAAAAAAAAc/RFSv-itGeuc/s1600/surface_tension_c_ph_784.jpg

6.2 Fluid dynamics: viscosity, laminar, turbulent flow, boundary layer

Internal friction in fluid flow /1

- Fluids will (try to) resist a change in shape, as will occur in fluid flow situations where different fluid elements have different velocities
- Note the **definition** of a fluid: ***a fluid is a substance that deforms continuously under the application of a shear stress*** *(sv: skjuvspänning)*
- Consider fluid flow between plates:
  - The **no-slip condition** says that at the wall the velocity of the fluid is the same as the wall velocity *), for a fixed wall $v_{fluid} = 0$ at the wall
  - Between the plates a **velocity profile** exists: it can be decribed as $v_x = v_x(y)$
  - Shear stresses, τ_{fluid}, arise due to velocity differences between different fluid elements

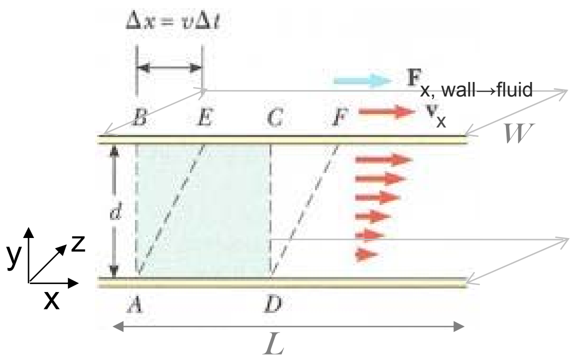


Picture T06

**) this applies always except for very low pressure gases, for example in the upper atmosphere*

Internal friction in fluid flow /2

- For a fluid between plates with width W (m), distance d (m) the shear force $\underline{F} = (F_x,F_y,F_z) = (F_x,0,0)$ *(unit: N)* to pull the fluid at velocity $\underline{v} = (v_x,v_y,v_z) = (v_x,0,0)$ gives a **shear stress** τ_{yx} *(unit: N/m²)* in the fluid at y = d that is equal to:



$v_x = 0$ @ $y = 0$

$$\frac{F_{x,\ fluid\rightarrow wall}}{surface} = \frac{-F_{x,wall\rightarrow fluid}}{W\cdot L} = \tau_{yx}\Big|_{y=d} = -\eta\frac{dv_x}{dy} \approx -\eta\frac{\Delta v_x}{\Delta y}$$

with τ_{yx} as stress in direction "x" in a plane for constant "y"

- This defines the **dynamic viscosity** η *(unit: Pa.s = kg.m⁻¹.s⁻¹)*

SIGN:

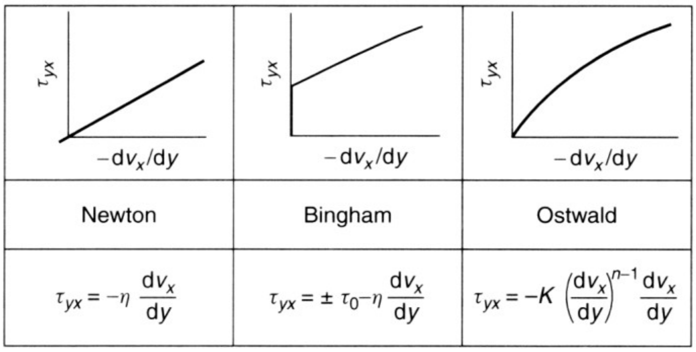
- ***! Note: τ_{yx} at $y = y_0$ is the shear stress of fluid elements with $y < y_0$ on the fluid elements with $y > y_0$. As a result $F_x > 0$ if $dv_x/dy < 0$!***

Picture: http://www.physics.uc.edu/~sitko/CollegePhysicsIII/9-Solids&Fluids/Solids&Fluids.htm

Internal friction in fluid flow /3

- The linear relation between τ_{yx} and dv_x/dy is referred to as **Newton's Law** which holds for so-called ***Newtonian fluids***
- For ***non-Newtonian fluids***, other relations between shear force and velocity gradient hold, for example Bingham fluids (toothpaste, clay) or pseudo-plastic (Ostwald) fluids (blood, yoghurt). For those, *viscosity is a function of the velocity gradient*: $\tau_{yx} = \eta(dv_x/dy)\cdot dv_x/dy$

Newtonian vs non-Newtonian fluids



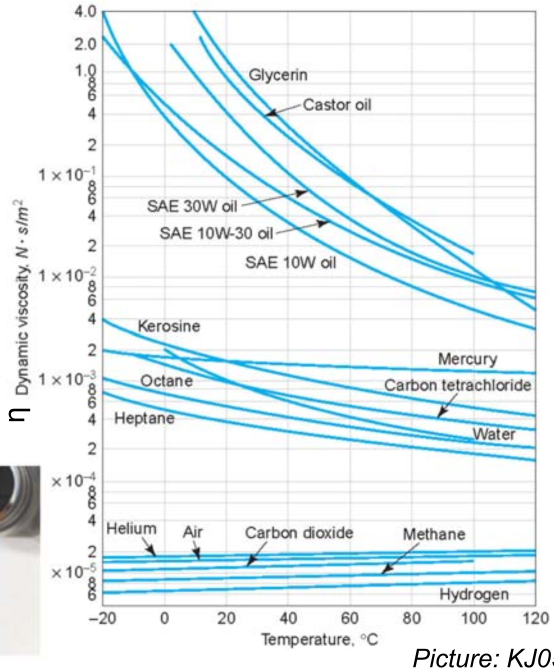
Picture: BMH99

- **Note:** The flow of a fluid between plates, or in a tube or on a surface doesn't necessarily require moving walls: **usually the driving force is gravity, or a static pressure difference**

Viscosity

- **Viscosity** *(sv: viskositet)* is a measure of a fluid's resistance to flow; it describes the internal friction of a moving fluid.
- More specifically, it defines the rate of momentum transfer in a fluid as a result of a velocity gradient.
- **Dynamic viscosity** η (unit: Pa.s) is related to a **kinematic viscosity**, ν (unit: m^2/s) via fluid density ρ (kg/m^3) as: $\nu = \eta/\rho$

Picture T06



Picture: KJ05

Internal friction in fluid flow /5

- Concentration, c, temperature, T, and energy, E, are scalars, and their gradient is a **vector** such as dT/dx or ∇T = (∂T/ ∂x, ∂T/ ∂y, ∂T/ ∂z), etc.
- Velocity is a vector $\underline{v}$, for example $\underline{v} = (v_x, v_y, v_z)$ and it's gradient is a (second order) **tensor** with elements such as dv_x/dy (gradient of v_x in y-direction)

$$\nabla \underline{v} = \begin{pmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_y}{\partial x} & \frac{\partial v_z}{\partial x} \\ \frac{\partial v_x}{\partial y} & \frac{\partial v_y}{\partial y} & \frac{\partial v_z}{\partial y} \\ \frac{\partial v_x}{\partial z} & \frac{\partial v_y}{\partial z} & \frac{\partial v_z}{\partial z} \end{pmatrix}$$

note :

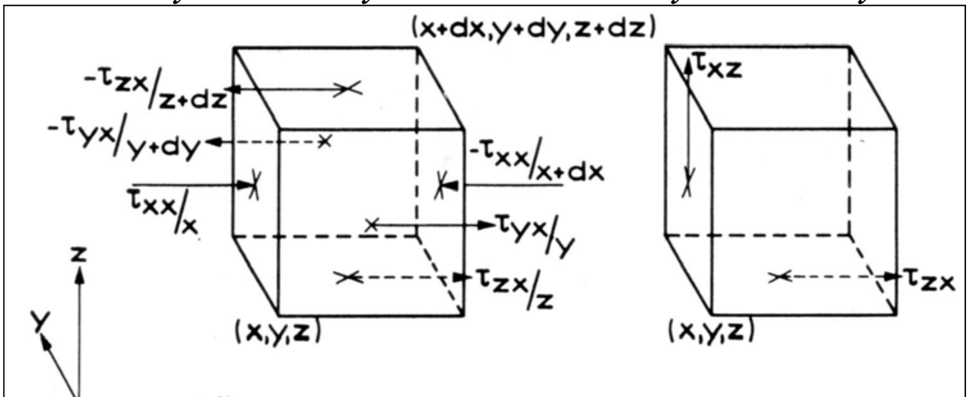
$$\nabla.\underline{v} = \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$

Gradients of a scalar property give a vector (or 1st order tensor); gradients of a vector property give a 2nd order tensor, etc.

Internal friction in fluid flow /6

- $\nabla \underline{v}$ results in **3 compressive stresses** *(sv: tryckspänningar)* τ_{xx}, τ_{yy} and τ_{zz} and **6 shear stresses** *(sv: skjuvspänningar)* τ_{xy}, τ_{xz}, τ_{yz}, τ_{zx}, τ_{yx} and τ_{zy}:

$$\tau_{yx} = -\eta \frac{dv_x}{dy} = -\nu \frac{d\rho v_x}{dy}; \quad \tau_{yz} = -\nu \frac{dv_z}{dy} = -\nu \frac{d\rho v_z}{dy}; \quad \text{etc.}$$



τ_{yx} is in x-direction in plane of constant y

Picture: SSJ84

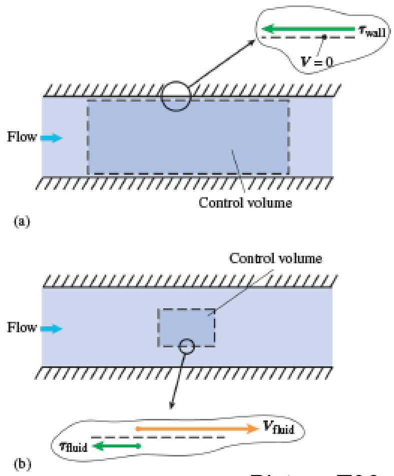
Viscous work

Vector/tensor calculations like this are beyond this course

- The shear stresses can be expressed as **tensor** $\underline{\underline{\tau}}$, resulting in a viscous shear force on a certain area A that is equal to $\underline{F}_{visc} = \underline{\underline{\tau}} \cdot \underline{A}$, with $\underline{A} = A\underline{n}$ with normal vector $\underline{n}$

- If the velocity $\underline{v}$ at surface $\underline{A}$ the rate of **viscous work** done by the fluid at surface A equals $\dot{W}_{visc} = \underline{F}_{visc} \cdot \underline{v} = \underline{\underline{\tau}} \cdot \underline{A} \cdot \underline{v}$,
which for a certain volume element of control volume (inside which $\underline{v}$ and $\underline{\underline{\tau}}$ can vary) with total outside surface A gives the rate of work done:

$$\dot{W}_{visc} = \int_A (\underline{\underline{\tau}} \cdot \underline{v}) \cdot d\underline{A}$$



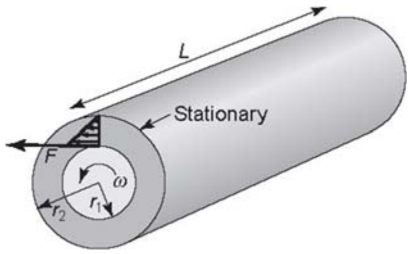
Picture T06

The friction work is dissipated as HEAT

- Note: at the wall $\underline{v}$ = 0 so no work is done; also at points where velocity and shear are perpendicular $\underline{\underline{\tau}} \cdot \underline{v}$ = 0 and no work is done.

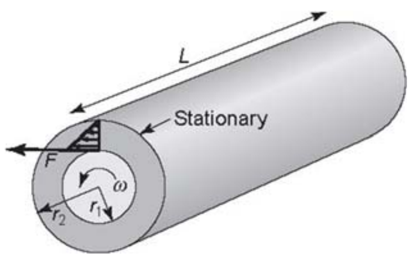
Example: shear stress concentric cylinders /1

- Oil with viscosity η = 0.05 Pa·s fills a 0.4 mm gap between two cylinders of which the inner one rotates whilst the outer one is fixed.
- The diameter of the inner cylinder is 8 cm, the length is 20 cm.
- Question: How much power is required to rotate the inner cylinder at 300 rpm?



Picture: KJ05
Question ÖS96-4.1

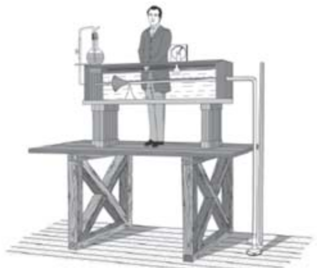
Example: shear stress concentric cylinders /2



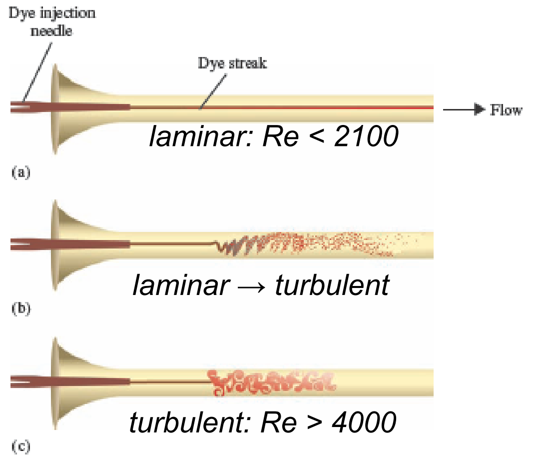
*) The space between the two cylinders is very small and may be treated as a flat plate

Picture: KJ05
Question ÖS96-4.1

Laminar ↔ turbulent fluid flow



Osborne Reynolds's dye-streak experiment (1883) for measuring laminar → turbulent flow transition



Pictures: T06

- For circular tube flow, the laminar → turbulent flow transition occurs at **Reynolds number Re** 2100 - 2300, with the **dimensionless number** defined as $\boxed{\mathbf{Re = \rho \langle v\rangle \cdot d/\eta}}$

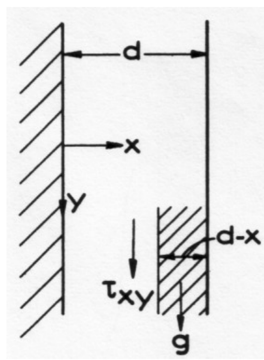
  for ρ = fluid's density (kg/m^3), $\langle v\rangle$ = fluid's **average** velocity (m/s), d = tube diameter (m) and η = fluid's dynamic viscosity (Pa·s)

Example: a liquid film on a vertical wall /1

- A stationary laminar flow of water (at 1200 kg/h) runs down a vertical surface (with width W = 1 m).

  Give
  - the expression for the shear stress distribution,
  - the expression for the velocity profile, and
  - the expression for volumetric flow rate $\dot{V}$ (m^3/s)

  and calculate
  - film thickness d
  - velocity $\langle v_y\rangle$ averaged over the film thickness
  - maximum velocity $v_{y,max}$



Data: dynamic viscosity for water $\eta = 10^{-3}$ Pa.s
density for water $\rho = 1000$ kg/m^3
gravity $g = 9.8$ m/s^2

Source: SSJ84

Example: a liquid film on a vertical wall /2

Answer: For this steady-state process:

- The vertical force balance for a volume element with length dy as shown gives $F_{gravity} = F_{shear}$

$$\rho \cdot (d - x) \cdot W \cdot dy \cdot g + \tau_{xy} \cdot W \cdot dy = 0 \Rightarrow \rho \cdot (d - x) \cdot g + \tau_{xy} = 0$$

with $\tau_{xy} = -\eta \dfrac{dv_y}{dx} = -\rho \cdot (d-x) \cdot g \Rightarrow \dfrac{dv_y}{dx} = \dfrac{\rho \cdot (d-x) g}{\eta}$, *integrating :*

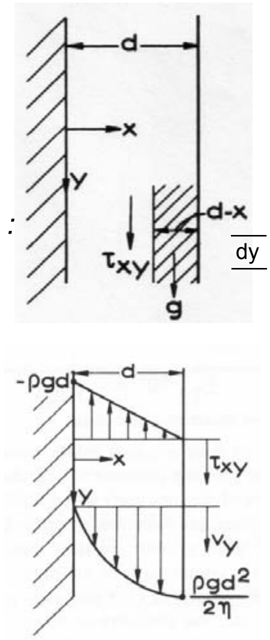
$$v_y(x) = \int_0^x \frac{dv_y}{dx} dx = \int_0^x \frac{\rho \cdot (d-x) g}{\eta} dx = \frac{\rho \cdot g}{\eta} \cdot (xd - ½x^2)$$

with $v_y = v_{y,max}$ @ x = d: $v_{y,max} = ½\rho g d^2/\eta$

For the average velocity <v> with $\dot{V}$ = <v>·d·W:

$$< v_y > = \frac{1}{d} \int_0^d v_y(x) \cdot dx = \frac{1}{d} \int_0^d \frac{\rho g}{\eta} \cdot (xd - ½x^2) \cdot dx = \frac{\rho g d^2}{3\eta}$$

$$\textit{and} < v_y > = \frac{\dot{V}}{W \cdot d} \quad \textit{gives} \quad d = \sqrt[3]{\frac{3\eta \dot{V}}{\rho g}}$$



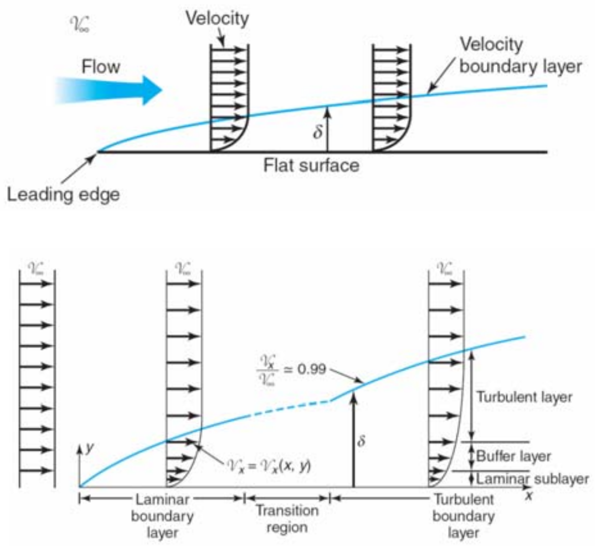
Source: SSJ84

The data gives: d = 0.47 mm, $<v_y>$ = 0.71 m/s; $v_{y,max}$ = 1.07 m/s

Boundary layers

- At the interface of a surface* and a flowing medium, a **<u>thin</u>** (~ 0.01 - 1 mm) layer of fluid is created in which the velocity increases from v = 0 at the interface to the free-flow velocity v = v_∞ (or 0.99·v_∞)
- In this **boundary layer** *(sv: gränsskikt)* all the **thermal and/or viscous effects** of the surface are concentrated
- The boundary layer can develop from **laminar to turbulent** flow

* *This can be a solid surface or another flowing medium*



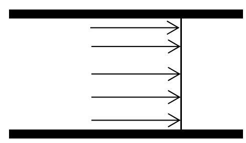
Growth of the velocity boundary layer on a flat surface.

Pictures: KJ05

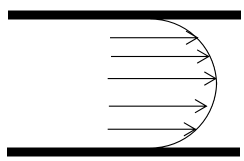
6.3 Fluid dynamics: internal flows / tube flow

Internal flows; velocity profiles

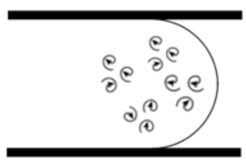
- Fluid flow in a tube or other confinement *(sv: inspärrning)* will show:
  - zero velocity (the no-slip condition) at the walls; and
  - maximum velocity furthest from the walls (*i.e.* at a tube flow centre line or at a free surface)
- The velocity profile is the result of viscous friction, and for turbulent flow, "**eddy**" currents (→ so-called "eddy viscosity": $\eta = \eta_{viscous} + \eta_{eddy}$)
- In many applications a plug flow idealisation may be used described by an **average velocity <v>**



Plug flow idealisation



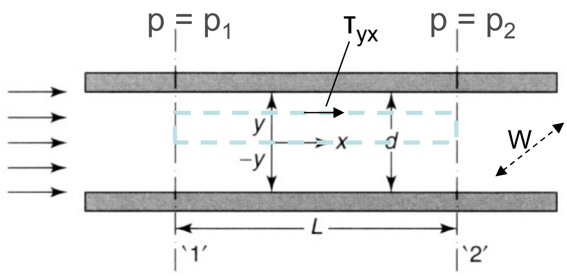
Velocity profile due to viscous friction



Velocity profile due to turbulent "eddies"

Laminar flow between two plates /1

- For a steady-state fluid flow between two stagnant parallel plates, the forces on a volume element between point "1" and "2" and between y = centre line and y = y are (for plate width W) :



Picture: BMH99

$$@\,\text{"1" pressure force} = p_1 \cdot y \cdot W\ ; \quad @\,\text{"2" pressure force} = -p_2 \cdot y \cdot W$$

$$\text{shear force on volume element} = -\tau_{yx} \cdot L \cdot W$$

$$\text{The force balance gives } p_1 \cdot y - p_2 \cdot y - \tau_{yx} \cdot L = 0 \Rightarrow \tau_{yx} = \frac{p_1 - p_2}{L} \cdot y$$

$$\text{With } \tau_{yx} = -\eta \cdot \frac{dv_x}{dy} \Rightarrow \frac{dv_x}{dy} = -\frac{p_1 - p_2}{\eta \cdot L} \cdot y \quad \text{with} \quad v_x = 0\ @\ y = \pm \tfrac{1}{2} d$$

τ_{yx} acts on fluid y > y, so -τ_{yx} acts on fluid y < y which is the fluid element

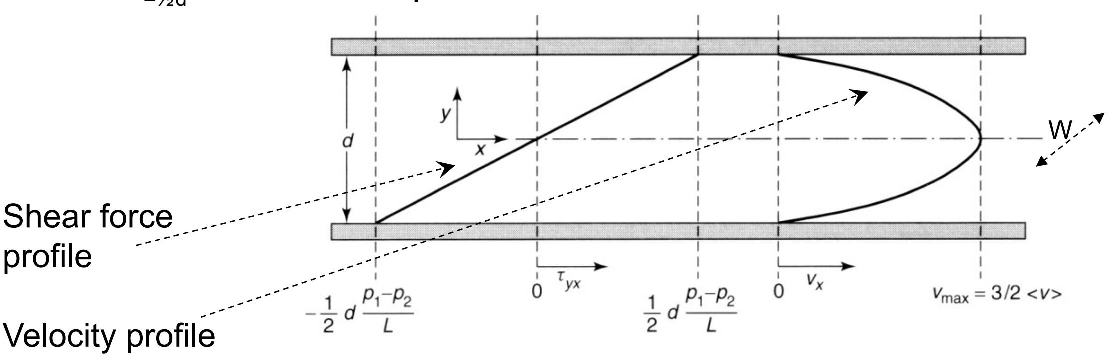
Laminar flow between two plates /2

Calculation of the velocity profile and maximum velocity :

$$\text{Integration : } v_x = \frac{p_1 - p_2}{2 \cdot \eta \cdot L}\left(\frac{d^2}{4} - y^2\right) \quad \text{and} \quad v_{x,max} = \frac{p_1 - p_2}{8 \cdot \eta \cdot L} d^2 \quad @\ y = 0$$

Calculation of the flow rate $\dot{V}(m^3/s)$:

$$\dot{V} = W \cdot \int_{-\frac{1}{2}d}^{\frac{1}{2}d} v_x dy = W \cdot \frac{p_1 - p_2}{12 \cdot \eta \cdot L} d^3 = W \cdot d \cdot \langle v_x \rangle\ , \quad \Rightarrow \langle v_x \rangle = \frac{2}{3} v_{x,max}$$



Picture: BMH99

Stationary laminar tube flow

$$@\text{"1" pressure force} = p_1 \cdot \pi \cdot r^2 \;;$$

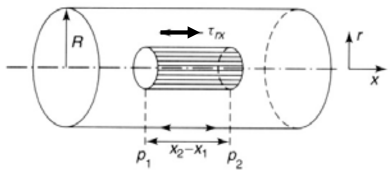
$$@\text{"2" pressure force} = -p_2 \cdot \pi \cdot r^2$$

$$\text{shear force } \underline{\text{on}} \text{ volume element} = -\tau_{rx} \cdot (x_2 - x_1) \cdot 2\pi \cdot r$$

$$\text{Force balance: } \; p_1 \cdot \pi \cdot r^2 - p_2 \cdot \pi \cdot r^2 - \tau_{rx} \cdot (x_2 - x_1) \cdot 2\pi \cdot r = 0$$

$$\Rightarrow \tau_{rx} = \frac{p_1 - p_2}{2 \cdot (x_2 - x_1)} \cdot r = ½ r \cdot \left(\frac{-dp}{dx} \right)$$

$$\text{With } \tau_{rx} = -\eta \cdot \frac{dv_x}{dr} \Rightarrow \frac{dv_x}{dr} = \frac{-½ r}{\eta} \cdot \left(\frac{-dp}{dx} \right) \quad \text{with } v_x = 0 \;@\; r = R$$



at the centre line:

$$\tau_{rx} = 0$$

at the wall:

$$\tau_{rx} = ½ R \cdot \left(-\frac{dp}{dx} \right) = \frac{4 \cdot \langle v_x \rangle \cdot \eta}{R}$$

$$= \tau_{fluid \to wall} = -\tau_{wall \to fluid}$$

Velocity profile and maximum velocity:

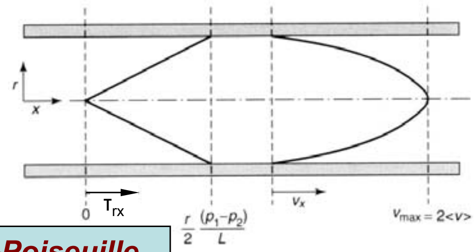
$$\text{Integration: } \; v_x(r) = \frac{1}{4 \cdot \eta} \cdot \left(-\frac{dp}{dx} \right) \cdot \left(R^2 - r^2 \right) \quad \text{and}$$

$$v_{x,\,max} = \frac{1}{4 \cdot \eta} \cdot \left(-\frac{dp}{dx} \right) \cdot R^2 \quad @\; r = 0$$

Calculation of the flow rate $\dot{V}(m^3/s)$ *:*

$$\dot{V} = \int_0^R 2\pi r \cdot v_x dr = \frac{\pi R^4}{8 \cdot \eta} \cdot \left(-\frac{dp}{dx} \right) =$$

$$= \pi R^2 \cdot \langle v_x \rangle \;, \quad \Rightarrow \langle v_x \rangle = ½ v_{x,\,max}$$



*"**Hagen-Poiseuille** relationship"*

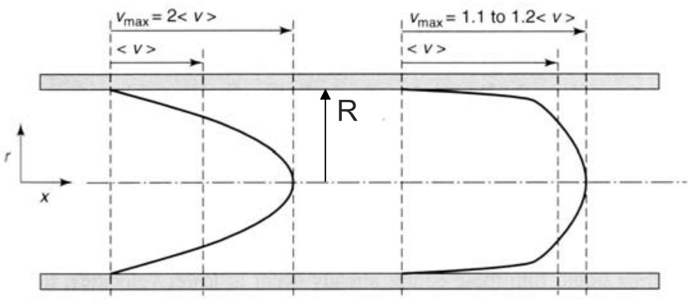
Pictures: BMH99

Tube flow velocity profiles

- **Laminar and turbulent tube flows** show different velocity profiles

  Laminar:
  $v_x(r) = (1 - r^2/R^2) \cdot v_{max}$
  cross-sectional average velocity <v> = $½ \cdot v_{max}$

  Turbulent:
  $v_x(r) \approx (1 - r/R)^{1/7} \cdot v_{max}$
  cross-sectional average velocity <v> = $0.875 \cdot v_{max}$



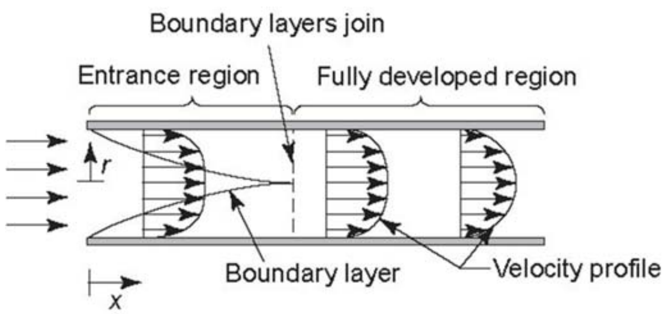
- The **cross-sectional average velocity <v>** is used in dimensional analysis or the resulting dimensionless groups (Re, and others)

$$\langle v \rangle = \frac{\int_0^R v_x(r) \cdot 2 \cdot \pi \cdot r \cdot dr}{\pi R^2} = \frac{\dot{V}(m^3/s)}{\pi R^2 (m^2)}$$

Picture: BMH99

Tube flow entrance region

- Flow entering a tube requires a certain distance to produce a "**developed flow"** with a constant boundary layer: **the entrance region**
- For the entrance region in **laminar tube flow**, the **Graetz number** quantifies for the boundary layer build-up *(see also section 5.2 - Convective heat transfer)*



Picture: KJ05

- The entrance length L_{ent} for a hydrodynamically developed tube flow (tube diameter D) is

$L_{ent} \approx 0.065 \cdot Re \cdot D$

for laminar flow Re < 2100

$L_{ent} \approx 4.4 \cdot Re^{1/6} \cdot D$

for turbulent flow Re > 4000

6.4 Fluid dynamics: pressure drop & energy dissipation in tube systems

Tube systems /1

- In a tube system, **pressure drop losses** <u>resulting from fluid internal friction and wall friction</u> in straight and curved tube sections, valves, inlet/outlet sections, diameter changes *etc. etc.* **must be compensated for by adding mechanical energy** via pumps, compressors, turbines, ventilators *(sv: pumpar, kompressorer, turbiner, fläktar)* etc.

- <u>Additional effects that must be compensated</u> for are **kinetic energy** (if flow velocities change) and **potential energy** (for non-horizontal tube sections)

Picture: http://www.raneng.com/Arco%20EH.htm

Tube systems /2

- For a flow tube system from point "1" at height z_1, average velocity $\langle v\rangle_1$, pressure p_1, volume flow $\dot{V}_1$, to point "2" at height z_2, velocity $\langle v\rangle_2$, pressure p_2, volume flow $\dot{V}_2$, pumping power *(sv: pumpeffekt)* P_{pump} compensates for flow friction losses P_{losses} :

Picture: http://www.pharmaceutical-technology.com/contractors/water_treatment/fischer/fischer2.html

General energy balance with heat input $\dot{Q}$, *work input* $\dot{W}$, *potential and kinetic energy and "flow work" :*

$$\dot{m}_1 \cdot (u_1 + gz_1 + ½\langle v\rangle_1^2) + p_1\dot{V}_1 + \dot{Q} + \dot{W} = \dot{m}_2 \cdot (u_2 + gz_2 + ½\langle v\rangle_2^2) + p_2\dot{V}_2$$

For isothermal flows, no heat effect ($\dot{Q} = 0$), *no work* ($\dot{W} = 0$) *:*

$$\dot{m}_1 \cdot (gz_1 + ½\langle v\rangle_1^2) + p_1\dot{V}_1 = \dot{m}_2 \cdot (gz_2 + ½\langle v\rangle_2^2) + p_2\dot{V}_2$$

With work input to compensate for flow friction losses P_{losses}

for example $\dot{W} = P_{pump} = P_{losses}$ ($= -\dot{Q}$, *but assuming* $\dot{Q} \approx 0$) *:*

$$\dot{m}_1 \cdot (gz_1 + ½\langle v\rangle_1^2) + p_1\dot{V}_1 + P_{pump} = \dot{m}_2 \cdot (gz_2 + ½\langle v\rangle_2^2) + p_2\dot{V}_2 + P_{losses}$$

Tube systems /3

- Flow through pipes and conduits *(sv: rör, ledning, kanal)* with height z_1, velocity v_1, pressure p_1, volume flow $\dot{V}_1$ → height z_2, velocity v_2, pressure p_2, volume flow $\dot{V}_2$

Picture: http://www.chemicals-technology.com/contractors/pipes/fischer/fischer2.html

$$\dot{m}_1 \cdot (u_1 + gz_1 + ½\langle v\rangle_1^2) + p_1\dot{V}_1 + P_{pump} = \dot{m}_2 \cdot (u_2 + gz_2 + ½\langle v\rangle_2^2) + p_2\dot{V}_2 + P_{losses}$$

Special case 1: for an isothermal inviscid fluid (negligible viscosity),

$\rightarrow \mathrm{P_{pump}} = \mathrm{P_{losses}} \approx 0;$ this gives Bernouilli's equation:

$$\dot{m}_1 \cdot (gz_1 + ½\langle v\rangle_1^2) + p_1\dot{V}_1 = \dot{m}_2 \cdot (gz_2 + ½\langle v\rangle_2^2) + p_2\dot{V}_2$$

Tube systems /4

- Flow through pipes and conduits *(sv: rör, ledning, kanal)* with height z_1, velocity v_1, pressure p_1, volume flow $\dot{V}_1$ → height z_2, velocity v_2, pressure p_2, volume flow $\dot{V}_2$

Picture: http://www.chemicals-technology.com/contractors/pipes/fischer/fischer2.html

$$\dot{m}_1 \cdot (u_1 + gz_1 + ½\langle v\rangle_1^2) + p_1\dot{V}_1 + P_{pump} = \dot{m}_2 \cdot (u_2 + gz_2 + ½\langle v\rangle_2^2) + p_2\dot{V}_2 + P_{losses}$$

Special case 2: correcting for velocity profiles in stream cross - section:

$$\dot{m}_1 \cdot (gz_1 + ½\xi_1\langle v\rangle_1^2) + p_1\dot{V}_1 + P_{pump} = \dot{m}_2 \cdot (gz_2 + ½\xi_2\langle v\rangle_2^2) + p_2\dot{V}_2 + P_{losses}$$

with kinetic energy correction factor ξ, for stream cross – sectional area A:

ξ

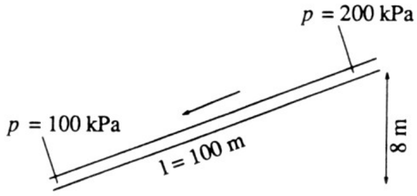
$$\xi = \frac{\dot{E}_{kinetic}}{½ \cdot \dot{m} \cdot \langle v\rangle^2} = \frac{\int_A ½ \cdot \dot{m} \cdot v^2 dA}{½ \cdot \rho \cdot A \cdot \langle v\rangle^3} = \frac{½ \cdot \rho \int_A v^3 dA}{½ \cdot \rho \cdot A \cdot \langle v\rangle^3} = \frac{\frac{1}{A}\int_A v^3 dA}{\langle v\rangle^3}$$

See also CEWR10 p. 222

$\xi \approx 2$ for laminar flows, and $\xi \approx 1.05 – 1.10$ for turbulent flows

Example: friction losses (ÖS96-4.6)

- 1 liter/s ethanol (density ρ = 791 kg/m^3) is pumped through a tube (diameter d = 25 mm) with a downwards slope. Pressure is measured at 2 points 100 m apart, as shown. Calculate the friction losses per meter tube, P_{losses} /l (W/m)



Picture: ÖS96

Tube systems /5

- For a tubing network *(sv: rörsystem)*, a design calculation can involve
  - **Calculation of power losses**, primarily pressure drop losses that must be compensated for with pumps *etc.* in a given process tubing situation
  - **Calculation of flow velocities** or volume streams that will result when applying a certain pumping power to a certain tube system flow situation
  - **Calculation of tube diameters**, lengths and tubing lay-out for a certain process situation, often based on given pumps or pressure drop data *etc.*

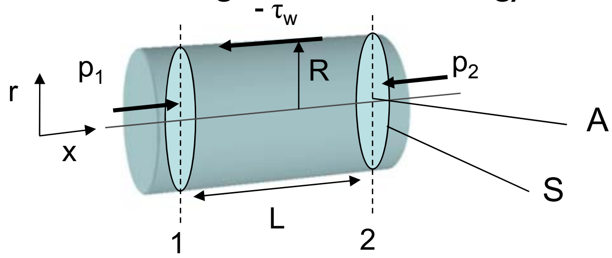
Picture http://www.pipecuff.com/

Sometimes iterative calculations are needed: $P_{pump} \rightarrow p_2$ and v_2 ; $\rightarrow$ adjust $p_2 \rightarrow$ new value for P_{pump} etc.

(see also ÖS96 p. 41)

Pressure drop /1

- The pressure drop in a tube flow system can be predicted ***if the shear force at the wall τ_w is known***
- For example for **laminar** tube flow (tube diameter d = 2R, flow direction "x"), (-dp/dx) = $-2\cdot\tau_w/R$ where $\tau_w = \tau_{fluid \rightarrow wall}$ can be related to dv_x/dr, but for **turbulent** flow such **information** is **not available**
- Force analysis shows 3 forces acting on a flow volume element: surface forces (pressure and surface shear), and body force (gravity). These can **change the kinetic energy** $E_k = ½mv^2$ and **potential energy** E_p = mgz. For a horizontal tube the body forces cannot change, but surface forces will change the kinetic energy.



Volume element with length L (m), cross-section A (m²), circumference S (m), density ρ (kg/m³)

Pressure drop /2 friction factor

- The surface shear force acting on the surface of a moving fluid element can be expressed as

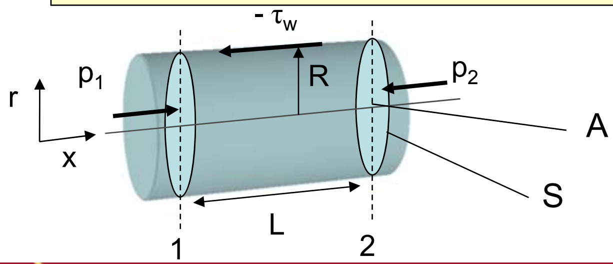
$$\tau_w = \textbf{friction factor} \cdot (E_{kinetic}/\text{volume}) = f \cdot ½\rho <v>^2$$

= *dynamic pressure or "thrust" (sv: stöt)*

- For flow in a horizontal tube with radius R the force balance at the wall for length section L gives

$p_1\cdot A - p_2\cdot A - \tau_w\cdot S\cdot L = 0$, with $\tau_w = \tau_{fluid \rightarrow wall} = -\tau_{wall\rightarrow fluid}$

$$\rightarrow \boxed{(p_1 - p_2) = \tau_w \cdot L\cdot S/A = f\cdot ½\cdot\rho\cdot<v>^2\cdot L\cdot S/A = -\Delta p}$$



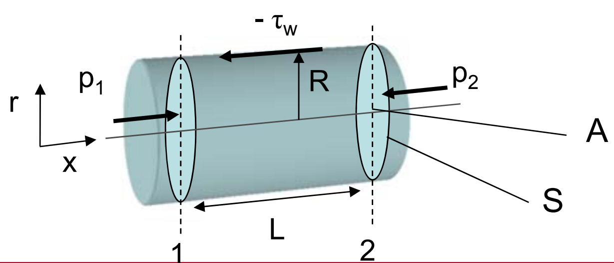
with for a round tube cross-section $A = \pi R^2$, circumference $S = 2\pi R$

Pressure drop /3 friction factor

Be careful with literature data !

- This defines the **Fanning friction factor *f*;**
  also used is **Darcy or Blasius friction factor ζ = 4*f***

- The group ½·ρ·<v>² (unit: N/m²) follows also from dimensional analysis, reasoning that $\tau_w = \tau_w(\rho, \eta, \langle v_x \rangle, \text{geometry})$, which for a tube with diameter D gives $\tau_w = \tau_w(\rho, \eta, \langle v_x \rangle, D)$.
- It is found that
  $\tau_w / (\rho \cdot \langle v \rangle^2) = f(\mathrm{Re})$,
  which is usually written
  as $\boxed{\boldsymbol{\tau_w = \tfrac{1}{2} \cdot f \cdot \rho \cdot \langle v \rangle^2}}$



Hydraulic diameter

- The ratio **A/S** (unit: m) is a characteristic dimension of the tube, pipe, duct or channel known as hydraulic radius, while **4·A/S** is known as **hydraulic diameter** D_h *(see Figure below)* *with* ***A* = *cross-sectional area*** *(sv: tvärsnitt);* ***S* = *perimeter*** *(sv: omkrets)* ***touched by fluid***

- <u>For example</u> for a round tube with diameter D, completely filled with fluid: D_h = D;
  for a square channel with width W, fluid height H:
  $D_h = 4 \cdot A/S =$
  $4 \cdot (H \cdot W)/(2H+W)$

Flow situation	Hydraulic diameter $D_h = 4A/S$	A
Circular pipe	D	$\frac{\pi}{4} D^2$
Concentric pipe or slit	$D_2 - D_1 = 2\delta$	$\frac{\pi}{4}(D_2^2 - D_1^2)$
Rectangular pipe	$\frac{2WB}{W+B}$	WB
Open channel	$\frac{4WH}{W+2H}$	WH
Open channel	$\frac{2H}{\sqrt{2}}$	H^2
Half-filled	D	$\frac{\pi}{8} D^2$
Liquid film in a tube	4δ	$\delta \pi D$

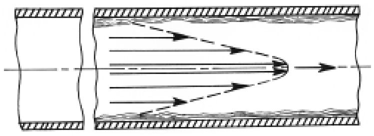
Picture: BMH99

Pressure drop /4 laminar tube flow

- Thus for the pressure drop for flow in a tube or duct with hydraulic diameter $D_h = 4 \cdot A/S$:

  $$(p_1 - p_2) = -\Delta p = \tau_w \cdot L \cdot (4/D_h) = 4f \cdot ½ \cdot \rho \cdot \langle v \rangle^2 \cdot L/D_h$$

- For a **laminar flow in a round tube** (Hagen - Poisseuille flow, with D_h = diameter $D = 2R$):



  $- \tau_w = \tau_{wall \to fluid} = ½R \cdot (-\Delta p/L)$

  $\rightarrow - \tau_w = 4\eta \langle v \rangle /R = 8\eta \langle v \rangle /D = f \cdot ½ \cdot \rho \cdot \langle v \rangle^2$

  $\rightarrow$ $\boxed{f = 16\eta/(\rho \langle v \rangle d) = 16 / Re \ ; \quad 4f = \zeta = 64 / Re}$

  with $Re < 2100$

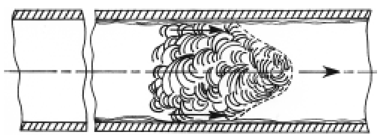
- ***For non-circular ducts another proportionality constant is needed !***

Picture: http://www.mbamanufacturing.com/Publication/LamTurbFlow.htm

Pressure drop /5 turbulent tube flow

- Pressure drop for flow in a tube or duct with hydraulic diameter $D_h = 4 \cdot A/S$: $(p_1 - p_2) = -\Delta p = \tau_w \cdot L \cdot (4/D_h) = 4f \cdot ½ \cdot \rho \cdot \langle v \rangle^2 \cdot L/D_h$

- For a **turbulent flow in a tube of duct** it is found that $f \sim Re^{-0.25 \ldots 0}$ (less direct influence of viscosity than in laminar flow) and $\Delta p \sim v^{1.75..2}$



- **For smooth pipes**

  $\boxed{f = 0.0791 \cdot Re^{-0.25} \ ; \quad 4f = \zeta = 0.316 \cdot Re^{-0.25}}$

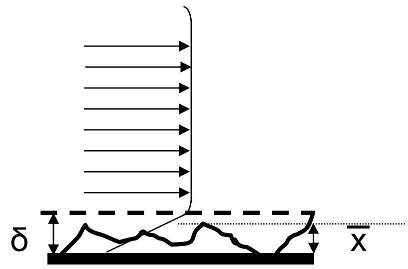
  (Blasius' equation) with $4000 < Re < 10^5$

  can be used for any cross-sectional shape using characteristic diameter = hydraulic diameter D_h

Picture: http://www.mbamanufacturing.com/Publication/LamTurbFlow.htm

Pressure drop /6 wall roughness

- For rough pipes, **wall surface roughness** *(sv: väggskrovlighet)* $\overline{x}$ is important if it is of the same order as the thickness of the laminar boundary layer, δ;
- Important at great wall roughness or high Re numbers.
- Roughness data is found in tables
- Important is the **relative roughness** $\overline{x}$/D, with tube diameter D
- Not important for laminar flows
- The friction factor f or ζ can be read from a **friction factor chart** or **Moody chart** as function of Re and relative wall roughness



APPROXIMATION for MOODY CHART

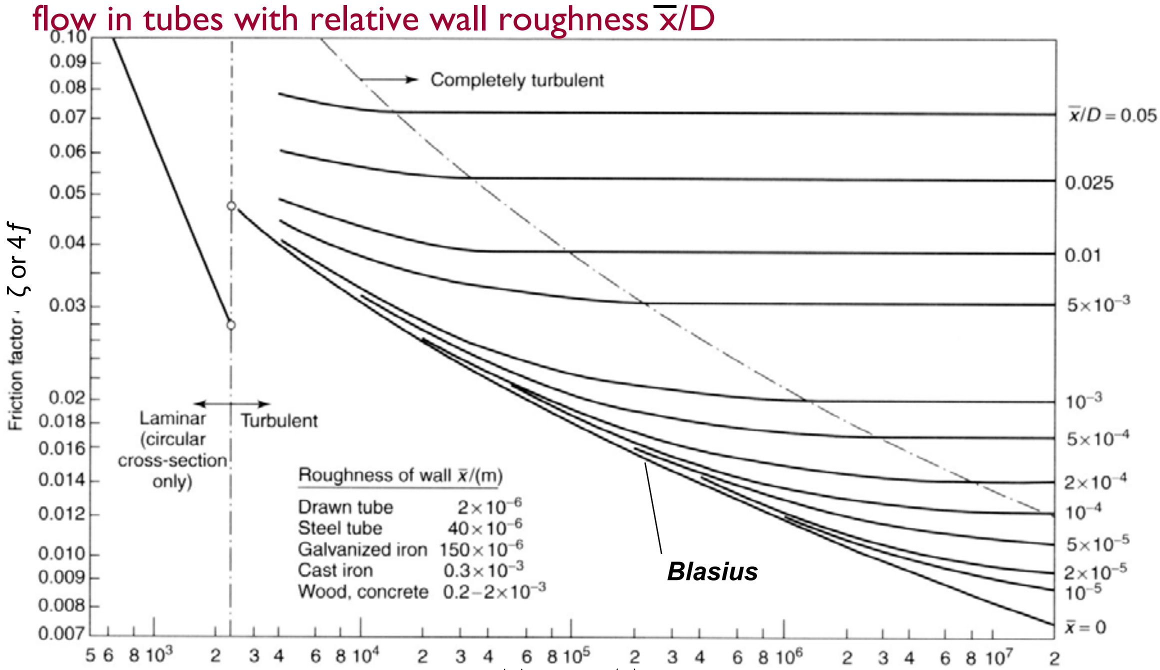
$$4f = \zeta = \frac{0.25}{\left({}^{10}\log\left(\frac{\overline{x}}{3.7D} + \frac{5.74}{Re_D^{0.9}} \right) \right)^2}$$

$$5000 \le Re_D \le 10^8 \quad \text{and } 10^{-6} \le \frac{\overline{x}}{D} \le 10^{-2}$$

Tube flow friction factor

MOODY CHART

flow in tubes with relative wall roughness $\overline{x}$/D

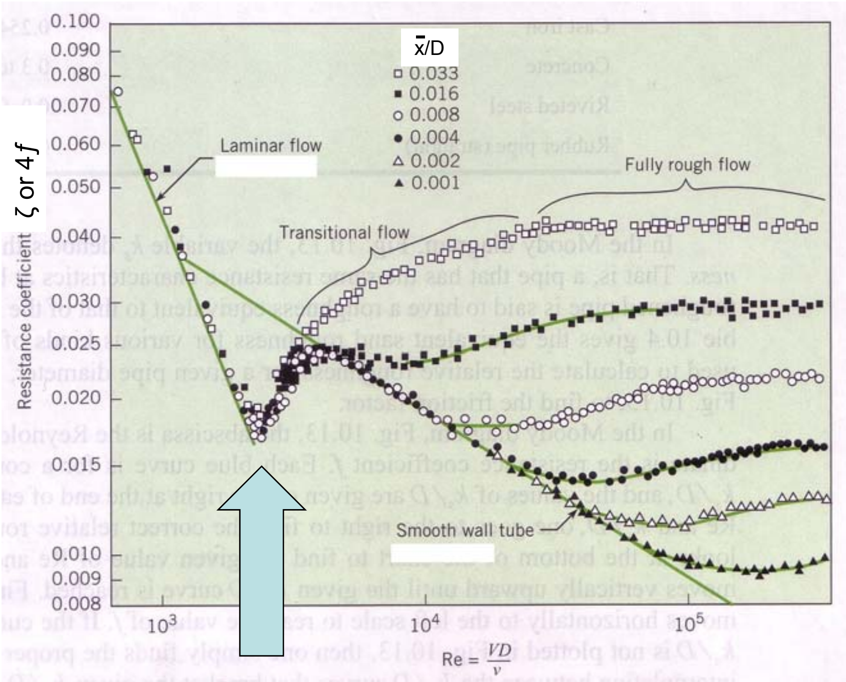


$$Re = \frac{\rho\langle v\rangle}{\eta}\frac{4A}{S} = \frac{\rho\langle v\rangle D_{hydraulic}}{\eta}$$

Picture: BMH99

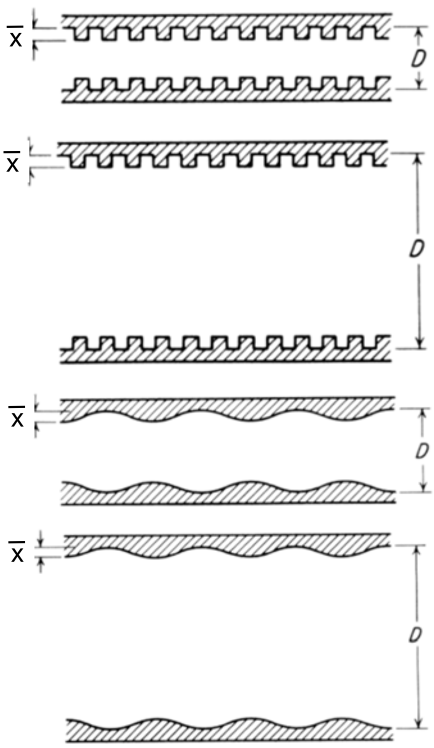
Tube flow friction factor

flow in tubes with relative wall roughness $\bar{x}$/D - ***the transition region***



Picture: CEWR10

Wall roughness data

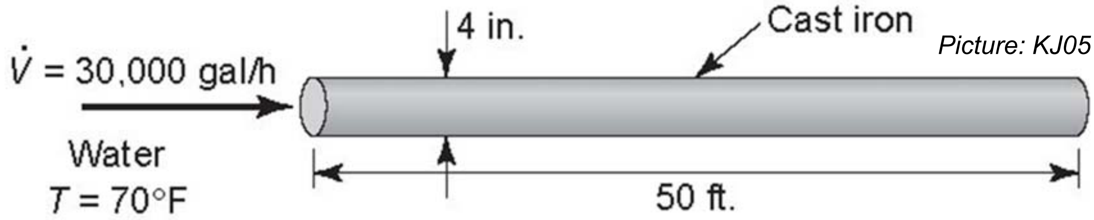


Material	Condition	Roughness Height, $\bar{x}$ (mm)	Uncertainty (%)
Steel	Sheet metal, new	0.05	±60
	Stainless, new	0.002	±50
	Commercial, new	0.046	±30
	Riveted	3.0	±70
	Rusted	2.0	±50
Iron	Cast, new	0.26	±50
	Wrought, new	0.046	±20
	Galvanized, new	0.15	±40
	Asphalted cast	0.12	±50
Brass	Drawn, new	0.002	±50
Plastic	Drawn tubing	0.0015	±60
Glass	—	Smooth	
Concrete	Smoothed	0.04	±60
	Rough	2.0	±50
Rubber	Smoothed	0.01	±60
Wood	Stave	0.5	±40

← Relative wall roughness, small or large diameter tubes

Table: T06
Pictures: MSH93

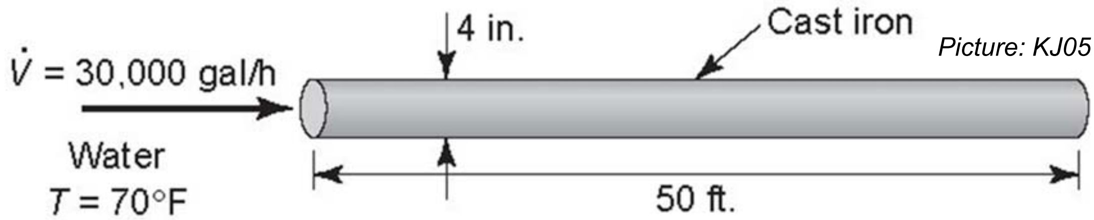
Example: pipe flow friction /1



Picture: KJ05

- A horizontal cast-iron pipe with diameter 4" carries 30000 (US) gal/h water at 70°F. Pipe length is 50 ft. <u>Calculate</u> the pressure drop. The water's density is 62.2 lbm/ft^3; dynamic viscosity is $65.8 \cdot 10^{-5}$ lbm/(ft · s)

Example: pipe flow friction /2



Picture: KJ05

Pressure drop /7 Fittings and valves

- Pressure drop across a tube section can be expressed as
  $-\Delta p = 4f \cdot ½ \cdot \rho \cdot \langle v \rangle^2 \cdot L/D_h = \zeta \cdot ½ \cdot \rho \cdot \langle v \rangle^2 \cdot L/D_h$
- Similarly, for the sudden local pressure drop caused over a very short distance by, for example
  - A change in tube diameter, or a bend or curve, or a T-junction
  - A valve *(sv: ventil, klaff)* or other fitting *(sv: rörelement)*
  - An inlet or outlet (sharp or smooth)

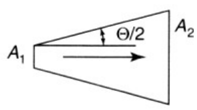
For these, pressure drop can be expressed as

$$\boxed{\begin{array}{l} -\Delta p = K_w \cdot ½ \cdot \rho \cdot \langle v \rangle^2 \\ -\Delta p = \zeta' \cdot ½ \cdot \rho \cdot \langle v \rangle^2 \end{array}}$$ or

with coefficients K_w or ζ' independent of flow Reynolds number for $Re > 10^5$

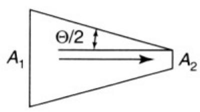
Picture: http://www.chicagobrassworks.com/gs.htm

Friction loss factors K_w (or ζ') for flow tube elements / 1 of 4

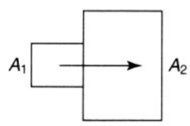


$K_w = k\left(1 - \frac{A_1}{A_2}\right)^2$

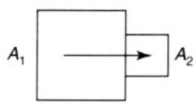
Θ =	< 10°	10°	20°	30°	40°	50°	60°	70°	80°	90°
k =	0	0.17	0.41	0.71	0.90	1.03	1.12	1.13	1.10	1.05



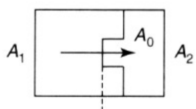
Θ =	10°	20°	30°	40°	50°	60°	70°	80°
K_w =	0.16	0.20	0.24	0.28	0.31	0.32	0.34	0.35



$K_w = \left(1 - \frac{A_1}{A_2}\right)^2$ *(corrected!)*



$K_w = 0.45\left(1 - \frac{A_2}{A_1}\right)$

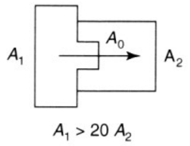


$\frac{A_0}{A_1}$ =	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
K_w =	226	47.8	17.5	7.8	3.75	1.80	0.80	0.30	0.06

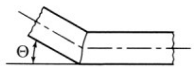
Downstream velocity, $Re > 10^5$

Picture: BMH99

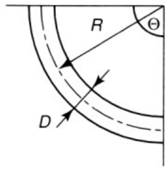
Friction loss factors K_w (or ζ′) for flow tube elements / 2 of 4



A_0/A_2 =	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
K_w =	232	51	20	9.6	5.3	3.1	1.9	1.2	0.73	0.48

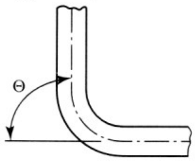


Θ =	20°	40°	60°	80°	90°	100°	120°	140
K_w =	0.05	0.14	0.36	0.74	0.98	1.26	1.86	2.43



$$K_W = \left(0.131 + 0.163\left(\frac{D}{R}\right)^{3.5}\right)\frac{\Theta}{90^\circ}$$

K_w (referring to downstream velocity for Re > 10^5)

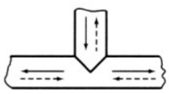


Θ =	15°	30°	45°	60°	90°
K_w =	0.02	0.11	0.26	0.50	1.20

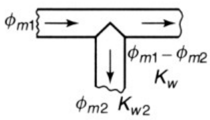
Downstream velocity, Re > 10^5

Picture: BMH99

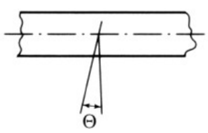
Friction loss factors K_w (or ζ′) for flow tube elements / 3 of 4



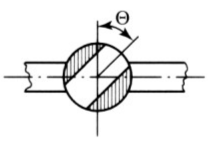
—— $K_w \simeq 2$ – – – $K_w \sim 0.75$



ϕ_{m2}/ϕ_{m1} =	0	0.2	0.4	0.6	0.8	1.0
K_w =	0.95	0.88	0.89	0.95	1.10	1.28
K_{w2} =	–	−0.08	−0.05	0.07	0.21	0.35



Θ =	20°	30°	40°	50°	60°	70°	80°	85°
K_w =	751	118	33	11	4	1.5	0.52	0.24



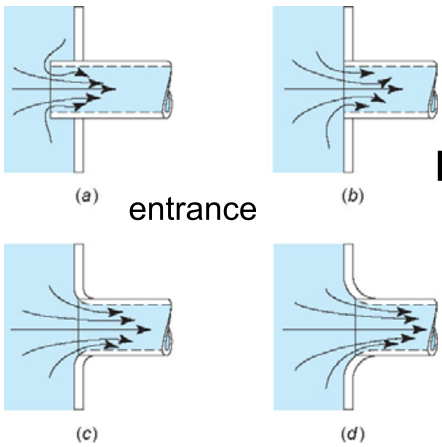
Θ =	25°	30°	40°	50°	60°	70°	80°	85°
K_w =	486	206	53	17	5.5	1.6	0.29	0.05

Gate valve	Fraction closed	0	1/8	2/8	3/8	4/8	5/8	6/8	7/8
	K_w =	0.05	0.07	0.26	0.81	2.1	5.5	17	98

Downstream velocity, Re > 10^5

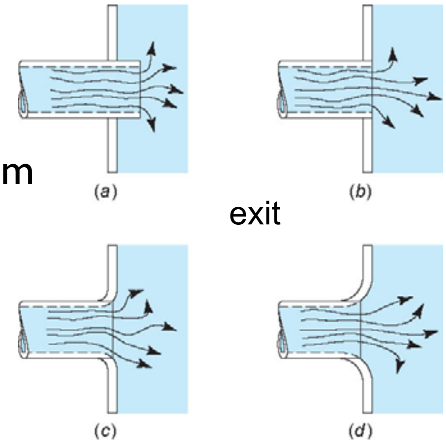
Picture: BMH99

Friction loss factors K_w (or ζ') for flow tube elements / 4 of 4



entrance

Downstream velocity, Re > 10^5



exit

Entrance / exit flow conditions & loss coefficient:

(a) reentrant,	entrance K_w = 0.8,	exit K_w = 1.0
(b) sharp-edged,	entrance K_w = 0.5,	exit K_w = 1.0
(c) slightly rounded,	entrance K_w = 0.2,	exit K_w = 1.0
(d) well-rounded,	entrance K_w = 0.04,	exit K_w = 1.0

Pictures: KJ05

Tube elements: example

- Friction coefficients K_w or ζ' for several tube sections and fitting elements:

a) Bend 90°, R/d =1 ζ' = 0.5

b) Sharp bend 90° ζ' = 0.98 or elbow ζ' = 1.2

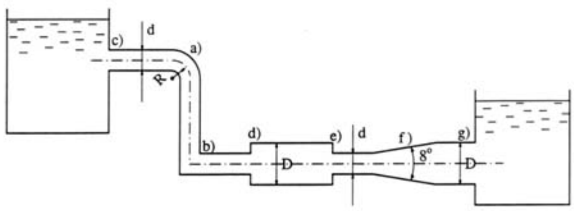
c) Tube inlet, sharp ζ' = 0.5 or smooth ζ' = 0.20

d) Diameter increase, sharp $\zeta' = (1-d^2/D^2)^2$

e) Diameter decrease, sharp $\zeta' = 0.45\cdot(1-d^2/D^2)$

f) Diameter increase, diffusor with $\theta/2 < 10°$ $\zeta' \approx 0$

g) Tube outlet, turbulent ζ' = 1 or laminar ζ' = 2



For this set-up if for example D = 80 mm, d = 50 mm, for turbulent flow: $\sum\zeta'$ = 0.50 + 0.50 + 0.98 + 0.37 + 0.27 + 0 + 1.1 = 3.72 for the fittings, bends and diameter changes only.

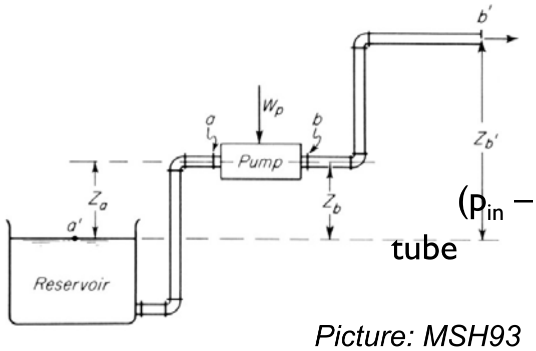
Picture: ÖS96

Pressure drop, pressure loss, power loss, energy dissipation /1

- For fluid flow with viscous friction through a channel the power loss (energy dissipation) P_{loss} *(sv: effekförlust)* can be related to pressure loss $-\Delta p_{loss}$ for a given volume stream $\dot{V}$:

$$\boxed{-\Delta p_{loss} = \frac{P_{losses}}{\dot{V}}}$$

(unit: Pa) which is equal to (p_{in} – p_{out}) **only** for a horizontal tube without diameter changes.



Picture: MSH93

- For the energy equation for a tube system (with $\dot{Q}$ = 0), dividing by $\dot{V}$ (noting that $\dot{m} = \rho \cdot \dot{V}$ requires ρ = constant) this gives

$$\boxed{\rho g(z_1 - z_2) + ½(\xi_1 \rho \langle v \rangle_1^2 - \xi_2 \rho \langle v \rangle_2^2) + (p_1 - p_2) + (-\Delta p)_{pump} = (-\Delta p)_{losses}}$$

Pressure drop, pressure loss, power loss, energy dissipation /2

- If density changes are significant (typical for gases) then $\dot{V}_1 \neq \dot{V}_2$ and that must be accounted for:

$$\boxed{-\Delta p_{loss} = \frac{P_{losses}}{\dot{V}} = \frac{P_{losses}}{\dot{m}} \int_1^2 d\rho = \int_1^2 -dp_{loss}}$$

and

$$\boxed{g(z_1 - z_2) + ½(\xi_1 \langle v \rangle_1^2 - \xi_2 \langle v \rangle_2^2) + \int_1^2 \frac{-dp}{\rho} + (-\Delta p)_{pump} = (-\Delta p)_{losses}}$$

- With pressure drop Δp ~ shear force it follows that **Δp ~ velocity for laminar flow**, and **Δp ~ velocity$^{1.75....2}$ for turbulent flow**. Note: for laminar: Δp ~ v with 4*f* ~ 1/Re ~1/v
- With viscous work ~ shear force × velocity, $P_{loss} \sim \Delta p \cdot \dot{V} \sim$ velocity·Δp this gives **P_{loss} ~ velocity² for laminar flow**, and **P_{loss} ~ velocity$^{2.75....3}$ for turbulent flow**.

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Pressure drop, pressure loss, power loss, energy dissipation /3

- For the power loss (energy dissipation) for a flow channel with **total** pressure losses Δp_{loss}, composed of
  - Δp_{loss} (ζ, L, D) for the straigth sections and
  - Δp_{loss} (ζ') for the fittings, valves, diameter changes, in-/outlet, ... :

$$4f = \zeta = \frac{-\Delta p_{losses}}{½\rho\langle v\rangle^2}\cdot\frac{D_h}{L} = \frac{P_{losses}}{½\rho\langle v\rangle^2\cdot\dot{V}}\cdot\frac{D_h}{L} = \frac{P_{losses}}{½\dot{m}\langle v\rangle^2}\cdot\frac{D_h}{L} \quad \textit{for tube sections}$$

$$K_w = \zeta' = \frac{-\Delta p_{losses}}{½\rho\langle v\rangle^2} = \frac{P_{losses}}{½\rho\langle v\rangle^2\cdot\dot{V}} \quad \textit{for valves, fittings, diameter changes, ...}$$

which gives for the total tubing system including fittings *etc:*

$$-\Delta p_{loss} = ½\rho\langle v\rangle^2\cdot\left(\zeta\cdot\frac{L}{D_h}+\sum\zeta'\right) \quad \textit{and} \quad P_{losses} = ½\rho\dot{V}\langle v\rangle^2\cdot\left(\zeta\cdot\frac{L}{D_h}+\sum\zeta'\right)$$

Note: kinetic energy correction factor ξ is now included in ζ or $4f$!!!!

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Calculation of volume flow or tube diameter

- Calculation of pressure drop -Δp or power loss P_{loss} from flow channel diameters and friction factors is relatively straight-forward; <u>more complicated,</u> however, is to determine volume stream $\dot{V}$ or channel diameter D_h based on $-\Delta p$ or P_{loss}

- An **iterative procedure** can be used, using $\dot{V} = A\cdot\langle v\rangle$ for flow cross-section A and the expressions given above; for tube system based on a round tube with $A = ¼\pi D^2$ this gives

$$\dot{V} = \sqrt{\frac{\pi^2(-\Delta p)_{loss}D^4}{8\rho\left(\zeta\frac{L}{D}+\sum\zeta'\right)}} \quad \textit{and} \quad D = \sqrt[4]{\frac{8\rho\dot{V}^2\left(\zeta\frac{L}{D}+\sum\zeta'\right)}{\pi^2(-\Delta p)_{loss}}}$$

where ζ (or $4f$) and ζ' (or K_w) are functions of <v>, D and/or Re !

(see also ÖS96 p. 48)

Example: old exam question /question

- Calculate what the inner diameter d (in m) of a well heat-insulated steel tube should be for transporting $\dot{m}$ = 3,2 kg/s steam with temperature 180°C and pressure 300 kPa (density ρ = 1,464 kg/m^3, dynamic viscosity η = $15{,}1\times10^{-6}$ Pa · s), if the pressure drop in straight tube sections may not be more than 250 Pa per meter. Wall roughness is $k = \bar{x}$ = 0,4 mm.
- Note that for round tubes: $$\mathrm{Re} = \frac{4\cdot\dot{m}}{\pi\cdot\eta\cdot d}$$
- Advice: develop an expression d = f(<v>, ζ, ...) and iterate a few times to find a result for d (m).

Example: old exam question /answer

Calculation of volume flow or tube diameter

- Two expressions for this are given in CEWR10, p. 332

$$\dot{V} = -2.22 \cdot D^{5/2} \cdot \frac{(-\Delta p)_{loss}}{\rho \cdot L} \cdot {}^{10}\log\left(\frac{\bar{x}}{3.7 \cdot D} + \frac{1.78 \cdot \eta}{D^{3/2} \cdot \rho \cdot \sqrt{\frac{(-\Delta p)_{loss}}{\rho \cdot L}}}\right)$$

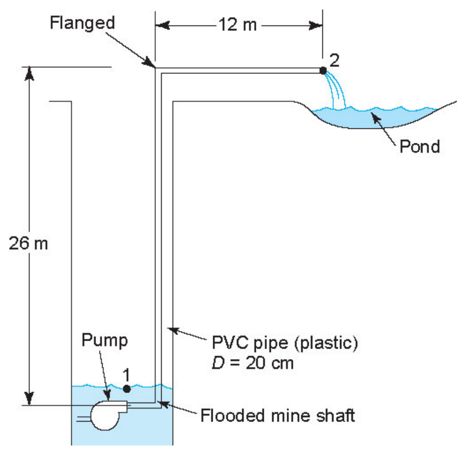
$$D = 0.66 \cdot \left((\bar{x})^{1.25} \cdot \left(\frac{L \cdot \dot{V}^2 \cdot \rho}{(-\Delta p)_{loss}}\right)^{4.75} + \frac{\eta}{\rho} \cdot \dot{V}^{9.4} \cdot \left(\frac{L \cdot \rho}{(-\Delta p)_{loss}}\right)^{5.2} \right)^{0.04}$$

$$\textit{for } \mathrm{Re} > 3000, \quad \frac{\bar{x}}{D} < 0.02$$

which should be used with caution.

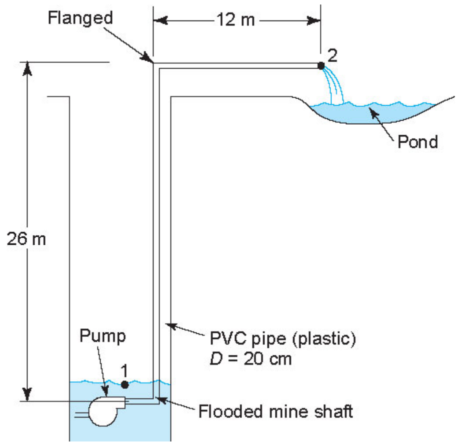
Example: water pumping system /1

- A pump is used to remove water from a mine shaft – see Figure. How much pump power P_{pump} (in kW) is needed to remove water at a rate of 65.0 kg/s?
  Assume an ideal pump (efficiency 100%).
  Assume density ρ = 997 kg/m³, viscosity η=1.12·10⁻³ Pa·s



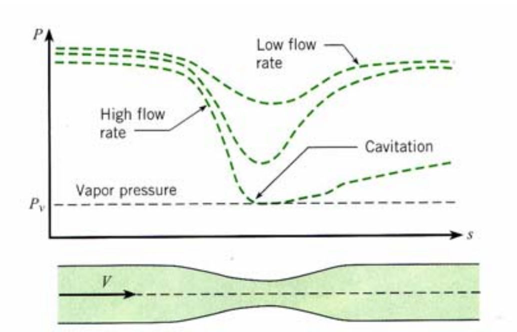
Picture: KJ05

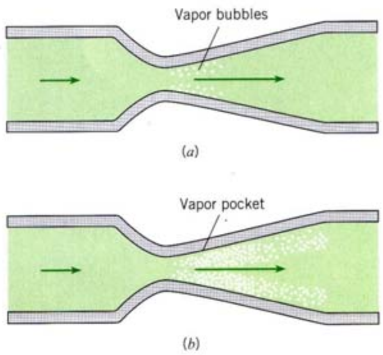
Example: water pumping system /2



Picture: KJ05

Cavitation





Pictures: CEWR10

- Cavitation occurs if fluid pressure is reduced to the vapour pressure (at the given temperature) so that **boiling** occurs.
- The formation and collapse of bubbles gives shock waves, noise, and other problematic dynamic effects that can result in reduced performance, failure and damage.
- Typically occurs at high velocity locations in, for example, pumps or valves, but can damage also tube walls.

6.5 Flow systems with negligible losses, flow measurement

Flow systems with negligible losses /1

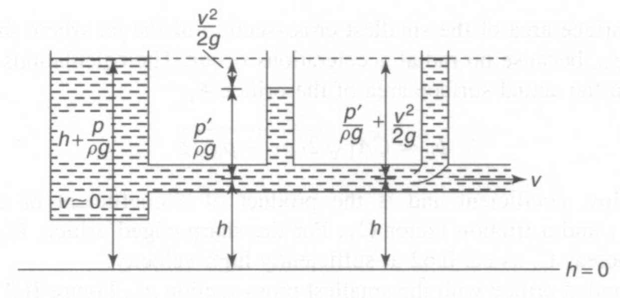
- Often the energy dissipation P_{loss} can be neglected in comparison with the (mechanical) energy changes in a flow system.
- If the fluid density can be considered constant this gives the **Bernouilli's equation**, which can be written as

$$\boxed{\frac{p}{\rho g} + z + \frac{½v^2}{g} = constant}$$

where the three terms

(unit: m) are referrred to as

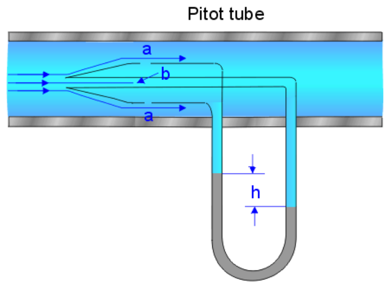
- **pressure head,**
- **static head** and
- **velocity head**



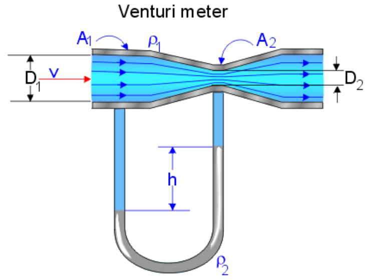
Picture: BMH99

Flow systems with negligible losses /2

- This is used when measuring fluid velocities with a so-called **Pitot tube**: in the Figure →
  $p_{@b} - p_{@a} = ½\rho\langle v\rangle^2 = \rho g h$



- In a **venturi flowmeter**, the pressure difference between main flow and the throat as shown in Figure → equals
  $p_{@A1} - p_{@A2} = ½\rho\langle v\rangle^2_{@A2} - ½\rho\langle v\rangle^2_{@A1}$
  (which gives $p_{@1} > p_{@2}$!)
  with $\langle v\rangle_1 \cdot A_1 = \langle v\rangle_2 \cdot A_2$ and
  $p_{@A1} - p_{@A2} = \rho h g$ the flow $\dot{V}$ at A_2 can be calculated for a liquid:



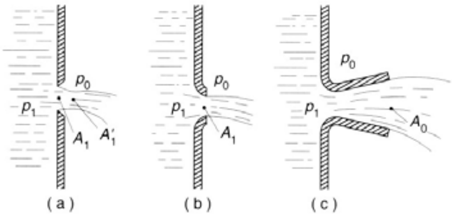
$$\dot{V} = A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho}} \Big/ \sqrt{1 - \frac{A_2^2}{A_1^2}}$$

For a gas: (ideal, adiabatic process): use $p \cdot \rho^{-\gamma}$ = constant, $\gamma = c_p/c_v$

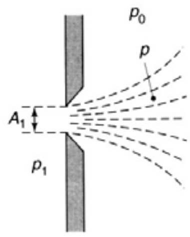
Pictures: http://www1.uts.com/Physics/flowmetering/flowmeter.htm

Flow systems with negligible losses /3

- For flow of **liquid** from an **orifice** *(sv: mynning, öppning)* friction losses can be neglected
- At some distance from the opening, (at cross-sectional area A_1), the velocity is much smaller than the velocity $\langle v\rangle$ in the opening (area A_1'):



$p_0 + ½\rho\langle v\rangle^2 \approx p_1$ this gives
$\langle v\rangle \approx \sqrt{(2(p_1 - p_0)/\rho)}$
$\dot{V} = A_1'\langle v\rangle = C_f A_1 \sqrt{(2(p_1 - p_0)/\rho)}$
with friction factor C_f



$C_f \approx 1$ for a sharp edge (a),
$C_f \approx 0.95$-0.99 for a rounded outlet (b).
For a diffusor (c) with angle < 8°,
$\dot{V} = C_f A_0 \sqrt{(2(p_1 - p_0)/\rho)}$ with $C_f \approx 1$

For a gas :
(ideal, adiabatic process):
$p_0 < p$ in jet $< p_1$
use $p \cdot \rho^{-\gamma}$ = constant, $\gamma = c_p/c_v$

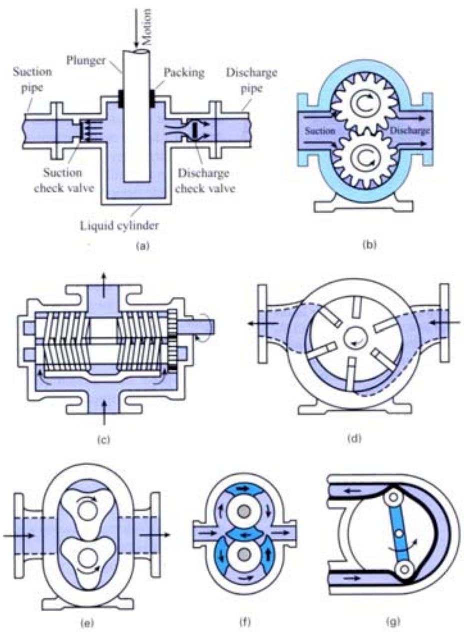
Pictures: BMH99

6.6 Pumps, compressors, fans

Pumps, compressors, fans /1

- **Creating a flow and/or increasing the pressure** of a fluid, or **compensating for pressure losses** is accomplished with **pumps** *(sv: pumpar)* for liquids, or with **compressors** or **fans** *(sv: kompressorer, fläktar)* for gases

- Usually a fan creates flow with minimal pressure change; if a fan creates a higher outlet pressure then it is generally referred to as a **blower** *(sv: bläster)*



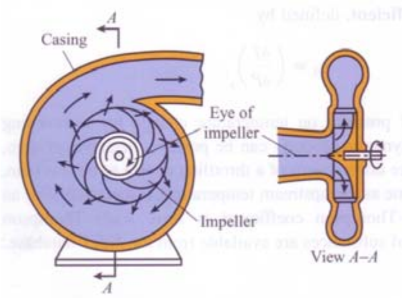
Positive-displacement pumps

Picture: T06

Pumps, compressors, fans /2

- Pumps, compressors and fans can be divided into two major categories:
  - **Positive displacement devices** based on ”pushing” the fluid through the device *(see previous slide)*
  - **Dynamic devices** based on transfer of energy as momentum *(sv: rörelsemängd)* from rotary blades or vanes, or from a high-speed fluid stream *(<u>for example</u>, centrifugal pumps and rotodynamic compressors and fans)*

Centrifugal pump



pictures: TO6

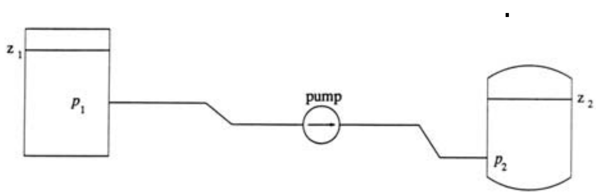
Pumps /1

- The general relation between pump (or compressor) power and the pressure difference Δp_{pump} *(sv: uppfordringstryck)* **for a given flow tubing system situation** follows from the mechanical energy balance ($\dot{Q}$ = 0, no heat transfer or significant temperature changes), assuming also that $\Delta\dot{E}_{kinetic}$ = 0:

$$-\Delta p_{pump} = (p_2 - p_1) + \rho g (z_2 - z_1) + ½\rho\langle v\rangle^2 \cdot \left(\zeta \cdot \frac{L}{D_h} + \sum \zeta'\right) \quad \textit{with} \quad \langle v\rangle^2 = \frac{\dot{V}^2}{A^2}$$

$$-\Delta p_{pump} = \frac{P_{pump}}{\dot{V}} = \frac{\dot{H}_2 - \dot{H}_1}{\dot{V}} = \rho(h_2 - h_1) = \rho g \cdot \Delta z_{pump}, \quad \textit{with "pump head"}\ \Delta z_{pump}$$

- The **pump head** (unit: m) is the pressure rise across the pump equivalent height fluid

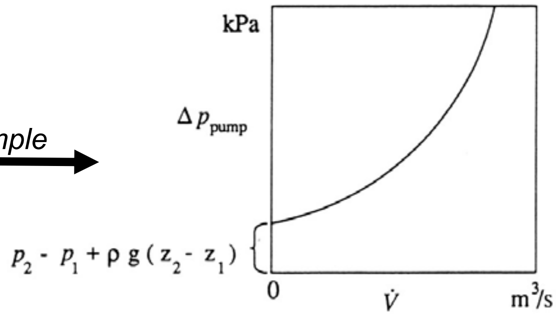


Picture: ÖS96

Pumps /2

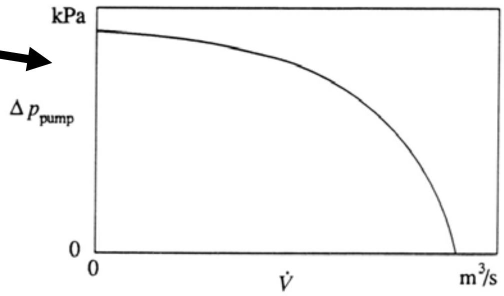
- The relation between $-\Delta p_{pump}$ and $\dot{V}$ is a **characteristic for the flow tubing system** *(sv: rörledningskarakteristika)*

for example



- For the pump itself, the **pump characteristic** *(sv: pumpkarakteristika)* gives the performance $-\Delta p_{pump}$ *versus* $\dot{V}$

for example

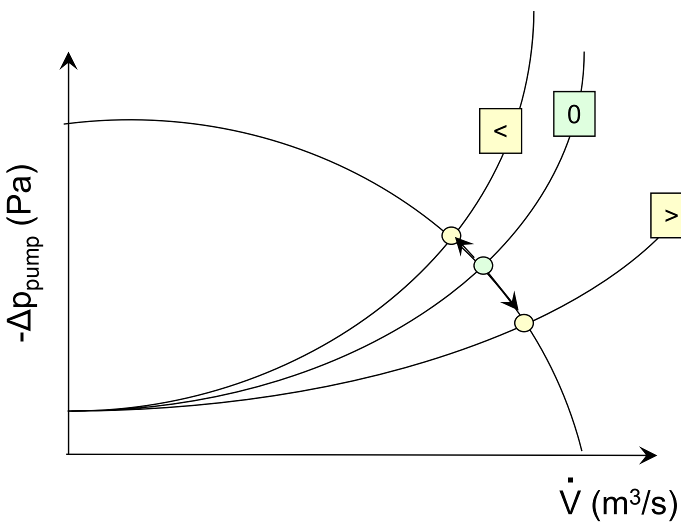


- Combining the two lines in one diagram gives the **working point** *(sv: arbetspunkt)* at the point where the lines cross

Pictures: ÖS96

Pumps /3

- Changing the flow resistance in the tube network will give another system characteristic line
- The new working point will give another fluid flow throughput



For example, closing or opening a valve

Pumps /4

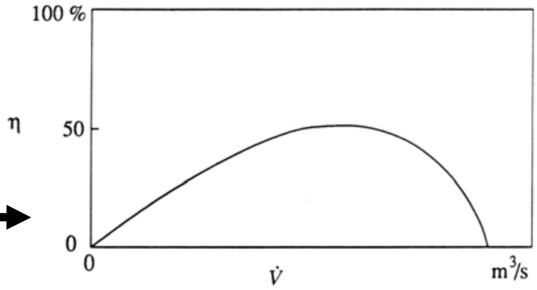
- The pump itself generates a viscous friction effect in the fluid, and as a result not all pump power P_{pump} will be available to give a pressure increase $-\Delta p_{pump}$ in flow $\dot{V}$. The **pump efficiency** *(sv: pumpverkningsgrad)* η_{pump} quantifies for this:

$$\boxed{\eta_{pump} = \frac{P_{pump}}{\text{Power input}} = \frac{-\Delta p_{pump} \cdot \dot{V}}{\dot{W}_{in}}}$$

Picture: ÖS96

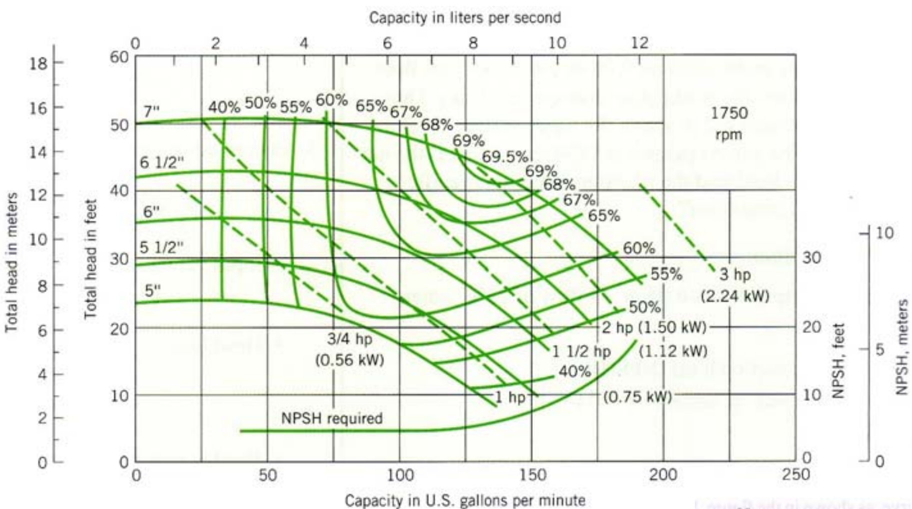
- For a given pump the efficiency depends on the fluid that is pumped and the volume stream $\dot{V}$

for example →

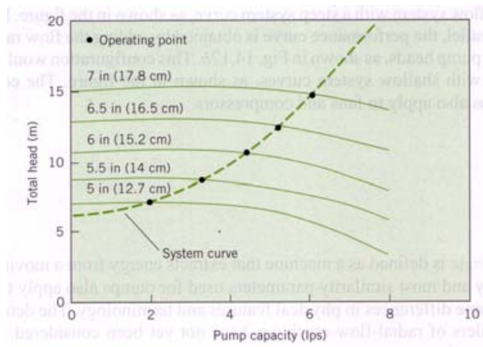


Pump characteristics & performance

- examples

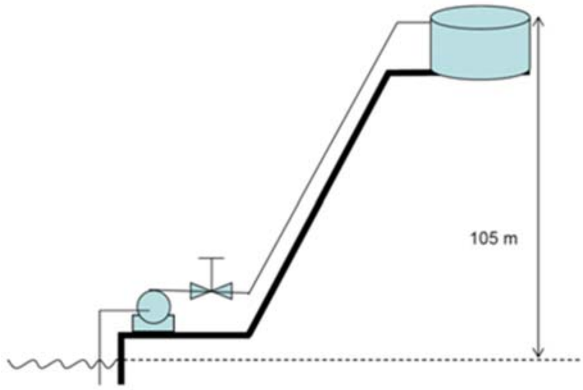


Picture: CEWR10



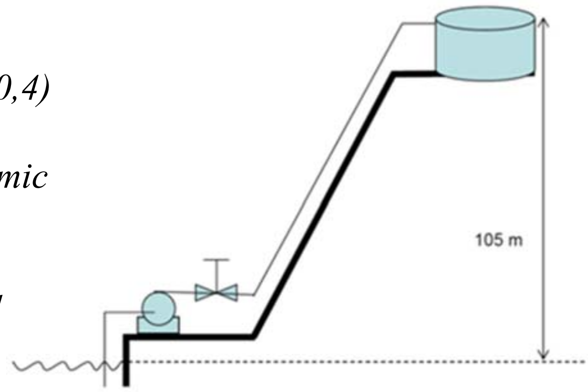
Old exam question /q 1 of 2

- *Cooling water must be pumped from a reservoir up to a process through a tube system with a few bends and a valve as shown in the figure. At both liquid surfaces the pressure equals ambient atmospheric pressure. The cooling water (20°C) flow is 135 m^3/h. The height difference between reservoir and process is 105 m and the total tube length is 166 m, of which 16 m is upstream ("before") of the pump.*
- *a) What tube diameter must be chosen so that the flow velocity does not exceed 2 m/s, and what is the Reynolds number of the flow then?*
- *..... continues*



Old exam question /q 2 of 2

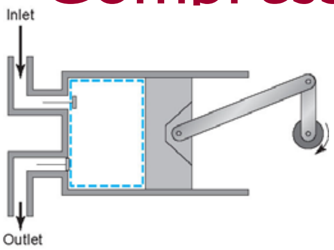
- *b) What pressure head should the pump be able to produce so that the flow objective is achieved?*
- *c) Calculate the pump power that is needed for a pump with an efficiency of 80%.*
- *d) Is there a risk of so-called "cavitation" somewhere in this tube system?*
- *Assume that the friction coefficient for the valve is $\zeta' = 2,0$, assume two 45° elbow bends ($\zeta' = 0,4$) and an 90° elbow bend ($\zeta' = 0,9$). Density water = 1000 kg/m^3; dynamic viscosity water = 0,001 Pa·s. Water vapour pressure at 20 °C is 2336,8 Pa. Assume the tube wall roughness to be = $4,7 \cdot 10^{-4}$ m.*



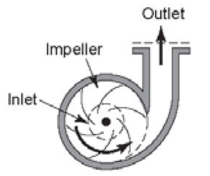
Old exam question /a 1 of 2

Old exam question /a 2 of 2

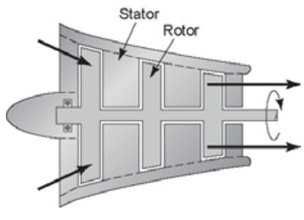
Compressors



(*a*) Reciprocating compressor



(*b*) Centrifugal compressor



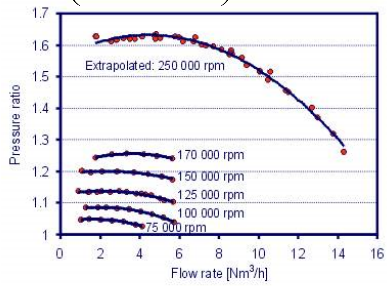
(c) Axial flow compressor

Pictures: JK05

- A compressor increases pressure of a gas
- Most important are (a) reciprocating, (b) centrifugal and (c) axial flow types
- **If** $p_{out} \le 1.1 \times p_{in}$ the calculations are similar to those for a pump, **otherwise** gas compressibility must be considered → calculate as polytropic process with

$$\boxed{P_{compr} = \dot{W}_{in} \cdot \eta_{compr} = \dot{H}_{out} - \dot{H}_{in}}$$

$$\dot{W}_{in} = P_{compressor,theor} = \frac{\kappa}{\kappa-1} \cdot \dot{V}_{in} \cdot p_{in} \cdot \left(\left(\frac{p_{out}}{p_{in}}\right)^{\frac{\kappa-1}{\kappa}} - 1 \right), \quad \text{and } \kappa = \frac{c_p}{c_v}$$



A compressor characteristic

Picture: http://www.mech.kuleuven.be/micro/topics/turbine/

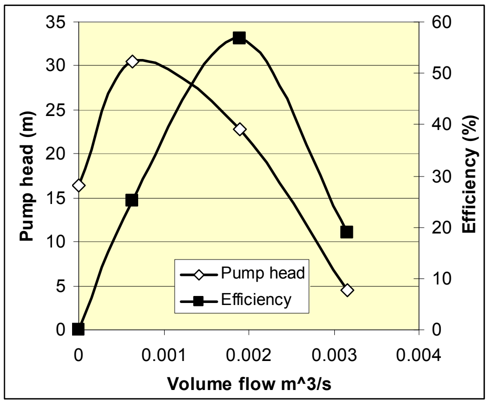
Example: pump /1

- A 1 hp (746 W) electrical motor drives a centrifugal pump for which the catalogue gives some *tabelised data.*
- Calculate the pumping power and efficiency for pumping water (ρ = 996 kg/m³) with this pump, and plot these as a function of the flow rate $\dot{V}$.

Flow (m³/s)	Pump head (m)
3.16×10^{-3}	4.6
1.89×10^{-3}	22.9
0.63×10^{-3}	30.5
0	16.5

Source: T06

Example: pump /2



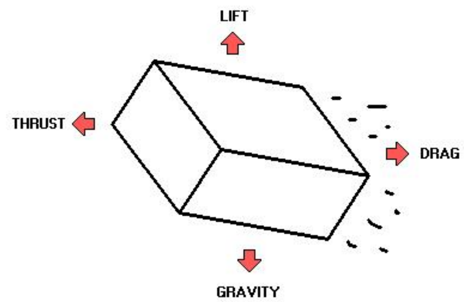
Source: T06

6.7 Fluid dynamics: external flows

Fluid flow around objects

- In the cases of
  - **an object moving through a fluid**
  - **a fluid flow around an object**

  the **velocity difference** generates forces
- Forces acting parallel to the flow direction are **drag forces**; forces acting perpendicular to the flow direction are **lift forces**
  - The flow field around an object can be divided in two parts: the **boundary layer*** where the viscous forces are active, and the **free-stream velocity** (or the stagnant surrounding fluid)



* See section 6.2

Picture: http://www.weirdrichard.com/images/forces.jpg

Flow around a flat plate / I

- For flow along a flat plate, the forces on the plate are friction forces. The **shear stress on each side of the surface** is

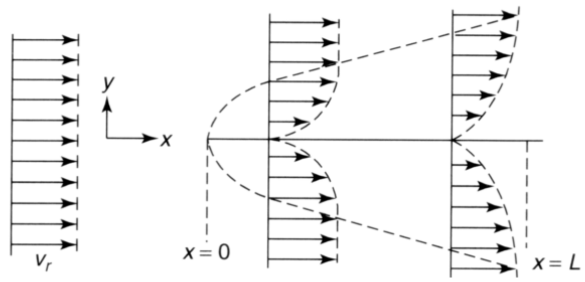
$$\tau_{yx}\big|_{y=0} = \tau_w = \eta_{fluid}\frac{v_r}{\delta} = 0.664\cdot\left(\frac{v_r x \rho_{fluid}}{\eta_{fluid}}\right)^{-½}\cdot ½\rho_{fluid}v_r^2, \quad \text{for} \quad Re_x = \frac{v_r \cdot x \cdot \rho_{fluid}}{\eta_{fluid}} < 3\times 10^5$$

  with (laminar) boundary layer thickness δ and **relative velocity** v_r
- The **drag force** on each side of a plate with length L and width b is then given by

$$F_{drag} = F_D = b\int_0^L \tau_w dx =$$

$$= 1.33\left(\frac{v_r L \rho_{fluid}}{\eta_{fluid}}\right)^{-½}\cdot bL\cdot ½\rho_{fluid}v_r^2$$

$$for \quad Re_L = \frac{v_r\cdot L\cdot \rho_{fluid}}{\eta_{fluid}} < 3\times 10^5$$



- The pressure ½ρv² is known as THRUST *(sv: stöt)*

Picture: BMH99

Flow around a flat plate /2

- This defines the (length-averaged) **drag coefficient C_D** as

$$\boxed{F_D = C_D \cdot A \cdot ½\rho v_r^2} \quad \text{with } C_D = \frac{1.33}{\sqrt{Re_L}} \quad \text{for} \quad Re_L < 3 \times 10^5$$

where A (m^2) is the area (one side) of the plate

- For turbulent cases, experimental results give

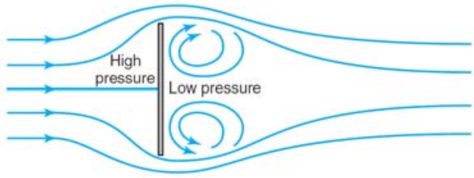
$$C_D = \frac{0.074}{Re_L^{1/5}} \text{ for } 10^5 < Re_L < 10^7 \text{ ; } C_D = \frac{0.445}{{}^{10}\log(Re_L)^{2.58}} \text{ for } 10^7 < Re_L < 10^9$$

- For a flat surface with a laminar region followed by a turbulent region, a "composite" drag composition can be calculated with

$$C_D = \frac{0.074}{Re_L^{1/5}} - \frac{1740}{Re_L}$$

Picture: KJ05

- For a flate plate **perpendicular** to fluid the drag coefficient equals ~2 , largely independent of Re-number



Flow around cylinders, spheres /1

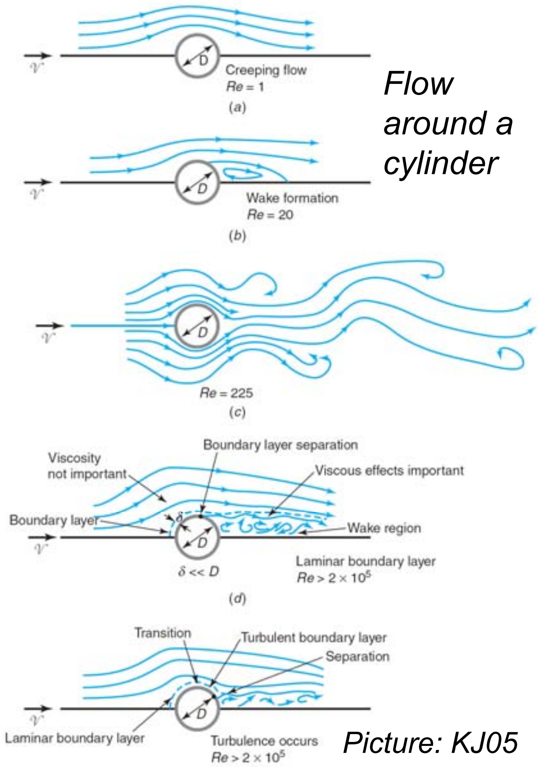
- For a general surface area A⊥ (m^2) perpendicular to the flow, the **drag force** is

$$\boxed{F_D = C_D \cdot A_\perp \cdot ½\rho v_r^2}$$

(where $½\rho v_r^2$ is actually the pressure difference between the front and the back of the object)

- With increasing Re-numbers, **boundary layer separation** occurs, and

a **wake region** *(sv: köl(vatten))* arises where kinetic energy is only partly converted into pressure



Flow around a cylinder

Picture: KJ05

Flow around cylinders, spheres /2

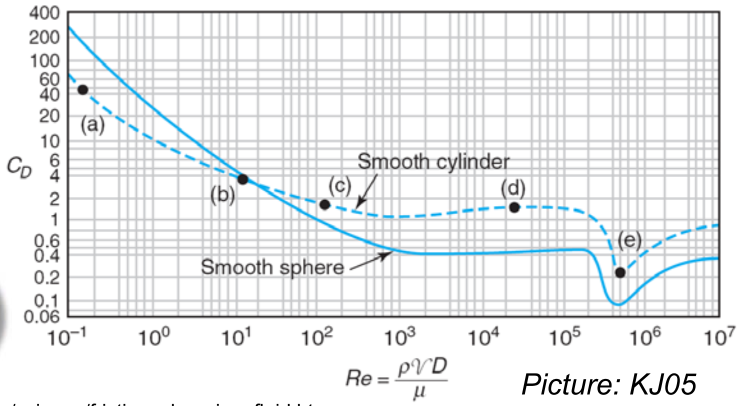
- For spherical particles the drag coefficient equals

$$C_D = \frac{24}{Re}$$
for Re << 1 or < 0.2
$$C_D = \frac{24}{Re}\left(1 + \frac{3}{16}Re\right)$$
for 0.2 < Re < 2
$$C_D = \frac{24}{Re}\left(1 + \frac{1}{6}Re^{2/3}\right)$$
for 2 < Re < 800
$$C_D = 0.44$$
for $800 < Re < 10^5$

- For flow at Re <0.1 around a sphere, the relation C_D=24/Re follows also from **Stokes' law**

$$\boxed{F_{drag} = 3\pi\eta v_r D}$$

for a sphere with diameter D and relative velocity v_r

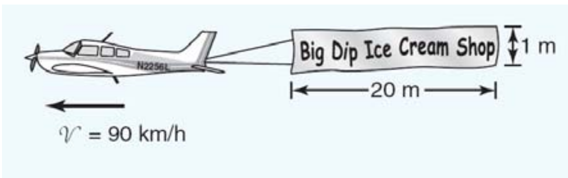


Picture: KJ05

Picture: http://www.school-for-champions.com/science/friction_changing_fluid.htm

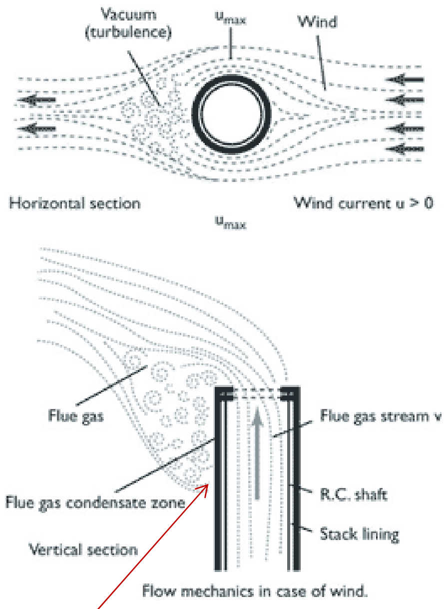
Example: drag on a flat plate

- An advertising banner (1 m x 20 m) is towed behind an aeroplane at 90 km/h, in air at 32°C.
- Calculate the power (in kW) needed to pull the banner.

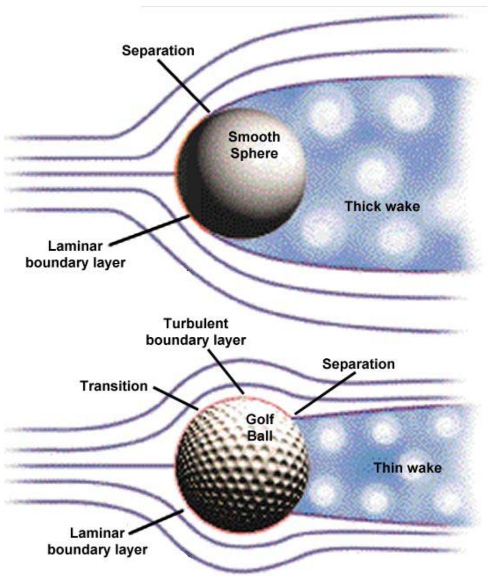


Boundary layer separation examples

Picture: http://www.structural.net/news/Media_coverage/media_plant_services.html



erosion, corrosion



Picture: http://www.aerospaceweb.org/question/aerodynamics/q0215.shtml

Sources #6

- BMH99: Beek, W.J., Muttzall, K.M.K., van Heuven, J.W. "Transport phenomena" Wiley, 2nd edition (1999)
- BSL60: R.B. Bird, W.E. Stewart, E.N. Lightfoot "Transport phenomena" Wiley (1960)
- CEWR10: C.T. Crowe, D.F. Elger, B.C. Williams, J.A. Roberson "Engineering Fluid Mechanics", 9th ed., Wiley (2010)
- **KJ05: D. Kaminski, M. Jensen "Introduction to Thermal and Fluids Engineering", Wiley (2005)**
- MSH93: W.L. McCabe, J.C. Smith. P. Harriott "Unit operations of chemical engineering" 5th ed. McGraw-Hill (1993)
- SSJ84: J.M. Smith, E. Stammers, L.P.B.M Janssen "Fysische Transportverschijnselen I" TU Delft, D.U.M. (1984) *(in Dutch)*
- **T06: S.R. Turns "Thermal – Fluid Sciences", Cambridge Univ. Press (2006)**
- ÖS96: G. Öhman, H. Saxén "Värmeteknikens grunder", Åbo Akademi University (1996)

Picture: http://www.ecotrust.org/copperriver/crks_cd/content/pages/photographs/images/